Architecture & Civil Engineering

First edition | 10 units | 50 lessons | A video for every lesson

FIRST EDITION

Course guide

This course teaches the two professions that shape the built world, the architect and the civil engineer, as one subject. Over ten units the student learns to draw like an architect (scale, plans, elevations, sections, perspective and 3D models), to read the history of architecture as a history of structural ideas, and to calculate like an engineer: loads and equilibrium, trusses, arches and the catenary, stress and strain, bending and deflection, cables and buckling, wind on towers, effective stress, footings and piles, earth pressure and dams, surveying, water treatment, runoff, pavements and traffic. Every lesson has worked examples and a practice set with answers, and every unit ends with a 100-point test; a midterm and a final close each half of the year.

Every lesson follows the same path. **The story** is a real moment from history: Vitruvius writing for Augustus, Imhotep's Step Pyramid, Wren's hidden cone in the dome of St Paul's, Brunelleschi's mirror, Paxton's blotting-paper sketch, the snow on the Hartford roof, Bessemer and Eads, the Roeblings at Brooklyn, the Quebec Bridge, Fazlur Khan's tubes, Terzaghi's clay, the Leaning Tower of Pisa, Hoover Dam, Bazalgette's sewers, McAdam's roads and the Washington Monument. Then **how they figured it out**: what failed, what they measured and what they changed. Then **the physics**, with equations and worked examples, and **the engineering**: how the idea becomes a safe, useful and beautiful building or public work. Finally the student repeats the discovery in a build: chalk lintels, block arches, buttressed walls, truss models with string for tension members, card beams, sand foundations, model dams, a water level and a traffic count, always with a prediction, a fair test, a data table and an improvement round. The last unit is a design studio in which the student designs a small house or chapel for a real client, from program to presentation.

God made an orderly world that can be measured, and He made people in His image to be makers. The Greek word Paul uses for a “wise masterbuilder,” *architekton*, gives us our word architect. Building is a way of serving neighbors: a roof that stays up under snow, a bridge that holds, water that is clean, an exit that opens. This course teaches that service with honesty and care: checking assumptions, testing materials, listening to warnings and counting the cost before beginning. The old Latin saying *Fabricando fit faber*, “by making, one becomes a maker,” is the method of the whole course.

> Videos were chosen lesson by lesson from trusted teaching sources: Practical Engineering (Grady Hillhouse), The Efficient Engineer, 30X40 Design Workshop (architect Eric Reinholdt), Structures with Prof. H, Engineer4Free, Smarthistory, Animagraffs, Sabin Civil Engineering (Lesics), The B1M, English Heritage, the Institution of Civil Engineers, the Texas Society of Professional Engineers, the North Carolina School of Science and Mathematics, university lecturers and others. Every video was checked in preview for content and accuracy. Many run 10–20 minutes, and a few longer architecture videos are best watched in two parts. Several open with a short sponsor message; skip it. Videos on failures (the St. Francis Dam, the Quebec Bridge, Ronan Point) use historic photographs and describe the loss of life plainly; watch those together.

Start with Lesson 1.1 →

Who it is for

Written for students in grades 9–12 who have completed Algebra I and geometry. Right-triangle trigonometry is reviewed where it is first needed (Lesson 1.2), and logarithms and the hyperbolic cosine are introduced briefly where used. Students work fairly independently with a parent or mentor who checks calculations, joins the reviews in the design studio and supervises any use of craft knives, hot glue and plaster. No kits are required: every build uses craft-store, hardware-store or kitchen materials, and the free online modeling programs SketchUp Free or Tinkercad. Optional kits and tools are listed in each unit.

How to use this course

1. **The weekly rhythm.** The course has 50 lessons in ten units, each taught in three sessions of about 50 minutes: (1) the video, the story and core ideas, and the worked examples; (2) the build and the engineer's notebook; (3) the practice set and corrections. After each unit comes a test day; review days and exams follow Units 5 and 10. At five sessions a week the course fills about 33 weeks, leaving room for review, field trips and the design presentation.
2. **Story, then physics, then engineering.** Read the core ideas carefully, then close the page and tell the story back to a parent: what problem the builders faced, what they learned and how. Then work every worked example on paper before looking at the steps.
3. **Equations with units.** Write the formula, substitute numbers with their units, and box the answer with its unit. Check whether the size of the answer makes sense. Most mistakes in engineering are unit mistakes.
4. **Practice sets and corrections.** Each lesson has 8–12 practice problems from easy to challenge, with answers. Do them without looking, then check, and rework every one you missed in the correction session. The challenge problem is optional but worth trying.
5. **Picture study.** Each lesson has one public-domain picture: a measured drawing from the Historic American Buildings Survey or Engineering Record, a historic photograph, an engraving or a painting. Look at it quietly for two minutes, put it away, describe it, then work through the "look for" items and questions, which point to the structure and design.
6. **The engineer's notebook.** Use a graph-paper notebook all year. For every build: a dated prediction, a labeled sketch, a data table with units, a graph, a comparison of measurement with prediction, and a note on what you would change. Honest records matter more than perfect results: "a just weight is his delight" (Proverbs 11:1).
7. **Builds and safety.** Read the safety note before each build. An adult handles craft knives, hot-glue guns and plaster. Wear safety glasses for anything that snaps, breaks or falls during load tests. Never go near real dams, storm drains, highways or construction sites for a build; measure traffic only from a safe place well back from the road; call 811 before digging.

8. **Unit tests, midterm and final.** Each unit ends with a 100-point test mixing calculations, concepts and one short essay. The midterm (Units 1–5) and final (Units 1–10) are 100-point exams of 100 and 120 minutes. Calculators and an architect's scale are allowed.
9. **The design studio.** Unit 10 is a five-step design project: program, site, schematic design, cost and code, and presentation. Choose a real client if you can (your family, your church, a neighbor) and present the design to them at the end. It makes a fine portfolio piece.
10. **Optional kits and software.** KEVA planks, LEGO Architecture sets, Pitsco balsa supplies, Froebel blocks, an Arduino for a model traffic signal and the free Bridge Design Contest software are listed where they fit. Every lesson can be completed without them.

Scope and sequence

UNIT 1 · WEEKS 1–4

Architects and Civil Engineers

The professions and their roots: Vitruvius's firmness, commodity and delight; Imhotep and the pyramids; Wren and the dome of St Paul's; licensure and the St. Francis Dam; and the design and construction team, with Nehemiah's wall.

1.1 Vitruvius: Firmness, Commodity and Delight

1.2 Imhotep and the Pyramids

1.3 Christopher Wren and St Paul's

1.4 The Professions Today: Architects and Engineers

1.5 The Design and Construction Team

UNIT 2 · WEEKS 4–7

Architectural Drawing

The language of building: scale and the architect's scale, floor plans, elevations and sections, Brunelleschi's perspective, and 3D modeling from Froebel's blocks to SketchUp.

2.1 Scale and the Architect's Scale

2.2 Floor Plans

2.3 Elevations and Sections

2.4 Perspective Drawing: Brunelleschi's Experiment

2.5 Modeling in 3D: From Froebel's Blocks to SketchUp

UNIT 3 · WEEKS 7–10

The History of Architecture

Architecture as structural history: Greek orders and Roman concrete, Gothic vaults and buttresses, Palladio's proportions, the Crystal Palace, and modern frames and Fallingwater's cantilevers.

3.1 Greek Orders and Roman Engineering

3.2 Romanesque and Gothic Cathedrals

3.3 The Renaissance and Palladio

3.4 The Industrial Age: Iron, Glass and the Crystal Palace

3.5 Modern Architecture: The Frame, the Skyscraper and Fallingwater

UNIT 4 · WEEKS 10–13

Structures I: Forces and Loads

Forces and loads: dead, live, snow, wind and seismic loads; statics; trusses by the method of joints; arches and the catenary; and load paths and progressive collapse.

4.1 Loads: Dead, Live, Wind, Snow and Seismic

4.2 Statics and Equilibrium

4.3 Trusses by the Method of Joints

4.4 Arches and the Catenary: Hooke, Poleni and Gaudí

4.5 Load Paths: From Roof to Foundation

UNIT 5 · WEEKS 13–17

Structures II: Materials and Members

Materials and members: timber and the balloon frame, masonry and the Monadnock, steel and the stress-strain curve, reinforced and prestressed concrete, and beams in bending and deflection.

5.1 Timber: The Balloon Frame and Wood as a Structural Material

5.2 Masonry: The Monadnock and the Limits of Brick

5.3 Steel: Bessemer, Eads and the Stress-Strain Curve

5.4 Concrete: Roman, Modern, Reinforced and Prestressed

5.5 Beams: Bending and Deflection from Galileo to Today

UNIT 6 · WEEKS 17–20

Bridges

Bridges: types and spans at the Forth Bridge, the Iron Bridge and Telford, the Brooklyn Bridge and Golden Gate, the Quebec and Tacoma Narrows failures, and a model bridge competition.

6.1 Bridge Types and Spans: The Forth Bridge

6.2 The Iron Bridge and Thomas Telford

6.3 The Brooklyn Bridge and the Golden Gate

6.4 Failures: The Quebec Bridge and Tacoma Narrows

6.5 The Bridge Competition: Design, Analyze, Build and Test

UNIT 7 · WEEKS 20–23

Tall Buildings

Tall buildings: the steel frame, Otis's elevator, wind and Fazlur Khan's tubes, the Empire State Building and Burj Khalifa, and fire safety and building codes.

7.1 The Home Insurance Building and the Steel Frame

7.2 Elevators and Otis's Safety Brake

7.3 Wind and the Tube: Fazlur Khan

7.4 From the Empire State Building to the Burj Khalifa

7.5 Fire Safety and Building Codes

UNIT 8 · WEEKS 23–26

Foundations and Geotechnical Engineering

Foundations and geotechnical engineering: soils and effective stress, footings and piles, the Leaning Tower of Pisa, retaining walls and slopes, and Hoover Dam.

8.1 Soils and Site Investigation: Terzaghi's Science

8.2 Shallow and Deep Foundations

8.3 The Leaning Tower of Pisa

8.4 Retaining Walls and Slopes

8.5 Dams: Hoover Dam

UNIT 9 · WEEKS 27–30

Water and Transportation

Water and transportation: surveying from the groma to GPS, water treatment, Bazalgette's sewers and stormwater, roads and pavements, and railways and traffic engineering.

9.1 Surveying: From the Roman Groma to GPS

9.2 Water Supply and Treatment

9.3 Wastewater and Stormwater: Bazalgette's London

9.4 Roads and Pavement: The Romans and McAdam

9.5 Railways and Traffic Engineering

UNIT 10 · WEEKS 30–33

Design Studio

Design studio: the client and program, site planning, designing a small house or chapel, codes, cost and schedule, and presenting the design with drawings and a model.

10.1 The Client and the Program

10.2 Site Planning

10.3 Designing a Small House or Chapel

10.4 Codes, Cost and Schedule: Counting the Cost

10.5 Presenting the Design: Drawings and a Model

Weekly pacing calendar

WEEK	PLAN
1	• Day 1: Lesson 1.1 Vitruvius: Firmness, Commodity and Delight: video, teaching and worked examples • Day 2: Lesson 1.1: build and notebook

WEEK	PLAN
	• Day 3: Lesson 1.1: practice set and corrections • Day 4: Lesson 1.2 Imhotep and the Pyramids: video, teaching and worked examples • Day 5: Lesson 1.2: build and notebook *Five sessions a week of about 50 minutes. Families on a four-day week can combine the practice day of one lesson with the teaching day of the next.*
2	• Day 1: Lesson 1.2: practice set and corrections • Day 2: Lesson 1.3 Christopher Wren and St Paul's: video, teaching and worked examples • Day 3: Lesson 1.3: build and notebook • Day 4: Lesson 1.3: practice set and corrections • Day 5: Lesson 1.4 The Professions Today: Architects and Engineers: video, teaching and worked examples
3	• Day 1: Lesson 1.4: build and notebook • Day 2: Lesson 1.4: practice set and corrections • Day 3: Lesson 1.5 The Design and Construction Team: video, teaching and worked examples • Day 4: Lesson 1.5: build and notebook • Day 5: Lesson 1.5: practice set and corrections
4	• Day 1: Unit 1 test • Day 2: Lesson 2.1 Scale and the Architect's Scale: video, teaching and worked examples • Day 3: Lesson 2.1: build and notebook • Day 4: Lesson 2.1: practice set and corrections • Day 5: Lesson 2.2 Floor Plans: video, teaching and worked examples
5	• Day 1: Lesson 2.2: build and notebook • Day 2: Lesson 2.2: practice set and corrections • Day 3: Lesson 2.3 Elevations and Sections: video, teaching and worked examples • Day 4: Lesson 2.3: build and notebook • Day 5: Lesson 2.3: practice set and corrections
6	• Day 1: Lesson 2.4 Perspective Drawing: Brunelleschi's Experiment: video, teaching and worked examples • Day 2: Lesson 2.4: build and notebook • Day 3: Lesson 2.4: practice set and corrections • Day 4: Lesson 2.5 Modeling in 3D: From Froebel's Blocks to SketchUp: video, teaching and worked examples

WEEK	PLAN
	• Day 5: Lesson 2.5: build and notebook
7	• Day 1: Lesson 2.5: practice set and corrections • Day 2: Unit 2 test • Day 3: Lesson 3.1 Greek Orders and Roman Engineering: video, teaching and worked examples • Day 4: Lesson 3.1: build and notebook • Day 5: Lesson 3.1: practice set and corrections
8	• Day 1: Lesson 3.2 Romanesque and Gothic Cathedrals: video, teaching and worked examples • Day 2: Lesson 3.2: build and notebook • Day 3: Lesson 3.2: practice set and corrections • Day 4: Lesson 3.3 The Renaissance and Palladio: video, teaching and worked examples • Day 5: Lesson 3.3: build and notebook
9	• Day 1: Lesson 3.3: practice set and corrections • Day 2: Lesson 3.4 The Industrial Age: Iron, Glass and the Crystal Palace: video, teaching and worked examples • Day 3: Lesson 3.4: build and notebook • Day 4: Lesson 3.4: practice set and corrections • Day 5: Lesson 3.5 Modern Architecture: The Frame, the Skyscraper and Fallingwater: video, teaching and worked examples
10	• Day 1: Lesson 3.5: build and notebook • Day 2: Lesson 3.5: practice set and corrections • Day 3: Unit 3 test • Day 4: Lesson 4.1 Loads: Dead, Live, Wind, Snow and Seismic: video, teaching and worked examples • Day 5: Lesson 4.1: build and notebook
11	• Day 1: Lesson 4.1: practice set and corrections • Day 2: Lesson 4.2 Statics and Equilibrium: video, teaching and worked examples • Day 3: Lesson 4.2: build and notebook • Day 4: Lesson 4.2: practice set and corrections • Day 5: Lesson 4.3 Trusses by the Method of Joints: video, teaching and worked examples
12	• Day 1: Lesson 4.3: build and notebook

WEEK	PLAN
	• Day 2: Lesson 4.3: practice set and corrections • Day 3: Lesson 4.4 Arches and the Catenary: Hooke, Poleni and Gaudí: video, teaching and worked examples • Day 4: Lesson 4.4: build and notebook • Day 5: Lesson 4.4: practice set and corrections
13	• Day 1: Lesson 4.5 Load Paths: From Roof to Foundation: video, teaching and worked examples • Day 2: Lesson 4.5: build and notebook • Day 3: Lesson 4.5: practice set and corrections • Day 4: Unit 4 test • Day 5: Lesson 5.1 Timber: The Balloon Frame and Wood as a Structural Material: video, teaching and worked examples
14	• Day 1: Lesson 5.1: build and notebook • Day 2: Lesson 5.1: practice set and corrections • Day 3: Lesson 5.2 Masonry: The Monadnock and the Limits of Brick: video, teaching and worked examples • Day 4: Lesson 5.2: build and notebook • Day 5: Lesson 5.2: practice set and corrections
15	• Day 1: Lesson 5.3 Steel: Bessemer, Eads and the Stress-Strain Curve: video, teaching and worked examples • Day 2: Lesson 5.3: build and notebook • Day 3: Lesson 5.3: practice set and corrections • Day 4: Lesson 5.4 Concrete: Roman, Modern, Reinforced and Prestressed: video, teaching and worked examples • Day 5: Lesson 5.4: build and notebook
16	• Day 1: Lesson 5.4: practice set and corrections • Day 2: Lesson 5.5 Beams: Bending and Deflection from Galileo to Today: video, teaching and worked examples • Day 3: Lesson 5.5: build and notebook • Day 4: Lesson 5.5: practice set and corrections • Day 5: Unit 5 test
17	• Day 1: Review day for the midterm • Day 2: Midterm exam

WEEK	PLAN
	• Day 3: Lesson 6.1 Bridge Types and Spans: The Forth Bridge: video, teaching and worked examples • Day 4: Lesson 6.1: build and notebook • Day 5: Lesson 6.1: practice set and corrections
18	• Day 1: Lesson 6.2 The Iron Bridge and Thomas Telford: video, teaching and worked examples • Day 2: Lesson 6.2: build and notebook • Day 3: Lesson 6.2: practice set and corrections • Day 4: Lesson 6.3 The Brooklyn Bridge and the Golden Gate: video, teaching and worked examples • Day 5: Lesson 6.3: build and notebook
19	• Day 1: Lesson 6.3: practice set and corrections • Day 2: Lesson 6.4 Failures: The Quebec Bridge and Tacoma Narrows: video, teaching and worked examples • Day 3: Lesson 6.4: build and notebook • Day 4: Lesson 6.4: practice set and corrections • Day 5: Lesson 6.5 The Bridge Competition: Design, Analyze, Build and Test: video, teaching and worked examples
20	• Day 1: Lesson 6.5: build and notebook • Day 2: Lesson 6.5: practice set and corrections • Day 3: Unit 6 test • Day 4: Lesson 7.1 The Home Insurance Building and the Steel Frame: video, teaching and worked examples • Day 5: Lesson 7.1: build and notebook
21	• Day 1: Lesson 7.1: practice set and corrections • Day 2: Lesson 7.2 Elevators and Otis's Safety Brake: video, teaching and worked examples • Day 3: Lesson 7.2: build and notebook • Day 4: Lesson 7.2: practice set and corrections • Day 5: Lesson 7.3 Wind and the Tube: Fazlur Khan: video, teaching and worked examples
22	• Day 1: Lesson 7.3: build and notebook • Day 2: Lesson 7.3: practice set and corrections

WEEK	PLAN
	• Day 3: Lesson 7.4 From the Empire State Building to the Burj Khalifa: video, teaching and worked examples • Day 4: Lesson 7.4: build and notebook • Day 5: Lesson 7.4: practice set and corrections
23	• Day 1: Lesson 7.5 Fire Safety and Building Codes: video, teaching and worked examples • Day 2: Lesson 7.5: build and notebook • Day 3: Lesson 7.5: practice set and corrections • Day 4: Unit 7 test • Day 5: Lesson 8.1 Soils and Site Investigation: Terzaghi's Science: video, teaching and worked examples
24	• Day 1: Lesson 8.1: build and notebook • Day 2: Lesson 8.1: practice set and corrections • Day 3: Lesson 8.2 Shallow and Deep Foundations: video, teaching and worked examples • Day 4: Lesson 8.2: build and notebook • Day 5: Lesson 8.2: practice set and corrections
25	• Day 1: Lesson 8.3 The Leaning Tower of Pisa: video, teaching and worked examples • Day 2: Lesson 8.3: build and notebook • Day 3: Lesson 8.3: practice set and corrections • Day 4: Lesson 8.4 Retaining Walls and Slopes: video, teaching and worked examples • Day 5: Lesson 8.4: build and notebook
26	• Day 1: Lesson 8.4: practice set and corrections • Day 2: Lesson 8.5 Dams: Hoover Dam: video, teaching and worked examples • Day 3: Lesson 8.5: build and notebook • Day 4: Lesson 8.5: practice set and corrections • Day 5: Unit 8 test
27	• Day 1: Lesson 9.1 Surveying: From the Roman Groma to GPS: video, teaching and worked examples • Day 2: Lesson 9.1: build and notebook • Day 3: Lesson 9.1: practice set and corrections • Day 4: Lesson 9.2 Water Supply and Treatment: video, teaching and worked examples • Day 5: Lesson 9.2: build and notebook
28	• Day 1: Lesson 9.2: practice set and corrections

WEEK	PLAN
	• Day 2: Lesson 9.3 Wastewater and Stormwater: Bazalgette's London: video, teaching and worked examples • Day 3: Lesson 9.3: build and notebook • Day 4: Lesson 9.3: practice set and corrections • Day 5: Lesson 9.4 Roads and Pavement: The Romans and McAdam: video, teaching and worked examples
29	• Day 1: Lesson 9.4: build and notebook • Day 2: Lesson 9.4: practice set and corrections • Day 3: Lesson 9.5 Railways and Traffic Engineering: video, teaching and worked examples • Day 4: Lesson 9.5: build and notebook • Day 5: Lesson 9.5: practice set and corrections
30	• Day 1: Unit 9 test • Day 2: Lesson 10.1 The Client and the Program: video, teaching and worked examples • Day 3: Lesson 10.1: build and notebook • Day 4: Lesson 10.1: practice set and corrections • Day 5: Lesson 10.2 Site Planning: video, teaching and worked examples
31	• Day 1: Lesson 10.2: build and notebook • Day 2: Lesson 10.2: practice set and corrections • Day 3: Lesson 10.3 Designing a Small House or Chapel: video, teaching and worked examples • Day 4: Lesson 10.3: build and notebook • Day 5: Lesson 10.3: practice set and corrections
32	• Day 1: Lesson 10.4 Codes, Cost and Schedule: Counting the Cost: video, teaching and worked examples • Day 2: Lesson 10.4: build and notebook • Day 3: Lesson 10.4: practice set and corrections • Day 4: Lesson 10.5 Presenting the Design: Drawings and a Model: video, teaching and worked examples • Day 5: Lesson 10.5: build and notebook
33	• Day 1: Lesson 10.5: practice set and corrections • Day 2: Unit 10 test • Day 3: Review day for the final

WEEK	PLAN
	• Day 4: Final exam

Materials

Engineer's notebook (graph-paper composition book) and a scientific calculator
For predictions, data, graphs, calculations with units and reflections, all year.

Architect's scale (triangular), drafting pencils, eraser, a 30/60° and a 45° triangle, a compass, graph paper and tracing paper
For the drawing unit and the design studio.

Free software: SketchUp Free or Tinkercad (a parent sets up the account)
For 3D modeling in Lesson 2.5 and the design studio.

Foam board, card stock, corrugated cardboard, balsa or basswood strips and sheets, craft sticks, drinking straws, wooden skewers, string and thread
The main building materials for models and load tests.

Low-temperature hot-glue gun, white glue and wood glue, tape, scissors, a craft knife and cutting mat (adult use)
For assembling models.

Kitchen scale (grams), tape measure (a 30 m or 100 ft tape is ideal), ruler, protractor, stopwatch or phone
Measuring is the heart of every build.

Pennies, washers, a paper cup on a string loop, books, a bucket, sand and rice for loads
Standard loads for load tests.

Sidewalk chalk, plaster of Paris or sand-mortar mix, stiff wire, a dust mask and gloves
For stone lintels (Lesson 3.1) and reinforced beams (Lesson 5.4). An adult mixes plaster or mortar.

A tub or box of sand, some clay or garden soil, gravel, a long plastic tub or rain gutter
For the soils, foundation, retaining-wall, dam and road builds.

About 6 m of clear vinyl tubing, food coloring, potassium alum (from the spice aisle), a 2-liter bottle, coffee filters
For the water level (Lesson 9.1) and the water-treatment test (Lesson 9.2).

A box fan with a guard, a hair dryer, a kitchen thermometer
For wind tests on towers and decks and the fireproofing test.

Safety glasses

For every load test that can snap, crack or fall.

Optional: KEVA planks, a LEGO Architecture set, Pitsco balsa supplies, a Froebel block set, an Arduino starter kit

Never required. See the kits listed in each unit.

Grading

Grades come from practice sets, builds, the engineer's notebook, unit tests, the midterm and the final. A calculation earns full credit when the formula, the substitution with units and the answer with its unit are all shown. A build that fails but is honestly recorded, analyzed and improved earns full credit.

PART	%
Practice sets and corrections	**15**
Builds (fair tests, measurement, improvement round) and the design studio	**25**
Engineer's notebook	**10**
Unit tests	**30**
Midterm exam	**10**
Final exam	**10**

Scale: A 90–100, B 80–89, C 70–79, D 60–69. Below 60: review and retake the test or redo the build with help.

- Grade practice sets on completion and corrections: every missed problem reworked correctly earns back its credit.
- In tests, give partial credit for a correct setup with units even if the arithmetic slips; a wrong formula or a missing unit is the more serious error.
- Grade builds on the process: prediction, fair test, careful measurement, comparison with calculation, and an honest improvement round.

- The Unit 10 design studio counts as the builds for that unit and makes an excellent transcript portfolio piece. Keep each unit's portfolio project in one folder.
- For a transcript, this course is a full-year, one-credit elective in engineering or applied science (architecture and civil engineering), with laboratory and design work.

A note to parents

You do not need to be an engineer or architect to guide this course. Read the story with your student, ask them to explain the physics back to you, and check that every calculation has units. The practice answers are given; your job is to make sure your student works honestly before checking, and reworks mistakes. In the builds, be the lab partner and safety officer: hold the tape, handle the craft knife and hot glue, and ask the two best questions in engineering: "What do you predict, and why?" and "How do you know?"

Expect some builds to fail, and welcome it. Chalk will snap, arches will fold, walls will tip and model dams will leak. The course is built to show that failure carefully studied is how builders have always learned, from the Bent Pyramid to the Tacoma Narrows Bridge. Several lessons tell of real disasters, such as the St. Francis Dam, the Knickerbocker Theatre, the Quebec Bridge and Ronan Point, with the loss of life stated plainly but without gruesome detail; the emphasis is always on what engineers learned and the responsibility they carry for the people who trust their work.

The design studio in the last unit is a good time to involve your family or church as clients. Students take the work far more seriously when a real person will hear their presentation. Throughout, the course treats building as a vocation of service, in which skill, honesty and care for neighbors go together. *Fabricando fit faber*: by making, one becomes a maker.

UNIT 1

Architects and Civil Engineers

Every building and public work begins with people: someone who needs it, someone who designs it, someone who checks it and many who build it. This unit introduces the two professions at the heart of this course, the architect and the civil engineer, through four great builders of the past and the team that builds today. You will meet Vitruvius, whose three tests of a good building (firmness, commodity and delight) still frame every design review; Imhotep and the pyramid builders, whose real genius was organization; and Christopher Wren, astronomer turned architect, who hid a brick cone inside the dome of St Paul's.

Then you will see how the professions are organized today, and why. The failure of the St. Francis Dam in 1928 shows why engineers are licensed and why every design must be independently checked. Nehemiah's wall shows how a project team divides its work. Along the way you will start the habits of the whole year: calculate with units, test honestly, and record everything in your engineer's notebook.

PREREQUISITES

- Algebra I: solving equations, ratios and proportions, scientific notation
- Geometry: area and volume of prisms, pyramids, cones and spheres; right-triangle trigonometry (tangent) is reviewed in Lesson 1.2
- Comfort with SI units (meters, kilograms, newtons) and US customary units (feet, pounds)

MASTERY CHECKLIST

- ☐ I can explain Vitruvius's firmness, commodity and delight and use them to critique a building.
- ☐ I can calculate weight from density and volume and bearing pressure from force and area.
- ☐ I can compute the volume of a pyramid and convert a seked to a slope angle.
- ☐ I can explain Wren's three-part dome and why domes and cones need a tension ring.
- ☐ I can describe the work and licensing of architects and civil engineers and calculate a factor of safety.
- ☐ I can find a project's critical path and estimate a budget with fees and contingency.

OPTIONAL KITS FOR THIS UNIT

- **KEVA planks**: All ages. Identical maple planks that stack without glue; excellent for stepped pyramids (Lesson 1.2), arches and domes, and for the two-ways building test in Lesson 1.5.
- **LEGO Architecture**: Ages 12 and up for most sets. Models of famous buildings and skylines; useful for studying proportion and massing and as reference models through the year. Never required.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

1.1	**Vitruvius: Firmness, Commodity and Delight**	3 sessions of 50 minutes
1.2	**Imhotep and the Pyramids**	3 sessions of 50 minutes

1.3	**Christopher Wren and St Paul's**	3 sessions of 50 minutes
1.4	**The Professions Today: Architects and Engineers**	3 sessions of 50 minutes
1.5	**The Design and Construction Team**	3 sessions of 50 minutes

Unit 1 test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation; correct setup earns partial credit. Answer short-answer questions in complete sentences. The test is worth 100 points.

1. **5 pts**
   Name Vitruvius's three tests of a good building, in Latin or English, and explain each in one sentence.

2. **6 pts**
   A sandstone block measures 1.0 m × 0.80 m × 0.50 m and has a density of 2,300 kg/m^3. Find its mass and weight.

3. **7 pts**
   A column carries 180 kN and rests on soil that can safely carry 120 kPa. What is the minimum area of its footing, and the side of a square footing (rounded up to the next 0.1 m)?

4. **7 pts**
   A pyramid has a square base 60 m on a side and a height of 40 m. Find its volume and the slope angle of its faces.

5. **5 pts**
   Convert a seked of 6 palms (per cubit of 7 palms) to a slope angle.

6. **6 pts**
   Why did the builders of the Bent Pyramid change its angle partway up, and what did the next pyramid show they had learned?

7. 6 pts

A project needs 1,200,000 blocks placed in 15 years of 300 working days each. How many blocks must be placed per day?

8. 7 pts

Describe the three parts of Wren's dome at St Paul's and the job of each.

9. 6 pts

Why did Wren put an iron chain around the base of the brick cone?

10. 7 pts

A brick ring 6.0 m in diameter and 0.46 m thick carries 3,000 kN. Find the compressive stress in MPa.

11. 6 pts

A cable holds 45 kN and its working load is 15 kN. Find the factor of safety. If the load is raised to 25 kN, what is the new factor of safety?

12. 6 pts

The water behind a dam rises from 30 m to 36 m deep. By what percent does the horizontal water force increase?

13. 6 pts

Distinguish the work of an architect, a structural engineer and a geotechnical engineer on a new library.

14. 7 pts

Tasks: A (4 weeks); B (6 weeks, after A); C (3 weeks, after A); D (5 weeks, after C); E (2 weeks, after B and D). Find the critical path, the duration and the float of B.

15. 5 pts

A building costs $2,500,000 to construct. With design fees of 9% and a contingency of 8%, what total budget is needed?

16. 8 pts

Short essay. Explain what the St. Francis Dam failure taught the engineering profession. Include at least two physical causes of the failure and two changes that followed.

ANSWER KEY

1. Firmitas (firmness): it stands safely and lasts. Utilitas (commodity): it serves its purpose well. Venustas (delight): it is beautiful and a pleasure to see and use.
2. $V = 0.40\ \mathrm{m}^3$; $m = 920$ kg; $W = 920 \times 9.81 \approx 9030$ N ≈ 9.0 kN.
3. $A = 180 \div 120 = 1.5\ \mathrm{m}^2$; side $= \sqrt{1.5} \approx 1.22$, so 1.3 m.
4. $V = \frac{1}{3}(60)^2(40) = 48{,}000\ \mathrm{m}^3$; $\tan\theta = 40/30$, $\theta \approx 53.1^\circ$.
5. $\tan\theta = 7/6 \approx 1.167$, $\theta \approx 49.4^\circ$.
6. The steep lower part and its foundation were probably cracking and settling, so they lowered the angle to reduce the load. The Red Pyramid was built at the gentler angle (about 43°) from the start.
7. $1{,}200{,}000 \div 4500 \approx 267$ blocks per day.
8. The brick inner dome forms the ceiling seen from inside; the hidden brick cone carries the heavy stone lantern and cross down to the drum; the outer dome, a timber frame covered in lead, gives the tall shape seen across London and keeps out the weather.
9. A loaded cone pushes outward at its base, creating hoop tension. Brick is weak in tension, so the iron chain acts like a barrel hoop and holds the base from spreading.
10. $A \approx \pi(6.0)(0.46) \approx 8.67\ \mathrm{m}^2$; $\sigma \approx 3 \times 10^6 \div 8.67 \approx 0.35$ MPa.
11. $45 \div 15 = 3.0$; $45 \div 25 = 1.8$.
12. $(36/30)^2 = 1.44$, a 44% increase.
13. The architect designs the library's plan, form, spaces and materials and coordinates the team; the structural engineer designs its frame, floors and foundations to carry the loads; the geotechnical engineer investigates the soil and rock and recommends the foundation type and bearing capacity.
14. A–B–E = 12 weeks; A–C–D–E = 14 weeks. Critical path A–C–D–E, 14 weeks. E starts at week 12; B can finish at week 10, so B has 2 weeks of float.
15. $2{,}500{,}000 \times 1.17 = \$2{,}925{,}000$.
16. Model answer: the dam failed because it stood on weak rock (an ancient landslide in schist on one side and conglomerate that softened when wet on the other) and because water seeping under it created uplift; the design had been raised without widening its base, and no independent geologist or engineer had reviewed it. The disaster led California to create a state dam-safety program and to require licensing of civil engineers, and it is taught everywhere as a lesson

that reputation is no substitute for independent checking and site investigation, and that engineers must hold public safety paramount.

PORTFOLIO PIECE: A BUILDING BIOGRAPHY

Choose a public building in your town, such as a church, library, courthouse or school. Find out who designed it, who built it and when (a cornerstone, a historical society or the building's own office can help). Measure its main floor by pacing or with a tape measure, draw a simple floor plan to scale, and compute the floor area.

Write a two-page report that tells the building's story and the team that built it, then gives a Vitruvian critique with one paragraph each for firmness, commodity and delight. Finish with one improvement you would propose and a rough cost estimate with a contingency.

LOOK FOR	WHAT IT MEANS
Research	Names the architect, builder and date where they can be found, and cites where the information came from.
Measurement and drawing	Plan is drawn to a stated scale with dimensions; floor area is computed correctly with units.
Vitruvian critique	Firmness, commodity and delight are each discussed with specific observations, not general praise.
Proposal and estimate	A clear improvement is proposed with a reasoned cost estimate including fees and contingency.
Presentation	The report is neat, organized and written in complete sentences, with sketches labeled.

1.1 Vitruvius: Firmness, Commodity and Delight

OBJECTIVES

- Explain Vitruvius's three tests of a good building (firmitas, utilitas, venustas) and apply them to a real building.
- Calculate the weight of building materials from their volume and density, and the bearing pressure under a column.
- Use a weighted decision matrix to compare design options fairly.

> **Warm-up.** Pick the building you know best: your house, your church or your library. In your notebook, write one sentence for each question. What keeps it standing? What does it let people do well, and what does it make awkward? Is there anything about it that is a pleasure to look at? You have just made your first Vitruvian critique.

WATCH

Architectural Structures – an introduction

Angus Macdonald · 11:02

https://www.youtube.com/watch?v=ol8nHA8AXl4

> **WHEN TO WATCH**
> After the story and before the physics. Listen for the ways structure and architecture can be related: hidden, shown, or the structure as the whole idea.

Note: A narrated slide lecture by a structural engineer who taught architecture students at the University of Edinburgh for many years. It is a small channel, but the teaching is careful and fits this lesson exactly.

KEY TERMS

firmitas (firmness)

Structural soundness: the building stands, carries its loads safely and lasts.

utilitas (commodity)

Usefulness: the building serves its purpose well, with the right rooms, light, access and comfort.

venustas (delight)

Beauty: proportion, order and fitness that give pleasure to the people who use and see the building.

bearing pressure

The force a column or wall presses onto what is beneath it, divided by the contact area: $p = F/A$, in pascals (N/m^2).

density

Mass per unit volume, $\rho = m/V$, measured in kg/m^3. Stone is about 2,300–2,700 kg/m^3; water is 1,000 kg/m^3.

decision matrix

A table that scores each design option against weighted criteria so a choice can be made honestly and explained to others.

CORE IDEAS

The story. In the first century BC, a Roman named Marcus Vitruvius Pollio, who had built machines for Julius Caesar's army and worked on public buildings, wrote a book for the emperor Augustus. He called it *De architectura*, and it is the only complete book on architecture to survive from the ancient world. In ten short books Vitruvius covered almost everything a builder needed: the education of an architect, bricks, lime and sand, the orders of columns, temples, theaters, baths, harbors, houses, water supply and aqueducts, sundials and lifting machines. In Book I he wrote that every building should be made with regard to *firmitas, utilitas, venustas*. In 1624 the English diplomat Sir Henry Wotton translated the three words as "commodity, firmness and delight," and architects have quoted them ever since.

How they figured it out. Vitruvius was not a theorist working from equations. He collected what generations of Greek and Roman builders had learned by doing: that lime mortar made with volcanic ash from near Puteoli (pozzolana) hardens even under water; that foundations must be dug down to solid ground; that a temple's columns look and stand best at certain proportions. His book was nearly forgotten for centuries. In about 1414 the Italian scholar Poggio Bracciolini found a copy in the library of the monastery of St Gall, and printed editions followed from about 1486. Renaissance architects such as Alberti, Bramante and Palladio studied Vitruvius closely and measured the Roman ruins to check him. His three tests have outlasted every style.

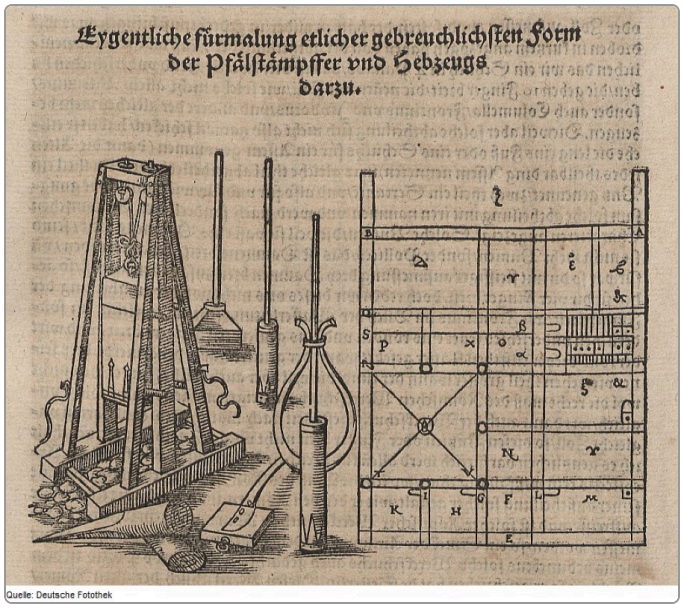
The physics. Firmness is the part we can calculate. Every building material has weight, and that weight must travel down through the structure to the ground. The weight of a block is its mass times the acceleration of gravity, and its mass is its

density times its volume: $W = mg = \rho V g$, with $g = 9.81\ \mathrm{m/s^2}$. When a column stands on the ground, it presses with a **bearing pressure** $p = F/A$. A stone column that weighs 120 kN and stands on a base 0.80 m square presses on the soil with $p = 120{,}000\ \mathrm{N} \div 0.64\ \mathrm{m^2} \approx 190\ \mathrm{kPa}$. If the soil cannot take that pressure, the column sinks. Vitruvius told builders to dig foundations "down to the solid," and in soft ground to drive wooden piles. That is firmness: knowing the loads and giving them a safe path to the earth.

The engineering. Commodity and delight cannot be calculated as neatly, but they can be judged honestly. Engineers and architects do this with a **decision matrix**: list the criteria (strength, cost, usefulness, beauty, maintenance), give each a weight, score each option, and multiply. The matrix does not make the decision for you, but it makes you say what you value and why, and it lets the client see your reasoning. A good design does not trade one of the three tests away for the others. A beautiful building that leaks, a strong one that nobody can use, or a useful one that is ugly will each be replaced sooner than it should be. The best buildings, like the best tools, are sound, serviceable and a pleasure to see.

Today. Vitruvius also wrote that an architect should know drawing, geometry, history, music, medicine, law and astronomy, because a building touches every part of life. That is still true. In this course you will be both architect and engineer: you will draw, calculate, build models and test them, and judge every design by all three of Vitruvius's tests. The Greek word for "master builder," *architekton*, is the word Paul uses of himself in 1 Corinthians 3:10.

PICTURE STUDY



Pile Drivers and Hoisting Gear, Unknown woodcut artist, 1575 · Woodcut · Deutsche Fotothek, Saxon State and University Library, Dresden

Image: Wikimedia Commons, public domain

This woodcut from a 1575 German book of building shows the tools that made Vitruvius's advice about foundations possible: tall frames for driving wooden piles and hand rammers for packing the ground. The heading reads, roughly, "A true picture of the most common pile rammers and the hoisting gear for them."

Look closely

- Find the tall frame on the left. Where is the heavy weight, and how would the crew lift it before letting it fall?
- Find the hand rammers. What part of the job do you think they were used for?
- Look at the grid on the right. It is a foundation plan. What might the letters mark?
- Notice that no part of the building itself is shown, only what goes under it. Why would a builder's book spend a whole page on this?

Talk about it

1. Which of Vitruvius's three tests does this picture serve, and how?
2. Why would driving piles help a heavy wall stand on soft, wet ground?

3. Builders in 1575 could not calculate bearing pressure. How do you think they decided how many piles to drive?

BUILD

The Vitruvian Pavilion: A Three-Test Design

You will design and build a small card pavilion (a roof on supports), then test it against all three of Vitruvius's tests: firmness by loading it, commodity by checking that it does its job, and delight by having others judge its proportions. Then you will improve it and score it again.

You will need

- 10 index cards (3 × 5 in), a sheet of card stock, tape and scissors
- A ruler and a kitchen scale
- A small toy figure about 5 cm tall (or a 5 cm paper cutout) for scale
- Pennies or small bags of rice for loads (2.5 g per US penny)
- Graph paper and your engineer's notebook

Steps

1. Write the design brief in your notebook: the pavilion must cover a floor at least 12 cm × 12 cm, stand at least 8 cm tall inside so the figure fits, have an opening at least 5 cm wide, carry at least 200 g on its roof, and use no more than 10 index cards plus tape.
2. Sketch three different ideas (for example: four columns and a flat roof; walls with an arched opening; a folded or pitched roof). Label each with its main structural idea.
3. Build a simple decision matrix. Use weights firmness 0.4, commodity 0.35, delight 0.25 and score each sketch from 1 to 10 as a prediction. Choose one to build.
4. Build it. Weigh the finished pavilion and record its mass in grams.
5. **Firmness test.** Set a flat card on the roof and add pennies in groups of 10 until the roof sags more than 1 cm or a support buckles. Record the largest load it carried, in grams, and how it failed.
6. **Commodity test.** Measure the floor area, inside height and opening width. Check each against the brief. Can the figure walk in and stand up?

7. **Delight test.** Ask two or three people, without telling them your scores, to rate its appearance from 1 to 10 and say one thing they like. Average the ratings.
8. Compute the actual weighted score and the strength-to-weight ratio (load carried ÷ pavilion mass).
9. **Improve.** Change one thing to raise your weakest score, rebuild, and test all three again. Record both rounds side by side.

Tips

- Folding a card into an angle or a tube makes a much stiffer column than a flat card.
- Load the roof gently and in the center first; then try one test with the load off center to see whether the pavilion tips.
- Honest scoring matters more than a high score. A low score with a clear reason is good engineering.

Check your work

- ☐ I can name and explain Vitruvius's three tests.
- ☐ My notebook shows a decision matrix, test data and an improvement round.
- ☐ I can explain how my improvement changed each of the three scores.

ENGINEER'S NOTEBOOK

1. Draw your pavilion in plan (from above) and elevation (from the front), with the dimensions in centimeters.
2. Make a data table for both rounds: mass (g), load carried (g), strength-to-weight ratio, floor area (cm^2), height (cm), opening (cm), delight average, weighted score.
3. Write a one-paragraph Vitruvian critique of a building in your town: one sentence each for firmness, commodity and delight, and one for what you would change.

KEY FORMULAS

$$\rho = \frac{m}{V}$$

$$W = mg = \rho V g$$

$$p = \frac{F}{A}$$

$$\text{weighted score} = \sum(\text{weight} \times \text{score})$$

> **From history.** Vitruvius dedicated his book to Augustus, who later boasted (according to the historian Suetonius) that he had found Rome a city of brick and left it a city of marble. The best-known illustrations of Vitruvius were not in his book at all; they were added by Renaissance editors who tried to draw what his words described, sometimes guessing wrongly, which is why scholars still argue about some of his machines.

WORKED EXAMPLES

Example 1. A limestone block for a Roman wall measures 1.20 m × 0.60 m × 0.50 m. Limestone has a density of about 2,500 kg/m^3. Find the block's mass and weight.

Show solution

1. Volume: $V = (1.20\ \text{m})(0.60\ \text{m})(0.50\ \text{m}) = 0.36\ \text{m}^3$.
2. Mass: $m = \rho V = (2500\ \text{kg/m}^3)(0.36\ \text{m}^3) = 900\ \text{kg}$.
3. Weight: $W = mg = (900\ \text{kg})(9.81\ \text{m/s}^2) = 8830\ \text{N} \approx 8.8\ \text{kN}$.

Answer: $m = 900$ kg, $W \approx 8.8$ kN (about the weight of a small car).

Example 2. A stone column weighs 96 kN and carries another 144 kN from the roof. It stands on a square footing 1.0 m on a side. What bearing pressure does it put on the soil? A firm clay can safely carry about 150 kPa. Is the footing big enough? If not, what size would be?

Show solution

1. Total force: $F = 96 + 144 = 240\ \text{kN} = 240{,}000\ \text{N}$.
2. Pressure: $p = F/A = 240{,}000\ \text{N} \div 1.0\ \text{m}^2 = 240{,}000\ \text{Pa} = 240\ \text{kPa}$.
3. Compare: 240 kPa is more than the 150 kPa the clay can safely carry, so the footing is too small.
4. Required area: $A = F/p = 240\ \text{kN} \div 150\ \text{kPa} = 1.6\ \text{m}^2$, so the side is $\sqrt{1.6} \approx 1.27$ m. Use a 1.3 m square footing.

Answer: 240 kPa, too much; a footing about 1.3 m square (1.6 m^2 or more) is needed.

Example 3. A church is choosing between three designs for a new entrance porch. Criteria and weights: firmness 0.4, commodity 0.35, delight 0.25. Scores out of 10: Design A (8, 6, 9), Design B (9, 8, 5), Design C (7, 9, 7). Which design scores best?

Show solution

1. A: $0.4(8) + 0.35(6) + 0.25(9) = 3.2 + 2.1 + 2.25 = 7.55$.
2. B: $0.4(9) + 0.35(8) + 0.25(5) = 3.6 + 2.8 + 1.25 = 7.65$.
3. C: $0.4(7) + 0.35(9) + 0.25(7) = 2.8 + 3.15 + 1.75 = 7.70$.
4. C scores highest, but the three are close. A close result is a signal to look again: could Design C be made stronger, or Design B more beautiful?

Answer: Design C (7.70), narrowly ahead of B (7.65) and A (7.55).

PRACTICE

1. Easy Give the Latin word and the English translation for each of Vitruvius's three tests.

   Show answer

   Firmitas, firmness; utilitas, commodity (usefulness); venustas, delight (beauty).

2. Easy A granite block is 0.50 m × 0.50 m × 1.00 m. Granite's density is 2,700 kg/m^3. Find its mass.

   Show answer

   $V = 0.25\ \mathrm{m}^3$, so $m = 2700 \times 0.25 = 675\ \mathrm{kg}$.

3. Easy Convert a force of 18,500 N to kilonewtons, and a pressure of 0.25 MPa to kilopascals.

   Show answer

   18.5 kN; 250 kPa.

4. Medium A brick has a mass of 2.3 kg and measures 0.215 m × 0.1025 m × 0.065 m. Find its density.

   Show answer

   $V = 0.215 \times 0.1025 \times 0.065 \approx 0.001432\ \mathrm{m}^3$, so
   $\rho \approx 2.3 \div 0.001432 \approx 1610\ \mathrm{kg/m}^3$.

5. Medium A wall 6.0 m long, 3.0 m high and 0.40 m thick is built of stone with density 2,400 kg/m^3. Find its weight in kilonewtons.

Show answer

$V = 7.2\ \text{m}^3$, $m = 17{,}280$ kg, $W = 17{,}280 \times 9.81 \approx 169{,}500\ \text{N} \approx 170\ \text{kN}$.

6. Medium The wall in Problem 5 sits on a strip footing 6.0 m long and 0.60 m wide. What bearing pressure does it exert?

Show answer

$A = 3.6\ \text{m}^2$, $p = 169.5\ \text{kN} \div 3.6\ \text{m}^2 \approx 47\ \text{kPa}$.

7. Medium The Roman foot (pes) was about 0.296 m. A temple Vitruvius describes is 120 Roman feet long. How long is it in meters and in modern feet (1 ft = 0.3048 m)?

Show answer

$120 \times 0.296 = 35.5$ m; $35.5 \div 0.3048 \approx 117$ ft.

8. Hard A column load of 300 kN must rest on soil that can safely carry 200 kPa. What is the smallest square footing that will work? Give the side to the nearest 0.1 m, rounding up.

Show answer

$A = 300 \div 200 = 1.5\ \text{m}^2$; side $= \sqrt{1.5} \approx 1.22$ m, so use 1.3 m.

9. Hard Redo Worked Example 3 with the weights changed to firmness 0.5, commodity 0.3, delight 0.2. Which design wins now? What does this tell you about decision matrices?

Show answer

A: 4.0 + 1.8 + 1.8 = 7.6; B: 4.5 + 2.4 + 1.0 = 7.9; C: 3.5 + 2.7 + 1.4 = 7.6. B wins. The weights carry the designer's values; when options are close, changing the weights can change the winner, so the weights must be chosen honestly and explained.

10. Hard Conceptual. Give an example of a building that is firm and useful but lacks delight, and explain why delight still matters to a client or a town.

Show answer

Model answer: a plain metal warehouse or a concrete parking garage. Delight matters because buildings last for generations and shape the street; people care for and keep buildings they find beautiful, so a building without delight is often neglected or torn down early.

11. Challenge A cylindrical stone column is 0.60 m in diameter and 6.0 m tall (density 2,500 kg/m^3). Find the bearing pressure at its base from its own weight alone. Then show that this pressure depends only on the height, not the diameter.

Show answer

$V = \pi(0.30)^2(6.0) \approx 1.70\ \mathrm{m}^3$, $W \approx 41.6\ \mathrm{kN}$, $A \approx 0.283\ \mathrm{m}^2$, $p \approx 147\ \mathrm{kPa}$. In general $p = \rho V g/A = \rho(Ah)g/A = \rho g h = 2500 \times 9.81 \times 6.0 \approx 147\ \mathrm{kPa}$, so the area cancels.

COMMON MISTAKES

- Mixing up mass (kg) and weight (N). Multiply mass by 9.81 m/s^2 to get weight in newtons.
- Forgetting to square both dimensions when finding an area, or to convert cm to m before computing pressure.
- Treating delight as a matter of taste only. Proportion, order, light and fitness for purpose can be discussed and judged with reasons.

> **Make it a game · Three-Test Verdict.** Pick any building you can see from a window or remember well. In ninety seconds, judge it on firmitas, utilitas and venustas, giving one concrete reason for each. Your parent argues the opposite on one test. Then estimate the weight of a column's load: volume times density, and the bearing pressure beneath it. Closest estimate wins.

> **Scripture.** “According to the grace of God which is given unto me, as a wise masterbuilder, I have laid the foundation, and another buildeth thereon. But let every man take heed how he buildeth thereupon.” (1 Corinthians 3:10, KJV)
>
> The Greek word Paul uses for “masterbuilder” is *architekton*, the root of our word architect. The warning to take heed how we build applies to buildings as well as lives.

1.2 Imhotep and the Pyramids

OBJECTIVES

- Describe how Egyptian builders moved from the mastaba to the Step Pyramid to the true pyramid, and what problems they solved along the way.
- Compute the volume and mass of a pyramid and convert the Egyptian seked into a slope angle.
- Estimate the labor and logistics of a great building project from simple rates.

> **Warm-up.** Pour a cup of dry sand, sugar or rice into a heap on a tray. Without measuring, guess the angle of the slope. Now imagine the heap made of stacked stone blocks instead. Could it be steeper? Why? Write your guess and keep the heap for the build.

WATCH

The INCREDIBLE Ancient Engineering That Built the Pyramids

Be Smart · 7:01

https://www.youtube.com/watch?v=52V9jmrgSbI

> **WHEN TO WATCH**
> After the story. Write down three engineering problems the video says the builders had to solve.

Note: Opens with a short sponsor message; skip it.

BACKUP VIDEO

The Inventor of the First Pyramid | Lost Treasures of Egypt

National Geographic · 4:01

https://www.youtube.com/watch?v=brMc05lOIFg

With the story of Imhotep and the Step Pyramid.

KEY TERMS

mastaba

A flat-topped, rectangular Egyptian tomb building with sloping sides, the starting point for the first pyramid.

step pyramid

A pyramid built as a stack of shrinking platforms, like Djoser's pyramid at Saqqara.

seked

The Egyptian measure of slope: how many palms the face runs horizontally for each cubit (7 palms) it rises.

angle of repose

The steepest slope at which a heap of loose material (sand, gravel) stays without sliding; for dry sand about 30–35°.

logistics

The planning of how materials, tools, workers and food arrive where and when they are needed.

CORE IDEAS

The story. About 2650 BC, the Egyptian king Djoser's chief official and builder, Imhotep, did something no one had done before: he built a great monument entirely of cut stone. His name is carved on the base of a statue of Djoser found at Saqqara, which lists his titles. Imhotep's first design was a mastaba, a low flat-topped tomb. He enlarged it, then set smaller mastabas on top, one after another, until it became a pyramid of six steps about 62 m high. A century later, builders for King Khufu raised the Great Pyramid at Giza, about 146.6 m tall with a base about 230 m on each side. For nearly four thousand years it was the tallest structure on earth.

How they figured it out. The Egyptians learned by building, and they did not always get it right the first time. Djoser's steps were built of layers that lean inward, which keeps the stone in compression and makes a stable heap. Under the next king, Sneferu, builders tried for smooth sides. The so-called Bent Pyramid at Dahshur starts at a steep angle of about 54° and then changes to about 43° partway up, probably because the structure and its foundation were cracking and settling. Sneferu's next pyramid, the Red Pyramid, was built at the gentler angle from the start. Khufu's builders then reached about 52° with careful workmanship. They leveled the base so accurately that it is flat to within about 2 cm, and they aligned the

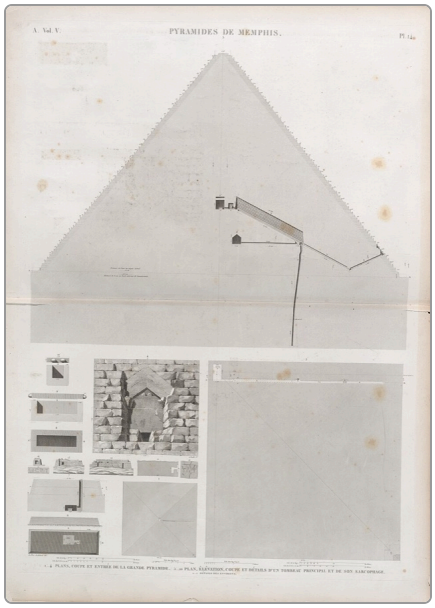
sides to north, south, east and west within a small fraction of a degree, probably by sighting stars. A papyrus logbook kept by an inspector named Merer, found in 2013 at Wadi al-Jarf, records crews ferrying limestone from the Tura quarries across the Nile to Giza by boat and canal.

The physics. A pyramid is the most stable shape for a heap of heavy blocks: its weight is lowest where it is widest, and every stone presses down and inward on the ones beneath. The volume of a pyramid is $V = \frac{1}{3}b^2h$ for a square base of side b and height h. The Egyptians described the slope of a face by its **seked**: the horizontal run in palms for a rise of one cubit (7 palms). The Great Pyramid has a seked of 5½ palms, so $\tan\theta = 7/5.5$ and $\theta \approx 51.8^\circ$. Loose sand slumps at its angle of repose, about 30–35°, but cut and fitted stone, held by friction and its own weight, can stand much steeper. Moving the stone took force: dragging a sledge needs a pull about equal to the friction coefficient times the weight, $F = \mu W$. A wall painting in the tomb of Djehutihotep shows a man pouring water in front of a sledge carrying a colossal statue; experiments in 2014 showed that damp sand can cut the pulling force roughly in half.

The engineering. The real achievement of the pyramids was organization. Several million blocks had to be quarried, shaped, moved and set in one king's reign, about 20 years, which means hundreds of blocks every working day. Archaeologists have found the village where the workers lived near Giza, with bakeries and cemeteries; the crews were organized into named gangs and divisions, fed and housed by the state. Ramps, sledges, levers, copper tools, boats and an enormous supply system were all part of the machine. Today we call this project management and logistics, and they are as much a part of civil engineering as the shape of the structure.

Today. Engineers still estimate big projects the way you will in this lesson: quantity (how much material), rate (how fast it can be placed), and duration (how long the job will take). A pyramid, a dam and a skyscraper are all answered the same way.

PICTURE STUDY



Plans and Section of the Great Pyramid, Engineers and artists of Napoleon's Commission of Sciences and Arts, 1809 · Engraving · *Description de l'Égypte*; New York Public Library

Image: Wikimedia Commons, public domain

In 1798–1801, French engineers and scholars measured the monuments of Egypt and published their drawings in an enormous book, the *Description de l'Égypte*. This plate is a measured section: the pyramid sliced through its center so you can see the passages inside.

Look closely

- Find the long sloping passages. Which way do they run, and where do they lead?
- Find the faces of the pyramid. Use a protractor on a printout or your screen to estimate their slope. Is it close to 52°?
- Look at the small drawings below. Which are plans and which are sections or elevations?
- Notice the scale bar or measurements on the plate. Why would the engineers include them?

Talk about it

1. Why is a section drawing more useful to an engineer than a picture of the outside?

2. The passages are open spaces inside a mass of stone. What must the stones above a passage do to keep it from collapsing?
3. These drawings were made about 4,300 years after the pyramid was built. What can measured drawings teach us that the builders never wrote down?

BUILD

Steeper than Sand: Slopes, Sekeds and a Stepped Model

You will measure the angle of repose of loose materials, then build a stepped pyramid from blocks to the Great Pyramid's seked and see how much steeper fitted blocks can stand. Finally, you will estimate the labor for your model the way the Egyptians' overseers had to.

You will need

- Dry sand, rice or sugar, a tray and a funnel or paper cone
- A protractor, ruler and calculator
- About 100 sugar cubes, LEGO bricks or small wooden blocks of one size
- A stopwatch
- Graph paper

Steps

1. Pour each loose material slowly through the funnel onto the tray from a low height until the heap stops growing taller. Measure the heap's height and the radius of its base in three directions. Compute the angle with $\theta = \tan^{-1}(h/r)$. Repeat twice and average.
2. Lightly dampen one material and repeat. Record whether the angle increases.
3. Convert the Great Pyramid's seked (5½ palms per cubit) to a rise-to-run ratio. For your blocks, work out how far each layer must step in so that the overall slope matches (for blocks 1 cm tall, each step should run in about 0.79 cm).
4. **Predict.** Will your stepped model be stable at 51.8°? Could it stand steeper? Write your prediction.
5. Build a square stepped pyramid with a base at least 7 blocks wide. Time how long it takes and count the blocks.
6. Measure its height and base, compute its overall slope angle, and compare with 51.8° and with the angle of repose.
7. Try a steeper version (each layer stepping in less). Tap the table gently beside it ten times. Record at what slope it begins to shed blocks.

8. Compute your building rate (blocks per minute). Use it to estimate how long you alone would need for 2.3 million blocks, and how many builders working like you would be needed to finish in 20 years of 300 ten-hour days.
9. **Improve.** Change your method (presorting blocks, working with a partner, building one layer at a time) and measure the new rate.

Tips

- Measure the height of the heap at its peak with a ruler held straight up, and the radius from the peak's center to the edge.
- For the steepness test, tap the same way each time so the test is fair.
- Keep a tidy table: material, dry or damp, height, radius, angle.

Check your work

☐ I can convert a seked to a slope angle.

☐ My data compare the angle of repose with the slope of my stepped model.

☐ I can estimate labor from a measured rate and explain why the pyramids needed such organization.

ENGINEER'S NOTEBOOK

1. A labeled section sketch of the Step Pyramid (a stack of mastabas) and of the Great Pyramid, with heights and slopes marked.
2. Your angle-of-repose table and the average angle for each material.
3. Your rate calculation, with each step and its units, and one sentence on what it tells you about the size of the workforce.

KEY FORMULAS

$$V = \tfrac{1}{3}b^2h$$

$$\tan\theta = \frac{\text{rise}}{\text{run}} = \frac{7\text{ palms}}{\text{seked}}$$

$$F_{\text{pull}} = \mu W$$

$$\text{rate} = \frac{\text{quantity}}{\text{time}}$$

From history. The Greek writer Herodotus visited Egypt about 2,000 years after the Great Pyramid was built and repeated stories that 100,000 men worked on it in three-month shifts. Modern archaeologists, working from the size of the workers' village and bakery at Giza, think a permanent force of perhaps 20,000 to 30,000 people is more likely, with more at the height of the work.

WORKED EXAMPLES

Example 1. The Great Pyramid's base is about 230.3 m on a side and its original height about 146.6 m. Find its volume and estimate its mass if it were solid limestone at 2,500 kg/m^3.

Show solution

1. $V = \frac{1}{3}b^2h = \frac{1}{3}(230.3\text{ m})^2(146.6\text{ m})$.
2. $(230.3)^2 = 53{,}038\text{ m}^2$, and $53{,}038 \times 146.6 = 7{,}775{,}400\text{ m}^3$; divide by 3: $V \approx 2.59 \times 10^6\text{ m}^3$.
3. $m = \rho V = (2500\text{ kg/m}^3)(2.59 \times 10^6\text{ m}^3) \approx 6.5 \times 10^9\text{ kg}$, about 6.5 million tonnes.
4. This is an overestimate: the pyramid was built over a knoll of natural bedrock and has passages and chambers inside.

Answer: $V \approx 2.6$ million m^3; mass roughly 6.5 million tonnes.

Example 2. The Great Pyramid's faces have a seked of 5½ palms. What is the slope angle? Use it to check the height from the base: is $h = (b/2)\tan\theta$ close to 146.6 m?

Show solution

1. $\tan\theta = 7/5.5 = 1.2727$.
2. $\theta = \tan^{-1}(1.2727) \approx 51.8^\circ$.
3. Half the base: $b/2 = 115.15$ m. Height: $h = 115.15 \times 1.2727 \approx 146.6$ m.
4. The seked and the measured base give exactly the measured height, which shows how consistently the builders kept their slope.

Answer: $\theta \approx 51.8^\circ$; the computed height, 146.6 m, matches.

Example 3. Suppose the Great Pyramid used 2.3 million blocks set over 20 years, working 300 days a year and 10 hours a day. How many blocks per day and how many minutes per block is that?

Show solution

1. Working days: $20 \times 300 = 6000$ days.
2. Blocks per day: $2{,}300{,}000 \div 6000 \approx 383$ blocks/day.
3. Blocks per hour: $383 \div 10 \approx 38$; minutes per block: $60 \div 38 \approx 1.6$ min.
4. That is one block set roughly every minute and a half, all day, for twenty years. Many crews must have worked at once.

Answer: About 380 blocks a day, one every 1.6 minutes.

Example 4. A 2.5-tonne block rides on a wooden sledge over damp sand with a friction coefficient of about 0.25. If one worker can pull steadily with about 300 N, how many workers are needed on level ground?

Show solution

1. Weight: $W = mg = (2500\ \text{kg})(9.81\ \text{m/s}^2) = 24{,}500$ N.
2. Pull needed: $F = \mu W = 0.25 \times 24{,}500 = 6130$ N.
3. Workers: $6130 \div 300 \approx 20.4$, so 21 workers.

Answer: About 21 workers (more on a ramp or on dry sand).

PRACTICE

1. Easy What did Imhotep build, and why was it new?

   Show answer

   The Step Pyramid of Djoser at Saqqara (about 2650 BC), the first great monument built entirely of cut stone, made by stacking mastaba-like layers.

2. Easy A seked of 7 palms means the face runs 7 palms for each 7 palms of rise. What angle is that?

   Show answer

   $\tan\theta = 1$, so $\theta = 45^\circ$.

3. Easy Find the volume of a pyramid with a square base 10 m on a side and a height of 6 m.

Show answer

$V = \frac{1}{3}(10)^2(6) = 200\ \mathrm{m}^3$.

4. Medium The Red Pyramid has a slope of about 43°. What seked (palms per cubit of rise) does that correspond to?

Show answer

$\text{seked} = 7/\tan 43^\circ \approx 7/0.933 \approx 7.5$ palms.

5. Medium Djoser's Step Pyramid is about 62 m tall with a base of about 109 m × 121 m. Estimate its volume as if it were a smooth pyramid on a rectangular base, $V = \frac{1}{3}(\text{base area})h$.

Show answer

$V \approx \frac{1}{3}(109 \times 121)(62) \approx 273{,}000\ \mathrm{m}^3$ (its steps make the real volume a little larger).

6. Medium A heap of dry sand is 12 cm tall with a base radius of 19 cm. Find its angle of repose.

Show answer

$\theta = \tan^{-1}(12/19) \approx 32^\circ$.

7. Medium Merer's crew may have carried about 30 blocks per boat trip, making about two round trips every week. How many blocks would 40 such crews deliver in a year of 50 weeks?

Show answer

$30 \times 2 \times 50 \times 40 = 120{,}000$ blocks a year.

8. Hard A 2.5 t block is dragged up a ramp sloping at 5° on damp sand ($\mu = 0.25$). The pull must overcome friction, about $\mu W \cos\theta$, plus the slope component $W \sin\theta$. Find the total pull and the number of 300 N workers.

Show answer

$W = 24{,}500$ N; friction $\approx 0.25(24{,}500)(0.996) \approx 6100$ N; slope part $= 24{,}500 \sin 5^\circ \approx 2140$ N; total ≈ 8240 N, about 28 workers.

9. Hard The Great Pyramid's base is level to within about 2 cm over 230 m. Express that as a slope in millimeters per meter and as a percent.

Show answer

$20\ \text{mm} \div 230\ \text{m} \approx 0.087\ \text{mm/m}$, or about 0.009%.

10. Hard Conceptual. Why might the builders of the Bent Pyramid have changed its slope partway up? What does that tell you about how engineering knowledge grows?

Show answer

Model answer: the steep lower part and its foundation were probably cracking or settling, so they reduced the angle to lower the load and the risk. Engineering knowledge grows by careful observation of problems and changing the design, then carrying the lesson forward (the Red Pyramid used the gentler angle from the start).

11. Challenge Show that if every dimension of a pyramid is doubled, its volume (and weight) grows 8 times, but its base area grows only 4 times. What happens to the average pressure on the ground under it?

Show answer

$V \propto b^2 h$, so V grows $2^2 \times 2 = 8$ times; base area b^2 grows 4 times. Average pressure $= W/A$ grows $8/4 = 2$ times. Bigger structures press harder on their foundations, which is why a giant pyramid needs firm rock beneath it.

COMMON MISTAKES

- Forgetting the ⅓ in the pyramid volume formula. A pyramid holds one-third as much as a box with the same base and height.
- Using the full base instead of half the base when finding the slope from height. The face rises over half the base width.
- Thinking the pyramids were built by slaves under the lash. The evidence from the workers' village points to organized, fed and housed crews of laborers and craftsmen.

> **Make it a game · Seked Challenge.** Sand-slope duel. Build a pile of sugar or rice and measure how steep it will stand. Predict the angle of repose first, then compare. Next, convert a seked of 5½ palms to an angle and say which pyramid stage, mastaba, step or true, a builder would choose. Closest prediction scores. Best of three.

1.3 Christopher Wren and St Paul's

OBJECTIVES

- Tell how Christopher Wren rebuilt St Paul's Cathedral after the Great Fire and explain his three-part dome.
- Explain why domes and cones push outward at their base and how a tension ring (Wren's iron chain) resists it.
- Compute compressive stress in a masonry element and estimate quantities from simple geometry.

> **Warm-up.** Push down on the top of an upturned paper cup and watch its rim. Now cut a cone from paper, stand it on the table and press its tip. What does the bottom edge try to do? Hold it with a rubber band around the base and press again.

WATCH

Designing St Paul's - The Wren Office Drawings

St Paul's Cathedral · 5:29

https://www.youtube.com/watch?v=skX-RXJnaWU

> **WHEN TO WATCH**
> After the story. Watch how the dome design changed from drawing to drawing, and note what the drawings show that the finished building hides.

KEY TERMS

dome

A curved roof shaped like part of a sphere or a similar surface; it carries load mostly in compression but pushes outward near its base.

drum

The cylindrical wall on which a dome sits, often pierced with windows.

lantern

A small tower with openings on top of a dome, letting light in and crowning the building.

hoop tension

The pull that develops around the lower part of a dome or the base of a cone as it tries to spread outward.

compressive stress

Force per unit area pressing on a material, $\sigma = F/A$, in pascals; masonry is strong in compression.

CORE IDEAS

The story. On 2 September 1666 a fire began in a bakery on Pudding Lane in London. In four days it destroyed about 13,200 houses, 87 parish churches and the old Gothic cathedral of St Paul's. Christopher Wren, then 34, was a professor of astronomy at Oxford and a founding member of the Royal Society, a brilliant mathematician who had turned to architecture only a few years before. He was named one of the commissioners for rebuilding the City, became Surveyor of the King's Works in 1669, and was given the task of rebuilding the cathedral and 51 of the city's churches. Work on St Paul's began in 1675 and the building was declared complete in 1711. Wren lived to see it finished. His son had a Latin epitaph placed over his tomb in the crypt: "Reader, if you seek his monument, look around you."

How they figured it out. Wren's design went through many stages. His favorite, a huge Greek-cross plan built as a large oak model in 1673–74 (the Great Model, still kept at the cathedral), was rejected by the clergy as too unlike a traditional cathedral. His approved "Warrant Design" of 1675 had a long nave and a strange tiered spire, but Wren was allowed to make changes as the work went on, and he quietly reworked the design for decades. The great problem was the dome. Wren wanted a tall, majestic outer dome to rule the London skyline, a lower inner dome that would look right from the floor, and a heavy stone lantern on top. No single masonry dome could do all three. His answer, worked out with his friend Robert Hooke, was to build three structures nested inside one another.

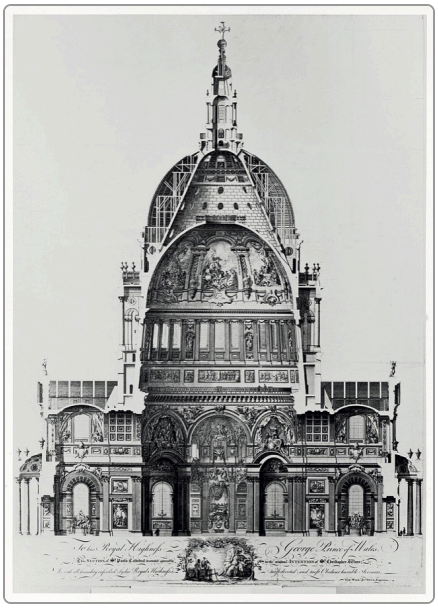
The physics. The inner dome is brick, shaped to look well from below. Above it, hidden from view, stands a cone of brick about 18 inches (0.46 m) thick. The cone carries the stone lantern and the cross, several hundred tonnes, straight down to the drum. Over the cone is a light timber frame covered with lead: the outer dome you see from the street. A cone is a good shape for a heavy point load at its top, because the load runs down its straight sides in compression. But the sloping sides push outward at the base, just as a pile of books leaning against each other will slide apart at the bottom. That outward push creates **hoop tension** around the base, and

masonry is weak in tension. Wren bound the base of the cone with a great iron chain set into the stonework, like a barrel hoop.

The engineering. Wren's dome is a lesson in letting each part do one job well: brick for compression, iron for tension, timber and lead for a light weatherproof shell. The stress in the brick is modest. Even with several hundred tonnes on top, the cone's large ring of brick keeps the compressive stress well below what good brick and mortar can carry. The result has stood for three centuries, through London's weather and the Second World War, when volunteers of the St Paul's Watch kept fires off the roof. Wren, the son of a clergyman and a devout Anglican, gave London more than fifty churches, and he is remembered as both an architect and a scientist.

Try it. In the build you will make cones and domes from card, load their tops and see the base spread, then add a "chain" and measure how much more they carry.

PICTURE STUDY



Section of the Dome of St Paul's Cathedral, Engraving after the drawings of Christopher Wren's office, 1755 · Engraving · London

Image: Wikimedia Commons, public domain

This engraving slices through the middle of St Paul's to show what no visitor can see from inside or out: the three structures of Wren's dome. Look at it as an engineer reading a section.

Look closely

- Find the inner dome, the one visible from the cathedral floor. How high is it compared with the outer dome?
- Find the cone between the two domes. Follow it up to the lantern. What is it carrying?
- Find the outer dome. What is between it and the cone?
- Find where the cone meets the drum. This is where Wren's iron chain was set. Why there?

Talk about it

1. Why did Wren need three structures instead of one?
2. Which parts of the dome are in compression? Where is there tension?
3. If the iron chain were removed, what would the base of the cone try to do?

> **Safety first.** Wear safety glasses for the loading test; a card cone can snap and fling a coin. Keep fingers out from under the loading cup.

BUILD

The Cone, the Dome and the Chain

You will build card cones and domes, load them at the top like Wren's lantern, and measure how the base spreads and when they fail. Then you will add a "chain" of string or a rubber band around the base and repeat the test.

You will need

- Card stock (several sheets of the same weight), tape, scissors and a compass
- String, thin rubber bands or a strip of paper tape for the tension ring
- A paper cup and pennies or rice for loading; a kitchen scale
- A ruler and a sheet of graph paper to mark the base outline

Steps

1. Cut identical circles of card stock 20 cm in diameter. Make cones by cutting a 90° wedge from a circle and taping the edges together. Make at least six.
2. Stand a cone on graph paper and trace its base. Set a small paper cup on its tip (tape a coin to the tip to make a flat seat if needed).
3. **Predict.** How much load will a plain cone carry? How much more with a tie around its base?
4. Add pennies to the cup in groups of 5. After each group, mark the base outline again and measure how far the base has spread. Record load and spread until the cone buckles.
5. Test two more plain cones the same way and average the failure loads.
6. Now tie a string tightly around the base of three new cones (or slip a rubber band over). Test them the same way.
7. Make a shallow "dome" from strips of card taped into a hemisphere of the same diameter and test it with and without a tie.
8. Graph base spread (mm) against load (g) for tied and untied cones on the same axes.
9. **Improve.** Try one change to carry more load with the same card: a second card cone inside the first, a deeper cone (cut a smaller wedge), or ribs. Test

and compare.

Tips

- Load slowly and in the center; an off-center load makes the cone fail early.
- Set the cones on a smooth table so the base can spread freely; on a rough surface friction acts like a weak tie.
- Count the pennies' mass (2.5 g each) carefully and record failure as the last load held.

Check your work

- ☐ My data show how much the base spread with and without a tie.
- ☐ I can explain why a dome or cone needs a tension ring at its base.
- ☐ I can describe Wren's three-part dome and the job of each part.

ENGINEER'S NOTEBOOK

1. Draw Wren's dome in section and label inner dome, cone, outer dome, lantern, drum and chain. Draw arrows for compression and for the outward push at the base.
2. Your load-versus-spread table and graph for tied and untied cones.
3. Explain in three or four sentences how your test matches what Wren and Hooke understood about the cone.

KEY FORMULAS

$\sigma = \dfrac{F}{A}$

$A_{\text{ring}} \approx \pi D t$ (thin ring of diameter D and thickness t)

$A_{\text{hemisphere}} = 2\pi r^2$

$\text{real size} = \text{drawing size} \times \text{scale factor}$

From history. Robert Hooke, Wren's friend and assistant surveyor after the fire, was the first to state (in 1675, hidden in a Latin anagram) that the ideal shape of an arch is a hanging chain turned upside down. Wren and Hooke discussed this as the dome was being designed. You will meet Hooke's idea again in Lesson 4.4.

Example 1. Suppose the lantern and cross weigh about 700 tonnes and are carried by the brick cone. If the top of the cone is a ring 8.0 m in diameter and 0.46 m thick, find the compressive stress in the brick there.

Show solution

1. Load: $F = mg = (700{,}000\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) \approx 6.87 \times 10^6\ \mathrm{N}$.
2. Ring area: $A \approx \pi D t = \pi(8.0\ \mathrm{m})(0.46\ \mathrm{m}) \approx 11.6\ \mathrm{m}^2$.
3. Stress: $\sigma = F/A = 6.87 \times 10^6 \div 11.6 \approx 5.9 \times 10^5\ \mathrm{Pa} \approx 0.59\ \mathrm{MPa}$.
4. Good brick masonry can carry several megapascals in compression, so this is a comfortable stress.

Answer: About 0.6 MPa, well within the strength of brick masonry.

Example 2. St Paul's is about 111 m tall to the top of the cross. How tall would it be on a drawing at a scale of 1:500? At 1:200?

Show solution

1. At 1:500, every 500 m of building is 1 m of drawing:
   $111\ \mathrm{m} \div 500 = 0.222\ \mathrm{m} = 22.2\ \mathrm{cm}$.
2. At 1:200: $111\ \mathrm{m} \div 200 = 0.555\ \mathrm{m} = 55.5\ \mathrm{cm}$.

Answer: 22.2 cm at 1:500; 55.5 cm at 1:200.

Example 3. Estimate the lead covering of the outer dome. Treat it as a hemisphere of radius 17 m covered with lead sheet 3 mm thick (density of lead 11,340 kg/m^3).

Show solution

1. Area: $A = 2\pi r^2 = 2\pi(17\text{ m})^2 \approx 1816\text{ m}^2$.
2. Volume of lead: $V = At = 1816\text{ m}^2 \times 0.003\text{ m} \approx 5.45\text{ m}^3$.
3. Mass: $m = \rho V = 11{,}340 \times 5.45 \approx 61{,}800$ kg, about 62 tonnes.
4. This is why the outer dome is a light timber frame: the lead alone is heavy, and a masonry outer dome of that height would have been far heavier.

Answer: Roughly 60 tonnes of lead (an estimate; the real dome is not a perfect hemisphere).

PRACTICE

1. Easy Name the three main structures of the dome of St Paul's and the material of each.

   Show answer

   The inner dome (brick), the hidden cone (brick), and the outer dome (timber frame covered with lead).

2. Easy In what year did the Great Fire of London begin, and when was St Paul's declared complete?

   Show answer

   1666; 1711.

3. Easy A drawing at 1:100 shows a column 9.0 cm tall. How tall is the real column?

   Show answer

   $9.0 \times 100 = 900\text{ cm} = 9.0\text{ m}$.

4. Medium A brick pier has a cross section 1.2 m × 1.2 m and carries 2,400 kN. Find the compressive stress in MPa.

   Show answer

   $A = 1.44\text{ m}^2$; $\sigma = 2.4 \times 10^6 \div 1.44 \approx 1.67 \times 10^6\text{ Pa} = 1.67\text{ MPa}$.

5. Medium A thin brick ring 10 m in diameter and 0.46 m thick carries 5,000 kN. Find the stress.

Show answer

$A \approx \pi(10)(0.46) \approx 14.5\ \mathrm{m}^2$; $\sigma \approx 5 \times 10^6 \div 14.5 \approx 0.34\ \mathrm{MPa}$.

6. Medium The inner dome of St Paul's spans about 31 m. Find the area of the floor beneath it, treating it as a circle.

Show answer

$A = \pi(15.5)^2 \approx 755\ \mathrm{m}^2$.

7. Medium Why did Wren's clergy reject the Great Model design, and what does that tell you about the client's role in architecture?

Show answer

They thought the Greek-cross plan was too unlike a traditional cathedral, with no long nave for processions and worship. The client's needs and traditions (commodity) shape the design as much as the architect's ideas.

8. Hard A cone carries a vertical load P at its tip, and its sides slope at angle α from the vertical. The force along the sides has a horizontal component of about $P \tan \alpha$ in total pushing outward around the base. For $P = 6900$ kN and $\alpha = 20^\circ$, estimate the total outward push.

Show answer

$P \tan 20^\circ = 6900 \times 0.364 \approx 2510$ kN, which a tension ring (the chain) must help resist.

9. Hard Using the result of Problem 8, a ring tie around a circle of radius R that resists a total outward push H spread evenly carries a tension of about $T = H/(2\pi)$. Find T, and the cross-sectional area of wrought iron needed if it may safely carry 100 MPa.

Show answer

$T \approx 2510 \div 6.283 \approx 400$ kN; $A = 400{,}000 \div 100 \times 10^6 = 0.004\ \mathrm{m}^2 = 40\ \mathrm{cm}^2$ (for example a bar about 6.3 cm square).

10. Hard Conceptual. Why is an outer dome of timber and lead better for St Paul's than an outer dome of stone the same size?

Show answer

A timber frame is far lighter, so it adds little load to the drum and piers below and creates little outward thrust; the lead keeps out the weather. A stone dome that tall would have been enormously heavy and would push outward hard on the drum.

11. Challenge In your build, the untied cones failed at an average of 380 g and the tied cones at 950 g. By what percent did the tie increase the strength? Give one reason your result might differ from a real masonry cone.

Show answer

$(950 - 380)/380 \approx 1.50$, a 150% increase. A card cone fails by buckling of thin sheets and is affected by the table's friction and the tape seam, while a masonry cone is thick, heavy and fails by cracking in tension at the base.

COMMON MISTAKES

- Thinking the dome you see from inside is the same dome you see from outside. At St Paul's they are different structures with a cone between them.
- Forgetting that domes and cones push outward at their base. Without a ring or buttress, the base spreads and cracks.
- Using the full area of a circle when a load is carried only by a thin ring of material. Use $\pi D t$ for a thin ring.

> **Make it a game · Dome Teach-Back.** Teach your parent, in sixty seconds, how Wren's three-part dome works and why a dome pushes outward at its base. Use a bowl or hat as a prop. Your parent slips in one wrong fact about the drum, lantern or iron chain. Catch it for a point. Swap, then explain hoop tension in a single sentence.

1.4 The Professions Today: Architects and Engineers

OBJECTIVES

- Distinguish the work of the architect, the civil engineer and the main civil-engineering specialties, and describe how each is licensed.
- Explain why licensure and codes of ethics exist, using the St. Francis Dam failure as a case study.
- Calculate a factor of safety and show how a small change in a design can change the loads a lot.

> **Warm-up.** List every person you think was involved in building your house or church, from the first idea to moving in. Try for at least ten. Circle the ones who needed a license.

WATCH

The Value of Professional Engineer (PE) License

Texas Society of Professional Engineers (TSPE) · 7:53

https://www.youtube.com/watch?v=EdCFFImNlSc

> **WHEN TO WATCH**
> After the core ideas. Write down the steps to becoming a licensed engineer as they are described.

BACKUP VIDEO

The Story of the St. Francis Dam

Santa Clarita · 6:42

https://www.youtube.com/watch?v=YWf6H3l4T4E

With the story of the St. Francis Dam.

KEY TERMS

architect

A licensed professional who designs buildings: their plan, form, spaces, materials and how they meet codes and the client's needs.

civil engineer

An engineer who designs public works and structures: buildings' frames, bridges, roads, dams, water and sewer systems. Specialties include structural, geotechnical, transportation, water resources and environmental engineering.

professional engineer (PE)

An engineer licensed by a state after an accredited degree, the Fundamentals of Engineering (FE) exam, about four years of supervised experience and the Principles and Practice of Engineering (PE) exam.

factor of safety

The ratio of what a part can carry to what it must carry:
$FS = \text{capacity}/\text{demand}$.

code of ethics

A profession's written rules of conduct. Engineering codes begin by requiring engineers to hold paramount the safety, health and welfare of the public.

CORE IDEAS

The story. In 1926 the City of Los Angeles finished the St. Francis Dam, a curved concrete gravity dam in San Francisquito Canyon, designed and built under William Mulholland, the city's self-taught and widely admired chief engineer. Just before midnight on March 12, 1928, with the reservoir nearly full, the dam failed. The flood traveled about 54 miles to the Pacific and took the lives of at least 431 people. That morning Mulholland had inspected a leak and judged it safe. At the inquest he took full responsibility for the dam.

How they figured it out. Investigators found that the dam had been built on poor ground. One side of the canyon was schist with weak layers and the remains of an ancient landslide, and the other was a conglomerate rock that softened and crumbled when soaked. Water seeping under the dam pushed up on its base (uplift),

and the dam's height had been raised twice during design without widening its base. No independent engineer or geologist had checked the design. California answered in 1929 with a state dam-safety program and a law requiring civil engineers to be licensed. Wyoming had passed the first engineering licensure law in the nation in 1907, after its state engineer saw too many water-rights maps drawn by people who did not know what they were doing. By the 1940s every state licensed engineers.

The physics. Engineers never design a part to carry exactly the load it will see. They divide its capacity by its demand to get a **factor of safety**, $FS = C/D$, and they require it to be well above 1 because loads, materials and calculations are all uncertain. Some loads grow much faster than the size of the structure. The water pressing on a dam grows with the square of the water depth, $F \propto H^2$: raise a dam from 175 ft to 195 ft (11% taller) and the push of the water grows by $(195/175)^2 \approx 1.24$, or 24%. The uplift under the dam grows too. A design change that looks small can use up much of the factor of safety.

The engineering. Today the two professions divide the work this way. The **architect** leads the design of a building: its plan, form, spaces, light, materials and code compliance, and coordinates the engineers. Architects earn an accredited degree, complete supervised experience and pass the Architect Registration Examination. The **civil engineer** designs what keeps buildings and public works standing and working: the structural engineer designs frames and foundations; the geotechnical engineer studies the soil and rock (exactly what was missing at St. Francis); transportation engineers design roads and traffic; water-resources and environmental engineers design dams, pipes and treatment plants. A licensed engineer's seal on a drawing is a public promise that a qualified person checked it. The National Society of Professional Engineers' code begins by telling engineers to hold paramount the safety, health and welfare of the public.

Today. Engineering is a vocation of service. Many people trust the work of engineers every day without ever knowing their names: when they drive over a bridge, drink from a tap or sit in a church balcony. Licensure, peer review and honest checking are ways of loving those neighbors.

PICTURE STUDY

The St. Francis Dam after the Failure, Unknown photographer, March 1928 · Photograph · Wikimedia Commons

Image: Wikimedia Commons, public domain

Days after the failure, photographers recorded the canyon. The center section of the dam still stood alone, nicknamed "the Tombstone," while both wings were gone. Engineers studied photographs like this one to understand how the dam failed.

Look closely

- Find the standing center section. Why might this part have survived while the sides did not?
- Look at the canyon walls where the dam was attached. What do you notice about the rock and the slopes?
- Find the empty reservoir basin behind. How much water do you think it held?
- Look for the remains of the dam's wings. Where did the pieces go?

Talk about it

1. The concrete of the center section held. What does that suggest about whether the failure began in the concrete or in the ground?
2. What kind of specialist engineer should have studied this site before the dam was built?

3. How does this photograph help explain why states began to license engineers?

Safety first. Wear safety glasses when loading paper beams to failure. Keep feet and hands out from under the hanging load.

BUILD

Designer and Checker: A Peer-Reviewed Paper Beam

Real engineering designs are checked by a second engineer. In this build you will design a paper beam to carry a required load with a factor of safety of 2, write your calculation, have someone check it independently, and then test to failure to find the true factor of safety.

You will need

- Printer paper or card stock, tape and a ruler
- Two stacks of books as supports, 30 cm apart
- A paper cup or small bag hung from a string loop, and pennies or rice for loads; a kitchen scale
- A partner (parent or sibling) to act as checker

Steps

1. **The requirement.** The beam must span 30 cm and carry a service load of 250 g at midspan with a factor of safety of at least 2, so it must be able to hold at least 500 g. Use no more than two sheets of paper.
2. Test one flat sheet across the gap to learn what plain paper carries. Record the result.
3. Design your beam (folded channel, tube, triangle, I-shape). Write a short design note: shape, dimensions, and your reasoning for why it should carry 500 g.
4. **Check.** Give the design note and beam to your checker before testing. The checker writes down any questions or concerns and predicts the failure load.
5. Answer the checker's questions in writing and make any changes you agree are needed. Record what was changed and why.
6. Test: add load in steps of 50 g, watching for sag. Record the load at which the beam fails and how it failed.
7. Calculate the actual factor of safety: failure load ÷ 250 g. Did it meet the requirement?

8. **Improve.** Redesign to meet the requirement with less paper, or to raise the factor of safety, and test again. Compare both beams.

Tips

- Hang the load from a loop around the beam at midspan so it pulls straight down.
- Fold creases sharply with the edge of a ruler; crisp folds make stiffer beams.
- A checker who finds a problem is helping, not criticizing. Record the review honestly.

Check your work

- ☐ I can calculate a factor of safety from a test.
- ☐ My notebook shows the design note, the checker's comments and my responses.
- ☐ I can explain why licensure and independent checking protect the public.

ENGINEER'S NOTEBOOK

1. A table comparing the architect, the structural engineer, the geotechnical engineer and the contractor: what each does, what training and license each needs, and one decision each makes on a building project.
2. Your design note, the checker's comments and the test results, with the calculated factor of safety.
3. A short paragraph: what might have been different at the St. Francis Dam if an independent geologist and engineer had reviewed the site and design?

KEY FORMULAS

$$FS = \frac{\text{capacity}}{\text{demand}}$$

$$\frac{F_2}{F_1} = \left(\frac{H_2}{H_1}\right)^2 \text{ (water force on a dam)}$$

$$1 \text{ ft} = 0.3048 \text{ m}$$

From history. William Mulholland had built the Los Angeles Aqueduct (1913), one of the great water projects of its time, and was widely admired. After the St. Francis Dam failure he retired. His story is often taught to engineering students because it shows that experience and reputation are no substitute for independent checking, especially of the ground beneath a structure.

Example 1. A steel hanger can carry 60 kN before it yields. It must support a walkway load of 24 kN. Find its factor of safety. If the design is changed so the hanger carries two walkways (48 kN), what is the new factor of safety?

Show solution

1. $FS = C/D = 60 \text{ kN} \div 24 \text{ kN} = 2.5$.
2. With two walkways: $FS = 60 \div 48 = 1.25$.
3. A change that doubles the demand halves the factor of safety. A designer who reviews only the original calculation would never see the problem; every change must be checked.

Answer: 2.5 originally; 1.25 after the change.

Example 2. A concrete dam is first designed to hold water 175 ft deep. The design is raised so the water can be 195 ft deep. By what percent does the horizontal force of the water increase?

Show solution

1. Force is proportional to the square of depth: $F_2/F_1 = (H_2/H_1)^2$.
2. $(195/175)^2 = (1.114)^2 \approx 1.242$.
3. The water force increases by about 24%, while the height increased by only about 11%.

Answer: About 24% more force.

Example 3. A student plans to be licensed as a civil engineer. Lay out a realistic timeline from the start of college, assuming a four-year accredited degree, the FE exam in the last semester, and four years of supervised experience before the PE exam.

Show solution

1. Years 1–4: accredited engineering degree; take the FE exam near graduation and become an Engineer in Training (EIT).
2. Years 5–8: work under licensed engineers, keeping a record of experience.
3. End of year 8: take the PE exam in a discipline such as civil (structural, geotechnical, transportation or water resources).
4. After passing and approval by the state board: licensed PE, about eight years after starting college.

Answer: About eight years from the first day of college to the PE license.

PRACTICE

1. Easy A rope can hold 9.0 kN and carries 3.0 kN. Find its factor of safety.

   Show answer

   $FS = 9.0 \div 3.0 = 3.0.$

2. Easy Which kind of engineer studies the soil and rock under a structure?

   Show answer

   A geotechnical engineer.

3. Easy Convert 195 ft to meters.

   Show answer

   $195 \times 0.3048 \approx 59.4 \text{ m}.$

4. Medium A beam must carry 12 kN with a factor of safety of 2.5. What capacity must it have?

   Show answer

   $C = FS \times D = 2.5 \times 12 = 30 \text{ kN}.$

5. Medium The water behind a dam rises from 40 m to 50 m deep. By what factor does the water's horizontal force increase?

Show answer

$(50/40)^2 = 1.5625$, an increase of about 56%.

6. Medium List the four main steps to becoming a licensed professional engineer in the United States.

Show answer

An accredited engineering degree; passing the FE exam (becoming an EIT); about four years of supervised experience; passing the PE exam and being approved by a state board.

7. Medium Give two physical reasons the St. Francis Dam failed.

Show answer

Weak foundation rock (schist with an old landslide on one side; conglomerate that softened when wet on the other), and water seeping under the dam that created uplift; the design had also been raised in height without widening the base.

8. Hard A floor beam was designed with FS = 2.0 for a 10 kN load. The owner later adds a library of heavy shelving, and the load becomes 16 kN. What is the new factor of safety, and what should the engineer advise?

Show answer

Capacity $= 20$ kN; new $FS = 20 \div 16 = 1.25$. The engineer should advise strengthening the beam or limiting the load, because 1.25 leaves too little margin for uncertainty.

9. Hard A dam's base width is usually made larger as the dam gets taller. If water force grows as H^2 and the dam's weight (which resists sliding) grows roughly as $H \times$ base width, what must happen to the base width to keep the same ratio of weight to water force?

Show answer

Weight $\propto H \cdot B$ and force $\propto H^2$, so the ratio is $\propto B/H$. To keep it constant, the base width must grow in proportion to the height.

10. Hard Conceptual. Why is an engineer's seal on a drawing a matter of ethics, not just paperwork?

Show answer

Model answer: by sealing a drawing the engineer personally certifies that it was prepared or checked by a qualified person and is safe; the public cannot check the design themselves and must trust that promise, so a seal is a matter of honesty and responsibility.

11. Challenge A paper beam in your test failed at 640 g. The required service load was 250 g. A checker had predicted 480 g. Find the actual factor of safety and the percent error of the checker's prediction. Is an underestimate or an overestimate the safer kind of error?

Show answer

$FS = 640 \div 250 = 2.56$. Error $= (480 - 640)/640 = -25\%$. An underestimate is safer: the real beam is stronger than predicted.

COMMON MISTAKES

- Thinking architects and engineers do the same job. Architects lead the design of the building; engineers design its structure, foundations and systems, and both are licensed separately.
- Treating a factor of safety as wasted strength. It covers real uncertainty in loads, materials, workmanship and calculation.
- Assuming force grows in proportion to size. Many loads, like water pressure on a dam, grow with the square of depth.

Make it a game · Who Signs the Plan?. Role-card drill. Write architect, civil engineer, structural, geotechnical and transportation on cards, plus short job descriptions. Shuffle and match them against the clock, then say how each is licensed and why the St. Francis Dam failure made ethics matter. Beat your best time in a second round.

Scripture. "When thou buildest a new house, then thou shalt make a battlement for thy roof, that thou bring not blood upon thine house, if any man fall from thence." (Deuteronomy 22:8, KJV)

One of the oldest building codes in the world: a builder is responsible for the safety of the people who will use what he builds.

1.5 The Design and Construction Team

OBJECTIVES

- Name the members of a design and construction team and the phases of a project from programming to construction administration.
- Compare design-bid-build and design-build delivery.
- Build a simple schedule, find its critical path and estimate a project budget with fees and contingency.

> **Warm-up.** Read Nehemiah 3:1-5 aloud. Count how many different groups are named in those five verses and note how each one's work is described. What does this tell you about how the work was organized?

WATCH

Structural Engineer vs Architect - Design Meeting

30X40 Design Workshop · 25:44

https://www.youtube.com/watch?v=29-xtjX8rAk

> **WHEN TO WATCH**
> After the core ideas. You can watch it in two parts. List each decision the architect and the engineer make together, and who leads each one.

Note: An architect, Eric Reinholdt, and his structural engineer work through the structure of a real house at the drawing table. Long, but an unusually honest view of how design teams actually work.

BACKUP VIDEO

What Is Design Build vs. Design Bid Build?

Jason Schroeder · 12:22

https://www.youtube.com/watch?v=uASYzkjeeEM

With the engineering paragraph. The speaker strongly prefers design-build; notice his reasons and think about what the other side would say.

KEY TERMS

owner (client)

The person or organization that wants the building, pays for it and sets its purpose and budget.

general contractor

The company that builds the project, hiring and coordinating subcontractors (concrete, steel, plumbing, electrical and others).

design-bid-build

A delivery method in which the owner hires an architect to complete the design, then contractors bid to build it, and the lowest qualified bidder builds.

design-build

A delivery method in which one team, under one contract, both designs and builds the project.

critical path

The longest chain of tasks that depend on each other in a schedule. Any delay on it delays the whole project.

contingency

Money (or time) set aside in a budget for the unknown problems every project meets.

CORE IDEAS

The story. In about 445 BC, Nehemiah, the cupbearer of the Persian king Artaxerxes, received permission to go to Jerusalem and rebuild its broken walls. He first inspected the ruins by night (Nehemiah 2:12-15), then organized the work. Chapter 3 of his book lists about forty groups, each assigned its own section of wall or gate: priests, goldsmiths, perfumers, rulers of districts, families, even daughters. Many repaired the part of the wall in front of their own houses. When enemies threatened, half worked while half stood guard, and the builders kept their tools in one hand. The wall was finished in fifty-two days (Nehemiah 6:15).

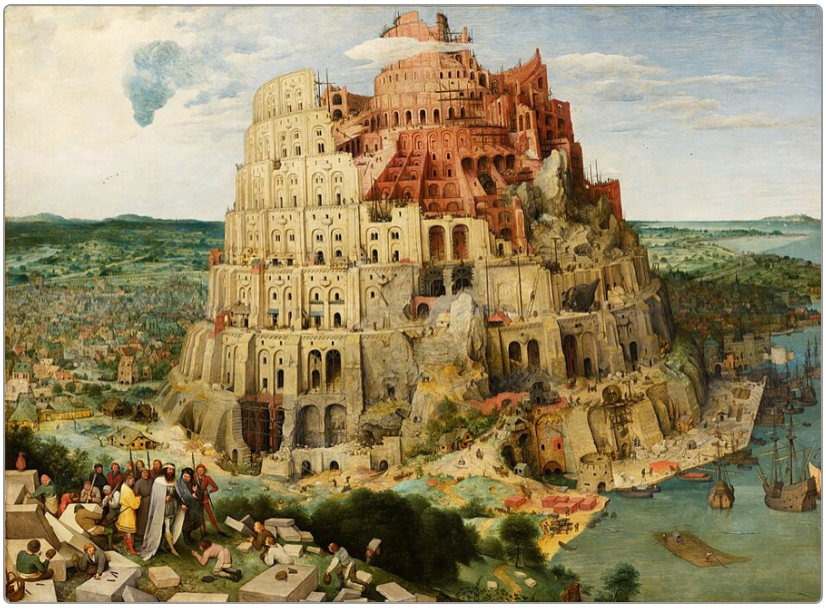
How they figured it out. Nehemiah did what every project manager still does. He surveyed the existing conditions before planning. He secured materials in advance: a letter from the king for timber from the royal forest (2:8). He broke an enormous job into many small sections that could be built at the same time by teams who each owned their piece; engineers today call this a work breakdown structure. He matched the work to the people, kept everyone informed, and dealt with risks as they came. And he kept the goal in front of the team, recording that the people had a mind to work.

The physics. In project planning, time behaves like a chain of dependencies. Some tasks can happen at once; others must wait (you cannot frame a wall until the foundation is poured, or roof a building before its walls stand). Draw the tasks as a network and add up the durations along every path. The longest path is the **critical path**, and it sets the shortest possible schedule. Tasks off the critical path have **float**: they can slip a little without delaying the finish. Cost works the same way, as a sum of quantities times unit prices, plus fees and a **contingency** for the unknown.

The engineering. A modern building project has a team much like Nehemiah's. The **owner** sets the purpose and budget. The **architect** leads the design and coordinates the **engineers**: structural, civil (site, grading and drainage), mechanical, electrical and plumbing (often called MEP), and sometimes geotechnical, fire protection and others. The **general contractor** builds it with many **subcontractors**, and the local **building official** reviews the drawings and inspects the work. The design passes through phases: programming (what the building must contain), schematic design, design development, construction documents, bidding or negotiation, and construction administration, when the architect answers questions (RFIs), reviews shop drawings (submittals), and checks the work. In **design-bid-build**, the owner holds separate contracts with the designer and the builder; competition keeps prices honest, but the builder joins late. In **design-build**, one team designs and builds under one contract, which can be faster and encourages cooperation, but the owner gives up some independent oversight. Either way, success depends on clear communication and every member doing his part faithfully.

Try it. In this lesson's build, you will plan a small project with a schedule and roles, then build it two ways and measure which organization works better.

PICTURE STUDY



The Tower of Babel, Pieter Bruegel the Elder, 1563 · Oil on oak panel · Kunsthistorisches Museum, Vienna

Image: Wikimedia Commons, public domain

Bruegel painted the story of Genesis 11 as a sixteenth-century construction site, full of the cranes, scaffolds and crafts of his own day. Look at it as a builder would. Then remember why the project failed: its builders wanted to make a name for themselves, and their work fell into confusion.

Look closely

- Find at least three cranes or lifting machines. How are they powered?
- Find the harbor. How is stone arriving at the site?
- Look closely at the tiers. Are they all finished to the same level? What does that tell you about the order of work?
- Find the foreground group. Who seems to be in charge, and who is doing the work?

Talk about it

1. What would a project manager worry about on a site like this one?
2. Some arches lean and some parts look unstable. Can you find places where the painter shows the structure failing?

3. Compare the builders of Babel (Genesis 11:4) with Nehemiah's builders. What made the difference between them?

Safety first. Have an adult handle the hot-glue gun if you use one; use low-temperature glue and keep a bowl of cool water nearby for burns.

BUILD

Two Ways to Build: Prefabrication and the Assembly Team

Solomon's temple was built of stone made ready before it was brought to the site (1 Kings 6:7). In this build you will construct the same small structure twice: once by yourself doing every step in order, and once with a planned team and prefabricated parts. You will schedule both, time both, and compare.

You will need

- About 120 craft sticks and white glue or a low-temperature glue gun
- A stopwatch, graph paper and a pencil
- One or two helpers
- A simple design: a craft-stick tower 4 identical square frames high, each frame braced with one diagonal (or a small bridge of identical panels)

Steps

1. Draw the structure in your notebook and list every task: cutting or sorting sticks, gluing each frame, adding diagonals, stacking and joining.
2. Estimate a duration for each task and draw a network diagram showing which tasks must wait for others. Find the critical path for building alone and for building with a team.
3. **Round 1, one builder.** Build the whole structure yourself, start to finish. Time each task and the total.
4. **Round 2, team with prefabrication.** Assign roles: one person prepares and sorts sticks, one glues frames, one adds diagonals, and one assembles (you may combine roles). Build the frames ahead of time as prefabricated parts, then assemble. Time each task and the total.
5. Measure the quality of each structure: check that the frames are square with a set square or a card corner, and that the tower stands plumb (hang a string with a weight beside it).
6. Load-test both: add a book at a time to the top until one sags or tilts. Record the loads.

7. Make a Gantt chart (a bar chart of tasks against time) for both rounds from your measurements.
8. **Improve.** Identify the task that held up the team in Round 2 (the bottleneck on the critical path). Reorganize and run a third round, or explain in writing how you would.

Tips

- Let glue set between steps in both rounds the same way, so the test is fair.
- Write down every surprise as it happens; these are the RFIs of your project.
- The goal is not only speed. A fast build that is out of square is a failure.

Check your work

☐ I can draw a network diagram and find the critical path.

☐ My notebook compares the planned and actual schedules for both rounds.

☐ I can explain the roles on a real design and construction team.

ENGINEER'S NOTEBOOK

1. An organization chart for a real building project (owner, architect, engineers, contractor, subcontractors, building official) with one sentence on each role.
2. Your network diagram, critical path and Gantt charts for both rounds, with planned and actual times.
3. A one-paragraph comparison of design-bid-build and design-build: one advantage and one risk of each.

KEY FORMULAS

$$\text{project duration} = \text{length of the critical path}$$
$$\text{float} = \text{latest start} - \text{earliest start}$$
$$\text{total budget} = \text{construction cost} \times (1 + \text{fee rate} + \text{contingency rate})$$

From history. Medieval cathedrals were built by teams led by a master mason who was both designer and builder, much like design-build today. The split between the architect who designs and the contractor who builds grew up in the 1700s and 1800s, and the modern design-build movement brings some of the old unity back.

Example 1. A small chapel project has these tasks: A, site work (3 weeks); B, foundation (5 weeks, after A); C, order roof trusses (2 weeks, after A); D, walls (4 weeks, after B); E, roof (2 weeks, after C and D). Find the critical path, the shortest duration and the float of task C.

Show solution

1. Paths: A–B–D–E: $3 + 5 + 4 + 2 = 14$ weeks. A–C–E: $3 + 2 + 2 = 7$ weeks.
2. The longest path is A–B–D–E, so it is the critical path and the project takes 14 weeks.
3. Task E cannot start until D finishes at week 12. C can finish as early as week 5, so it can slip up to $12 - 5 = 7$ weeks without delaying E.

Answer: Critical path A–B–D–E, 14 weeks; task C has 7 weeks of float.

Example 2. A parish hall is estimated to cost \$1,200,000 to build. Design fees are 8% of construction cost and the owner sets a 10% contingency. What total budget should the owner plan for?

Show solution

1. Fees: $0.08 \times \$1{,}200{,}000 = \$96{,}000$.
2. Contingency: $0.10 \times \$1{,}200{,}000 = \$120{,}000$.
3. Total: $\$1{,}200{,}000 + \$96{,}000 + \$120{,}000 = \$1{,}416{,}000$, or $\$1{,}200{,}000 \times 1.18$.

Answer: \$1,416,000.

Example 3. Suppose a city wall 2,400 m long is divided among 42 work groups, and the work must be done in 52 days. How long a section does each group build, and at what rate?

Show solution

1. Section length: $2400 \text{ m} \div 42 \approx 57 \text{ m}$ per group.
2. Rate: $57 \text{ m} \div 52 \text{ days} \approx 1.1 \text{ m}$ of wall per day per group.
3. A single group building the whole wall at that rate would need $2400 \div 1.1 \approx 2200$ days, about six years. Dividing the work is what makes 52 days possible.

Answer: About 57 m per group at about 1.1 m per day.

PRACTICE

1. Easy Who leads the design of a building, and who builds it?

   Show answer

   The architect leads the design; the general contractor (with subcontractors) builds it.

2. Easy List the main design phases in order.

   Show answer

   Programming, schematic design, design development, construction documents, bidding or negotiation, construction administration.

3. Easy Construction costs $500,000 and the contingency is 5%. How much is the contingency?

   Show answer

   $25,000.

4. Medium Tasks: A (2 days); B (4 days, after A); C (3 days, after A); D (1 day, after B and C). Find the critical path and duration.

   Show answer

   A–B–D, $2 + 4 + 1 = 7$ days (A–C–D is 6 days).

5. Medium In Problem 4, how much float does task C have?

   Show answer

   D starts at day 6; C can finish as early as day 5, so C has 1 day of float.

6. Medium What is an RFI and who answers it?

Show answer

A Request for Information: a written question from the contractor about the drawings or specifications, answered by the architect or engineer during construction.

7. Medium A house costs $180 per square foot to build. What is the construction cost of a 1,850 ft^2 house? Add a 7% design fee and a 10% contingency.

Show answer

$1850 \times 180 = \$333{,}000$; total $= 333{,}000 \times 1.17 = \$389{,}610$.

8. Hard In Worked Example 1, the foundation (task B) takes 2 weeks longer than planned because of bad soil. What is the new duration? If instead the truss order (task C) were 4 weeks late, what would happen?

Show answer

B is on the critical path, so the project grows to 16 weeks. C has 7 weeks of float, so a 4-week delay does not change the 14-week finish.

9. Hard Give one advantage and one disadvantage of design-bid-build for a church building committee spending donated money.

Show answer

Advantage: competitive bids on a complete design give a clear, fair price and an independent architect who checks the builder for the owner. Disadvantage: it is usually slower, and the builder's knowledge of cost and construction comes in only after the design is finished, which can lead to redesign if bids come in high.

10. Hard Conceptual. Nehemiah 3 shows many families repairing the wall in front of their own houses. Why is that a wise way to assign work?

Show answer

Model answer: each group had a clear, bounded section; they lived beside it, so they had a strong motive to build it well and could work without traveling; responsibility for each section was obvious, and many sections could proceed at once.

11. **Challenge** A project's critical path is 40 weeks. The owner wants it in 36 weeks. Crashing (adding workers) can shorten critical task X by up to 3 weeks at $8,000 per week and critical task Y by up to 2 weeks at $12,000 per week. What is the cheapest way to save 4 weeks, assuming the path stays critical?

    Show answer

    Shorten X by 3 weeks ($24,000) and Y by 1 week ($12,000): total $36,000. Using the cheaper task first gives the least cost.

COMMON MISTAKES

- Adding up all the task durations to get the schedule. Tasks that run at the same time overlap; only the critical path sets the duration.
- Leaving contingency out of a budget. Every real project meets surprises, especially underground.
- Thinking the team ends when the drawings are done. The architect and engineers stay involved through construction to answer questions and check the work.

Make it a game · Project Phase Relay. Write each project phase, from programming to construction administration, on a separate scrap. Your parent shuffles. Rebuild the order in thirty seconds, then name who on the team acts in each phase. Finish with a one-minute debate: design-bid-build versus design-build for building a tool shed. Your parent picks the winner.

Scripture. "So built we the wall; and all the wall was joined together unto the half thereof: for the people had a mind to work." (Nehemiah 4:6, KJV)

A great project is a team effort. Each group built its own section, and the sections joined into one wall.

UNIT 1

Architects and Civil Engineers

Every building and public work begins with people: someone who needs it, someone who designs it, someone who checks it and many who build it. This unit introduces the two professions at the heart of this course, the architect and the civil engineer, through four great builders of the past and the team that builds today. You will meet Vitruvius, whose three tests of a good building (firmness, commodity and delight) still frame every design review; Imhotep and the pyramid builders, whose real genius was organization; and Christopher Wren, astronomer turned architect, who hid a brick cone inside the dome of St Paul's.

Then you will see how the professions are organized today, and why. The failure of the St. Francis Dam in 1928 shows why engineers are licensed and why every design must be independently checked. Nehemiah's wall shows how a project team divides its work. Along the way you will start the habits of the whole year: calculate with units, test honestly, and record everything in your engineer's notebook.

PREREQUISITES

- Algebra I: solving equations, ratios and proportions, scientific notation
- Geometry: area and volume of prisms, pyramids, cones and spheres; right-triangle trigonometry (tangent) is reviewed in Lesson 1.2
- Comfort with SI units (meters, kilograms, newtons) and US customary units (feet, pounds)

MASTERY CHECKLIST

- ☐ I can explain Vitruvius's firmness, commodity and delight and use them to critique a building.
- ☐ I can calculate weight from density and volume and bearing pressure from force and area.
- ☐ I can compute the volume of a pyramid and convert a seked to a slope angle.
- ☐ I can explain Wren's three-part dome and why domes and cones need a tension ring.
- ☐ I can describe the work and licensing of architects and civil engineers and calculate a factor of safety.
- ☐ I can find a project's critical path and estimate a budget with fees and contingency.

OPTIONAL KITS FOR THIS UNIT

- **KEVA planks**: All ages. Identical maple planks that stack without glue; excellent for stepped pyramids (Lesson 1.2), arches and domes, and for the two-ways building test in Lesson 1.5.
- **LEGO Architecture**: Ages 12 and up for most sets. Models of famous buildings and skylines; useful for studying proportion and massing and as reference models through the year. Never required.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

1.1	**Vitruvius: Firmness, Commodity and Delight**	3 sessions of 50 minutes
1.2	**Imhotep and the Pyramids**	3 sessions of 50 minutes

1.3	**Christopher Wren and St Paul's**	3 sessions of 50 minutes
1.4	**The Professions Today: Architects and Engineers**	3 sessions of 50 minutes
1.5	**The Design and Construction Team**	3 sessions of 50 minutes

Unit 1 test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation; correct setup earns partial credit. Answer short-answer questions in complete sentences. The test is worth 100 points.

1. **5 pts**
   Name Vitruvius's three tests of a good building, in Latin or English, and explain each in one sentence.

2. **6 pts**
   A sandstone block measures 1.0 m × 0.80 m × 0.50 m and has a density of 2,300 kg/m^3. Find its mass and weight.

3. **7 pts**
   A column carries 180 kN and rests on soil that can safely carry 120 kPa. What is the minimum area of its footing, and the side of a square footing (rounded up to the next 0.1 m)?

4. **7 pts**
   A pyramid has a square base 60 m on a side and a height of 40 m. Find its volume and the slope angle of its faces.

5. **5 pts**
   Convert a seked of 6 palms (per cubit of 7 palms) to a slope angle.

6. **6 pts**
   Why did the builders of the Bent Pyramid change its angle partway up, and what did the next pyramid show they had learned?

7. 6 pts

A project needs 1,200,000 blocks placed in 15 years of 300 working days each. How many blocks must be placed per day?

8. 7 pts

Describe the three parts of Wren's dome at St Paul's and the job of each.

9. 6 pts

Why did Wren put an iron chain around the base of the brick cone?

10. 7 pts

A brick ring 6.0 m in diameter and 0.46 m thick carries 3,000 kN. Find the compressive stress in MPa.

11. 6 pts

A cable holds 45 kN and its working load is 15 kN. Find the factor of safety. If the load is raised to 25 kN, what is the new factor of safety?

12. 6 pts

The water behind a dam rises from 30 m to 36 m deep. By what percent does the horizontal water force increase?

13. 6 pts

Distinguish the work of an architect, a structural engineer and a geotechnical engineer on a new library.

14. 7 pts

Tasks: A (4 weeks); B (6 weeks, after A); C (3 weeks, after A); D (5 weeks, after C); E (2 weeks, after B and D). Find the critical path, the duration and the float of B.

15. 5 pts

A building costs $2,500,000 to construct. With design fees of 9% and a contingency of 8%, what total budget is needed?

16. 8 pts

Short essay. Explain what the St. Francis Dam failure taught the engineering profession. Include at least two physical causes of the failure and two changes that followed.

ANSWER KEY

1. Firmitas (firmness): it stands safely and lasts. Utilitas (commodity): it serves its purpose well. Venustas (delight): it is beautiful and a pleasure to see and use.
2. $V = 0.40\ \mathrm{m}^3$; $m = 920$ kg; $W = 920 \times 9.81 \approx 9030$ N ≈ 9.0 kN.
3. $A = 180 \div 120 = 1.5\ \mathrm{m}^2$; side $= \sqrt{1.5} \approx 1.22$, so 1.3 m.
4. $V = \frac{1}{3}(60)^2(40) = 48{,}000\ \mathrm{m}^3$; $\tan\theta = 40/30$, $\theta \approx 53.1^\circ$.
5. $\tan\theta = 7/6 \approx 1.167$, $\theta \approx 49.4^\circ$.
6. The steep lower part and its foundation were probably cracking and settling, so they lowered the angle to reduce the load. The Red Pyramid was built at the gentler angle (about 43°) from the start.
7. $1{,}200{,}000 \div 4500 \approx 267$ blocks per day.
8. The brick inner dome forms the ceiling seen from inside; the hidden brick cone carries the heavy stone lantern and cross down to the drum; the outer dome, a timber frame covered in lead, gives the tall shape seen across London and keeps out the weather.
9. A loaded cone pushes outward at its base, creating hoop tension. Brick is weak in tension, so the iron chain acts like a barrel hoop and holds the base from spreading.
10. $A \approx \pi(6.0)(0.46) \approx 8.67\ \mathrm{m}^2$; $\sigma \approx 3 \times 10^6 \div 8.67 \approx 0.35$ MPa.
11. $45 \div 15 = 3.0$; $45 \div 25 = 1.8$.
12. $(36/30)^2 = 1.44$, a 44% increase.
13. The architect designs the library's plan, form, spaces and materials and coordinates the team; the structural engineer designs its frame, floors and foundations to carry the loads; the geotechnical engineer investigates the soil and rock and recommends the foundation type and bearing capacity.
14. A–B–E = 12 weeks; A–C–D–E = 14 weeks. Critical path A–C–D–E, 14 weeks. E starts at week 12; B can finish at week 10, so B has 2 weeks of float.
15. $2{,}500{,}000 \times 1.17 = \$2{,}925{,}000$.
16. Model answer: the dam failed because it stood on weak rock (an ancient landslide in schist on one side and conglomerate that softened when wet on the other) and because water seeping under it created uplift; the design had been raised without widening its base, and no independent geologist or engineer had reviewed it. The disaster led California to create a state dam-safety program and to require licensing of civil engineers, and it is taught everywhere as a lesson

that reputation is no substitute for independent checking and site investigation, and that engineers must hold public safety paramount.

PORTFOLIO PIECE: A BUILDING BIOGRAPHY

Choose a public building in your town, such as a church, library, courthouse or school. Find out who designed it, who built it and when (a cornerstone, a historical society or the building's own office can help). Measure its main floor by pacing or with a tape measure, draw a simple floor plan to scale, and compute the floor area.

Write a two-page report that tells the building's story and the team that built it, then gives a Vitruvian critique with one paragraph each for firmness, commodity and delight. Finish with one improvement you would propose and a rough cost estimate with a contingency.

LOOK FOR	WHAT IT MEANS
Research	Names the architect, builder and date where they can be found, and cites where the information came from.
Measurement and drawing	Plan is drawn to a stated scale with dimensions; floor area is computed correctly with units.
Vitruvian critique	Firmness, commodity and delight are each discussed with specific observations, not general praise.
Proposal and estimate	A clear improvement is proposed with a reasoned cost estimate including fees and contingency.
Presentation	The report is neat, organized and written in complete sentences, with sketches labeled.

UNIT 2

Architectural Drawing

Drawings are the language of building. Before a single stone is cut, the architect and engineer must describe the whole building so exactly that hundreds of people who have never met can build it. In this unit you will learn that language: scale and the architect's scale, floor plans, elevations and sections, perspective, and three-dimensional modeling on paper, with blocks and in free software.

Each skill comes with its story: Thomas Jefferson drawing Monticello on graph paper; the monks who drew the plan of St Gall twelve centuries ago; the architects of the Historic American Buildings Survey measuring the nation's old buildings; Brunelleschi and his mirror; and Frank Lloyd Wright, whose mother's gift of kindergarten blocks shaped the way he saw buildings. By the end you will be able to measure a real building and draw it in plan, elevation, section and perspective, and build it as a computer model.

PREREQUISITES

- Unit 1: units and conversions, area and volume
- Geometry: similar triangles, the Pythagorean theorem, area of composite shapes
- Right-triangle trigonometry: the tangent ratio
- A computer with a web browser for SketchUp Free or Tinkercad in Lesson 2.5

MASTERY CHECKLIST

- ☐ I can read and draw at ¼″ = 1′-0″, ⅛″ = 1′-0″, 1:50 and 1:100, and I know that areas scale by the square of the scale factor.
- ☐ I can draw and read a dimensioned floor plan with correct conventions and compute net and gross areas.
- ☐ I can draw elevations and sections that line up with the plan and calculate roof pitch, rafter length and stair risers.
- ☐ I can explain Brunelleschi's perspective experiment and use

$y' = fY/Z$.

- ☐ I can build a simple building model in SketchUp Free or Tinkercad and produce plan, elevation and perspective views from it.

OPTIONAL KITS FOR THIS UNIT

- **SketchUp Free (web)**: Free browser-based 3D modeling from Trimble; a parent creates the account. Used in Lesson 2.5, the portfolio, and Unit 10. Not a kit, but the main tool of this unit.
- **Tinkercad**: Free, very simple browser-based 3D modeling from Autodesk, good for students new to CAD; a parent sets up the account for students under 13. An alternative to SketchUp in Lesson 2.5.
- **Froebel Gifts (Froebel USA)**: Reproductions of Friedrich Froebel's kindergarten block sets, the gifts that shaped Frank Lloyd Wright. Lovely for Lesson 2.5 and for younger siblings; plain wooden cubes work just as well.
- **KEVA planks**: Identical maple planks for block models of buildings in Lesson 2.5 and for model-making all year.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

2.1	**Scale and the Architect's Scale**	3 sessions of 50 minutes
2.2	**Floor Plans**	3 sessions of 50 minutes
2.3	**Elevations and Sections**	3 sessions of 50 minutes
2.4	**Perspective Drawing: Brunelleschi's Experiment**	3 sessions of 50 minutes
2.5	**Modeling in 3D: From Froebel's Blocks to SketchUp**	3 sessions of 50 minutes

Unit 2 test

Time allowed: 50 minutes. Calculators and an architect's scale are allowed. Show your work and units for every calculation. Use a ruler for any drawing. The test is worth 100 points.

1. **5 pts**

   What is the scale factor and ratio of ⅛″ = 1′-0″?

2. 6 pts

On a ¼″ = 1′-0″ plan, a room measures 3⅜ in × 4¼ in. Find its real dimensions and area.

3. 6 pts

A wall 7.35 m long is drawn at 1:50. How long is it on paper?

4. 6 pts

A 1:100 model has a roof area of 240 cm^2. What is the real roof area in m^2?

5. 5 pts

What is a cut plane, and how are the walls it cuts shown on a plan?

6. 6 pts

Check this dimension string: 5′-3″, 2′-10″, 8′-0″, 3′-11″; overall 20′-0″.

7. 6 pts

A house is 44 ft × 28 ft outside. Its rooms total 1,010 ft^2. Find the gross area and efficiency.

8. 7 pts

A gable roof on a building 32 ft wide has a 9:12 pitch. Find the roof angle, the rise and the rafter length (no overhang).

9. 7 pts

A stair must rise 10′-3″. With a maximum riser of 7¾ in and 10 in treads, find the number of risers, the riser height and the total run.

10. 7 pts

From 35 m away, the top of a steeple is sighted at 42° with an eye height of 1.5 m. How tall is the steeple?

11. 6 pts

What is the difference between an elevation and a section, and why does a builder need both?

12. 6 pts

Describe Brunelleschi's perspective experiment in three or four sentences.

13. **7 pts**

A column 9.0 m tall is 36 m from your eye and the picture plane is 0.60 m away. How tall is its image? How tall would it be at 54 m?

14. **6 pts**

In two-point perspective, where do the vanishing points lie, and what happens to vertical lines?

15. **6 pts**

A chapel is a box 36 ft × 22 ft × 14 ft with a gable roof rising 9 ft along its length. Find its enclosed volume.

16. **8 pts**

Short essay. How did Froebel's blocks, Monge's descriptive geometry and Sketchpad each change the way designers think about buildings in three dimensions?

ANSWER KEY

1. $12 \div 0.125 = 96$; 1:96.
2. 13.5 ft $\times$ 17 ft (13′-6″ × 17′-0″); area 229.5 ft^2.
3. $7350 \div 50 = 147$ mm.
4. $240 \times 10{,}000 = 2{,}400{,}000\ \text{cm}^2 = 240\ \text{m}^2$.
5. The imaginary horizontal slice about 4 ft above the floor; walls cut by it are drawn with heavy lines or filled in (poché).
6. 18′ + 24″ = 20′-0″. It is correct.
7. Gross 1232 ft^2; efficiency $1010 \div 1232 \approx 82\%$.
8. $\theta = \tan^{-1}(0.75) \approx 36.9^\circ$; run 16 ft, rise 12 ft; rafter $= \sqrt{256 + 144} = 20$ ft.
9. $123 \div 7.75 = 15.9$, so 16 risers of $123 \div 16 \approx 7.69$ in; 15 treads, run 150 in = 12′-6″.
10. $35 \tan 42^\circ + 1.5 \approx 31.5 + 1.5 = 33.0$ m.
11. An elevation shows the outside face; a section cuts through to show floors, roof structure, ceiling heights and foundations inside. The builder needs the elevation for the exterior appearance and openings and the section for heights and how the building is put together.
12. About 1415–1420, Brunelleschi painted the Florence Baptistery on a small panel from inside the cathedral doorway and drilled a hole at the point opposite his eye. A viewer looked through the hole from the back at a mirror held in front of the painting, and saw the painting reflected; lowering the mirror showed the real building from the same point. The two matched, proving his geometric method of perspective.
13. $0.60 \times 9.0 \div 36 = 0.15$ m; at 54 m, 0.10 m.
14. Both vanishing points lie on the horizon line (at eye height); vertical lines stay vertical and parallel.
15. $36 \times 22 \times 14 = 11{,}088$ plus $\frac{1}{2}(22)(9)(36) = 3564$, total 14,652 ft^3.
16. Model answer: Froebel's blocks trained children such as Frank Lloyd Wright to compose buildings from simple masses and planes; Monge showed that orthographic views define any three-dimensional shape exactly, which became the basis of engineering and architectural drawing; Sketchpad began computer-aided design, leading to modeling programs such as SketchUp in which a designer builds one model and draws every view from it.

PORTFOLIO PIECE: A MEASURED DRAWING SET

Measure your home, your church or another small building you can access. Produce a set of drawings like a HABS sheet: a dimensioned floor plan at ¼″ = 1′-0″ (or 1:50), two elevations and one section at a matching scale, all lined up, and a hand-drawn two-point perspective.

Then build the building as a model in SketchUp Free or Tinkercad from your drawings and produce a perspective view from the model. Compare it with your hand perspective and write a half-page reflection on what each method showed you.

LOOK FOR	WHAT IT MEANS
Measurement	Dimensions were measured carefully and checked (strings add up; height checked two ways).
Drafting conventions	Correct line weights, door swings, symbols, scale, north arrow, title block and section marks.
Projection	Plan, elevations and section line up and agree; perspective has a correct horizon and vanishing points.
Computer model	The model is built to true dimensions, organized in groups, and its views agree with the drawings.
Reflection	Thoughtful comparison of hand drawing and computer modeling, with specific examples.

2.1 Scale and the Architect's Scale

OBJECTIVES

- Read and use the architect's scale (for example ¼″ = 1′-0″) and metric scales (1:50, 1:100).
- Convert between drawing size and real size, and explain why areas scale by the square of the scale factor.
- Make an accurate scaled drawing of a real room and check it by measuring it back.

> **Warm-up.** Hold a ruler at arm's length in front of a doorway across the room. How many centimeters on the ruler does the door appear to be? Now measure the real door. You have just found a scale factor. Write both numbers and the ratio.

WATCH

How to Read an Architectural Scale | Beginner

Training Hands Academy · 6:13

https://www.youtube.com/watch?v=94fBiFHy5gM

> **WHEN TO WATCH**
> Before the worked examples. Pause after each reading he makes and try it yourself on your own scale.

BACKUP VIDEO

How to Read an Architect's Scale

WeldNotes · 7:09

https://www.youtube.com/watch?v=PrbjWgyZlFc

If the scale is still confusing: a simple animated explanation of how feet and inches are read from opposite ends.

KEY TERMS

scale
The fixed ratio between a drawing and the real object, written as ¼″ = 1′-0″ or 1:48.

architect's scale
A triangular ruler with eleven scales (from 3⁄32″ = 1′-0″ to 3″ = 1′-0″, plus a full-size 16ths scale), each read in feet, with the inches subdivided at one end.

engineer's scale

A scale divided into 10, 20, 30, 40, 50 or 60 parts per inch, used for site plans and civil drawings (for example 1″ = 20′).

dimension

A number written on a drawing that gives the true size; a builder reads the dimension, never measures the drawing, when one is given.

scale factor

The number of real units represented by one drawing unit: for ¼″ = 1′-0″ it is 48.

CORE IDEAS

The story. In 1768, a young Virginia lawyer named Thomas Jefferson began leveling the top of a small mountain near Charlottesville for a house he called Monticello. He had never studied with an architect. He taught himself from books, above all *The Four Books of Architecture* by the Italian Andrea Palladio, and he drew his own designs. His first elevation of Monticello, drawn about 1771, shows a two-story porch with columns and a pediment, borrowed from Palladio's villas. Jefferson often drew on graph paper, letting each little square stand for a fixed length, so that every wall and window could be laid out to scale and checked by counting squares. Over the next fifty years he rebuilt and redrew Monticello many times, and later designed the Virginia State Capitol and the University of Virginia.

How they figured it out. A building is too big to draw full size, so architects agree on a **scale**: a fixed ratio between the drawing and the building. Medieval masons laid out details full size on plaster "tracing floors," but by the Renaissance architects drew smaller, with a scale bar on the sheet. Jefferson's graph paper was a homemade scale. Today architects in the United States use the triangular **architect's scale**, which has eleven common scales on its six edges, and engineers use the **engineer's scale** for site plans. Most of the world uses metric ratios such as 1:50 and 1:100.

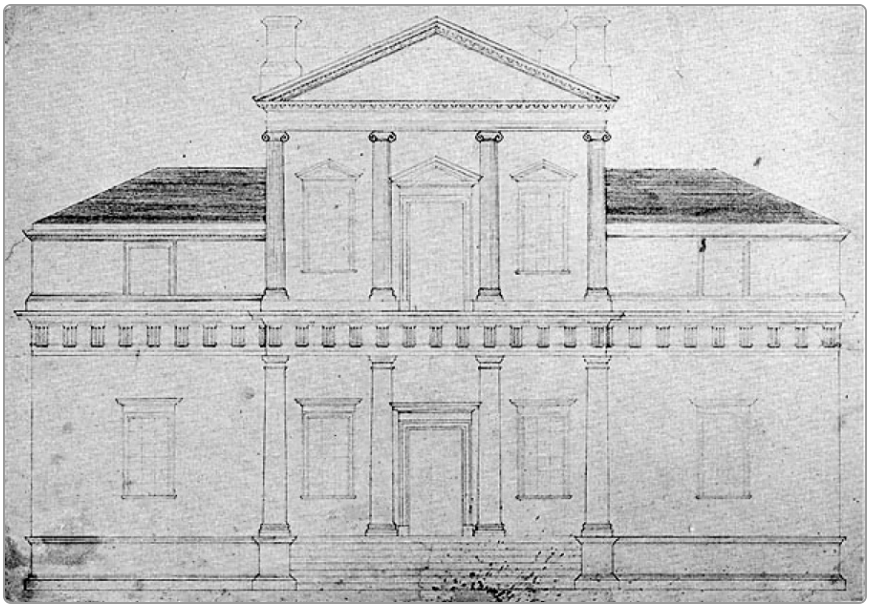
The physics. Scale is a proportion. At ¼″ = 1′-0″, one quarter inch on paper stands for twelve inches of building, so the scale factor is $12 \div \frac{1}{4} = 48$ and the ratio is 1:48. To find a real length, multiply the drawing length by the scale factor; to draw a real length, divide by it. **Areas scale by the square** of the factor and **volumes by the cube.** A room that measures 3 in × 4 in on a ¼″ drawing is 12 ft × 16 ft, or 192 ft^2, while the

drawing itself covers only 12 in^2: the area ratio is $48^2 = 2304$. This matters for models, too: a 1:50 model of a beam has $1/50$ the length but only $1/125{,}000$ of the weight, which is why a small model can look strong when the real structure would not be.

The engineering. Choose the scale to fit the job. Site plans use engineer's scales such as 1″ = 20′ or 1:500. Floor plans of houses are usually drawn at ¼″ = 1′-0″ (1:50 in metric countries), larger buildings at ⅛″ = 1′-0″ (1:100). Details of a window or a stair are drawn at 1½″ or 3″ = 1′-0″ (1:10 or 1:5). Every sheet states its scale, and many add a bar scale so that the drawing can still be read after it is copied larger or smaller. And every important size is written as a dimension, because paper stretches and copies distort; builders are taught never to scale a drawing when a dimension is given.

Try it. In the build you will measure a real room, draw it at ¼″ = 1′-0″, and then test your drawing by having someone else read it with a scale and compare with the real room.

PICTURE STUDY



First Elevation of Monticello, Thomas Jefferson, about 1771 · Ink drawing · Massachusetts Historical Society

Image: Wikimedia Commons, public domain

This is Jefferson's own drawing of the first version of his house, made when he was in his late twenties. He drew it from what he had learned in Palladio's books, and he kept changing the house for the rest of his life.

Look closely

- Find the two stories of columns in the center. Which columns look thicker, the lower or the upper?
- Count the windows on each side of the center. Is the drawing symmetrical?
- Find the triangular pediment. Estimate its slope.
- Look for any lines that show the drawing was laid out carefully (a ground line, center line or construction lines).

Talk about it

1. This is an elevation, not a picture. What does an elevation show that a perspective sketch does not?
2. If you knew the house was 60 ft wide, how could you find the height of the columns from this drawing?
3. Why would a self-taught architect draw his house to scale before building it?

BUILD

A Measured Room at ¼″ = 1′-0″

You will measure a real room, draw its floor plan at ¼″ = 1′-0″ with an architect's scale, and test how accurate your drawing is by having a partner read it back.

You will need

- An architect's scale (a triangular one is best) and a tape measure
- Graph paper with ¼ in squares, or plain paper, a sharp pencil and an eraser
- A partner
- A calculator

Steps

1. Choose a room with at least one door, two windows and a closet or alcove. Sketch it freehand first, large and loose, with every wall segment.
2. Measure every wall segment, the width of each door and window, and the distance from each corner to the nearest opening. Write the measurements on the sketch in feet and inches. Measure the overall length and width as a check.
3. Check your measurements: do the segments along each wall add up to the overall length? If they differ by more than 1 in, measure again.
4. Draw the plan at ¼″ = 1′-0″ using the architect's scale. Draw walls as double lines about 5 in apart (real size), show door swings as quarter circles, and windows as thin double lines in the wall.
5. Add dimension strings along two sides, a title, the scale and a north arrow.
6. **Predict.** How close do you think a stranger could measure from your drawing? Write a prediction in inches.
7. **Test.** Give the drawing (without your measurement sketch) to your partner. Ask them to use the scale to read five lengths you choose. Compare each with the real measurement and compute the error in inches and percent.
8. Also draw the same room at ⅛″ = 1′-0″. Which scale gave smaller errors? Why?

9. **Improve.** Redraw the wall with the largest error, find the cause (a measuring slip, a drawing slip, a thick pencil line) and record it.

Tips

- Measure in feet and inches, not inches only, to match the architect's scale.
- Keep your pencil sharp; a line 1⁄32 in thick is 1½ in thick in real life at ¼″ scale.
- Hold the tape tight and level; a sagging tape reads long.

Check your work

☐ I can read lengths at ¼″, ⅛″ and 1″ = 1′-0″ on the architect's scale.

☐ My plan is drawn to scale with dimensions, a title, a scale and a north arrow.

☐ My notebook shows the measured errors and what caused the largest one.

ENGINEER'S NOTEBOOK

1. Your freehand measurement sketch and your finished plan (attach or copy into the notebook).
2. An error table: five lengths, real measurement, partner's reading, error (in) and error (%), for both scales.
3. A short paragraph explaining why builders follow written dimensions instead of measuring the drawing.

KEY FORMULAS

$$\text{scale factor } k = \frac{\text{real length}}{\text{drawing length}} \text{ (same units)}$$

$$\text{real} = k \times \text{drawing}, \quad \text{drawing} = \text{real} \div k$$

$$A_{\text{real}} = k^2 A_{\text{drawing}}, \quad V_{\text{real}} = k^3 V_{\text{model}}$$

$$\tfrac{1}{4}'' = 1'\text{-}0'' \;\Rightarrow\; k = 48$$

From history. Jefferson's early drawings for Monticello and many of his other architectural drawings survive at the Massachusetts Historical Society. When he designed the University of Virginia, he sent drawings of each pavilion to the builders with instructions that the dimensions be followed exactly, and he checked the work himself from his house on the hill.

Example 1. On a plan drawn at ¼″ = 1′-0″, a wall measures 3⅝ in. How long is the real wall?

Show solution

1. Each ¼ in stands for 1 ft, so each inch stands for 4 ft.
2. Real length: $3.625 \text{ in} \times 4 \text{ ft/in} = 14.5 \text{ ft}$.
3. Write it as feet and inches: $14.5 \text{ ft} = 14'\text{-}6''$.

Answer: 14′-6″

Example 2. A church hall is 18.6 m long. How long will it be on a 1:100 plan, and on a 1:50 plan?

Show solution

1. 1:100: $18.6 \text{ m} \div 100 = 0.186 \text{ m} = 186 \text{ mm}$.
2. 1:50: $18.6 \text{ m} \div 50 = 0.372 \text{ m} = 372 \text{ mm}$.
3. The 1:50 drawing is twice as long, so it shows more detail but needs a bigger sheet.

Answer: 186 mm at 1:100; 372 mm at 1:50.

Example 3. A room is drawn 2½ in × 3¼ in at ⅛″ = 1′-0″. Find its real dimensions and floor area, and the ratio of real area to drawing area.

Show solution

1. At ⅛″ = 1′-0″, each inch stands for 8 ft, so $k = 96$.
2. Real size: $2.5 \times 8 = 20$ ft and $3.25 \times 8 = 26$ ft.
3. Area: $20 \times 26 = 520 \text{ ft}^2$.
4. Area ratio: $k^2 = 96^2 = 9216$. Check: drawing area is $2.5 \times 3.25 = 8.125 \text{ in}^2$, and $8.125 \times 9216 = 74{,}880 \text{ in}^2 = 520 \text{ ft}^2$ (because $1 \text{ ft}^2 = 144 \text{ in}^2$).

Answer: 20 ft × 26 ft, 520 ft²; area ratio 9,216.

PRACTICE

1. Easy What is the scale factor of ½″ = 1′-0″? Write it as a ratio.

   Show answer

   $12 \div 0.5 = 24$, so 1:24.

2. Easy On a ¼″ = 1′-0″ plan, a door is drawn ¾ in wide. How wide is the real door?

   Show answer

   $0.75 \times 4 = 3$ ft, a 3′-0″ door.

3. Easy A 4.2 m wall is drawn at 1:50. How long is it on paper?

   Show answer

   $4200 \text{ mm} \div 50 = 84 \text{ mm}$.

4. Medium On a ⅛″ = 1′-0″ plan, a hall measures 5⅜ in. How long is it?

   Show answer

   $5.375 \times 8 = 43$ ft.

5. Medium A site plan at 1″ = 20′ shows a driveway 3.6 in long. How long is it in feet and in meters?

   Show answer

   $3.6 \times 20 = 72 \text{ ft} \approx 21.9 \text{ m}$.

6. Medium How long will a 26′-9″ wall be on a ¼″ = 1′-0″ drawing?

   Show answer

   $26.75 \text{ ft} \div 4 = 6.6875 \text{ in} = 6\frac{11}{16} \text{ in}$.

7. Medium A rectangular room is 6.0 cm × 9.0 cm on a 1:100 plan. Find its area in square meters.

   Show answer

   $6.0 \text{ m} \times 9.0 \text{ m} = 54 \text{ m}^2$.

8. Hard A model of a house is built at 1:48. The model's walls weigh 0.40 kg. If the real walls were made of the same material, about how much would they weigh?

   Show answer

   Weight scales as k^3: $0.40 \times 48^3 = 0.40 \times 110{,}592 \approx 44{,}000$ kg.

9. Hard A plan was photocopied at 85% and is now read with an architect's scale set to ¼″ = 1′-0″. A wall that is really 20 ft long will read as how long? What should the drawing have had to prevent this error?

Show answer

The drawn 5 in becomes $5 \times 0.85 = 4.25$ in, which reads as 17 ft. A bar scale (which shrinks with the copy) and written dimensions prevent the error.

10. Hard Conceptual. Why might a 1:50 paper model of a bridge hold far more (in proportion) than the real bridge would?

Show answer

Model answer: weight grows with the cube of size but strength of members grows roughly with the square (their cross-sectional area), so a small model is relatively much stronger compared with its own weight; scale models of structures can mislead unless the loads are scaled carefully.

11. Challenge Jefferson drew on graph paper with 10 squares to the inch, letting each square equal 2 ft. What is his scale as a ratio, and what would it be on an architect's scale? Does it match any standard architect's scale?

Show answer

1 in = 20 ft, so $k = 240$ and the ratio is 1:240. That is 0.05 in per ft, or about $\frac{1}{20}$″ = 1′-0″; it is not one of the standard architect's scales (it matches an engineer's 1″ = 20′ scale).

COMMON MISTAKES

- Reading the architect's scale from the wrong end. Each edge has two scales that run in opposite directions; the inch subdivisions are at the end of the one you are using.
- Forgetting to square the scale factor for areas (and cube it for volumes).
- Measuring a drawing when a written dimension is given. The dimension governs.

> **Make it a game · Scale Snap Quiz.** Rapid-fire scale drill. Your parent calls a real dimension, like a 12 ft wall, and you give its length at ¼″ = 1′-0″ and at 1:50 within ten seconds. Then answer: if the scale factor doubles, what happens to area? Ten correct in a row earns a personal-record mark. Record the streak and try to beat it.

2.2 Floor Plans

OBJECTIVES

- Read and draw the conventions of a floor plan: the cut plane, walls, doors, windows, stairs, symbols, labels and dimension strings.
- Compute room and building areas, including compound shapes and net-to-gross ratios.
- Check a plan for practical needs: door clearances, hall widths and circulation.

> **Warm-up.** Without looking, sketch the floor plan of your home's main floor from memory in two minutes. Then walk through and mark everything you got wrong. What did you forget most often: doors, closets, or the thickness of walls?

WATCH

How to Read House Plans for Construction

Essential Craftsman · 16:06

https://www.youtube.com/watch?v=aOpKMyjLW5g

> **WHEN TO WATCH**
> After the core ideas. Make a list of every symbol and abbreviation he explains, with a small sketch of each.

Note: A builder with decades of experience explains a real set of house plans. There is a short channel promotion near the start.

KEY TERMS

floor plan

A horizontal section through a building, cut about 4 ft (1.2 m) above the floor and viewed from above.

cut plane

The imaginary horizontal slice of a plan. Walls cut by it are drawn with heavy lines (or filled, called poché); things below it are drawn lighter.

dimension string

A line of chained dimensions along a wall, with an overall dimension outside it; the parts must add to the whole.

door swing

A quarter-circle arc drawn on the plan showing which way a door opens and the space it needs.

circulation

The paths people use to move through a building: halls, stairs, doorways and the clear space within rooms.

net and gross area

Gross area is measured to the outside of exterior walls; net (usable) area is the floor inside rooms, without walls, shafts and circulation.

CORE IDEAS

The story. The oldest surviving architectural plan in Europe is a drawing on five pieces of parchment sewn together, about 112 × 77 cm, kept in the abbey library of St Gall in Switzerland. It was drawn in red ink between about 819 and 826 and sent to Gozbert, abbot of St Gall, probably from the nearby monastery of Reichenau. It shows an entire ideal monastery: a great church, a cloister, dormitory, refectory and kitchen, a school, guest houses, an infirmary, workshops, barns, a mill, a medicinal herb garden and a cemetery, about forty buildings in all, with more than three hundred Latin labels telling what each room was for. The monastery was never built exactly as drawn, but the plan shows how carefully the monks thought about the life that would happen inside.

How they figured it out. The St Gall plan is already a true plan: walls drawn as lines seen from above, with doors, benches, beds and tables placed in each room. Scholars still debate whether it was drawn to one consistent scale. Over the following centuries architects settled on the rules used today. A plan is a horizontal **section** cut about 4 ft above the floor and viewed from above. Whatever the cut passes through (walls, columns, window sills) is drawn with a heavy line; whatever lies below (the floor pattern, steps, counters) is drawn lighter; things overhead (beams, upper cabinets) are dashed. Doors are drawn open with the arc of their swing, so you can see the space they need.

The physics. Plans are geometry. A room's area is found by breaking it into rectangles and triangles. Dimension strings follow a strict rule: the chain of small dimensions along a wall must add up exactly to the overall dimension, and a good

drafter always checks the sum. Architects also track **gross area** (to the outside face of the exterior walls) and **net area** (usable room area). The ratio of net to gross is the building's efficiency; for houses it is often about 80–85%, because walls, halls and stairs take the rest.

The engineering. A good plan works for the people who use it. Halls in a house must be at least 3 ft (0.9 m) wide under the model residential code; a standard door is 2′-8″ to 3′-0″ wide; a wheelchair needs a clear circle about 5 ft (1.5 m) across to turn around. The plan must show how people move from the entrance to every room without passing awkwardly through others, where the light comes from, and which spaces should be near each other: kitchen beside dining, quiet rooms away from noisy ones. On construction drawings the plan also carries the information builders need: wall types, room names and numbers, door and window marks keyed to schedules, and references to sections and details on other sheets.

Try it. In the build you will draw a measured plan of a floor of your home and test its circulation with furniture cut to scale.

PICTURE STUDY

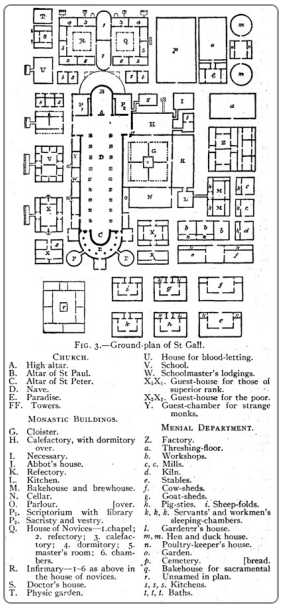


FIG. 3.—Ground-plan of St Gall.

CHURCH.

A. High altar.
B. Altar of St Paul.
C. Altar of St Peter.
D. Nave.
E. Paradise.
FF. Towers.

MONASTIC BUILDINGS.

G. Cloister.
H. Calefactory, with dormitory over.
I. Necessary.
J. Abbot's house.
K. Refectory.
L. Kitchen.
M. Bakehouse and brewhouse.
N. Cellar.
O. Parlour.
P_1. Scriptorium with library [over.
P_2. Sacristy and vestry.
Q. House of Novices—1. chapel; 2. refectory; 3. calefactory; 4. dormitory; 5. master's room; 6. chambers.
R. Infirmary—1–6 as above in the house of novices.
S. Doctor's house.
T. Physic garden.
U. House for blood-letting.
V. School.
W. Schoolmaster's lodgings.
X_1X_1. Guest-house for those of superior rank.
X_2X_2. Guest-house for the poor.
Y. Guest-chamber for strange monks.

MENIAL DEPARTMENT.

Z. Factory.
a. Threshing-floor.
b. Workshops.
c, c. Mills.
d. Kiln.
e. Stables.
f. Cow-sheds.
g. Goat-sheds.
h. Pig-sties. *i*. Sheep-folds.
k, k, k. Servants' and workmen's sleeping-chambers.
l. Gardener's house.
m, m. Hen and duck house.
n. Poultry-keeper's house.
o. Garden.
p. Cemetery. [bread.
q. Bakehouse for sacramental
r. Unnamed in plan.
s, s, s. Kitchens.
t, t, t. Baths.

The Plan of St Gall (redrawn, with key), Redrawing of the ninth-century plan from an early twentieth-century reference work, original about 819–826; redrawn about 1900 · Line engraving · Original in the Abbey Library of St Gall, Switzerland; redrawing via Wikimedia Commons

Image: Wikimedia Commons, public domain

The original plan, drawn in red ink on parchment about 1,200 years ago, is too faded to read in a small picture, so here is a clear redrawing with a key. It shows how monks arranged a whole monastery for a life of prayer, study, work and hospitality.

Look closely

- Find the church (D is the nave). Find its altars and the rounded ends.
- Find the cloister (G). Which buildings open onto it? Use the key.
- Find the infirmary, the school and the guest houses. Where are they placed relative to the church?
- Find the workshops, mills, barns and animal sheds. Why are they at the edges?

Talk about it

1. Which rooms did the planner place close together, and why?
2. How is this drawing like a modern floor plan, and how is it different?
3. If you had to build from this plan, what information would you still need?

BUILD

A Measured Plan and a Circulation Test

You will draw a measured floor plan of one floor of your home (or church hall) at ¼″ = 1′-0″, compute its areas, and test how well it works by placing scaled furniture and tracing the paths people walk.

You will need

- Tape measure, architect's scale, ¼-inch graph paper or drafting paper, pencils
- Card stock and scissors for furniture cutouts
- Colored pencils
- A partner to hold the tape

Steps

1. Sketch the floor freehand, then measure every room: wall lengths, openings, wall thicknesses (measure at a doorway), and overall outside dimensions.
2. Draw the plan at ¼″ = 1′-0″ with heavy cut walls, door swings, windows, stairs (with an arrow and UP or DN), and lighter lines for counters and fixtures.
3. Add dimension strings on two sides. Check that each string adds to its overall dimension.
4. Label every room and compute its net area. Compute the gross area and the efficiency.
5. Cut out the main furniture at the same scale (a bed, a sofa, a table with chairs) and place it on the plan.
6. **Circulation test.** With colored pencils, trace the path from the front door to the kitchen, from each bedroom to a bathroom, and from the kitchen to the dining table. Measure each path's length using the scale.
7. Check clearances: is every hall at least 3 ft wide? Is there at least 3 ft beside the bed and around the table? Mark every place that fails.
8. **Improve.** Propose one change to the plan (moving a door, swapping two rooms, removing a wall) that shortens the most-used path or fixes a

clearance. Draw it and measure the new path.

Tips

- Measure wall thickness at a door opening, where you can see the whole wall.
- Draw the walls first and add doors and windows by measuring from the corners.
- Use a template or a compass for the door swings so they are true quarter circles.

Check your work

- ☐ My plan uses correct line weights and conventions and is dimensioned.
- ☐ I computed room areas, gross area and efficiency correctly.
- ☐ My circulation test identifies at least one problem and proposes a measured improvement.

ENGINEER'S NOTEBOOK

1. A legend of plan symbols you learned: walls, doors (swing, sliding, pocket), windows, stairs, fixtures, column, north arrow, section cut mark.
2. A table of rooms with dimensions and net area, the gross area and the efficiency.
3. Before-and-after sketches of your proposed improvement, with the path lengths.

KEY FORMULAS

$$A_{\text{rectangle}} = \ell w, \quad A_{\text{triangle}} = \tfrac{1}{2}bh$$

$$\text{efficiency} = \frac{\text{net area}}{\text{gross area}} \times 100\%$$

$$\sum(\text{partial dimensions}) = \text{overall dimension}$$

From history. In 1979 Walter Horn and Ernest Born of the University of California published a three-volume study of the St Gall plan, measuring every line, and they argued it was drawn to a consistent scale. Others disagree. In the 2010s a group in Germany began building a version of the monastery from the plan using only medieval tools and methods, a project called Campus Galli.

Example 1. An L-shaped living room is 22 ft along its long wall and 18 ft along the short end. A 9 ft × 7 ft rectangle is missing from one corner (the L's notch). Find its area.

Show solution

1. Area of the full rectangle: $22 \times 18 = 396$ ft^2.
2. Area of the notch: $9 \times 7 = 63$ ft^2.
3. L-shaped area: $396 - 63 = 333$ ft^2.
4. Check by adding two rectangles instead: $22 \times 11 = 242$ plus $13 \times 7 = 91$ gives 333 ft^2.

Answer: 333 ft^2

Example 2. A wall's dimension string reads 3′-4″, 2′-8″, 6′-0″, 3′-0″ and 4′-10″, and the overall dimension reads 20′-0″. Is the string correct?

Show solution

1. Add the feet: $3 + 2 + 6 + 3 + 4 = 18$ ft.
2. Add the inches: $4 + 8 + 0 + 0 + 10 = 22$ in $= 1$′-10″.
3. Total: 19′-10″, which does not match the 20′-0″ overall. One of the partial dimensions is 2 in short, and the drafter must find which before the drawing is issued.

Answer: No: the parts add to 19′-10″, 2 in short of the overall 20′-0″.

Example 3. A small house measures 40 ft × 30 ft to the outside of its walls. Its rooms add up to 1,000 ft^2 of usable floor. Find the gross area and the efficiency.

Show solution

1. Gross area: $40 \times 30 = 1200$ ft^2.
2. Efficiency: $1000 \div 1200 \times 100\% \approx 83\%$.
3. The other 200 ft^2 is walls, the hall, the stair and closets that were not counted as rooms.

Answer: 1,200 ft^2 gross; about 83% efficient.

PRACTICE

1. **Easy** At about what height is a floor plan cut?

   Show answer

   About 4 ft (1.2 m) above the floor.

2. **Easy** A bedroom is 12′-0″ × 13′-6″. Find its area.

   Show answer

   $12 \times 13.5 = 162\ \text{ft}^2$.

3. **Easy** On a plan, what does a dashed line usually mean?

   Show answer

   Something above the cut plane (a beam, an upper cabinet, a roof overhang) or hidden from view.

4. **Medium** A kitchen is 14 ft × 12 ft with a 4 ft × 3 ft pantry closet built into one corner. What is the kitchen's net floor area?

   Show answer

   $168 - 12 = 156\ \text{ft}^2$.

5. **Medium** Check this dimension string: 4′-6″, 3′-0″, 7′-9″, 2′-8″, overall 18′-0″.

   Show answer

   $16' + 23'' = 17'$-$11''$, 1 in short; the string is wrong.

6. **Medium** A room has a trapezoid shape on plan with parallel walls of 10 m and 14 m, 6 m apart. Find its area.

   Show answer

   $\frac{1}{2}(10 + 14)(6) = 72\ \text{m}^2$.

7. **Medium** A church hall is 24 m × 15 m gross. Its usable rooms total 290 m². Find the efficiency.

   Show answer

   $290 \div 360 \approx 0.806$, about 81%.

8. Hard A 3′-0″ door swings into a hall 3′-6″ wide. When it is open against the hall's far wall, how much clear width is left for a person walking past? What would you change?

Show answer

A 3 ft door opened across a 3′-6″ hall leaves only about 6 in, so it blocks the hall. Swing the door into the room instead, or use a pocket door.

9. Hard A designer wants a house with 1,500 ft^2 of usable rooms and expects 82% efficiency. What gross area should be planned? If the house is to be 30 ft deep, how long must it be?

Show answer

$1500 \div 0.82 \approx 1829$ ft^2; length $\approx 1829 \div 30 \approx 61$ ft.

10. Hard Conceptual. Why does a plan show door swings and not just openings?

Show answer

Model answer: the swing shows the space the door needs and which way it opens, so the designer can check that it does not hit furniture, other doors or block a hall, and so the builder hangs it correctly; exit doors must swing the right way for safety.

11. Challenge Suppose the St Gall plan's church is drawn 30 cm long and was meant to be about 200 feet of 0.296 m, what scale would the drawing be? Is that close to a scale an architect today might use?

Show answer

Real length ≈ 59.2 m; scale $= 59.2 \div 0.30 \approx 197$, about 1:200, close to modern 1:200 (or 1⁄16″ = 1′-0″, which is 1:192).

COMMON MISTAKES

- Drawing walls as single lines. Real walls have thickness, and rooms lose area to them.
- Letting a dimension string fail to add up to the overall dimension.
- Forgetting circulation space. A room can hold its furniture and still be unusable if there is no clear path through it.

Make it a game · Plan From Memory. Beat the clock. Sketch your home's floor plan from memory in three minutes: walls, doors with swings, windows, stairs and labels. Then walk through and check it. Score one point per correct item and subtract one per miss. Compute the net area of one room, then explain the difference from gross area.

Scripture. “shew them the form of the house, and the fashion thereof, and the goings out thereof, and the comings in thereof” (Ezekiel 43:11, KJV)

Ezekiel is told to write out the plan of the temple, its form and its ways in and out, so that the people can see it. A floor plan does the same.

2.3 Elevations and Sections

OBJECTIVES

- Read and draw exterior elevations and building sections, and explain how they relate to the plan through orthographic projection.
- Calculate roof pitch, heights and stair geometry from drawings and building codes.
- Measure the height of a real building with a clinometer and trigonometry.

> **Warm-up.** Stand facing the front of your house or church. Draw only what you see straight on, with no perspective: as if you were infinitely far away and the building were flat. Which features were hard to draw without them getting smaller toward the edges?

WATCH

basic architectural drawings 1: plan, elevation, section

BAU Studio 1 · 11:21

https://www.youtube.com/watch?v=pzKHy5RFBao

> **WHEN TO WATCH**
> Before the build. Draw along with him on paper: plan, two elevations and a section of the same little house.

Note: A lecturer at a Dutch technical university draws a small house on a dark board. Simple and clear.

KEY TERMS

orthographic projection

Drawing an object by projecting it straight onto flat planes at right angles, so true lengths are shown without perspective. Plans, elevations and sections are orthographic.

elevation

An orthographic view of one face of a building, usually named for the direction it faces: north, south, east, west.

section

A vertical slice through a building showing what you would see if it were cut and the near part removed: floors, walls, roof, stairs, foundations and ceiling heights.

section mark

A symbol on the plan showing where a section is cut and which way you are looking, with a reference to the sheet where it is drawn.

roof pitch

The slope of a roof, written as rise in inches per 12 in of run, such as 6:12.

CORE IDEAS

The story. In the winter of 1933, during the Great Depression, many American architects had no work. Charles E. Peterson, a young architect with the National Park Service, proposed putting them to work measuring and drawing the nation's historic buildings before they were lost. Within weeks about a thousand architects and draftsmen were at work. The program became the Historic American Buildings Survey (HABS), and later the Historic American Engineering Record (HAER) for bridges, mills and machines. Its measured drawings of more than 40,000 sites are kept in the Library of Congress, free for anyone to use. Every HABS drawing set includes plans, elevations and sections, drawn to the same standards architects use for new buildings.

How they figured it out. The idea of showing an object in several flat views at right angles is old. Renaissance architects drew plans, elevations and sections, and the French mathematician Gaspard Monge made it a science called **descriptive geometry** in the 1790s, teaching it to engineers so they could design fortifications, bridges and machines exactly. In **orthographic projection**, imagine the building inside a glass box and project every point straight onto each side of the box. The top of the box shows the plan, the sides show the elevations, and a slice through the middle shows a section. Because the views are projected, points line up between them: a window shown in the plan sits directly above or beside the same window in the elevation, and the drafter carries lines from one view to the next.

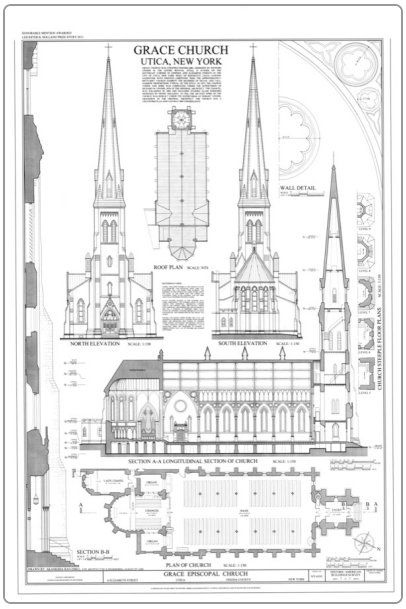
The physics. Elevations and sections show vertical dimensions: floor-to-floor heights, ceiling heights, sill and head heights of windows, roof slopes and the depth of foundations. Roof slope is given as **pitch**, rise per 12 in of run: a 6:12 roof rises 6 in

for every 12 in, an angle of $\tan^{-1}(6/12) \approx 26.6^\circ$. A rafter's length follows from the Pythagorean theorem. Stairs are governed by code: in houses a riser may be at most 7¾ in and a tread at least 10 in deep, so a stair rising 9 ft needs $108 \div 7.75 = 13.9$, or 14 risers, and 13 treads. The section is where an architect proves that the stair fits, the headroom works (at least 6 ft 8 in), and the beams and ducts fit between floors.

The engineering. On a set of construction drawings, the plan carries **section marks** showing where each section is cut, and the elevations and sections carry level marks (finished floor, top of plate, ridge) that the builder uses to set heights. The structural engineer uses sections to show how the roof is framed and how loads come down to the foundation; you will draw load paths on sections in Unit 4. Surveyors and historians use the same drawings in reverse: they measure an existing building and draw its elevations and sections, as HABS still does today.

Try it. In the build you will measure a real building's height with a homemade clinometer, then draw its elevation and a section to scale.

PICTURE STUDY



Elevations, Plan and Section of Grace Church, Utica, New York, Historic American Buildings Survey, 2010 · Measured drawing · Library of Congress, Prints and Photographs Division

Image: Wikimedia Commons, public domain

This single sheet from the Historic American Buildings Survey shows a nineteenth-century Gothic Revival church in several orthographic views at once. Look for how the views line up with each other.

Look closely

- Find the two elevations. Which is north and which is south? How do they differ?
- Find the longitudinal section. What can you see in the section that the elevations hide (the roof trusses, the floor, the inside of the tower)?
- Find the floor plan. Trace a line from a buttress in the plan up to the same buttress in an elevation.
- Find the scale and the title block. What information does the title block give?

Talk about it

1. Why would a surveyor draw all of these views on one sheet?
2. Using the scale, estimate the height of the spire. How could you check it at the church itself?
3. What does the section tell a structural engineer that the plan does not?

BUILD

Measuring Up: Elevation and Section of a Real Building

You will measure the height of a real building with a homemade clinometer, check it a second way, and then draw an elevation and a section of the building to scale.

You will need

- A protractor, a drinking straw, thread and a washer or nut (to make a clinometer)
- A tape measure (a 100 ft or 30 m tape is best), a calculator
- Graph paper or drafting paper, an architect's or metric scale, pencils
- A partner

Steps

1. Make a clinometer: tape the straw along the protractor's straight edge, and hang the thread with the washer from the protractor's center hole. When you sight through the straw, the angle of elevation is 90° minus the angle the thread marks (or read it directly if your protractor is labeled from 0 at the center).
2. Practice on something you can measure directly, such as a doorway or a wall of known height. Record the error.
3. Choose a building with a gable or tower. Measure a distance of 15–30 m (50–100 ft) from its face along level ground. Measure your eye height.
4. Sight the ridge or top three times and average the angles. Compute the height with $h = d \tan\theta + h_{\text{eye}}$.
5. **Check another way.** Count brick courses or siding boards and multiply by their spacing, or measure from a second distance. Compare the two results and compute the percent difference.
6. Measure the widths of the building, its doors and windows (pace or tape them), and estimate window heights by counting siding or brick courses.
7. Draw the front elevation at a suitable scale (⅛″ = 1′-0″ or 1:100). Add level marks for the floor, the eaves and the ridge.

8. Draw a section through the building as you imagine it inside, showing floors, ceilings, the roof slope and a stair if there is one. Check your stair against the code rule (7¾ in maximum riser, 10 in minimum tread).
9. **Improve.** Repeat the height measurement from a better position (farther away, or at the level of the base) and see whether the two methods now agree more closely.

Tips

- Measuring from farther away usually reduces the error, because a small angle mistake makes less difference.
- Stand on level ground with the building's base; if the ground slopes, measure the angle to the base too and add the two heights.
- Never stand in a street to take a measurement.

Check your work

- ☐ I can measure a building's height with a clinometer and check it a second way.
- ☐ My elevation and section are drawn to scale, aligned and labeled with levels.
- ☐ I can calculate roof pitch and stair geometry from a section.

ENGINEER'S NOTEBOOK

1. A labeled diagram of the glass-box idea: an object inside a box, with its plan, front, side and rear views unfolded.
2. Your clinometer data: three angles, the average, the distance, the height, the check method and the percent difference.
3. Your elevation and section of the building, aligned so that heights carry across from one to the other.

KEY FORMULAS

$$\text{pitch} = \frac{\text{rise}}{12}, \quad \theta = \tan^{-1}\left(\frac{\text{rise}}{\text{run}}\right)$$

$$L_{\text{rafter}} = \sqrt{\text{run}^2 + \text{rise}^2}$$

$$n_{\text{risers}} = \left\lceil \frac{\text{total rise}}{\text{max riser}} \right\rceil$$

$h = d \tan\theta + h_{\text{eye}}$ (height by clinometer)

From history. HABS was started as a temporary relief program for unemployed architects, but it was so useful that it never stopped. Students and professionals still produce HABS and HAER drawings each summer, and the Peterson Prize, named for Charles Peterson, is awarded each year for the best measured drawings by students.

WORKED EXAMPLES

Example 1. A house is 28 ft wide with a gable roof of 8:12 pitch. The ridge is centered. Find the roof angle, the rise from the top of the walls to the ridge, and the rafter length (not counting the overhang).

Show solution

1. Angle: $\theta = \tan^{-1}(8/12) \approx 33.7^\circ$.
2. Run of each rafter: half the width, 14 ft.
3. Rise: 14 ft $\times \frac{8}{12} \approx 9.33$ ft $= 9'$-4″.
4. Rafter: $L = \sqrt{14^2 + 9.33^2} = \sqrt{196 + 87.1} \approx 16.8$ ft.

Answer: About 33.7°, a rise of 9′-4″, and rafters about 16.8 ft long.

Example 2. A stair must rise 9′-1⅛″ (109⅛ in) from floor to floor. Using a maximum riser of 7¾ in and a tread of 10 in, find the number of risers, the actual riser height, the number of treads and the total run.

Show solution

1. Minimum risers: $109.125 \div 7.75 = 14.08$, so 15 risers (always round up).
2. Actual riser: $109.125 \div 15 = 7.275$ in, a little over 7¼ in.
3. Treads: one fewer than risers, $15 - 1 = 14$ treads.
4. Total run: 14×10 in $= 140$ in $= 11'$-8″.

Answer: 15 risers of about 7¼ in, 14 treads, total run 11′-8″.

Example 3. Standing 20.0 m from a church tower, you sight its top with a clinometer at 52°. Your eye is 1.5 m above the ground. How tall is the tower?

Show solution

1. Height above eye: $20.0 \text{ m} \times \tan 52^\circ = 20.0 \times 1.280 = 25.6 \text{ m}$.
2. Add eye height: $25.6 + 1.5 = 27.1 \text{ m}$.
3. On a 1:100 elevation the tower would be drawn 271 mm tall.

Answer: About 27 m tall.

PRACTICE

1. Easy What is the difference between an elevation and a section?

   Show answer

   An elevation shows an outside face of a building as seen straight on; a section shows a vertical slice through it, revealing floors, walls and roof inside.

2. Easy What angle is a 12:12 roof pitch?

   Show answer

   45°.

3. Easy A roof has a pitch of 4:12 and a run of 15 ft. What is its rise?

   Show answer

   $15 \times 4/12 = 5 \text{ ft}$.

4. Medium Find the angle of a 7:12 roof and the length of a rafter with a 12 ft run.

   Show answer

   $\theta = \tan^{-1}(7/12) \approx 30.3^\circ$; rise $= 7$ ft; $L = \sqrt{144 + 49} \approx 13.9 \text{ ft}$.

5. Medium A stair rises 8′-6″. Using a 7¾ in maximum riser, how many risers are needed, and what is the actual riser height?

   Show answer

   $102 \div 7.75 = 13.2$, so 14 risers of $102 \div 14 \approx 7.29$ in.

6. Medium From 25 m away, the top of a steeple is sighted at 48° with an eye height of 1.6 m. How tall is it?

Show answer

$$25 \tan 48^\circ + 1.6 \approx 27.8 + 1.6 = 29.4 \text{ m}.$$

7. Medium On a section at ¼″ = 1′-0″, the floor-to-ceiling height measures 2 in. What is the real ceiling height? Does it meet a 7 ft minimum?

Show answer

$2 \times 4 = 8$ ft, which meets the 7 ft minimum.

8. Hard A clinometer reading of 40° is off by 1° (it should be 41°). At a distance of 30 m, how much error in height does this cause?

Show answer

$$30(\tan 41^\circ - \tan 40^\circ) = 30(0.8693 - 0.8391) \approx 0.91 \text{ m}.$$

9. Hard A stair has 14 risers of 7.3 in and 13 treads of 10 in. Find its total rise, total run and slope angle. What is the steepest angle the code limits (7¾ in riser, 10 in tread) allow?

Show answer

Rise $14 \times 7.3 = 102.2$ in; run $13 \times 10 = 130$ in; angle $\tan^{-1}(7.3/10) \approx 36.1^\circ$. The code limit is $\tan^{-1}(7.75/10) \approx 37.8^\circ$.

10. Hard Conceptual. Why must elevations and sections line up with the plan when drawn on the same sheet?

Show answer

Model answer: because they are projections of the same building, every point appears in each view along shared lines; lining them up lets the drafter carry heights and positions from one view to another and lets the reader check that the views agree.

11. Challenge From point A you sight a tower's top at 35°. You walk 20 m closer and sight it at 50°. Ignoring eye height, find the tower's height without measuring the distance to it.

Show answer

Let x be the distance from the closer point. $h = x \tan 50^\circ = (x + 20) \tan 35^\circ$. So $1.1918x = 0.7002x + 14.00$, $x \approx 28.5$ m, and $h \approx 28.5 \times 1.1918 \approx 33.9$ m.

COMMON MISTAKES

- Drawing elevations in perspective. An elevation shows every window at its true size, no matter how far it is from the center.
- Rounding the number of stair risers down. Always round up, then recompute the actual riser height.
- Forgetting to add your eye height to a clinometer measurement.

> **Make it a game · Elevation Whodunit.** Your parent sketches a plain elevation of a house without telling you which one. Use a tape to measure the real house or a toy, and decide which elevation matches. Then calculate roof pitch from rise and run, and stair rise from floor height. Swap roles with your own sketch. Fewest guesses wins.

2.4 Perspective Drawing: Brunelleschi's Experiment

OBJECTIVES

- Explain Brunelleschi's perspective experiment and Alberti's method, and the ideas of the picture plane, horizon line and vanishing point.
- Use similar triangles to calculate how large an object appears on a picture plane.
- Construct one-point and two-point perspective drawings of a simple building.

> **Warm-up.** Look down a long hallway, a road or a row of fence posts. Hold a pencil at arm's length and line it up with the edges of the floor or road. Where do the lines seem to meet? Is that point above, below or at the level of your eyes?

WATCH

Linear Perspective: Brunelleschi's Experiment

Smarthistory · 4:16

https://www.youtube.com/watch?v=bkNMM8uiMww

> **WHEN TO WATCH**
> After the story. Watch how the mirror and the hole in the panel let a viewer compare the painting with the real building.

KEY TERMS

linear perspective

A method of drawing that projects a three-dimensional scene onto a flat surface as seen from one eye point, so that parallel lines meet at vanishing points.

picture plane

The imaginary transparent surface between the eye and the scene on which the picture is made.

horizon line

The line at the viewer's eye height where all horizontal vanishing points lie.

vanishing point

The point on the picture where a set of parallel lines in the scene appear to meet.

similar triangles

Triangles with equal angles and proportional sides; they give the size of an object's image: $y\prime = fY/Z$.

CORE IDEAS

The story. Sometime around 1415–1420, the Florentine goldsmith and architect Filippo Brunelleschi stood inside the doorway of the unfinished cathedral of Florence and painted, on a small square panel, the Baptistery of San Giovanni across the square. He drilled a small hole through the panel at the point directly opposite his eye. A viewer held the panel with its painted side facing away, looked through the hole from the back, and held up a mirror at arm's length. In the mirror he saw the painting. Lower the mirror, and he saw the real Baptistery from exactly the same spot. According to Brunelleschi's biographer Antonio Manetti, the painting and the building matched so well that viewers could hardly tell which they were seeing. The same Brunelleschi would soon build the great dome of the cathedral.

How they figured it out. Painters before Brunelleschi had made buildings look deep by experience, but their lines did not meet consistently. Brunelleschi probably used his skill as a surveyor and architect, working out his picture from a plan and an elevation, so that every point was where geometry put it. In 1435 the scholar and architect Leon Battista Alberti wrote the first book on the method, *On Painting*. He told painters to think of the picture as an open window through which the scene is seen, and he showed how to construct a floor of squares that shrinks correctly toward a single point.

The physics. Perspective is projection through a point, the eye. Imagine a straight line from each point of the scene to your eye; where that line passes through the picture plane, you draw the point. By **similar triangles**, an object of height Y at distance Z from the eye appears on a picture plane at distance f from the eye with height $y\prime = fY/Z$. Double the distance and the image is half as tall. Lines that are parallel in the scene meet at a **vanishing point**; for horizontal lines, it lies on the **horizon line** at the height of the eye. In one-point perspective you face one wall head-on and its receding lines meet at one point; in two-point perspective you look at a corner and each set of horizontal lines has its own vanishing point. A camera

and your eye work the same way: the lens is the eye point and the sensor or retina is the picture plane.

The engineering. Architects use perspective to show clients what a building will really look like, because most people cannot imagine a building from plans and elevations alone. Before computers, architects constructed perspectives by hand from the plan and elevation, exactly as Alberti taught; today computer models produce them, but the geometry inside the software is the same. Perspective also matters for design itself: the Greeks bowed the floor of the Parthenon upward slightly and leaned its columns inward, probably so that it would look straight and stable to a person standing below. An architect must design not only what a building is but how it will be seen.

Try it. In the build you will repeat Alberti's window: trace a real building onto a clear sheet from one fixed eye point and test the similar-triangles formula with real measurements.

PICTURE STUDY

The Ideal City, Unknown Italian painter (formerly attributed to Piero della Francesca), about 1480 · Painting on wood panel · Galleria Nazionale delle Marche, Urbino

Image: Wikimedia Commons, public domain

Two generations after Brunelleschi's experiment, a painter at the court of Urbino painted this calm, empty square as a showpiece of perfect perspective and classical architecture. No one knows for certain who painted it or what it was for.

Look closely

- Find the lines of the pavement and the cornices of the buildings on each side. Follow them with a ruler. Where do they meet?
- Find the horizon line. How high is it compared with the doors of the buildings? How tall is the viewer?
- Compare the size of the nearest windows with the farthest. How much smaller do they become?
- Look at the round temple in the center. Is it drawn in one-point perspective too?

Talk about it

1. Why do you think the painter left the square empty of people?
2. If the nearest palace door and a farther door are the same real size, how could you use the picture to estimate how far apart they are?
3. How is this painting like an architect's presentation drawing of a planned town?

BUILD

Alberti's Window: Tracing and Testing Perspective

You will trace a real building through a clear sheet from a fixed eye point, locate the horizon and vanishing points, and test the formula $y\prime = fY/Z$ with real measurements. Then you will construct a two-point perspective of a simple building by hand.

You will need

- A clear plastic sheet (a sheet protector or transparency) taped in a window or a picture frame with the glass
- A dry-erase or fine permanent marker, a ruler and tape measure
- A piece of card with a small hole as a fixed eye point, held in place on a stand or a stack of books
- Drawing paper, a long ruler and pencils

Steps

1. Fix the eye-hole card so that the hole is at a measured distance f from the clear sheet (about 40–50 cm). Measure f carefully.
2. Looking through the hole with one eye, trace the outline of a building, a fence or a row of windows seen through the sheet. Include the ground line and the roof line.
3. Extend the traced horizontal edges with a ruler until they meet. Mark the vanishing point(s) and draw the horizon line through them. Measure the height of your eye hole above the floor and compare with the horizon height on the sheet.
4. Choose an object you can measure: a door or a window of known height Y. Measure its distance Z from the eye. **Predict** its traced height with $y\prime = fY/Z$.
5. Measure the traced height and compare with your prediction. Compute the percent difference.
6. Repeat with a second object at a different distance and check that $y\prime Z$ is about the same for objects of the same height.

7. Now construct a two-point perspective of a small gabled building on paper: draw the horizon, two vanishing points far apart, the near vertical corner, and project the walls and roof.
8. **Improve.** Move the vanishing points closer together and redraw. What happens to the building's look? Record which looks most natural and why.

Tips

- Keep your head still; the method works only from one fixed eye point.
- Use one eye; two eyes see from two points.
- Put the vanishing points for two-point perspective far apart (off the page if needed) to avoid a distorted, fish-eye look.

Check your work

☐ I can explain the picture plane, horizon line and vanishing point.

☐ My test compares predicted and measured image heights.

☐ I can construct one- and two-point perspectives of a simple building.

ENGINEER'S NOTEBOOK

1. A diagram in plan view of Brunelleschi's experiment: the panel, the hole, the mirror, the viewer and the Baptistery, with sight lines.
2. A similar-triangles diagram showing Y, Z, f and $y\prime$, and your measured data table.
3. Your two-point perspective of a small building, with construction lines left visible.

KEY FORMULAS

$y\prime = \dfrac{fY}{Z}$ (image height on the picture plane)

$\dfrac{y\prime_1}{y\prime_2} = \dfrac{Z_2}{Z_1}$ (same object at two distances)

$\text{horizon height} = \text{eye height}$

From history. Brunelleschi's panel is lost, but painters soon followed his method. Within a few years Masaccio painted a fresco in Santa Maria Novella in Florence with a barrel-vaulted chapel drawn in exact perspective, and Alberti's book spread the method across Europe.

Example 1. A bell tower 30 m tall stands 120 m away. You hold a sheet of glass 0.50 m from your eye. How tall will the tower's image be on the glass (ignoring your eye height)?

Show solution

1. Use similar triangles: $y\prime = fY/Z$.
2. $y\prime = (0.50\ \text{m})(30\ \text{m}) \div 120\ \text{m} = 0.125\ \text{m}$.
3. The tower will appear 12.5 cm tall on the glass.

Answer: 12.5 cm

Example 2. A row of identical columns, each 6.0 m tall, runs straight away from you. The first is 10 m away and appears 30 cm tall on your picture plane. How tall does the column 40 m away appear? Where is the eye if the picture plane is the same?

Show solution

1. Image height is inversely proportional to distance: $y\prime_2 = y\prime_1(Z_1/Z_2)$.
2. $y\prime_2 = 30\ \text{cm} \times (10/40) = 7.5\ \text{cm}$.
3. From the first column: $f = y\prime Z/Y = 0.30 \times 10 \div 6.0 = 0.50\ \text{m}$; the picture plane is half a meter from the eye.

Answer: 7.5 cm; the picture plane is 0.50 m from the eye.

Example 3. In a one-point perspective drawing, the horizon line is drawn 5.5 ft above the ground line at the scale of the front wall. A doorway on the front wall is 7 ft tall. Where does the horizon cross the doorway, and what does that tell you about the viewer?

Show solution

1. The horizon line is always at the viewer's eye height.
2. On the front wall (drawn at true scale), the horizon crosses the doorway 5.5 ft above the floor, $5.5/7 \approx 79\%$ of its height.
3. The viewer is a standing adult whose eyes are about 5½ ft above the ground. A horizon drawn near the ground would show a child's or a bird's-eye-of-the-ground view; one drawn high above the roof would be an aerial view.

Answer: At 5.5 ft, about four-fifths of the way up the door; the viewer is a standing adult.

PRACTICE

1. Easy What is a vanishing point?

   Show answer

   The point in a perspective picture where lines that are parallel in the scene appear to meet.

2. Easy Where is the horizon line drawn in a perspective?

   Show answer

   At the viewer's eye height.

3. Easy A tree 8 m tall is 40 m away; the picture plane is 0.5 m from the eye. How tall is its image?

   Show answer

   $0.5 \times 8 \div 40 = 0.10 \text{ m} = 10 \text{ cm}.$

4. Medium The same tree is 100 m away. How tall is its image now?

   Show answer

   $0.5 \times 8 \div 100 = 0.04 \text{ m} = 4 \text{ cm}.$

5. Medium A doorway 2.1 m tall appears 7.0 cm tall on a picture plane 0.40 m from the eye. How far away is it?

Show answer

$Z = fY/y\prime = 0.40 \times 2.1 \div 0.070 = 12$ m.

6. Medium In two-point perspective, which lines stay vertical and which converge?

Show answer

Vertical edges stay vertical; the two sets of horizontal edges converge to the two vanishing points on the horizon.

7. Medium A camera lens acts as the eye and its sensor is 50 mm behind it. A cathedral front 45 m tall fills 30 mm of the sensor. How far away is the camera?

Show answer

$Z = 50 \text{ mm} \times 45 \text{ m} \div 30 \text{ mm} = 75$ m.

8. Hard Two identical columns appear 12 cm and 4 cm tall in a photograph. The nearer one is 15 m away. How far apart are the columns along the line of sight?

Show answer

Image height ∝ 1/Z: $Z_2 = 15 \times 12/4 = 45$ m; they are 30 m apart.

9. Hard Why does a tall building photographed from the street seem to lean backward, and how do architectural photographers correct it?

Show answer

Tilting the camera up makes the vertical edges converge toward a third vanishing point in the sky. Photographers keep the camera level and shift the lens (a shift or tilt-shift lens), or correct the picture afterward, so verticals stay vertical.

10. Hard Conceptual. Why did Brunelleschi's experiment need a peephole at a fixed point?

Show answer

Model answer: a perspective is correct only from the single eye point used to construct it; the peephole put the viewer's eye exactly where Brunelleschi's had been, so the painting and the real building could be compared fairly.

11. **Challenge** In a one-point perspective, floor tiles 0.5 m deep run away from a viewer whose eye is 1.5 m above the floor and 0.5 m from the picture plane. The near edge of the first tile is 1.0 m from the eye. How far below the horizon line, on the picture plane, are the edges at 1.0 m, 1.5 m and 2.0 m? Do the gaps between them shrink evenly?

    Show answer

    Below-horizon distance $= f \times 1.5/Z$: at 1.0 m, 0.75 m; at 1.5 m, 0.50 m; at 2.0 m, 0.375 m. The gaps are 0.25 m then 0.125 m, so they shrink faster than evenly (as $1/Z$).

COMMON MISTAKES

- Putting the horizon line at the top of the building. It belongs at eye height.
- Making the image shrink in proportion to distance subtracted, not divided. Size goes as $1/Z$: twice as far, half as big.
- Placing two-point vanishing points too close together, which makes buildings look stretched and distorted.

> **Make it a game · Vanishing Point Duel.** Draw a horizon line and one vanishing point on paper. Each of you has two minutes to sketch a box receding into the distance, then compare. Next, use similar triangles to calculate how tall a six-foot figure appears on a picture plane one foot away from your eye. Closest answer wins the point.

2.5 Modeling in 3D: From Froebel's Blocks to SketchUp

OBJECTIVES

- Tell how Frank Lloyd Wright's childhood blocks and the history of descriptive geometry and CAD shaped the way architects think in three dimensions.
- Build a simple building model in SketchUp Free or Tinkercad from measured dimensions, and generate plans, elevations and views from it.
- Compute volumes, surface areas and model sizes, and use isometric projection.

> **Warm-up.** Take a handful of identical blocks (LEGO, sugar cubes or wooden cubes). In three minutes, build the smallest building you can that has a door, a window and a roof. Sketch it from the front, the side and the top. Which view was hardest to draw?

WATCH

SketchUp Tutorial for Beginners - Learn SketchUp in 10 MINUTES | (SketchUp Free 2022)

MasterSketchUp · 10:16

https://www.youtube.com/watch?v=zpdMZK6LlDE

> **WHEN TO WATCH**
> At the start of the build session, with SketchUp Free open in another window. Pause and copy each tool as it is shown.

BACKUP VIDEO

Can You Learn the Design DNA of Frank Lloyd Wright with Froebel Blocks?

Froebel USA · 7:14

https://www.youtube.com/watch?v=T4bUNv8DJ34

With the story. An architecture professor teaches design with Froebel blocks, as Wright learned as a boy.

KEY TERMS

Froebel gifts

Sets of wooden blocks, balls and tablets designed by the German educator Friedrich Froebel for the first kindergartens (1830s–1840s), used to teach form, proportion and composition.

descriptive geometry

Gaspard Monge's method (1790s) of representing three-dimensional objects exactly with two or more orthographic views.

CAD

Computer-aided design: software for drawing and modeling objects precisely, beginning with Ivan Sutherland's Sketchpad in 1963.

isometric drawing

A pictorial drawing in which the three axes are drawn 120° apart and lengths along them are drawn at the same scale.

component (or group)

In a 3D model, a set of geometry treated as one object that can be copied; editing one copy of a component changes them all.

CORE IDEAS

The story. In 1876, at the Centennial Exposition in Philadelphia, a schoolteacher named Anna Lloyd Wright saw a display of the Froebel kindergarten gifts and bought a set for her nine-year-old son, Frank. The gifts were simple: smooth maple cubes, cylinders and spheres, blocks that divided a cube into smaller cubes and prisms, and colored paper squares on a grid. Frank Lloyd Wright later wrote that the feel of those maple blocks had stayed in his fingers all his life. His houses of the early 1900s, such as the Robie House in Chicago (1909–1910), are compositions of long horizontal planes and blocks stacked and slid past each other. Its roof cantilevers far beyond its supports, carried by steel beams hidden inside.

How they figured it out. Architects have always needed ways to think in three dimensions. Masons built wood or plaster models; Wren built the Great Model of St Paul's. In the 1790s Gaspard Monge showed how two orthographic views define any shape exactly, and engineers have drawn that way ever since. In 1963 Ivan

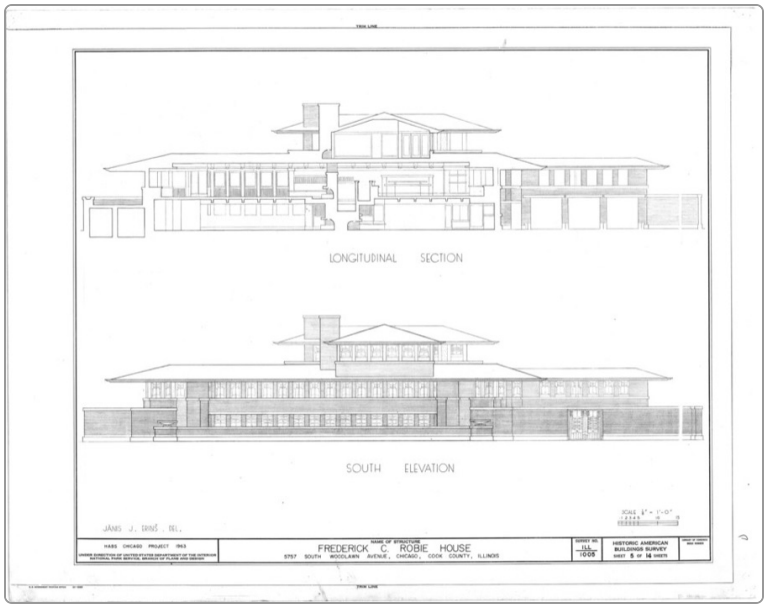
Sutherland at MIT wrote Sketchpad, the first interactive drawing program, in which a user drew on a screen with a light pen and the computer kept the geometry exact. Computer modeling grew from there. In 2000 a small company in Boulder, Colorado, released SketchUp, a modeling program built around one simple idea: draw a flat shape, then push or pull it into a solid, almost like stacking Froebel blocks. A free version runs in a web browser today.

The physics. A 3D model is geometry you can measure. Its volume is the sum of the volumes of its pieces; its surface area tells you how much siding, roofing or paint the building needs; and every view (plan, elevation, section, perspective) is a projection of the same model, so they always agree. An **isometric drawing** shows the three axes 120° apart with true lengths along each axis, so it is easy to draw and measure, though not quite what the eye sees. (In a true isometric projection, lengths along the axes appear shortened by a factor of about 0.816; isometric *drawings* skip the shortening for convenience.) A physical model at 1:100 has 1/100 the length, 1/10,000 the area and 1/1,000,000 the volume of the building.

The engineering. Today architects and engineers build a complete digital model of a building before construction, often called a building information model (BIM), in which each wall, beam and duct knows its size and material. The software can list quantities, check that pipes do not run through beams, and print every drawing. But the computer only does what the designer tells it. Good modelers work like Wright with his blocks: they start with simple masses, think about proportion and composition, and add detail only when the big idea is right. They also name and organize parts into groups and components so the model can be changed easily.

Try it. In the build you will model a small chapel twice: once with blocks on the table and once in SketchUp Free or Tinkercad, then compare them and produce a plan, an elevation and a perspective from your computer model.

PICTURE STUDY



Longitudinal Section and South Elevation of the Robie House, Historic American Buildings Survey, 1960s · Measured drawing · Library of Congress, Prints and Photographs Division

Image: Wikimedia Commons, public domain

HABS architects measured Frank Lloyd Wright's Robie House in Chicago and drew it in plans, sections and elevations. On this sheet, notice how the house is made of long horizontal layers, like blocks stacked and slid past each other.

Look closely

- Find the long roofs in the elevation. How far do they reach beyond the walls below them?
- Find the section. Where are the floors, and where would a person stand in each room?
- Look for places where one block-like mass sits on another but is shifted sideways.
- Find the chimney mass in the center. How does it tie the horizontal layers together?

Talk about it

1. How does this house remind you of building with blocks?
2. A roof that reaches far beyond its support is a cantilever. What must be inside the roof to carry it?

3. If you modeled this house in SketchUp, what simple shapes would you start with?

BUILD

One Chapel, Two Models

You will design a small chapel (or garden pavilion) with blocks on the table, measure it, and then build the same design in SketchUp Free or Tinkercad. From the computer model you will produce a plan, an elevation and a perspective, and compare the two ways of modeling.

You will need

- Identical blocks: LEGO, KEVA planks, wooden cubes or sugar cubes
- A ruler, graph paper and a calculator
- A computer with SketchUp Free (app.sketchup.com) or Tinkercad in a web browser; a parent sets up any account
- Optional: card stock and glue for a final physical model at ¼″ = 1′-0″

Steps

1. Write a short brief: a chapel for about 30 people, with an entrance porch, a main room with windows on both long sides, and a raised area at the far end.
2. Build the design with blocks. Decide what one block represents (for example, 2 ft). Try at least two arrangements and photograph each.
3. Measure the chosen block model and convert it to real dimensions. Record the length, width, wall height, roof height and window sizes in a table.
4. In SketchUp Free (or Tinkercad), draw the floor rectangle to real dimensions, push it up to the wall height, and add the roof, porch and windows. Use groups or components for repeated windows.
5. Use the software's views to export or screenshot a plan (top view), a front elevation (front view, parallel projection) and a perspective.
6. Use the software's measuring or entity info tools to find the floor area and the volume. Compare with your hand calculation from the block model.
7. **Test.** Ask someone to look at the three views and the perspective and describe the chapel. Did they understand it? Which view helped most?
8. **Improve.** Change one design idea in the computer model (roof pitch, window rhythm, porch shape) and compare the before and after

perspectives. Record which you prefer and why, using Vitruvius's three tests.

Tips

- Type exact dimensions in SketchUp instead of dragging by eye.
- Group each part as you finish it so it does not stick to the next part.
- Save versions as you go (Chapel-1, Chapel-2) so you can go back.

Check your work

- ☐ I built the same design as a block model and as a computer model.
- ☐ My plan, elevation and perspective come from one model and agree.
- ☐ My hand-calculated volume and area agree with the software within a few percent, or I can explain why not.

ENGINEER'S NOTEBOOK

1. Photographs or sketches of your block arrangements, with a sentence about why you chose one.
2. An isometric drawing of your chapel on isometric or plain paper, drawn at a stated scale.
3. A comparison table: block model vs. computer model, with dimensions, area, volume, time taken, and one strength and one weakness of each method.

KEY FORMULAS

$V = \sum V_{\text{pieces}}, \quad V_{\text{prism}} = A_{\text{base}} h$

$A_{\text{model}} = A_{\text{real}}/k^2, \quad V_{\text{model}} = V_{\text{real}}/k^3$

isometric axes at 30° to the horizontal; true isometric factor $\sqrt{2/3} \approx 0.816$

From history. Friedrich Froebel, who designed the gifts, also coined the word "kindergarten," a children's garden. He was a devout Lutheran who believed that children learn about the order of God's creation by handling simple, well-made forms. Many architects of the early twentieth century, not only Wright, played with Froebel blocks as children.

Example 1. A simple chapel is a box 30 ft long, 20 ft wide and 12 ft high to the eaves, with a gable roof rising 8 ft to the ridge along its length. Find the volume enclosed.

Show solution

1. Box: $30 \times 20 \times 12 = 7200$ ft^3.
2. Roof: a triangular prism with a triangle of base 20 ft and height 8 ft, $\frac{1}{2}(20)(8) = 80$ ft^2, times length 30 ft: 2400 ft^3.
3. Total: $7200 + 2400 = 9600$ ft^3.

Answer: 9,600 ft^3

Example 2. Find the roof area of the chapel in Worked Example 1 (no overhangs), and the number of bundles of shingles if one bundle covers 33.3 ft^2 and you add 10% for waste.

Show solution

1. Each roof slope runs from eave to ridge: $\sqrt{10^2 + 8^2} = \sqrt{164} \approx 12.81$ ft.
2. Two slopes, each 30 ft long: $2 \times 30 \times 12.81 \approx 768.4$ ft^2.
3. With 10% waste: $768.4 \times 1.10 \approx 845$ ft^2.
4. Bundles: $845 \div 33.3 \approx 25.4$, so 26 bundles.

Answer: About 768 ft^2 of roof; 26 bundles.

Example 3. You will build a physical model of the chapel at 1:48 (¼″ = 1′-0″). What are the model's length, width and ridge height in inches, and what fraction of the real volume does it hold?

Show solution

1. Length: 30 ft $\div$ 4 ft/in $= 7.5$ in; width: $20 \div 4 = 5$ in.
2. Ridge height: $(12 + 8) \div 4 = 5$ in.
3. Volume ratio: $1/48^3 = 1/110{,}592$.

Answer: 7.5 in × 5 in, 5 in to the ridge; 1/110,592 of the real volume.

PRACTICE

1. **Easy** What was Sketchpad, and when was it made?

   Show answer

   The first interactive computer drawing program, written by Ivan Sutherland at MIT in 1963.

2. **Easy** At what angles to the horizontal are the two receding axes drawn in an isometric drawing?

   Show answer

   30° (the three axes are 120° apart, with one vertical).

3. **Easy** A box-shaped room is 5 m × 4 m × 3 m. Find its volume.

   Show answer

   $60\ \text{m}^3$.

4. **Medium** A tower is a cylinder 4 m in diameter and 18 m tall topped by a cone 6 m tall. Find its volume.

   Show answer

   Cylinder: $\pi(2)^2(18) \approx 226.2\ \text{m}^3$; cone: $\frac{1}{3}\pi(2)^2(6) \approx 25.1\ \text{m}^3$; total $\approx 251\ \text{m}^3$.

5. **Medium** A model at 1:100 has a floor area of 150 cm^2. What is the real floor area in m^2?

   Show answer

   $150 \times 100^2 = 1{,}500{,}000\ \text{cm}^2 = 150\ \text{m}^2$.

6. **Medium** In true isometric projection, a 10 ft edge along an axis appears how long (before applying the drawing scale)?

   Show answer

   $10 \times 0.816 \approx 8.16\ \text{ft}$.

7. **Medium** A gable roof on a house 24 ft wide has a 6:12 pitch and is 40 ft long. Find the roof area (no overhangs).

   Show answer

   Run 12 ft, rise 6 ft, slope length $\sqrt{144 + 36} \approx 13.42$ ft; area $= 2 \times 40 \times 13.42 \approx 1073\ \text{ft}^2$.

8. Hard The Robie House roof cantilevers about 20 ft beyond its last support. Why would a wood beam be a poor choice for this, and what did Wright use?

Show answer

A long cantilever bends and deflects a great deal; wood of a practical size would sag too much and creep over time. Wright used steel beams hidden in the roof, which are much stiffer and stronger for their depth.

9. Hard Your block model uses cubes 2.5 cm on a side, each representing 2 ft. What is the model's scale as a ratio? Your model chapel is 14 blocks long. How long is the real chapel?

Show answer

2 ft = 60.96 cm, so 1:24.4 (about 1:24). The chapel is $14 \times 2 = 28$ ft long.

10. Hard Conceptual. Why do modelers use components for repeated objects like windows?

Show answer

Model answer: all copies of a component share one definition, so changing one changes all of them; this saves time, keeps the design consistent and lets the software count how many there are.

11. Challenge A chapel's nave is a box 40 ft × 24 ft × 16 ft with a semicircular apse of radius 12 ft (same 16 ft height) on one end, under a flat ceiling. Find the floor area and volume. How many people could it seat at 7 ft^2 per person if one-third of the floor is aisles and the apse?

Show answer

Floor: $40 \times 24 + \frac{1}{2}\pi(12)^2 \approx 960 + 226 = 1186\ \text{ft}^2$; volume $\approx 1186 \times 16 \approx 18{,}980\ \text{ft}^3$. Seating area $\approx \frac{2}{3}(1186) \approx 791\ \text{ft}^2$, so about 113 people.

COMMON MISTAKES

- Dragging shapes by eye in CAD instead of typing exact dimensions.
- Forgetting to group parts, so that geometry sticks together and cannot be moved or changed cleanly.
- Thinking an isometric drawing is a perspective. Isometric lines stay parallel and do not meet at vanishing points.

> **Make it a game · Block Builder Memory.** Gather ten building blocks, cups or books. Your parent builds a small structure and hides it after ten seconds. Rebuild it from memory, then draw its plan and an isometric view. Add one point for each correct dimension and each right name for a Froebel gift or CAD term. Swap roles for round two.

UNIT 2

Architectural Drawing

Drawings are the language of building. Before a single stone is cut, the architect and engineer must describe the whole building so exactly that hundreds of people who have never met can build it. In this unit you will learn that language: scale and the architect's scale, floor plans, elevations and sections, perspective, and three-dimensional modeling on paper, with blocks and in free software.

Each skill comes with its story: Thomas Jefferson drawing Monticello on graph paper; the monks who drew the plan of St Gall twelve centuries ago; the architects of the Historic American Buildings Survey measuring the nation's old buildings; Brunelleschi and his mirror; and Frank Lloyd Wright, whose mother's gift of kindergarten blocks shaped the way he saw buildings. By the end you will be able to measure a real building and draw it in plan, elevation, section and perspective, and build it as a computer model.

PREREQUISITES

- Unit 1: units and conversions, area and volume
- Geometry: similar triangles, the Pythagorean theorem, area of composite shapes
- Right-triangle trigonometry: the tangent ratio
- A computer with a web browser for SketchUp Free or Tinkercad in Lesson 2.5

MASTERY CHECKLIST

- ☐ I can read and draw at ¼″ = 1′-0″, ⅛″ = 1′-0″, 1:50 and 1:100, and I know that areas scale by the square of the scale factor.
- ☐ I can draw and read a dimensioned floor plan with correct conventions and compute net and gross areas.
- ☐ I can draw elevations and sections that line up with the plan and calculate roof pitch, rafter length and stair risers.
- ☐ I can explain Brunelleschi's perspective experiment and use

$y' = fY/Z$.

- ☐ I can build a simple building model in SketchUp Free or Tinkercad and produce plan, elevation and perspective views from it.

OPTIONAL KITS FOR THIS UNIT

- **SketchUp Free (web)**: Free browser-based 3D modeling from Trimble; a parent creates the account. Used in Lesson 2.5, the portfolio, and Unit 10. Not a kit, but the main tool of this unit.
- **Tinkercad**: Free, very simple browser-based 3D modeling from Autodesk, good for students new to CAD; a parent sets up the account for students under 13. An alternative to SketchUp in Lesson 2.5.
- **Froebel Gifts (Froebel USA)**: Reproductions of Friedrich Froebel's kindergarten block sets, the gifts that shaped Frank Lloyd Wright. Lovely for Lesson 2.5 and for younger siblings; plain wooden cubes work just as well.
- **KEVA planks**: Identical maple planks for block models of buildings in Lesson 2.5 and for model-making all year.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

2.1	**Scale and the Architect's Scale**	3 sessions of 50 minutes
2.2	**Floor Plans**	3 sessions of 50 minutes
2.3	**Elevations and Sections**	3 sessions of 50 minutes
2.4	**Perspective Drawing: Brunelleschi's Experiment**	3 sessions of 50 minutes
2.5	**Modeling in 3D: From Froebel's Blocks to SketchUp**	3 sessions of 50 minutes

Unit 2 test

Time allowed: 50 minutes. Calculators and an architect's scale are allowed. Show your work and units for every calculation. Use a ruler for any drawing. The test is worth 100 points.

1. **5 pts**

   What is the scale factor and ratio of ⅛″ = 1′-0″?

2. 6 pts

On a ¼″ = 1′-0″ plan, a room measures 3⅜ in × 4¼ in. Find its real dimensions and area.

3. 6 pts

A wall 7.35 m long is drawn at 1:50. How long is it on paper?

4. 6 pts

A 1:100 model has a roof area of 240 cm^2. What is the real roof area in m^2?

5. 5 pts

What is a cut plane, and how are the walls it cuts shown on a plan?

6. 6 pts

Check this dimension string: 5′-3″, 2′-10″, 8′-0″, 3′-11″; overall 20′-0″.

7. 6 pts

A house is 44 ft × 28 ft outside. Its rooms total 1,010 ft^2. Find the gross area and efficiency.

8. 7 pts

A gable roof on a building 32 ft wide has a 9:12 pitch. Find the roof angle, the rise and the rafter length (no overhang).

9. 7 pts

A stair must rise 10′-3″. With a maximum riser of 7¾ in and 10 in treads, find the number of risers, the riser height and the total run.

10. 7 pts

From 35 m away, the top of a steeple is sighted at 42° with an eye height of 1.5 m. How tall is the steeple?

11. 6 pts

What is the difference between an elevation and a section, and why does a builder need both?

12. 6 pts

Describe Brunelleschi's perspective experiment in three or four sentences.

13. **7 pts**

A column 9.0 m tall is 36 m from your eye and the picture plane is 0.60 m away. How tall is its image? How tall would it be at 54 m?

14. **6 pts**

In two-point perspective, where do the vanishing points lie, and what happens to vertical lines?

15. **6 pts**

A chapel is a box 36 ft × 22 ft × 14 ft with a gable roof rising 9 ft along its length. Find its enclosed volume.

16. **8 pts**

Short essay. How did Froebel's blocks, Monge's descriptive geometry and Sketchpad each change the way designers think about buildings in three dimensions?

ANSWER KEY

1. $12 \div 0.125 = 96$; 1:96.
2. 13.5 ft × 17 ft (13′-6″ × 17′-0″); area 229.5 ft^2.
3. $7350 \div 50 = 147$ mm.
4. $240 \times 10{,}000 = 2{,}400{,}000\ \text{cm}^2 = 240\ \text{m}^2$.
5. The imaginary horizontal slice about 4 ft above the floor; walls cut by it are drawn with heavy lines or filled in (poché).
6. 18′ + 24″ = 20′-0″. It is correct.
7. Gross 1232 ft^2; efficiency $1010 \div 1232 \approx 82\%$.
8. $\theta = \tan^{-1}(0.75) \approx 36.9^\circ$; run 16 ft, rise 12 ft; rafter $= \sqrt{256 + 144} = 20$ ft.
9. $123 \div 7.75 = 15.9$, so 16 risers of $123 \div 16 \approx 7.69$ in; 15 treads, run 150 in = 12′-6″.
10. $35 \tan 42^\circ + 1.5 \approx 31.5 + 1.5 = 33.0$ m.
11. An elevation shows the outside face; a section cuts through to show floors, roof structure, ceiling heights and foundations inside. The builder needs the elevation for the exterior appearance and openings and the section for heights and how the building is put together.
12. About 1415–1420, Brunelleschi painted the Florence Baptistery on a small panel from inside the cathedral doorway and drilled a hole at the point opposite his eye. A viewer looked through the hole from the back at a mirror held in front of the painting, and saw the painting reflected; lowering the mirror showed the real building from the same point. The two matched, proving his geometric method of perspective.
13. $0.60 \times 9.0 \div 36 = 0.15$ m; at 54 m, 0.10 m.
14. Both vanishing points lie on the horizon line (at eye height); vertical lines stay vertical and parallel.
15. $36 \times 22 \times 14 = 11{,}088$ plus $\frac{1}{2}(22)(9)(36) = 3564$, total 14,652 ft^3.
16. Model answer: Froebel's blocks trained children such as Frank Lloyd Wright to compose buildings from simple masses and planes; Monge showed that orthographic views define any three-dimensional shape exactly, which became the basis of engineering and architectural drawing; Sketchpad began computer-aided design, leading to modeling programs such as SketchUp in which a designer builds one model and draws every view from it.

PORTFOLIO PIECE: A MEASURED DRAWING SET

Measure your home, your church or another small building you can access. Produce a set of drawings like a HABS sheet: a dimensioned floor plan at ¼″ = 1′-0″ (or 1:50), two elevations and one section at a matching scale, all lined up, and a hand-drawn two-point perspective.

Then build the building as a model in SketchUp Free or Tinkercad from your drawings and produce a perspective view from the model. Compare it with your hand perspective and write a half-page reflection on what each method showed you.

LOOK FOR	WHAT IT MEANS
Measurement	Dimensions were measured carefully and checked (strings add up; height checked two ways).
Drafting conventions	Correct line weights, door swings, symbols, scale, north arrow, title block and section marks.
Projection	Plan, elevations and section line up and agree; perspective has a correct horizon and vanishing points.
Computer model	The model is built to true dimensions, organized in groups, and its views agree with the drawings.
Reflection	Thoughtful comparison of hand drawing and computer modeling, with specific examples.

UNIT 3

The History of Architecture

Every great style of architecture grew from a structural idea and a new material. In this unit you will walk through five of them. The Greeks perfected post and lintel in stone; the Romans invented concrete and spanned the Pantheon's dome. Gothic builders raised stone frameworks with pointed arches and flying buttresses so that churches could be filled with light. Palladio studied the Roman ruins and taught the world proportion. Paxton built the Crystal Palace from factory-made iron and glass in a matter of months. And modern architects used steel and reinforced concrete frames to free walls and reach out in cantilevers, sometimes, as at Fallingwater, farther than the concrete could bear without help.

In each lesson you will do what the builders did: test a structural idea with a model, measure what happens and improve it. Chalk lintels will snap and block arches will stand; walls will tip unless buttressed; frames will lean unless braced; and cantilevers will sag and creep.

PREREQUISITES

- Units 1–2: weight, density, stress, factor of safety, scale drawings
- Algebra: ratios, square roots and proportional reasoning ($y \propto x^2$, $y \propto 1/x$)
- Geometry: volume of spheres and shells; circles

MASTERY CHECKLIST

- ☐ I can explain why stone lintels need short spans and how arches, vaults, domes and concrete let the Romans span much more.
- ☐ I can calculate the thrust of an arch with

$H = wL^2/(8h)$

and check a buttress against overturning.

- ☐ I can use Palladio's ratios and the arithmetic, geometric and harmonic means in a design.
- ☐ I can calculate quantities, rates and thermal expansion for a prefabricated iron-and-glass building and explain why frames need bracing.
- ☐ I can calculate the moment and deflection of a cantilever and explain creep and post-tensioning using Fallingwater.

OPTIONAL KITS FOR THIS UNIT

- **KEVA planks**: Identical maple planks, excellent for the wall-and-buttress push test (Lesson 3.2), arches and Palladian façades.
- **LEGO Architecture**: Ages 12 and up for most sets. Several sets model famous buildings and show their massing and proportions; useful for comparing styles. Never required.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

3.1 Greek Orders and Roman Engineering

3 sessions of 50 minutes

3.2	**Romanesque and Gothic Cathedrals**	3 sessions of 50 minutes
3.3	**The Renaissance and Palladio**	3 sessions of 50 minutes
3.4	**The Industrial Age: Iron, Glass and the Crystal Palace**	3 sessions of 50 minutes
3.5	**Modern Architecture: The Frame, the Skyscraper and Fallingwater** 3 sessions of 50 minutes	

Unit 3 test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation. Answer short-answer questions in complete sentences. The test is worth 100 points.

1. **5 pts**
   Identify the Doric, Ionic and Corinthian orders by their capitals, and say which is stoutest.

2. **6 pts**
   A stone lintel carries 90 kN at midspan with a 2.4 m span. Estimate its capacity at 3.6 m (ignore self-weight). Why is the real capacity even lower?

3. **7 pts**
   Describe three things the Romans did to make the Pantheon's dome stand.

4. **6 pts**
   A dome has an inner radius of 15 m and an average thickness of 2.0 m. Estimate its volume as a hemispherical shell and its weight at 2,000 kg/m^3.

5. **7 pts**
   An arch carries 25 kN/m over 16 m. Find the horizontal thrust for a rise of 8 m and for a rise of 12 m.

6. 7 pts

A buttress pier 2.4 m wide weighs 1,000 kN. A thrust of 80 kN acts 10 m above its base. Find the factor of safety against overturning.

7. 6 pts

Explain why Gothic cathedrals could have huge windows while Romanesque churches could not.

8. 6 pts

Find the arithmetic, geometric and harmonic means of 20 ft and 30 ft.

9. 5 pts

Why did Palladio's *Four Books* have such a great influence?

10. 6 pts

The Crystal Palace used about 293,000 panes of 49 in × 10 in. Estimate the total glass area in ft^2.

11. 6 pts

An iron girder line 300 m long warms by 30 °C. How much does it grow? ($\alpha = 12 \times 10^{-6}/^{\circ}\mathrm{C}$)

12. 6 pts

Why does a frame of columns and beams need diagonal bracing?

13. 6 pts

A cantilever 3.5 m long carries 14 kN/m. Find the moment at the support.

14. 7 pts

A uniformly loaded cantilever deflects 8 mm. Find its deflection if its length increases by 30%. With a creep factor of 2, what would the long-term deflection of the longer cantilever be?

15. 6 pts

What is post-tensioning, and why was it used at Fallingwater?

16. **8 pts**

Short essay. Choose two of the styles in this unit (Greek, Roman, Gothic, Renaissance, Industrial, Modern) and explain how each one's look came from its structural system and materials.

ANSWER KEY

1. Doric: plain cushion capital (stoutest); Ionic: scroll volutes; Corinthian: acanthus leaves (most slender).
2. $90 \times 2.4/3.6 = 60$ kN. The lintel's own weight adds bending that grows with the square of the span.
3. Any three: graded the concrete from heavy aggregate low down to light pumice near the top; made the dome thick at the base and thin at the top; added stepped rings outside the base; used coffers to remove weight; built relieving arches in the drum; left an oculus to remove the heaviest top part.
4. $\frac{2}{3}\pi(17^3 - 15^3) = \frac{2}{3}\pi(4913 - 3375) \approx 3221$ m^3; mass $\approx 6.44 \times 10^6$ kg, weight $\approx 63{,}200$ kN.
5. $wL^2/8 = 800$; $H = 800/8 = 100$ kN and $800/12 \approx 66.7$ kN.
6. $M_r = 1000 \times 1.2 = 1200$; $M_o = 800$; $FS = 1.5$.
7. Gothic ribbed vaults gathered the roof's weight onto a few piers, pointed arches reduced the thrust, and flying buttresses carried the remaining thrust outside, so the walls between piers carried little load and could be glass. Romanesque barrel vaults pushed along the whole wall, which had to be thick and solid.
8. $A = 25$, $G = \sqrt{600} \approx 24.5$, $H = 1200/50 = 24$ ft.
9. It gave clear rules, proportions and woodcuts of buildings with dimensions, so builders anywhere, such as Inigo Jones in England and Thomas Jefferson in Virginia, could study and imitate his designs.
10. $293{,}000 \times 490/144 \approx 997{,}000$ ft^2.
11. $12 \times 10^{-6} \times 300 \times 30 = 0.108$ m ≈ 11 cm.
12. Sideways loads such as wind make a rectangular frame rack (lean); diagonals form triangles, which cannot change shape without stretching a member, so the frame stays square.
13. $14 \times 3.5^2/2 = 85.75$ kN·m.
14. $8 \times 1.3^4 \approx 8 \times 2.856 \approx 22.8$ mm; long-term $\approx 3 \times 22.8 \approx 68.5$ mm.
15. Stretching high-strength steel cables along or through concrete members and anchoring them, to squeeze the concrete and counter bending. At Fallingwater it was used in 2001–2002 to stop the main cantilevers, which had sagged about 7 in through cracking and creep, from sagging further.
16. Model answer: e.g., Greek temples have closely spaced columns and deep lintels because stone is weak in tension and lintels can span only a short distance;

Gothic cathedrals have tall pointed arches, ribbed vaults, flying buttresses and walls of glass because the structure was gathered into a stone framework that carried the vault's thrust outside; modern buildings have open plans, glass walls and cantilevers because steel and reinforced concrete frames carry all the load.

PORTFOLIO PIECE: A STYLE AND STRUCTURE STUDY

Choose one historic building from this unit or near your home that belongs to a clear style. Draw its section to scale, showing how its structure carries the loads to the ground, and build a model of its key structural idea (an arch, a buttressed wall, a braced frame, a cantilever, a coffered dome).

Test the model with a measured load and compare it with a simple calculation. Write a two-page report explaining how the building's structural system created its style, with your drawing, test data and photographs.

LOOK FOR	WHAT IT MEANS
Historical understanding	Accurately describes the building, its date, builders and style, and the materials and methods available to them.
Structural explanation	Clearly explains the load path and the structural idea, with a labeled section drawing.
Model and test	The model represents the key idea; the test is fair, measured and recorded.
Calculation	A relevant calculation (thrust, overturning, deflection, quantity) is done correctly with units and compared with the test.
Writing	Organized, clear, in complete sentences, with figures labeled.

3.1 Greek Orders and Roman Engineering

OBJECTIVES

- Identify the Doric, Ionic and Corinthian orders and explain the structural limits of post-and-lintel construction in stone.
- Explain how the Romans used the arch, the vault, the dome and concrete to span much larger spaces, using the Pantheon as the case study.
- Calculate how a lintel's strength depends on its span and how lighter materials reduce the weight of a dome.

> **Warm-up.** Lay a stick of sidewalk chalk across two books 5 cm apart and press down in the middle until it snaps. Now try it with the books 10 cm apart. Which broke more easily? Look at the broken faces: where did the crack start, at the top or the bottom?

WATCH

Rome's Pantheon: The Secrets of the World's Largest Dome of Its Time I SLICE HISTORY

SLICE History · 12:57

https://www.youtube.com/watch?v=YPumOC0P1Us

> **WHEN TO WATCH**
> After the story. Watch for how the builders changed the concrete from the bottom of the dome to the top.

KEY TERMS

post and lintel

A structure of upright posts (columns) carrying horizontal beams (lintels). A stone lintel bends, putting its bottom in tension, so stone spans must be short.

classical orders

The systems of column, capital and entablature with set proportions: Doric (stout, plain capital), Ionic (scroll volutes), Corinthian (acanthus leaves, most slender).

entasis

The slight outward swelling of a column's shaft, a visual refinement used by Greek builders.

opus caementicium

Roman concrete: lime mortar, often with volcanic ash (pozzolana), mixed with chunks of stone or brick (the aggregate).

oculus

A circular opening at the top of a dome, such as the 8 m "eye" of the Pantheon.

coffer

A sunken panel in a ceiling or dome, which removes weight while leaving ribs of material.

CORE IDEAS

The story. Between 447 and 432 BC the Athenians built the Parthenon, a marble temple of Doric columns about 10.4 m tall. About 550 years later, around AD 113–125, the emperor Hadrian rebuilt a temple in Rome called the Pantheon. Its porch still has Greek-style Corinthian columns carrying a stone beam, but behind the porch stands a round drum of brick-faced concrete holding up a dome 43.3 m across, with an open eye at its top. For more than 1,300 years it was the largest dome in the world, and it is still the largest unreinforced concrete dome. In AD 609 it was given to the church and dedicated as Santa Maria ad Martyres, and because it has been in use ever since, it is the best-preserved great building of ancient Rome.

How they figured it out. Greek builders perfected post and lintel. They knew that marble is immensely strong when squeezed but cracks when bent, so they kept the spans between columns short and the lintels deep. They refined the look with great care: the Parthenon's base curves upward by several centimeters toward the middle of each side, its columns lean slightly inward and swell gently (entasis), and the corner columns are a little thicker. The Romans took a different path. They used the arch, which turns load into compression, and stretched it into the barrel vault and spun it into the dome. And they developed concrete: lime and volcanic ash from near Puteoli, mixed with broken stone, poured in layers behind brick facing. For the Pantheon they graded the concrete, using heavy stone in the foundations, then brick and tufa, and light volcanic pumice and scoria near the top, and they made the dome about 6 m thick at its base thinning to about 1.2 m at the oculus.

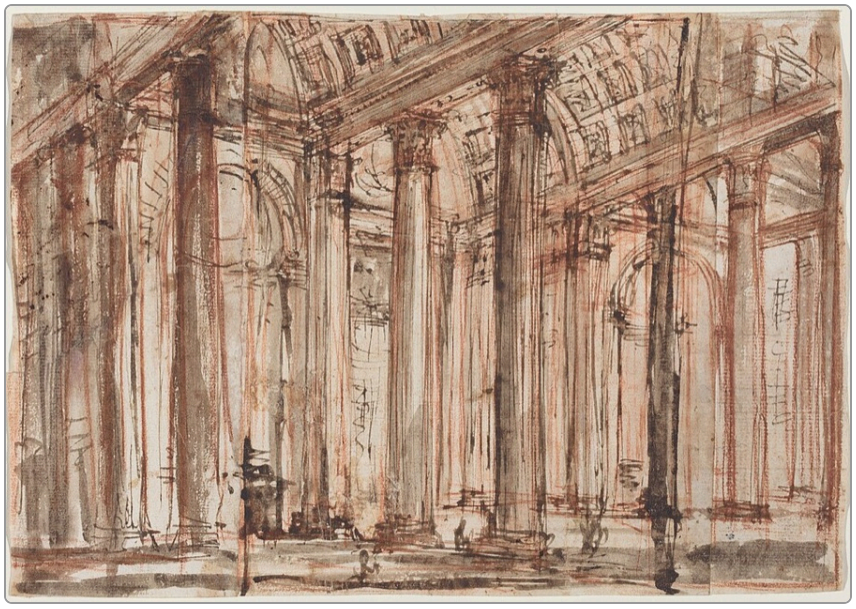
The physics. A beam bends under load: its top is squeezed (compression) and its bottom stretched (tension). Stone's tensile strength is perhaps a tenth or less of its

compressive strength, so a stone lintel cracks from the bottom. A lintel's capacity for a load at midspan falls in proportion to its span ($P \propto 1/L$ for the same section), and its own weight adds bending that grows with the square of the span. That sets a practical limit of a few meters for stone. An arch or dome, shaped well, carries its load mostly in compression, so it can span far more. But a hemispherical dome under its own weight is in compression only near the top. Below about 52° from the crown, the circles of latitude (the hoops) are pulled into tension and the dome tends to crack in vertical lines. The Romans countered this by making the lower dome very thick, adding stepped rings of masonry outside its base, and lightening the upper dome so there was less weight to push outward.

The engineering. The Pantheon is a lesson in using each material for what it does well. The concrete is light where weight hurts and heavy where it helps. The 140 coffers on the underside remove weight while leaving thick ribs. Relieving arches of brick buried in the drum steer loads around the niches to the solid piers. The result is so well balanced that, although the drum has cracks, the dome has stood for nineteen centuries. Classical columns, meanwhile, became the vocabulary of Western architecture: when you see Doric, Ionic or Corinthian columns on a bank, a courthouse or a church, you are seeing the Greek orders Vitruvius described.

Try it. In the build you will test chalk "stone" lintels of different spans to failure, then build an arch from blocks and see how much more it carries.

PICTURE STUDY



The Portico of the Pantheon, Giovanni Battista Piranesi, 1750s–early 1760s · Pen and brown ink with red chalk · National Gallery of Art, Washington

Image: Wikimedia Commons, public domain

Piranesi, the great eighteenth-century etcher of Rome, sketched the Pantheon's porch from inside its forest of columns. The porch is Greek in spirit, post and lintel; the round building behind it is Roman, arch and dome.

Look closely

- Count the columns you can see. How closely are they spaced compared with their thickness?
- Find the capitals at the tops of the columns. Which order are they?
- Look up at the ceiling of the porch. What shapes do you see, and why might they be there?
- Find the doorway or the back wall. How does the drawing make the porch feel deep?

Talk about it

1. Why do the columns of the porch have to be so close together?
2. Behind this porch is a dome 43 m across with no columns at all. What structural idea makes that possible?

3. Why do you think the builders put a Greek-style porch on a Roman domed building?

Safety first. Wear safety glasses when snapping chalk or loading block arches; pieces can fly. Keep fingers out from under the loads.

BUILD

Stone Beams and Stone Arches

Sidewalk chalk behaves much like stone: strong in compression, weak in tension. You will test chalk lintels at different spans to find how strength depends on span, then build a block arch and compare the loads it can carry.

You will need

- A box of identical thick sidewalk chalk sticks
- Two stacks of books or blocks for supports, a ruler
- A paper cup hung on a string loop, and pennies or sand for loads; a kitchen scale
- For the arch: wooden blocks, sugar cubes, or foam pieces cut into wedges; a cardboard form (centering) to build the arch on

Steps

1. Measure a chalk stick's diameter and length. Set the supports 4 cm apart and hang the cup from the middle of the stick with a loop of string.
2. **Predict.** If 4 cm holds a certain load, what will 8 cm hold? Write your prediction using $P \propto 1/L$.
3. Add pennies (or sand) slowly until the chalk snaps. Weigh the cup and load. Record the failure load. Repeat with three sticks and average.
4. Repeat at spans of 6 cm, 8 cm and 10 cm. Look at each broken face and note where the crack began.
5. Graph failure load against span and against $1/L$. Is the second graph close to a straight line through the origin?
6. Build an arch of at least seven wedge-shaped blocks over a cardboard form spanning 10 cm. Set heavy blocks or books against its feet. Remove the form.
7. Load the arch at its crown with the cup and pennies until it fails, or until it carries several times what the 10 cm chalk lintel held. Record the load and how it failed.
8. **Improve.** Remove the books at the feet and test again. Then try loading the arch off center. What does the arch need that the lintel did not?

Tips

- Hang the load gently; a sudden drop breaks chalk early.
- Make the arch's joints tight; gaps let the blocks rotate and the arch fold.
- Record every result, even odd ones, and say what might have caused them.

Check your work

- ☐ My data show how a stone lintel's strength falls as its span grows.
- ☐ I can explain why the Greeks spaced columns closely and how the Romans spanned larger spaces.
- ☐ I can explain why an arch needs something to hold its feet from spreading.

ENGINEER'S NOTEBOOK

1. Sketch a Doric, an Ionic and a Corinthian capital, with the column proportions noted.
2. Your chalk-lintel data table and the graph of failure load against $1/L$.
3. A section sketch of the Pantheon showing the drum, the stepped rings, the coffered dome and the oculus, with notes on where the concrete was heavy and where it was light.

KEY FORMULAS

$P_{\text{fail}} \propto \dfrac{1}{L}$ (lintel of fixed section, load at midspan)

$V_{\text{sphere}} = \frac{4}{3}\pi r^3, \quad V_{\text{shell}} \approx \frac{2}{3}\pi(r_o^3 - r_i^3)$ (hemisphere)

$W = \rho V g$

$\text{slenderness} = \dfrac{\text{column height}}{\text{diameter}}$

From history. Roman concrete has lasted remarkably well, especially in seawater harbors, where its volcanic ash reacts with seawater to grow new minerals that strengthen it over time. Researchers in 2023 also found that Romans often mixed in quicklime, leaving small lime lumps that can help heal cracks when water seeps in. You will compare Roman and modern concrete in Lesson 5.4.

Example 1. A marble lintel carries 120 kN at midspan when the columns are 2.0 m apart. With the same lintel, about what load could it carry if the columns were spread to 3.0 m and 4.0 m? (Ignore the lintel's own weight.)

Show solution

1. For the same section, $P \propto 1/L$, so $P_2 = P_1(L_1/L_2)$.
2. At 3.0 m: $120 \times (2.0/3.0) = 80$ kN.
3. At 4.0 m: $120 \times (2.0/4.0) = 60$ kN.
4. Its own weight also grows with length and adds bending that grows with the square of the span, so in reality the loss is even greater. That is why Greek columns are close together.

Answer: About 80 kN at 3.0 m and 60 kN at 4.0 m (less, counting self-weight).

Example 2. Model the Pantheon dome as a hemispherical shell with inner radius 21.65 m and an average thickness of 3.0 m. Estimate the volume of concrete, and its weight if it were all ordinary concrete (2,200 kg/m^3) or all pumice concrete (1,350 kg/m^3).

Show solution

1. Outer radius: $r_o = 21.65 + 3.0 = 24.65$ m.
2. Shell volume: $V \approx \frac{2}{3}\pi(24.65^3 - 21.65^3) = \frac{2}{3}\pi(14{,}978 - 10{,}148) \approx 10{,}100$ m^3.
3. Ordinary concrete: $m = 2200 \times 10{,}100 \approx 2.2 \times 10^7$ kg, about 22,000 t.
4. Pumice concrete: $m = 1350 \times 10{,}100 \approx 1.4 \times 10^7$ kg, about 14,000 t. Using light aggregate saves roughly 40% of the weight (the Romans used it only in the upper parts, where it helps most).

Answer: About 10,000 m^3; roughly 22,000 t if dense, 14,000 t if all light concrete.

Example 3. The Parthenon's Doric columns are about 10.4 m tall and 1.9 m in diameter at the base. Find their slenderness. A Corinthian column of the same height is about 10 diameters tall. What would its diameter be?

Show solution

1. Doric slenderness: $10.4 \div 1.9 \approx 5.5$.
2. Corinthian: $d = 10.4 \div 10 = 1.04$ m.
3. The Corinthian column is about half as thick, so it has roughly a quarter of the cross-sectional area. It suits lighter loads or a different look, not heavy stone lintels.

Answer: About 5.5 diameters; a Corinthian column would be about 1.0 m thick.

PRACTICE

1. Easy How can you tell the three Greek orders apart by their capitals?

   Show answer

   Doric: plain, cushion-like capital; Ionic: scroll volutes; Corinthian: acanthus leaves.

2. Easy Why does a stone lintel crack on its bottom face?

   Show answer

   Under load the beam bends; its bottom is stretched (tension), and stone is weak in tension.

3. Easy The Pantheon's dome is 43.3 m across. How many feet is that?

   Show answer

   $43.3 \div 0.3048 \approx 142$ ft.

4. Medium A chalk lintel failed at 300 g with a 5 cm span. Predict the failure load at 7.5 cm.

   Show answer

   $300 \times 5/7.5 = 200$ g.

5. Medium The Pantheon's oculus is about 8.2 m in diameter. What is its area? About what fraction of the floor (43.3 m diameter) does it equal?

Show answer

Oculus $\pi(4.1)^2 \approx 52.8\ \mathrm{m}^2$; floor $\pi(21.65)^2 \approx 1472\ \mathrm{m}^2$; ratio about 3.6%.

6. Medium An Ionic column is 9 diameters tall. If it is 0.8 m thick, how tall is it?

Show answer

7.2 m.

7. Medium The inside of the Pantheon would just hold a sphere 43.3 m across. Find the volume of that sphere.

Show answer

$\frac{4}{3}\pi(21.65)^3 \approx 42{,}500\ \mathrm{m}^3$.

8. Hard A stone lintel's own weight creates bending that grows with the square of its span. If a 2 m lintel's self-weight uses 10% of its strength, what fraction does a 4 m lintel of the same section use? What is left for the roof load, compared with the 2 m lintel?

Show answer

Self-weight moment grows by $2^2 = 4$, but strength stays the same, so it uses about 40%. Left for roof load: 60% of the strength, applied over twice the span, so the lintel can carry far less roof (the moment from a central load doubles with span), roughly a third as much as the 2 m lintel.

9. Hard Explain why the Romans used heavy aggregate in the Pantheon's foundations but light pumice near the top of the dome.

Show answer

Weight low down helps: it steadies the drum and resists the outward thrust of the dome. Weight high up hurts: it adds to the load and the outward push the lower dome and drum must resist, so the top was made as light as possible.

10. Hard Conceptual. The Parthenon's base curves upward a few centimeters toward the middle, and its columns lean slightly inward. Give one structural or practical reason and one visual reason for such refinements.

Show answer

Model answer: a crowned base sheds rainwater; inward-leaning columns slightly brace the building. Visually, a perfectly flat base and vertical columns can appear to sag or splay to the eye from below, so the refinements make the temple look straight, stable and alive.

11. Challenge For a thin hemispherical dome under its own weight, the hoop force changes from compression to tension at the angle ϕ from the crown where $\cos^2\phi + \cos\phi - 1 = 0$. Solve for ϕ.

Show answer

Let $c = \cos\phi$: $c^2 + c - 1 = 0$, $c = (-1 + \sqrt{5})/2 \approx 0.618$, so $\phi \approx 51.8^\circ$ from the crown.

COMMON MISTAKES

- Thinking stone is weak. Stone is very strong in compression; it is weak only in tension and bending.
- Assuming a dome is in compression everywhere. A hemispherical dome's lower part is pulled apart in its hoops.
- Confusing the orders by their shafts. Look at the capital.

> **Make it a game · Span It.** Challenge with household materials. Bridge a gap between two books using a flat index card, then fold it into an arch and compare how much weight in coins it carries. Predict first. Explain why the Romans' arch beat post and lintel, and point to the Pantheon's oculus on a sketch. Best prediction wins.

3.2 Romanesque and Gothic Cathedrals

OBJECTIVES

- Explain how Romanesque and Gothic builders carried stone vaults: thick walls, pointed arches, ribbed vaults and flying buttresses.
- Calculate the horizontal thrust of an arch and show why a taller (pointed) arch thrusts less.
- Test how a buttress and a pinnacle help a wall resist a sideways push.

> **Warm-up.** Lean two books against each other like a tent on a smooth table. Slowly push down on the top. What happens to the bottoms? Now put a heavy object against each bottom edge and try again. You have just made a buttress.

WATCH

Engineering and the Flying Buttress

purdueMET · 12:20

https://www.youtube.com/watch?v=vNYWtczxi8w

> **WHEN TO WATCH**
> After the story, before the worked examples. Draw his free-body diagram of the buttress in your notebook as he draws it.

Note: A Purdue University engineering-technology professor explains the flying buttress with statics on a whiteboard.

BACKUP VIDEO

Gothic architecture explained

Smarthistory · 4:31

https://www.youtube.com/watch?v=G94jFWH8NSM

With the story: a short introduction to the parts of a Gothic cathedral.

KEY TERMS

barrel vault

A continuous half-cylinder of masonry; it pushes outward along its whole length, so it needs thick, continuous walls.

pointed arch

An arch made of two arcs meeting at a point. It can be made any height for a given span and pushes outward less than a round arch.

ribbed vault

A vault built on a frame of stone ribs that gathers its weight onto a few points at the corners of each bay.

flying buttress

An arched strut that carries the outward thrust of a high vault across the side aisles to a heavy pier outside.

thrust

The horizontal push of an arch or vault at its supports, which must be resisted or the arch spreads and falls.

pinnacle

A heavy stone spire on top of a buttress; its weight turns the combined force down into the buttress.

CORE IDEAS

The story. In the early 1140s Abbot Suger of the abbey of Saint-Denis, near Paris, rebuilt the east end of his church so that it would be filled with light. His masons used pointed arches and ribbed vaults on slender columns, and the walls between became windows of colored glass. The new choir was consecrated in 1144, and the Gothic style spread across Europe. Builders raced upward: Notre-Dame in Paris, Chartres, Reims, Amiens. At Beauvais the choir vault was raised to about 48 m, the highest Gothic vault ever built. In 1284, twelve years after it was finished, part of it collapsed. The builders rebuilt it with extra piers between the old ones, doubling the number of supports, and the choir has stood since.

How they figured it out. Romanesque churches of the 1000s and 1100s used round arches and barrel vaults like the Romans, which push outward all along the wall; their walls had to be very thick with small windows. Gothic builders made three changes. The **pointed arch** could rise to any height and pushed out less. The **ribbed vault** gathered the weight of each bay onto four corner points. And the **flying buttress** carried the outward push of the high vaults over the aisles to tall piers outside. Medieval masters had no theory of statics. They worked by geometry and

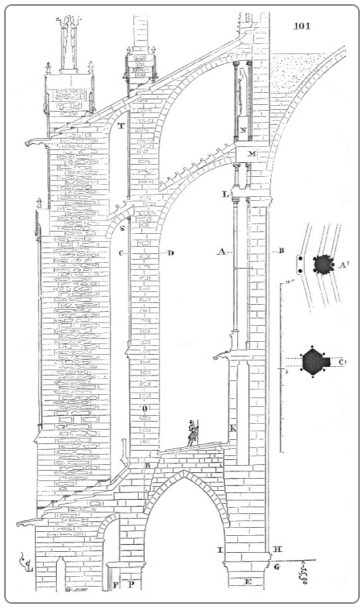
proportion, built, watched for cracks, and adjusted. Failures like Beauvais taught them where the limits were.

The physics. An arch carrying a load pushes both down and out on its supports. For an arch carrying a uniform load w per meter over span L with rise h, the horizontal thrust is about $H = \frac{wL^2}{8h}$. Doubling the rise halves the thrust. That is why a tall pointed arch thrusts much less than a round one of the same span. The thrust must go somewhere. A buttress is a heavy pier: the vault's thrust tries to overturn it, but the buttress's own weight, acting at its center, resists. The resultant of the thrust and the weight must pass through the base of the pier, ideally through its middle third, or the pier will tip and crack. Adding weight on top (a **pinnacle**) bends the resultant more steeply downward and keeps it inside the base.

The engineering. A Gothic cathedral is a framework of stone: piers, ribs and buttresses carry the loads, and the walls between them are glass. Its builders achieved more with less material than anyone before them, but the system is delicate: if a buttress settles or a pier leans, forces shift. Today engineers use the same statics, and computer models, to check old cathedrals, and when Notre-Dame of Paris burned in 2019, engineers had to brace the flying buttresses with timber centering before the vaults could be repaired. The Gothic builders' goal was not height for its own sake but light: churches filled with light, a picture of the light of the world.

Try it. In the build you will push sideways on the top of a model wall and measure how much push it takes to tip it, with and without a buttress and a pinnacle.

PICTURE STUDY



Flying Buttress of the Choir of Beauvais Cathedral, Eugène-Emmanuel Viollet-le-Duc, 1854–1868 · Engraving · *Dictionnaire raisonné de l'architecture française*, Paris

Image: Wikimedia Commons, public domain

Viollet-le-Duc was the great nineteenth-century restorer of French cathedrals. In his dictionary of architecture he drew how Gothic buildings actually work. This section shows the flying buttresses of Beauvais, the tallest Gothic choir ever built.

Look closely

- Find the high wall on the left where the vault would push outward. Where do the flying buttresses touch it?
- Count the flying buttresses. Why might the builders have used two, one above the other?
- Find the tall outer pier and its pinnacle. What does the pinnacle add?
- Look at the small plans to the right. What do they show about the shape of the piers?

Talk about it

1. Trace with your finger the path of the vault's thrust from the wall to the ground.
2. Why do the flying buttresses lean, instead of being horizontal?

3. Part of this choir collapsed in 1284. What does a picture like this tell you about how delicate the system is?

Safety first. Wear safety glasses; block towers and walls can fall. Keep the test on a low table so falling pieces do not drop far.

BUILD

Push Test: Walls, Buttresses and Pinnacles

You will build a model wall of blocks, push sideways on its top with a measured force (as a vault would), and find the force that tips it. Then you will add a buttress, then a pinnacle, and compare.

You will need

- Identical blocks (wooden blocks, KEVA planks, LEGO bricks or sugar cubes)
- String, a paper cup and pennies; a pencil or smooth dowel taped to the table edge as a pulley
- A ruler and a kitchen scale

Steps

1. Build a wall 2 blocks thick, 4 blocks long and 10 blocks high on a flat table. Weigh the blocks you used.
2. Tie a string around the wall near its top, run it horizontally over the pulley at the table edge, and hang the cup.
3. **Predict.** Using $FS = W(b/2)/(Hy)$ with $FS = 1$, estimate the hanging load that will tip the wall. (Measure b, the wall's thickness, and y, the height of the string.)
4. Add pennies slowly until the wall tips or slides. Record the load (convert grams to newtons with 0.0098 N per gram). Repeat twice.
5. Rebuild the wall and add a buttress: a block pier against the outer face (the side away from the pull, toward which it tips) 3 blocks deep and the full height. Test again.
6. Add a pinnacle: extra blocks stacked on top of the buttress. Test again.
7. Try a flying buttress: a separate tall pier a short distance away, connected to the wall top by a leaning strut (a ruler or stick). Test it.
8. Make a table comparing the tipping loads and calculate how many times stronger each version is than the plain wall.

9. **Improve.** Using the same number of blocks as your best version, design an even better arrangement and test it.

Tips

- Keep the string horizontal so the push is purely sideways.
- Build carefully with tight joints so the wall acts as one piece.
- If the wall slides instead of tipping, put a strip of rubber shelf liner under it.

Check your work

- ☐ I measured the tipping load for the plain, buttressed and pinnacled walls.
- ☐ I can compare my measured result with the prediction from moments.
- ☐ I can explain how Gothic builders used buttresses and pinnacles to open walls for windows.

ENGINEER'S NOTEBOOK

1. A labeled cross section of a Gothic cathedral: nave, aisles, vault, clerestory windows, flying buttresses, piers and pinnacles. Draw arrows for the thrust path.
2. A free-body diagram of a buttress pier with the thrust H, the weight W and the ground reaction.
3. Your push-test table and a bar chart of tipping loads.

KEY FORMULAS

$H = \dfrac{wL^2}{8h}$ (uniformly loaded parabolic arch)

$V = \dfrac{wL}{2}$ (vertical reaction at each support)

$M_{\text{resist}} = W \cdot \dfrac{b}{2}, \quad M_{\text{overturn}} = H \cdot y$

$FS_{\text{overturning}} = \dfrac{M_{\text{resist}}}{M_{\text{overturn}}}$

From history. The thirteenth-century French master mason Villard de Honnecourt left a sketchbook of drawings: plans of cathedral towers, machines such as a water-powered saw, and geometric tricks for laying out arches. It is one of the very few records of how medieval builders thought. Much of their knowledge was passed on orally in the masons' lodges and kept as trade secrets.

Example 1. A vault carries 40 kN per meter of span over a span of 12 m. Compare the thrust for a round arch (rise 6 m) and a pointed arch (rise 10 m).

Show solution

1. Round: $H = wL^2/(8h) = (40)(12)^2 \div (8 \times 6) = 5760 \div 48 = 120$ kN.
2. Pointed: $H = 5760 \div (8 \times 10) = 72$ kN.
3. The vertical reaction is the same in both: $V = wL/2 = 240$ kN at each side.
4. The pointed arch pushes out 40% less, so its buttresses can be lighter.

Answer: 120 kN for the round arch and 72 kN for the pointed arch.

Example 2. A buttress pier is 2.0 m wide at the base, 15 m tall and weighs 900 kN. The vault's thrust of 72 kN acts on it 12 m above the base. What is the factor of safety against overturning about the outer edge?

Show solution

1. Resisting moment about the outer edge:
   $M_r = W(b/2) = 900 \times 1.0 = 900$ kN·m.
2. Overturning moment: $M_o = Hy = 72 \times 12 = 864$ kN·m.
3. $FS = 900 \div 864 \approx 1.04$. That is barely above 1: the pier would be on the edge of tipping, and its edge would crush long before.

Answer: FS ≈ 1.04, far too low; the buttress must be wider, heavier or both.

Example 3. A pinnacle weighing 300 kN is added on top of the buttress in Example 2, centered on it, and the base is widened to 2.6 m (the pier's weight becomes 1,170 kN). Find the new factor of safety.

Show solution

1. Total weight: $1170 + 300 = 1470$ kN, acting at the center, 1.3 m from the outer edge.
2. $M_r = 1470 \times 1.3 = 1911$ kN·m.
3. $M_o = 864$ kN·m (unchanged).
4. $FS = 1911 \div 864 \approx 2.2$.

Answer: FS ≈ 2.2. The pinnacle and the wider base more than double the margin.

PRACTICE

1. Easy Name three structural inventions of Gothic builders.

   Show answer

   The pointed arch, the ribbed vault and the flying buttress.

2. Easy Why did Romanesque churches have thick walls and small windows?

   Show answer

   Their round barrel vaults pushed outward along the whole length of the wall, so the walls had to be thick and continuous to resist the thrust.

3. Easy An arch carries 30 kN/m over 10 m with a rise of 5 m. Find the thrust.

   Show answer

   $H = 30 \times 100 \div 40 = 75$ kN.

4. Medium For the arch in Problem 3, what rise would cut the thrust to 50 kN?

   Show answer

   $h = wL^2/(8H) = 3000 \div 400 = 7.5$ m.

5. Medium Find the vertical reaction at each support of the arch in Problem 3.

   Show answer

   $V = wL/2 = 150$ kN.

6. Medium A pier weighs 600 kN and is 1.8 m wide. A thrust of 50 kN acts 9 m above its base. Find the factor of safety against overturning.

Show answer

$M_r = 600 \times 0.9 = 540$; $M_o = 450$; $FS = 1.2$.

7. Medium In Problem 6, how heavy a pinnacle would raise the factor of safety to 2.0?

Show answer

Need $M_r = 900$ kN·m, so total weight $= 900 \div 0.9 = 1000$ kN: a 400 kN pinnacle.

8. Hard The resultant of a pier's weight W and a thrust H at height y meets the base a distance $e = Hy/W$ from the center. For the pier in Problem 6, find e, and say whether it lies within the middle third of the 1.8 m base.

Show answer

$e = 50 \times 9 \div 600 = 0.75$ m. The middle third extends $1.8/6 = 0.30$ m each side of center, so 0.75 m is far outside it: the inner edge would lift and the outer edge crush.

9. Hard Beauvais's choir was rebuilt with twice as many piers. If each bay's vault thrust was carried by its own pair of buttresses, how did halving the bay width change the thrust each buttress had to carry?

Show answer

Each buttress collects the thrust from a bay half as wide, so it carries about half as much thrust (the thrust per meter of wall is unchanged, but each buttress serves half the length).

10. Hard Conceptual. Why are flying buttresses built as arches leaning down and away from the wall, instead of horizontal beams?

Show answer

Model answer: an arched, sloping strut carries the thrust in compression along its line down toward the pier, where it combines with the pier's weight; a horizontal stone beam would have to resist bending from its own weight and would be weak in tension.

11. **Challenge** Show that for the same load and span, a pointed arch of rise h_2 has thrust $H_2 = H_1(h_1/h_2)$. A round arch of span 14 m has rise 7 m and thrust 140 kN. Find the thrust of a pointed arch of rise 12 m, and the percent reduction.

Show answer

$H \propto 1/h$, so $H_2 = 140 \times 7/12 \approx 81.7$ kN, a reduction of about 42%.

COMMON MISTAKES

- Thinking flying buttresses hold the wall up from falling inward. They resist the vault's outward push.
- Forgetting that the vertical load on an arch's supports does not change with its rise; only the horizontal thrust does.
- Putting all of a buttress's weight at its edge when computing moments. A uniform pier's weight acts at its center.

Make it a game · Taller Means Less Thrust. Draw two arches on paper with the same span: one rounded, one pointed. Predict which pushes outward less at the base, then explain with a sketch of the force triangle. Your parent plays skeptic and asks why. Then name from memory the five parts that carry a Gothic vault. Hit all five for a record.

Scripture. "Honour and majesty are before him: strength and beauty are in his sanctuary." (Psalm 96:6, KJV)

Gothic builders tried to make churches that showed both strength and beauty: a stone framework that stands, and walls of light.

3.3 The Renaissance and Palladio

OBJECTIVES

- Tell how Andrea Palladio studied Roman buildings and published *The Four Books of Architecture*, and how his villas spread across Europe and America.
- Use proportions, ratios and the arithmetic, geometric and harmonic means to design rooms the way Palladio did.
- Design and build a façade model using a consistent proportional system.

> **Warm-up.** Draw five rectangles of the same area but different shapes (very long, square, and in between). Ask three people which one looks "best" for a doorway, a window and a room. Record their answers. Do they agree?

WATCH

Why Palladio's Work Still Resonates

National Building Museum · 2:00

https://www.youtube.com/watch?v=xHNTv8ct5qM

> **WHEN TO WATCH**
> After the story paragraph. Howard Burns, one of the leading Palladio scholars, explains Palladio's systematic approach and why architects still copy him. Then study the plan and elevation in the picture study to see the system for yourself.

KEY TERMS

Renaissance

The revival of classical learning and art in Italy from about 1400, spreading through Europe in the 1500s.

proportion

A relationship of size among parts, such as a room twice as long as it is wide (1:2).

arithmetic mean

$\frac{a+b}{2}$, the halfway value. Palladio used it, with the geometric and harmonic means, to set the heights of rooms.

geometric mean

$\sqrt{ab}$, the value x for which $a : x = x : b$.

harmonic mean

$\frac{2ab}{a+b}$, the mean that relates to musical harmony; it is always the smallest of the three means.

Palladian window

A window with a tall arched center opening flanked by two lower square-headed openings, also called a serliana.

CORE IDEAS

The story. Andrea di Pietro was born in Padua in 1508 and trained as a stonemason. In his twenties, while working in Vicenza, he was noticed by the scholar Gian Giorgio Trissino, who took him to Rome to measure the ancient ruins and gave him a new name, Palladio. Palladio spent years in Rome with a measuring rod and a sketchbook, drawing baths, temples and theaters in plan, elevation and section. Back in the Veneto he designed palaces, churches and, above all, villas: country houses at the center of working farms for Venetian families. In 1570 he published *I quattro libri dell'architettura* (The Four Books of Architecture), illustrated with clear woodcuts of his own buildings and the Roman ones he had measured, with every room's dimensions written in.

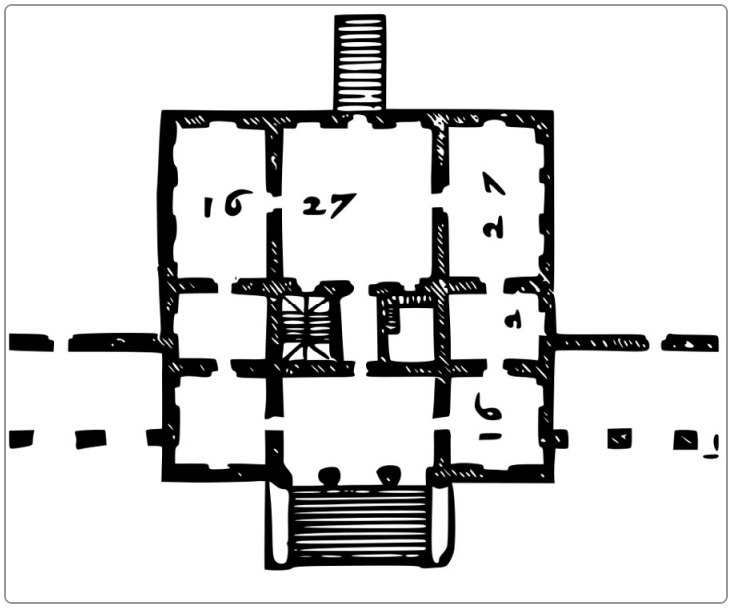
How they figured it out. Renaissance architects believed that beauty comes from order and proportion, as in music, where strings whose lengths are in simple ratios (1:2, 2:3, 3:4) sound in harmony. Palladio chose room shapes from a short list of ratios: the square (1:1), and rectangles of 3:4, 2:3, 3:5, 1:√2 and 1:2. He set the height of a vaulted room from its length and width using one of three means, arithmetic, geometric or harmonic. He then made each room related to its neighbors, so the whole plan read as one composition. His Villa Emo at Fanzolo (about 1559) is a simple central block with long arcaded farm wings (barchesse) for storing grain and tools: beautiful and entirely practical.

The physics. Palladio's proportions are mathematics you can compute. For a room a by b, the arithmetic mean is $\frac{a+b}{2}$, the geometric mean $\sqrt{ab}$, and the harmonic mean $\frac{2ab}{a+b}$. For a room 12 by 18 feet, those are 15, about 14.7 and 14.4 feet. Notice that the harmonic mean is always the smallest and the arithmetic the largest. A system of proportion also helps structurally: when rooms line up and walls stack over walls, loads travel straight down; spans stay within what timber floors and brick vaults can carry; and repeating dimensions simplify construction. In Book III of the *Four Books*, Palladio even published designs for timber truss bridges, among the earliest truss drawings in print.

The engineering. Palladio's book made him the most imitated architect in history. Inigo Jones brought his ideas to England in the early 1600s; in the 1700s Palladian country houses spread across Britain; and in America, Thomas Jefferson called the *Four Books* his bible of architecture and built Monticello and the University of Virginia on Palladian principles. The lesson for designers today is that a clear system, a set of proportions and a module, makes a design coherent, makes it easier to build, and gives it a calm beauty that people recognize even when they cannot say why.

Try it. In the build you will design a small façade and room plan with Palladian proportions, then survey people to see which proportions they prefer.

PICTURE STUDY



Plan of the Villa Emo, from The Four Books of Architecture, Andrea Palladio, 1570 · Woodcut (redrawn) · *I quattro libri dell'architettura*, Venice

Image: Wikimedia Commons, public domain

In his book, Palladio drew each villa in a simple plan with the dimensions of its rooms written in. This is the Villa Emo, a farmhouse for a Venetian family. The dashed lines on each side are the long arcaded farm wings.

Look closely

- Find the numbers written in the rooms. They are dimensions in the feet of Vicenza. Which rooms are squares and which are rectangles?
- Find the central hall. How is it placed in relation to the entrance stair?
- Compare the left and right halves of the plan. Is it symmetrical?
- Find the small square room with a stair drawn inside it. Why might it be placed in the middle?

Talk about it

1. Choose two room dimensions on the plan and find their ratio. Is it one of Palladio's favorite ratios?
2. Why would Palladio write the dimensions on the plan instead of only drawing it to scale?

3. This villa was the center of a working farm. Where would the grain and tools have been kept?

BUILD

A Palladian Façade and a Proportion Survey

You will design the plan and front façade of a small villa or parish house using Palladio's ratios and means, build a card model of the façade, and then test with a survey whether people prefer proportioned rectangles.

You will need

- Graph paper, a ruler, a compass and a calculator
- Card stock, a craft knife (adult) or scissors, glue
- Ten rectangles cut from card, all with the same area (about 60 cm^2) but different ratios: 1:1, 3:4, 2:3, 1:√2, 3:5, 1:2, 1:3 and three "random" ratios
- A few volunteers for the survey

Steps

1. Choose a module, for example 2 ft. Lay out a plan with a central hall and side rooms whose ratios are all from Palladio's list (1:1, 3:4, 2:3, 3:5, 1:√2, 1:2). Write the dimensions in each room.
2. Compute the arithmetic, geometric and harmonic means for each room and choose a height for each. Record which mean you used.
3. Design the front façade: a central portico with an even number of columns (so there is a space, not a column, in the middle), windows whose height-to-width ratio is from the list, and a pediment.
4. Draw the façade at a scale of 1:50 and build it as a card model.
5. **Predict.** Which of the ten survey rectangles do you think people will choose as most pleasing for a window? For a room?
6. Run the survey: show each volunteer the ten rectangles in random order and ask them to pick the most pleasing for a window and for a room. Record their choices.
7. Tally the results. Did the Palladian ratios win more often than the random ones? Calculate the percentage.
8. **Improve.** Change the façade's weakest proportion (the one that looks least settled to you or your volunteers) and compare the two versions side by side.

Tips

- Use the same module for walls, windows and columns so that everything lines up.
- Do not tell volunteers which rectangles are Palladian; let them choose freely.
- Ten volunteers give a better answer than three, but even a small survey is honest data if you record it fairly.

Check your work

- ☐ My plan uses Palladian ratios, and I computed the three means correctly.
- ☐ My façade model is built to scale with a consistent module.
- ☐ I collected and tallied survey data and wrote an honest conclusion.

ENGINEER'S NOTEBOOK

1. Your villa plan with dimensions, ratios and chosen heights.
2. A table of the three means for each room, and which you chose.
3. Your survey tally and a bar chart, with one paragraph on what it shows and what it does not prove.

KEY FORMULAS

$$A = \frac{a+b}{2}, \quad G = \sqrt{ab}, \quad H = \frac{2ab}{a+b}$$

$H \le G \le A$, with $G^2 = A \cdot H$

$$\text{ratio} = \frac{\text{long side}}{\text{short side}}$$

From history. Palladio's Basilica in Vicenza is an old town hall that he wrapped in a new two-story arcade of stone. To make each arch fit bays of slightly different widths, he used the motif that now bears his name: an arch flanked by two narrow square-topped openings, whose side openings could be stretched or shrunk to fit. It is a fine example of an architect solving an engineering problem with a beautiful idea.

Example 1. A vaulted room is 16 ft wide and 27 ft long. Find the arithmetic, geometric and harmonic means as possible heights.

Show solution

1. Arithmetic: $A = (16 + 27)/2 = 21.5$ ft.
2. Geometric: $G = \sqrt{16 \times 27} = \sqrt{432} \approx 20.8$ ft.
3. Harmonic: $H = 2(16)(27)/(16 + 27) = 864/43 \approx 20.1$ ft.
4. Check: $A \cdot H = 21.5 \times 20.09 \approx 432 = G^2$. Note that 16 : 27 is close to 3 : 5 (16 : 26.7).

Answer: 21.5 ft, about 20.8 ft, and about 20.1 ft.

Example 2. A villa's central hall is a square 30 ft on a side. The side rooms are to be 18 ft wide, with lengths in the ratio 3:5 (width:length). How long are they? What ratio is the whole width of hall plus two side rooms to the hall's depth?

Show solution

1. Side room length: $18 \times 5/3 = 30$ ft, the same as the hall depth, so the rooms line up.
2. Total width: $18 + 30 + 18 = 66$ ft; depth 30 ft.
3. Ratio: $66 : 30 = 11 : 5 = 2.2$.

Answer: 30 ft long; the whole block is 66 ft by 30 ft, a ratio of 11:5.

Example 3. Palladio's rooms of ratio $1 : \sqrt{2}$ have a special property: cut one in half across its length and each half has the same shape. Show this for a room 10 ft by 14.14 ft.

Show solution

1. Original ratio: $14.14 \div 10 = 1.414 \approx \sqrt{2}$.
2. Cut across the long side: each half is 10×7.07 ft.
3. New ratio: $10 \div 7.07 = 1.414 \approx \sqrt{2}$. The halves have the same proportion. (Modern A4 and A3 paper uses the same idea.)

Answer: Each half is 7.07 ft × 10 ft, again in the ratio 1 : √2.

PRACTICE

1. Easy When was *The Four Books of Architecture* published, and what was in it?

   Show answer

   1570, in Venice: Palladio's rules for the orders and building, woodcuts of his own buildings with dimensions, and Roman buildings he had measured.

2. Easy Find the arithmetic mean of 12 and 20.

   Show answer

   16.

3. Easy Find the geometric mean of 9 and 25.

   Show answer

   $\sqrt{225} = 15$.

4. Medium Find the harmonic mean of 12 and 24.

   Show answer

   $2(12)(24)/36 = 16$.

5. Medium A room is 18 ft by 24 ft. What ratio is that in lowest terms? Find the three means.

   Show answer

   3:4; arithmetic 21 ft, geometric $\sqrt{432} \approx 20.8$ ft, harmonic $864/42 \approx 20.6$ ft.

6. Medium A window is 3 ft wide. How tall is it if its ratio is 1:2? If it is 1:√2?

   Show answer

   6 ft; $3 \times 1.414 \approx 4.24$ ft.

7. Medium Show that $G^2 = A \cdot H$ for $a = 4$ and $b = 9$.

   Show answer

   $A = 6.5$, $H = 72/13 \approx 5.538$, $A \cdot H = 36$, and $G = \sqrt{36} = 6$, so $G^2 = 36$.

8. Hard A portico has 6 columns 2.0 ft in diameter, with the spaces between them equal to 2¼ diameters (Vitruvius's "eustyle" spacing). How wide is the portico from the outside of the first column to the outside of the last?

   Show answer

   6 columns × 2.0 = 12 ft; 5 spaces × 4.5 ft = 22.5 ft; total 34.5 ft.

9. Hard Why does a portico usually have an even number of columns?

Show answer

So that there is an open space, not a column, on the center line, lining up with the door and the axis of the building.

10. Hard Conceptual. Give one practical (structural or construction) benefit of designing with a module and consistent proportions.

Show answer

Model answer: walls and columns stack and line up, so loads go straight down; repeated dimensions mean repeated parts (beams, windows, joists) that are cheaper and quicker to build; spans stay within what the materials can carry.

11. Challenge Prove that the harmonic mean of two different positive numbers is less than their geometric mean. (Hint: start from $(\sqrt{a}-\sqrt{b})^2 > 0$.)

Show answer

$(\sqrt{a}-\sqrt{b})^2 > 0$ gives $a+b > 2\sqrt{ab}$. Then $H = \dfrac{2ab}{a+b} < \dfrac{2ab}{2\sqrt{ab}} = \sqrt{ab} = G$.

COMMON MISTAKES

- Mixing up the means. Harmonic is smallest, arithmetic largest, geometric in between.
- Choosing pleasing proportions for each part separately without relating them. Palladio related every room to its neighbors.
- Placing a column on the center line of a portico. Use an even number of columns.

> **Make it a game · Proportion Hunt.** Scavenger hunt. Find three rectangles in your home whose proportions are close to 2:3, 3:4 or 1:1, measure them, and label each by ratio. Then compute the arithmetic, geometric and harmonic means of 6 and 12. Your parent picks which ratio they would choose for a dining room. Defend your choice.

3.4 The Industrial Age: Iron, Glass and the Crystal Palace

OBJECTIVES

- Tell how Joseph Paxton designed the Crystal Palace and how it was prefabricated and built in months.
- Calculate quantities, building rates and thermal expansion for a modular iron-and-glass building.
- Build a modular braced frame and test how diagonal bracing makes it stiff.

> **Warm-up.** Look at a greenhouse, a shopping-mall skylight or a picture of a train station roof. How many identical parts can you count? Why might a builder want all the parts to be the same?

WATCH

The Crystal Palace and iron in architecture

Harvard Online · 5:27

https://www.youtube.com/watch?v=lx2KIYFhD4M

> **WHEN TO WATCH**
> After the story. Write down two ways the Crystal Palace was new.

BACKUP VIDEO

The Largest Building In The World (in 1851): The London History Show

J. Draper · 18:54

https://www.youtube.com/watch?v=lunN7Ob_R9k

For more of the story of the Great Exhibition, Paxton and the building.

KEY TERMS

prefabrication

Making building parts in a factory to standard sizes and assembling them on site.

module

A standard dimension that sets the grid of a building; the Crystal Palace's was 24 ft (and 8 ft for the glazing).

cast iron

Iron poured into molds; strong in compression but brittle in tension. Used for the Crystal Palace's columns.

wrought iron

Iron worked by hammering and rolling; tougher and better in tension than cast iron. Used for trusses and tie rods.

bracing

Diagonal members that keep a frame of columns and beams from leaning sideways (racking).

thermal expansion

The growth of a material when it warms: $\Delta L = \alpha L \Delta T$.

CORE IDEAS

The story. In 1850 Britain planned a Great Exhibition of the Works of Industry of All Nations, to open in Hyde Park, London, on 1 May 1851. A design competition drew 245 entries, all rejected, and the committee's own design, a vast brick hall with a dome, would need millions of bricks and could not be finished in time. Joseph Paxton, head gardener to the Duke of Devonshire at Chatsworth, had built huge glass houses, including one for the giant Amazon water lily, whose ribbed leaves inspired his roof. In June 1850, during a railway meeting, he sketched an idea on a sheet of blotting paper: a building made entirely of identical iron and glass parts. Within about two weeks he had drawings, and the committee accepted a bid from the contractors Fox, Henderson and Company. The building, nicknamed the Crystal Palace, was about 1,848 ft (563 m) long and covered roughly 19 acres. The iron-and-glass structure went up in roughly six months, the Exhibition opened on schedule, and six million visitors came.

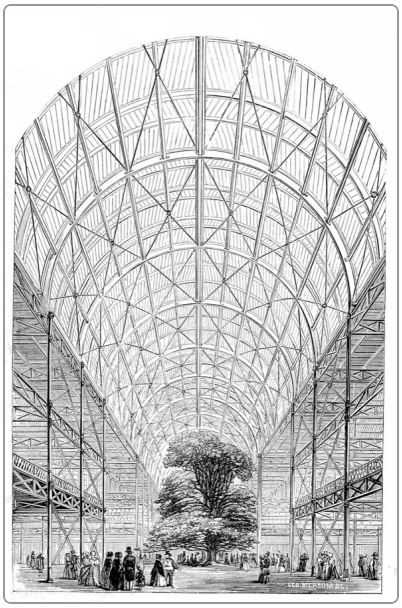
How they figured it out. Speed came from the system. Everything was laid out on a grid with a 24 ft module. There were a few standard parts: hollow cast-iron columns (which also carried rainwater down to drains), cast-iron and wrought-iron girders, wooden gutters and sash bars, and panes of glass 49 in × 10 in made by Chance Brothers, about 293,000 of them. Parts were made in factories, brought by rail and bolted together. Glaziers worked from small wheeled trolleys that ran in the roof gutters, and contemporary accounts say a team of about 80 glaziers once set more than 18,000 panes in a single week. To answer fears that the galleries would collapse under crowds, the builders tested a section of gallery floor with men jumping on it and soldiers marching over it in step.

The physics. A frame of columns and beams joined by bolts can carry vertical loads well, but sideways forces such as wind tend to make it lean, like a bookshelf pushed at the top. Diagonal **bracing** stops this, because a triangle cannot change shape without changing the length of a side; the Crystal Palace had diagonal wrought-iron tie rods in many bays. Iron and glass also expand when they warm, by $\Delta L = \alpha L \Delta T$. For iron, $\alpha \approx 12 \times 10^{-6}$ per °C, so a 563 m line of iron girders warming by 25 °C grows by about 17 cm. A modular building with many joints and some give in its wooden gutters can absorb that; a rigid one would crack its glass.

The engineering. The Crystal Palace was the first great prefabricated building and showed what the Industrial Age could do: standard parts, mass production, railways for delivery, and a design made for fast assembly. It led to the iron-and-glass railway stations, market halls and exhibition buildings of the later 1800s, and its ideas lead on to the steel frame of the skyscraper (Unit 7) and to modern modular construction. It also showed a weakness: iron loses strength in fire. After the Exhibition the palace was taken down and rebuilt, larger, at Sydenham in south London, where it burned down in 1936.

Try it. In the build you will design a module, prefabricate the parts for several bays of a frame, assemble it against the clock, and test its stiffness with and without diagonal bracing.

PICTURE STUDY



The Transept of the Crystal Palace, Engraver for *The Crystal Palace and Its Contents*, 1851 · Wood engraving · *The Crystal Palace and Its Contents*, London

Image: Wikimedia Commons, public domain

The transept was the tallest part of the Crystal Palace, a glass barrel vault about 108 ft high, built so that the old elm trees of Hyde Park could stay where they were. Look at it as a builder: count the parts.

Look closely

- Find the curved ribs of the vault. Are they all the same? How far apart are they?
- Find the columns and the galleries. How do the galleries rest on the columns?
- Look for the diagonal lines in the side bays. What are they for?
- Find the tree. How does the building make room for it?

Talk about it

1. How many times can you see the same part repeated in this picture?
2. Why would a building made of identical parts be faster to build?
3. Iron loses strength when it gets very hot. What does this suggest about the dangers of a building like this?

> **Safety first.** If you use a hot-glue gun, an adult should handle it or supervise; use low-temperature glue. Wear safety glasses for the stiffness test.

BUILD

A Prefabricated, Braced Frame

You will design a module, prefabricate identical parts for a three-bay frame, assemble it and time yourself, then test how far it leans under a sideways push with and without diagonal bracing.

You will need

- Drinking straws or wooden coffee stirrers, pipe cleaners or paper clips for joints (or craft sticks and glue)
- Thread or thin string for diagonal ties; clear plastic wrap for “glass”
- A ruler, a stopwatch, a paper cup, string, pennies and a pencil taped to the table edge as a pulley
- Graph paper

Steps

1. Choose a module (for example 10 cm). Draw a three-bay frame in elevation and plan: columns at each module, beams joining the tops, and a simple roof.
2. List every part and how many of each you need. Calculate the totals before you cut anything.
3. **Prefabricate.** Cut and prepare all the parts first, sorting them into piles. Time this stage.
4. **Assemble.** Build the frame and time this stage separately. Record any part that did not fit and why.
5. Fix the base to a sheet of cardboard. Tie a string to the top corner and run it horizontally over the pulley to the cup.
6. Add pennies in steps of 5 and measure the sideways lean of the top with a ruler after each step, up to a lean of 2 cm or 50 pennies.
7. Add diagonal thread ties across the end bays (an X in each), pulled snug. Repeat the test.
8. Graph lean against load for both cases. How many times stiffer is the braced frame?

9. **Improve.** Use the same amount of thread to brace in a different pattern (one diagonal per bay, or only the middle bay). Test and compare. Finally, cover the roof with plastic wrap "glass."

Tips

- Use the same joint method everywhere, as a factory would.
- Thread ties work only in tension; that is why an X is used, so one tie is always pulled whichever way the wind blows.
- Keep the string horizontal and measure the lean at the same point each time.

Check your work

- ☐ I prefabricated identical parts and timed fabrication and assembly separately.
- ☐ My lean-versus-load graph shows the effect of bracing.
- ☐ I can explain how Paxton's system made a vast building possible in months.

ENGINEER'S NOTEBOOK

1. Your parts list with quantities per module and in total, and your fabrication and assembly times.
2. The lean-versus-load graph for braced and unbraced frames, with the stiffness ratio.
3. A sketch of Paxton's ridge-and-furrow roof and his hollow columns used as drainpipes, with labels.

KEY FORMULAS

$$\text{quantity} = (\text{number of modules}) \times (\text{parts per module})$$
$$\Delta L = \alpha L \Delta T, \quad \alpha_{\text{iron}} \approx 12 \times 10^{-6} \,/\,^{\circ}\text{C}$$
$$\text{rate} = \frac{\text{quantity}}{\text{time}}$$

From history. Paxton was knighted for the Crystal Palace. In 1849 he was the first to make the giant Amazon water lily, Victoria regia, flower in England, and he built a special glass house for it at Chatsworth. He liked to show how strong its huge floating leaves were by standing his young daughter Annie on one.

Example 1. Each pane of the Crystal Palace was 49 in × 10 in, and there were about 293,000 panes. Estimate the total area of glass in square feet and square meters.

Show solution

1. Area of one pane: $49 \times 10 = 490 \text{ in}^2 = 490 \div 144 \approx 3.40 \text{ ft}^2$.
2. Total: $293{,}000 \times 3.40 \approx 997{,}000 \text{ ft}^2$, about a million square feet.
3. In metric: $997{,}000 \times 0.0929 \approx 92{,}600 \text{ m}^2$, about 9 hectares of glass.

Answer: About 1,000,000 ft^2, or about 93,000 m^2.

Example 2. If 80 glaziers set 18,000 panes in a 6-day week of 10-hour days, how many panes did each glazier set per hour? At that rate, how many weeks would 293,000 panes take?

Show solution

1. Glazier-hours: $80 \times 6 \times 10 = 4800$.
2. Rate: $18{,}000 \div 4800 = 3.75$ panes per glazier-hour, about one every 16 minutes.
3. Weeks: $293{,}000 \div 18{,}000 \approx 16.3$ weeks.

Answer: About 3.75 panes per glazier per hour; about 16 weeks for all the glass at that pace.

Example 3. A line of iron girders 563 m long warms from 5 °C on a winter morning to 30 °C in summer sun. How much longer does it get?

Show solution

1. $\Delta T = 30 - 5 = 25\ ^\circ\text{C}$.
2. $\Delta L = \alpha L \Delta T = (12 \times 10^{-6})(563 \text{ m})(25) \approx 0.169 \text{ m}$.
3. About 17 cm (6.6 in) over the whole building, which the many joints must take up, a few millimeters at each one.

Answer: About 17 cm.

PRACTICE

1. Easy Who designed the Crystal Palace, and what was his first career?

Show answer

Joseph Paxton, a gardener (head gardener at Chatsworth) who designed large glass houses.

2. Easy What is the difference between cast iron and wrought iron in how they are used?

Show answer

Cast iron is strong in compression but brittle in tension, so it was used for columns; wrought iron is tougher and good in tension, so it was used for ties and girders.

3. Easy How many 24 ft modules fit in 1,848 ft?

Show answer

77.

4. Medium If each 24 ft × 24 ft bay of roof needed 168 panes, how many panes did 1,700 roof bays need?

Show answer

$168 \times 1700 = 285{,}600$ panes.

5. Medium A steel beam 40 m long heats up by 35 °C. How much does it grow? ($\alpha = 12 \times 10^{-6}/^{\circ}\mathrm{C}$)

Show answer

$12 \times 10^{-6} \times 40 \times 35 = 0.0168\ \mathrm{m} \approx 17\ \mathrm{mm}$.

6. Medium The Crystal Palace covered about 19 acres. Convert to square feet and square meters (1 acre = 43,560 ft^2).

Show answer

$19 \times 43{,}560 \approx 828{,}000\ \mathrm{ft}^2 \approx 76{,}900\ \mathrm{m}^2$.

7. Medium A factory can cast 40 columns a day. How many working days for 3,300 columns?

Show answer

$3300 \div 40 = 82.5$, so 83 days.

8. Hard Glass expands with $\alpha \approx 9 \times 10^{-6}/^{\circ}\mathrm{C}$ and iron with $12 \times 10^{-6}/^{\circ}\mathrm{C}$. Over a 2.0 m length warmed 40 °C, how much more does the iron grow than the glass? Why does this matter for how glass is held?

Show answer

Iron: 0.96 mm; glass: 0.72 mm; difference about 0.24 mm. If glass were clamped tightly to iron, the difference would stress and crack it, so glass must sit in a flexible bed (putty or gaskets) with a little room to move.

9. Hard A frame without bracing leaned 18 mm under a load that made the braced frame lean 1.5 mm. How many times stiffer was the braced frame? Explain why.

Show answer

$18 \div 1.5 = 12$ times stiffer. The diagonal ties form triangles, which cannot change shape unless a side stretches, so the frame cannot rack.

10. Hard Conceptual. Name two lessons from the Crystal Palace that still guide construction today, and one weakness it showed.

Show answer

Model answer: prefabrication of standard parts and design on a modular grid speed construction and lower cost; testing structures before use builds confidence. Weakness: unprotected iron loses strength in fire (the rebuilt palace burned in 1936).

11. Challenge A row of 76 identical bays shares one line of girders. If the girders grow 17 cm in total on a hot day and every joint between bays takes an equal share, how much must each joint move? If the gutters are timber that swells and shrinks too, why might that help?

Show answer

There are 76 bays and 77 lines of columns, so 76 joints along the run: $170 \div 76 \approx 2.2$ mm per joint. Flexible timber parts and slightly loose bolted joints can take up a few millimeters each, so no single part is overstressed.

COMMON MISTAKES

- Thinking prefabrication means the parts are simple. The design of the system, the joints and the tolerances is what makes it work.

- Forgetting that a frame needs bracing for sideways loads even if it carries vertical loads easily.
- Ignoring thermal movement in long buildings. Large structures need joints that let them grow and shrink.

> **Make it a game · Frame It Fast.** Build a frame from straws, craft sticks or rolled paper, with and without a diagonal brace. Predict which resists a sideways push, then test. Say whether Paxton's modules were cast or wrought iron, and why. Calculate how many modules fit a hall of length 1,848 feet at 24 feet. Point for each correct step.

3.5 Modern Architecture: The Frame, the Skyscraper and Fallingwater

OBJECTIVES

- Explain how steel and reinforced concrete frames freed walls and plans, from Sullivan's skyscrapers to Le Corbusier's Dom-ino and Wright's Fallingwater.
- Calculate the moment and deflection of a cantilever and show how strongly deflection depends on length.
- Measure cantilever deflection and creep in a model and relate them to the Fallingwater repairs.

> **Warm-up.** Hold a ruler flat on the edge of a table with 10 cm sticking out, and press a coin on the tip. Measure how far it bends. Now let 20 cm stick out and repeat. Did doubling the length double the bend?

WATCH

Why Fallingwater Is Falling Apart (Architect Explains)

Dan Fouché · 19:15

https://www.youtube.com/watch?v=xbZuU4TX2GM

> **WHEN TO WATCH**
> After the core ideas. Note what he says happened to the cantilevers, and how they were repaired.

Note: An architect walks through Fallingwater with lively collage-style graphics. A small channel, but the account of the cantilevers and repairs is detailed and clear.

KEY TERMS

frame structure

A structure of columns and beams (steel or reinforced concrete) that carries all the loads, so walls need not.

free plan

A plan in which interior walls can go anywhere, because columns, not walls, carry the floors.

cantilever

A beam or slab supported at one end only, reaching out into space.

deflection

How far a structural member bends under load.

creep

The slow, continuing deformation of a material under a constant load over months and years; concrete creeps significantly.

post-tensioning

Stretching high-strength steel cables inside or along concrete after it hardens, then anchoring them, to squeeze the concrete and lift or stiffen it.

CORE IDEAS

The story. In 1896 the Chicago architect Louis Sullivan wrote that in a tall office building "form ever follows function," and his steel-framed skyscrapers showed it. In 1914–15 the Swiss-French architect Le Corbusier drew the Dom-ino house: concrete floor slabs on slender columns, with a stair, and nothing else. Walls could go anywhere. Then in 1935 the Pittsburgh merchant Edgar Kaufmann asked Frank Lloyd Wright for a weekend house near a waterfall on Bear Run in western Pennsylvania. Wright put the house over the waterfall. Built in 1936–38, Fallingwater's reinforced concrete terraces reach out from a stone core over the stream like trays balanced on a hand.

How they figured it out. Kaufmann's engineers checked Wright's calculations and were worried; the contractor quietly added more steel reinforcing than Wright had specified. When the wooden formwork under the main living-room terrace was removed, the cantilever sagged and cracked, and it kept sagging for decades as the concrete crept. By the 1990s the tip of the main cantilever had dropped by about 7 in (18 cm). Engineers measured it, built models and found that the concrete beams were not strong enough for the bending they carried. In 2001–2002, a firm of structural engineers threaded high-strength steel cables through the concrete beams beneath the floors, tensioned them and anchored them, a method called **post-tensioning**. The sagging stopped. The house is now owned by the Western Pennsylvania Conservancy and visited by many thousands of people each year.

The physics. A cantilever carries all its load back to its one support, so the bending there is large. For a cantilever of length L carrying a uniform load w per meter, the bending moment at the support is $M = \frac{wL^2}{2}$, and the tip deflection is $\delta = \frac{wL^4}{8EI}$, where E is the material's stiffness and I (the second moment of area) measures the shape's depth. For a point load P at the tip, $\delta = \frac{PL^3}{3EI}$. Deflection grows with the fourth power of the length under uniform load: double the reach and the sag grows

16 times. Concrete also **creeps**. Under a steady load its long-term deflection can grow to two or three times the deflection on the first day.

The engineering. Modern frames made the open plan, the glass wall and the cantilever possible, and the best modern architecture uses them honestly and beautifully. But a frame or cantilever must be designed for strength and for stiffness, now and over its whole life, including creep. Fallingwater is an example of a design of great delight whose firmness was underestimated; its rescue is an example of careful investigation and modern engineering serving a beloved building. Firmness, commodity and delight belong together.

Today. In Unit 5 you will learn to compute I and the stresses in beams, and in Unit 7 you will see how steel frames grew into skyscrapers.

PICTURE STUDY

Fallingwater: The Cantilevered Terraces, Historic American Buildings Survey, 1985 · Photograph · Library of Congress, Prints and Photographs Division

Image: Wikimedia Commons, public domain

HABS photographers documented Fallingwater in 1985, before the cantilevers were repaired. The terraces seem to float over the stream; look for what actually holds them up.

Look closely

- Find the stone wall. Where do the terraces begin, and how far do they reach beyond it?
- Look under the terraces. Can you see any posts or beams supporting their outer edges?
- Find the waterfall. How does the house relate to it?
- Look at the thick parapets around the terraces. Could they be part of the structure?

Talk about it

1. Why is a cantilever more likely to sag than a beam supported at both ends?
2. The parapets act like deep beams. Why would depth make a beam stiffer?
3. How does this house score on firmness, commodity and delight?

BUILD

Cantilevers: Length, Depth and Creep

You will measure how the tip deflection of a cantilever depends on its length and depth, compare with $\delta \propto L^3$ for a point load, and watch a plastic cantilever creep over a day.

You will need

- A plastic 30 cm ruler, a strip of foam board and a strip of thin cardboard
- A heavy book or clamp to hold the strip on the table edge
- Coins or a small bag of rice of known mass; a kitchen scale
- A second ruler held vertically to measure deflection; a phone camera helps

Steps

1. Clamp the plastic ruler flat on the table edge under a heavy book with 10 cm projecting. Measure the height of the tip above the floor with no load.
2. Hang a fixed load (for example 50 g) at the tip and measure the new tip height. The difference is the deflection. Repeat for projecting lengths of 15, 20 and 25 cm.
3. **Predict.** If 10 cm gives a deflection δ_1, what do you expect at 20 cm with $\delta \propto L^3$?
4. Make a table and graph δ against L^3. Is it close to a straight line?
5. **Depth test.** Turn the ruler on edge (so it is deep instead of flat) at 20 cm and load it the same way. Compare the deflection with the flat ruler.
6. Test foam board and cardboard strips the same way. Which is stiffest for its weight?
7. **Creep test.** Load a plastic ruler at 20 cm with a heavier load that bends it noticeably. Measure the deflection at 1 minute, 10 minutes, 1 hour and the next day. Graph deflection against time.
8. **Improve.** Design a “terrace” cantilever of foam board that reaches 25 cm and deflects less than 5 mm under 100 g, using a parapet (a deep edge strip) or a second layer. Test it.

Tips

- Clamp the strip firmly; a loose clamp adds deflection.
- Measure from the same point at the tip each time.
- Leave the creep test where no one will bump it.

Check your work

☐ My data show how deflection grows with length, and I compared it with L^3.

☐ I measured creep over time and can explain what it means for concrete.

☐ I can explain why Fallingwater's cantilevers sagged and how post-tensioning repaired them.

ENGINEER'S NOTEBOOK

1. A labeled sketch of a cantilever with its load, support, moment at the support and deflected shape.
2. Your length-test table and graph of δ against L^3.
3. Your creep graph and two sentences on what it shows.

KEY FORMULAS

$M_{\text{max}} = \dfrac{wL^2}{2}$ (cantilever, uniform load)

$\delta_{\text{tip}} = \dfrac{wL^4}{8EI}$ (uniform load), $\delta_{\text{tip}} = \dfrac{PL^3}{3EI}$ (point load at tip)

$\delta_{\text{long-term}} \approx (1 + C_c)\,\delta_{\text{initial}}$ (C_c: creep factor, often 1–2 for concrete)

From history. When Wright first showed Kaufmann the drawings, Kaufmann was surprised: he had expected a house with a view of the waterfall, not a house on top of it. Wright reportedly told him he wanted him to live with the waterfall, not just to look at it. Kaufmann's son later gave the house to the Western Pennsylvania Conservancy in 1963.

Example 1. A concrete terrace cantilevers 4.0 m and carries 12 kN per meter of its length (its weight plus people). Find the bending moment at the support. If the reach were 5.0 m, what would the moment be?

Show solution

1. $M = wL^2/2 = 12 \times 4.0^2 \div 2 = 96$ kN·m.
2. At 5.0 m: $M = 12 \times 25 \div 2 = 150$ kN·m.
3. A 25% longer reach increases the moment by $150/96 \approx 1.56$, or 56%.

Answer: 96 kN·m at 4.0 m; 150 kN·m at 5.0 m.

Example 2. A cantilever deflects 12 mm under a uniform load. How much would it deflect if it were 1.5 times as long, with the same section and load per meter?

Show solution

1. $\delta \propto L^4$, so $\delta_2 = \delta_1(1.5)^4$.
2. $(1.5)^4 = 5.0625$.
3. $\delta_2 = 12 \times 5.06 \approx 61$ mm.

Answer: About 61 mm, five times as much.

Example 3. A concrete cantilever's tip sags 1.8 in when the formwork is removed. With a creep factor of 2.0, what long-term deflection should the engineer expect? Compare with Fallingwater's roughly 7 in.

Show solution

1. Long-term: $\delta \approx (1 + C_c)\delta_0 = (1 + 2.0)(1.8) = 5.4$ in.
2. Fallingwater's main cantilever reached about 7 in, which suggests its initial sag, its creep or both were larger, consistent with the cracking found in its beams.

Answer: About 5.4 in.

PRACTICE

1. Easy What does a frame structure allow an architect to do with walls?

   Show answer

   Walls no longer carry the floors, so they can be placed freely, opened up or made of glass (curtain walls).

2. Easy Who wrote that "form ever follows function," and about what kind of building?

   Show answer

   Louis Sullivan, in 1896, about the tall office building.

3. Easy A cantilever 3.0 m long carries 10 kN/m. Find the moment at the support.

   Show answer

   $10 \times 9 \div 2 = 45$ kN·m.

4. Medium A cantilever under a tip load deflects 4 mm. If its length doubles, what is the new deflection?

   Show answer

   $\delta \propto L^3$, so $4 \times 8 = 32$ mm.

5. Medium A uniformly loaded cantilever deflects 10 mm. If its length is reduced to 80%, what is the deflection?

   Show answer

   $10 \times 0.8^4 \approx 4.1$ mm.

6. Medium A 2.5 m cantilever carries a tip load of 8 kN plus its own weight of 3 kN/m. Find the moment at the support.

   Show answer

   $8 \times 2.5 + 3 \times 2.5^2/2 = 20 + 9.375 = 29.4$ kN·m.

7. Medium A beam's stiffness for bending depends on I, which for a rectangle is $bh^3/12$. If a ruler is 30 mm wide and 3 mm thick, by what factor is it stiffer turned on edge?

   Show answer

   Flat: $I = 30 \times 27/12 = 67.5$ mm^4; on edge: $3 \times 27{,}000/12 = 6750$ mm^4; 100 times stiffer.

8. Hard A concrete cantilever deflects 25 mm when first loaded. The engineer allows a long-term deflection of no more than $L/180$. For $L = 5.4$ m and a creep factor of 2.0, does it pass?

Show answer

Limit $= 5400/180 = 30$ mm; long-term $= 3 \times 25 = 75$ mm. It fails badly.

9. Hard Explain in your own words how post-tensioning stops a concrete cantilever from sagging further.

Show answer

Model answer: steel cables are stretched along the beams and anchored; their pull squeezes the concrete and, because they are placed to curve, pushes upward against the sagging, closing cracks and reducing the tension and bending the concrete must resist, so creep and further sag stop.

10. Hard Conceptual. Why must a structural engineer check stiffness (deflection) as well as strength?

Show answer

Model answer: a member can be strong enough not to break yet bend so much that floors slope, glass cracks, doors jam, people feel it bounce, or (in concrete) it creeps and cracks over time; serviceability is part of firmness.

11. Challenge Two cantilevers carry the same uniform load. A is 4 m long and 400 mm deep; B is 6 m long. Using $\delta \propto L^4/h^3$ (same width and material), how deep must B be to deflect no more than A?

Show answer

L^4/h^3 equal: $h_B^3 = h_A^3(6/4)^4 = 400^3 \times 5.0625$, so
$h_B = 400 \times 5.0625^{1/3} \approx 400 \times 1.717 \approx 687$ mm.

COMMON MISTAKES

- Assuming deflection grows in proportion to length. For a cantilever it grows with the cube (point load) or fourth power (uniform load).
- Forgetting creep. Concrete keeps deflecting for years under sustained load.
- Thinking a member that has not broken is safe. Excessive deflection is a structural problem too.

> **Make it a game · Cantilever Length Duel.** Slide a ruler off a table edge, then load the end with coins or a small eraser. Guess in advance how far it can overhang before it tips or bends, and write a number. Then double the overhang and predict the new deflection: it grows far more than double. Closest predictions earn pretend points.

UNIT 3

The History of Architecture

Every great style of architecture grew from a structural idea and a new material. In this unit you will walk through five of them. The Greeks perfected post and lintel in stone; the Romans invented concrete and spanned the Pantheon's dome. Gothic builders raised stone frameworks with pointed arches and flying buttresses so that churches could be filled with light. Palladio studied the Roman ruins and taught the world proportion. Paxton built the Crystal Palace from factory-made iron and glass in a matter of months. And modern architects used steel and reinforced concrete frames to free walls and reach out in cantilevers, sometimes, as at Fallingwater, farther than the concrete could bear without help.

In each lesson you will do what the builders did: test a structural idea with a model, measure what happens and improve it. Chalk lintels will snap and block arches will stand; walls will tip unless buttressed; frames will lean unless braced; and cantilevers will sag and creep.

PREREQUISITES

- Units 1–2: weight, density, stress, factor of safety, scale drawings
- Algebra: ratios, square roots and proportional reasoning ($y \propto x^2$, $y \propto 1/x$)
- Geometry: volume of spheres and shells; circles

MASTERY CHECKLIST

- ☐ I can explain why stone lintels need short spans and how arches, vaults, domes and concrete let the Romans span much more.
- ☐ I can calculate the thrust of an arch with

$H = wL^2/(8h)$

and check a buttress against overturning.

- ☐ I can use Palladio's ratios and the arithmetic, geometric and harmonic means in a design.
- ☐ I can calculate quantities, rates and thermal expansion for a prefabricated iron-and-glass building and explain why frames need bracing.
- ☐ I can calculate the moment and deflection of a cantilever and explain creep and post-tensioning using Fallingwater.

OPTIONAL KITS FOR THIS UNIT

- **KEVA planks**: Identical maple planks, excellent for the wall-and-buttress push test (Lesson 3.2), arches and Palladian façades.
- **LEGO Architecture**: Ages 12 and up for most sets. Several sets model famous buildings and show their massing and proportions; useful for comparing styles. Never required.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

3.1 Greek Orders and Roman Engineering

3 sessions of 50 minutes

3.2	**Romanesque and Gothic Cathedrals**	3 sessions of 50 minutes
3.3	**The Renaissance and Palladio**	3 sessions of 50 minutes
3.4	**The Industrial Age: Iron, Glass and the Crystal Palace**	3 sessions of 50 minutes
3.5	**Modern Architecture: The Frame, the Skyscraper and Fallingwater** 3 sessions of 50 minutes	

Unit 3 test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation. Answer short-answer questions in complete sentences. The test is worth 100 points.

1. **5 pts**
   Identify the Doric, Ionic and Corinthian orders by their capitals, and say which is stoutest.

2. **6 pts**
   A stone lintel carries 90 kN at midspan with a 2.4 m span. Estimate its capacity at 3.6 m (ignore self-weight). Why is the real capacity even lower?

3. **7 pts**
   Describe three things the Romans did to make the Pantheon's dome stand.

4. **6 pts**
   A dome has an inner radius of 15 m and an average thickness of 2.0 m. Estimate its volume as a hemispherical shell and its weight at 2,000 kg/m^3.

5. **7 pts**
   An arch carries 25 kN/m over 16 m. Find the horizontal thrust for a rise of 8 m and for a rise of 12 m.

6. 7 pts

A buttress pier 2.4 m wide weighs 1,000 kN. A thrust of 80 kN acts 10 m above its base. Find the factor of safety against overturning.

7. 6 pts

Explain why Gothic cathedrals could have huge windows while Romanesque churches could not.

8. 6 pts

Find the arithmetic, geometric and harmonic means of 20 ft and 30 ft.

9. 5 pts

Why did Palladio's *Four Books* have such a great influence?

10. 6 pts

The Crystal Palace used about 293,000 panes of 49 in × 10 in. Estimate the total glass area in ft^2.

11. 6 pts

An iron girder line 300 m long warms by 30 °C. How much does it grow? ($\alpha = 12 \times 10^{-6}/^{\circ}\mathrm{C}$)

12. 6 pts

Why does a frame of columns and beams need diagonal bracing?

13. 6 pts

A cantilever 3.5 m long carries 14 kN/m. Find the moment at the support.

14. 7 pts

A uniformly loaded cantilever deflects 8 mm. Find its deflection if its length increases by 30%. With a creep factor of 2, what would the long-term deflection of the longer cantilever be?

15. 6 pts

What is post-tensioning, and why was it used at Fallingwater?

16. **8 pts**

Short essay. Choose two of the styles in this unit (Greek, Roman, Gothic, Renaissance, Industrial, Modern) and explain how each one's look came from its structural system and materials.

ANSWER KEY

1. Doric: plain cushion capital (stoutest); Ionic: scroll volutes; Corinthian: acanthus leaves (most slender).
2. $90 \times 2.4/3.6 = 60$ kN. The lintel's own weight adds bending that grows with the square of the span.
3. Any three: graded the concrete from heavy aggregate low down to light pumice near the top; made the dome thick at the base and thin at the top; added stepped rings outside the base; used coffers to remove weight; built relieving arches in the drum; left an oculus to remove the heaviest top part.
4. $\frac{2}{3}\pi(17^3 - 15^3) = \frac{2}{3}\pi(4913 - 3375) \approx 3221$ m^3; mass $\approx 6.44 \times 10^6$ kg, weight $\approx 63{,}200$ kN.
5. $wL^2/8 = 800$; $H = 800/8 = 100$ kN and $800/12 \approx 66.7$ kN.
6. $M_r = 1000 \times 1.2 = 1200$; $M_o = 800$; $FS = 1.5$.
7. Gothic ribbed vaults gathered the roof's weight onto a few piers, pointed arches reduced the thrust, and flying buttresses carried the remaining thrust outside, so the walls between piers carried little load and could be glass. Romanesque barrel vaults pushed along the whole wall, which had to be thick and solid.
8. $A = 25$, $G = \sqrt{600} \approx 24.5$, $H = 1200/50 = 24$ ft.
9. It gave clear rules, proportions and woodcuts of buildings with dimensions, so builders anywhere, such as Inigo Jones in England and Thomas Jefferson in Virginia, could study and imitate his designs.
10. $293{,}000 \times 490/144 \approx 997{,}000$ ft^2.
11. $12 \times 10^{-6} \times 300 \times 30 = 0.108$ m ≈ 11 cm.
12. Sideways loads such as wind make a rectangular frame rack (lean); diagonals form triangles, which cannot change shape without stretching a member, so the frame stays square.
13. $14 \times 3.5^2/2 = 85.75$ kN·m.
14. $8 \times 1.3^4 \approx 8 \times 2.856 \approx 22.8$ mm; long-term $\approx 3 \times 22.8 \approx 68.5$ mm.
15. Stretching high-strength steel cables along or through concrete members and anchoring them, to squeeze the concrete and counter bending. At Fallingwater it was used in 2001–2002 to stop the main cantilevers, which had sagged about 7 in through cracking and creep, from sagging further.
16. Model answer: e.g., Greek temples have closely spaced columns and deep lintels because stone is weak in tension and lintels can span only a short distance;

Gothic cathedrals have tall pointed arches, ribbed vaults, flying buttresses and walls of glass because the structure was gathered into a stone framework that carried the vault's thrust outside; modern buildings have open plans, glass walls and cantilevers because steel and reinforced concrete frames carry all the load.

PORTFOLIO PIECE: A STYLE AND STRUCTURE STUDY

Choose one historic building from this unit or near your home that belongs to a clear style. Draw its section to scale, showing how its structure carries the loads to the ground, and build a model of its key structural idea (an arch, a buttressed wall, a braced frame, a cantilever, a coffered dome).

Test the model with a measured load and compare it with a simple calculation. Write a two-page report explaining how the building's structural system created its style, with your drawing, test data and photographs.

LOOK FOR	WHAT IT MEANS
Historical understanding	Accurately describes the building, its date, builders and style, and the materials and methods available to them.
Structural explanation	Clearly explains the load path and the structural idea, with a labeled section drawing.
Model and test	The model represents the key idea; the test is fair, measured and recorded.
Calculation	A relevant calculation (thrust, overturning, deflection, quantity) is done correctly with units and compared with the test.
Writing	Organized, clear, in complete sentences, with figures labeled.

UNIT 4

Structures I: Forces and Loads

Firmness begins with forces. In this unit you will learn the core of structural engineering: what loads a building must carry, how to keep it in equilibrium, how forces divide among the members of a truss, why arches take the shapes they do, and how every load must find a path to the ground.

The stories are of engineers learning by thought and by failure: Stevin's chain of balls, the Forth Bridge's human model, Whipple's truss book, Hooke's anagram, Poleni's chain and Gaudí's upside-down models, the snow-loaded roofs at Hartford and the Knickerbocker, and the gas explosion at Ronan Point. You will solve real statics problems, analyze trusses by the method of joints, measure reactions on kitchen scales, and build models that stand, sag, carry and collapse in ways you can predict.

PREREQUISITES

- Algebra: solving linear equations and systems of two equations
- Right-triangle trigonometry: sine, cosine and tangent
- Units 1–3: weight, pressure, stress, arch thrust ($H = wL^2/8h$)
- A scientific calculator with e^x (or cosh)

MASTERY CHECKLIST

- ☐ I can identify and estimate dead, live, snow, wind and seismic loads and combine them with load factors.
- ☐ I can draw free-body diagrams and solve for reactions using

$\sum F$ and

0

$\sum M = 0$.

- ☐ I can find every member force of a simple truss by the method of joints and say which are in tension and which in compression.
- ☐ I can explain Hooke's hanging-chain principle and calculate with the catenary and parabola.
- ☐ I can trace load paths, compute tributary areas and column loads, and explain progressive collapse.

OPTIONAL KITS FOR THIS UNIT

- **KEVA planks**: Identical maple planks for arch-building and stacking tests in Lessons 4.4 and 4.5.
- **Pitsco Education (structures supplies)**: Sells balsa and basswood strips, bridge and truss kits and structure testers used in schools; useful for Lesson 4.3 and Unit 6. Every build here also works with straws, string and craft sticks.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

4.1	**Loads: Dead, Live, Wind, Snow and Seismic**	3 sessions of 50 minutes
4.2	**Statics and Equilibrium**	3 sessions of 50 minutes
4.3	**Trusses by the Method of Joints**	3 sessions of 50 minutes
4.4	**Arches and the Catenary: Hooke, Poleni and Gaudí**	3 sessions of 50 minutes
4.5	**Load Paths: From Roof to Foundation**	3 sessions of 50 minutes

Unit 4 test

Time allowed: 50 minutes. Calculators are allowed. Draw free-body diagrams where they help. Show your work and units for every calculation. The test is worth 100 points.

1. **5 pts**
   Define dead load and live load and give two examples of each.

2. **6 pts**
   A floor has $D = 20$ psf and $L = 40$ psf. Find the factored load ($1.2D + 1.6L$) and the total on a 15 ft × 20 ft room.

3. **6 pts**
   Find the wind pressure for a 120 mph wind, and the force on a wall 30 ft by 10 ft.

4. **5 pts**

   Why did the Hartford Civic Center roof fall even though the snow load that night was below its supposed design load?

5. **7 pts**

   A 9 m beam has a 30 kN load 3 m from the left support and a 15 kN load 7 m from the left support. Find the reactions.

6. **6 pts**

   A 16 ft beam carries 500 lb/ft over its full length. Find the reactions.

7. **7 pts**

   A 1,000 N lamp hangs from two cables, each at 40° above the horizontal on opposite sides. Find the tension in each.

8. **8 pts**

   A triangular truss spans 10 m with its apex 2.5 m high and carries 20 kN at the apex. Find the force in each rafter and in the tie, with T or C.

9. **8 pts**

   For the three-panel truss of Lesson 4.3 (span 12 m, panels 4 m, height 3 m) with a single 12 kN load at C (the joint 8 m from A), find the reactions and the forces in FD, CD and CF.

10. **5 pts**

    Check whether a truss with 11 members, 7 joints and 3 reactions is statically determinate.

11. **6 pts**

    A chain hangs with $a = 15$ cm between supports 30 cm apart. Find its sag with the catenary and with the parabola approximation.

12. **6 pts**

    A parabolic arch spans 24 m, rises 6 m and carries 25 kN/m. Find the horizontal thrust and the vertical reaction at each support.

13. **5 pts**

    State Hooke's principle and explain why it works.

14. **7 pts**

A five-story building has interior columns on a 7 m × 7 m grid. Each floor is 9 kPa factored and the roof 6 kPa. Find the ground-floor interior column load.

15. **5 pts**

What is an alternate load path, and why do building rules require one?

16. **8 pts**

Short essay. Using Ronan Point and Hartford, explain why engineers must check their assumptions and design for what can go wrong, not only for the expected loads.

ANSWER KEY

1. Dead load: permanent weight of the structure and fixed parts (e.g., floor framing, roofing, walls). Live load: people and movable contents (e.g., a congregation, furniture, stored books).
2. $24 + 64 = 88$ psf; $88 \times 300 = 26{,}400$ lb.
3. $0.00256 \times 14{,}400 \approx 36.9$ psf; $36.9 \times 300 \approx 11{,}100$ lb.
4. Its long compression members were not braced against buckling as the design model assumed; some had likely buckled already, and the excess deflection during construction was a missed warning.
5. Moments about left: $9R_R = 90 + 105 = 195$, $R_R \approx 21.7$ kN; $R_L = 45 - 21.7 \approx 23.3$ kN.
6. Total 8,000 lb; 4,000 lb each.
7. $2T \sin 40^\circ = 1000$, $T = 1000 \div 1.286 \approx 778$ N.
8. Rafter length $\sqrt{25 + 6.25} \approx 5.59$ m, $\sin\theta \approx 0.447$, $\cos\theta \approx 0.894$. Rafters: $10 \div 0.447 \approx 22.4$ kN C; tie $22.4 \times 0.894 \approx 20.0$ kN T.
9. $R_A = 12(4)/12 = 4$ kN, $R_D = 8$ kN. Joint D: $F_{FD}(0.6) = 8$, $F_{FD} \approx 13.3$ kN C; $F_{CD} = 13.3 \times 0.8 \approx 10.7$ kN T. Joint F: vertical, $F_{CF} = F_{FD}(0.6) = 8$ kN T (no diagonal meets F other than FD in this layout).
10. $m + r = 14 = 2j$; determinate.
11. Catenary: $15(\cosh 1 - 1) \approx 8.15$ cm; parabola $225/30 = 7.5$ cm.
12. $H = 25 \times 576 \div 48 = 300$ kN; $V = 25 \times 12 = 300$ kN.
13. A hanging chain inverted gives the ideal arch. The chain can only pull, so its shape puts every link in pure tension under its loads; the mirror shape puts every part of a rigid arch in pure compression under the same loads.
14. $A = 49$ m^2; floors $5 \times 441 = 2205$; roof 294; total 2499 kN.
15. A second route for loads to reach the ground if a member is lost, through ties, continuity or redundancy; it prevents a local failure from becoming a progressive collapse, as at Ronan Point.
16. Model answer: at Hartford the design assumed bracing that did not exist and engineers dismissed unexpected deflections; at Ronan Point the designers assumed walls would never be removed, so the panels were not tied. In both, the structure carried its normal loads but had hidden weaknesses. Engineers must check that their models match reality, heed warning signs, and give structures

robustness (redundancy and ties) so that accidents and errors do not become disasters.

PORTFOLIO PIECE: A LOAD PATH AND TRUSS REPORT

Choose a real structure you can visit: a roof truss in an attic or garage, a pedestrian truss bridge, or a picnic shelter. Sketch it, measure it, and draw its load path from the roof or deck to the ground.

Estimate its loads (dead, live and snow for your town), analyze its main truss or frame with the method of joints or with beam reactions, and identify the most heavily loaded member. Then build a scale model of the truss, predict and test which members are in tension, and write a two-page report with your drawings, calculations and test results.

LOOK FOR	WHAT IT MEANS
Observation	Accurate measured sketch of a real structure with members and connections identified.
Loads	Reasonable dead, live and snow loads with sources, combined with load factors.
Analysis	Correct free-body diagrams, reactions and member forces with T or C and units.
Model test	Model built to scale; tension and compression predictions tested and discussed honestly.
Report	Clear, organized writing with labeled drawings and a conclusion about the structure's weakest link.

4.1 Loads: Dead, Live, Wind, Snow and Seismic

OBJECTIVES

- Name and estimate the loads a building must carry: dead, live, snow, wind and seismic.
- Calculate loads in pounds per square foot and kilopascals and combine them with load factors.
- Load-test a model roof with a uniform “snow” load and relate the result to real snow depths.

> **Warm-up.** Stand on a bathroom scale, then lift one foot. Does the reading change? Now crouch and stand up quickly. What happens to the reading? Write one sentence about the difference between a load that sits still and one that moves.

WATCH

Projected Loads and Snow Loads - Intro to Structural Analysis

Structures with Prof. H · 14:02

https://www.youtube.com/watch?v=t5k2IEd5ffo

> **WHEN TO WATCH**
> After Worked Example 2. Follow his example of dead plus snow load on a roof truss with your calculator.

Note: A university structures professor works a real snow-load problem on a tablet. Technical; watch the first half closely and the rest as preview of Lesson 4.3.

BACKUP VIDEO

Engineer Explains: Wind loads on Structures

Brendan Hasty · 7:04

https://www.youtube.com/watch?v=8l_-ztybeqo

With the wind-load paragraph.

KEY TERMS

dead load (D)

The permanent weight of the structure and everything fixed to it: framing, floors, roofing, walls, finishes, fixed equipment.

live load (L)

The weight of people, furniture and movable contents, set by building codes by use: about 40 psf for homes, 50 psf for offices, 100 psf for corridors and assembly areas with movable seats.

snow load (S)

The weight of snow on a roof, found from the ground snow load for the location and adjusted for roof shape, exposure and drifting.

wind load (W)

Pressure and suction on a building from wind; it grows with the square of wind speed.

seismic load (E)

The sideways inertia forces from earthquake ground shaking, roughly proportional to the building's weight.

load combination

A code rule that adds the loads with factors for safety, such as $1.2D + 1.6L$.

CORE IDEAS

The story. On the evening of January 17, 1978, about 5,000 people watched a college basketball game at the Hartford Civic Center in Connecticut. The arena's roof was a steel space frame, a three-dimensional lattice of steel bars about 300 by 360 feet, assembled on the ground and jacked into place in 1973. Snow and freezing rain fell all night. At about 4:20 a.m., hours after the crowd had gone home, the whole roof collapsed into the empty arena. No one was hurt. Fifty-six years earlier, on January 28, 1922, the flat roof of the Knickerbocker Theatre in Washington, D.C., had given way under heavy snow during a film, and 98 people died. Both failures taught engineers to respect snow and to check their assumptions.

How they figured it out. Investigators of the Hartford collapse found that the snow load that night was less than the roof was supposedly designed for. The problem was the design itself. The long top members of the space frame were in compression, and they were not braced against buckling as the computer model had assumed. Some of them had probably already buckled before that night, and during

construction the roof had deflected about twice as much as predicted, a warning sign that was explained away. The snow merely finished what the design had started. After Washington's 1922 disaster, codes began requiring roofs to be designed for local snow loads, and today maps in the national standard (ASCE 7) give the ground snow load for every part of the country.

The physics. Engineers express loads as force per unit area: pounds per square foot (psf) in the United States, kilonewtons per square meter (kPa) elsewhere. **Dead load** is the weight of the building itself; a wood-framed floor weighs about 10–15 psf. **Live load** comes from people and contents. A crowd packed tightly weighs about 100 psf, which is why corridors and assembly rooms are designed for it. **Snow** weighs from about 5 lb/ft^3 when fresh and fluffy to 20–30 lb/ft^3 when wet and packed, so 2 ft of wet snow can weigh 50 psf. A simple rule from the code for a flat roof is $p_f \approx 0.7 p_g$, where p_g is the ground snow load. **Wind** pressure is the kinetic energy of moving air stopped against a wall: $q = \frac{1}{2}\rho V^2$, which in US units is about $q = 0.00256\, V^2$ (psf, with V in mph). Double the wind speed and the pressure quadruples. **Seismic** force comes from Newton's second law: when the ground shakes, the building's mass must be accelerated, so the base shear is about $V = C_s W$, a fraction of the building's weight.

The engineering. Loads are uncertain, so codes multiply them by **load factors** and check several **combinations**, because the worst case is not always everything at once. Common strength combinations include $1.4D$, $1.2D + 1.6L$, $1.2D + 1.6S + L$ (in some uses $0.5L$), and $0.9D + 1.0W$, which checks whether wind can lift or overturn a light building. Dead load is known best and gets the smallest factor; live, snow and wind loads are less certain and get larger factors. The engineer designs each member for the worst combination it may see.

Try it. In the build you will load a model roof panel uniformly with rice as "snow," measure how it sags, and convert your loads into psf and real snow depths.

PICTURE STUDY

The Knickerbocker Theatre after the Snowstorm, National Photo Company, January 1922 · Glass-plate photograph · Library of Congress

Image: Wikimedia Commons, public domain

On January 28, 1922, the flat roof of this Washington theater collapsed under about 28 inches of snow, and 98 people lost their lives. Photographs like this one were studied by engineers and city officials, and they led to stricter rules for roof design. We look at it soberly, as engineers who must not repeat such mistakes.

Look closely

- Find the walls. Are they still standing? What does that tell you about which part failed?
- Look at the snow on top of the walls. Estimate its depth.
- Find pieces of the roof structure in the debris. What were they made of?
- Look at the stage at the far end. What is different about the roof there?

Talk about it

1. The walls stood but the roof fell. What load was the roof carrying that the walls were not?
2. Twenty-eight inches of snow is about 2.3 ft. If the snow weighed 12 lb/ft^3, what load in psf was on the roof?

3. What do you think a city should change after a failure like this?

Safety first. Wear safety glasses for the roof test, and keep the loaded panel low over a tray so spilled rice and falling pieces do not go far.

BUILD

Snow on the Roof: A Uniform Load Test

You will build a small flat roof panel, load it uniformly with rice as model "snow," measure its sag at each step, and find its failure load. Then you will convert your loads into psf and compare a deeper roof structure with a shallow one.

You will need

- Foam board or corrugated cardboard; card stock; tape and glue
- Two books or blocks as supports, 30 cm apart
- Rice or dry beans and a measuring cup; a kitchen scale
- A ruler held vertically or a phone camera on a stand to measure sag
- A shallow tray to catch spills

Steps

1. Build a roof panel 30 cm wide and 35 cm long from one layer of card stock, spanning 30 cm between the supports. Measure the loaded area: span × width.
2. Build a second panel of the same card stock stiffened underneath with two or three folded card "joists" running across the span.
3. **Predict.** How many grams of rice will each panel carry before it sags 2 cm? Which will carry more, and how many times more?
4. Weigh out rice in 100 g cups. Spread each cup evenly over the panel (a uniform load). After each cup, measure the sag at midspan.
5. Keep loading until the sag reaches 3 cm or the panel fails. Record the load and the way it failed.
6. Convert each load to pressure: $p = \text{weight} \div \text{area}$. Use $1\ \text{g} \rightarrow 0.00981\ \text{N}$, find Pa, then psf ($1\ \text{psf} = 47.9\ \text{Pa}$).
7. Graph sag against pressure for both panels. Find the slope (stiffness) of each.
8. Calculate what depth of real wet snow (about 320 kg/m^3, or 20 lb/ft^3) would produce your failure pressure on a full-size roof. (The model is not to scale; this is just to compare numbers.)

9. **Improve.** Using no more than one extra sheet of card, redesign the stiffened panel to carry at least twice the load at 2 cm of sag. Test it.

Tips

- Spread the rice evenly with a spoon or a piece of card; piles make point loads.
- Measure sag at the same point each time, the center of the span.
- Note when the panel begins to wrinkle or buckle on top: that is compression failure.

Check your work

- ☐ I can convert a model load into a pressure in Pa and psf.
- ☐ My graph compares the stiffness of two panels.
- ☐ I can explain how snow, live, wind and seismic loads are found and combined.

ENGINEER'S NOTEBOOK

1. A table of typical loads: dead loads for floors and roofs, live loads by use, ground snow load for your town (look up a snow-load map or ask your local building department), and a design wind speed.
2. Your load-test data table, the sag-pressure graph and the stiffness of each panel.
3. A paragraph on the Hartford Civic Center: what failed, why, and what warning sign was missed.

KEY FORMULAS

$p = \dfrac{F}{A}$ (psf or kPa), $\quad 1\ \text{kPa} \approx 20.9\ \text{psf}$

$p_f \approx 0.7\, p_g$ (flat roof snow, simplified)

$q = \frac{1}{2}\rho V^2 \approx 0.613\, V^2$ (Pa, V in m/s), $\quad q \approx 0.00256\, V^2$ (psf, V in mph)

$U = 1.2D + 1.6L, \quad U = 1.2D + 1.6S + L, \quad U = 0.9D + 1.0W$

$V_{\text{base}} = C_s W$

From history. The worst snow failures often happen in late winter after rain falls on old snow, which then soaks it like a sponge and freezes. Building officials in snowy towns sometimes ask owners of large flat roofs to clear snow after heavy storms, and some modern arenas have sensors that measure the roof's deflection to warn when snow loads are getting high.

Example 1. A church hall floor is 30 ft × 40 ft. Its dead load is 15 psf and its live load (assembly, movable seats) is 100 psf. Find the total factored load using $1.2D + 1.6L$.

Show solution

1. Area: $30 \times 40 = 1200$ ft^2.
2. Factored pressure: $1.2(15) + 1.6(100) = 18 + 160 = 178$ psf.
3. Total: $178 \times 1200 = 213{,}600$ lb ≈ 214 kips (1 kip = 1,000 lb).

Answer: 178 psf factored, about 214,000 lb over the floor.

Example 2. A town has a ground snow load of 30 psf. Estimate the flat-roof snow load. A roof is 50 ft × 80 ft. What total snow weight might it carry? What depth of wet snow (20 lb/ft^3) does the roof load represent?

Show solution

1. Roof snow: $p_f \approx 0.7 \times 30 = 21$ psf.
2. Total: $21 \times (50 \times 80) = 21 \times 4000 = 84{,}000$ lb.
3. Depth: 21 psf $\div$ 20 lb/ft^3 $= 1.05$ ft, about 13 in of wet snow.

Answer: About 21 psf; 84,000 lb total; equivalent to about a foot of wet snow.

Example 3. Find the wind pressure for a 90 mph wind and a 130 mph wind. A wall 20 ft wide and 12 ft tall faces the wind. What force does each wind put on it (pressure × area, ignoring code coefficients)?

Show solution

1. 90 mph: $q = 0.00256(90)^2 = 0.00256 \times 8100 \approx 20.7$ psf.
2. 130 mph: $q = 0.00256(130)^2 = 0.00256 \times 16{,}900 \approx 43.3$ psf.
3. Wall area: $20 \times 12 = 240$ ft^2. Forces: $20.7 \times 240 \approx 4970$ lb and $43.3 \times 240 \approx 10{,}400$ lb.
4. A 44% faster wind more than doubles the force, because pressure goes as V^2: $(130/90)^2 \approx 2.09$.

Answer: About 20.7 psf and 43.3 psf; about 5,000 lb and 10,400 lb on the wall.

Example 4. A two-story building weighs 1,800 kN. For its site and type, the code gives a seismic coefficient $C_s = 0.12$. Find the design base shear.

Show solution

1. $V = C_s W = 0.12 \times 1800 \text{ kN} = 216 \text{ kN}$.
2. This sideways force, about one-eighth of the weight, must be carried by walls, braces or frames down to the foundation.

Answer: 216 kN

PRACTICE

1. Easy Classify each as dead, live, snow, wind or seismic: (a) roof shingles, (b) people at a dinner, (c) a blizzard, (d) a gust, (e) ground shaking.

   Show answer

   (a) dead, (b) live, (c) snow, (d) wind, (e) seismic.

2. Easy Convert 50 psf to kPa (1 kPa ≈ 20.9 psf).

   Show answer

   $50 \div 20.9 \approx 2.4 \text{ kPa}$.

3. Easy A ground snow load is 40 psf. Estimate the flat-roof snow load.

   Show answer

   $0.7 \times 40 = 28 \text{ psf}$.

4. Medium Find the wind pressure for a 110 mph wind.

   Show answer

   $0.00256 \times 12{,}100 \approx 31.0 \text{ psf}$.

5. Medium In metric, find the wind pressure for a 40 m/s wind using $q = 0.613V^2$.

   Show answer

   $0.613 \times 1600 \approx 981 \text{ Pa} \approx 0.98 \text{ kPa}$.

6. Medium An office floor has $D = 60$ psf and $L = 50$ psf. Which is larger: $1.4D$ or $1.2D + 1.6L$?

   Show answer

   $1.4D = 84$ psf; $1.2D + 1.6L = 72 + 80 = 152$ psf, the larger.

7. Medium Two feet of wet snow at 25 lb/ft^3 lie on a roof. What is the load in psf? Is it more or less than a 40 psf design snow load?

Show answer

$2 \times 25 = 50$ psf, more than 40 psf: the roof would be overloaded.

8. Hard A light warehouse roof has a dead load of 12 psf. Wind creates an upward suction of 18 psf on it. Check the combination $0.9D + 1.0W$ (taking uplift as negative). What does the result mean?

Show answer

$0.9(12) - 18 = 10.8 - 18 = -7.2$ psf. The net is upward: the roof must be tied down to its walls and foundations, or it will be lifted off.

9. Hard A wind speed increases from 100 mph to 150 mph. By what factor does the wind pressure increase?

Show answer

$(150/100)^2 = 2.25$.

10. Hard A building weighs 6,000 kN and has a seismic coefficient of 0.08. Find the base shear. If its weight is reduced 20% by using lighter materials, what is the new base shear?

Show answer

480 kN; with 4800 kN, 384 kN. Lighter buildings attract smaller earthquake forces.

11. Hard Conceptual. Why does dead load get a smaller load factor (1.2) than live load (1.6)?

Show answer

Model answer: the dead load can be calculated accurately from the drawings and materials, while live loads (people, furniture, storage) are uncertain and can exceed what is expected, so they need a larger margin.

12. **Challenge** The Knickerbocker roof carried about 28 in of snow. If the snow weighed 15 lb/ft^3 and the roof's dead load was 20 psf, what was the total load in psf? If the roof had been designed for a total of 50 psf with a factor of safety of 1.5 (ultimate capacity 75 psf), how close was it to failure?

Show answer

Snow: $(28/12) \times 15 = 35$ psf; total 55 psf. It was above the 50 psf design load and at about 73% of its 75 psf ultimate capacity; any weakness in the members or connections, or uneven drifting, could make up the rest.

COMMON MISTAKES

- Forgetting that wind pressure goes as the square of speed.
- Adding all loads at full value. Codes use factored combinations; the worst case may be a combination without every load.
- Using ground snow load directly for a roof without adjusting it, or forgetting drifts against higher walls.

Make it a game · Load Roll Call. Name the five loads a building carries in sixty seconds, with a typical value for each in pounds per square foot. Your parent shuffles notes about a pretend roof, like six inches of snow, and you calculate the load combination with a load factor. Points for each correct term and number. Try to beat your streak.

Scripture. "Therefore whosoever heareth these sayings of mine, and doeth them, I will liken him unto a wise man, which built his house upon a rock: And the rain descended, and the floods came, and the winds blew, and beat upon that house; and it fell not: for it was founded upon a rock." (Matthew 7:24-25, KJV)

Jesus names the loads every builder must plan for: rain, floods and winds. A wise builder designs for them before they come.

4.2 Statics and Equilibrium

OBJECTIVES

- State the conditions of equilibrium, $\sum F_x = 0$, $\sum F_y = 0$ and $\sum M = 0$, and draw free-body diagrams.
- Calculate support reactions for beams with point and uniform loads.
- Resolve forces into components to find tensions and compressions in cables and struts.

> **Warm-up.** Lay a ruler across your finger and balance it. Put a coin near one end. Where must your finger move to balance it again? Try two coins on one side and one on the other. Can you predict where to put your finger before you try?

WATCH

Static Equilibrium: Support Reactions

North Carolina School of Science and Mathematics · 15:46

https://www.youtube.com/watch?v=IqUXdQDumLQ

> **WHEN TO WATCH**
> Before the worked examples. Copy his table of supports (roller, pin, fixed) and the reactions each provides.

BACKUP VIDEO

Statics: Lesson 29 - 2D Reaction at Supports, Example Problem

Jeff Hanson · 13:46

https://www.youtube.com/watch?v=bPbQeptNxEo

After Worked Example 2, for another fully worked reactions problem at the whiteboard.

KEY TERMS

equilibrium

The state of a body at rest (or moving steadily): the sum of all forces and all moments on it is zero.

free-body diagram

A sketch of one body, cut free from its surroundings, with every force acting on it drawn as an arrow.

moment

The turning effect of a force about a point: force × perpendicular distance, $M = Fd$, in N·m or ft·lb.

reaction

A force (or moment) that a support exerts on a structure to hold it in equilibrium.

pin and roller

A pin support resists vertical and horizontal force; a roller resists only force perpendicular to its surface. A fixed support also resists moment.

components

The horizontal and vertical parts of a force at an angle: $F_x = F\cos\theta$, $F_y = F\sin\theta$.

CORE IDEAS

The story. In 1586 Simon Stevin, an engineer of Bruges working in the Netherlands, published a book on statics, *De Beghinselen der Weeghconst* (The Elements of the Art of Weighing). On its title page he drew a puzzle: a loop of fourteen equal balls draped over a triangle whose two slopes have lengths in the ratio 2:1. Four balls lie on the long slope and two on the short, and the rest hang beneath. Will the chain slide? Stevin argued that it cannot, or it would turn forever by itself, which is impossible. The eight hanging balls balance each other, so the four balls on the long slope must exactly balance the two on the short. He wrote over the picture: *Wonder en is gheen wonder*, "A wonder, yet no wonder." From this he worked out the law of the inclined plane and the rule for combining forces at angles.

How they figured it out. Archimedes had found the law of the lever in the third century BC: two weights balance when weight × distance is equal on each side. Stevin added forces at angles, and later Pierre Varignon (1687) and Isaac Newton put statics on its modern footing. Engineers have used it ever since to find the forces in every beam, cable and column. A famous illustration came in 1887, when the engineers of Scotland's Forth Bridge, John Fowler and Benjamin Baker, explained their cantilever design with a "human model": two men sitting on chairs with their arms in tension and sticks in compression, holding up a third man in the middle, with brick counterweights at the ends.

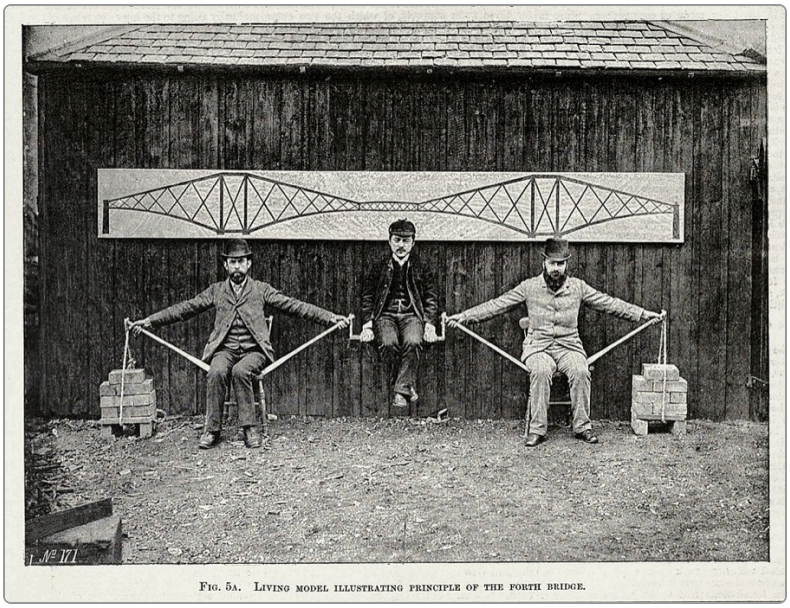
The physics. A body is in **equilibrium** when nothing accelerates it, so by Newton's first law the forces and moments on it must sum to zero: $\sum F_x = 0$, $\sum F_y = 0$, $\sum M = 0$. To use this, draw a **free-body diagram**: the structure alone, with each load and each unknown support reaction drawn as an arrow. Then write the three

equations and solve. Taking moments about a support is the key trick, because the reactions at that support have no moment about it, leaving one unknown. For a beam of span L with a load P at distance a from support A, $R_B = Pa/L$ and $R_A = P - R_B$. A uniform load w over the whole span acts like a single load wL at the center.

The engineering. Every structural calculation begins with statics. Before an engineer can choose a beam size, a column, a bolt or a footing, she must know the forces on it, and those come from equilibrium. The Forth Bridge's human model shows the idea perfectly: the load in the middle goes into the arms (tension) and sticks (compression), down through the men into the chairs, and the counterweights keep the whole thing from tipping. If any one of the three equations is not satisfied, something moves, and in a building, something moving means something breaking.

Try it. In the build you will put a plank across two kitchen scales, place loads on it, and check that the scale readings match your equilibrium calculations.

PICTURE STUDY



The Human Cantilever Model of the Forth Bridge, Unknown photographer, 1887 · Photograph · Wikimedia Commons

Image: Wikimedia Commons, public domain

To explain their cantilever bridge over the Firth of Forth, engineers John Fowler and Benjamin Baker posed this living model. The man in the middle is Kaichi Watanabe, a Japanese engineer then studying in Scotland. Every force in the bridge can be found in this one photograph.

Look closely

- Find the men's arms. Are they being pulled or pushed? Find the sticks. Are they pulled or pushed?
- Find the counterweights of bricks at the outer ends. What would happen without them?
- Find where all the loads finally go down to the ground.
- Find the short beam in the middle. What holds it up?

Talk about it

1. Draw a free-body diagram of one seated man and his chair. What forces act on it?
2. If the middle man weighs 700 N and is shared equally, how much does each side carry?

3. Why is this a good way to explain a structure to people who are not engineers?

Safety first. Keep the plank low and the weights small (books or water bottles). Keep fingers and toes clear in case the plank slips off a scale.

BUILD

Reactions on the Kitchen Scales

You will rest a board on two kitchen scales, place loads at measured positions, predict the reading on each scale from the equilibrium equations, and compare with the actual readings.

You will need

- A straight board or a long ruler/meter stick (60–100 cm)
- Two digital kitchen scales (or bathroom scales for heavier loads); two small blocks so the board rests on a narrow edge over each scale
- Weights: water bottles, cans or books of known mass
- A tape measure and a calculator

Steps

1. Set a narrow block on each scale and rest the board on the blocks. Measure the span between the block edges, L. Zero the scales with only the blocks on them.
2. Read both scales with the board alone. Do they add to the board's weight? Each should be about half.
3. Place one weight at a measured distance a from the left support. **Predict** both readings with $R_B = W_{\text{load}} a/L$ (plus half the board's weight on each) before you look.
4. Read the scales. Compute the error in grams and percent.
5. Repeat with the weight at four other positions, including one beyond the right support (an overhang). What happens to the left reading?
6. Place two weights at once and predict and check again.
7. Graph the right-scale reading against a for a single weight. It should be a straight line. Find its slope and compare with W_{load}/L.
8. **Improve.** Find the source of your largest error (where exactly the board touches the block, the board's own weight, scale accuracy) and change the setup to reduce it. Repeat one measurement.

Tips

- Use narrow edges (a pencil taped to each block) so the support point is well defined.
- Measure distances to the center of each weight.
- Kitchen scales need a moment to settle; read them after the board is still.

Check your work

- ☐ I can draw a free-body diagram and write the three equilibrium equations.
- ☐ My predicted and measured reactions agree within a few percent, or I can explain why not.
- ☐ I can find cable tensions by resolving forces into components.

ENGINEER'S NOTEBOOK

1. Free-body diagrams of the board for each test, with the loads, distances and reactions labeled.
2. A table of predicted and measured reactions with the percent error, and the graph of the right reaction against position.
3. A free-body diagram of the human cantilever model, showing tension and compression members.

KEY FORMULAS

$$\sum F_x = 0, \quad \sum F_y = 0, \quad \sum M = 0$$
$$M = Fd_\perp$$
$$R_B = \frac{Pa}{L}, \quad R_A = P - R_B$$
$$F_x = F\cos\theta, \quad F_y = F\sin\theta$$

From history. Stevin was a practical engineer as well as a mathematician. He designed windmills and sluices for draining Dutch polders, advised Prince Maurice of Nassau on fortifications, and wrote a small book that taught Europeans to use decimal fractions, which he argued would save merchants and surveyors endless trouble.

Example 1. A beam spans 6.0 m between a pin at A and a roller at B. It carries a 12 kN load 2.0 m from A and a 6 kN load 4.5 m from A. Find the reactions.

Show solution

1. Take moments about A (counterclockwise positive): $R_B(6.0) - 12(2.0) - 6(4.5) = 0$.
2. $6R_B = 24 + 27 = 51$, so $R_B = 8.5$ kN.
3. Vertical forces: $R_A + R_B = 12 + 6 = 18$, so $R_A = 9.5$ kN.
4. Check with moments about B: $R_A(6.0) = 12(4.0) + 6(1.5) = 48 + 9 = 57$, and $57/6 = 9.5$. ✓

Answer: $R_A = 9.5$ kN, $R_B = 8.5$ kN.

Example 2. A 20 ft floor beam carries a uniform load of 400 lb/ft over its whole length and a 2,000 lb point load 5 ft from the left support. Find the reactions.

Show solution

1. Uniform load total: $400 \times 20 = 8000$ lb, acting at the center, 10 ft from each end.
2. Moments about the left support: $R_R(20) = 8000(10) + 2000(5) = 80{,}000 + 10{,}000 = 90{,}000$, so $R_R = 4500$ lb.
3. Vertical: $R_L = 8000 + 2000 - 4500 = 5500$ lb.

Answer: $R_L = 5500$ lb, $R_R = 4500$ lb.

Example 3. A 600 N sign hangs from a ring held by two cables: one at 30° above the horizontal to the left, one at 45° above the horizontal to the right. Find the tension in each cable.

Show solution

1. Horizontal: $T_1 \cos 30^\circ = T_2 \cos 45^\circ$, so $0.866\,T_1 = 0.707\,T_2$ and $T_2 = 1.225\,T_1$.
2. Vertical: $T_1 \sin 30^\circ + T_2 \sin 45^\circ = 600$, so $0.5\,T_1 + 0.707(1.225\,T_1) = 600$.
3. $0.5\,T_1 + 0.866\,T_1 = 1.366\,T_1 = 600$, giving $T_1 \approx 439$ N.
4. $T_2 = 1.225 \times 439 \approx 538$ N. The steeper cable carries more.

Answer: $T_1 \approx 439$ N (30° cable), $T_2 \approx 538$ N (45° cable).

PRACTICE

1. Easy What are the three equations of equilibrium in two dimensions?

   Show answer

   $\sum F_x = 0, \sum F_y = 0, \sum M = 0.$

2. Easy A 10 kN load sits at the center of a 4 m beam. Find the reactions.

   Show answer

   5 kN at each support.

3. Easy Find the moment of a 50 N force applied 0.30 m from a bolt, perpendicular to the wrench.

   Show answer

   15 N·m.

4. Medium A 5 m beam has a 20 kN load 1.5 m from the left support. Find the reactions.

   Show answer

   $R_R = 20(1.5)/5 = 6$ kN; $R_L = 14$ kN.

5. Medium A 12 ft beam carries 250 lb/ft over its whole length. Find the reactions.

   Show answer

   Total 3,000 lb; 1,500 lb at each end.

6. Medium A 400 N force acts at 35° above the horizontal. Find its horizontal and vertical components.

Show answer

$F_x = 400\cos 35^\circ \approx 328$ N; $F_y = 400\sin 35^\circ \approx 229$ N.

7. Medium A weight of 900 N hangs from the middle of a cable whose two halves each make 20° with the horizontal. Find the tension.

Show answer

$2T\sin 20^\circ = 900$, $T = 900 \div (2 \times 0.342) \approx 1316$ N.

8. Hard A beam 8 m long is supported at A (left end) and at B, 6 m from A, with a 2 m overhang. A 10 kN load hangs at the tip of the overhang. Find the reactions.

Show answer

Moments about A: $R_B(6) = 10(8)$, $R_B \approx 13.3$ kN up;
$R_A = 10 - 13.3 = -3.3$ kN, so A must hold the beam down with 3.3 kN.

9. Hard In the human cantilever, suppose each arm pulls at 30° above the horizontal and carries half the middle man's 700 N weight vertically. Find the tension in each arm and the horizontal pull it puts on the man's shoulders.

Show answer

$T\sin 30^\circ = 350$, $T = 700$ N; horizontal $= 700\cos 30^\circ \approx 606$ N.

10. Hard Conceptual. In Problem 7, what happens to the cable tension as the cable is pulled tighter and the angle gets smaller? Could the cable ever be perfectly straight?

Show answer

Model answer: $T = W/(2\sin\theta)$ grows without limit as $\theta \to 0$, so a loaded cable can never be perfectly straight; tightening it to nearly straight makes the tension enormous. That is why suspension cables sag.

11. **Challenge** Stevin's triangle has slopes of lengths 2 and 1 (ratio) with 4 and 2 balls on them. Show that the force along each slope balances, using the inclined-plane rule: the force along a slope from weight W is $W \sin\theta$, and for a triangle of height h, $\sin\theta = h/\ell$.

Show answer

Long slope: $4w(h/2\ell_0)$; short slope: $2w(h/\ell_0)$ where ℓ_0 is the short length. Both equal $2wh/\ell_0$, so the chain is balanced.

COMMON MISTAKES

- Leaving a force off the free-body diagram, often the weight of the member itself or a reaction.
- Using the full length instead of the perpendicular distance when computing a moment.
- Forgetting that a uniform load acts at its center when replaced by a single resultant.

Make it a game · Balance the Beam. Lay a yardstick across two kitchen scales or two books, and place a coin pile at a chosen spot. Before measuring, write the support reactions from the sum of moments. Check against the scales. The closest prediction scores. Then draw the free-body diagram from memory and label every force and its direction.

4.3 Trusses by the Method of Joints

OBJECTIVES

- Explain why trusses are built of triangles and why their members carry only tension or compression.
- Find the forces in every member of a simple truss by the method of joints.
- Build a truss in which predicted tension members are replaced by string, and test it.

> **Warm-up.** Make a square from four straws joined at the corners with pins or pipe cleaners. Push one corner sideways. Now add one diagonal straw and push again. Why does the diagonal make such a difference?

WATCH

Truss analysis by method of joints explained

Engineer4Free · 5:11

https://www.youtube.com/watch?v=08ChRvOt9iI

> **WHEN TO WATCH**
> After the core ideas, before Worked Example 2. Pause at each joint and write the two equations yourself.

BACKUP VIDEO

Understanding and Analysing Trusses

The Efficient Engineer · 17:40

https://www.youtube.com/watch?v=Hn_iozUo9m4

For a fuller, animated explanation of truss types, joints and the method of sections.

KEY TERMS

truss

A structure of straight members joined at their ends into triangles, carrying loads applied at the joints.

two-force member

A member loaded only at its two ends, so the force in it acts along its length: pure tension or pure compression.

method of joints

Finding member forces by applying $\sum F_x = 0$ and $\sum F_y = 0$ at each joint in turn, starting where there are only two unknowns.

chord and web

The top and bottom members of a truss are the chords; the verticals and diagonals between them are the web.

zero-force member

A member that carries no force under a particular loading, though it may be needed for other loads or to brace the truss.

CORE IDEAS

The story. In the 1830s and 1840s American railroads needed bridges by the thousand, and builders such as William Howe and Thomas and Caleb Pratt patented truss designs of timber and iron. But they sized the members by experience and guesswork, and bridges failed. Squire Whipple, a surveyor and instrument maker from upstate New York who had worked on the Erie Canal, patented an iron bowstring truss in 1841 and in 1847 published *A Work on Bridge Building*, the first book to show how to calculate the force in every member of a truss from statics. The Pennsylvania Railroad engineer Herman Haupt published a similar analysis in 1851. From then on, truss bridges could be designed, not guessed.

How they figured it out. Whipple's insight was that if the members of a truss are joined by pins and loaded only at the joints, each member can push or pull only along its own length. Every joint is a tiny problem in equilibrium, like the ring holding the sign in Lesson 4.2. Solve one joint, carry the known force to the next, and you can walk across the whole truss. He checked his methods by testing, and his bowstring trusses were built across the Erie Canal and elsewhere; a few still stand.

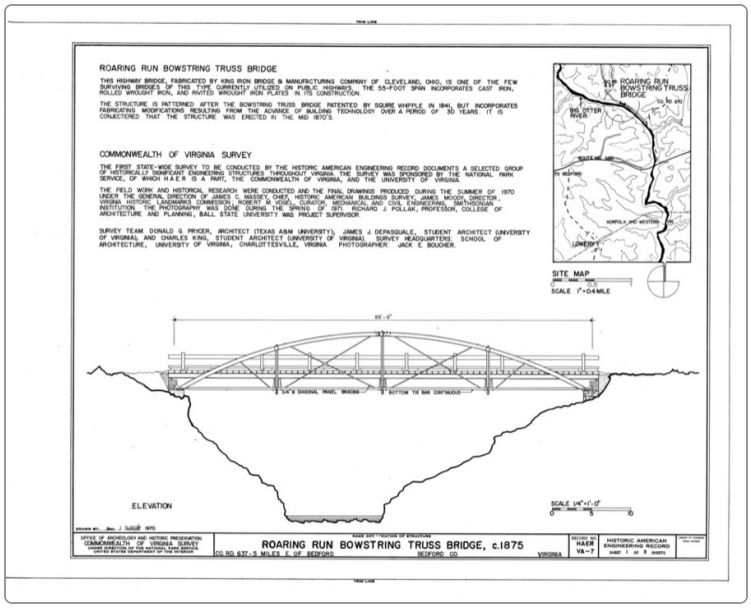
The physics. A triangle cannot change shape unless one of its sides changes length, so a framework of triangles is rigid even with pinned joints. In an ideal truss each member is a **two-force member**: the force in it is pure **tension** (pulling, T) or pure **compression** (pushing, C). To use the **method of joints**: (1) find the support reactions from the whole truss; (2) start at a joint with only two unknown members; (3) assume each unknown member is in tension (pulling away from the joint); (4) write $\sum F_x = 0$ and $\sum F_y = 0$ and solve; a negative answer means compression; (5)

move to the next joint. For a simple triangle of span L and height h loaded by P at its apex, each sloping member carries $C = \dfrac{P/2}{\sin\theta}$ and the bottom tie carries $T = C\cos\theta$.

The engineering. Knowing which members are in tension and which are in compression lets engineers size each one correctly. Tension members can be slender rods, cables or flat bars, because pulling straightens them. Compression members must be stocky or braced, because pushing makes them buckle, as at Hartford. That is why the Pratt truss, with its long diagonals in tension and its short verticals in compression, was well suited to iron and steel, and why the Howe truss, with diagonals in compression, suited timber diagonals with iron vertical rods. Roof trusses in nearly every house built today are designed by computer with exactly this method.

Try it. In the build you will analyze a truss, predict which members are in tension, replace those members with string, and see whether it still stands.

PICTURE STUDY



Roaring Run Bowstring Truss Bridge, Historic American Engineering Record, 1970s · Measured drawing · Library of Congress, Prints and Photographs Division

Image: Wikimedia Commons, public domain

This small iron bridge in Virginia, built about 1875 by the King Iron Bridge Company of Cleveland, follows the bowstring pattern Squire Whipple patented in 1841. HAER engineers measured and drew it as one of the few survivors of its kind.

Look closely

- Find the curved top chord (the bow) and the straight bottom chord (the string). Which do you think is in compression?
- Find the vertical and diagonal members between them. How many panels does the truss have?
- Read the history at the top of the sheet. What materials is the bridge made of?
- Find the scale and the span. About how long is the bridge?

Talk about it

1. Why does a bow shape suit a truss carrying a load along its whole length?
2. If the bottom chord is in tension, what material would you choose for it, and why?

3. Why do you think HAER chose to document this small bridge?

Safety first. Wear safety glasses during load tests. Pins and sharp straw ends can poke; handle them carefully.

BUILD

String Where It Pulls: A Truss Prediction Test

You will analyze a truss on paper, predict every member's force as tension or compression, and then build it with stiff straws (or craft sticks) for compression members and string for tension members. If your analysis is right, the truss will stand and carry load.

You will need

- Stiff drinking straws or craft sticks, thread or thin string, pins, paper clips or pipe cleaners for joints
- Tape, a ruler and graph paper
- A paper cup, string and pennies or rice for loads; two books as supports

Steps

1. Choose the truss of Worked Example 2 (three panels) or a Warren truss of your own, and draw it to scale with the loads at the bottom joints.
2. Solve every member force by the method of joints. Mark each member T (tension), C (compression) or 0.
3. **Predict.** Which members could be replaced by string? Write your list.
4. Build the truss about 30 cm long with straws for the C members and tightly tied string for the T members. Leave zero-force members out for now.
5. Set it on the supports and hang equal loads from the two bottom joints. Does it stand? If not, find which member collapsed and recheck your analysis.
6. Load it gradually until something fails. Record the load and which member failed first, and whether it buckled (compression) or snapped or slipped (tension).
7. Now load only one bottom joint. Does the truss still stand without its middle diagonal? Explain using your analysis.
8. **Improve.** Add the middle diagonal and strengthen the member that failed first (double a straw, or brace a long compression member at its middle). Retest and compare.

Tips

- Tie string tightly; a slack string lets the truss sag before it takes load.
- Joints must be at the ends of members, and loads must hang at joints, for the analysis to apply.
- Long compression members buckle easily; keep them short or brace them.

Check your work

- ☐ I solved every member force by the method of joints.
- ☐ My string-and-straw truss stood, or I found and fixed the error in my analysis.
- ☐ I can explain why a zero-force member may still be needed.

ENGINEER'S NOTEBOOK

1. Your truss drawn to scale with every member force labeled T, C or 0, and the free-body diagram of each joint you solved.
2. A record of the load tests: load, member that failed, and how it failed.
3. Sketch a Pratt truss and a Howe truss under the same loads, and mark which diagonals are in tension in each.

KEY FORMULAS

At each joint: $\sum F_x = 0, \sum F_y = 0$

Simple triangle (apex load P): $C = \dfrac{P/2}{\sin\theta}, \quad T = C\cos\theta$

Determinacy check: $m + r = 2j$ (m members, r reactions, j joints)

From history. Squire Whipple was largely ignored in his lifetime and spent his last years in modest circumstances, but engineers came to regard his book as the foundation of American bridge engineering. Herman Haupt, who published a similar analysis, later ran military railroads during the Civil War and built a trestle so quickly from green timber that Abraham Lincoln described it as made of beanpoles and cornstalks.

Example 1. A simple roof truss is a triangle spanning 8.0 m with its apex 3.0 m above the supports. A 12 kN load acts at the apex. Find the force in each member.

Show solution

1. Reactions: by symmetry, 6.0 kN up at each support.
2. Geometry: each rafter has run 4.0 m and rise 3.0 m, so its length is 5.0 m, $\sin\theta = 0.6$, $\cos\theta = 0.8$.
3. Left support joint, vertical: $F_{\text{rafter}} \sin\theta = 6.0$, so $F_{\text{rafter}} = 6.0 \div 0.6 = 10.0$ kN. The rafter pushes down on the joint: compression.
4. Horizontal: the tie must balance the rafter's horizontal push: $F_{\text{tie}} = 10.0 \times 0.8 = 8.0$ kN, pulling: tension.

Answer: Each rafter: 10.0 kN compression; the bottom tie: 8.0 kN tension.

Example 2. A three-panel truss spans 12 m with bottom joints A, B, C, D (A and D are supports) every 4 m, and top joints E above B and F above C, 3 m high. Members: AB, BC, CD, AE, EF, FD, BE, CF and a diagonal EC. Loads of 10 kN hang at B and at C. Find all member forces.

Show solution

1. Reactions: by symmetry, 10 kN up at A and at D.
2. Joint A: AE slopes up at $\sin\theta = 3/5$, $\cos\theta = 4/5$. Vertical: $F_{AE}(0.6) + 10 = 0$, so $F_{AE} = -16.7$ kN (compression). Horizontal: $F_{AB} + F_{AE}(0.8) = 0$, so $F_{AB} = 13.3$ kN (tension).
3. Joint B: vertical: $F_{BE} - 10 = 0$, so $F_{BE} = 10$ kN (tension). Horizontal: $F_{BC} = F_{AB} = 13.3$ kN (tension).
4. Joint E (each member taken as pulling away from E): vertical: $-0.6F_{AE} - F_{BE} - 0.6F_{EC} = 0$, that is $10 - 10 - 0.6F_{EC} = 0$, so $F_{EC} = 0$. Horizontal: $-0.8F_{AE} + F_{EF} + 0.8F_{EC} = 0$, so $F_{EF} = 0.8F_{AE} = -13.3$ kN (compression).
5. By symmetry: $F_{FD} = -16.7$ (C), $F_{CD} = 13.3$ (T), $F_{CF} = 10$ (T). The middle diagonal EC carries zero under this symmetric load: the shear in the middle panel is zero.

Answer: Bottom chord 13.3 kN T in all three panels; top chord EF 13.3 kN C; end diagonals 16.7 kN C; verticals 10 kN T; middle diagonal 0.

Example 3. Check whether the truss in Worked Example 2 is statically determinate.

Show solution

1. Count: members $m = 9$, joints $j = 6$, reactions $r = 3$ (two at the pin, one at the roller).
2. $m + r = 12$ and $2j = 12$. They are equal, so the truss can be solved by statics alone.

Answer: Yes: $m + r = 2j = 12$.

PRACTICE

1. Easy Why are trusses made of triangles?

Show answer

A triangle cannot change shape unless a side changes length, so a framework of triangles is rigid even with pinned joints.

2. Easy In an ideal truss, why does each member carry only tension or compression?

Show answer

It is loaded only at its two pinned ends, so the force must act along the line joining them.

3. Easy Check determinacy: a truss has 7 members, 5 joints and 3 reactions.

Show answer

$m + r = 10 = 2j$; determinate.

4. Medium A triangular truss spans 6 m with its apex 2 m high and carries 9 kN at the apex. Find the forces.

Show answer

Rafter length $\sqrt{9+4} \approx 3.606$ m, $\sin\theta = 0.555$; rafters $4.5/0.555 \approx 8.11$ kN C; tie $8.11 \times (3/3.606) \approx 6.75$ kN T.

5. Medium In Problem 4, what happens to the rafter and tie forces if the apex is raised to 4 m?

Show answer

Rafter length 5 m, $\sin\theta = 0.8$: rafters $4.5/0.8 \approx 5.63$ kN C; tie $5.63 \times 0.6 \approx 3.38$ kN T. A steeper truss has smaller forces.

6. Medium At a joint, a member at 45° carries 14.1 kN compression and one horizontal member and one vertical member are unknown. If there is no external load at the joint, find the other two.

Show answer

The 45° force has components $14.1 \times 0.707 \approx 10$ kN each way, so the horizontal and vertical members each carry 10 kN (tension or compression depending on the directions) to balance it.

7. Medium Which members of a Pratt truss under downward loads are usually in tension: the diagonals or the verticals?

Show answer

The diagonals (the verticals are in compression).

8. Hard In the truss of Worked Example 2, a single 12 kN load hangs at B only. Find the reactions and the forces in AE, AB and BE.

Show answer

$R_A = 12(8)/12 = 8$ kN, $R_D = 4$ kN. Joint A: $F_{AE} = -8/0.6 \approx -13.3$ (C), $F_{AB} = 13.3 \times 0.8 \approx 10.7$ (T). Joint B: $F_{BE} = 12$ (T).

9. Hard For the load in Problem 8, find the force in diagonal EC. (Hint: the shear in the middle panel equals $R_D = 4$ kN.)

Show answer

The diagonal's vertical component must carry the 4 kN shear: $F_{EC}(0.6) = 4$, $F_{EC} \approx 6.7$ kN, and with this geometry it is compression.

10. Hard Conceptual. Why might a designer keep a zero-force member in a truss?

Show answer

Model answer: it carries force under other loads (moving trains, wind, uneven snow), and it braces long compression members against buckling, shortening their effective length.

11. Challenge A Warren truss has equilateral triangles with sides 3 m. A load of 20 kN sits at the middle bottom joint of a two-triangle-wide span (three bottom joints, A, B, C and two top joints). Find the force in each diagonal next to the support A.

Show answer

Reaction at A is 10 kN. The diagonal from A slopes at 60°: $F \sin 60^\circ = 10$, $F \approx 11.5$ kN C; the bottom chord AB carries $11.5 \cos 60^\circ \approx 5.8$ kN T.

COMMON MISTAKES

- Starting at a joint with three or more unknowns. Start where there are only two.
- Mixing up signs. Assume tension for every unknown; a negative answer means compression.
- Forgetting the support reactions before analyzing the first joint.

> **Make it a game · Pull or Push?.** Draw a small truss on paper and number its joints. Your parent calls a member; you must say within ten seconds whether it is in tension or compression and the force from the method of joints. A wrong answer loses a point. Then explain why every member only carries axial force. Switch and compete.

4.4 Arches and the Catenary: Hooke, Poleni and Gaudí

OBJECTIVES

- Explain Hooke's principle that a hanging chain, inverted, gives the ideal shape of an arch.
- Describe the catenary $y = a\cosh(x/a)$ and the parabola, and when each is the right shape.
- Make a hanging-chain model, compare it with the equations, and build an arch to its shape.

> **Warm-up.** Hold a necklace or light chain by its two ends and let it hang. Move your hands closer and farther apart. Sketch the shapes. Now imagine the picture turned upside down. Where have you seen that shape in a building?

WATCH

3. Arches and Chains

Engineering Models · 9:27

https://www.youtube.com/watch?v=JlL6ZHChhQE

> **WHEN TO WATCH**
> After the story. Watch how the model arch fails when its line of thrust leaves the stones.

KEY TERMS

catenary

The curve of a uniform chain hanging under its own weight: $y = a\cosh(x/a)$.

funicular shape

The shape a cable or chain takes under a given set of loads; inverted, it is an arch in pure compression for those loads.

line of thrust

The path of the compressive force through an arch. If it stays within the masonry (ideally the middle third), the arch stands.

parabola

The funicular shape for a load spread evenly along the horizontal, like a bridge deck hung from a cable.

hyperbolic cosine

$\cosh x = \frac{1}{2}(e^x + e^{-x})$; it describes the catenary.

CORE IDEAS

The story. In 1675 Robert Hooke, curator of experiments to the Royal Society and Christopher Wren's friend, was working on the dome of St Paul's. He announced that he had found the true shape of an arch, but he hid it as a Latin anagram. After his death the answer was published: *Ut pendet continuum flexile, sic stabit contiguum rigidum inversum*, "As hangs the flexible line, so, but inverted, will stand the rigid arch." In 1748 the Italian engineer Giovanni Poleni used the idea to study cracks in the dome of St Peter's in Rome. He hung a chain loaded with weights proportioned to slices of the dome and showed that the line of thrust stayed inside the dome's thickness, so it was safe, provided iron rings were added around its base. A century and a half later, in Barcelona, Antoni Gaudí designed churches upside down, with strings and little bags of lead shot hanging from the ceiling of his workshop, then photographed them and turned the photos over.

How they figured it out. A chain can only pull; it cannot push or bend. Hanging under its own weight, it takes exactly the shape in which every link is in pure tension. Turn that shape over, and every block of a rigid arch is in pure compression, the condition masonry loves. Mathematicians soon found the equation: in 1691 Leibniz, Huygens and Johann Bernoulli independently showed that a uniform hanging chain follows the hyperbolic cosine, the **catenary**. If the load is not the chain itself but something hung evenly along the horizontal, such as a bridge deck, the shape is a **parabola**. Gaudí's hanging models found the right shape for any combination of loads, without equations.

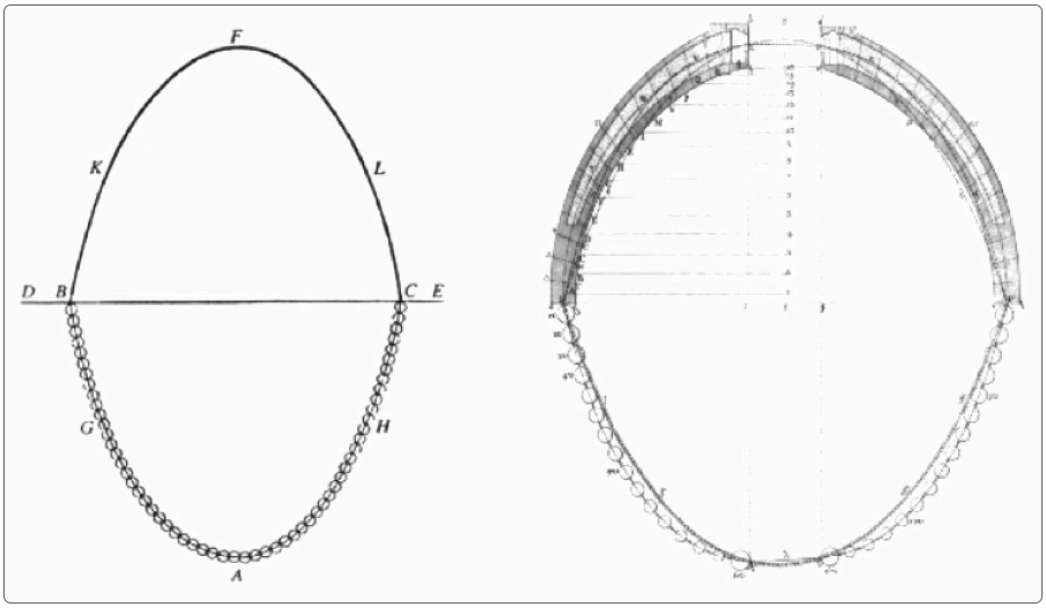
The physics. The catenary is $y = a\cosh(x/a)$, where $a = H/w$ is the ratio of the horizontal tension to the weight per unit length. Turned over, a catenary arch of span L and rise h satisfies $h = a\left(\cosh\frac{L}{2a} - 1\right)$. Near its bottom the catenary looks

like a parabola, $y \approx \frac{x^2}{2a}$, but it is steeper at the ends. For a parabolic arch carrying a uniform load w over span L and rise h, the horizontal thrust is $H = \dfrac{wL^2}{8h}$, as you saw in Lesson 3.2. A semicircular arch is not funicular for its own weight; its line of thrust wanders toward the outside near the haunches, which is why round arches need heavy fill or buttressing there. An arch stands as long as its line of thrust stays within its thickness.

The engineering. The Gateway Arch in St. Louis (1965), 630 ft tall and 630 ft wide, is a weighted catenary: its legs are thicker at the bottom, so the curve was adjusted to match. Gaudí's Sagrada Família in Barcelona, still under construction, is built from his funicular studies, now checked with computers. And engineers analyze old masonry bridges and vaults today by finding whether a line of thrust can fit inside the stone, exactly as Poleni did. The hanging chain is one of the simplest and most beautiful tools in engineering: let gravity find the shape.

Try it. In the build you will hang a chain, measure it, compare it with the catenary and parabola, and build an arch on the inverted shape.

PICTURE STUDY



The Hanging Chain and the Dome of St Peter's, Giovanni Poleni, 1748 · Engraving · *Memorie istoriche della gran cupola del Tempio Vaticano*, Padua

Image: Wikimedia Commons, public domain

Poleni was asked whether the cracked dome of St Peter's in Rome was about to fall. He answered with a hanging chain. On the left, the chain hangs below the line and the arch stands above it as its mirror image; on the right, Poleni draws the chain's curve through the thickness of the real dome.

Look closely

- Find the chain on the left. How do the beads show its shape?
- Compare the arch above the line with the chain below it. Are they mirror images?
- On the right, find the dome's two shells. Does the curved line stay inside the stonework?
- Look at where the dome meets its drum. What might Poleni recommend there?

Talk about it

1. Why does a chain show the shape of a perfect arch?
2. What would it mean if the curve passed outside the dome's stone?

3. Poleni recommended iron rings around the dome. What force would they resist?

Safety first. Wear safety glasses when loading block arches, and keep fingers clear. Hang the chain from something secure.

BUILD

Let Gravity Find the Shape

You will hang a chain against a board, trace and measure it, compare the measurements with the catenary and the parabola, then build an arch of blocks on the inverted curve and compare it with a semicircular arch of the same span.

You will need

- A light, even chain (a ball chain or necklace) about 60 cm long, two pushpins
- A sheet of poster board or cardboard on a wall, a pencil, a ruler
- Cardboard and a craft knife (adult) to cut an arch form
- Blocks: sugar cubes, foam pieces or small wooden blocks, and a paper cup with pennies for loading
- A calculator or spreadsheet

Steps

1. Pin the chain's ends to the board at the same height, 40 cm apart. Let it hang still. Trace its curve carefully.
2. Draw a horizontal axis through the lowest point. Measure the curve's height y above the low point at $x = 0, \pm 5, \pm 10, \pm 15, \pm 20$ cm.
3. Find a from the sag: solve $h = a(\cosh(20/a) - 1)$ by trying values of a on a calculator until it matches your measured sag.
4. Compute the catenary $y = a(\cosh(x/a) - 1)$ and the parabola $y = x^2/(2a)$ at each x. Make a table and plot all three curves.
5. Cut the inverted catenary from cardboard as a form. Build an arch of blocks over it, with heavy blocks or books against the feet. Remove the form.
6. Build a semicircular arch of the same span and similar blocks the same way.
7. **Predict.** Which arch will carry more at its crown? Which will carry more with a load off center, one-quarter of the way across?
8. Load each arch at the crown, then (rebuilt) at the quarter point, until it fails. Record the loads and where the arch hinged open.

9. **Improve.** Hang a weight from the chain at its quarter point and trace the new shape. Build an arch to that shape and test it with a load at the quarter point.

Tips

- A ball chain gives a smooth, even curve.
- Measure y from the lowest point, and keep the board flat against the wall.
- When an arch fails, it usually opens at four hinges; look for where the joints gap.

Check your work

- ☐ My measurements match the catenary more closely than the parabola, or I can explain why not.
- ☐ I tested catenary and semicircular arches under centered and off-center loads.
- ☐ I can explain why an arch shaped to its loads is in pure compression.

ENGINEER'S NOTEBOOK

1. Your traced chain with measurements, the table of measured, catenary and parabola heights, and the plot.
2. Sketches of how each arch failed, with the hinge points marked.
3. A drawing of Gaudí's method: a hanging model and the arch it becomes when turned over.

KEY FORMULAS

$y = a\cosh\left(\frac{x}{a}\right), \quad a = \frac{H}{w}$

$h = a\left(\cosh\frac{L}{2a} - 1\right)$ (sag or rise)

$y \approx \frac{x^2}{2a}$ (parabola, near the bottom)

$H = \frac{wL^2}{8h}$ (parabolic arch or cable)

From history. Hooke's anagram appeared at the end of his 1676 book on telescopes, along with another anagram that hid his law of springs ("as the extension, so the force"). It was a common way for scientists of the time to claim a discovery before they were ready to publish it in full.

Example 1. A chain hangs with $a = 10$ cm between supports 20 cm apart (at $x = \pm 10$ cm). How deep is the sag? Compare with the parabola approximation.

Show solution

1. Catenary: $h = a(\cosh(L/2a) - 1) = 10(\cosh 1 - 1)$.
2. $\cosh 1 = \frac{1}{2}(e + e^{-1}) = \frac{1}{2}(2.718 + 0.368) = 1.543$, so $h = 10(0.543) = 5.43$ cm.
3. Parabola: $h \approx x^2/(2a) = 100/20 = 5.0$ cm.
4. The parabola underestimates the sag by about 8% at this fairly deep curve; for shallow curves the two are almost the same.

Answer: Catenary 5.43 cm; parabola 5.0 cm.

Example 2. A masonry arch of span 10 m and rise 4 m carries a uniform load of 30 kN per meter (including its own weight, treated as uniform along the span). Find its horizontal thrust and the total force at each support.

Show solution

1. Thrust: $H = wL^2/(8h) = 30 \times 100 \div 32 = 93.75$ kN.
2. Vertical reaction: $V = wL/2 = 150$ kN.
3. Total: $R = \sqrt{H^2 + V^2} = \sqrt{93.75^2 + 150^2} = \sqrt{8789 + 22{,}500} \approx 177$ kN, at $\tan^{-1}(150/93.75) \approx 58^\circ$ below horizontal.

Answer: $H \approx 94$ kN; total about 177 kN at each support, at about 58° to the horizontal.

Example 3. Gaudí hung a string model at a scale of 1:10 with weights of 1 g for every 1 kg of real load. A string segment in the model carries 150 g of tension. What real force does it represent?

Show solution

1. Weights scale by 1,000: every 1 g stands for 1 kg.
2. Tension in the model: 150 g, so the real force corresponds to 150 kg, or $150 \times 9.81 \approx 1470$ N.
3. Because every force in the model is scaled by the same factor, the shape is exactly right; only the magnitudes are scaled.

Answer: About 1,470 N (the weight of 150 kg) of compression in the real arch.

PRACTICE

1. Easy State Hooke's principle in your own words.

   Show answer

   The shape of a hanging chain, turned upside down, is the shape of an arch that stands in pure compression under the same loads.

2. Easy What shape does a cable take when it carries a load spread evenly along the horizontal, like a bridge deck?

   Show answer

   A parabola.

3. Easy Compute $\cosh 0$ and $\cosh 2$.

   Show answer

   $\cosh 0 = 1$; $\cosh 2 = \frac{1}{2}(e^2 + e^{-2}) \approx 3.762$.

4. Medium A catenary has $a = 20$ m. How high above its lowest point is it at $x = 10$ m? Compare with the parabola.

   Show answer

   $20(\cosh 0.5 - 1) = 20(1.1276 - 1) \approx 2.55$ m; parabola $100/40 = 2.5$ m.

5. Medium An arch spans 20 m with a rise of 5 m and carries 18 kN/m. Find the thrust.

Show answer

$18 \times 400 \div 40 = 180$ kN.

6. Medium For the arch in Problem 5, find the vertical reaction and the total reaction at each support.

Show answer

$V = 180$ kN; $R = \sqrt{180^2 + 180^2} \approx 255$ kN at 45°.

7. Medium The Gateway Arch is 630 ft tall and 630 ft wide at its base. What is its height-to-span ratio? Is it a shallow or steep arch?

Show answer

1:1, very steep; a steep arch has a small thrust for its height.

8. Hard A chain spans 40 cm and sags 12 cm. Show that $a \approx 18.4$ cm fits $h = a(\cosh(L/2a) - 1)$.

Show answer

$20/18.4 \approx 1.087$; $\cosh 1.087 \approx 1.651$; $18.4 \times 0.651 \approx 12.0$ cm. ✓

9. Hard Why does a semicircular arch need heavy fill above its haunches (the parts between the crown and the springing)?

Show answer

A semicircle is not the funicular shape for its own weight; its line of thrust moves toward the outside face near the haunches, and the arch tends to open there. Weight placed over the haunches pulls the line of thrust back inside the masonry.

10. Hard Conceptual. Why did Gaudí not need to solve any equations to find his arch shapes?

Show answer

Model answer: his hanging string models, with weights in proportion to the real loads, found the funicular shapes automatically by gravity; every string was in pure tension, so the inverted shape was in pure compression.

11. **Challenge** Show that for small x/a, $a(\cosh(x/a) - 1) \approx \dfrac{x^2}{2a}$, using $\cosh u \approx 1 + \dfrac{u^2}{2}$.

Show answer

$$a(\cosh(x/a) - 1) \approx a\left(1 + \frac{x^2}{2a^2} - 1\right) = \frac{x^2}{2a}.$$

COMMON MISTAKES

- Thinking the catenary and the parabola are the same curve. They are close only when the sag is small.
- Using a semicircle as if it were the ideal arch. It is beautiful but not funicular for its own weight.
- Forgetting that an arch's shape is ideal only for one load pattern; moving loads need thickness or bracing.

> **Make it a game · Hang the Chain.** Hang a necklace, string or small chain between two pins, trace its curve onto paper, then flip the drawing: that is an ideal arch. Predict whether the shape is a parabola or a catenary before checking the lesson. Explain, in one minute, how Hooke, Poleni and Gaudí used this trick. Teach it back.

4.5 Load Paths: From Roof to Foundation

OBJECTIVES

- Trace the load path of a building from the roof and floors through beams, columns and walls to the foundation.
- Calculate tributary areas and accumulate column and footing loads floor by floor.
- Explain progressive collapse and alternate load paths using the Ronan Point case, and test a model with and without ties.

> **Warm-up.** Put your hand flat on a table and press down. Trace in words where that force goes: through the tabletop, into the legs, into the floor, into the joists, and then where? Keep going until you reach the ground.

WATCH

How do structures carry wind and seismic loads? An Intro to Lateral Force Resisting Systems

Structures with Prof. H · 4:41

https://www.youtube.com/watch?v=yFX4JLe3JJM

> **WHEN TO WATCH**
> After the engineering paragraph. Notice how sideways loads need their own path: braces, rigid frames or shear walls.

BACKUP VIDEO

The Ronan Point Tower Disaster 1968 | Plainly Difficult Documentary

Plainly Difficult · 16:30

https://www.youtube.com/watch?v=lIvD2agDBnM

With the story, for a fuller account of Ronan Point and what changed afterward.

KEY TERMS

load path

The route a load travels through a structure, from where it is applied down to the ground.

tributary area

The area of floor or roof whose load is carried by a particular beam, column or footing.

gravity and lateral systems

The members that carry vertical loads (slabs, beams, columns, bearing walls) and those that carry sideways loads (braces, rigid frames, shear walls).

alternate load path

A second way for loads to reach the ground if one member is lost, provided by ties, continuity and redundancy.

progressive collapse

A chain reaction in which the failure of one member causes neighbors to fail, spreading through a structure out of proportion to the first damage.

CORE IDEAS

The story. Early on the morning of May 16, 1968, a woman on the 18th floor of Ronan Point, a new 22-story apartment tower in east London, struck a match to light her stove. A leaking gas fitting had filled the kitchen with gas, and the small explosion blew out a load-bearing wall panel at the corner of the building. Ronan Point was built from large precast concrete panels stacked like a house of cards, each floor resting on the wall panels below. With one panel gone, the floors above it fell, and their weight crushed the floors below, all the way to the ground. One whole corner of the tower collapsed. Four people died. Because it was early morning, many of the living rooms in the collapsed corner were empty; it could have been far worse.

How they figured it out. The inquiry found that the explosion was not large; the problem was that the building had no second way to carry its loads. The panels were joined by little more than their weight and some bolts and mortar. Nothing tied them together into a whole. Britain changed its building regulations in 1970 to require buildings to be designed against **disproportionate collapse**: either the structure must be tied together so that it can bridge over a lost member, or key elements must be strong enough to survive accidental damage. Similar rules now apply in many countries.

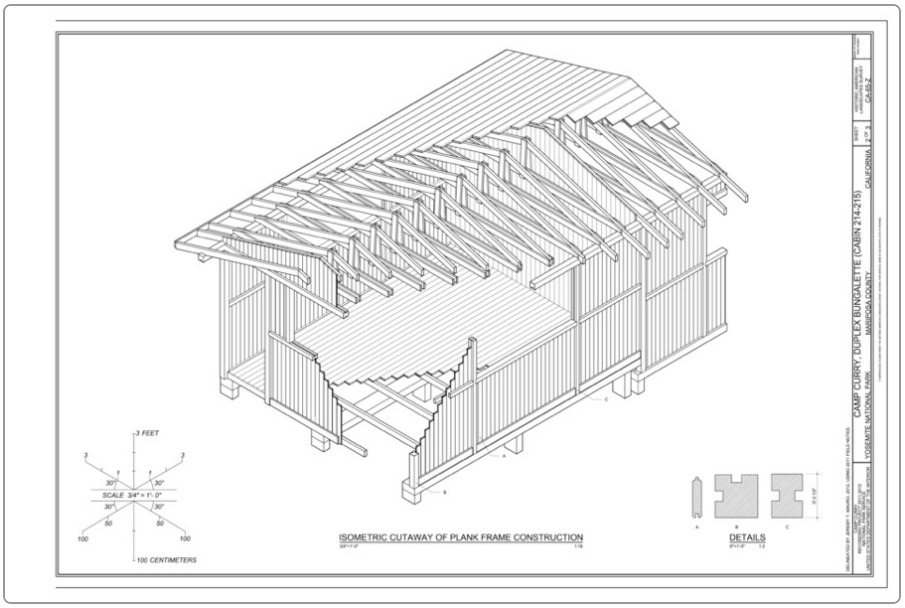
The physics. Every load must have a **load path** to the ground, and every link in the path must be strong enough. A typical path: the roof deck or floor slab spans to **beams**; beams carry it to **girders**; girders deliver it to **columns** (or bearing walls); columns carry it down, collecting load from every floor, to **footings**, which spread it into the soil. To find how much each member carries, engineers use **tributary areas**: a column in a grid of bays carries the floor area halfway to each neighboring

column. An interior column in a grid of 6 m × 8 m bays has a tributary area of 48 m^2 per floor; an edge column about half that; a corner column about a quarter. A column at the bottom of a five-story building carries five floors' worth plus the roof. Sideways loads (wind and earthquakes) need their own path through braces, rigid frames or shear walls, as you saw in the video.

The engineering. Good structural design makes the load path clear, direct and continuous: columns stacked over columns, walls over walls, no surprise loads landing on members not designed for them. It also builds in **redundancy**, so that if one member is lost, the loads can find another way. Steel and concrete frames with continuous beams and well-tied connections can often bridge over a lost column; the floors hang briefly as catenaries or cantilevers until help comes. Engineers today check important buildings by imagining a column removed and asking: what happens next?

Try it. In the build you will make a stacked card "tower," remove one wall panel, and see what happens; then you will tie the panels together and try again.

PICTURE STUDY



Isometric Cutaway of Plank Frame Construction, Camp Curry, Yosemite, Historic American Landscapes Survey, 2012 · Measured drawing · Library of Congress, Prints and Photographs Division

Image: Wikimedia Commons, public domain

This drawing cuts away part of a simple cabin in Yosemite National Park to show how it is built. Every piece of the load path is visible, from the roof boards to the blocks on the ground.

Look closely

- Start at the roof boards. What do they rest on?
- Follow the rafters down. What carries them at their lower ends?
- Find the floor. What supports the floor boards, and what supports those?
- Find the posts and blocks at the bottom. How is the load spread into the ground?

Talk about it

1. Trace with your finger the path a heavy snow load would take from the roof to the ground.
2. If one floor joist broke, could the floor boards carry the load to the neighboring joists? Why?
3. Look at the isometric scale in the corner. How is it different from a perspective view?

> **Safety first.** Wear safety glasses for the collapse test, and build the model tower on the floor or a low table.

BUILD

Remove a Wall: Progressive Collapse and Ties

You will build a model tower of card wall panels and floor plates stacked without ties, remove one lower wall panel, and record what happens. Then you will tie the panels together and repeat.

You will need

- Index cards or card stock for walls and floors (all the same size)
- Clear tape, thin string or dental floss for ties
- Coins or small washers to load each floor equally
- A ruler and a phone to film the test

Steps

1. Design a tower of at least five stories: each story has four card walls standing on edge (folded slightly into an L at the corners so they stand) and a card floor plate on top.
2. Place the same number of coins on each floor as a "live load." Count and record them.
3. **Predict.** If you slide out one corner wall on the second story, how many floors will collapse?
4. Gently slide out one corner wall on the second story with a straightedge. Film it. Record how many floors fell and how far the damage spread.
5. Rebuild the tower the same way, but tie it together: tape the walls to the floor plates at every joint, and run a string tie around each floor's edge (like a belt) taped in place.
6. Remove the same wall. Record what happens. Did the floor above hang from its ties or bridge over the gap?
7. Try a third version with a different alternate load path, for example a strong beam (a folded card) along the edge of each floor.
8. **Improve.** Using the least extra tape and string, find a tying scheme that keeps all floors standing when one wall is removed. Record what you used.

Tips

- Build all versions with the same cards and loads so the test is fair.
- Slide the wall out slowly, without bumping anything else.
- Watch the film in slow motion to see how the collapse spread.

Check your work

- ☐ I can trace the load path of a building and compute tributary areas and column loads.
- ☐ My tests compare an untied and a tied tower after removing a wall.
- ☐ I can explain why Ronan Point collapsed and what changed in building rules.

ENGINEER'S NOTEBOOK

1. A floor plan of a building on a column grid with the tributary area of an interior, an edge and a corner column shaded, and their loads calculated.
2. A section drawing of a small building with arrows showing the load path from roof to soil.
3. Your collapse-test results for each version, with sketches of the damage.

KEY FORMULAS

$A_{\text{trib}} = \left(\frac{L_1}{2} + \frac{L_2}{2}\right)\left(\frac{W_1}{2} + \frac{W_2}{2}\right)$ (interior column)
$P_{\text{column}} = \sum_{\text{floors}} q_u A_{\text{trib}}, \quad q_u = 1.2D + 1.6L$
$w_{\text{beam}} = q \times (\text{tributary width})$

From history. Ronan Point was repaired and reoccupied, but residents never trusted it, and in 1986 it was carefully taken apart panel by panel. The engineers who dismantled it found many joints poorly packed with mortar, and even some stuffed with newspaper, confirming that workmanship mattered as much as design.

Example 1. A floor has columns on a grid of 6.0 m × 8.0 m bays. The floor dead load is 5.0 kPa and live load 2.5 kPa. Find the factored load per floor on an interior column.

Show solution

1. Tributary area: halfway to each neighbor in both directions: $6.0 \times 8.0 = 48\ \mathrm{m}^2$.
2. Factored pressure: $1.2(5.0) + 1.6(2.5) = 6.0 + 4.0 = 10.0\ \mathrm{kPa}$.
3. Load per floor: $10.0 \times 48 = 480\ \mathrm{kN}$.

Answer: 480 kN per floor.

Example 2. The building in Example 1 has four floors and a roof (roof factored load 6.0 kPa). Find the factored load on the interior column at the ground floor, and on the corner column.

Show solution

1. Interior column: four floors × 480 kN = 1920 kN, plus roof $6.0 \times 48 = 288$ kN; total 2208 kN.
2. Corner column: tributary area $3.0 \times 4.0 = 12\ \mathrm{m}^2$ per level.
3. Corner: $4 \times (10.0 \times 12) + 6.0 \times 12 = 480 + 72 = 552$ kN, a quarter of the interior column.

Answer: Interior about 2,210 kN; corner about 550 kN.

Example 3. Floor joists span 14 ft between beams and are spaced 16 in apart. The floor load is 50 psf (dead plus live, unfactored). Find the load per foot on one joist and the reaction at each end.

Show solution

1. Tributary width of one joist: its spacing, $16 \text{ in} = 1.333 \text{ ft}$.
2. Load per foot: $w = 50 \times 1.333 \approx 66.7 \text{ lb/ft}$.
3. Total on the joist: $66.7 \times 14 \approx 933$ lb; each end reaction is half, about 467 lb, which goes into the beam.

Answer: About 66.7 lb/ft; about 467 lb at each end.

PRACTICE

1. Easy List the parts of a typical load path from a floor to the soil.

   Show answer

   Slab (or deck) → beams/joists → girders → columns or bearing walls → footings → soil.

2. Easy Find the tributary area of an interior column on a 5 m × 7 m grid.

   Show answer

   35 m^2.

3. Easy What is progressive collapse?

   Show answer

   A failure that spreads from one damaged member to its neighbors, so the collapse is far larger than the first damage.

4. Medium An edge column (not a corner) on a 5 m × 7 m grid has what tributary area?

   Show answer

   About half an interior one: $2.5 \times 7 = 17.5 \text{ m}^2$ (or 5×3.5, depending on which edge).

5. **Medium** A floor has $D = 4$ kPa and $L = 3$ kPa. Find the factored load on a column with a tributary area of 36 m^2.

   Show answer

   $(4.8 + 4.8) \times 36 = 9.6 \times 36 = 345.6$ kN.

6. **Medium** Joists 12 in apart span 12 ft and carry 55 psf. Find the load per foot and the end reaction of one joist.

   Show answer

   55 lb/ft; total 660 lb; 330 lb at each end.

7. **Medium** A beam carries joists from a 10 ft strip of floor on each side (tributary width 10 ft) at 60 psf. Find the load per foot on the beam.

   Show answer

   $60 \times 10 = 600$ lb/ft.

8. **Hard** A six-story building has interior columns with 64 m^2 tributary area. Each floor is 10 kPa factored and the roof 7 kPa. Find the ground-floor column load. If the footing soil can safely carry 300 kPa (factored for this check), what area of footing is needed?

   Show answer

   $6 \times 640 + 448 = 3840 + 448 = 4288$ kN; area $= 4288 \div 300 \approx 14.3$ m^2, about 3.8 m square.

9. **Hard** In a frame with continuous beams, if an interior ground-floor column is removed, the beams above must span twice as far. By what factor does the bending moment in a uniformly loaded beam increase when its span doubles?

   Show answer

   $M = wL^2/8$, so doubling L multiplies M by 4. The beams must be strong enough (or tied well enough to act as catenaries) to survive that.

10. Hard Conceptual. Why did Ronan Point collapse so far beyond the panel that was blown out?

Show answer

Model answer: the floors above rested on that one panel with no ties or continuity to carry their weight another way, so they fell; their falling weight and impact then overloaded the floors below, which also had no alternate load path, and the collapse ran down the whole corner.

11. Challenge Two designs carry a 4,000 kN roof-and-floor load. Design A uses four columns; design B uses nine columns on a grid three by three, with the same total load. Ignoring edge effects, compare the load on the most heavily loaded column and discuss the redundancy of each.

Show answer

A: about 1,000 kN each. B: an interior column of a 3 × 3 grid with equal bays carries about four times a corner column; with tributary shares 4:2:1 (interior: edge: corner) and 1 interior, 4 edge, 4 corner columns, $4 + 8 + 4 = 16$ shares, so the interior carries $4000 \times 4/16 = 1000$ kN, edges 500 kN, corners 250 kN. B has more columns, so losing one leaves more alternate paths: it is more redundant.

COMMON MISTAKES

- Forgetting to add the loads from every floor above when sizing a column.
- Giving edge and corner columns the same tributary area as interior ones.
- Assuming a structure that carries its normal loads is robust. Robustness means surviving the loss of a member.

> **Make it a game · Trace the Load.** Draw a three-floor building as a stack of boxes. Your parent picks a point on the roof; you trace the load path down to the footing, naming each element. Then compute the tributary area of one column and how much load accumulates floor by floor. Wrong path costs a point. Swap and compete.

UNIT 4

Structures I: Forces and Loads

Firmness begins with forces. In this unit you will learn the core of structural engineering: what loads a building must carry, how to keep it in equilibrium, how forces divide among the members of a truss, why arches take the shapes they do, and how every load must find a path to the ground.

The stories are of engineers learning by thought and by failure: Stevin's chain of balls, the Forth Bridge's human model, Whipple's truss book, Hooke's anagram, Poleni's chain and Gaudí's upside-down models, the snow-loaded roofs at Hartford and the Knickerbocker, and the gas explosion at Ronan Point. You will solve real statics problems, analyze trusses by the method of joints, measure reactions on kitchen scales, and build models that stand, sag, carry and collapse in ways you can predict.

PREREQUISITES

- Algebra: solving linear equations and systems of two equations
- Right-triangle trigonometry: sine, cosine and tangent
- Units 1–3: weight, pressure, stress, arch thrust ($H = wL^2/8h$)
- A scientific calculator with e^x (or cosh)

MASTERY CHECKLIST

- ☐ I can identify and estimate dead, live, snow, wind and seismic loads and combine them with load factors.
- ☐ I can draw free-body diagrams and solve for reactions using

$\sum F$ and

0

$\sum M = 0$.

- ☐ I can find every member force of a simple truss by the method of joints and say which are in tension and which in compression.
- ☐ I can explain Hooke's hanging-chain principle and calculate with the catenary and parabola.
- ☐ I can trace load paths, compute tributary areas and column loads, and explain progressive collapse.

OPTIONAL KITS FOR THIS UNIT

- **KEVA planks**: Identical maple planks for arch-building and stacking tests in Lessons 4.4 and 4.5.
- **Pitsco Education (structures supplies)**: Sells balsa and basswood strips, bridge and truss kits and structure testers used in schools; useful for Lesson 4.3 and Unit 6. Every build here also works with straws, string and craft sticks.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

4.1	**Loads: Dead, Live, Wind, Snow and Seismic**	3 sessions of 50 minutes
4.2	**Statics and Equilibrium**	3 sessions of 50 minutes
4.3	**Trusses by the Method of Joints**	3 sessions of 50 minutes
4.4	**Arches and the Catenary: Hooke, Poleni and Gaudí**	3 sessions of 50 minutes
4.5	**Load Paths: From Roof to Foundation**	3 sessions of 50 minutes

Unit 4 test

Time allowed: 50 minutes. Calculators are allowed. Draw free-body diagrams where they help. Show your work and units for every calculation. The test is worth 100 points.

1. **5 pts**
   Define dead load and live load and give two examples of each.

2. **6 pts**
   A floor has $D = 20$ psf and $L = 40$ psf. Find the factored load ($1.2D + 1.6L$) and the total on a 15 ft × 20 ft room.

3. **6 pts**
   Find the wind pressure for a 120 mph wind, and the force on a wall 30 ft by 10 ft.

4. **5 pts**

   Why did the Hartford Civic Center roof fall even though the snow load that night was below its supposed design load?

5. **7 pts**

   A 9 m beam has a 30 kN load 3 m from the left support and a 15 kN load 7 m from the left support. Find the reactions.

6. **6 pts**

   A 16 ft beam carries 500 lb/ft over its full length. Find the reactions.

7. **7 pts**

   A 1,000 N lamp hangs from two cables, each at 40° above the horizontal on opposite sides. Find the tension in each.

8. **8 pts**

   A triangular truss spans 10 m with its apex 2.5 m high and carries 20 kN at the apex. Find the force in each rafter and in the tie, with T or C.

9. **8 pts**

   For the three-panel truss of Lesson 4.3 (span 12 m, panels 4 m, height 3 m) with a single 12 kN load at C (the joint 8 m from A), find the reactions and the forces in FD, CD and CF.

10. **5 pts**

    Check whether a truss with 11 members, 7 joints and 3 reactions is statically determinate.

11. **6 pts**

    A chain hangs with $a = 15$ cm between supports 30 cm apart. Find its sag with the catenary and with the parabola approximation.

12. **6 pts**

    A parabolic arch spans 24 m, rises 6 m and carries 25 kN/m. Find the horizontal thrust and the vertical reaction at each support.

13. **5 pts**

    State Hooke's principle and explain why it works.

14. **7 pts**

A five-story building has interior columns on a 7 m × 7 m grid. Each floor is 9 kPa factored and the roof 6 kPa. Find the ground-floor interior column load.

15. **5 pts**

What is an alternate load path, and why do building rules require one?

16. **8 pts**

Short essay. Using Ronan Point and Hartford, explain why engineers must check their assumptions and design for what can go wrong, not only for the expected loads.

ANSWER KEY

1. Dead load: permanent weight of the structure and fixed parts (e.g., floor framing, roofing, walls). Live load: people and movable contents (e.g., a congregation, furniture, stored books).
2. $24 + 64 = 88$ psf; $88 \times 300 = 26{,}400$ lb.
3. $0.00256 \times 14{,}400 \approx 36.9$ psf; $36.9 \times 300 \approx 11{,}100$ lb.
4. Its long compression members were not braced against buckling as the design model assumed; some had likely buckled already, and the excess deflection during construction was a missed warning.
5. Moments about left: $9R_R = 90 + 105 = 195$, $R_R \approx 21.7$ kN; $R_L = 45 - 21.7 \approx 23.3$ kN.
6. Total 8,000 lb; 4,000 lb each.
7. $2T \sin 40^\circ = 1000$, $T = 1000 \div 1.286 \approx 778$ N.
8. Rafter length $\sqrt{25 + 6.25} \approx 5.59$ m, $\sin\theta \approx 0.447$, $\cos\theta \approx 0.894$. Rafters: $10 \div 0.447 \approx 22.4$ kN C; tie $22.4 \times 0.894 \approx 20.0$ kN T.
9. $R_A = 12(4)/12 = 4$ kN, $R_D = 8$ kN. Joint D: $F_{FD}(0.6) = 8$, $F_{FD} \approx 13.3$ kN C; $F_{CD} = 13.3 \times 0.8 \approx 10.7$ kN T. Joint F: vertical, $F_{CF} = F_{FD}(0.6) = 8$ kN T (no diagonal meets F other than FD in this layout).
10. $m + r = 14 = 2j$; determinate.
11. Catenary: $15(\cosh 1 - 1) \approx 8.15$ cm; parabola $225/30 = 7.5$ cm.
12. $H = 25 \times 576 \div 48 = 300$ kN; $V = 25 \times 12 = 300$ kN.
13. A hanging chain inverted gives the ideal arch. The chain can only pull, so its shape puts every link in pure tension under its loads; the mirror shape puts every part of a rigid arch in pure compression under the same loads.
14. $A = 49$ m^2; floors $5 \times 441 = 2205$; roof 294; total 2499 kN.
15. A second route for loads to reach the ground if a member is lost, through ties, continuity or redundancy; it prevents a local failure from becoming a progressive collapse, as at Ronan Point.
16. Model answer: at Hartford the design assumed bracing that did not exist and engineers dismissed unexpected deflections; at Ronan Point the designers assumed walls would never be removed, so the panels were not tied. In both, the structure carried its normal loads but had hidden weaknesses. Engineers must check that their models match reality, heed warning signs, and give structures

robustness (redundancy and ties) so that accidents and errors do not become disasters.

PORTFOLIO PIECE: A LOAD PATH AND TRUSS REPORT

Choose a real structure you can visit: a roof truss in an attic or garage, a pedestrian truss bridge, or a picnic shelter. Sketch it, measure it, and draw its load path from the roof or deck to the ground.

Estimate its loads (dead, live and snow for your town), analyze its main truss or frame with the method of joints or with beam reactions, and identify the most heavily loaded member. Then build a scale model of the truss, predict and test which members are in tension, and write a two-page report with your drawings, calculations and test results.

LOOK FOR	WHAT IT MEANS
Observation	Accurate measured sketch of a real structure with members and connections identified.
Loads	Reasonable dead, live and snow loads with sources, combined with load factors.
Analysis	Correct free-body diagrams, reactions and member forces with T or C and units.
Model test	Model built to scale; tension and compression predictions tested and discussed honestly.
Report	Clear, organized writing with labeled drawings and a conclusion about the structure's weakest link.

UNIT 5

Structures II: Materials and Members

A structure is only as good as what it is made of. In this unit you will study the four great structural materials, timber, masonry, steel and concrete, through the people who learned to use them: the Chicago carpenters of the balloon frame, Burnham and Root and the massive walls of the Monadnock, Bessemer and Eads, and Aspdin, Monier, Hennebique and Freyssinet. For each you will learn how it behaves: where it is strong and weak, how it changes with water, heat and time, and how engineers design with it.

The unit ends where Galileo began the science of structures: with the beam. You will calculate bending moments, stresses and deflections, size a real floor joist, and test model beams to find out why depth is everything. These are the calculations you will use in bridges, towers and foundations for the rest of the course.

PREREQUISITES

- Unit 4: loads, reactions and statics
- Algebra: formulas with powers (h^2, h^3, L^4) and solving for a variable
- Unit conversions between SI and US units (MPa, psi, ksi, kN, lb)

MASTERY CHECKLIST

- ☐ I can explain how wood's grain and moisture affect its strength and check a stud for compression and bearing.
- ☐ I can calculate the base stress in a masonry wall and the thickness needed for a given height and load.
- ☐ I can use $\sigma = P/A$, $\varepsilon = \delta/L$, and $\delta = PL/AE$, and read a stress-strain curve.
- ☐ I can explain the water-cement ratio and calculate the moment capacity of a simple reinforced concrete beam.
- ☐ I can calculate M, I, S, bending stress and deflection, and size a joist for strength and stiffness.

OPTIONAL KITS FOR THIS UNIT

- **Pitsco Education (balsa and basswood)**: Precision balsa and basswood strips and sheets for the grain test (5.1) and beam tests (5.5), and the bridge competition in Unit 6. Craft-store balsa also works.
- **KEVA planks**: Useful for masonry-bond walls in Lesson 5.2 and for testing a plank flat versus on edge in Lesson 5.5.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

5.1	**Timber: The Balloon Frame and Wood as a Structural Material**	3 sessions of 50 minutes
5.2	**Masonry: The Monadnock and the Limits of Brick**	3 sessions of 50 minutes
5.3	**Steel: Bessemer, Eads and the Stress-Strain Curve**	3 sessions of 50 minutes
5.4	**Concrete: Roman, Modern, Reinforced and Prestressed**	3 sessions of 50 minutes
5.5	**Beams: Bending and Deflection from Galileo to Today**	3 sessions of 50 minutes

Unit 5 Test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation. Use actual (not nominal) lumber sizes. The test is worth 100 points.

1. **5 pts**
   Why did the platform frame replace the balloon frame?

2. **6 pts**
   A 2×4 stud carries 2,100 lb. Find the stress in the stud and the bearing stress on the plate. Is it acceptable with limits of 1,350 psi and 625 psi?

3. **6 pts**
   Find the self-weight stress at the base of a 45 m brick wall ($\gamma = 19\ \mathrm{kN/m^3}$).

4. 7 pts

The wall in Problem 3 also carries 360 kN/m from its floors. With an allowable stress of 1.4 MPa, what is the minimum thickness?

5. 5 pts

Why was the Monadnock the end of tall load-bearing masonry in Chicago?

6. 7 pts

A steel tie 20 mm in diameter and 4.0 m long carries 45 kN. Find the stress, the stretch ($E = 200$ GPa), and the factor of safety against a 250 MPa yield.

7. 6 pts

Sketch and label a stress-strain curve for mild steel, and explain what ductility means for safety.

8. 5 pts

Why did Eads insist on testing every batch of steel?

9. 6 pts

A mix has 360 kg of cement per cubic meter and a water-cement ratio of 0.45. How much water does it have? What happens to the strength if the ratio rises to 0.60?

10. 8 pts

A concrete beam is 300 mm wide with $d = 550$ mm, $A_s = 1200\ \text{mm}^2$, $f_y = 415$ MPa and $f\prime_c = 30$ MPa. Find a and M_n.

11. 5 pts

What does prestressing do for a concrete beam?

12. 6 pts

A 5 m beam carries 8 kN/m. Find the maximum moment. A rectangular wood beam 90 mm wide must keep its stress below 10 MPa. What depth is needed?

13. 6 pts

A 2×12 (1½ in × 11¼ in) joist spans 16 ft at 16 in on center with 50 psf. Find the bending stress.

14. **7 pts**

A beam deflects 8 mm. Find its deflection if (a) its depth is doubled, (b) its span is doubled (same load per meter).

15. **7 pts**

A plank 240 mm × 40 mm is used as a beam. How many times stronger and stiffer is it on edge than flat?

16. **8 pts**

Short essay. Compare timber, masonry, steel and reinforced concrete: for each, give one great strength, one weakness, and how engineers design around the weakness.

ANSWER KEY

1. In the balloon frame the long studs leave open cavities from basement to attic that spread fire; the platform frame caps each story with a floor platform, which blocks fire and is easier to build with shorter lumber.
2. $2100 \div 5.25 = 400$ psi for both; acceptable.
3. 855 kPa.
4. $t = 360 \div (1400 - 855) \approx 0.66$ m.
5. At sixteen stories its walls had to be about 6 ft thick at the base, using up valuable floor space; taller walls would be impractical or impossible, while steel frames could carry the loads on slender columns.
6. $A \approx 314\ \text{mm}^2$; $\sigma \approx 143$ MPa; $\delta = 143 \times 4000/200{,}000 \approx 2.9$ mm; $FS \approx 1.75$.
7. Straight elastic line to the yield point, a yield plateau, strain hardening up to the ultimate strength, then necking to fracture. Ductility means steel stretches a lot before breaking, giving warning and redistributing load.
8. Early steel was not uniform, and he would not trust the mill's claims; testing ensured each piece met his specification, which made the bridge safe and pushed the industry toward reliable standards.
9. 162 kg of water; the strength drops substantially (more water leaves more voids).
10. $a = 498{,}000 \div (0.85 \times 30 \times 300) \approx 65.1$ mm;
    $M_n = 498{,}000 \times (550 - 32.6) \approx 2.58 \times 10^8$ N·mm $= 258$ kN·m.
11. Tensioned tendons squeeze the concrete in advance, so service loads must first overcome that compression before tension can develop; the beam stays uncracked and deflects less.
12. $M = 8 \times 25/8 = 25$ kN·m; $S = 25 \times 10^6/10 = 2.5 \times 10^6\ \text{mm}^3 = 90h^2/6$,
    $h^2 \approx 166{,}700$, $h \approx 408$ mm.
13. $w = 66.7$ lb/ft; $M = 66.7 \times 256/8 \approx 2134$ ft·lb $= 25{,}600$ in·lb;
    $S = 1.5 \times 126.6/6 \approx 31.6\ \text{in}^3$; $\sigma \approx 810$ psi.
14. (a) $8/8 = 1$ mm; (b) $8 \times 16 = 128$ mm.
15. Depth ratio 6: 6 times stronger, 36 times stiffer.
16. Model answer: timber is light and strong along the grain but weak across it and vulnerable to moisture, fire and rot (designers check bearing, keep it dry, protect or oversize it for fire); masonry is durable, fire-resistant and strong in compression but weak in tension and heavy (designers keep it in compression, bond it, reinforce it, and limit height); steel is strong, stiff and ductile but rusts

and weakens in fire (paint, galvanize, fireproof); reinforced concrete is moldable, durable and fire-resistant but weak in tension and prone to cracking (steel on the tension side, cover, control joints, prestressing).

PORTFOLIO PIECE: A MATERIALS TEST REPORT AND A DESIGNED BEAM

Choose two of this unit's builds and write them up as a formal test report: purpose, materials, procedure, data, graphs, results compared with theory, and sources of error. Then design a beam of balsa or basswood to span 40 cm and carry a 2 kg center load with a factor of safety of 2, using your measured strength and E to size it.

Build it, predict its deflection under 2 kg, test it to failure, and compare the measured deflection and failure load with your predictions. Report its efficiency: failure load divided by beam weight.

LOOK FOR	WHAT IT MEANS
Test reports	Two complete reports with clear procedures, data tables with units, graphs and honest error discussion.
Design calculation	Beam sized from measured properties with moment, section modulus and deflection calculated correctly.
Prediction vs result	Predicted deflection and failure load compared with measurements, with percent differences and explanations.
Efficiency	Beam weight, failure load and efficiency recorded; design choices (such as an I-shape) explained.
Presentation	Report is neat, organized and written in complete sentences.

5.1 Timber: The Balloon Frame and Wood as a Structural Material

OBJECTIVES

- Explain how the balloon frame changed building in America, and how wood's grain, moisture and species govern its strength.
- Calculate stresses in wood studs and posts, including bearing across the grain, and compare wood's strength-to-weight with steel.
- Test wood strips along and across the grain and record the difference.

> **Warm-up.** Take a wooden craft stick. Try to snap it by bending it across its length, then try to split it along its length with your thumbnails. Which is easier? Look closely at the broken ends and the split faces. What do you see?

WATCH

Balloon Framing! How it started, why it happened, how it matters.

Brent Hull · 11:07

https://www.youtube.com/watch?v=FcZYsgr4x38

> **WHEN TO WATCH**
> After the story. List the three things he says made balloon framing possible.

Note: A lecture by a historic-preservation builder and millwork craftsman, filmed in his shop.

BACKUP VIDEO

Wood's properties: Introduction and concepts

WoodSolutions Campus · 3:18

https://www.youtube.com/watch?v=SrWcYWzJJJ8

With the physics paragraph: a short overview of wood's structural, durability and fire properties.

KEY TERMS

balloon frame

A light wood frame of sawn studs running the full height of the building from sill to roof, fastened with machine-made nails (1830s onward).

platform frame

The modern method: each story is framed as a floor platform, and the walls of the next story are built on it.

grain

The direction of the wood fibers. Wood is far stronger and stiffer along the grain than across it.

moisture content

The weight of water in wood as a percentage of its dry weight. Wood shrinks and weakens as it gets wetter, up to about 30%.

nominal size

The name of a lumber size, such as 2×4; its actual dressed size is smaller, 1½ in × 3½ in.

mass timber

Large engineered wood panels and beams, such as cross-laminated timber (CLT) and glulam, used in tall wood buildings.

CORE IDEAS

The story. In 1833 Chicago was a muddy village of a few hundred people. That year a carpenter named Augustine Taylor built St. Mary's, the town's first Catholic church, by a new method. Instead of heavy posts and beams cut and joined by skilled timber framers, he used slender sawn boards, 2 by 4 inches and the like, set close together like the ribs of a basket and nailed with cheap machine-made nails. Skeptics said such a light frame would blow away like a balloon, and the name stuck. But the balloon frame was fast, cheap and needed fewer skilled hands. By 1850 Chicago had nearly 30,000 people, and light wood framing spread west with the railroads. Most American houses are still built of light wood frames today.

How they figured it out. The balloon frame depended on three industrial changes: steam-powered sawmills that cut standard lumber, nail-making machines that made nails cheap, and railroads that carried both across the country. Builders learned its strengths and weaknesses by experience. The long studs ran unbroken from the foundation to the roof, so a fire starting in the basement could rush up the hollow wall cavities to the attic. By the 1900s builders changed to the **platform frame**, in

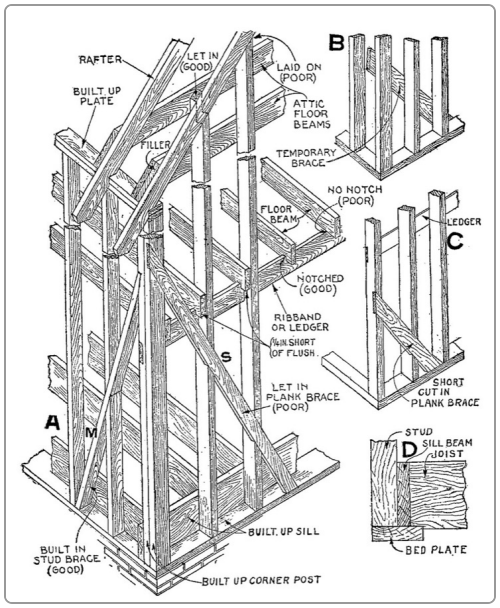
which each floor is a platform that caps the walls below and blocks the path of fire, and codes now require fire-blocking in walls.

The physics. Wood is a natural composite of long cellulose fibers bound together, like a bundle of drinking straws glued side by side. Along the **grain** it is strong and stiff; across the grain it is weak and soft, and it splits easily. A typical design value for Douglas fir framing lumber is about 1,350 psi in compression along the grain, but only about 625 psi in compression across the grain, where a stud bears on the horizontal plate beneath it. Wood also changes with water: below about 30% moisture it shrinks as it dries, more across the grain than along it, and it is weaker and creeps more when wet. Kept dry, wood lasts for centuries; kept wet, fungi rot it. For its weight, wood is remarkably strong: along the grain, good timber is about as strong per kilogram as structural steel, or stronger.

The engineering. Timber design means respecting the grain. Engineers check studs and posts for crushing and buckling along the grain, plates and sills for crushing across the grain, beams for bending (Lesson 5.5), and connections for splitting near bolts and nails. They also design for moisture, keeping wood dry with roofs, flashing and ventilation, and for fire with protective gypsum board or by making members thick enough to char on the outside while the inside keeps its strength. Today engineered timber such as glulam and cross-laminated timber (CLT), made of boards glued in layers, lets wood reach new heights: the Ascent tower in Milwaukee, completed in 2022, stands 25 stories on a mass-timber structure.

Try it. In the build you will hang loads from thin wood strips cut along the grain and across it, and compare their strengths.

PICTURE STUDY



The Balloon Frame, Illustration from an early twentieth-century carpentry handbook, early 1900s · Line engraving · Wikimedia Commons

Image: Wikimedia Commons, public domain

Carpentry handbooks taught builders the balloon frame with drawings like this one, which labels each part and marks good and poor ways of joining them. Look at it as a carpenter and as an engineer.

Look closely

- Find the long studs. Where do they begin and end?
- Find the ribbon (or ledger) board that carries the second-floor joists. How is it attached to the studs?
- Find the diagonal braces. What job do they do?
- Find the labels “good” and “poor.” What is the difference between the good and poor details?

Talk about it

1. Why might a fire spread quickly up the cavities between these studs?
2. Which parts of this frame carry load along the grain, and which across it?
3. How does this light frame differ from Paxton’s iron frame in Lesson 3.4, and how is it similar?

> **Safety first.** An adult should cut thin wood strips with a craft knife or fine saw. Wear safety glasses for the breaking tests.

BUILD

With the Grain and Against It

You will cut thin strips from a sheet of balsa or basswood both along and across the grain, load them in tension and in bending until they break, and measure how much stronger the wood is along its fibers. Then you will see how wetting changes it.

You will need

- A sheet of balsa or basswood about 1.5–2 mm thick
- A metal ruler and a craft knife (adult) or a fine-tooth saw
- A paper cup, string, binder clips and pennies or sand; a kitchen scale
- Two books as supports for bending tests; a cup of water

Steps

1. Cut six strips 1 cm wide and 10 cm long with the grain running along the strip, and six with the grain running across the strip.
2. Measure each strip's width and thickness with a ruler and calculate its cross-sectional area in mm^2.
3. **Predict.** How many times stronger will the along-grain strips be in tension?
4. **Tension test.** Clamp a strip at the top with a binder clip hung from a hook, clip the cup to the bottom, and add sand slowly until it breaks. Record the load. Test three of each kind.
5. Compute the breaking stress, $\sigma = F/A$, in MPa for each, and average them.
6. **Bending test.** Span strips of each kind across a 6 cm gap and hang the cup at midspan. Record the breaking load.
7. Soak two along-grain strips in water for 30 minutes, blot them, and test them in tension. Compare with the dry results.
8. Make a table and a bar chart of average strengths for along-grain, across-grain and wet strips.
9. **Improve.** Glue two across-grain strips to one along-grain strip in a three-layer sandwich (a tiny cross-laminated panel) and test it in bending both ways. What does cross-lamination do?

Tips

- Cut carefully; a nick in the edge makes a strip fail early.
- Add load slowly and steadily.
- Keep track of which strips are which with a pencil mark.

Check your work

- ☐ My data show the difference between strength along and across the grain.
- ☐ I can calculate bearing stress across the grain and explain why it often controls in wood framing.
- ☐ I can explain why the platform frame replaced the balloon frame.

ENGINEER'S NOTEBOOK

1. A labeled sketch of a balloon frame and a platform frame side by side, with the fire path marked in each.
2. Your test data table with areas, loads and stresses in MPa, and the bar chart.
3. A short paragraph on how moisture affects wood and how builders keep it dry.

KEY FORMULAS

$\sigma = \dfrac{P}{A}$ (compression or tension)

$\sigma_{\perp} = \dfrac{P}{A_{\text{bearing}}}$ (bearing across the grain)

$\text{specific strength} = \dfrac{\sigma_{\text{strength}}}{\rho}$

2×4 actual size $1\frac{1}{2}'' \times 3\frac{1}{2}''$, $A = 5.25\ \text{in}^2$

From history. Writers spread the balloon frame as much as builders did: the farm writer Solon Robinson praised it in the 1850s, and carpentry manuals such as William Bell's *Carpentry Made Easy* (1858) taught it step by step. Historians still debate whether Augustine Taylor or the businessman George Washington Snow first used it in Chicago.

Example 1. A wall of 2×4 studs at 16 in on center carries a roof and floor load of 800 lb per foot of wall. Find the load per stud, the compressive stress in the stud, and the bearing stress on the plate. Compare with 1,350 psi along the grain and 625 psi across the grain.

Show solution

1. Each stud carries 16 in of wall: $800 \times (16/12) \approx 1067$ lb.
2. Area of a 2×4: $1.5 \times 3.5 = 5.25$ in^2.
3. Stress in the stud (along the grain): $1067 \div 5.25 \approx 203$ psi, well under 1,350 psi.
4. Bearing on the plate (across the plate's grain): also about 203 psi on 5.25 in^2, under 625 psi. Both pass, with bearing the closer check.

Answer: About 1,067 lb per stud; 203 psi in the stud and on the plate; both acceptable.

Example 2. A 6×6 post (actual 5½ in × 5½ in) carries 18,000 lb and sits on a 2×6 sill plate. Check bearing across the grain at 625 psi. How large a steel bearing plate would spread the load enough?

Show solution

1. Post area: $5.5 \times 5.5 = 30.25$ in^2.
2. Bearing stress on the sill: $18{,}000 \div 30.25 \approx 595$ psi, just under 625 psi.
3. The margin is thin; a bearing plate would help. For, say, 500 psi: $A = 18{,}000 \div 500 = 36$ in^2, a 6 in × 6 in plate (if the sill is wide enough), or the post should bear directly on concrete.

Answer: 595 psi, barely acceptable; a 6 in square bearing plate lowers it to 500 psi.

Example 3. Compare strength per unit weight along the grain: clear Douglas fir (strength about 50 MPa in compression, density 530 kg/m^3) and structural steel (yield 250 MPa, density 7,850 kg/m^3).

Show solution

1. Wood: 50×10^6 Pa $\div$ 530 kg/m^3 $\approx$ 94,000 N·m/kg.
2. Steel: $250 \times 10^6 \div 7850 \approx 32{,}000$ N·m/kg.
3. Wood is about three times stronger for its weight in this comparison, though steel is far stronger for its size, much stiffer, uniform, and equally strong in every direction.

Answer: About 94 kN·m/kg for fir versus 32 kN·m/kg for mild steel.

PRACTICE

1. Easy What are the actual dimensions of a 2×6? Find its cross-sectional area.

   Show answer

   $1\frac{1}{2}'' \times 5\frac{1}{2}''$; area 8.25 in^2.

2. Easy Why is wood stronger along the grain than across it?

   Show answer

   Its long fibers carry load along their length; across the grain, only the weaker bonds between fibers resist.

3. Easy What three industrial changes made the balloon frame possible?

   Show answer

   Steam sawmills producing standard lumber, machine-made nails, and railroads to ship them.

4. Medium A 2×6 stud carries 3,000 lb. Find the compressive stress.

   Show answer

   $3000 \div 8.25 \approx 364$ psi.

5. Medium A 4×4 post (3½ in × 3½ in) bears on a wood beam with 625 psi allowed across the grain. What is the most load it can deliver?

   Show answer

   $12.25 \times 625 \approx 7660$ lb.

6. Medium A wall carries 1,200 lb/ft with 2×4 studs. What stud spacing (in inches) keeps each stud's load under 1,600 lb?

Show answer

$1600/1200 = 1.333$ ft $= 16$ in maximum.

7. Medium A green timber 9.25 in wide shrinks about 4% across its width as it dries. How much narrower does it become?

Show answer

$0.04 \times 9.25 = 0.37$ in, about ⅜ in.

8. Hard A balsa strip 10 mm wide and 1.6 mm thick broke in tension at 2.4 kg along the grain. Another, cut across the grain, broke at 0.15 kg. Find both stresses in MPa and their ratio.

Show answer

$A = 16$ mm^2. Along: $2.4 \times 9.81/16 \approx 1.47$ MPa; across: $0.15 \times 9.81/16 \approx 0.092$ MPa; ratio 16.

9. Hard A glulam column 6¾ in × 9 in carries 60,000 lb. Find the stress. If its allowable compressive stress (including buckling) is 1,200 psi, does it pass?

Show answer

$A = 60.75$ in^2; $\sigma \approx 988$ psi; it passes.

10. Hard Conceptual. Why did codes require fire-blocking after balloon framing, and how does a platform frame help?

Show answer

Model answer: in a balloon frame the stud cavities run unbroken from basement to attic, so fire and hot gases rise through them like chimneys; a platform frame caps each story with a floor platform, and fire-blocks close the cavities at each level.

11. **Challenge** A CLT floor panel is 175 mm thick and weighs 470 kg/m^3. A concrete slab of equal strength might be 200 mm thick at 2,400 kg/m^3. Compare their weights per square meter. How does that change the loads on columns and foundations of a 12-story building with 1,000 m^2 floors?

    Show answer

    CLT: $0.175 \times 470 \approx 82$ kg/m^2; concrete: $0.200 \times 2400 = 480$ kg/m^2. Difference ≈ 398 kg/m^2 per floor; over 12 floors of 1,000 m^2, about 4.8×10^6 kg (roughly 47,000 kN) less weight for columns, foundations and earthquake design to carry.

COMMON MISTAKES

- Using nominal dimensions (2 × 4) instead of actual ones (1½ × 3½) in calculations.
- Forgetting to check bearing across the grain, which often governs before the stud itself fails.
- Ignoring moisture. Wet wood is weaker, creeps more, shrinks as it dries and can rot.

> **Make it a game · Grain Match.** Memory match with homemade cards. Write terms like balloon frame, platform frame, grain, moisture, nominal size and mass timber, and match them to explanations on a second set. Flip pairs from memory. Then explain in one sentence why wood is stronger along the grain than across it. Beat your own fewest flips.

5.2 Masonry: The Monadnock and the Limits of Brick

OBJECTIVES

- Explain how masonry carries load, why bearing walls must thicken as buildings get taller, and how brick bonds tie walls together.
- Calculate the stress at the base of a masonry wall from its self-weight and floor loads, and the thickness needed.
- Test how bond pattern affects a model brick wall's resistance to a sideways push.

> **Warm-up.** Look at a brick wall outside. Sketch the pattern of the bricks in a small area. Are all the bricks laid with their long side showing, or do some show their short ends? Why might a mason mix them?

WATCH

Pushing the limits, The Monadnock Building

Smarthistory · 4:14

https://www.youtube.com/watch?v=c2hie9nlx4M

> **WHEN TO WATCH**
> After the story. Notice how thick the walls are at the bottom and how the building's shape curves outward at the top.

BACKUP VIDEO

Monadnock Building: Building Tall in the 19th Century

Chicago Architecture Center · 1:58

https://www.youtube.com/watch?v=3EJidEJa9dA

A short animation of how Chicago moved from masonry to steel frames.

KEY TERMS

masonry
Construction of units (brick, stone, concrete block) laid in mortar. Strong in compression, weak in tension.

bearing wall
A wall that carries the floors and roof above it down to the foundation.

bond

The pattern in which bricks overlap so that the joints do not line up, tying the wall together: running, English and Flemish bonds.

header and stretcher

A header is a brick laid with its end showing, tying two wythes (layers) together; a stretcher shows its long side.

unit weight

Weight per unit volume, γ; for brick masonry about 120 lb/ft^3 (19 kN/m^3).

CORE IDEAS

The story. Between 1889 and 1891, the Chicago architects Daniel Burnham and John Root built the Monadnock Building, sixteen stories of dark brown brick on Jackson Boulevard. It was one of the tallest office buildings in the world and the last great skyscraper in Chicago to stand on load-bearing masonry walls. To carry sixteen floors, its walls are about 6 ft (1.8 m) thick at the bottom, narrowing as they rise. The ground floor's windows sit in deep tunnels of brick. The building has almost no ornament; its walls simply flare outward at the base and curve out at the top like a papyrus stalk. Within two years a southern addition was built on a steel frame, with far thinner walls, and the age of tall masonry was over.

How they figured it out. Builders had always thickened walls toward the bottom of tall towers, by rule of thumb. Chicago's building rules of the 1880s set minimum thicknesses that grew with the number of stories, adding inches of wall for each floor above. Engineers could also calculate it: the stress at the bottom of a wall is its own weight plus everything it carries, divided by its area. As a building grows taller, the weight of the wall itself becomes the biggest load, and the extra thickness takes up valuable floor space on the lowest, most rentable floors. The steel frame solved that problem by carrying the loads on slender columns.

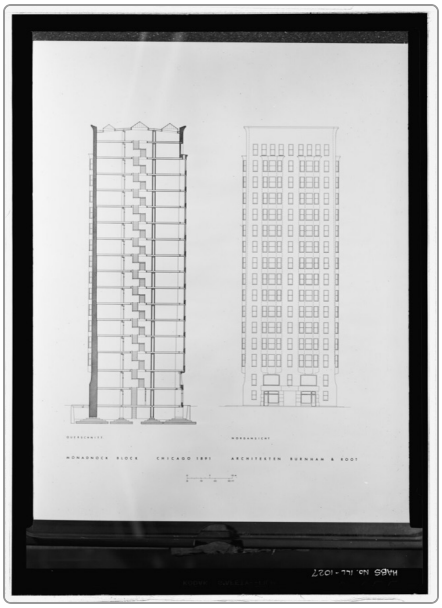
The physics. The compressive stress at the base of a uniform wall of height h from its own weight is $\sigma = \gamma h$, independent of its thickness. For brick at 19 kN/m^3, a 60 m wall presses on its base with about 1.1 MPa from its own weight alone. The floors add a line load w (force per meter of wall), so the base stress is $\sigma = \dfrac{w}{t} + \gamma h$ for a wall of thickness t. If the masonry may safely carry σ_{allow}, the wall must be at least $t = \dfrac{w}{\sigma_{\text{allow}} - \gamma h}$ thick. As γh approaches the allowable stress, the required thickness

shoots up: a masonry wall has a height limit set by its own weight. Masonry is weak in tension, so walls must also be thick enough that wind cannot bend them into tension, and they must be **bonded** so the units act together.

The engineering. Masonry is durable, fire-resistant, beautiful and made from earth. Its weaknesses are tension, earthquakes and its own weight. Modern masonry is often **reinforced**: steel bars in grouted cells of concrete block let walls resist bending and shaking. Brick today is usually a veneer, a single layer tied to a steel or wood frame behind it. The art of the mason remains: bonds that interlock, mortar joints that are full, flashing that sheds water, and arches and lintels over every opening. The Monadnock still stands, a monument to the limits of a material.

Try it. In the build you will build walls of toy bricks in stack bond and running bond and measure how much sideways push each can take.

PICTURE STUDY



Cross Section and Elevation of the Monadnock Building, Dieter Sengler, for the Historic American Buildings Survey, 1964 · Measured drawing (photocopy) · Library of Congress, Prints and Photographs Division

Image: Wikimedia Commons, public domain

A HABS draftsman drew the Monadnock in section and elevation. In the section you can see what the street hides: how thick the walls are at the bottom and how they narrow toward the top.

Look closely

- In the section, compare the wall thickness at the ground floor with the wall thickness at the top.
- Count the floors. Are their heights all the same?
- In the elevation, find the bay windows. Where do they begin and end?
- Look at the very top and the very bottom of the elevation. How does the wall curve?

Talk about it

1. Why must a masonry bearing wall be thickest at the bottom?
2. What did those thick lower walls cost the owners in rentable space?
3. How would a steel frame change this section drawing?

BUILD

Bond Matters: Stack Bond vs. Running Bond

You will build model brick walls in different bonds from toy bricks, push them sideways at the top with a measured load, and find which bond resists best. Then you will compute the self-weight stress in your tallest wall.

You will need

- LEGO bricks (2×4 bricks are ideal) or sugar cubes with a little frosting "mortar"
- String, a paper cup, pennies, and a pencil taped to the table edge as a pulley
- A ruler and a kitchen scale

Steps

1. Build a wall 8 bricks long and 10 courses high in **stack bond** (every brick directly over the one below, joints lined up). Do not press the bricks together tightly; set them just snug so you test the bond, not the clutch.
2. Build a second wall of the same size in **running bond** (each course offset by half a brick).
3. If you can, build a third wall two bricks thick with headers tying the layers (English or Flemish bond).
4. **Predict.** Which will resist the most sideways push before it cracks or tips?
5. Tie a string to a brick near the middle of the top course, run it over the pulley, and add pennies slowly. Record the load when the wall first cracks (a joint opens) and when it fails.
6. Repeat each test twice and average.
7. Weigh one brick and measure its size. Compute the unit weight of your "masonry" and the self-weight stress at the base of a 10-course wall and of a wall as tall as the Monadnock built of the same bricks.
8. **Improve.** Using the same number of bricks, design the stiffest wall you can (try pilasters, a zigzag plan, or buttresses). Test it.

Tips

- Build on a flat surface and keep all walls equally snug.
- Pull at the same height and spot each time.
- Watch where the first crack appears and sketch it.

Check your work

- ☐ My tests compare at least two bond patterns fairly.
- ☐ I can calculate the self-weight stress at the base of a wall and the thickness needed to carry floors.
- ☐ I can explain why the Monadnock was the end of tall bearing-wall buildings.

ENGINEER'S NOTEBOOK

1. Sketches of running, stack, English and Flemish bonds with headers and stretchers labeled.
2. Your push-test data and sketches of the crack patterns.
3. A graph of required wall thickness against building height for a wall carrying 30 kN/m per floor (use the formula in the lesson) from 2 to 16 stories.

KEY FORMULAS

$\sigma_{\text{self}} = \gamma h$

$\sigma_{\text{base}} = \dfrac{w}{t} + \gamma h$ (per meter of wall)

$t_{\min} = \dfrac{w}{\sigma_{\text{allow}} - \gamma h}$

$\gamma_{\text{brick}} \approx 19\ \text{kN/m}^3 \approx 120\ \text{lb/ft}^3$

From history. John Root died suddenly in 1891, just as the Monadnock was finished; Daniel Burnham went on to plan the 1893 World's Columbian Exposition and the 1909 Plan of Chicago. Burnham is remembered for the advice, "Make no little plans," though historians are not sure he said it in exactly those words.

Example 1. A brick wall stands 60 m tall (about the Monadnock's height). Find the stress at its base from its own weight alone ($\gamma = 19\ \mathrm{kN/m^3}$).

Show solution

1. $\sigma = \gamma h = 19\ \mathrm{kN/m^3} \times 60\ \mathrm{m} = 1140\ \mathrm{kN/m^2}$.
2. That is 1,140 kPa, or about 1.14 MPa, no matter how thick the wall is.

Answer: About 1.14 MPa.

Example 2. The same 60 m wall also carries sixteen floors, each delivering 30 kN per meter of wall. If the masonry may safely carry 1.5 MPa, how thick must the wall be at the base?

Show solution

1. Floor line load: $w = 16 \times 30 = 480\ \mathrm{kN/m}$.
2. Stress left for the floors: $1500 - 1140 = 360\ \mathrm{kPa}$.
3. Thickness: $t = w/360 = 480 \div 360 \approx 1.33\ \mathrm{m}$.
4. The real walls are about 1.8 m thick, which also allows for window openings (which reduce the wall's area), wind, and the cautious allowable stresses of the day.

Answer: At least about 1.3 m (the real walls are about 1.8 m).

Example 3. How tall could a brick wall be before its own weight alone used up an allowable stress of 1.5 MPa?

Show solution

1. Set $\gamma h = \sigma_{\text{allow}}$: $h = 1500\ \mathrm{kPa} \div 19\ \mathrm{kN/m^3} \approx 79\ \mathrm{m}$.
2. At that height the wall could carry nothing else; real bearing-wall buildings must stop well short of it.

Answer: About 79 m.

PRACTICE

1. Easy Why do masons offset the joints of each course?

   Show answer

   So that each brick bridges the joint below; the wall acts as one piece and cracks cannot run straight up a joint.

2. Easy Find the self-weight stress at the base of a 10 m brick wall ($\gamma = 19\ \mathrm{kN/m^3}$).

   Show answer

   190 kPa.

3. Easy Convert 120 lb/ft^3 × 50 ft into psf and psi.

   Show answer

   6000 psf ÷ 144 ≈ 41.7 psi.

4. Medium A wall 0.30 m thick carries a floor line load of 90 kN/m and is 12 m tall. Find its base stress.

   Show answer

   $90/0.30 + 19 \times 12 = 300 + 228 = 528\ \mathrm{kPa}$.

5. Medium For Problem 4, if the allowable stress is 0.8 MPa, what is the least thickness?

   Show answer

   $t = 90 \div (800 - 228) \approx 0.157$ m; 0.30 m is ample.

6. Medium A wall 1.8 m thick and 1 m long is pierced by a window 0.9 m wide in each 1.8 m of length. By what factor does the window raise the stress in the remaining piers?

   Show answer

   Half the wall is left, so the stress doubles (factor 2) in the piers between windows.

7. Medium How tall can a brick wall be before self-weight alone reaches 2.0 MPa?

   Show answer

   $2000 \div 19 \approx 105$ m.

8. Hard A ten-story building's wall carries 25 kN/m from each floor and is 40 m tall. With an allowable stress of 1.2 MPa, find the minimum base thickness. Repeat for a twenty-story, 80 m wall.

Show answer

10 stories: $250/(1200 - 760) \approx 0.57$ m. 20 stories: $500/(1200 - 1520) < 0$: impossible; the wall's own weight exceeds the allowable stress.

9. Hard Explain why the required thickness in Problem 8 rises so quickly as the building gets taller.

Show answer

The term γh eats up the allowable stress, leaving less and less for the floors, so $t = w/(\sigma_{\text{allow}} - \gamma h)$ grows rapidly and becomes infinite when $\gamma h = \sigma_{\text{allow}}$.

10. Hard Conceptual. Why is unreinforced masonry dangerous in earthquakes, and what does reinforcing do?

Show answer

Model answer: shaking bends walls sideways, causing tension that unreinforced masonry cannot resist, so walls crack and fall; steel bars grouted into the wall carry the tension and hold it together.

11. Challenge A tapered wall is 1.8 m thick at the base and 0.45 m thick at the top of a 60 m height, narrowing uniformly. Find its average thickness, its weight per meter of length, and the self-weight stress at the base. Compare with a uniform wall.

Show answer

Average $= 1.125$ m; weight $= 19 \times 1.125 \times 60 \approx 1283$ kN per meter; base stress $= 1283 \div 1.8 \approx 713$ kPa, compared with 1,140 kPa for a uniform wall: tapering leaves more capacity for the floors.

COMMON MISTAKES

- Thinking a thicker wall reduces its own self-weight stress. A uniform wall's self-weight stress depends only on its height.
- Forgetting that openings concentrate load in the piers between them.
- Ignoring tension: masonry must not be bent by wind or shaking without reinforcement.

Make it a game · Brick Bond Builders. Using sugar cubes, dominoes or wooden blocks, build a wall in stack bond and another in running bond. Predict which resists a sideways nudge. Then say aloud why a masonry wall must thicken at the base as it rises, and calculate the stress at the base of a wall from its weight. Closest prediction wins.

Scripture. “And they said one to another, Go to, let us make brick, and burn them throughly. And they had brick for stone, and slime had they for morter.” (Genesis 11:3, KJV)

The oldest account of brickmaking describes fired brick set in bitumen mortar, the materials of ancient Mesopotamia. Babel was not undone by its bricks but by the builders’ purpose.

5.3 Steel: Bessemer, Eads and the Stress-Strain Curve

OBJECTIVES

- Tell how the Bessemer process made cheap steel and how the Eads Bridge proved it in a great structure.
- Read a stress-strain curve and use Hooke's law, $\sigma = E\varepsilon$ and $\delta = PL/AE$, to calculate stress and stretch.
- Measure a stress-strain curve for a plastic strip and identify its elastic region, yield and necking.

> **Warm-up.** Bend a steel paper clip slightly and let go. Does it spring back? Now bend it much farther. Does it spring back now? Bend it back and forth a few times. What happens? Write down what you think is going on inside the metal.

WATCH

An Introduction to Stress and Strain

The Efficient Engineer · 10:01

https://www.youtube.com/watch?v=aQf6Q8t1FQE

> **WHEN TO WATCH**
> After the story, before the worked examples. Sketch the stress-strain diagram as he draws it.

KEY TERMS

steel

Iron with a small, controlled amount of carbon (about 0.1–1%), much stronger and tougher than wrought iron and less brittle than cast iron.

Bessemer process

Making steel by blowing air through molten pig iron so the oxygen burns out carbon and impurities (patented 1856).

strain

Stretch per unit length, $\varepsilon = \delta/L$ (no units).

modulus of elasticity (E)

Stiffness of a material, the slope of the elastic part of the stress-strain curve: about 200 GPa (29,000 ksi) for steel.

yield strength

The stress at which a metal begins to deform permanently; about 250 MPa (36 ksi) for A36 steel and 345 MPa (50 ksi) for A992.

ductility

The ability to stretch a great deal before breaking, giving warning; steel is ductile, cast iron is brittle.

CORE IDEAS

The story. In August 1856 the English inventor Henry Bessemer described a remarkable discovery to the British Association: if air is blown through a bath of molten pig iron, the oxygen burns away the carbon and silicon, the heat of that burning keeps the metal molten, and in about twenty minutes the iron becomes steel. Before that, steel was made a few pounds at a time and was costly; within a few decades it was made by the ton for rails, ships and bridges. In St. Louis, James Buchanan Eads, a self-taught engineer who had made his name salvaging sunken riverboats, was building a bridge across the Mississippi. Completed in 1874, the Eads Bridge carried trains and wagons on three arches of steel tubes, spans of about 502, 520 and 502 ft. It was the first major bridge built of steel, and it still carries trains today.

How they figured it out. Bessemer's first steel was brittle until Robert Mushet showed that adding a little manganese removed the oxygen and sulfur problems, and Sidney Gilchrist Thomas and Percy Gilchrist found in 1878 how to remove phosphorus from common iron ores. Eads, meanwhile, refused to trust the steel makers' word. He insisted on testing samples of every batch in a testing machine and rejected steel that did not meet his specifications. The steel company objected, but the tests forced better, more uniform steel, and modern materials specifications began in part with his demands.

The physics. When a steel bar is pulled, it stretches. **Stress** is the force per area, $\sigma = P/A$, and **strain** is the stretch per length, $\varepsilon = \delta/L$. Up to the elastic limit, stress is proportional to strain (Hooke's law): $\sigma = E\varepsilon$, with $E \approx 200$ GPa for every steel, strong or weak. Combining these, a bar of length L stretches $\delta = \dfrac{PL}{AE}$. Past the **yield** point the steel stretches a long way at nearly constant stress (the yield plateau), then hardens and becomes stronger until, at its ultimate strength, it begins to neck down and break. Mild steel can stretch 20% or more before breaking. That ductility is a gift: steel structures bend, sag and warn before they fail.

The engineering. Engineers design steel so that, under service loads, it stays in the elastic region with a margin below yield, and so that, if overloaded, it yields and bends rather than snapping. Steel is made to standards (ASTM A36, A992 and others) that guarantee its yield and ultimate strengths, and mills must certify each heat of steel, as Eads demanded. Steel is very strong for its size and very stiff, but it is heavy, it rusts unless protected, and it loses strength in fire, so it must be painted or galvanized and fireproofed. With steel came the skyscraper (Unit 7) and the long-span bridge (Unit 6).

Try it. In the build you will measure a stress-strain curve for a strip of plastic, which yields and necks like steel but at forces you can apply by hand.

PICTURE STUDY

A Bessemer Steel Converter, Engraving in *Harper's Weekly*, 1889 · Wood engraving · Library of Congress
Image: Wikimedia Commons, public domain

This engraving shows a Bessemer converter at work in an American steel mill, a great egg-shaped vessel of molten iron with air blasting through it. The flame is the carbon and impurities burning away.

Look closely

- Find the mouth of the converter and the column of flame. What is burning?
- Find the trunnions (pivots) on which the converter turns. Why must it tip?
- Find the workers. How big is the converter compared with them?
- Look for pipes or ducts. Where does the air come in?

Talk about it

1. Why does blowing air through iron make it hotter instead of cooling it?
2. Why was cheap steel so important to bridges and buildings?
3. Why did Eads insist on testing every batch of steel instead of trusting the mill?

Safety first. Wear safety glasses when stretching strips to failure; they can snap back. Never put weights where they can fall on feet.

BUILD

A Stress-Strain Curve You Can Make

Steel is too strong to stretch by hand, but strips of plastic from a grocery bag or a milk-jug ring behave in the same way: they stretch elastically, yield, neck down and finally break. You will measure a full stress-strain curve and find the modulus, yield and ultimate strength of your strip.

You will need

- Strips cut from a plastic grocery bag or thin plastic sheet, about 2 cm wide and 15 cm long
- Two binder clips, a hook or rod to hang from, a paper cup, and sand or water for loads; a kitchen scale
- A ruler with millimeter marks and a fine marker
- A micrometer or a stack-and-divide method for thickness (measure a stack of 10 layers and divide by 10)

Steps

1. Measure the strip's width and thickness and find its area in mm^2.
2. Mark two lines 10 cm apart on the strip as your gauge length L.
3. Clip the top to a hook and the cup to the bottom. Add load in small steps (20 g of sand at a time). After each step, measure the distance between the marks.
4. Record load and length at each step. Keep going until the strip necks (narrows in one spot) and breaks.
5. Compute stress ($\sigma = P/A$, in MPa) and strain ($\varepsilon = \Delta L/L$) for each step.
6. Plot the stress-strain curve. Mark the elastic region, the yield point (where the curve bends over), the ultimate (highest) stress, and the breaking point.
7. Find E from the slope of the straight early part. How does it compare with steel's 200,000 MPa?
8. **Unload test.** With a fresh strip, load it to about half the yield load and unload. Does it return to its original length? Repeat beyond yield. Does it return now?

9. **Improve.** Repeat the test with the strip cut in the other direction of the bag. Plastic sheet is often stronger one way. Compare the curves.

Tips

- Add load gently; a sudden drop gives a false reading.
- Read lengths quickly but after the strip has settled; plastic creeps.
- A phone photo of the ruler beside the marks at each step makes reading easier.

Check your work

- ☐ My stress-strain curve shows an elastic region, yield, ultimate strength and fracture.
- ☐ I can calculate stress, strain and elongation in a steel member.
- ☐ I can explain why ductility makes steel structures safer.

ENGINEER'S NOTEBOOK

1. A labeled sketch of the stress-strain curve for mild steel: elastic region, yield plateau, strain hardening, ultimate strength, necking, fracture.
2. Your data table and stress-strain plot, with E, yield and ultimate stress marked.
3. A paragraph comparing cast iron, wrought iron and steel in strength, ductility and use.

KEY FORMULAS

$$\sigma = \frac{P}{A}, \quad \varepsilon = \frac{\delta}{L}$$

$$\sigma = E\varepsilon \text{ (elastic)}$$

$$\delta = \frac{PL}{AE}$$

$$FS = \frac{\sigma_{\text{yield}}}{\sigma_{\text{actual}}}$$

From history. Eads had never built a bridge before. To sink the foundations of his piers to bedrock beneath the Mississippi's shifting sand, he used compressed-air caissons, and many workers suffered from the then-mysterious decompression sickness, called "the bends." His physician, Dr. Alphonse Jaminet, studied it and recommended slower decompression, knowledge the Brooklyn Bridge builders would soon need (Lesson 6.3).

Example 1. A steel rod 25 mm in diameter and 3.0 m long carries a tension of 60 kN. Find the stress, strain and stretch ($E = 200$ GPa). If it is A36 steel ($\sigma_y = 250$ MPa), what is the factor of safety against yielding?

Show solution

1. Area: $A = \pi(12.5\text{ mm})^2 \approx 491\text{ mm}^2$.
2. Stress: $\sigma = 60{,}000\text{ N} \div 491\text{ mm}^2 \approx 122\text{ MPa}$ (1 N/mm² = 1 MPa).
3. Strain: $\varepsilon = \sigma/E = 122 \div 200{,}000 \approx 6.11 \times 10^{-4}$.
4. Stretch: $\delta = \varepsilon L = 6.11 \times 10^{-4} \times 3000\text{ mm} \approx 1.83\text{ mm}$.
5. Factor of safety: $250 \div 122 \approx 2.0$.

Answer: 122 MPa, strain 0.00061, stretch 1.8 mm, FS ≈ 2.0.

Example 2. A steel hanger must carry 200 kN with stress no more than 150 MPa. What area is needed? What diameter of round rod gives that area?

Show solution

1. $A = P/\sigma = 200{,}000 \div 150 \approx 1333\text{ mm}^2$.
2. $d = \sqrt{4A/\pi} = \sqrt{4 \times 1333/3.1416} \approx 41.2\text{ mm}$.
3. Choose the next standard size up, for example a 42 mm or 45 mm rod.

Answer: At least 1,333 mm², a rod about 42 mm in diameter.

Example 3. In US units: a W-shape steel column has $A = 14.4\text{ in}^2$ and is 12 ft long. It carries 300 kips. Find its stress and how much it shortens ($E = 29{,}000$ ksi).

Show solution

1. Stress: $300 \div 14.4 \approx 20.8$ ksi.
2. Shortening:
   $\delta = PL/(AE) = 300 \times 144 \div (14.4 \times 29{,}000) = 43{,}200 \div 417{,}600 \approx 0.103\text{ in}$.
3. About a tenth of an inch. In a 60-story building, the columns shorten several inches, and engineers set each floor a little high to make up for it.

Answer: 20.8 ksi; shortens about 0.10 in.

PRACTICE

1. **Easy** What does the Bessemer process remove from pig iron, and how?

   Show answer

   Excess carbon (and silicon and other impurities), by blowing air through the molten iron so oxygen burns them out.

2. **Easy** A bar of area 400 mm^2 carries 50 kN. Find the stress.

   Show answer

   125 MPa.

3. **Easy** A 2.0 m bar stretches 1.2 mm. Find the strain.

   Show answer

   $1.2 \div 2000 = 6.0 \times 10^{-4}$.

4. **Medium** A steel bar has a strain of 0.0010. Find its stress ($E = 200$ GPa). Is A36 steel still elastic?

   Show answer

   200 MPa; yes, it is below 250 MPa.

5. **Medium** A 6.0 m steel tie of area 1,200 mm^2 carries 180 kN. How much does it stretch?

   Show answer

   $\delta = 180{,}000 \times 6000 \div (1200 \times 200{,}000) = 4.5$ mm.

6. **Medium** What area of A992 steel ($\sigma_y = 345$ MPa) carries 500 kN with a factor of safety of 1.67 against yield?

   Show answer

   Allowable $= 345/1.67 \approx 207$ MPa; $A = 500{,}000/207 \approx 2420$ mm^2.

7. **Medium** Convert 50 ksi to MPa (1 ksi = 6.895 MPa).

   Show answer

   ≈ 345 MPa.

8. Hard An aluminum bar ($E = 70$ GPa) and a steel bar of the same size carry the same load. Which stretches more, and by what factor?

Show answer

Aluminum, by $200/70 \approx 2.9$ times.

9. Hard A 40-story building has steel columns 160 m tall in total, working at an average stress of 140 MPa. Estimate how much they shorten under load.

Show answer

$\delta = \sigma L/E = 140 \times 160{,}000 \text{ mm} \div 200{,}000 = 112 \text{ mm}$, about 11 cm.

10. Hard Conceptual. Why is a ductile material safer for a building's frame than a brittle one, even if the brittle one is stronger?

Show answer

Model answer: a ductile material yields and deforms visibly before it breaks, giving warning and redistributing load to other members; a brittle material breaks suddenly without warning.

11. Challenge Two steel cables, one 10 m long and one 20 m long, of the same area, share a load equally hung between them on a rigid beam so that both must stretch the same amount. Which carries more of the load, and what fraction?

Show answer

For equal stretch, $P_1 L_1 = P_2 L_2$, so $P_1/P_2 = L_2/L_1 = 2$: the shorter cable carries twice as much, two-thirds of the load.

COMMON MISTAKES

- Forgetting to convert units: 1 N/mm^2 = 1 MPa; 1 GPa = 1,000 MPa.
- Thinking stronger steels are stiffer. All steels have about the same E; stronger steels only yield at higher stress.
- Using the diameter instead of the radius in $A = \pi r^2$.

> **Make it a game · Stress-Strain Story.** Two truths and a tall tale about Bessemer, Eads and the stress-strain curve. Each of you states three facts; the other guesses the false one. Then draw a stress-strain curve from memory and label yield, ultimate and fracture. Use δ = PL/AE to stretch a made-up rod. Best of three rounds.

5.4 Concrete: Roman, Modern, Reinforced and Prestressed

OBJECTIVES

- Trace concrete from Roman pozzolana to Portland cement, reinforced concrete (Monier, Hennebique) and prestressed concrete (Freyssinet).
- Calculate concrete mix quantities, explain the water-cement ratio, and find the moment capacity of a simple reinforced concrete beam.
- Cast and test small beams with and without reinforcement, and with the reinforcement on the wrong side.

> **Warm-up.** Look at a concrete sidewalk. Find the cracks. Are they random or regular? Are there grooves cut on purpose? Why would someone cut grooves in a sidewalk?

WATCH

Was Roman Concrete Better?

Practical Engineering · 9:34

https://www.youtube.com/watch?v=qL0BB2PRY7k

> **WHEN TO WATCH**
> After the story. Note the main differences he finds between Roman and modern concrete.

BACKUP VIDEO

What is Prestressed Concrete?

Practical Engineering · 8:46

https://www.youtube.com/watch?v=P13Mau2VUWw

With the engineering paragraph on prestressing.

KEY TERMS

Portland cement

A fine powder of burned limestone and clay that hardens by reacting with water (hydration); patented by Joseph Aspdin in 1824.

aggregate

The sand (fine) and gravel or crushed stone (coarse) that make up most of concrete's volume.

water-cement ratio

The weight of water divided by the weight of cement; lower ratios make stronger, denser concrete (about 0.40–0.55 for structural concrete).

reinforced concrete

Concrete with steel bars (rebar) placed where tension occurs, so the steel carries tension and the concrete compression.

prestressed concrete

Concrete squeezed in advance by tensioned steel tendons, so that loads must first overcome the squeeze before tension can crack it.

curing

Keeping new concrete moist and warm enough while it gains strength, usually for at least a week.

CORE IDEAS

The story. After Rome fell, the secret of its concrete was largely lost for over a thousand years. In 1824 Joseph Aspdin, a bricklayer in Leeds, England, patented a cement made by burning limestone and clay, which he called Portland cement because it looked like the stone of the Isle of Portland. Concrete made with it was strong in compression but cracked when bent. In 1867 a Paris gardener, Joseph Monier, patented flower pots and tubs of concrete reinforced with iron mesh, and in the following years he patented reinforced beams, pipes and bridges. In 1892 the French builder François Hennebique patented a complete system of reinforced concrete columns, beams and slabs, with bent bars and stirrups, and licensed it around the world. In 1928 the engineer Eugène Freyssinet patented prestressed concrete, and after the Second World War it rebuilt the bridges of Europe.

How they figured it out. Engineers learned concrete by testing. In 1918 Duff Abrams at the Lewis Institute in Chicago published the results of thousands of tests showing that concrete's strength depends mainly on the **water-cement ratio**: extra water makes concrete easier to pour but leaves tiny voids when it evaporates, and weakens it. Engineers learned that steel and concrete work together because they expand almost equally with heat, because concrete grips the ridged bars, and because the alkaline concrete protects the steel from rust. They learned the hard way that if water and salt reach the steel through cracks, it rusts, swells and breaks the concrete apart.

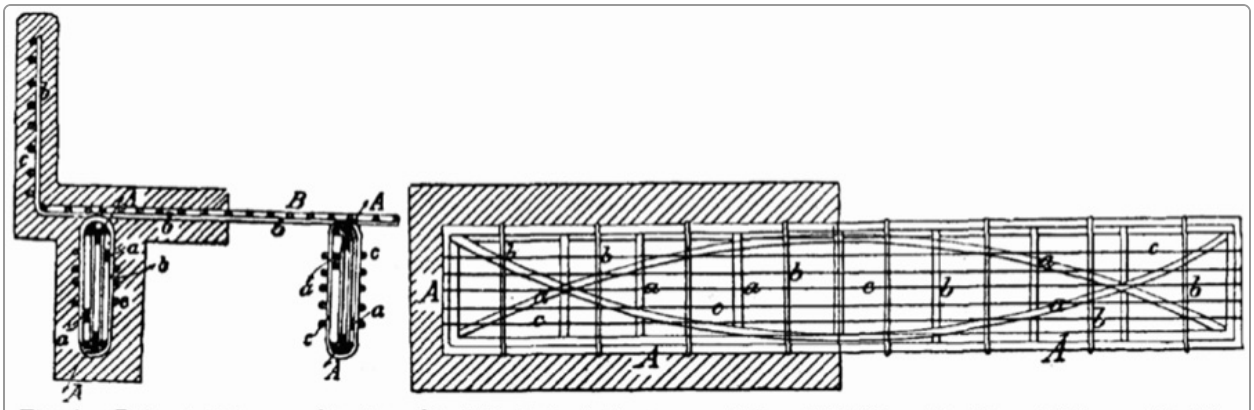
The physics. Concrete is strong in compression (typically 20–40 MPa, or 3,000–6,000 psi) but its tensile strength is only about a tenth of that. In a beam, the bottom is in

tension, so steel bars are placed near the bottom. The steel carries a tensile force $T = A_s f_y$; the concrete above carries an equal compression in a block of depth a, $C = 0.85 f'_c\, a\, b$. Setting $T = C$ gives $a = \dfrac{A_s f_y}{0.85 f'_c b}$, and the beam's moment capacity is $M_n = A_s f_y \left(d - \frac{a}{2}\right)$, where d is the depth to the steel. In **prestressed** concrete, tendons stretched to very high stress squeeze the concrete before it is loaded, so the beam stays in compression and uncracked under service loads, and it deflects much less.

The engineering. Concrete is the most used building material in the world: foundations, slabs, columns, dams, bridges and roads. A good concrete structure needs the right mix (cement, water, sand, gravel and sometimes fly ash or slag), steel in the right places with enough cover to protect it, proper placing and vibration to remove air pockets, and careful curing. Joints are cut in slabs so that shrinkage cracks happen in straight, planned lines instead of randomly. And the Roman lesson remains: modern researchers are studying Roman mixes and self-healing concretes to build structures that last centuries, not decades.

Try it. In the build you will cast small beams of plaster or mortar, some plain, some with wire near the bottom and some with wire near the top, and see what reinforcement does and why its placement matters.

PICTURE STUDY



Monier's Patent Drawing for a Reinforced Concrete Beam, Joseph Monier, 1878 · Patent drawing (as reproduced in a German handbook) · Wikimedia Commons

Image: Wikimedia Commons, public domain

Joseph Monier was a gardener, not an engineer, but he patented reinforced concrete pots in 1867 and beams in 1878. This drawing shows his idea for putting an iron cage inside a concrete beam. The caption, in German, reads "Monier's patent drawing for reinforced concrete beams of 1878 (cross section and long section)."

Look closely

- Find the cross section on the left. Where are the iron bars?
- Find the long section on the right. Follow the curved bars. Where are they low and where are they high?
- Find the crossing bars in the long section. What might they resist?
- Find the small letters. What do you think they label?

Talk about it

1. Where is a simply supported beam in tension? Do Monier's bars follow the tension?
2. Why might bars that curve up near the supports be useful?
3. Why must concrete cover the steel completely?

Safety first. Mix plaster or mortar outdoors or with good ventilation, wearing a dust mask and safety glasses; an adult mixes. Cement and mortar are caustic to skin, so wear gloves and wash any off promptly. Do not pour leftover plaster down a drain.

BUILD

Where Does the Steel Go?

You will cast small beams of plaster of Paris (or a sand-mortar mix): some plain, some with wire reinforcement near the bottom, and some with the same wire near the top. Testing them as simple beams shows why the steel must be on the tension side.

You will need

- Plaster of Paris or a bag of sand-mortar mix, water, a measuring cup and a kitchen scale
- A mold: a cardboard box or milk carton cut into troughs about 3 cm × 3 cm × 25 cm, lined with plastic wrap
- Reinforcement: lengths of stiff wire, straightened paper clips or galvanized fence wire
- Two supports 20 cm apart, a cup and sand for loading, safety glasses, gloves and a dust mask

Steps

1. Plan six beams: two plain, two with wire 5 mm above the bottom, two with wire 5 mm below the top. Label the molds.
2. Weigh the dry powder and water for each batch so that every beam has the same water ratio. Record it.
3. Pour the bottom layer, place the wire where planned (hold it with small chips of hardened plaster or bent tabs), and fill the mold. Tap the mold to release bubbles.
4. Cure: let plaster set a day and dry two days; keep mortar damp under plastic for a week.
5. **Predict.** Rank the three kinds of beam by strength.
6. Test each beam across the 20 cm supports with the cup hung at midspan by a string loop. Add sand slowly until failure. Record the load and watch how it fails.

7. Note especially what happens to the bottom-reinforced beams after the first crack: do they fall apart or hang together?
8. Make a bar chart of average failure loads and describe the failure of each kind.
9. **Improve.** Make one more batch with extra water (a higher water ratio) and one with less, cast plain beams, and compare their strength.

Tips

- Keep the wire straight and centered across the width.
- Test beams the same number of days after casting.
- A plain beam will snap suddenly; a well-reinforced one cracks, sags and holds.

Check your work

- ☐ My tests show that steel near the bottom (the tension side) adds strength and toughness, and steel near the top does little.
- ☐ I can calculate the moment capacity of a simple reinforced concrete beam.
- ☐ I can explain the water-cement ratio and why curing matters.

ENGINEER'S NOTEBOOK

1. A timeline of concrete: Roman pozzolana, Aspdin, Monier, Hennebique, Abrams, Freyssinet, with one sentence each.
2. Your beam test table, bar chart and sketches of each kind of failure.
3. A labeled cross section of a reinforced concrete beam showing the steel, the cover, the compression block, d, a and the forces T and C.

KEY FORMULAS

$$\mathrm{w/c} = \frac{m_{\text{water}}}{m_{\text{cement}}}$$

$$T = A_s f_y, \quad C = 0.85 f'_c\, a\, b$$

$$a = \frac{A_s f_y}{0.85 f'_c\, b}, \quad M_n = A_s f_y \left(d - \frac{a}{2}\right)$$

$$\rho_{\text{concrete}} \approx 2400\ \mathrm{kg/m^3}$$

From history. Hennebique's firm became one of the largest engineering businesses of its day, with licensed agents in many countries who used his patented system and sent their designs to his Paris office for checking. One of the first reinforced concrete bridges in the United States, the Alvord Lake Bridge in San Francisco's Golden Gate Park (1889), was built by Ernest Ransome, who patented twisted square bars that gripped the concrete better; it still stands.

WORKED EXAMPLES

Example 1. A rectangular concrete beam is 300 mm wide with steel at an effective depth of 500 mm. It has three bars with a total area of 900 mm², $f_y = 415$ MPa (Grade 60), and the concrete has $f'_c = 28$ MPa. Find its nominal moment capacity.

Show solution

1. Steel force: $T = A_s f_y = 900 \times 415 = 373{,}500$ N.
2. Compression block depth:
   $a = 373{,}500 \div (0.85 \times 28 \times 300) = 373{,}500 \div 7140 \approx 52.3$ mm.
3. Lever arm: $d - a/2 = 500 - 26.2 = 473.8$ mm.
4. $M_n = 373{,}500 \times 473.8 \approx 1.77 \times 10^8$ N·mm $= 177$ kN·m.

Answer: $M_n \approx 177$ kN·m.

Example 2. A mix uses 350 kg of cement per cubic meter of concrete. For a water-cement ratio of 0.45, how much water is needed? If a worker adds another 30 kg of water to make it easier to pour, what is the new ratio?

Show solution

1. Water: $0.45 \times 350 = 157.5$ kg (about 158 liters).
2. With 30 kg more: $187.5 \div 350 \approx 0.54$.
3. By Abrams's law, raising the ratio from 0.45 to 0.54 can cut the strength by roughly a fifth or more, which is why adding water at the site is forbidden without the engineer's approval.

Answer: 157.5 kg of water; the new ratio is about 0.54, a weaker concrete.

Example 3. A garage slab is 20 ft × 24 ft and 4 in thick. How many cubic yards of concrete are needed (add 5% for waste)? What does it weigh at 150 lb/ft^3?

Show solution

1. Volume: $20 \times 24 \times (4/12) = 160$ ft^3.
2. In cubic yards: $160 \div 27 \approx 5.93$ yd^3; with 5% waste, 6.22, so order about 6¼ yd^3.
3. Weight: $160 \times 150 = 24{,}000$ lb, 12 tons.

Answer: About 6¼ cubic yards; about 24,000 lb.

PRACTICE

1. Easy Why is steel placed near the bottom of a simply supported concrete beam?

   Show answer

   The bottom is in tension under load, and concrete is weak in tension; steel carries the tension.

2. Easy What did Duff Abrams discover about concrete strength?

   Show answer

   It depends mainly on the water-cement ratio: less water (for the same cement) gives stronger concrete.

3. Easy Find the volume of a footing 2.0 m × 2.0 m × 0.5 m and its mass at 2,400 kg/m^3.

   Show answer

   2.0 m^3; 4800 kg.

4. Medium A mix has 300 kg of cement and 150 kg of water per cubic meter. Find the water-cement ratio.

   Show answer

   0.50.

5. Medium Find the tensile force capacity of four bars totaling 1,140 mm^2 with $f_y = 415$ MPa.

   Show answer

   $1140 \times 415 = 473{,}100$ N ≈ 473 kN.

6. Medium A sidewalk 4 ft wide and 60 ft long is 4 in thick. How many cubic yards of concrete are needed?

Show answer

$4 \times 60 \times (1/3) = 80\ \text{ft}^3 \approx 2.96\ \text{yd}^3$.

7. Medium Concrete's compressive strength is 30 MPa. Estimate its tensile strength.

Show answer

About a tenth: roughly 3 MPa.

8. Hard A beam is 250 mm wide with $d = 450$ mm, $A_s = 1000\ \text{mm}^2$, $f_y = 415$ MPa and $f\prime_c = 25$ MPa. Find a and M_n.

Show answer

$a = 415{,}000 \div (0.85 \times 25 \times 250) \approx 78.1$ mm;
$M_n = 415{,}000 \times (450 - 39.1) \approx 1.71 \times 10^8$ N·mm $= 171$ kN·m.

9. Hard For the beam in Problem 8, what happens to M_n if the steel is doubled to 2,000 mm^2? Does it double?

Show answer

$a \approx 156$ mm; $M_n = 830{,}000 \times (450 - 78) \approx 3.09 \times 10^8$ N·mm $= 309$ kN·m, less than double, because the compression block deepens and the lever arm shrinks. (Codes also limit steel so the beam stays ductile.)

10. Hard Conceptual. Why must reinforcing steel have enough concrete cover, and what happens if salt water reaches it?

Show answer

Model answer: the cover protects the steel from moisture, oxygen and fire, and the concrete's alkalinity keeps the steel from rusting; if salt water reaches it through cracks or thin cover, the steel rusts, expands and cracks off the concrete (spalling), weakening the structure.

11. **Challenge** A prestressed beam has a tendon force of 900 kN acting at the center of a 300 mm × 600 mm section. Find the uniform compressive stress it creates. If loads later create a bending tension of 3.5 MPa at the bottom, is the bottom still in compression?

    Show answer

    $\sigma = 900{,}000 \div 180{,}000 = 5.0$ MPa compression; $5.0 - 3.5 = 1.5$ MPa, so the bottom remains in compression and uncracked.

COMMON MISTAKES

- Adding water to make concrete flow more easily. It weakens the concrete.
- Putting reinforcement in the middle or the top of a simply supported beam. It belongs on the tension side.
- Letting new concrete dry out too fast. It needs moist curing to gain strength.

> **Make it a game · Mix and Recall.** Quiz yourself on a recipe card: the proportions for a concrete mix, the water-cement ratio, and which way the steel goes in a beam and why. Your parent covers the card and quizzes you. Then trace concrete's history from Roman pozzolana to Freyssinet in under a minute. Aim for no missed names.

5.5 Beams: Bending and Deflection from Galileo to Today

OBJECTIVES

- Explain how a beam resists bending with compression on one side and tension on the other, and why depth matters so much.
- Calculate bending moment, bending stress ($\sigma = M/S$), moment of inertia and deflection for rectangular beams.
- Size a floor joist for strength and stiffness and test model beams to confirm the effect of depth.

> **Warm-up.** Take a thin plastic ruler and lay it flat across two books; press the middle. Turn it on edge and press again. How many times harder is it to bend on edge? Guess a number before you read on.

WATCH

Understanding Stresses in Beams

The Efficient Engineer · 14:48

https://www.youtube.com/watch?v=f08Y39UiC-o

> **WHEN TO WATCH**
> After the story. Stop at the derivation of the bending-stress formula and copy it into your notebook.

BACKUP VIDEO

Understanding the Deflection of Beams

The Efficient Engineer · 22:14

https://www.youtube.com/watch?v=MvBqCeZllpQ

With Worked Example 3, on deflection. The first half is enough for this lesson.

KEY TERMS

bending moment (M)

The internal turning effect in a beam that bends it; for a simply supported beam with a uniform load, $M_{\max} = wL^2/8$ at midspan.

moment of inertia (I)

A measure of how a cross section's area is spread from the neutral axis; for a rectangle, $I = bh^3/12$.

deflection limit

A code limit on how much a beam may sag, such as span/360 under live load for floors.

neutral axis

The line through a beam's cross section where the bending stress is zero, between the compressed and stretched sides.

section modulus (S)

$S = I/c$, where c is the distance to the outer fiber; for a rectangle, $S = bh^2/6$. Bending stress is $\sigma = M/S$.

CORE IDEAS

The story. In 1638, old, blind and under house arrest, Galileo Galilei published his last great book, *Dialogues Concerning Two New Sciences*. The first of the new sciences was the strength of materials. Galileo drew a beam built into a wall with a weight hanging from its end, and asked how its strength depends on its size and shape. He found the right answer to the most important question: a beam's strength grows with its width but with the *square* of its depth, which is why joists stand on edge. He also saw that strength does not keep up with weight as things grow larger, so that a giant animal or an enormous wooden ship would break under its own weight. That is the square-cube law you met in Lesson 2.1.

How they figured it out. Galileo assumed that the whole section of the beam was in tension, pivoting about its bottom edge, and so he got the number in his formula wrong by a factor of three. The French physicist Edme Mariotte, testing wood and glass rods around 1680, showed that one side of a beam is stretched and the other squeezed. Over the next 150 years, Antoine Parent, Charles-Augustin de Coulomb (1773) and finally Claude-Louis Navier (1826) worked out that the stress varies in a straight line from compression on one face, through zero at the **neutral axis**, to

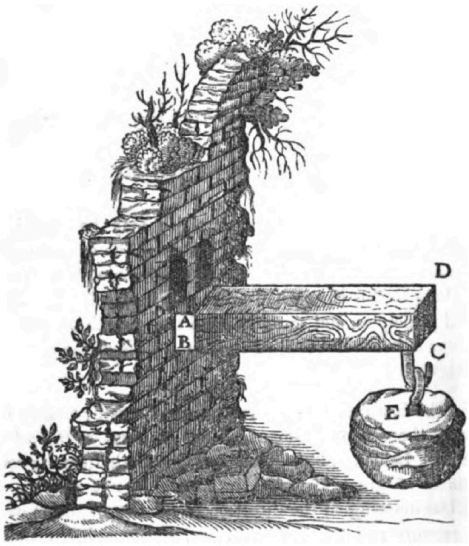
tension on the other. Navier's lectures to French engineers became the basis of every beam calculation since.

The physics. Under load, a simply supported beam bends into a smile: the top shortens (compression) and the bottom stretches (tension). The largest bending moment for a uniform load w over span L is $M = \frac{wL^2}{8}$; for a point load P at midspan, $M = \frac{PL}{4}$. The stress at the outer fibers is $\sigma = \frac{Mc}{I} = \frac{M}{S}$. For a rectangle b wide and h deep, $I = \frac{bh^3}{12}$ and $S = \frac{bh^2}{6}$: double the depth and the beam is four times as strong and eight times as stiff. Deflection for a uniform load is $\delta = \frac{5wL^4}{384EI}$, so it grows with the fourth power of the span. That is why material placed far from the neutral axis does the most work, and why steel beams are shaped like an I, with flanges at top and bottom joined by a thin web.

The engineering. Engineers size every beam for two things: **strength** (the bending stress must stay below what the material can safely carry) and **stiffness** (the deflection must stay below a limit, such as span/360 for a floor under live load, so floors do not bounce, crack ceilings or tilt furniture). They also check shear near the supports, bearing at the ends, and that a deep, thin beam will not twist sideways (lateral buckling). Span tables for joists, which a carpenter can read without calculating, are built on exactly these formulas. Galileo would recognize the problem; he simply could not yet finish the solution.

Try it. In the build you will test beams of the same material in different orientations and depths, measure strength and deflection, and compare with bh^2 and bh^3.

PICTURE STUDY



(From the *Discorsi*, Leiden 1638.)

Galileo's Cantilever Beam, Illustration from Galileo Galilei's *Discorsi*, 1638 · Woodcut · *Discorsi e dimostrazioni matematiche intorno a due nuove scienze*, Leiden

Image: Wikimedia Commons, public domain

This woodcut from Galileo's last book is the first illustration of a structural engineering problem in a science book. A beam is built into a wall and loaded at its end. Galileo asked: how strong is it, and how does its strength depend on its size?

Look closely

- Find the letters A and B at the wall. Galileo imagined the beam pivoting about the bottom edge at B. Where would it break?
- Find the weight E hanging at C. How does its distance from the wall matter?
- Look at the beam's cross section at the wall. Is the beam deeper than it is wide?
- Notice the ruined wall with plants growing on it. Why might the artist have drawn the wall this way?

Talk about it

1. Where is this beam in tension and where in compression?
2. If the beam were twice as deep, how much more weight could it carry? Twice as long?

3. Galileo got part of his answer wrong. Why is a wrong but useful first answer still valuable in science?

Safety first. Wear safety glasses for the break tests; balsa and plastic can snap and fly. Keep loads low over a soft surface.

BUILD

Depth Is Everything: Beam Tests

You will test beams of the same material in different orientations and with different depths, measure their deflection and breaking loads, and check whether strength grows as bh^2 and stiffness as bh^3. Then you will design a lightweight I-shaped beam.

You will need

- Balsa or basswood strips of one size (for example 3 mm × 10 mm × 300 mm), or identical plastic rulers
- Wood glue or white glue to laminate strips into deeper beams
- Two supports 25 cm apart, a cup on a string loop, sand or pennies; a kitchen scale
- A ruler held vertically or a phone camera to measure deflection

Steps

1. Test a single strip laid flat: hang loads at midspan in steps, measure deflection at each step, and keep going until it breaks.
2. Test a single strip on edge (support it so it cannot tip sideways, for example between two pins at each end). Measure deflection and breaking load.
3. Glue two strips into a beam twice as deep, and three strips into a beam three times as deep. Let the glue dry. Test each on edge.
4. **Predict** before each test using $S \propto bh^2$ for strength and $I \propto bh^3$ for stiffness.
5. Make a table: orientation or depth, b, h, bh^2, bh^3, breaking load, stiffness (load per mm of deflection, from the straight part of your data).
6. Plot breaking load against bh^2 and stiffness against bh^3. Are they close to straight lines?
7. Calculate E for the wood from one test using $\delta = PL^3/(48EI)$.
8. **Improve.** Using the same total amount of wood as the three-strip beam, glue up an I-beam (flat strips as flanges top and bottom, a strip on edge as the web). Test it. Compare its stiffness and strength per gram with the solid beam.

Tips

- Laminated beams must be glued well along their whole length, or the layers slide and act separately.
- Measure deflection at the same point each time and zero your reading before loading.
- A deep, thin beam can twist sideways; brace it lightly at the ends.

Check your work

☐ My data show strength rising roughly with

h^2 and stiffness with

h^3.

☐ I calculated

E for my material.

☐ I can size a joist for bending stress and deflection, and explain why I-beams are efficient.

ENGINEER'S NOTEBOOK

1. A labeled sketch of a bent beam showing compression on top, tension on the bottom, the neutral axis and the linear stress diagram across the depth.
2. Your beam-test table and the two graphs.
3. A comparison of the solid and I-shaped beams per gram of material, with a sentence explaining why the I-beam does better.

KEY FORMULAS

$M_{\max} = \dfrac{wL^2}{8}$ (uniform load), $\quad M_{\max} = \dfrac{PL}{4}$ (center point load)

$\sigma = \dfrac{Mc}{I} = \dfrac{M}{S}$

$I = \dfrac{bh^3}{12}, \quad S = \dfrac{bh^2}{6}$ (rectangle)

$\delta_{\max} = \dfrac{5wL^4}{384EI}$ (uniform load), $\quad \delta_{\max} = \dfrac{PL^3}{48EI}$ (center point load)

From history. Navier, who taught at the École des Ponts et Chaussées in Paris, also designed a suspension bridge over the Seine. It was nearly complete in 1826 when a burst water main softened the ground under one anchorage and the bridge had to be dismantled before it opened, an embarrassing reminder, as in Unit 8, that foundations matter as much as calculations.

WORKED EXAMPLES

Example 1. Floor joists are Douglas fir 2×10s (actual 1½ in × 9¼ in) at 16 in on center, spanning 14 ft. The floor load is 50 psf (10 dead + 40 live). Find the maximum bending stress and compare it with an allowable of about 1,000 psi.

Show solution

1. Load per joist: $w = 50 \text{ psf} \times (16/12) \text{ ft} \approx 66.7 \text{ lb/ft}$.
2. Moment: $M = wL^2/8 = 66.7 \times 14^2 \div 8 \approx 1633 \text{ ft·lb} = 19{,}600 \text{ in·lb}$.
3. Section modulus: $S = bh^2/6 = 1.5 \times 9.25^2 \div 6 \approx 21.4 \text{ in}^3$.
4. Stress: $\sigma = M/S = 19{,}600 \div 21.4 \approx 916$ psi, below about 1,000 psi. The joist passes for strength, with a modest margin.

Answer: About 916 psi; acceptable.

Example 2. Check the same joist for deflection under the 40 psf live load alone, with $E = 1{,}600{,}000$ psi and a limit of $L/360$.

Show solution

1. Live load per joist: $40 \times 1.333 = 53.3 \text{ lb/ft} = 4.44 \text{ lb/in}$; span $L = 168$ in.
2. Moment of inertia: $I = bh^3/12 = 1.5 \times 9.25^3 \div 12 \approx 98.9 \text{ in}^4$.
3. Deflection:
   $\delta = 5wL^4/(384EI) = 5(4.44)(168)^4 \div (384 \times 1.6 \times 10^6 \times 98.9) \approx 0.29$ in.
4. Limit: $168/360 \approx 0.47$ in. The joist passes.

Answer: About 0.29 in, under the 0.47 in limit.

Example 3. A wooden plank 200 mm wide and 50 mm thick is used as a beam. Compare its strength and stiffness laid flat and stood on edge.

Show solution

1. Flat: $b = 200$, $h = 50$: $S = 200 \times 50^2/6 \approx 83{,}300\ \text{mm}^3$, $I = 200 \times 50^3/12 \approx 2.08 \times 10^6\ \text{mm}^4$.
2. On edge: $b = 50$, $h = 200$: $S = 50 \times 200^2/6 \approx 333{,}000\ \text{mm}^3$, $I = 50 \times 200^3/12 \approx 3.33 \times 10^7\ \text{mm}^4$.
3. On edge it is $333{,}000/83{,}300 = 4$ times stronger and $3.33 \times 10^7/2.08 \times 10^6 = 16$ times stiffer: the ratio of depths (4) and its square (16).

Answer: On edge: 4 times stronger, 16 times stiffer.

PRACTICE

1. Easy Find the maximum moment in a 6 m beam carrying 10 kN/m.

   Show answer

   $10 \times 36 \div 8 = 45\ \text{kN·m}$.

2. Easy Find S and I for a beam 100 mm wide and 200 mm deep.

   Show answer

   $S = 100 \times 40{,}000 \div 6 \approx 6.67 \times 10^5\ \text{mm}^3$;
   $I = 100 \times 8 \times 10^6 \div 12 \approx 6.67 \times 10^7\ \text{mm}^4$.

3. Easy If a beam's depth is tripled, by what factors do its strength and stiffness change?

   Show answer

   Strength ×9, stiffness ×27.

4. Medium A 4 m beam has a 12 kN load at midspan. Find the maximum moment.

   Show answer

   $PL/4 = 12 \times 4 \div 4 = 12\ \text{kN·m}$.

5. Medium For Problem 1, what bending stress occurs in the beam of Problem 2?

   Show answer

   $\sigma = 45 \times 10^6 \div 6.67 \times 10^5 \approx 67.5\ \text{MPa}$.

6. Medium A 2×8 (1½ in × 7¼ in) joist carries a moment of 12,000 in·lb. Find its bending stress.

Show answer

$S = 1.5 \times 52.56 \div 6 \approx 13.1 \text{ in}^3$; $\sigma \approx 913$ psi.

7. Medium If the span of a uniformly loaded beam is increased by 20%, by what factors do the moment and the deflection increase?

Show answer

Moment ×1.44 (L^2); deflection ×2.07 (L^4).

8. Hard A steel beam ($E = 200$ GPa, $I = 1.2 \times 10^8 \text{ mm}^4$) spans 8 m with a uniform load of 15 kN/m. Find its midspan deflection and compare it with span/360.

Show answer

$\delta = 5(15)(8000)^4 \div (384 \times 200{,}000 \times 1.2 \times 10^8)$ with $w = 15$ N/mm:
$= 5 \times 15 \times 4.096 \times 10^{15} \div 9.216 \times 10^{15} \approx 33.3$ mm; limit $8000/360 \approx 22.2$ mm; it fails the deflection check.

9. Hard What depth of 1½ in-wide joist would carry a moment of 30,000 in·lb with a stress of 1,000 psi?

Show answer

$S = 30 \text{ in}^3 = 1.5h^2/6$, $h^2 = 120$, $h \approx 10.95$ in: use a 2×12 (11¼ in).

10. Hard Conceptual. Why is an I-shaped beam more efficient than a solid rectangle of the same weight?

Show answer

Model answer: bending stress is largest far from the neutral axis and zero at it; the I-beam puts most of its material in the flanges, far from the axis, where it contributes most to I and S, and uses only a thin web near the axis to hold the flanges apart and carry shear.

11. **Challenge** Galileo's mistaken model gave a beam's bending strength as $\sigma_t\, bh^2/2$, but the correct elastic result is $\sigma\, bh^2/6$. By what factor did Galileo overestimate the strength? Why might his mistake not have caused failures in practice?

Show answer

By a factor of 3. In practice, builders sized beams by experience and wide safety margins, and Galileo's formula gave correct proportions (strength $\propto bh^2$, so comparisons between beams were right) even though the absolute value was too high.

COMMON MISTAKES

- Using bh^2 when the problem asks for stiffness (it is bh^3) or vice versa.
- Mixing units in the deflection formula. Put everything in N and mm (or lb and in) before calculating.
- Checking strength but not deflection. Many floors pass for strength but fail for stiffness.

> **Make it a game · Depth Is Everything.** Stack five sheets of paper, loose, then glued, across two books. Predict how much the depth difference changes stiffness, then test with coins. Say why a deep beam resists bending, naming the neutral axis, compression and tension. Calculate the moment of inertia of a 2 by 4 in both orientations. Best prediction scores.

UNIT 5

Structures II: Materials and Members

A structure is only as good as what it is made of. In this unit you will study the four great structural materials, timber, masonry, steel and concrete, through the people who learned to use them: the Chicago carpenters of the balloon frame, Burnham and Root and the massive walls of the Monadnock, Bessemer and Eads, and Aspdin, Monier, Hennebique and Freyssinet. For each you will learn how it behaves: where it is strong and weak, how it changes with water, heat and time, and how engineers design with it.

The unit ends where Galileo began the science of structures: with the beam. You will calculate bending moments, stresses and deflections, size a real floor joist, and test model beams to find out why depth is everything. These are the calculations you will use in bridges, towers and foundations for the rest of the course.

PREREQUISITES

- Unit 4: loads, reactions and statics
- Algebra: formulas with powers (h^2, h^3, L^4) and solving for a variable
- Unit conversions between SI and US units (MPa, psi, ksi, kN, lb)

MASTERY CHECKLIST

- ☐ I can explain how wood's grain and moisture affect its strength and check a stud for compression and bearing.
- ☐ I can calculate the base stress in a masonry wall and the thickness needed for a given height and load.
- ☐ I can use $\sigma = P/A$, $\varepsilon = \delta/L$, and $\delta = PL/AE$, and read a stress-strain curve.
- ☐ I can explain the water-cement ratio and calculate the moment capacity of a simple reinforced concrete beam.
- ☐ I can calculate M, I, S, bending stress and deflection, and size a joist for strength and stiffness.

OPTIONAL KITS FOR THIS UNIT

- **Pitsco Education (balsa and basswood)**: Precision balsa and basswood strips and sheets for the grain test (5.1) and beam tests (5.5), and the bridge competition in Unit 6. Craft-store balsa also works.
- **KEVA planks**: Useful for masonry-bond walls in Lesson 5.2 and for testing a plank flat versus on edge in Lesson 5.5.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

5.1	**Timber: The Balloon Frame and Wood as a Structural Material**	3 sessions of 50 minutes
5.2	**Masonry: The Monadnock and the Limits of Brick**	3 sessions of 50 minutes
5.3	**Steel: Bessemer, Eads and the Stress-Strain Curve**	3 sessions of 50 minutes
5.4	**Concrete: Roman, Modern, Reinforced and Prestressed**	3 sessions of 50 minutes
5.5	**Beams: Bending and Deflection from Galileo to Today**	3 sessions of 50 minutes

Unit 5 Test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation. Use actual (not nominal) lumber sizes. The test is worth 100 points.

1. **5 pts**
   Why did the platform frame replace the balloon frame?

2. **6 pts**
   A 2×4 stud carries 2,100 lb. Find the stress in the stud and the bearing stress on the plate. Is it acceptable with limits of 1,350 psi and 625 psi?

3. **6 pts**
   Find the self-weight stress at the base of a 45 m brick wall ($\gamma = 19\ \mathrm{kN/m^3}$).

4. 7 pts

The wall in Problem 3 also carries 360 kN/m from its floors. With an allowable stress of 1.4 MPa, what is the minimum thickness?

5. 5 pts

Why was the Monadnock the end of tall load-bearing masonry in Chicago?

6. 7 pts

A steel tie 20 mm in diameter and 4.0 m long carries 45 kN. Find the stress, the stretch ($E = 200$ GPa), and the factor of safety against a 250 MPa yield.

7. 6 pts

Sketch and label a stress-strain curve for mild steel, and explain what ductility means for safety.

8. 5 pts

Why did Eads insist on testing every batch of steel?

9. 6 pts

A mix has 360 kg of cement per cubic meter and a water-cement ratio of 0.45. How much water does it have? What happens to the strength if the ratio rises to 0.60?

10. 8 pts

A concrete beam is 300 mm wide with $d = 550$ mm, $A_s = 1200\ \text{mm}^2$, $f_y = 415$ MPa and $f\prime_c = 30$ MPa. Find a and M_n.

11. 5 pts

What does prestressing do for a concrete beam?

12. 6 pts

A 5 m beam carries 8 kN/m. Find the maximum moment. A rectangular wood beam 90 mm wide must keep its stress below 10 MPa. What depth is needed?

13. 6 pts

A 2×12 (1½ in × 11¼ in) joist spans 16 ft at 16 in on center with 50 psf. Find the bending stress.

14. **7 pts**

A beam deflects 8 mm. Find its deflection if (a) its depth is doubled, (b) its span is doubled (same load per meter).

15. **7 pts**

A plank 240 mm × 40 mm is used as a beam. How many times stronger and stiffer is it on edge than flat?

16. **8 pts**

Short essay. Compare timber, masonry, steel and reinforced concrete: for each, give one great strength, one weakness, and how engineers design around the weakness.

ANSWER KEY

1. In the balloon frame the long studs leave open cavities from basement to attic that spread fire; the platform frame caps each story with a floor platform, which blocks fire and is easier to build with shorter lumber.
2. $2100 \div 5.25 = 400$ psi for both; acceptable.
3. 855 kPa.
4. $t = 360 \div (1400 - 855) \approx 0.66$ m.
5. At sixteen stories its walls had to be about 6 ft thick at the base, using up valuable floor space; taller walls would be impractical or impossible, while steel frames could carry the loads on slender columns.
6. $A \approx 314\ \text{mm}^2$; $\sigma \approx 143$ MPa; $\delta = 143 \times 4000/200{,}000 \approx 2.9$ mm; $FS \approx 1.75$.
7. Straight elastic line to the yield point, a yield plateau, strain hardening up to the ultimate strength, then necking to fracture. Ductility means steel stretches a lot before breaking, giving warning and redistributing load.
8. Early steel was not uniform, and he would not trust the mill's claims; testing ensured each piece met his specification, which made the bridge safe and pushed the industry toward reliable standards.
9. 162 kg of water; the strength drops substantially (more water leaves more voids).
10. $a = 498{,}000 \div (0.85 \times 30 \times 300) \approx 65.1$ mm;
    $M_n = 498{,}000 \times (550 - 32.6) \approx 2.58 \times 10^8$ N·mm $= 258$ kN·m.
11. Tensioned tendons squeeze the concrete in advance, so service loads must first overcome that compression before tension can develop; the beam stays uncracked and deflects less.
12. $M = 8 \times 25/8 = 25$ kN·m; $S = 25 \times 10^6/10 = 2.5 \times 10^6\ \text{mm}^3 = 90h^2/6$,
    $h^2 \approx 166{,}700$, $h \approx 408$ mm.
13. $w = 66.7$ lb/ft; $M = 66.7 \times 256/8 \approx 2134$ ft·lb $= 25{,}600$ in·lb;
    $S = 1.5 \times 126.6/6 \approx 31.6\ \text{in}^3$; $\sigma \approx 810$ psi.
14. (a) $8/8 = 1$ mm; (b) $8 \times 16 = 128$ mm.
15. Depth ratio 6: 6 times stronger, 36 times stiffer.
16. Model answer: timber is light and strong along the grain but weak across it and vulnerable to moisture, fire and rot (designers check bearing, keep it dry, protect or oversize it for fire); masonry is durable, fire-resistant and strong in compression but weak in tension and heavy (designers keep it in compression, bond it, reinforce it, and limit height); steel is strong, stiff and ductile but rusts

and weakens in fire (paint, galvanize, fireproof); reinforced concrete is moldable, durable and fire-resistant but weak in tension and prone to cracking (steel on the tension side, cover, control joints, prestressing).

PORTFOLIO PIECE: A MATERIALS TEST REPORT AND A DESIGNED BEAM

Choose two of this unit's builds and write them up as a formal test report: purpose, materials, procedure, data, graphs, results compared with theory, and sources of error. Then design a beam of balsa or basswood to span 40 cm and carry a 2 kg center load with a factor of safety of 2, using your measured strength and E to size it.

Build it, predict its deflection under 2 kg, test it to failure, and compare the measured deflection and failure load with your predictions. Report its efficiency: failure load divided by beam weight.

LOOK FOR	WHAT IT MEANS
Test reports	Two complete reports with clear procedures, data tables with units, graphs and honest error discussion.
Design calculation	Beam sized from measured properties with moment, section modulus and deflection calculated correctly.
Prediction vs result	Predicted deflection and failure load compared with measurements, with percent differences and explanations.
Efficiency	Beam weight, failure load and efficiency recorded; design choices (such as an I-shape) explained.
Presentation	Report is neat, organized and written in complete sentences.

UNIT 6

Bridges

Bridges are where structural engineering is most visible and most daring. In this unit you will survey the bridge types and choose among them, then follow the history of great bridges through the people who built them: Abraham Darby's Iron Bridge and Thomas Telford's aqueduct and chains; John, Washington and Emily Roebling at Brooklyn; Fowler and Baker at the Forth, building a monument to safety after the Tay disaster; and the Golden Gate.

You will also study two of the most important failures in engineering history, the Quebec Bridge and the Tacoma Narrows Bridge, and what engineers learned from them: buckling, checking, wind and humility. The unit ends with a bridge competition in which you design, analyze, build and test your own truss bridge and score it on efficiency.

PREREQUISITES

- Unit 4: statics, method of joints, arches and cables
- Unit 5: stress, E, moment of inertia
- Algebra and trigonometry: square roots, sine and cosine, solving for a variable

MASTERY CHECKLIST

- ☐ I can name six bridge types, explain how each carries load, and choose a type for a site with reasons.
- ☐ I can explain how cast and wrought iron were used in the Iron Bridge and Telford's works, and calculate aqueduct loads and arch thrust.
- ☐ I can calculate suspension cable tension from span, sag and load, and the steel needed.
- ☐ I can use Euler's formula and explain the Quebec and Tacoma Narrows failures and what changed afterward.
- ☐ I can design, analyze, build and test a truss bridge and compute its efficiency.

OPTIONAL KITS FOR THIS UNIT

- **Pitsco Education (balsa bridge supplies)**: Balsa and basswood strips, bridge kits and a structures tester used in school bridge competitions; fits Lessons 6.4 and 6.5. Craft-store balsa or craft sticks work too.
- **Bridge Design Contest (Engineering Encounters)**: Free bridge-design software and an annual online contest for students, based on the West Point Bridge Designer; a good companion to Lesson 6.5. A parent should review registration.
- **KEVA planks**: Useful for balanced-cantilever and arch models in Lessons 6.1 and 6.2.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

6.1	**Bridge Types and Spans: The Forth Bridge**	3 sessions of 50 minutes
6.2	**The Iron Bridge and Thomas Telford**	3 sessions of 50 minutes
6.3	**The Brooklyn Bridge and the Golden Gate**	3 sessions of 50 minutes
6.4	**Failures: The Quebec Bridge and Tacoma Narrows**	3 sessions of 50 minutes
6.5	**The Bridge Competition: Design, Analyze, Build and Test**	3 sessions of 50 minutes

Unit 6 test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation. Draw free-body diagrams where they help. The test is worth 100 points.

1. **6 pts**
   Match each bridge type to its main way of carrying load: beam, arch, suspension, cable-stayed.

2. **5 pts**
   How did the Tay Bridge disaster influence the design of the Forth Bridge?

3. **6 pts**

   A load of 40 kN hangs 50 m from a pier on a cantilever arm. What counterweight is needed 30 m on the other side?

4. **6 pts**

   Why is cast iron suited to arches but not to chains?

5. **6 pts**

   A canal trough 3.0 m wide holds water 1.4 m deep and spans 15 m between piers. Find the water load per meter and the midspan moment.

6. **8 pts**

   A suspension span of 600 m with a sag of 60 m carries 120 kN/m. Find H, V and $T_{\max}$.

7. **5 pts**

   For Problem 6, what area of steel is needed at 300 MPa?

8. **6 pts**

   Describe the roles of John, Washington and Emily Roebling in building the Brooklyn Bridge.

9. **6 pts**

   How did Washington Roebling respond when fraudulent wire was found in the cables?

10. **7 pts**

    Find the Euler buckling load of a pinned steel strut 30 mm × 30 mm and 3.0 m long ($E = 200$ GPa). What if it is braced at midheight?

11. **6 pts**

    What were the structural cause and the process failures of the 1907 Quebec Bridge collapse?

12. **6 pts**

    Why did the Tacoma Narrows Bridge fail, and what changed in bridge design afterward?

13. **6 pts**

Find the vortex-shedding frequency of a deck 3.0 m deep in a 15 m/s wind with St = 0.15.

14. **7 pts**

In a model Pratt truss loaded by P at the center, a 45° end diagonal carries $0.707P$ in compression. It is a balsa strip 3 mm × 3 mm and 6 cm long ($E = 3000\ \mathrm{MPa}$). Find the load P at which it buckles.

15. **6 pts**

A bridge of 32 g held 27 kg; another of 20 g held 19 kg. Which was more efficient?

16. **8 pts**

Short essay. Explain what the Quebec Bridge collapse taught engineers about checking assumptions, and how the iron ring keeps that lesson alive.

ANSWER KEY

1. Beam: bending (compression on top, tension below). Arch: compression with outward thrust. Suspension: deck hung from draped cables in tension, anchored at the ends. Cable-stayed: deck held by straight cables to towers, deck in compression.
2. The Tay Bridge blew down because wind was not properly allowed for; the Forth Bridge was designed for a large wind pressure (56 psf) and made visibly massive and stiff to restore public confidence, and Bouch's Forth suspension design was abandoned.
3. $40 \times 50/30 \approx 66.7$ kN.
4. Cast iron is strong in compression but brittle and weak in tension; arches are in compression, chains in tension.
5. $w = 9.81 \times 3.0 \times 1.4 \approx 41.2$ kN/m; $M = 41.2 \times 225/8 \approx 1159$ kN·m.
6. $H = 120 \times 360{,}000/480 = 90{,}000$ kN; $V = 36{,}000$ kN; $T \approx 96{,}900$ kN.
7. $96.9 \times 10^6/300 \approx 323{,}000$ mm^2 (0.32 m^2).
8. John designed it and died in 1869 after an accident during surveying; Washington became chief engineer, was disabled by caisson disease in 1872 and directed the work from his window; Emily carried his instructions, learned the engineering and acted as his link to the site, and was first to cross at the opening in 1883.
9. The bad wire could not be removed, so he calculated the loss and added about 150 extra wires to each cable, keeping a large factor of safety.
10. $I = 30^4/12 = 67{,}500$ mm^4; $P = \pi^2(200{,}000)(67{,}500)/3000^2 \approx 14{,}800$ N; braced at midheight, four times: about 59,200 N.
11. Cause: the lower compression chords buckled because their lacing was too light, with the dead load about 20% higher than computed after the span was lengthened. Process: the consulting engineer supervised from afar, did not recalculate the dead load, and warnings were not acted on in time; there was no independent check.
12. Its shallow, solid-girder deck was aerodynamically unstable and twisted in aeroelastic flutter in a moderate wind; afterward, suspension decks were given deep open trusses or streamlined boxes and long spans were tested in wind tunnels.
13. $0.15 \times 15/3.0 = 0.75$ Hz.

14. $P_{cr} = \pi^2(3000)(6.75)/60^2 \approx 55.5$ N; $P = 55.5/0.707 \approx 78.5$ N.
15. First: $27{,}000/32 \approx 844$; second: $19{,}000/20 = 950$. The second.
16. Model answer: at Quebec, a famous engineer's assumptions went unchecked: the span was lengthened without recalculating the dead load, the chord lacing was not proven, and warnings from the site were not acted on. Engineers learned that every assumption must be verified, that designs need independent review, that field warnings must stop the work, and that reputation is no substitute for calculation. The Ritual of the Calling of an Engineer and its iron ring, worn on the working hand, remind each new engineer of humility and of the public's trust.

PORTFOLIO PIECE: A BRIDGE DESIGN REPORT

Prepare a design report for your competition bridge as an engineer would for a client: the rules (design criteria), the options you considered with a decision matrix, your method-of-joints analysis and member schedule, your predicted failure load and first member to fail, construction photos, and test results for both versions.

Finish with an honest comparison of prediction and result, a description of the failure, and a page on what a real bridge of your design would need (materials, foundations, wind checks) to be built safely across a river near you.

LOOK FOR	WHAT IT MEANS
Design process	Clear criteria, at least three options, and a reasoned decision matrix.
Analysis	Correct member forces, member sizing for tension and buckling, and a predicted failure load and location.
Construction	Neat, square, accurately built bridge that meets the rules; mass recorded.
Testing and comparison	Failure load, efficiency and failure mode recorded and compared honestly with predictions; improvement round documented.
Real-world reflection	Thoughtful discussion of what a full-size version would require, drawing on the unit's case studies.

6.1 Bridge Types and Spans: The Forth Bridge

OBJECTIVES

- Name the main bridge types (beam, truss, arch, cantilever, suspension, cable-stayed), explain how each carries load, and give the span range each suits.
- Use a decision matrix and simple calculations (wind pressure, thermal movement, span-to-depth) to choose a bridge type for a site.
- Model a balanced cantilever and measure how counterweights and anchoring keep it in equilibrium.

> **Warm-up.** Think of three bridges you have crossed. For each, write what you think holds it up: a beam, an arch, a truss, cables or something else. Which was longest? Which looked strongest? Were they the same one?

WATCH

Every Kind of Bridge Explained in 15 Minutes

Practical Engineering · 17:35

https://www.youtube.com/watch?v=DX_zkaK5PaI

> **WHEN TO WATCH**
> After the story. Make a table as you watch: bridge type, how it carries load, typical span, one example.

KEY TERMS

beam (girder) bridge
A deck carried by beams that bend between supports; simplest and most common, for spans up to about 300 m with steel box girders, usually far less.

cantilever bridge
A bridge built out from each pier in balanced arms, often with a short suspended span between arm tips; good for long spans built without falsework.

suspension bridge
A deck hung from vertical hangers on two main cables draped over towers and anchored at each end; the longest spans, over 2,000 m.

cable-stayed bridge
A deck held up by straight cables running directly from towers to the deck; spans up to about 1,100 m.

span

The distance between supports; the main span of a bridge is its longest.

falsework

Temporary supports used to hold a bridge up until it can carry itself.

CORE IDEAS

The story. On the stormy night of December 28, 1879, the central spans of the new Tay Bridge in Scotland blew down while a train was crossing, and all aboard, about 75 people, were lost. The inquiry found that its designer, Sir Thomas Bouch, had made no proper allowance for wind. Bouch had also designed a suspension bridge to cross the Firth of Forth nearby; it was cancelled. The new design, by John Fowler and Benjamin Baker, was the opposite of slender: a steel cantilever bridge of three great double-diamond towers, with two main spans of 1,710 ft (521 m), designed for a wind pressure of 56 lb per square foot. Built from 1883 to 1890 from about 54,000 tons of steel, the Forth Bridge was for a time the longest-span bridge in the world, and it still carries trains today.

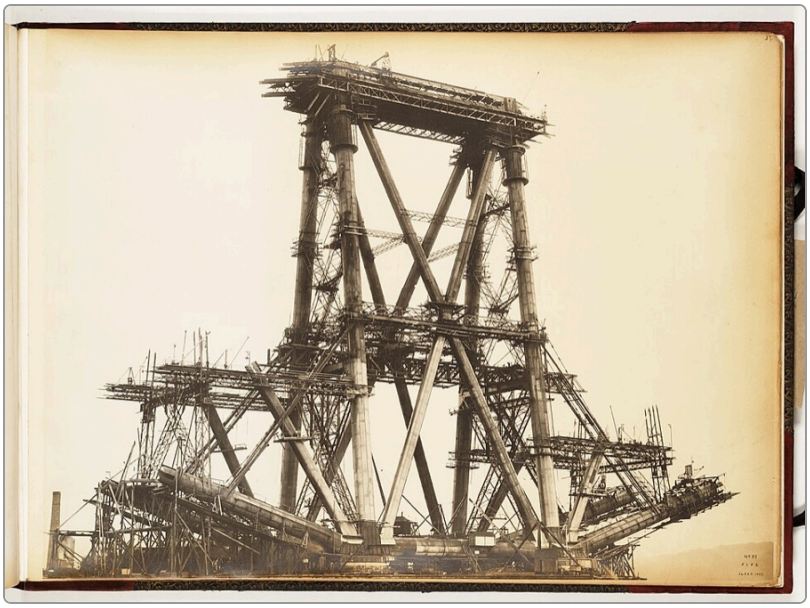
How they figured it out. Baker had to win back public confidence. He explained the cantilever principle with the human model you studied in Lesson 4.2, and he designed every member to be visibly massive: the tubular compression struts are 12 ft in diameter. The cantilevers were built outward from each tower in balance, each arm counterweighted by its partner, so no falsework was needed in the deep, tidal Firth. Engineers checked the wind loads and the steel's strength by test, and the design was so cautious that it became a symbol of reliability. (The bridge is so large that painting it became a British saying for an endless job.)

The physics. Every bridge type carries load to its supports in a different way. A **beam** bends: compression on top, tension below. A **truss** is a deep beam made of triangles, with members in pure tension or compression (Lesson 4.3). An **arch** works in compression and pushes outward on its abutments (Lesson 4.4). A **cantilever** balances arms on each side of a pier, like the human model. A **suspension** bridge hangs its deck from cables in tension, which pull inward on the towers and must be anchored into the ground. A **cable-stayed** bridge hangs its deck from straight cables to the towers, putting the deck itself in compression. The longer the span, the more the bridge's own weight dominates, so long spans need efficient forms that carry load in tension or compression rather than bending.

The engineering. Choosing a bridge type is a design decision. Engineers weigh the span (set by the river, the ships or the valley), the foundations available, construction methods (can falsework stand in the water?), wind and earthquake, cost, maintenance, and appearance. Roughly: beam bridges up to about 50–200 m, trusses and arches to about 500 m, cantilevers to about 550 m, cable-stayed to about 1,100 m, and suspension bridges beyond, with the Çanakkale Bridge in Turkey (2022) spanning 2,023 m. The Forth Bridge shows that a bridge must also earn the trust of the people who cross it.

Try it. In the build you will make a balanced cantilever model and measure the counterweight needed as the arm grows longer.

PICTURE STUDY



The Fife Cantilever of the Forth Bridge under Construction, Evelyn George Carey (photographer for the contractors), September 14, 1887 · Photograph · Wikimedia Commons (from the construction albums)

Image: Wikimedia Commons, public domain

The contractors hired a photographer to record every stage of the Forth Bridge. Here the Fife tower stands nearly complete, and the first huge diagonal strut has been raised to rail level. The arms that will reach across the water have not yet begun.

Look closely

- Find the vertical columns of the tower and the great diagonal tubes. Which do you think will be in compression?
- Find the lifting platform near the top. How might heavy steel parts have been raised?
- Look at the scaffold and walkways. How did the builders work so high above the water?
- Imagine the arms that will grow out from this tower. Which way will they reach?

Talk about it

1. Why can a cantilever be built without falsework standing in the water?
2. Why do you think Baker made the bridge look so massive?

3. What would happen to this tower if only one arm were built out and not the other?

Safety first. Wear safety glasses for the cantilever test, and keep the model low; counterweights can drop when it tips.

BUILD

A Balanced Cantilever

You will build one tower of a cantilever bridge with two arms from rulers or strips of wood, extend the arms in steps, and measure the counterweight and tie forces needed to keep it balanced, as Fowler and Baker did in their human model.

You will need

- A sturdy block or box as the tower (a pier)
- Two rulers or wooden strips as the arms, string for ties, tape
- Coins or washers as loads and counterweights; a kitchen scale
- A spring scale or luggage scale (optional) to measure tie tension

Steps

1. Set one ruler across the top of the block so that it balances at its middle (the pier). Mark the pier point.
2. Hang a load (say 20 g) at the tip of one arm, 10 cm from the pier. Find by experiment the counterweight needed at 10 cm on the other side, then at 5 cm. Compare with $W_1 x_1 = W_2 x_2$.
3. Now model the bridge: add a short "suspended span" (a strip of card) between the tip of this cantilever and a second tower, resting on both tips.
4. **Predict.** If you put a load in the middle of the suspended span, what fraction of it reaches each cantilever tip? What counterweight is needed at each anchor arm?
5. Load the suspended span and measure: does each tower stay balanced with your predicted counterweights?
6. Replace the counterweight with a string tie anchored to the table (a backstay). Measure its tension with a spring scale if you have one, or calculate it from moments.
7. Lengthen the arms in 2 cm steps and record the counterweight needed each time. Graph counterweight against arm length.

8. **Improve.** Design a version where the anchor arms are shorter than the river arms. How much heavier must the counterweights be? Test it.

Tips

- Use a pencil or a dowel taped to the top of the block as a sharp pivot.
- Measure distances from the pivot to the center of each weight.
- Keep the loads light; the point is balance, not breaking.

Check your work

☐ I can name six bridge types and explain how each carries load.

☐ My measurements agree with $W_1x_1 = W_2x_2$ within my measuring accuracy.

☐ I can explain how the Tay disaster shaped the Forth Bridge.

ENGINEER'S NOTEBOOK

1. A one-page chart of the six bridge types with a sketch, the main forces (T and C), the span range, and one famous example of each.
2. Your counterweight data and graph.
3. A short design memo choosing a bridge type for a real crossing near you, with a decision matrix.

KEY FORMULAS

$q \approx 0.00256\,V^2$ (psf, V in mph), $\quad F = qA$

$\Delta L = \alpha L \Delta T$

$\frac{L}{d}$ (span-to-depth ratio): about 15–25 for beams and trusses

Balanced cantilever: $W_1x_1 = W_2x_2$ about the pier

From history. Kaichi Watanabe, the man in the middle of the human model photograph, returned to Japan and became a leading engineer and businessman in its growing railway and construction industries. Benjamin Baker went on to help design the first Aswan Dam on the Nile.

Example 1. After the Tay disaster, the Forth Bridge was designed for a wind pressure of 56 psf. What wind speed does that correspond to with $q = 0.00256V^2$? Find the wind force on a girder face 300 ft long and 20 ft deep at that pressure.

Show solution

1. $V = \sqrt{q/0.00256} = \sqrt{56/0.00256} = \sqrt{21{,}875} \approx 148$ mph.
2. Area: $300 \times 20 = 6000$ ft^2.
3. Force: $F = 56 \times 6000 = 336{,}000$ lb, about 168 tons sideways on that one face.

Answer: About 148 mph; about 336,000 lb.

Example 2. A steel bridge 521 m long between expansion joints sees temperatures from −20 °C to 40 °C. How much must its joints allow it to move? ($\alpha = 12 \times 10^{-6}/^\circ\mathrm{C}$)

Show solution

1. $\Delta T = 40 - (-20) = 60\ ^\circ\mathrm{C}$.
2. $\Delta L = 12 \times 10^{-6} \times 521\ \mathrm{m} \times 60 \approx 0.375\ \mathrm{m}$.
3. The joints and bearings must allow about 38 cm of movement in a span this long.

Answer: About 0.38 m.

Example 3. A town must cross a river 180 m wide. Barges need the middle 120 m clear, foundations are good rock, and falsework is not allowed in the channel. Score three options with weights cost 0.35, constructability 0.30, maintenance 0.15, appearance 0.20: (A) steel truss on two river piers, scores 8, 6, 6, 5; (B) concrete arch springing from the banks, 6, 5, 8, 9; (C) cable-stayed, 5, 8, 7, 8.

Show solution

1. A: $0.35(8) + 0.30(6) + 0.15(6) + 0.20(5) = 2.8 + 1.8 + 0.9 + 1.0 = 6.5$.
2. B: $0.35(6) + 0.30(5) + 0.15(8) + 0.20(9) = 2.1 + 1.5 + 1.2 + 1.8 = 6.6$.
3. C: $0.35(5) + 0.30(8) + 0.15(7) + 0.20(8) = 1.75 + 2.4 + 1.05 + 1.6 = 6.8$.
4. The cable-stayed bridge scores best, mostly because it can be built out from its towers without falsework in the channel. The truss loses because its piers would sit in the barge channel unless the span were longer.

Answer: Option C, cable-stayed (6.8), then B (6.6) and A (6.5).

PRACTICE

1. Easy Which bridge type is used for the longest spans?

   Show answer

   The suspension bridge.

2. Easy In a suspension bridge, what are the main cables in: tension or compression? What are the towers in?

   Show answer

   Cables in tension; towers in compression.

3. Easy Convert the Forth Bridge's 1,710 ft main spans to meters.

   Show answer

   $1710 \times 0.3048 \approx 521$ m.

4. Medium A truss bridge spans 90 m with a span-to-depth ratio of 15. How deep is the truss?

   Show answer

   6 m.

5. Medium Find the wind pressure for an 80 mph wind and the force on a bridge face of 5,000 ft^2.

Show answer

$q \approx 16.4$ psf; $F \approx 82{,}000$ lb.

6. Medium A load of 30 N hangs 0.40 m from a pier on a cantilever. What counterweight is needed 0.25 m on the other side?

Show answer

$30 \times 0.40/0.25 = 48$ N.

7. Medium A concrete bridge 240 m long sees a 45 °C range of temperature ($\alpha = 10 \times 10^{-6}/^{\circ}\mathrm{C}$). How much movement must its joints allow?

Show answer

$10 \times 10^{-6} \times 240 \times 45 = 0.108$ m ≈ 11 cm.

8. Hard A suspended span of 100 m carrying 80 kN/m rests on the tips of two cantilevers. What force does it put on each tip? If each cantilever arm is 200 m long, what moment does this force create at the pier?

Show answer

Each tip carries $80 \times 100/2 = 4000$ kN; moment at the pier $= 4000 \times 200 =$ 800,000 kN·m (plus the arm's own weight).

9. Hard Why can a cantilever or cable-stayed bridge be built without falsework, but a traditional arch usually cannot?

Show answer

Cantilever and cable-stayed bridges are built outward from their supports in balanced pieces that carry themselves (held by counterweights or stays) at every stage; an arch cannot stand until it is complete and closed at the crown, so it needs centering or temporary ties.

10. Hard Conceptual. Why does a bridge's own weight matter more as the span grows?

Show answer

Model answer: the weight of the deck and structure grows with span (and the members get heavier to carry longer spans), and the bending or cable forces it causes grow roughly with the square of the span, while traffic load per meter stays the same; so at long spans most of the structure is carrying itself, and efficient forms working in tension or compression become essential.

11. Challenge Compare the bending moment in a 521 m simply supported span with uniform load w to the moment at the pier of a cantilever bridge whose arms reach 207 m from each pier toward the center, with a 107 m suspended span between them, under the same w.

Show answer

Simple span: $w(521)^2/8 \approx 33{,}930w$. Cantilever: the arm's own load $w(207)^2/2 \approx 21{,}420w$ plus the suspended span's tip reaction $(107w/2)(207) \approx 11{,}070w$, total $\approx 32{,}500w$. They are similar; the cantilever's real advantages are construction without falsework and that the moment is resisted over the deep tower, where the structure is deepest.

COMMON MISTAKES

- Thinking suspension and cable-stayed bridges are the same. Suspension decks hang from vertical hangers on draped main cables; cable-stayed decks are held by straight cables directly to the towers.
- Forgetting the horizontal pull of suspension cables, which must be anchored in massive anchorages.
- Choosing a bridge type by looks alone. Site, span, foundations and construction method usually decide.

> **Make it a game · Which Bridge Here?.** Your parent describes a site: a narrow gorge, a wide river, soft banks. You name the bridge type, the span range and the reason, and score points for sound reasons. Then compare your choices on a quick decision matrix. Next, draw a balanced cantilever and show where the counterweight sits. Three sites, best of three.

6.2 The Iron Bridge and Thomas Telford

OBJECTIVES

- Tell how the Iron Bridge (1779) and Thomas Telford's aqueducts and suspension bridge pioneered metal bridges.
- Explain the strengths of cast iron (compression) and wrought iron (tension) and match each to arches and chains.
- Calculate loads and thrusts for an iron arch and an aqueduct trough, and build a model arch of interlocking parts.

> **Warm-up.** Look at a wooden chair or table. How are the pieces joined: nails, screws, glue, or interlocking joints? Draw one joint. Now imagine making the same joint out of iron. Would it work?

WATCH

How England Was Made | Episode 2: Iron Bridge

English Heritage · 7:07

https://www.youtube.com/watch?v=O5F4rA8QXQY

> **WHEN TO WATCH**
> After the story. Notice the carpentry joints used in the iron.

BACKUP VIDEO

Engineering The Iron Bridge: Iron Casting From The 1700s

English Heritage · 3:57

https://www.youtube.com/watch?v=SHoY0thM8jE

With the engineering paragraph: craftsmen recreate eighteenth-century iron casting.

KEY TERMS

cast iron

Iron with 2–4% carbon, poured into molds; very strong in compression but brittle and weak in tension.

wrought iron

Nearly pure iron worked by hammering, tough and good in tension; used for chains, ties and plates before cheap steel.

aqueduct

A bridge that carries water, here a canal, across a valley.

rib

One of the curved members of an arch bridge; the Iron Bridge has five parallel ribs.

eyebar chain

A chain made of flat bars with holes (eyes) at each end, linked by pins, used for early suspension bridges.

CORE IDEAS

The story. In 1779, in the Severn Gorge in Shropshire, England, the ironmaster Abraham Darby III cast and raised the world's first major bridge made of iron. The bridge, designed with the architect Thomas Pritchard, spans about 100 ft (30.6 m) in a single semicircular arch of five cast-iron ribs, about 378 tons of iron in some 1,700 pieces. It opened to traffic on New Year's Day, 1781, and gave its name to the town that grew around it, Ironbridge. A generation later, a Scottish stonemason who had become Britain's greatest civil engineer, Thomas Telford, carried the Llangollen Canal 126 ft (38 m) above the River Dee on the Pontcysyllte Aqueduct (1805), a cast-iron trough on stone piers, and in 1826 opened the Menai Suspension Bridge to Anglesey, with a span of 577 ft (176 m) hung from wrought-iron chains.

How they figured it out. Darby's builders had no tradition of iron bridges, so they used what they knew: carpentry. The iron pieces are joined with dovetails, mortise-and-tenon joints and wedges, exactly as a carpenter would join oak. The shape, a round arch, kept the cast iron in compression, where it is strong. Telford learned from the Iron Bridge and improved on it: his later iron arches were lighter and flatter, and at Pontcysyllte he made the iron trough itself the structure, bolted from flanged plates and sealed with Welsh flannel and lead. For the Menai Bridge he tested the wrought-iron eyebars for strength before using them, and he raised each chain by hand-winches in a single day while crowds watched. Telford became the first president of the Institution of Civil Engineers in 1820.

The physics. Cast iron can carry several times more stress in compression than in tension, so it belongs in arches and columns. Wrought iron is much better in tension, so it belongs in chains and ties. An arch carries its load in compression and

pushes outward with a thrust $H \approx \dfrac{wL^2}{8h}$ (Lesson 4.4); a chain hangs in tension and pulls inward on its towers with the same formula. A canal aqueduct carries the weight of water, which is surprisingly heavy: each cubic meter weighs 9.8 kN, and Archimedes' principle means a floating boat adds nothing at all, because it displaces exactly its own weight of water.

The engineering. The Iron Bridge proved that metal could span rivers, and its builders learned by doing; the bridge has needed repairs because the gorge's slopes slowly squeeze its abutments together, and in the 1970s engineers built a concrete strut beneath the river to hold them apart. Telford's works show the next step: choosing each material for the job, testing it first, and designing for maintenance. Pontcysyllte still carries canal boats today, and it is a World Heritage Site. Cast iron itself later fell from favor for bridges after brittle failures, and wrought iron and then steel took its place.

Try it. In the build you will make an arch of interlocking parts that fit together without glue, like the Iron Bridge's joints, and test it.

PICTURE STUDY

The Cast Iron Bridge near Coalbrookdale, Michael Angelo Rooker, 1780s · Watercolor (engraved for publication) · Aberdeen Art Gallery and Museums

Image: Wikimedia Commons, public domain

Artists came from all over Britain to paint the new wonder of the Severn Gorge. Rooker's view, made a few years after the bridge opened, shows how light the iron arch looked compared with stone bridges of the same span.

Look closely

- Find the arch ribs. How does their shape compare with a semicircle?
- Find the stone abutments at each end. What job do they do for the arch?
- Look at the small circles in the spandrels (between the arch and the deck). Why might the designers have used rings there?
- Find the boats. How much headroom does the arch leave for river traffic?

Talk about it

1. Why does the round arch suit cast iron?
2. The gorge's slopes have slowly pushed the abutments inward. What would that do to the arch?
3. How would this view look different if the bridge were built of stone?

Safety first. If you cut parts with a craft knife, an adult should do it or supervise. Wear safety glasses when loading the arch.

BUILD

Joints without Glue: An Interlocking Arch

The Iron Bridge was assembled with carpentry joints. You will design and cut an arch of card or foam pieces that lock together with tabs and slots (no glue), build it over a form, and test how much it carries with and without restraint at its feet.

You will need

- Corrugated cardboard or foam board, a craft knife (adult) and a cutting mat
- A compass, ruler and protractor
- A cardboard form (centering) for the arch
- Books or bricks as abutments; a cup and pennies or sand for loading

Steps

1. Draw a semicircular arch of inner radius 12 cm and outer radius 15 cm. Divide it into 7 or 9 equal voussoirs (pieces).
2. Design a joint for each piece: a tab on one end and a matching slot on the next, or a dovetail. Make two identical arch "ribs."
3. Cut cross-pieces with slots that lock the two ribs together about 6 cm apart, as the Iron Bridge's five ribs are tied.
4. Assemble the arch over the form between two book abutments. Remove the form.
5. **Predict.** How much load will it carry at the crown? At the quarter point?
6. Load the crown with the cup until something slips or breaks. Record the load and where it failed.
7. Now push the abutments slightly toward each other (1 cm) before loading, as the gorge's slopes did to the Iron Bridge. What happens to the arch?
8. **Improve.** Redesign the joint that failed first, or add a deck and spandrel rings that stiffen the arch, and test again.

Tips

- Cut slots exactly as wide as the board is thick, for a snug fit.

- Label each piece and its position before you assemble.
- Load gently; card crushes at the joints first.

Check your work

- ☐ I built an arch of interlocking parts and tested it at the crown and quarter point.
- ☐ I can explain why cast iron suits arches and wrought iron suits chains.
- ☐ I can calculate the water load on an aqueduct and an arch's thrust.

ENGINEER'S NOTEBOOK

1. Sketches of the joints you designed, and of a dovetail and a mortise-and-tenon joint like those on the Iron Bridge.
2. Your load tests, with sketches of how the arch failed.
3. A timeline of Telford's major works with dates, spans and materials.

KEY FORMULAS

$H \approx \dfrac{wL^2}{8h}$ (arch thrust or cable pull)
$w_{\text{water}} = \gamma_w bd$ (load per meter of a trough of width b and depth d), $\gamma_w = 9.81\ \text{kN/m}^3$
$M = \dfrac{wL^2}{8}$ (simply supported trough span)

From history. Telford was born in 1757 to a shepherd's widow in the Scottish borders and apprenticed as a stonemason at fourteen. Besides bridges and canals he built over a thousand miles of roads, the Caledonian Canal and harbors around Britain, and the poet Robert Southey nicknamed him the Colossus of Roads. He is buried in Westminster Abbey.

Example 1. Estimate the Iron Bridge's arch thrust. Treat its weight, about 3,770 kN (384 tonnes), as a uniform load over its 30.6 m span, with a rise of about 15 m.

Show solution

1. Load per meter: $w = 3770 \div 30.6 \approx 123$ kN/m.
2. Thrust:
   $H = wL^2/(8h) = 123 \times 30.6^2 \div (8 \times 15) = 123 \times 936.4 \div 120 \approx 960$ kN.
3. Each abutment must resist roughly 960 kN of outward push, which is why abutment movement has been the bridge's main problem.

Answer: About 960 kN of thrust at each abutment (a rough estimate).

Example 2. A canal trough is 3.4 m wide and carries water 1.5 m deep. Find the water load per meter, and the bending moment in one 16 m span between piers treated as simply supported.

Show solution

1. Load: $w = 9.81 \times 3.4 \times 1.5 \approx 50.0$ kN/m.
2. Moment: $M = wL^2/8 = 50.0 \times 16^2 \div 8 = 1600$ kN·m.
3. A boat floating in the trough adds nothing: it pushes aside its own weight of water.

Answer: About 50 kN/m; about 1,600 kN·m per span (plus the trough's own weight).

Example 3. Cast iron of the period could safely carry roughly 100 MPa in compression but only about 20 MPa in tension. A rib section of 0.020 m^2 carries 900 kN of compression. Is it safe? If the same rib were bent so that part of it carried 900 kN of tension, would it be?

Show solution

1. Stress: $\sigma = 900{,}000\ \mathrm{N} \div 0.020\ \mathrm{m}^2 = 45 \times 10^6\ \mathrm{Pa} = 45\ \mathrm{MPa}$.
2. In compression: 45 MPa is under 100 MPa. Safe.
3. In tension: 45 MPa is more than double 20 MPa. It would crack. Cast iron must be kept in compression.

Answer: Safe in compression (45 MPa < 100 MPa); unsafe in tension (45 MPa > 20 MPa).

PRACTICE

1. Easy When was the Iron Bridge built, and who cast it?

   Show answer

   1779, by the ironmaster Abraham Darby III (opened 1781).

2. Easy Why did the Iron Bridge's builders use carpentry joints?

   Show answer

   No iron bridge tradition existed, so they joined iron the way they knew how to join timber: dovetails, mortise-and-tenon joints and wedges.

3. Easy How much does 1 m^3 of water weigh in kN?

   Show answer

   About 9.81 kN.

4. Medium An arch of span 40 m and rise 10 m carries 60 kN/m. Find its thrust.

   Show answer

   $60 \times 1600/80 = 1200\ \mathrm{kN}$.

5. Medium A trough 2.5 m wide carries water 1.2 m deep. Find the water load per meter.

   Show answer

   $9.81 \times 2.5 \times 1.2 \approx 29.4\ \mathrm{kN/m}$.

6. Medium A 50-tonne canal boat enters the aqueduct. By how much does the load on the aqueduct change? Explain.

Show answer

By zero: the boat displaces 50 tonnes of water, which spills or flows along the canal, so the total weight in the trough is unchanged.

7. Medium The Menai Bridge's span is 577 ft. Convert to meters.

Show answer

About 176 m.

8. Hard A wrought-iron chain may safely carry 75 MPa. The Menai chains together must carry a horizontal pull of 10,000 kN. What total chain area is needed? If there are 16 chains, what area does each need?

Show answer

$A = 10 \times 10^6 \div 75 \times 10^6 \approx 0.133\ \mathrm{m}^2$; each chain $\approx 0.0083\ \mathrm{m}^2 = 8300\ \mathrm{mm}^2$.

9. Hard If an arch's abutments are pushed closer together, its rise increases slightly and its joints crush at the crown and haunches. Why is this a problem for cast iron in particular?

Show answer

The arch deforms and some parts begin to bend, creating tension on one face; cast iron is brittle and weak in tension, so it cracks, as several Iron Bridge parts did.

10. Hard Conceptual. Why did Telford test his wrought-iron bars before using them?

Show answer

Model answer: iron quality varied from bar to bar, and a single weak link would break the chain; testing every bar (proof-loading) found the weak ones before they were built in, a habit that became standard engineering practice.

11. **Challenge** A cast-iron rib in compression has an area of 0.015 m^2 and is 30 m long, braced every 3 m. If the safe compression is 100 MPa, what force can it carry? If the bracing is removed so that a 6 m length can buckle, by what factor does its buckling resistance drop?

Show answer

Crushing capacity $= 100 \times 10^6 \times 0.015 = 1.5 \times 10^6$ N $= 1500$ kN. Buckling load goes as $1/L^2$, so doubling the unbraced length to 6 m cuts it to one-quarter.

COMMON MISTAKES

- Using cast iron in tension. It is brittle; keep it in compression.
- Thinking a floating boat adds weight to an aqueduct. It displaces its own weight of water.
- Forgetting that arches push outward and chains pull inward; both need massive abutments or anchorages.

> **Make it a game · Iron Matchmaker.** Draw a two-column table: cast iron and wrought iron. Sort ten bridge parts, like arch ribs, eyebar chains, aqueduct trough, into the right column from memory. Your parent checks and asks why. Then explain why Telford hung a bridge from chains. Each correct match earns a point; try to beat your last total.

6.3 The Brooklyn Bridge and the Golden Gate

OBJECTIVES

- Tell the stories of the Roebling family and the Brooklyn Bridge and of the Golden Gate Bridge, including the caissons, cable spinning and the honesty problem of the bad wire.
- Calculate suspension cable tension from span, sag and load, and the area of steel needed.
- Build a model suspension bridge and measure cable tension against sag.

> **Warm-up.** Hold a string between your hands with a small weight hung from its middle. Pull your hands apart until the string is nearly straight. What do you feel? Why can a suspension bridge cable never be perfectly straight?

WATCH

Golden Gate Bridge | The CRAZY Engineering behind it

Sabin Civil Engineering · 15:27

https://www.youtube.com/watch?v=E6tp8DCAJ-0

> **WHEN TO WATCH**
> After the Golden Gate part of the story. Watch for how the cable is spun and anchored.

Note: Animated explanation (made with the Lesics animation team). Includes a short sponsor segment.

BACKUP VIDEO

The Brooklyn Bridge Should Have Been Impossible in 1870

Engineering The Impossible · 15:20

https://www.youtube.com/watch?v=zmUKu1FJI1w

With the Brooklyn Bridge story: the caissons, the bends and the cables.

KEY TERMS

caisson

A huge open-bottomed box sunk into a riverbed, pressurized with air to keep water out while workers dig beneath it to reach firm ground for a foundation.

cable spinning

Building a suspension cable in place by carrying wires back and forth across the span on a spinning wheel, then compacting and wrapping them.

anchorage

The massive block of masonry or concrete at each end that holds the cables' pull.

caisson disease (the bends)

Illness caused when workers leave high-pressure air too quickly and dissolved nitrogen forms bubbles in the body.

sag

The vertical distance from the cable's ends to its lowest point; sag ratios are usually about 1:8 to 1:12 of the span.

deflection theory

An analysis method (Moisseiff and others) showing that a suspension bridge's cables help stiffen its deck, allowing lighter decks; pushed too far, it led to Tacoma Narrows.

CORE IDEAS

The story. John A. Roebling, a German-born engineer who had built suspension bridges at Pittsburgh, Cincinnati and Niagara Falls, designed a bridge between New York and Brooklyn with a main span of 1,595½ ft, half again longer than any before. In 1869, while he was surveying the site, a ferry crushed his foot, and he died of tetanus a few weeks later. His son Washington, 32, took over. Working in the compressed-air caissons beneath the East River, Washington was struck by caisson disease in 1872 and could no longer visit the site. For the next eleven years he directed the work from a window in Brooklyn, and his wife, Emily Warren Roebling, carried his instructions to the engineers, learned the mathematics of cables and stresses, and became the bridge's indispensable link. When the bridge opened on May 24, 1883, Emily was the first to cross it. Fifty-four years later, on May 27, 1937, the Golden Gate Bridge in San Francisco opened with a main span of 4,200 ft (1,280 m), the longest in the world at the time.

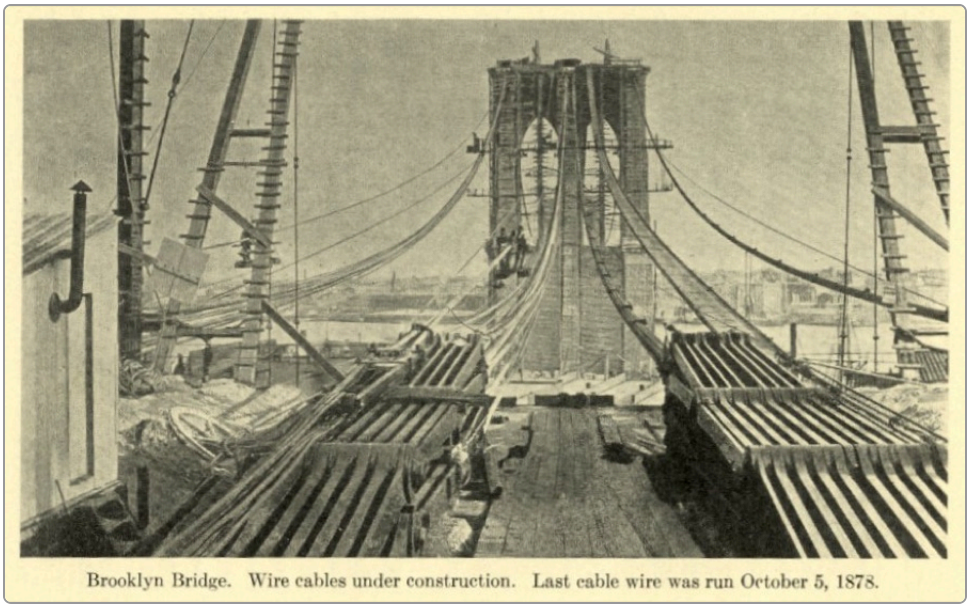
How they figured it out. Roebling's bridges were famous for their stiffness: he added diagonal stays from the towers and deep trusses in the deck, because he had studied earlier suspension bridges destroyed by wind. The cables were spun in place from steel wire, a first for a bridge this large. Partway through, Washington discovered that a contractor, J. Lloyd Haigh, had been substituting rejected, inferior wire for wire that had passed inspection. Much of the bad wire was already in the cables and could not be removed. Washington calculated the loss and added about 150 extra wires to each cable, and he insisted that the bridge was still six times stronger than it needed to be. At the Golden Gate, chief engineer Joseph Strauss, with Charles Ellis doing much of the structural design and Leon Moisseiff advising, spun two cables each 36⅜ in thick from 27,572 wires, and a safety net under the deck saved nineteen men who fell.

The physics. A cable carrying a uniform load w per meter of span hangs as a parabola. Its horizontal tension is $H = \dfrac{wL^2}{8d}$, where L is the span and d the sag; at each tower the cable also carries a vertical force $V = \dfrac{wL}{2}$, so the maximum tension is $T = \sqrt{H^2 + V^2}$. A flatter cable (smaller sag) has a much larger tension; that is why suspension bridges sag noticeably, usually about one-ninth to one-twelfth of the span. The cable's pull is resisted by the anchorages, which must be heavy enough not to slide or lift, and the towers carry the vertical component down in compression.

The engineering. A suspension bridge is a system: towers on deep foundations, cables of thousands of parallel wires, anchorages, hangers, and a stiffening deck that spreads loads and resists wind. The Brooklyn caissons were sunk 44 ft on the Brooklyn side and 78 ft on the Manhattan side, where Washington stopped above bedrock on firm sand rather than risk more lives. The wire fraud showed that inspection and honesty are part of engineering. And the Golden Gate showed that safety on the job can be engineered too.

Try it. In the build you will hang a model deck from a string cable, change the sag, and measure the cable tension with a spring scale.

PICTURE STUDY



Brooklyn Bridge: Wire Cables under Construction, Unknown photographer, 1878 · Photograph (halftone reproduction) · Wikimedia Commons

Image: Wikimedia Commons, public domain

This view from the anchorage shows the bridge in October 1878, when the last cable wire had just been run. The great granite tower with its pointed arches stands in the distance, and the cables sweep down and up toward it.

Look closely

- Count the main cables. Where do they rise and where do they dip?
- Find the temporary footbridge the workers used. How is it held up?
- Find the tower's arches. Why might a pointed (Gothic) shape suit a tower that is pressed down by the cables?
- Look at the anchorage in the foreground. What is it for?

Talk about it

1. Which parts are in tension and which in compression?
2. Why must the cables be anchored so heavily at each end?
3. Some of the wire in these cables was fraudulent. How did Washington Roebling make the bridge safe anyway?

Safety first. Wear safety glasses when tensioning strings. Keep loads small and clamp towers so they cannot tip onto hands.

BUILD

Sag and Tension: A Model Suspension Bridge

You will build a model suspension bridge with string cables over two towers, hang a deck from them, and measure how the cable tension changes as you change the sag, comparing with $H = wL^2/(8d)$.

You will need

- Two towers (stacks of books, blocks or wooden dowels clamped upright) about 60 cm apart
- Strong string or fishing line for cables and hangers; paper clips
- A deck of card strip with evenly spaced weights (coins taped every 5 cm)
- A spring scale or luggage scale to measure the backstay pull (or a cup and pennies over a pulley)

Steps

1. Tie a cable over the two towers and down to an anchor at each end. At one end, run the backstay to a spring scale, or over a pencil pulley to a cup of pennies, so you can measure its pull.
2. Hang the card deck from the cable with string hangers every 5 cm. Weigh the deck and its coins to find w (N per meter of span).
3. Adjust the cable to a sag of $L/8$. Measure the sag carefully and the backstay pull.
4. **Predict** the horizontal tension with $H = wL^2/(8d)$, and the backstay pull from the backstay's angle.
5. Repeat with sags of $L/6$, $L/10$ and $L/14$. Record sag and pull for each.
6. Graph pull against $1/d$. Is it close to a straight line?
7. Add a heavy "truck" at one quarter of the span and watch the deck. Then stiffen the deck with a folded card truss and repeat. What changes?
8. **Improve.** Add diagonal stays from the tower tops to the deck, as Roebling did, and test with the truck again.

Tips

- Keep the hangers equal so the deck stays level.

- Measure sag from the line joining the tower tops to the lowest point of the cable.
- Friction at the tower tops affects the reading; let the cable slide over a smooth pencil.

Check your work

☐ My data show tension rising as sag decreases, close to

$H \propto 1/d$.

☐ I can calculate cable tension and the steel area needed.

☐ I can tell how the Roeblings built the Brooklyn Bridge and how the wire fraud was handled.

ENGINEER'S NOTEBOOK

1. A labeled diagram of a suspension bridge: towers, main cables, hangers, deck, stiffening truss, anchorages, with arrows for the forces.
2. Your sag-tension data table and graph.
3. A paragraph on what the Haigh wire fraud teaches about inspection and honesty in engineering.

KEY FORMULAS

$$H = \frac{wL^2}{8d}$$

$$V = \frac{wL}{2}, \quad T_{\text{max}} = \sqrt{H^2 + V^2}$$

$$A_{\text{steel}} = \frac{T_{\text{max}}}{\sigma_{\text{allow}}}$$

From history. After the bridge opened, many people doubted it would hold. In May 1884 the showman P. T. Barnum led twenty-one elephants across it to prove its strength. Washington Roebling lived until 1926, and Emily, who earned a law certificate from New York University in 1899, became a sought-after speaker.

Example 1. Suppose a suspension bridge has a main span of 486 m, a sag of 39.6 m and a total load (deck, cables and traffic) of 70 kN per meter of span. Find the horizontal and maximum cable tension for all the cables together.

Show solution

1. $H = wL^2/(8d) = 70 \times 486^2 \div (8 \times 39.6) = 70 \times 236{,}196 \div 316.8 \approx 52{,}200$ kN.
2. $V = wL/2 = 70 \times 486/2 = 17{,}010$ kN.
3. $T_{\text{max}} = \sqrt{52{,}200^2 + 17{,}010^2} \approx 54{,}900$ kN.

Answer: $H \approx 52{,}200$ kN; $T_{\text{max}} \approx 54{,}900$ kN.

Example 2. For the cables in Example 1, the wire may safely carry 275 MPa (about a quarter of its breaking strength). What total area of steel is needed? If there are four cables, what area is in each?

Show solution

1. $A = T/\sigma = 54.9 \times 10^6$ N $\div$ 275 N/mm^2 $\approx$ 200,000 mm^2 (0.20 m^2).
2. Per cable: 200,000 $\div$ 4 = 50,000 mm^2, a solid circle about 252 mm across; with the gaps between round wires, the cable is somewhat larger.

Answer: About 0.20 m^2 total; about 50,000 mm^2 per cable.

Example 3. The Golden Gate's main span is 1,280 m with a sag of about 143 m. If the load were 200 kN/m for both cables together, find H and T_{max}. What happens to H if the sag were only 100 m?

Show solution

1. $H = 200 \times 1280^2 \div (8 \times 143) \approx 286{,}000$ kN.
2. $V = 200 \times 1280/2 = 128{,}000$ kN; $T = \sqrt{286{,}000^2 + 128{,}000^2} \approx 314{,}000$ kN.
3. With $d = 100$ m: $H = 286{,}000 \times 143/100 \approx 409{,}000$ kN, 43% more. A flatter cable needs much more steel.

Answer: About 286,000 kN and 314,000 kN; with less sag, H rises to about 409,000 kN.

PRACTICE

1. Easy What is a caisson, and why was it dangerous to work in?

   Show answer

   A pressurized box sunk into a riverbed so men could dig beneath it; returning too quickly from high pressure caused caisson disease (the bends).

2. Easy Who carried Washington Roebling's instructions to the bridge, and when did the bridge open?

   Show answer

   Emily Warren Roebling; May 24, 1883.

3. Easy A cable spans 100 m with sag 10 m and load 20 kN/m. Find H.

   Show answer

   $20 \times 10{,}000/80 = 2500$ kN.

4. Medium For Problem 3, find V and $T_{\max}$.

   Show answer

   $V = 1000$ kN; $T = \sqrt{2500^2 + 1000^2} \approx 2693$ kN.

5. Medium If the sag in Problem 3 is reduced to 5 m, what is H?

   Show answer

   5,000 kN, double.

6. Medium A cable must carry 3,000 kN at 300 MPa. What area of steel is needed?

   Show answer

   $10{,}000\ \text{mm}^2$.

7. Medium The Golden Gate's cables each have 27,572 wires about 4.9 mm in diameter. Find the steel area of one cable.

   Show answer

   Each wire $\pi(2.45)^2 \approx 18.9\ \text{mm}^2$; total
   $\approx 27{,}572 \times 18.9 \approx 520{,}000\ \text{mm}^2 \approx 0.52\ \text{m}^2$.

8. Hard A backstay leaves the tower at 30° below horizontal and must balance a main-cable horizontal pull of 40,000 kN. Find the backstay tension and the vertical force it adds to the tower.

Show answer

$T\cos 30^\circ = 40{,}000$, $T \approx 46{,}200$ kN; vertical $= T\sin 30^\circ \approx 23{,}100$ kN down into the tower.

9. Hard Washington Roebling said the bridge would remain safe after the bad wire was found because it had been designed with a factor of safety of about six against the cable load. If bad wire reduced the cable strength by 10% and he added extra wires restoring 5%, what factor of safety remained?

Show answer

$6 \times (1 - 0.10 + 0.05) = 6 \times 0.95 = 5.7$, still very large.

10. Hard Conceptual. Why did Washington Roebling stop the Manhattan caisson on sand instead of going on to bedrock?

Show answer

Model answer: the deeper the caisson, the higher the air pressure and the worse the caisson disease; workers had already died. Tests showed the dense sand at about 78 ft could safely carry the tower, so he judged further risk to life unjustified, and the tower has stood on it ever since.

11. Challenge Show that for a parabolic cable, the slope at the tower is $\tan\theta = 4d/L$, and use it to check V/H from the formulas $H = wL^2/(8d)$ and $V = wL/2$.

Show answer

The parabola $y = 4dx^2/L^2$ (from the low point, x from $-L/2$ to $L/2$) has slope $8dx/L^2$, which at $x = L/2$ is $4d/L$. Also $V/H = (wL/2) \div (wL^2/8d) = 4d/L$. They agree, so the tension points along the cable.

COMMON MISTAKES

- Forgetting the vertical component when computing the cable tension at the tower.
- Thinking less sag is better. Flatter cables carry much greater tension.
- Overlooking the anchorages, which must resist the full horizontal pull.

Make it a game · Sag and Tension. Hang a string with a small bag of coins between two chairs. Predict how the tension changes when you tighten the string, then test by watching the sag. Calculate cable tension from span, sag and load for a pretend bridge. Then tell the Roebling or Golden Gate story in one minute without notes.

Scripture. “A false balance is abomination to the LORD: but a just weight is his delight.” (Proverbs 11:1, KJV)

The fraudulent wire was a false balance. Honest materials, honest tests and honest records are the foundation of every safe bridge.

6.4 Failures: The Quebec Bridge and Tacoma Narrows

OBJECTIVES

- Explain why the Quebec Bridge (1907) and the Tacoma Narrows Bridge (1940) failed, and what engineers changed afterward.
- Use Euler's buckling formula and the effect of span and weight changes to show how a design can lose its margin.
- Compare a vortex-shedding frequency with a structure's natural frequency, and test how deck shape affects wind response in a model.

> **Warm-up.** Push down on the ends of a dry spaghetti strand until it bows and snaps. Now try a strand half as long. How much harder was it? Then blow across a strip of paper held by one edge. What does it do?

WATCH

Ego in Engineering: The Quebec Bridge Collapse

Brick Immortar · 14:47

https://www.youtube.com/watch?v=e4DTMe0huXM

> **WHEN TO WATCH**
> After the Quebec story. List each warning sign that was missed.

Note: Uses historic photographs of the wreckage and describes the deaths plainly but without graphic detail.

BACKUP VIDEO

Why the Tacoma Narrows Bridge Collapsed

Practical Engineering · 8:47

https://www.youtube.com/watch?v=mXTSnZgrfxM

With the Tacoma Narrows paragraph; watch the film of the twisting deck and his wind-tunnel models.

KEY TERMS

buckling

Sudden sideways bending of a compression member at a load far below its crushing strength.

Euler buckling load

$P_{cr} = \dfrac{\pi^2 EI}{(KL)^2}$, the load at which a slender column buckles; it falls with the square of the unbraced length.

lacing

The small diagonal bars that tie together the separate plates of a built-up compression member so they act as one.

aeroelastic flutter

A self-feeding oscillation in which wind forces on a structure grow with the structure's own motion, until it twists apart.

vortex shedding

The alternating whirls of air that peel off a bluff body in wind, pushing it side to side at a frequency $f = \mathrm{St}\, V/D$.

natural frequency

The rate at which a structure vibrates when disturbed and left alone.

CORE IDEAS

The story. In 1907 the Quebec Bridge over the St. Lawrence River was to be the longest cantilever span in the world. Its consulting engineer, Theodore Cooper, was among the most respected bridge engineers in America, but he was in his late sixties and in poor health, and he supervised the work from New York, visiting the site only a few times. In 1900 he had lengthened the main span from 1,600 to 1,800 ft to shorten the piers in deep water, but the dead load was not recalculated; the steel being built weighed about 20% more than the stresses had been computed for. In August 1907 a young inspector, Norman McLure, reported that the lower compression chords near the south pier were bending. Messages went back and forth. On the afternoon of August 29, before Cooper's order to stop placing load reached the site, the south arm buckled and fell into the river. Seventy-five workers died, among them thirty-three Mohawk ironworkers from Kahnawake. Thirty-three years later, on November 7, 1940, the new Tacoma Narrows suspension bridge in Washington State, only four months old, twisted itself apart in a 42 mph wind. No person died; a dog left in a car was lost.

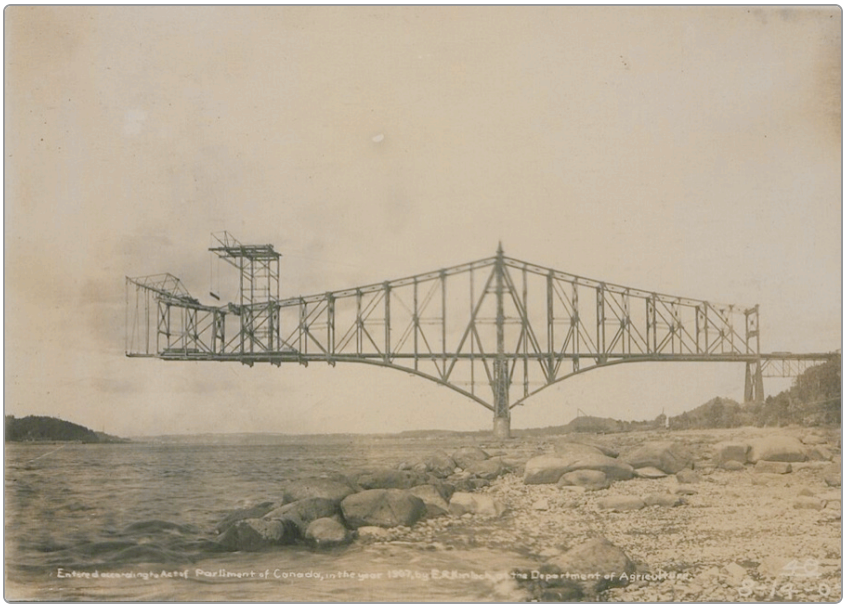
How they figured it out. A Royal Commission found that the Quebec chords failed because their lacing bars, which tied four heavy plate webs into one member, were far too light; the chord buckled, and the rest followed. The commission blamed the engineers' errors and the lack of checking. The bridge was redesigned with heavier members; in 1916 the center span fell during lifting when a casting broke, and thirteen more men died; and the bridge finally opened in 1917. It is still the longest cantilever span in the world, 549 m (1,800 ft). At Tacoma, film taken by Professor F. B. Farquharson showed the deck twisting in two halves, and wind-tunnel tests on models showed the culprit: the shallow solid-girder deck, very slender for its 2,800 ft span, was aerodynamically unstable. Once twisting began, the wind pushed harder with each cycle. This is **flutter**, not simple resonance. Engineers now test every long-span bridge in a wind tunnel.

The physics. A slender compression member buckles at $P_{cr} = \dfrac{\pi^2 EI}{(KL)^2}$. Its capacity depends on the stiffness I of the whole section, and if the lacing is too weak to make the parts act together, I collapses toward that of the separate plates. Also $P_{cr} \propto 1/L^2$: doubling the unbraced length cuts the capacity to a quarter. At Tacoma, wind flowing past the deck shed vortices at a frequency $f = \mathrm{St}\, V/D$ (St is the Strouhal number, about 0.1–0.2 for bluff shapes). For $V \approx 19$ m/s (42 mph) and a deck about 2.4 m deep, $f \approx 0.12 \times 19/2.4 \approx 1$ Hz, far from the deck's twisting frequency of about 0.2 Hz. So it was not resonance with vortices. The deck's own twisting changed the wind forces in a way that fed energy in: aeroelastic flutter.

The engineering. Both failures changed the profession. After Quebec, compression members were designed and checked far more carefully, and Canadian engineers created the Ritual of the Calling of an Engineer (1925), in which new engineers receive an iron ring worn on the little finger of the working hand, a reminder of humility and responsibility. After Tacoma, suspension bridges were given deep, open stiffening trusses or streamlined box decks, and every long span is tested in a wind tunnel before it is built. Failures teach when engineers study them honestly.

Try it. In the build you will test compression struts with and without lacing, and test model bridge decks of different shapes in front of a fan.

PICTURE STUDY



The Quebec Bridge, South Arm, 1907, Unknown photographer, 1907 · Photograph · Wikimedia Commons

Image: Wikimedia Commons, public domain

This photograph shows the south arm of the Quebec Bridge reaching out over the river in 1907, before the collapse. A large traveling crane at its tip lifts each new piece into place. The heavy lower chords near the pier carry the arm's weight in compression.

Look closely

- Find the tower over the pier and the arm reaching out to the left. Where is the arm deepest?
- Find the traveling crane at the tip. How does adding each new piece change the forces near the pier?
- Find the lower chords near the pier. Are they in tension or compression?
- Look at the anchor arm on the shore side. What is its job?

Talk about it

1. The lower chords near the pier buckled. Why would they be the most heavily loaded members?
2. What warning signs would an inspector see in a chord that is starting to buckle?

3. What should have happened when McLure reported the bending chords?

Safety first. Wear safety glasses for the strut tests; spaghetti and balsa snap and fly. Keep fingers away from fan blades and use a box fan with a grille.

BUILD

Lacing and Wind: Two Failure Tests

You will repeat the two lessons in miniature. First, you will build built-up compression struts with weak and strong lacing and find their buckling loads. Second, you will test model deck sections of different shapes in front of a fan and observe which twist.

You will need

- Balsa strips (or bundles of dry spaghetti), white glue, thread
- A kitchen scale used as a press: press down on a strut standing on the scale and read the force
- A box fan, card stock, a wooden skewer, tape, two bottles as supports
- A ruler and a phone camera for slow-motion video

Steps

1. **Struts.** Make three struts 15 cm long, each from four parallel balsa strips (or spaghetti) held about 1 cm apart: (A) tied only at the ends, (B) tied with thread every 5 cm, (C) braced with glued diagonal lacing along all four faces.
2. **Predict** the ranking and estimate how many times stronger C is than A.
3. Stand each strut on the kitchen scale, press down slowly on a block on top, and record the highest reading before it buckles. Test two of each.
4. Compare with Euler's formula for one strip alone and for four strips acting as one section.
5. **Decks.** Make three deck sections 30 cm long: a flat thin plate, an H shape (a flat plate with solid edge girders folded down, like Tacoma), and a streamlined box (a flattened tube with tapered edges, like modern decks).
6. Mount each on a skewer through its center as an axle between two bottles so it can twist, with a rubber band or tape giving it a little spring to return to level.
7. Set the fan in front and increase the speed. Film in slow motion. Record at which fan setting (if any) each deck begins to twist steadily, and how much.

8. **Improve.** Cut slots in the H-section deck, or add a fairing to its edges, and test again. Does it become more stable?

Tips

- Press the struts straight down; a tilted load buckles them early.
- Keep the fan the same distance from each deck.
- Write down what you see in the video frame by frame; flutter usually begins small and grows.

Check your work

- ☐ My strut tests show how lacing makes separate parts act as one compression member.
- ☐ My deck tests compare shapes fairly and identify the most stable.
- ☐ I can explain what engineers learned from Quebec and Tacoma Narrows.

ENGINEER'S NOTEBOOK

1. A labeled sketch of a built-up chord with lacing, and of the same chord buckled with failed lacing.
2. Your strut data and the comparison with Euler's formula.
3. Sketches of the three deck sections and your wind-test observations, with one paragraph on flutter.

KEY FORMULAS

$P_{cr} = \dfrac{\pi^2 EI}{(KL)^2}$ ($K = 1$ for pinned ends)

$I_{\text{rect}} = \dfrac{bh^3}{12}$

$f_{\text{vortex}} = \text{St}\,\dfrac{V}{D}$

$f_n = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ (simple oscillator)

From history. The Mohawk community of Kahnawake lost so many men at Quebec that its families asked that ironworkers no longer work all together on one job. Mohawk ironworkers went on to build many of New York's skyscrapers and bridges, and their skill on high steel is still famous.

Example 1. A designer computed a chord's stress for a dead load of 100 units, but the actual dead load is 20% higher, and live load adds 25 units. The member was sized with a factor of safety of 1.8 against the computed total (125 units). What is the real factor of safety?

Show solution

1. Computed total: $100 + 25 = 125$; capacity $= 1.8 \times 125 = 225$.
2. Actual total: $120 + 25 = 145$.
3. Real factor of safety: $225 \div 145 \approx 1.55$.
4. A 20% error in dead load quietly removed about 14% of the margin, and that was before the weak lacing made the chord weaker than assumed.

Answer: About 1.55 instead of 1.8.

Example 2. A balsa strut 3 mm × 3 mm in cross section and 50 mm long has $E = 3000$ MPa. Find its Euler buckling load, pinned at both ends. What if it were 100 mm long?

Show solution

1. $I = bh^3/12 = 3 \times 27/12 = 6.75\ \text{mm}^4$.
2. $P_{cr} = \pi^2(3000)(6.75) \div 50^2 \approx 199{,}900 \div 2500 \approx 80$ N (about 8 kg).
3. At 100 mm: $P_{cr} = 80 \div 4 \approx 20$ N.

Answer: About 80 N at 50 mm; about 20 N at 100 mm.

Example 3. Estimate the vortex-shedding frequency for the Tacoma Narrows deck in a 19 m/s wind with a depth of 2.4 m and a Strouhal number of 0.12. Compare it with the twisting frequency of about 0.2 Hz.

Show solution

1. $f = \mathrm{St}\, V/D = 0.12 \times 19 \div 2.4 \approx 0.95$ Hz.
2. That is almost five times the twisting frequency, so vortex shedding cannot explain the twisting by simple resonance.
3. The deck's twisting motion itself changed how the wind pushed on it, a feedback that grows: flutter.

Answer: About 1 Hz, not matching 0.2 Hz; the failure was flutter, not vortex resonance.

PRACTICE

1. Easy What was the main structural cause of the 1907 Quebec collapse?

   Show answer

   The lower compression chords near the south pier buckled because their lacing was too light (and the dead load had been underestimated).

2. Easy What is the iron ring, and what does it remind engineers of?

   Show answer

   A ring given in Canada's Ritual of the Calling of an Engineer (since 1925), worn on the little finger of the working hand, reminding engineers of humility and their responsibility to the public.

3. Easy If a column's unbraced length is tripled, by what factor does its buckling load change?

   Show answer

   It becomes one-ninth.

4. Medium Find the Euler buckling load of a steel rod ($E = 200$ GPa) 20 mm square and 2.0 m long, pinned at both ends.

   Show answer

   $I = 20^4/12 \approx 13{,}333\ \mathrm{mm}^4$; $P = \pi^2(200{,}000)(13{,}333)/2000^2 \approx 6580$ N.

5. Medium Four strips each 3 mm × 3 mm are glued into a solid 6 mm × 6 mm square. How much larger is its I than four separate strips?

Show answer

Solid: $6^4/12 = 108\ \text{mm}^4$; four separate: $4 \times 6.75 = 27\ \text{mm}^4$; four times larger.

6. Medium A sign 0.5 m deep stands in a 20 m/s wind. With St = 0.2, what is its vortex-shedding frequency?

Show answer

$0.2 \times 20/0.5 = 8\ \text{Hz}$.

7. Medium Why was the Quebec span lengthened from 1,600 ft to 1,800 ft, and what was not done?

Show answer

To shorten the piers in deep water and speed construction; the dead load (the weight of the larger structure) was not recalculated and checked.

8. Hard A bridge deck has a natural vertical frequency of 0.6 Hz and a depth of 2.4 m. With St = 0.12, at what wind speed would vortex shedding match it?

Show answer

$V = fD/\text{St} = 0.6 \times 2.4/0.12 = 12\ \text{m/s}$ (about 27 mph).

9. Hard Explain the difference between resonance and flutter in your own words.

Show answer

Model answer: resonance is a structure responding strongly to an outside push that happens to repeat at its natural frequency; flutter is a self-excited motion in which the structure's own movement changes the wind forces so that they add energy each cycle, growing until failure, even in steady wind.

10. Hard Conceptual. List three failures of process (not of steel) at Quebec.

Show answer

Model answer: the consulting engineer supervised from far away and seldom visited; the larger span's dead load was not recalculated; warnings about bending chords were not acted on immediately (load kept being added); designs were not independently checked.

11. **Challenge** A chord's buckling load with good lacing is P_1. With weak lacing, its parts act more like separate plates and its effective I drops to 30% of the laced value. If the chord carries $0.55P_1$, what is its factor of safety before and after the lacing fails?

Show answer

Before: $P_1/(0.55P_1) \approx 1.82$. After: $0.30P_1/(0.55P_1) \approx 0.55$, less than 1, so it buckles.

COMMON MISTAKES

- Blaming Tacoma Narrows on simple resonance. It was aeroelastic flutter.
- Treating a built-up member as one solid section without checking that its lacing can make the parts act together.
- Thinking a respected engineer's design does not need checking. Quebec proved otherwise.

Make it a game · Spot the Lost Margin. Failure investigation. Your parent reads three statements about the Quebec and Tacoma Narrows bridges, two accurate and one false. Name the false one and give the correct cause: buckling, lacing, flutter or vortex shedding. Then calculate how doubling a column's length changes its Euler load. Swap roles and keep score.

Scripture. "Where no counsel is, the people fall: but in the multitude of counsellors there is safety." (Proverbs 11:14, KJV)

At Quebec, one engineer's judgment went unchecked, and the bridge fell. Independent review and listening to warnings are forms of wise counsel.

6.5 The Bridge Competition: Design, Analyze, Build and Test

OBJECTIVES

- Design a model truss bridge to meet competition rules, choosing between Pratt, Howe and Warren trusses with reasons.
- Analyze the design by the method of joints, size its compression members against buckling, and predict its failure load.
- Build and test the bridge to failure and compute its structural efficiency.

> **Warm-up.** Look at the drawings of a Pratt truss and a Howe truss. In which one do the diagonals slope down toward the middle of the span? For a load in the middle, guess which diagonals are pulled in each one.

WATCH

Harvard Model Bridge Testing! Trusses and Beams

Paul Kassabian · 13:16

https://www.youtube.com/watch?v=oqUNMGr0lo8

> **WHEN TO WATCH**
> Before you design. Notice which bridges failed early and why, and what the winners did differently.

Note: A structural engineer who taught at Harvard's design school shows his students' bridges being load-tested to failure.

KEY TERMS

Pratt truss

A truss with vertical members in compression and diagonals sloping toward the center in tension (Thomas and Caleb Pratt, 1844).

Howe truss

A truss with diagonals in compression (timber) and verticals in tension (iron rods) (William Howe, 1840).

Warren truss

A truss of equilateral or isosceles triangles with no verticals, its diagonals alternating in tension and compression.

structural efficiency

Load carried divided by the structure's own weight (or mass); the competition score.

gusset

A plate (or card patch) that reinforces a joint where several members meet.

CORE IDEAS

The story. In the 1840s American railroads were building thousands of bridges, and inventors competed with patents. In 1840 William Howe, a Massachusetts builder, patented a truss of timber diagonals in compression with vertical iron rods in tension; the rods could be tightened with nuts to take up shrinkage in the wood. In 1844 Thomas Willis Pratt and his father Caleb patented the opposite arrangement: diagonals in tension and verticals in compression. With timber, the Howe truss was cheaper and became the standard. With iron and steel, which work beautifully in tension as long slender bars, the Pratt truss won: its long members (the diagonals) are in tension, where they cannot buckle, and its short members (the verticals) are in compression. Choosing the right truss for the material is the heart of good design, and engineering students around the world still learn it today in model bridge competitions.

How they figured it out. Early truss builders worked by trial and testing; after Whipple and Haupt (Lesson 4.3), they could calculate every member force. A designer compares candidates: for a given span and load, which arrangement keeps long members in tension and short ones in compression, uses the fewest joints, and is simplest to build? Then each member is sized: tension members for their strength, compression members against buckling. Joints are designed to carry the full force of the members meeting there. Finally, the design is built and tested, and the results are compared with the predictions.

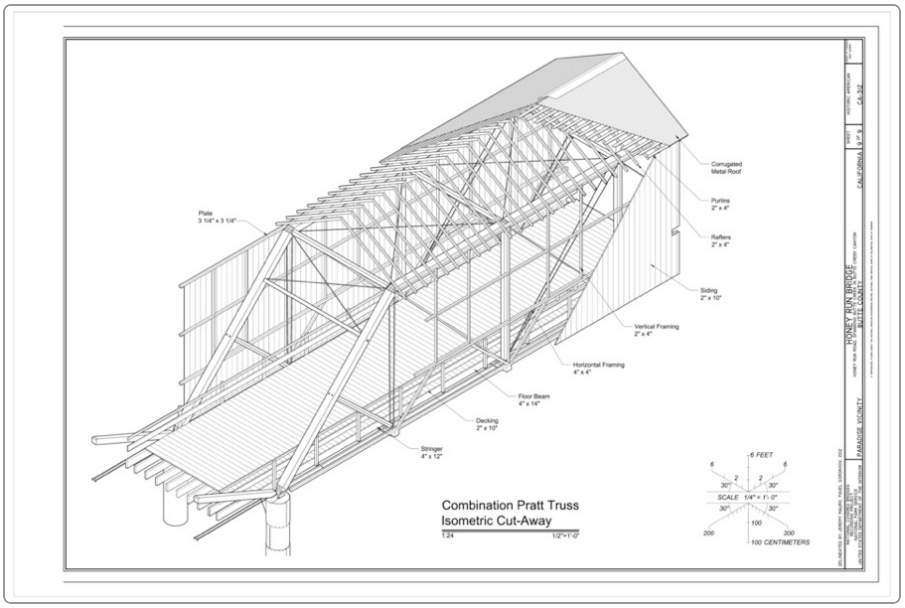
The physics. For a model bridge loaded at midspan, the method of joints gives every member force. Tension members fail by pulling apart, when $F = \sigma_t A$; compression members usually fail by buckling first, when $F = P_{cr} = \pi^2 EI/L^2$, so shorter compression members can be lighter. In balsa, which is roughly 10–20 MPa in

tension along the grain and has E of about 2,000–4,000 MPa, a 3 mm × 3 mm strip 5 cm long buckles at about 80 N, but at 10 cm only about 20 N. Glue joints are often the weakest link: the glue area at each joint must carry the member's force in shear.

The engineering. Competition rules imitate real constraints: a fixed span, a material limit, a loading point and a test to failure. The score, **efficiency** = load held ÷ bridge mass, rewards designs that put material only where it is needed. The best student balsa bridges hold well over a thousand times their own weight. Good competitors plan, analyze, build carefully with square, well-glued joints, test, study the failure, and improve. That is the whole design process in one project.

Try it. This lesson's build is the competition itself.

PICTURE STUDY



Combination Pratt Truss, Isometric Cutaway, Honey Run Bridge, Historic American Engineering Record, 2012 · Measured drawing · Library of Congress, Prints and Photographs Division

Image: Wikimedia Commons, public domain

The Honey Run Bridge in California (1886) was a covered "combination" bridge: heavy timbers where the truss is in compression and iron rods where it is in tension. HAER drew it in 2012 with part of its covering cut away. (The historic bridge was lost in a wildfire in 2018, which makes this drawing an important record.)

Look closely

- Find the heavy timber members. Are they the verticals, the diagonals or the chords?
- Find the slender iron rods. Which members are they?
- Find the roof and siding. Why would a timber bridge be covered?
- Find the labels. What sizes are the timbers?

Talk about it

1. In a Pratt truss, which members are in tension? Does the use of iron rods match?
2. Why does it make sense to use timber for compression and iron for tension?
3. What would you change if you were building this truss entirely of steel?

> **Safety first.** Wear safety glasses during load tests; balsa splinters and loads can drop. Load over a padded box or floor and keep feet clear. An adult handles any craft knife.

BUILD

The Bridge Competition

You will design, analyze, build and test a balsa (or craft-stick) truss bridge under competition rules, and compute its efficiency. Invite family members or friends to compete too.

You will need

- Balsa strips (for example 3 mm × 3 mm and 3 mm × 6 mm) or craft sticks; wood glue or cyanoacrylate (adult)
- Wax paper, pins and a printed full-size drawing to build on
- A kitchen scale (to 0.1 g if possible) to weigh the bridge
- A loading rig: a bucket hung from a loading block at midspan, with sand added slowly; or a bathroom scale for heavy craft-stick bridges

Steps

1. **Rules.** Clear span 30 cm; total length at most 35 cm; width at least 5 cm; mass at most 30 g of balsa (or 150 g of craft sticks); load applied at the center of the deck through a 5 cm loading block.
2. Sketch at least three truss options (Pratt, Howe, Warren). For each, use the method of joints to find the member forces for a center load P.
3. Size each member: tension members by strength, compression members by Euler buckling. Predict the failure load and which member will fail first.
4. Draw the chosen design full size, build two identical side trusses on wax paper over the drawing, then join them with cross-members and bracing. Keep joints tight and square.
5. Let the glue cure fully. Weigh the bridge.
6. Test: place it on the supports, apply the load slowly through the loading block, and film the test. Record the load at failure and which member failed first.
7. Compute the efficiency and compare the failure load and failure location with your prediction.

8. **Improve.** Using what you learned, build a second bridge that strengthens the member that failed and removes material from members that were overdesigned. Test it and compare efficiencies.

Tips

- Most bridges fail at a joint or by buckling a long compression member; brace those.
- Cross-bracing between the two trusses keeps the top chords from buckling sideways.
- Do not soak joints in glue; extra glue adds mass without much strength.

Check your work

- ☐ My design calculations predict the failure load and the first member to fail.
- ☐ I built and tested the bridge and computed its efficiency.
- ☐ I improved the design based on the failure and can explain why the second version did better (or did not).

ENGINEER'S NOTEBOOK

1. Your three truss options with member forces in terms of P, and the reasons for your choice.
2. A member schedule: each member's force, length, size, capacity (tension or buckling) and factor of safety.
3. Test records for both bridges: mass, failure load, efficiency, failure description and photos or sketches, and what you would change.

KEY FORMULAS

efficiency $= \dfrac{\text{load at failure}}{\text{mass of bridge}}$ (same units)

Tension member: $F_{\max} = \sigma_t A$

Compression member: $F_{\max} = \dfrac{\pi^2 EI}{L^2}$ (if less than $\sigma_c A$)

Joints: $\sum F_x = 0, \sum F_y = 0$

From history. Engineering Encounters' free online Bridge Design Contest, begun at the U.S. Military Academy at West Point in the early 2000s by Colonel Stephen Ressler, lets students anywhere design a virtual truss bridge, test it in a computer simulation and compete on cost. Millions of students have tried it.

Example 1. A model bridge of mass 28 g held 31.5 kg before it failed. Find its efficiency.

Show solution

1. Use the same units: 31.5 kg = 31,500 g.
2. Efficiency $= 31{,}500 \div 28 = 1125$.
3. The bridge held 1,125 times its own weight.

Answer: 1,125

Example 2. A Pratt truss spans 30 cm with 6 panels of 5 cm and a height of 5 cm. A load P is applied at the center bottom joint. The end diagonal is at 45°. Find the force in the end diagonal and the end bottom chord in terms of P. If the end diagonal is a 3 mm × 3 mm compression member 7.1 cm long ($E = 3000$ MPa), what P will buckle it?

Show solution

1. Each support carries $P/2$.
2. At the support joint, the end diagonal (rising at 45°) has a vertical component equal to $P/2$: $F_d \sin 45^\circ = P/2$, so $F_d = P/(2 \times 0.707) \approx 0.707P$ (compression).
3. The bottom chord balances its horizontal component: $F_b = 0.707P \times 0.707 = 0.5P$ (tension).
4. Buckling: $P_{cr} = \pi^2(3000)(6.75) \div 71^2 \approx 199{,}900 \div 5041 \approx 39.7$ N.
5. Set $0.707P = 39.7$ N: $P \approx 56$ N, a load of about 5.7 kg. To carry more, the end diagonal must be stiffer (doubled, or a deeper section).

Answer: End diagonal $0.707P$ (C), bottom chord $0.5P$ (T); the end diagonal buckles at $P \approx 56$ N (about 5.7 kg).

Example 3. A tension member in the same bridge carries $1.5P$. It is a 3 mm × 3 mm balsa strip with a tensile strength of 12 MPa. At what P does it break? Which member governs, this one or the end diagonal of Example 2?

Show solution

1. Area: 9 mm^2; capacity $= 12 \times 9 = 108$ N.
2. $1.5P = 108$, so $P = 72$ N.
3. The end diagonal buckles first, at $P \approx 56$ N, so it governs. Strengthening it is the best use of extra wood.

Answer: $P = 72$ N for the tension member; the end diagonal (56 N) governs.

PRACTICE

1. Easy In a Pratt truss under downward loads, which members are in tension?

   Show answer

   The diagonals (and the bottom chord); the verticals and top chord are in compression.

2. Easy A bridge of 45 g held 22 kg. Find its efficiency.

   Show answer

   $22{,}000/45 \approx 489$.

3. Easy Why did the Pratt truss suit iron and steel better than the Howe truss?

   Show answer

   Its long diagonals are in tension, where slender metal bars are very efficient and cannot buckle; its short verticals take the compression.

4. Medium A balsa strip 3 mm × 6 mm (6 mm in the direction it would bend) and 8 cm long has $E = 3000$ MPa. Find its buckling load about its stiff axis and its weak axis.

   Show answer

   Stiff: $I = 3 \times 216/12 = 54\ \text{mm}^4$, $P = \pi^2(3000)(54)/80^2 \approx 250$ N. Weak: $I = 6 \times 27/12 = 13.5\ \text{mm}^4$, $P \approx 62$ N. It buckles about the weak axis first unless braced.

5. Medium A tension member of 3 mm × 3 mm balsa (12 MPa) carries $0.8P$. At what P does it break?

Show answer

$108/0.8 = 135$ N.

6. Medium If a compression member's length is halved by adding a brace at its middle, by what factor does its buckling capacity increase?

Show answer

Four times.

7. Medium A Warren truss has 60° triangles. A center load of 100 N is carried equally by the two supports. Find the force in each end diagonal.

Show answer

$F \sin 60^\circ = 50$, $F \approx 57.7$ N (compression).

8. Hard Your bridge's top chord buckled sideways between joints 10 cm apart, at a predicted chord force of 25 N. You add cross-bracing that halves the unbraced length. Predict the new chord buckling force, and the new failure load if the chord force is $1.2P$.

Show answer

$25 \times 4 = 100$ N; $P = 100/1.2 \approx 83$ N (unless another member now fails first).

9. Hard A glue joint must carry 60 N. If the glue can carry about 1.5 MPa in shear, what glue area is needed? Is a simple butt joint of a 3 mm × 3 mm strip enough?

Show answer

$A = 60/1.5 = 40$ mm^2. A butt joint has only 9 mm^2, so a gusset (patch) or lapped joint is needed.

10. Hard Conceptual. Why might a bridge that held the most weight not win an efficiency competition?

Show answer

Model answer: efficiency divides the load by the bridge's mass; a heavy bridge can hold a lot but use material wastefully, while a lighter bridge with material only where needed can have a higher ratio.

11. **Challenge** Two designs: A has mass 24 g and is predicted to fail at 18 kg; B has mass 30 g and is predicted to fail at 25 kg. Which has the higher predicted efficiency? If B's weakest member can be strengthened by adding 2 g, raising its failure load to 32 kg, does that improve it?

    Show answer

    A: $18{,}000/24 = 750$; B: $25{,}000/30 \approx 833$; B is better. Improved B: $32{,}000/32 = 1000$, a clear improvement.

COMMON MISTAKES

- Making all members the same size. Size each for its own force.
- Ignoring buckling of long compression members and sideways buckling of the top chord.
- Building in a hurry. Most model bridges fail at a poorly made joint, not where the analysis predicts.

> **Make it a game · Pick the Truss.** Draw a Pratt, Howe and Warren truss from memory in sixty seconds. Mark which members are in tension and compression under a center load. Your parent covers one label and you fill it in. Then predict the failure load of a paper or craft-stick span with a reason. Closest prediction wins.

UNIT 6

Bridges

Bridges are where structural engineering is most visible and most daring. In this unit you will survey the bridge types and choose among them, then follow the history of great bridges through the people who built them: Abraham Darby's Iron Bridge and Thomas Telford's aqueduct and chains; John, Washington and Emily Roebling at Brooklyn; Fowler and Baker at the Forth, building a monument to safety after the Tay disaster; and the Golden Gate.

You will also study two of the most important failures in engineering history, the Quebec Bridge and the Tacoma Narrows Bridge, and what engineers learned from them: buckling, checking, wind and humility. The unit ends with a bridge competition in which you design, analyze, build and test your own truss bridge and score it on efficiency.

PREREQUISITES

- Unit 4: statics, method of joints, arches and cables
- Unit 5: stress, E, moment of inertia
- Algebra and trigonometry: square roots, sine and cosine, solving for a variable

MASTERY CHECKLIST

- ☐ I can name six bridge types, explain how each carries load, and choose a type for a site with reasons.
- ☐ I can explain how cast and wrought iron were used in the Iron Bridge and Telford's works, and calculate aqueduct loads and arch thrust.
- ☐ I can calculate suspension cable tension from span, sag and load, and the steel needed.
- ☐ I can use Euler's formula and explain the Quebec and Tacoma Narrows failures and what changed afterward.
- ☐ I can design, analyze, build and test a truss bridge and compute its efficiency.

OPTIONAL KITS FOR THIS UNIT

- **Pitsco Education (balsa bridge supplies)**: Balsa and basswood strips, bridge kits and a structures tester used in school bridge competitions; fits Lessons 6.4 and 6.5. Craft-store balsa or craft sticks work too.
- **Bridge Design Contest (Engineering Encounters)**: Free bridge-design software and an annual online contest for students, based on the West Point Bridge Designer; a good companion to Lesson 6.5. A parent should review registration.
- **KEVA planks**: Useful for balanced-cantilever and arch models in Lessons 6.1 and 6.2.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

6.1	**Bridge Types and Spans: The Forth Bridge**	3 sessions of 50 minutes
6.2	**The Iron Bridge and Thomas Telford**	3 sessions of 50 minutes
6.3	**The Brooklyn Bridge and the Golden Gate**	3 sessions of 50 minutes
6.4	**Failures: The Quebec Bridge and Tacoma Narrows**	3 sessions of 50 minutes
6.5	**The Bridge Competition: Design, Analyze, Build and Test**	3 sessions of 50 minutes

Unit 6 test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation. Draw free-body diagrams where they help. The test is worth 100 points.

1. **6 pts**
   Match each bridge type to its main way of carrying load: beam, arch, suspension, cable-stayed.

2. **5 pts**
   How did the Tay Bridge disaster influence the design of the Forth Bridge?

3. **6 pts**

   A load of 40 kN hangs 50 m from a pier on a cantilever arm. What counterweight is needed 30 m on the other side?

4. **6 pts**

   Why is cast iron suited to arches but not to chains?

5. **6 pts**

   A canal trough 3.0 m wide holds water 1.4 m deep and spans 15 m between piers. Find the water load per meter and the midspan moment.

6. **8 pts**

   A suspension span of 600 m with a sag of 60 m carries 120 kN/m. Find H, V and $T_{\max}$.

7. **5 pts**

   For Problem 6, what area of steel is needed at 300 MPa?

8. **6 pts**

   Describe the roles of John, Washington and Emily Roebling in building the Brooklyn Bridge.

9. **6 pts**

   How did Washington Roebling respond when fraudulent wire was found in the cables?

10. **7 pts**

    Find the Euler buckling load of a pinned steel strut 30 mm × 30 mm and 3.0 m long ($E = 200$ GPa). What if it is braced at midheight?

11. **6 pts**

    What were the structural cause and the process failures of the 1907 Quebec Bridge collapse?

12. **6 pts**

    Why did the Tacoma Narrows Bridge fail, and what changed in bridge design afterward?

13. **6 pts**

Find the vortex-shedding frequency of a deck 3.0 m deep in a 15 m/s wind with St = 0.15.

14. **7 pts**

In a model Pratt truss loaded by P at the center, a 45° end diagonal carries $0.707P$ in compression. It is a balsa strip 3 mm × 3 mm and 6 cm long ($E = 3000$ MPa). Find the load P at which it buckles.

15. **6 pts**

A bridge of 32 g held 27 kg; another of 20 g held 19 kg. Which was more efficient?

16. **8 pts**

Short essay. Explain what the Quebec Bridge collapse taught engineers about checking assumptions, and how the iron ring keeps that lesson alive.

ANSWER KEY

1. Beam: bending (compression on top, tension below). Arch: compression with outward thrust. Suspension: deck hung from draped cables in tension, anchored at the ends. Cable-stayed: deck held by straight cables to towers, deck in compression.
2. The Tay Bridge blew down because wind was not properly allowed for; the Forth Bridge was designed for a large wind pressure (56 psf) and made visibly massive and stiff to restore public confidence, and Bouch's Forth suspension design was abandoned.
3. $40 \times 50/30 \approx 66.7$ kN.
4. Cast iron is strong in compression but brittle and weak in tension; arches are in compression, chains in tension.
5. $w = 9.81 \times 3.0 \times 1.4 \approx 41.2$ kN/m; $M = 41.2 \times 225/8 \approx 1159$ kN·m.
6. $H = 120 \times 360{,}000/480 = 90{,}000$ kN; $V = 36{,}000$ kN; $T \approx 96{,}900$ kN.
7. $96.9 \times 10^6/300 \approx 323{,}000$ mm^2 (0.32 m^2).
8. John designed it and died in 1869 after an accident during surveying; Washington became chief engineer, was disabled by caisson disease in 1872 and directed the work from his window; Emily carried his instructions, learned the engineering and acted as his link to the site, and was first to cross at the opening in 1883.
9. The bad wire could not be removed, so he calculated the loss and added about 150 extra wires to each cable, keeping a large factor of safety.
10. $I = 30^4/12 = 67{,}500$ mm^4; $P = \pi^2(200{,}000)(67{,}500)/3000^2 \approx 14{,}800$ N; braced at midheight, four times: about 59,200 N.
11. Cause: the lower compression chords buckled because their lacing was too light, with the dead load about 20% higher than computed after the span was lengthened. Process: the consulting engineer supervised from afar, did not recalculate the dead load, and warnings were not acted on in time; there was no independent check.
12. Its shallow, solid-girder deck was aerodynamically unstable and twisted in aeroelastic flutter in a moderate wind; afterward, suspension decks were given deep open trusses or streamlined boxes and long spans were tested in wind tunnels.
13. $0.15 \times 15/3.0 = 0.75$ Hz.

14. $P_{cr} = \pi^2(3000)(6.75)/60^2 \approx 55.5$ N; $P = 55.5/0.707 \approx 78.5$ N.
15. First: $27{,}000/32 \approx 844$; second: $19{,}000/20 = 950$. The second.
16. Model answer: at Quebec, a famous engineer's assumptions went unchecked: the span was lengthened without recalculating the dead load, the chord lacing was not proven, and warnings from the site were not acted on. Engineers learned that every assumption must be verified, that designs need independent review, that field warnings must stop the work, and that reputation is no substitute for calculation. The Ritual of the Calling of an Engineer and its iron ring, worn on the working hand, remind each new engineer of humility and of the public's trust.

PORTFOLIO PIECE: A BRIDGE DESIGN REPORT

Prepare a design report for your competition bridge as an engineer would for a client: the rules (design criteria), the options you considered with a decision matrix, your method-of-joints analysis and member schedule, your predicted failure load and first member to fail, construction photos, and test results for both versions.

Finish with an honest comparison of prediction and result, a description of the failure, and a page on what a real bridge of your design would need (materials, foundations, wind checks) to be built safely across a river near you.

LOOK FOR	WHAT IT MEANS
Design process	Clear criteria, at least three options, and a reasoned decision matrix.
Analysis	Correct member forces, member sizing for tension and buckling, and a predicted failure load and location.
Construction	Neat, square, accurately built bridge that meets the rules; mass recorded.
Testing and comparison	Failure load, efficiency and failure mode recorded and compared honestly with predictions; improvement round documented.
Real-world reflection	Thoughtful discussion of what a full-size version would require, drawing on the unit's case studies.

UNIT 7

Tall Buildings

The skyscraper is the great American building type, and its story is a story of engineering. In this unit you will see how four inventions made tall buildings possible: the iron and steel frame (Jenney's Home Insurance Building), the safe elevator (Otis), structural systems that resist wind efficiently (Fazlur Khan's tubes), and modern concrete cores and wind engineering (the Burj Khalifa). Along the way you will meet the ironworkers of the Empire State Building, built in 410 days.

The unit ends with what keeps people safe in tall buildings and all buildings: fire-resistant construction, sprinklers and exits, and the building codes that grew out of hard lessons from the Great Fire of London onward. You will build and test model towers as walls and frames, elevators with counterweights and brakes, tubes in sideways load, tower shapes in front of a fan, and insulated columns in heat.

PREREQUISITES

- Units 4–5: loads, tributary areas, column loads, stress, bending and deflection
- Physics ideas: work, energy and power ($W = Fd$, $P = Fv$)
- Algebra: proportional reasoning with squares and powers

MASTERY CHECKLIST

- ☐ I can explain how the steel frame made the skyscraper possible and calculate column loads in a frame building.
- ☐ I can calculate elevator power and energy and explain how counterweights and safety brakes work.
- ☐ I can calculate wind force, overturning moment and drift for a tower and explain framed, trussed and bundled tubes.
- ☐ I can calculate construction rates, pumping pressure and vortex frequency, and explain the Burj Khalifa's buttressed core.
- ☐ I can compute occupant loads, exit widths and sprinkler demand, and explain how codes protect people from fire.

OPTIONAL KITS FOR THIS UNIT

- **LEGO Architecture**: Ages 12 and up for most sets. Skyline and landmark sets include famous towers; useful for studying massing and setbacks in Lesson 7.4. Never required.
- **KEVA planks**: Good for tall-tower stacking and wind tests in Lessons 7.3 and 7.4.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

7.1	**The Home Insurance Building and the Steel Frame**	3 sessions of 50 minutes
7.2	**Elevators and Otis's Safety Brake**	3 sessions of 50 minutes
7.3	**Wind and the Tube: Fazlur Khan**	3 sessions of 50 minutes
7.4	**From the Empire State Building to the Burj Khalifa**	3 sessions of 50 minutes
7.5	**Fire Safety and Building Codes**	3 sessions of 50 minutes

Unit 7 test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation. Answer short-answer questions in complete sentences. The test is worth 100 points.

1. **6 pts**
   What made the Home Insurance Building different from earlier tall buildings?

2. **6 pts**
   A 14-story frame building has columns on a 22 ft × 22 ft grid with 130 psf per floor. Find the ground-floor interior column load and the steel area at 20 ksi.

3. **5 pts**
   Why did early steel frames need fireproofing?

4. 5 pts

How did Otis's safety brake work?

5. 7 pts

A 1,400 kg car with a 1,200 kg load and a counterweight of 2,000 kg rises at 4 m/s. Find the motor power at 85% efficiency.

6. 6 pts

How much energy does it take to lift 1,000 kg to a height of 250 m?

7. 7 pts

A tower 50 m wide and 250 m tall has a uniform wind pressure of 1.6 kPa. Find the wind force, the overturning moment and the drift limit at $H/500$.

8. 6 pts

For the tower in Problem 7, if it is a tube 50 m deep, what force does each face carry?

9. 6 pts

Explain Fazlur Khan's tube idea and why it saves steel.

10. 6 pts

The Empire State Building rose 102 stories in 410 days. Find the average floors per week.

11. 6 pts

Find the minimum pressure to pump concrete (2,400 kg/m^3) to 450 m.

12. 6 pts

How does the Burj Khalifa's shape reduce wind effects?

13. 7 pts

A tower 40 m wide sways at 0.12 Hz. At what wind speed would vortex shedding match that sway (St = 0.15)?

14. 7 pts

A hall of 3,600 ft^2 is used for tables and chairs (15 ft^2/person). Find the occupant load, the exit width at 0.2 in per person and the minimum number of exits.

15. **6 pts**

A sprinkler system needs 0.20 gpm/ft^2 over 1,500 ft^2 for 90 minutes. Find the flow and the total water.

16. **8 pts**

Short essay. Explain how building codes grow out of disasters, using the Great Fire of London and at least one other example, and why codes are minimums rather than ideals.

ANSWER KEY

1. Its floors and much of its outer wall were carried by an internal frame of iron and (above the sixth floor) steel, so the walls no longer had to carry the building and grow thicker toward the bottom.
2. $14 \times 130 \times 484 \approx 880{,}900$ lb ≈ 881 kips; $A \approx 44$ in^2.
3. Steel loses much of its strength at fire temperatures, so it was insulated (with terra-cotta tile, later sprayed fireproofing or board) to stay cool enough to carry its loads during a fire.
4. The rope's tension kept a spring flexed; if the rope broke, the spring straightened and pushed pawls into ratchet teeth on the guide rails, stopping the car (it failed safe).
5. Imbalance 600 kg: $600 \times 9.81 \times 4 = 23{,}544$ W; $\div 0.85 \approx 27.7$ kW.
6. $1000 \times 9.81 \times 250 \approx 2.45$ MJ.
7. $F = 1.6 \times 50 \times 250 = 20{,}000$ kN; $M = 20{,}000 \times 125 = 2.5 \times 10^6$ kN·m; drift limit 0.50 m.
8. $2.5 \times 10^6/50 = 50{,}000$ kN.
9. Put closely spaced columns and deep beams (or diagonal braces) around the building's perimeter so the whole exterior acts as one hollow tube resisting wind like a cantilevered box beam; material at the perimeter is most effective against bending, so less steel per square foot is needed (the Hancock Center used about 30 lb/ft^2).
10. $102/(410/7) \approx 1.74$ floors per week.
11. $2400 \times 9.81 \times 450 \approx 10.6$ MPa.
12. Its Y-shaped plan and 26 spiraling setbacks change its width and shape with height, so wind vortices shed at different frequencies and cannot organize into a strong, rhythmic push; the tapering also reduces area high up.
13. $V = fB/\mathrm{St} = 0.12 \times 40/0.15 = 32$ m/s.
14. 240 people; 48 in; at least 2 exits (more than 49 occupants).
15. 300 gpm; 27,000 gallons.
16. Model answer: after the 1666 fire, the Rebuilding Act required brick and stone, set wall thicknesses and widened streets; after later fires (Chicago 1871, the Iroquois Theater 1903) codes added fireproofing, outward-swinging and unlocked exits, exit signs, sprinklers and egress widths. Each rule records a lesson. Codes are minimums because they must apply to all buildings and be

enforceable; good designers aim higher, and safety also depends on maintenance and on owners keeping exits clear and systems working.

PORTFOLIO PIECE: A TOWER DESIGN STUDY

Design a 30-story office or apartment tower for a site in a real city. Choose its structural system (frame, framed tube, braced tube or core with outriggers) and its shape, and justify the choice for wind. Compute the wind force, overturning moment and drift limit; the load on an interior ground-floor column; the elevators needed for morning traffic; and the stairs required for one floor's occupant load.

Build a 1:500 model (60 cm or so) of your tower, test it in front of a fan for sway, and write a three-page design report with plans, a section, your calculations and test results.

LOOK FOR	WHAT IT MEANS
Structural concept	System and shape chosen and justified with reference to wind and Khan's or the Burj Khalifa's ideas.
Calculations	Wind force, moment, drift limit, column load, elevator count and stair width computed correctly with units.
Model and test	Model built to scale and tested fairly; sway observations recorded and discussed.
Life safety	Exits, stairs and fire protection planned and checked against code numbers.
Report	Clear drawings, organized calculations and well-written conclusions.

7.1 The Home Insurance Building and the Steel Frame

OBJECTIVES

- Explain how the iron and steel frame made the skyscraper possible, using the Home Insurance Building (1885) as the case study.
- Compare the weight and floor space of bearing-wall and frame buildings, and calculate column loads in a frame.
- Build and test a bearing-wall model and a frame model of the same tower and compare strength, weight and window area.

> **Warm-up.** Look at a tall building near you, or a photograph of one. Can you tell from outside whether its walls hold it up or a frame inside does? What clues would you look for: thick walls, big windows, visible columns?

WATCH

Race for the Skies: Chicago vs. New York

The B1M · 11:43

https://www.youtube.com/watch?v=2e1ojG5f-Cc

> **WHEN TO WATCH**
> After the story. Note how the steel frame and the elevator together changed how tall buildings could rise.

KEY TERMS

frame construction

A building whose floors and walls are carried by a framework of columns and beams; the exterior wall is hung on the frame.

curtain wall

A non-load-bearing outer wall attached to the frame, carrying only its own weight and wind.

spandrel

The part of the exterior wall between the top of one window and the bottom of the window above; in a frame, it hides the floor beam.

fireproofing

Protection (originally terra-cotta tile and masonry, now sprayed fiber, board or concrete) that keeps a steel frame from losing strength in a fire.

bay

The rectangular area between four columns of a frame.

CORE IDEAS

The story. The Great Chicago Fire of October 1871 destroyed about 17,500 buildings and left a third of the city homeless. As Chicago rebuilt, land in the business district grew so valuable that owners wanted buildings taller than masonry walls could reasonably carry (Lesson 5.2). In 1884–85 the architect and engineer William Le Baron Jenney, who had studied engineering in Paris and served as an army engineer, built the Home Insurance Building at LaSalle and Adams Streets: ten stories and about 138 ft tall, later raised to twelve. Inside its stone and brick face stood a frame of cast-iron columns and wrought-iron beams, and above the sixth floor some of the beams were of the new Bessemer steel, the first structural steel used in a building. Much of the outer wall's weight was carried on the frame at each floor. It is often called the first skyscraper.

How they figured it out. The frame idea did not come from nowhere. Iron columns had been used inside mills and warehouses for decades, and Paxton's Crystal Palace (Lesson 3.4) was all frame. Jenney's contribution was to carry the outer walls on the frame as well, so the walls no longer had to grow thicker toward the bottom. City inspectors were so unsure of the new method that they reportedly halted construction for a time to check whether it was safe. When the building was taken down in 1931, a committee of architects and engineers examined its structure and concluded that it was indeed the first true frame building. Within a few years, Chicago architects such as Holabird and Roche, Burnham and Root, and Louis Sullivan were building all-steel frames with walls hung on them like curtains.

The physics. A bearing wall must carry its own weight and all the floors above, so its base stress is $\sigma = \dfrac{w}{t} + \gamma h$ and its thickness grows quickly with height. A frame gathers each floor's load into a few columns by tributary area (Lesson 4.5) and carries it down through slender steel, which is about ten times stronger in compression than brick masonry. The wall becomes a thin skin carrying only itself and the wind. A steel frame building weighs much less than a masonry building of the same size, which also lightens its foundations, and its walls can be mostly windows. But unprotected iron and steel lose much of their strength at the

temperatures of a building fire, so Chicago's frames were wrapped in hollow terra-cotta tile.

The engineering. The frame changed everything about tall buildings: plans freed from thick walls, large windows for daylight, faster construction, lighter foundations, and the possibility of going much higher, once the elevator (Lesson 7.2) made height useful. Engineers soon had to solve new problems: connecting beams to columns rigidly enough to resist wind, fireproofing every member, and protecting steel from rust in the walls. Sullivan's Wainwright Building in St. Louis (1891) showed how a steel-frame tower could look: a base, a tall shaft of repeated floors and a crowning top.

Try it. In the build you will build two model towers of the same size, one with bearing walls and one with a frame, and compare them.

PICTURE STUDY

The Home Insurance Building, Chicago, Unknown photographer, about 1920s · Photograph · Wikimedia Commons

Image: Wikimedia Commons, public domain

From the street, the Home Insurance Building looked like a masonry building. The great change was hidden inside: a frame of iron and steel that carried the floors and much of the wall.

Look closely

- Count the floors. Which floors look different, and why might they have been added later?
- Compare the width of the piers between windows with the width of the windows. How much of the wall is glass?
- Look at the ground floor. Are its piers much thicker than those above?
- Look at the neighboring buildings. Which look like bearing-wall buildings?

Talk about it

1. What clue in this photograph suggests that the walls are not carrying the whole building?
2. Why was fireproofing the iron and steel so important in Chicago in particular?

3. If you were a city inspector in 1884, what would you want to know before allowing this new kind of building?

Safety first. Use scissors and glue carefully; an adult handles any hot glue (low temperature). Wear safety glasses during load tests.

BUILD

Wall or Frame? Two Model Towers

You will build two towers of the same height and plan, one with card bearing walls and one with a frame of straws or sticks hung with card “curtain walls,” then compare their mass, window area and load capacity.

You will need

- Card stock and index cards; drinking straws or wooden skewers; tape and white glue
- A ruler, scissors and a kitchen scale
- Books or bags of rice for loading, a stiff card or board as a loading plate

Steps

1. Choose a plan: 10 cm × 10 cm, 5 floors of 6 cm each (30 cm tall). Both towers must have card floors at every level.
2. **Bearing-wall tower.** Build four card walls, with windows cut out wherever you like, that carry the floors.
3. **Frame tower.** Build four corner columns (and one middle column per side if you like) of straws or skewers, with card beams at each floor, and tape thin card or paper panels (curtain walls) to the outside.
4. Weigh each tower. Measure and calculate the window area as a percentage of the wall area.
5. **Predict.** Which will carry more load on its roof? Which will carry more per gram of tower?
6. Load each tower at the top with the plate and books until it fails or reaches a limit of 5 kg. Record the load and how it failed.
7. Compute strength-to-weight ratios. Make a comparison table.
8. **Improve.** Cut larger windows in each tower, as an architect would want, and test again (or predict, if the tower was destroyed). Which system loses less strength when you add windows?

Tips

- Keep both towers the same height and plan so the test is fair.

- Columns must be straight and plumb; a leaning column fails early.
- Join beams firmly to columns; the joints matter more than you think.

Check your work

- ☐ I built and tested both systems and compared strength, weight and window area.
- ☐ I can calculate column loads in a frame building.
- ☐ I can explain why the frame made the skyscraper possible.

ENGINEER'S NOTEBOOK

1. Section sketches of a bearing-wall building and a frame building of the same height, with wall thicknesses and load paths.
2. Your tower comparison table.
3. A short essay: why did the skyscraper begin in Chicago in the 1880s?

KEY FORMULAS

$P_{\text{column}} = n\,q\,A_{\text{trib}}$ (n floors)

$A_{\text{steel}} = \dfrac{P}{\sigma_{\text{allow}}}$

$\sigma_{\text{wall}} = \dfrac{w}{t} + \gamma h$

$\text{window ratio} = \dfrac{A_{\text{glass}}}{A_{\text{facade}}}$

From history. Jenney's office trained many of the architects of the Chicago School, including Daniel Burnham, William Holabird, Martin Roche and Louis Sullivan. A Chicago legend says Jenney got the idea of the frame when his wife set a heavy book on a small birdcage and it held, but the story cannot be confirmed.

Example 1. A ten-story frame building has columns on a 20 ft × 20 ft grid. Each floor (including the roof) has a total load of 150 psf. Find the load on an interior column at the ground floor, and the steel area needed at an allowable stress of 15,000 psi (typical of early steel design).

Show solution

1. Tributary area: $20 \times 20 = 400$ ft^2 per floor.
2. Load: $10 \times 150 \times 400 = 600{,}000$ lb $= 600$ kips.
3. Area: $600{,}000 \div 15{,}000 = 40$ in^2, for example a built-up column about 10 in × 10 in in outline with 1 in walls.

Answer: 600 kips; about 40 in^2 of steel.

Example 2. Compare a masonry bearing wall and a curtain wall for a ten-story, 140 ft building. The bearing wall needs to average 2.5 ft thick; the curtain wall of brick facing on the frame is 1.0 ft thick. Both weigh 120 lb/ft^3. Find the weight of each per foot of wall length.

Show solution

1. Bearing wall: $2.5 \times 140 \times 120 = 42{,}000$ lb per foot.
2. Curtain wall: $1.0 \times 140 \times 120 = 16{,}800$ lb per foot, and in a frame building its weight is carried floor by floor on the beams, not stacked to the ground.
3. The frame building's walls weigh 60% less, and the frame lets most of the wall be glass.

Answer: 42,000 lb/ft versus 16,800 lb/ft.

Example 3. A downtown lot is 100 ft × 140 ft. A bearing-wall design needs exterior walls 4 ft thick at the ground floor; a frame design needs 1 ft. How much ground-floor area does the frame gain?

Show solution

1. Bearing-wall interior: $(100 - 8)(140 - 8) = 92 \times 132 = 12{,}144\ \text{ft}^2$.
2. Frame interior: $(100 - 2)(140 - 2) = 98 \times 138 = 13{,}524\ \text{ft}^2$.
3. Gain: $13{,}524 - 12{,}144 = 1380\ \text{ft}^2$, more than 11% more rentable space on the most valuable floor.

Answer: About 1,380 ft^2 more.

PRACTICE

1. Easy What is a curtain wall?

   Show answer

   A non-load-bearing exterior wall hung on a building's frame, carrying only its own weight and wind.

2. Easy Who designed the Home Insurance Building, and when was it built?

   Show answer

   William Le Baron Jenney, 1884–85.

3. Easy A column has a tributary area of 300 ft^2 on each of 6 floors at 120 psf. Find its load.

   Show answer

   $6 \times 300 \times 120 = 216{,}000$ lb.

4. Medium The column in Problem 3 is steel with an allowable stress of 20 ksi. What area is needed?

   Show answer

   $216/20 = 10.8\ \text{in}^2$.

5. Medium A façade bay is 20 ft wide and 12 ft tall, with a window 16 ft × 8 ft. Find the window ratio.

   Show answer

   $128/240 \approx 53\%$.

6. Medium Steel can safely carry about 20,000 psi; brick masonry about 200 psi. How many times larger must a masonry pier be than a steel column for the same load?

Show answer

About 100 times the area.

7. Medium A twelve-story frame building's floor load is 140 psf, on a 25 ft × 25 ft grid. Find the ground-floor interior column load in kips.

Show answer

$12 \times 140 \times 625 = 1{,}050{,}000$ lb $= 1050$ kips.

8. Hard A steel frame building weighs about 100 psf of floor area per story in total and a masonry bearing-wall building about 200 psf. For a 10-story building of 14,000 ft^2 per floor, compare total weights. What does this mean for foundations?

Show answer

Frame: $10 \times 14{,}000 \times 100 = 14 \times 10^6$ lb; masonry: 28×10^6 lb. The frame building needs foundations carrying half as much, a big saving on Chicago's soft clay.

9. Hard Why did early frame buildings wrap their iron and steel in terra-cotta tile?

Show answer

Iron and steel lose much of their strength at fire temperatures (steel retains only about half its strength near 550 °C); hollow tile insulated the members so they stayed cool enough to carry the building during a fire.

10. Hard Conceptual. The frame let walls become mostly glass. Give one advantage and one new problem this created.

Show answer

Model answer: advantage, abundant daylight and views for offices; problem, glass walls must resist wind pressure and keep out water, lose heat in winter and gain it in summer, and must be attached so the frame's movements do not crack them.

11. **Challenge** A building has 16 columns on a 4 × 4 grid with 24 ft bays (so 3 bays each way). Find the tributary area of an interior, an edge and a corner column, and check that they add up to the total floor area.

Show answer

Interior $24 \times 24 = 576$ (4 columns); edge $12 \times 24 = 288$ (8 columns); corner $12 \times 12 = 144$ (4 columns). Sum:
$4(576) + 8(288) + 4(144) = 2304 + 2304 + 576 = 5184\ \text{ft}^2 = 72 \times 72$. ✓

COMMON MISTAKES

- Thinking the first skyscrapers were all glass. They had stone, brick or terra-cotta faces; the frame was hidden inside.
- Forgetting that the outer wall still has weight; in a frame it is carried by the frame at each floor.
- Ignoring fire. A steel frame must be fireproofed to be safe.

> **Make it a game · Wall or Frame?.** Compare two stacks of books: one thick bearing wall of books, one frame of a few books and cards. Predict which holds more weight per space used, then test with a small bag of coins. Calculate column loads for a bay of a frame building. Explain how Home Insurance Building made skyscrapers possible. Points per correct claim.

7.2 Elevators and Otis's Safety Brake

OBJECTIVES

- Tell how Elisha Otis's safety brake (1852–54) made passenger elevators trustworthy and tall buildings useful.
- Calculate the power and energy of an elevator and show how a counterweight saves energy.
- Build a model elevator with a counterweight and a safety brake, and measure the forces.

> **Warm-up.** Think about the tallest building you have been in. How long would it take to climb its stairs carrying groceries? Estimate the number of steps and the time. How would you feel about renting an office on the top floor without an elevator?

WATCH

Make Inventions: Otis Safety Elevator

Make: · 4:12

https://www.youtube.com/watch?v=sSjJjKcoNRk

> **WHEN TO WATCH**
> After the story. Watch how the spring and the ratchet teeth catch the car when the rope is cut.

Note: The maker re-creates Otis's rope-cutting test with a model and a safe test rig. Do not try a full-size version.

KEY TERMS

safety brake (safety gear)
A device that grips the guide rails and stops an elevator car if its rope breaks or it goes too fast; Otis's used a spring and ratchet teeth.

traction elevator
An elevator whose steel ropes pass over a grooved sheave driven by a motor, with the car on one end and a counterweight on the other.

counterweight

A weight on the other end of the ropes, usually equal to the car plus about 40–50% of its rated load, so the motor lifts only the difference.

governor

A speed-sensing device that trips the safety brake if the car overspeeds.

power

The rate of doing work, $P = W/t = Fv$, in watts.

CORE IDEAS

The story. In 1852, Elisha Graves Otis, a master mechanic at a bedstead factory in Yonkers, New York, was asked to build a hoist to lift heavy equipment to an upper floor. Hoists of the day fell whenever a rope broke, so Otis designed a safety device. In 1854, at the Exhibition of the Industry of All Nations in New York's Crystal Palace, he rode a platform high up an open shaft before a crowd and had the hoisting rope cut. The platform dropped only a few inches and stopped. Otis is said to have called out, "All safe, gentlemen, all safe." In 1857 the first passenger elevator with his safety brake was installed in the E. V. Haughwout store in New York, powered by steam. Otis died in 1861, but his sons built the business, and by the 1890s electric elevators were carrying people up the new skyscrapers.

How they figured it out. Otis's insight was to use the rope's own tension to hold the brake off. A wagon spring at the top of the car was kept flexed by the pull of the rope. If the rope broke, the spring snapped straight, pushing two pawls outward into ratchet teeth cut along the wooden guide rails, which stopped the car. The design failed safe: losing the rope applied the brake. Later engineers added the governor, a spinning wheel driven by a rope that runs with the car; if the car goes too fast, the governor trips the brake. In 1903 the Otis company introduced the gearless traction elevator, which made very tall buildings practical.

The physics. Lifting a mass m at steady speed v takes a force $F = mg$ and power $P = Fv = mgv$. A **counterweight** on the other end of the ropes balances the car and part of its load, so the motor lifts only the difference: $P = (m_{\text{car}} + m_{\text{load}} - m_{\text{cw}})gv$. When the car is empty going down or full going up, the imbalance is small; modern elevators even return energy to the building when the heavier side descends. Energy to lift a load a height h is $E = mgh$. A safety brake must stop a falling car within a

short distance, so it must absorb the car's kinetic energy $\frac{1}{2}mv^2$ with a braking force spread over a stopping distance.

The engineering. Elevators made the upper floors of tall buildings the most valuable instead of the least, and so they made skyscrapers worthwhile. But elevators also use floor space. A tall building needs many elevators in shafts that pass through every floor below, so designers group them in banks serving zones, use express elevators and sky lobbies, and stack double-deck cars. The core of a skyscraper, with its elevators, stairs and services, is often also its structural spine against wind. Elevator design today includes multiple ropes (any one can hold the car), overspeed governors, rail brakes, buffers in the pit and careful inspection.

Try it. In the build you will lift a model car with and without a counterweight, measure the force, and build a simple safety brake that catches the car when its string is released.

PICTURE STUDY

Otis Shows the Safety Elevator, 1854, Unknown illustrator, depicting 1854 · Illustration · Wikimedia Commons

Image: Wikimedia Commons, public domain

This illustration imagines the moment at the New York Crystal Palace when Otis had the rope cut and the platform held. Look for the parts of the safety system the artist included.

Look closely

- Find the platform and Otis on it. What is he doing with his hand?
- Find the guide rails on each side of the shaft. Where would the ratchet teeth be?
- Find the rope and the worker at the top. What has just happened?
- Look at the crowd. How do the artists show their reaction?

Talk about it

1. Why did Otis need to cut the rope in public, instead of just explaining his invention?
2. Explain how the brake used the rope's own tension to stay released.
3. What does "fail safe" mean, and why is it important for an elevator?

Safety first. Keep the model small and the falling weight light (a few coins), and test over a soft pad. Keep fingers clear of the ratchet when the car drops.

BUILD

A Counterweighted Elevator with a Safety Brake

You will build a model elevator shaft with a small car, measure the force needed to lift it with and without a counterweight, then add a spring-and-ratchet safety brake and test whether it catches the car when the lifting string is let go.

You will need

- A tall shoebox or two long boards as a shaft, with two guide rails of craft sticks
- Craft sticks with notches cut into them (ratchet teeth), a small box as the car, a bent hair clip or rubber band as the spring, and two small pawls (pieces of stick)
- String, a pulley (a spool on a pencil), coins for loads and counterweight
- A spring scale or a cup and pennies to measure the lifting force; a ruler and stopwatch

Steps

1. Build the shaft and guide rails. Cut notches every 1 cm along the inside face of each rail.
2. Make the car with two pawls on its sides held in by the string's pull through a flexed spring, as Otis did: when the string is tight, the pawls are pulled in; when it goes slack, the spring pushes them out into the notches.
3. Run the lifting string over the pulley. Load the car with 20 coins.
4. Measure the force to lift the car steadily with no counterweight (spring scale or pennies in a cup).
5. Add a counterweight equal to the empty car plus half the load. **Predict** the new lifting force, then measure it. Also measure lowering the car.
6. Time the car rising 30 cm at a steady speed and compute the power in each case: $P = Fv$.
7. **Safety test.** Raise the car, then let go of the lifting string (do not use the counterweight for this test). Measure how far the car drops before the brake catches. Repeat five times.

8. **Improve.** Change the spring, pawl shape or notch spacing to shorten the stopping distance, and test again.

Tips

- Make the pawls pivot freely; a stiff pivot slows the brake.
- Closely spaced notches stop the car sooner.
- Keep the string tight during normal running so the brake stays off.

Check your work

- ☐ I measured lifting forces with and without a counterweight and computed power.
- ☐ My safety brake catches the car, and I measured the stopping distance.
- ☐ I can explain how elevators made skyscrapers useful.

ENGINEER'S NOTEBOOK

1. A labeled diagram of Otis's safety brake and of a modern traction elevator (sheave, ropes, car, counterweight, governor, buffers).
2. Your force and power data with and without the counterweight.
3. Your stopping-distance data and improvements.

KEY FORMULAS

$P = Fv = mgv$

$P_{\text{motor}} = \dfrac{(m_{\text{car}} + m_{\text{load}} - m_{\text{cw}})\, g\, v}{\eta}$ (η = efficiency)

$E = mgh, \quad KE = \frac{1}{2}mv^2$

$F_{\text{brake}} \approx \dfrac{\frac{1}{2}mv^2}{d} + mg$ (average force over stopping distance d)

From history. The first electric elevator was shown by Werner von Siemens in 1880 in Germany. Today the fastest elevators, in towers in China, travel over 20 m/s (about 45 mph), and engineers have developed lightweight carbon-fiber hoist ropes because steel ropes become too heavy to lift themselves in very tall shafts.

Example 1. An elevator car has a mass of 1,200 kg and a rated load of 1,000 kg. The counterweight is the car plus 45% of the load. Find the motor power (ignoring losses) to lift a full car at 2.5 m/s, and compare it with having no counterweight.

Show solution

1. Counterweight: $1200 + 0.45(1000) = 1650$ kg.
2. Imbalance with a full car: $1200 + 1000 - 1650 = 550$ kg.
3. Power: $P = 550 \times 9.81 \times 2.5 \approx 13{,}500$ W $= 13.5$ kW.
4. Without a counterweight: $2200 \times 9.81 \times 2.5 \approx 54{,}000$ W, four times as much.

Answer: About 13.5 kW with the counterweight; about 54 kW without.

Example 2. How much energy does it take to lift 10 people (75 kg each) to the 80th floor of a building with floors 4.0 m apart? At 8 m/s, how long does the trip take (ignoring acceleration)?

Show solution

1. Height: $79 \times 4.0 = 316$ m (from the 1st to the 80th floor).
2. Energy: $E = mgh = 750 \times 9.81 \times 316 \approx 2.33 \times 10^6$ J, about 0.65 kWh.
3. Time: $316 \div 8 = 39.5$ s.

Answer: About 2.3 MJ (0.65 kWh); about 40 seconds.

Example 3. A 2,000 kg car is falling at 3.0 m/s when its safety brake engages and stops it in 0.30 m. Find the average braking force needed.

Show solution

1. Kinetic energy: $\frac{1}{2}(2000)(3.0)^2 = 9000$ J.
2. Force to remove it over 0.30 m: $9000 \div 0.30 = 30{,}000$ N.
3. Add the car's weight, which the brake must also hold while stopping: $2000 \times 9.81 = 19{,}620$ N.
4. Total: about 49,600 N, about 2.5 times the car's weight, so the passengers feel about 2.5 g briefly.

Answer: About 50 kN.

PRACTICE

1. Easy What happened at the 1854 Crystal Palace exhibition in New York?

   Show answer

   Otis rode a hoist platform up the shaft and had the rope cut; his safety brake caught the platform after a short drop.

2. Easy Find the power to lift 500 kg at 1.0 m/s.

   Show answer

   $500 \times 9.81 \times 1.0 \approx 4.9$ kW.

3. Easy Why is an elevator counterweight about equal to the car plus half its rated load?

   Show answer

   So the imbalance is never large: about half the rated load whether the car is empty or full, which keeps the motor small and saves energy.

4. Medium How much energy is needed to raise 900 kg by 120 m?

   Show answer

   $900 \times 9.81 \times 120 \approx 1.06$ MJ.

5. Medium A car of 1,000 kg with a 600 kg load and a 1,300 kg counterweight rises at 3 m/s. Find the motor power if the system is 80% efficient.

Show answer

Imbalance 300 kg: $300 \times 9.81 \times 3 = 8829$ W; $\div 0.8 \approx 11.0$ kW.

6. Medium An elevator travels 400 m at 10 m/s. Ignoring acceleration, how long does the trip take?

Show answer

40 s.

7. Medium In Problem 5, if the car comes down empty, which side is heavier, and by how much?

Show answer

The counterweight is heavier by $1300 - 1000 = 300$ kg; the motor must hold it back (and can regenerate energy).

8. Hard A car of 1,500 kg falls at 2.0 m/s when the brake engages. If the passengers should feel no more than 1.5 g of extra deceleration, what is the minimum stopping distance?

Show answer

Deceleration $1.5 \times 9.81 = 14.7\ \mathrm{m/s^2}$; $d = v^2/(2a) = 4/29.4 \approx 0.14$ m.

9. Hard A 60-story office tower has 3,000 occupants on its upper 50 floors who all arrive in the morning within 30 minutes. If each elevator round trip takes 3 minutes and carries 15 people, how many elevators are needed?

Show answer

Each elevator carries $15 \times 10 = 150$ people in 30 minutes; $3000/150 = 20$ elevators (in practice, zoned banks and express cars).

10. Hard Conceptual. Why did elevators change which floors of a building were most valuable?

Show answer

Model answer: before elevators, upper floors meant many stairs and were the least desirable and cheapest; with fast, safe elevators, upper floors offered light, quiet and views with no climbing, so they became the most valuable.

11. **Challenge** A building's elevators lift an average of 2,000 kg of people a net 150 m upward each hour of a 10-hour day (people coming down return some of the energy, ignore that). At 70% efficiency, how many kWh per day do they use? Compare with lifting the cars without counterweights (cars of 1,500 kg each making 40 trips a day each, 10 cars).

Show answer

People: $2000 \times 9.81 \times 150 \times 10 \approx 2.94 \times 10^7$ J, $\div 0.7 \approx 4.2 \times 10^7$ J ≈ 11.7 kWh. Cars without counterweights: $10 \times 40 \times 1500 \times 9.81 \times 150 \approx 8.8 \times 10^8$ J, $\div 0.7 \approx 350$ kWh: counterweights save most of the energy.

COMMON MISTAKES

- Forgetting the counterweight when computing elevator power.
- Confusing energy (joules) with power (watts).
- Thinking the safety brake is the rope. The brake works when the rope fails.

> **Make it a game · Counterweight Challenge.** Use a pulley or a smooth rod, string and two paper cups. Lift a cup of coins with and without a counterweight and feel the difference. Calculate the energy to lift an elevator car with and without one. Then explain Otis's safety brake in your own words. Score one point per correct step.

7.3 Wind and the Tube: Fazlur Khan

OBJECTIVES

- Explain why wind, not weight, governs the design of very tall buildings, and how Fazlur Khan's tube systems resist it efficiently.
- Calculate wind force, overturning moment and drift, and the column forces in a tube.
- Test model towers as frames, tubes and braced tubes in a sideways-load test.

> **Warm-up.** Hold a sheet of paper flat by its bottom edge and blow on it: it flops. Roll it into a tube, stand it up and blow: it stands. Fold it into a square tube. Why is a tube so much stiffer than the same paper flat?

WATCH

John Hancock Center: Fazlur Khan Braced Tube Invention | Skyscraper Revolution

Naveed Anwar · 7:19

https://www.youtube.com/watch?v=MmFlaKbWWzs

> **WHEN TO WATCH**
> After the story. Note the steel weight per square foot he quotes and why the X-braces save steel.

Note: A structural engineering professor's short lecture with slides. A small channel, but the content is expert and exactly on topic.

KEY TERMS

lateral load

A sideways load, from wind or earthquakes, that tries to bend and overturn a building.

drift

How far a building sways sideways at its top under wind; often limited to about height/400 to height/500.

framed tube

A building whose closely spaced exterior columns and deep spandrel beams form a stiff hollow tube that resists wind like a giant hollow beam.

trussed (braced) tube

A tube stiffened with large diagonal braces on its faces, like the John Hancock Center's X-braces.

bundled tube

Several tubes joined side by side, which can stop at different heights, like the Sears (Willis) Tower's nine tubes.

premium for height

The extra structural material per square foot needed as buildings get taller, mainly to resist wind.

CORE IDEAS

The story. Fazlur Rahman Khan was born in 1929 in Dhaka, in what is now Bangladesh, studied civil engineering there, and came to the United States in 1952 for graduate study at the University of Illinois. In 1955 he joined the Chicago office of Skidmore, Owings & Merrill (SOM). Khan saw that as buildings grew taller, the steel needed per square foot kept climbing, because the problem was no longer gravity but wind. He found a new way to build. The DeWitt-Chestnut apartments in Chicago (1965) were a **framed tube**: closely spaced columns all around the outside, tied by deep beams, so the whole perimeter acted as one hollow beam standing on end. The 100-story John Hancock Center (1969), designed with the architect Bruce Graham, added giant X-braces to its tapering tube. The Sears Tower (1973), now the Willis Tower, was a **bundled tube** of nine square tubes 75 ft on a side, rising to 1,450 ft, for twenty-five years the tallest building in the world.

How they figured it out. Khan thought about a tall building as a cantilever growing out of the ground and asked how to make that cantilever stiff with the least material. Material far from the center resists bending best (Lesson 5.5), so he put the columns at the outside. He studied the way a framed tube's faces carry wind (the columns at the corners carry more than those in the middle, an effect called shear lag) and countered it with diagonal braces or by bundling tubes. He used the early computers at SOM to analyze these structures. The Hancock Center used about 29.7 lb of steel per square foot of floor, roughly half what a conventional frame of that height would have needed. Khan was a devout man, generous with colleagues and students, and he died in 1982 while visiting Saudi Arabia; Chicago honored him with a street name and a sculpture at the Willis Tower.

The physics. Wind pressure q acts over the face of the building, so the total force is $F = qA = qBH$ for width B and height H. Acting at about mid-height, it creates an overturning moment at the base $M = F \cdot H/2 = \dfrac{qBH^2}{2}$, which grows with the square of the height (and faster, because wind is stronger higher up). A tube resists this moment like a hollow box beam: the windward face is stretched and the leeward face compressed, with forces of about M/D for a tube of depth D. The building also sways: its drift grows roughly with H^4 for a uniform cantilever, so stiffness, not strength, usually governs, and people at the top must not feel uncomfortable motion.

The engineering. Khan's tubes changed tall-building design worldwide. Today's supertall towers use concrete cores, outrigger trusses that tie the core to big perimeter columns, buttressed cores (Lesson 7.4) and tuned mass dampers, but the principle is Khan's: put the stiffness at the outside and let the whole building act together. Wind-tunnel tests of models check the pressures and motions before a tower is built.

Try it. In the build you will push sideways on model towers built as a plain frame, a framed tube and a braced tube, and measure their drift.

PICTURE STUDY

The Willis (Sears) Tower, Chicago, Carol M. Highsmith, between 1980 and 2006 · Photograph · Library of Congress (Carol M. Highsmith Archive)

Image: Wikimedia Commons, public domain

Carol Highsmith, who has given her photographs of America to the Library of Congress for public use, photographed the Willis Tower from the street. Its stepped outline shows Khan's bundled tube: nine square tubes that stop at different heights.

Look closely

- Count the setbacks where tubes stop. How many levels can you see?
- Look at the dark facade. Can you see the closely spaced columns that make up each tube's walls?
- Find the bands of darker or lighter floors near the setbacks. What might be at those levels?
- Compare the tower's width at the bottom and the top.

Talk about it

1. Why does stopping some tubes partway up help against wind?
2. If the whole tower is like a hollow beam standing on end, which faces are in tension and which in compression when wind blows from the west?
3. Why might the building's shape help "confuse" the wind?

> **Safety first.** Wear safety glasses for the pull tests. Keep the string pull gentle, and fix the tower bases to a board so they do not fall off the table.

BUILD

Frame, Tube, Braced Tube: A Drift Test

You will build three model towers of the same height and floor plan from straws or skewers and card, push each sideways at the top with a measured load, and measure the drift. Then you will compute the stiffness of each per gram of material.

You will need

- Drinking straws or bamboo skewers, card stock floors, tape or glue, thread for braces
- A base board, a ruler fixed vertically beside the tower top, string, a pencil pulley and pennies
- A kitchen scale

Steps

1. Build three towers 40 cm tall with a 10 cm × 10 cm plan and floors every 5 cm: (A) a frame with only four corner columns; (B) a framed tube with three columns per face (eight in all) and card spandrel strips at every floor tying them; (C) tube B plus thread X-braces on every face, like the Hancock Center.
2. Weigh each tower.
3. **Predict** the ranking and ratio of stiffnesses.
4. Fix a tower to the base. Tie a string at the top, run it horizontally over the pulley, and add pennies in steps of 5. Read the drift at the top on the ruler after each step, up to 2 cm or 50 pennies.
5. Graph load against drift for each tower and find its stiffness (slope).
6. Compute stiffness per gram for each. Which design is most efficient?
7. Test a bundled-tube version: two towers B taped side by side with one cut off at mid-height. Compare drift in each direction.
8. **Improve.** With the same thread, find the bracing pattern that gives tower B the greatest stiffness (X on every face, only the bottom half, every other face). Record your best.

Tips

- Keep the string horizontal and pull at the same height on each tower.
- Joints at the spandrels make the tube work; tape them well.
- Thread braces work only when tight.

Check your work

- ☐ My data compare frame, tube and braced tube stiffness fairly.
- ☐ I can compute wind force, overturning moment and drift limit for a tower.
- ☐ I can explain Fazlur Khan's tube idea and why it saved steel.

ENGINEER'S NOTEBOOK

1. Plan and elevation sketches of a framed tube, a trussed tube and a bundled tube, with the wind direction and the tension and compression faces marked.
2. Your drift data, graphs and stiffness-per-gram comparison.
3. A paragraph on the premium for height and how Khan reduced it.

KEY FORMULAS

$F = qBH$

$M_{\text{base}} \approx \frac{qBH^2}{2}$

$F_{\text{face}} \approx \frac{M}{D}$ (tension on the windward face, compression on the leeward face)

$\Delta_{\text{limit}} \approx \frac{H}{500}$

From history. Khan liked to say that a structural engineer should be able to see the whole building as one system. He was also an early user of computers in structural design, at a time when an analysis that now runs in seconds took hours on a mainframe. His daughter, Yasmin Sabina Khan, later wrote his biography.

Example 1. A tower is 40 m wide and 200 m tall. Treat the wind pressure as a uniform 1.5 kPa over the whole face. Find the wind force, the overturning moment at the base and the drift limit at $H/500$.

Show solution

1. Force: $F = qBH = 1.5 \times 40 \times 200 = 12{,}000$ kN.
2. Moment: $M = F \times H/2 = 12{,}000 \times 100 = 1.2 \times 10^6$ kN·m.
3. Drift limit: $200 \div 500 = 0.40$ m.

Answer: 12,000 kN; 1.2 million kN·m; 0.40 m.

Example 2. If the tower in Example 1 is a tube 40 m deep in the direction of the wind, estimate the total tension in the windward face and compression in the leeward face. If the leeward face has 20 columns, about how much extra force does each carry from wind?

Show solution

1. Face force: $M/D = 1.2 \times 10^6 \div 40 = 30{,}000$ kN.
2. Per column: $30{,}000 \div 20 = 1500$ kN of extra compression on the leeward side (and the same reduction, or tension, on the windward side).
3. (Shear lag makes the corner columns carry more than this average.)

Answer: About 30,000 kN per face; about 1,500 kN per column.

Example 3. Compare overturning moments for two towers of the same width and wind pressure, one 200 m tall and one 400 m tall.

Show solution

1. $M \propto H^2$, so doubling the height multiplies the moment by 4.
2. If the wind pressure also increases with height (as it does), the real increase is more than 4.
3. The taller tower's floor area only doubles, so the steel per square foot needed for wind roughly doubles: the premium for height.

Answer: At least 4 times the moment.

PRACTICE

1. Easy What is drift?

   Show answer

   The sideways movement of a building's top under lateral load (wind or earthquake).

2. Easy Name the three tube systems Khan developed and one building for each.

   Show answer

   Framed tube (DeWitt-Chestnut apartments), trussed tube (John Hancock Center), bundled tube (Sears/Willis Tower).

3. Easy Find the drift limit at $H/500$ for a 300 m tower.

   Show answer

   0.60 m.

4. Medium A tower 30 m wide and 150 m tall has a uniform wind pressure of 1.2 kPa. Find the wind force and base moment.

   Show answer

   $F = 5400$ kN; $M = 5400 \times 75 = 405{,}000$ kN·m.

5. Medium For Problem 4, if the tube is 30 m deep, what force does each face carry?

   Show answer

   $405{,}000/30 = 13{,}500$ kN.

6. Medium In US units, find the wind force on a building face 100 ft wide and 600 ft tall at an average pressure of 40 psf.

   Show answer

   $40 \times 60{,}000 = 2.4 \times 10^6$ lb.

7. Medium Why are tubes efficient at resisting wind?

   Show answer

   They place the structure at the building's perimeter, as far as possible from the center, where it gives the greatest resistance to bending, so the whole building acts like a hollow beam.

8. Hard A uniform cantilever's top deflection grows as H^4 (same section). If a 100 m tower drifts 0.15 m, what would a 150 m tower of the same section drift? Does it meet $H/500$?

Show answer

$0.15 \times 1.5^4 \approx 0.76$ m; limit $150/500 = 0.30$ m; it fails, so the taller tower needs a much stiffer section.

9. Hard The Hancock Center used about 29.7 lb of steel per square foot of floor area. If it has about 2.8 million ft^2 of floor area, how much steel is that in tons? If a conventional frame needed 50 lb/ft^2, how much steel did Khan's design save?

Show answer

$29.7 \times 2.8 \times 10^6 \approx 8.3 \times 10^7$ lb $\approx 41{,}600$ tons; conventional $\approx 70{,}000$ tons; saving about 28,000 tons.

10. Hard Conceptual. Why does stiffness, not strength, usually govern the design of a very tall building?

Show answer

Model answer: a structure strong enough not to fail may still sway too much; large drift cracks walls and windows and, above all, makes occupants uncomfortable or sick, so tall buildings are sized to limit motion, which requires more material than strength alone.

11. Challenge Wind pressure often increases with height roughly as $q = q_0(z/10)^{0.3}$. For a 300 m tower, how much greater is the pressure at the top than at 10 m? Why does this make the overturning moment grow faster than H^2?

Show answer

$(300/10)^{0.3} = 30^{0.3} \approx 2.77$ times greater. Pressure is largest high up, where its lever arm about the base is longest, so the moment grows faster than H^2 (roughly as $H^{2.3}$).

COMMON MISTAKES

- Thinking taller buildings are governed mainly by their weight. Above a few dozen stories, wind usually controls the structure.
- Forgetting that overturning moment grows with the square of height.
- Putting all the stiffness at the center. Material at the perimeter is far more effective against bending.

> **Make it a game · Drift Duel.** Build a tall tower from index cards or a spaghetti frame. Push at the top with one finger and measure the sideways drift. Add braces and predict how it changes. Explain why wind, not weight, governs very tall buildings, and which of the three tube systems Fazlur Khan used. Best prediction wins.

7.4 From the Empire State Building to the Burj Khalifa

OBJECTIVES

- Tell how the Empire State Building was built in 410 days and how the Burj Khalifa reached 828 m with a buttressed core.
- Calculate construction rates, concrete pumping pressure and the effect of shape on wind.
- Test tower shapes (prismatic, tapered and stepped) in a fan wind test and design the tallest stable paper tower.

> **Warm-up.** Look at a picture of the Empire State Building and one of the Burj Khalifa. Write three differences in their shapes. Why might the taller one have a more complicated shape?

WATCH

Burj Khalifa | All the Engineering Secrets of the Mega structure

Sabin Civil Engineering · 11:52

https://www.youtube.com/watch?v=SqFx0cBR0oo

> **WHEN TO WATCH**
> After the Burj Khalifa part of the story. Note the three reasons its shape helps against wind.

BACKUP VIDEO

Burj Khalifa Lecture Series, Supertallest: Wind Tunnel Testing

SKYSCRAPER MUSEUM · 2:19

https://www.youtube.com/watch?v=j9Nz2TTzb-c

A short clip from a lecture by the tower's structural engineer, William F. Baker of SOM, on wind-tunnel testing.

KEY TERMS

fast-track construction

Overlapping design and construction so that work begins before every drawing is finished, as at the Empire State Building.

setback

A step back in a tower's outline; the Burj Khalifa has 26 setbacks in a spiral pattern.

high-strength concrete

Concrete with compressive strength above about 50–60 MPa, used in the lower columns and walls of supertall towers.

buttressed core

The Burj Khalifa's structure: a hexagonal concrete core braced by three wings, each wall buttressing the others, like a three-legged stool.

vortex shedding

Alternating whirls of air that peel off a building and push it side to side; changing the shape with height keeps them from lining up.

CORE IDEAS

The story. In 1929 a group of investors decided to build the tallest building in the world on Fifth Avenue in New York. The architects Shreve, Lamb & Harmon produced drawings in about two weeks, and the builders Starrett Brothers and Eken organized the work like a factory. Excavation began in January 1930, steel began rising on March 17, and the Empire State Building opened on May 1, 1931, 410 days after construction began: 102 stories and 1,250 ft to the top of its mast. At times the frame rose four and a half floors a week. The photographer Lewis Hine climbed with the ironworkers to record their skill. Eighty years later, on January 4, 2010, the Burj Khalifa in Dubai opened, designed by the Chicago office of SOM (architect Adrian Smith, structural engineer William F. Baker). It stands 828 m (2,717 ft), more than twice as tall.

How they figured it out. At the Empire State Building, speed came from planning. Steel was fabricated in Pittsburgh to exact sizes and delivered by truck in the order it was needed, often installed within days of being rolled; a small railway on each floor carried materials; and every trade followed a precise schedule. The Burj Khalifa's

challenge was wind. Its engineers tested models in wind tunnels more than forty times and shaped the tower to “confuse the wind”: a Y-shaped plan of three wings around a central core, with the wings stepping back one after another in a spiral as the tower rises. Because the building’s width changes with height, the wind vortices that shed from it never organize into one strong rhythm. The concrete was pumped to a record height of about 600 m.

The physics. Supertall towers are dominated by wind and by the weight of their own structure. A **buttressed core** is efficient because each wing’s walls brace the core and the other wings, giving a very wide, stiff base. Wind vortices shed from a building at a frequency $f = \mathrm{St}\, V/B$, where B is the width; if that frequency matched the tower’s sway frequency, the tower would rock. Changing B with height spreads the shedding over many frequencies so they cannot all match. Building so tall also raises practical physics problems. Concrete pumped up 600 m needs a pressure of at least $\rho g h \approx 2400 \times 9.81 \times 600 \approx 14\ \mathrm{MPa}$ just to lift it, plus friction in the pipe. Columns shorten under load (Lesson 5.3), and concrete creeps, so floors are built slightly high to end up level.

The engineering. Tall buildings combine every part of this course: foundations (the Burj stands on a 3.7 m thick concrete raft on 192 piles about 1.5 m in diameter and 43 m long), materials, loads, wind, elevators, fire safety and construction management. They also raise questions of purpose. A tower can be a fine way to house many people and businesses on little land, or a monument to someone’s name. Engineers serve their neighbors by making it safe either way, and the best towers serve the city around them.

Try it. In the build you will test tower shapes in front of a fan and then build the tallest paper tower you can that passes a wind test.

PICTURE STUDY

Old-Timer Structural Worker, Empire State Building, Lewis W. Hine, 1930 · Photograph · National Archives (Works Progress Administration records)

Image: Wikimedia Commons, public domain

Lewis Hine, best known for photographs of working people, spent months climbing the Empire State Building's frame with the ironworkers. Here a veteran worker fastens a connection at the top of the frame, with the Chrysler Building behind him.

Look closely

- Find the column and the beam meeting at the connection. How are they joined?
- Count the rows of rivets or bolts. Why so many?
- Look at the worker's tools and gloves. What is he doing?
- Find the Chrysler Building. About how high above the street is the worker?

Talk about it

1. What does this connection have to carry: gravity, wind or both?
2. Why would a fast schedule depend on workers like this one?
3. What does Hine's photograph say about the dignity of work?

Safety first. Keep fingers away from the fan blades and use a fan with a guard. Wear safety glasses for wind tests in case a tower snaps.

BUILD

Confusing the Wind: Tower Shapes and a Height Challenge

You will build three paper towers of the same height and volume but different shapes, test their sway and stability in front of a box fan, and then use what you learn to build the tallest tower you can from ten sheets of paper that stays standing in the wind.

You will need

- Printer paper or card stock, tape, scissors and a ruler
- A box fan, a stopwatch and a phone for slow-motion video
- A base board and a little modeling clay to fix towers down
- Optional: a strip of ribbon on a stick to show the wind direction and gusts

Steps

1. Build three towers 60 cm tall with about the same amount of paper: (A) a straight square prism; (B) a tapered tower (wide base, narrow top); (C) a stepped tower with three or four setbacks, like the Burj Khalifa, in a Y-shaped plan if you can.
2. Fix each to the base board and place it 1 m in front of the fan.
3. **Predict.** Which will sway least? Which will rock side to side across the wind?
4. Run the fan on low, medium and high. Film each tower and measure the top's sway along the wind and across the wind (use a ruler behind it).
5. Count the sway cycles in 10 seconds to find each tower's sway frequency.
6. Compare: which shape had the smallest sway? Which shook across the wind most?
7. **Height challenge.** Using ten sheets of paper and 1 m of tape, build the tallest tower that stays standing for 30 seconds in front of the fan on medium. Measure its height.
8. **Improve.** Study how your challenge tower moved and failed, and build a second version. Record the height gained.

Tips

- Tapered and stepped towers have less area high up, where wind is strongest.

- A wide base and a heavy foundation keep a tower from tipping.
- Watch for across-wind rocking; it is the vortex effect.

Check your work

- ☐ My tests compare three tower shapes in the same wind.
- ☐ I can calculate construction rates, pumping pressure and vortex frequency.
- ☐ I can explain how the Burj Khalifa's shape and core resist wind.

ENGINEER'S NOTEBOOK

1. Plan sketches of the Empire State Building (a rectangular slab with setbacks) and the Burj Khalifa (a Y with a hexagonal core).
2. Your sway data for each shape and the height-challenge results.
3. A timeline of the tallest buildings from 1885 to today, with heights in meters and the structural system of each.

KEY FORMULAS

$\text{rate} = \dfrac{\text{floors}}{\text{time}}$

$p = \rho g h$ (pressure to lift a fluid a height h)

$f_{\text{vortex}} = \text{St}\,\dfrac{V}{B}$

$t = \dfrac{h}{v}$ (elevator or pump time)

From history. The Empire State Building's mast was designed as a mooring mast for airships, with a plan for passengers to walk down a gangplank onto the 102nd floor. A few tests showed that the winds at the top made it far too dangerous, and the idea was abandoned.

Example 1. The Empire State Building's 102 stories went up in 410 days of construction. What was its average rate in floors per week? If at its fastest the frame rose 4.5 floors per week, how long would 102 floors take at that pace?

Show solution

1. Weeks: $410 \div 7 \approx 58.6$ weeks.
2. Average rate: $102 \div 58.6 \approx 1.74$ floors per week (counting all work, not just steel).
3. At 4.5 floors per week: $102 \div 4.5 \approx 22.7$ weeks for the frame alone.

Answer: About 1.7 floors per week overall; about 23 weeks at the peak pace.

Example 2. Find the minimum pressure needed to pump concrete (density 2,400 kg/m^3) to a height of 600 m, ignoring friction. Compare with sea-level air pressure (101 kPa).

Show solution

1. $p = \rho g h = 2400 \times 9.81 \times 600 \approx 1.41 \times 10^7 \text{ Pa} = 14.1 \text{ MPa}$.
2. That is about 140 times atmospheric pressure, before adding pipe friction, which roughly doubled the needed pressure in practice.

Answer: About 14 MPa (about 140 atmospheres) before friction.

Example 3. A tower 60 m wide is struck by a 30 m/s wind. With St = 0.15, find the vortex-shedding frequency. Higher up, the same tower is 30 m wide. What is the frequency there? Why does this help?

Show solution

1. Bottom: $f = 0.15 \times 30/60 = 0.075$ Hz.
2. Top: $f = 0.15 \times 30/30 = 0.15$ Hz.
3. The vortices at different heights shed at different rates, so they cannot all push the tower in step with its own sway frequency; the forces partly cancel instead of building up.

Answer: 0.075 Hz at the base, 0.15 Hz higher up; the mismatch prevents a strong, organized sway.

PRACTICE

1. Easy How long did the Empire State Building take to build, and how tall is it to the top of its mast?

   Show answer

   410 days; 1,250 ft (381 m) to the top of the mast (1,454 ft with today's antenna).

2. Easy What is a buttressed core?

   Show answer

   A central core braced by wings whose walls buttress each other, as in the Burj Khalifa's Y-shaped plan.

3. Easy Convert 828 m to feet.

   Show answer

   About 2,717 ft.

4. Medium A tower's frame rises one floor every 3 days. How long for 160 floors?

   Show answer

   480 days.

5. Medium What pressure is needed to lift water ($\rho = 1000\ \mathrm{kg/m^3}$) to 500 m?

   Show answer

   $1000 \times 9.81 \times 500 \approx 4.9$ MPa.

6. Medium An elevator rises 504 m at 10 m/s. How long does it take, ignoring acceleration?

Show answer

50.4 s.

7. Medium A tower 45 m wide in a 25 m/s wind has St = 0.14. Find the vortex frequency.

Show answer

$0.14 \times 25/45 \approx 0.078$ Hz.

8. Hard A tower sways at 0.10 Hz. At what wind speed would vortices from its 50 m-wide base match that frequency (St = 0.15)? If the top 200 m is 25 m wide, what speed would match there?

Show answer

$V = fB/\mathrm{St}$: base $0.10 \times 50/0.15 \approx 33$ m/s; top ≈ 17 m/s. The critical speeds differ, so no single wind speed excites the whole tower.

9. Hard The Burj Khalifa's raft is 3.7 m thick. Suppose it covers 3,500 m^2. Find its volume and the weight of its concrete (2,400 kg/m^3).

Show answer

12,950 m^3; mass $\approx 3.1 \times 10^7$ kg; weight $\approx 305{,}000$ kN.

10. Hard Conceptual. Why was the Empire State Building built so fast, and what risks does such speed create?

Show answer

Model answer: careful planning, steel made to exact sizes and delivered just in time, on-site railways and coordinated trades. Risks: design errors not caught when construction outruns design, worker safety, and little time to correct mistakes; fast-track work requires excellent coordination and checking.

11. **Challenge** A supertall tower's concrete columns are 600 m tall in total, working at an average stress of 15 MPa with $E = 40$ GPa. Estimate their elastic shortening. If creep later doubles it, how much might the top floors drop without correction?

Show answer

$\delta = \sigma L / E = 15 \times 600{,}000 \text{ mm} \div 40{,}000 = 225 \text{ mm}$; with creep about 450 mm, so floors are built progressively higher to compensate.

COMMON MISTAKES

- Thinking the Burj Khalifa is mainly steel. Its main structure is reinforced concrete; steel forms the spire.
- Forgetting the shortening of tall columns and the need to build floors slightly high.
- Assuming a smooth, simple shape is best in wind. For supertall towers, varying the shape breaks up vortices.

Make it a game · Height Race Estimate. Estimation duel. Guess how many days the Empire State Building took, and how many floors per week that implies. Each of you calculates it from 410 days and 102 floors. Then sketch a buttressed core and setbacks of the Burj Khalifa. Closest guess earns a point; explain how shape confuses the wind.

Scripture. "And they said, Go to, let us build us a city and a tower, whose top may reach unto heaven; and let us make us a name, lest we be scattered abroad upon the face of the whole earth." (Genesis 11:4, KJV)

The oldest story of a tall tower is a warning against building to make a name for ourselves. Height is not wrong, but purpose matters: we build to serve.

7.5 Fire Safety and Building Codes

OBJECTIVES

- Trace building codes from the London Rebuilding Act of 1667 to modern codes, and explain fire-resistance ratings, sprinklers and egress.
- Calculate occupant loads, exit widths and sprinkler water demand.
- Test how insulation protects a “steel” member in a fire model and plan the egress for a real building.

> **Warm-up.** Walk through your home or church and find every way out of the main room. Time how long it takes to get outside from the farthest corner. Which exit would you use if the main door were blocked?

WATCH

What Is a Means of Egress?

CCPIA · 9:18

https://www.youtube.com/watch?v=idPJ1mdMQAY

> **WHEN TO WATCH**
> After the core ideas. List the three parts of a means of egress he names and give an example of each from your own building.

Note: A training video for building inspectors, filmed in a hotel; practical and clear.

KEY TERMS

building code

A law setting minimum standards for the safety of buildings: structure, fire, exits, accessibility, plumbing and more (in the U.S., usually based on the International Building Code).

fire-resistance rating

The time (in hours) a wall, floor or column can withstand a standard fire test without collapse or letting fire through.

means of egress

The continuous path out of a building: the exit access (to an exit), the exit itself (protected stairs, exit doors) and the exit discharge (to a public way).

occupant load

The number of people a space is designed for, found by dividing its floor area by an occupant load factor.

automatic sprinkler

A water nozzle held closed by a heat-sensitive glass bulb or metal link that opens by itself in a fire.

compartmentation

Dividing a building into fire-rated compartments so a fire is contained where it starts.

CORE IDEAS

The story. The Great Fire of London of September 1666 began in a bakery in Pudding Lane and, driven by an east wind through streets of timber houses, burned for four days. It destroyed some 13,200 houses, 87 churches and St Paul's Cathedral (Lesson 1.3). Remarkably few deaths were recorded, but a hundred thousand people were left homeless. Within days, Christopher Wren and others submitted plans for a new city. Wren's plan, with broad avenues radiating from piazzas, was never carried out, because property owners rebuilt on their old lots. But Parliament passed the Rebuilding Act of 1667: new buildings had to be of brick or stone, not timber; houses were divided into four classes by the size of street, each with set wall thicknesses and story heights; and some streets were widened. It was one of the first modern building codes. Soon afterward, Nicholas Barbon started one of the first fire insurance offices, which had its own firefighters.

How they figured it out. Codes have grown by learning from disasters, often bitterly: the Chicago fire of 1871, theater fires such as the Iroquois in 1903 (Grade 8), factory fires, and others. Each taught a lesson: noncombustible construction; fireproofing for steel; exit doors that open outward and are never locked; enough exits for the crowd; exit signs and emergency lighting; and automatic sprinklers, patented by Henry Parmelee in 1874 and improved by Frederick Grinnell. Fire scientists now test full-size assemblies in furnaces that follow a standard time-temperature curve, so a wall rated for two hours has been shown to hold back that fire for two hours.

The physics. Fire spreads by heat: conduction through materials, convection of hot gases rising through open stairs and shafts (as in the balloon frame, Lesson 5.1), and radiation across gaps. Steel loses about half its strength near 550 °C, and building

fires can exceed 1,000 °C, so steel is insulated with spray-applied fireproofing, gypsum board or concrete, which slow heating so the steel stays cool enough for the rated time. Concrete and masonry resist fire well. Heavy timber chars on its surface, and the char insulates the wood inside. Sprinklers fight fire with water's large heat capacity and heat of vaporization; one sprinkler opens over the fire, usually controlling it while it is small. The **means of egress** follows from simple rates: exits must be wide enough to let the whole occupant load out in a few minutes, so the code gives an exit width per person, about 0.2 in per person for doors and level paths and 0.3 in per person for stairs.

The engineering. A building code is a written summary of hard lessons and good judgment, a way of loving your neighbor by law. Architects and engineers use it from the first sketch: choosing the construction type and fire ratings, laying out compartments, placing at least two exits from larger spaces far enough apart that one fire cannot block both, limiting travel distance to an exit, designing sprinkler and alarm systems, and keeping exits clear and lighted. Codes are minimums. Good designers often do better, and good owners maintain what was built: an exit chained shut or a sprinkler valve closed can undo the best design.

Try it. In the build you will test how insulation slows the heating of a metal "column" and plan the exits for a real building.

PICTURE STUDY

Wren's Plan for Rebuilding London after the Great Fire, After Sir Christopher Wren, 1666 (later engraving) · Engraving · Royal Museums Greenwich

Image: Wikimedia Commons, public domain

Within days of the Great Fire, Wren drew this plan for a new London. The shaded area shows what burned. His broad, straight streets were never built, but the fire led to one of the first modern building codes.

Look closely

- Find the River Thames and the quay Wren proposed along it.
- Find the places where several avenues meet in a square. What might stand there?
- Compare the width of Wren's new streets with the old lanes that burned. How would wide streets help in a fire?
- Find St Paul's. How did Wren place it in his plan?

Talk about it

1. Why do you think property owners refused this plan?
2. Wide streets are firebreaks. What else in a city slows a fire's spread?
3. The 1667 Act required brick and stone. Why was that a better answer than wide streets alone?

> **Safety first.** An adult runs the heat test, using a hair dryer or a pan of hot water, never an open flame. Do not touch the heated metal; use an oven mitt and a thermometer.

BUILD

Fireproofing a Column and Planning the Exits

In two parts: first you will measure how insulation slows the heating of a metal "column" exposed to heat, as fireproofing protects steel; second you will plan the means of egress for a real building and check it against the code numbers.

You will need

- Two identical metal rods or large bolts (or two metal soup cans filled with sand), a kitchen or probe thermometer
- Insulation: several layers of paper towel, cardboard, or a wrap of drywall-joint compound or plaster (adult mixes)
- A hair dryer on high, set at a fixed distance (or a pan of hot water)
- A stopwatch, a tape measure and graph paper

Steps

1. **Part A.** Leave one rod bare and wrap the other in insulation of a measured thickness. Tape the thermometer probe to the middle of each (under the insulation for the wrapped one).
2. **Predict** how long each will take to warm by 20 °C.
3. With an adult, heat both from the same distance with the hair dryer, and record each temperature every 30 seconds for 10 minutes.
4. Graph temperature against time for both. Find the time each takes to rise 20 °C. Repeat with a thicker insulation layer.
5. **Part B.** Measure the main room of your church, school or home and draw its plan at ¼″ = 1′-0″ with every exit door.
6. Compute the occupant load (15 ft^2/person for tables and chairs, 7 ft^2/person for chairs only), the exit width needed at 0.2 in per person, and the number of exits needed (2 if more than 49 people).
7. Measure the travel distance from the farthest point to the nearest exit and check it against 200 ft (unsprinklered assembly) or 250 ft (sprinklered).
8. **Improve.** If the room fails any check, propose a change (another door, a wider door, a limit on seating) and draw it.

Tips

- Keep the heat source at exactly the same distance for both rods.
- Measure exit widths as the clear opening, not the frame.
- Look for exit signs, door hardware and emergency lights and note them on your plan.

Check your work

- ☐ My graph shows how insulation delays heating, as fireproofing protects steel.
- ☐ I computed the occupant load, exit width and number of exits for a real room.
- ☐ I can explain how building codes grew out of disasters such as the Great Fire of London.

ENGINEER'S NOTEBOOK

1. A timeline of fire-safety lessons: London 1666, Chicago 1871, Parmelee's sprinkler 1874, the Iroquois Theater 1903, modern codes.
2. Your heating graph and the delay time for each insulation thickness.
3. Your egress plan with occupant load, exit widths, travel distances and any proposed improvements.

KEY FORMULAS

$$N = \frac{A}{\text{occupant load factor}}$$

$W_{\text{exit}} = N \times 0.2$ in (doors, level paths), $W_{\text{stair}} = N \times 0.3$ in

$Q = \text{density} \times \text{area}$ (sprinkler water demand, gpm)

$Q_{\text{heat}} = mc\Delta T$

From history. A tall stone column called the Monument, designed by Wren and Robert Hooke, was built near Pudding Lane between 1671 and 1677 to commemorate the Great Fire. Its height, 202 ft, equals its distance from the spot where the fire began. Hooke also designed it to be used as a giant scientific instrument, with a shaft for experiments with pendulums and telescopes.

Example 1. A fellowship hall of 4,200 ft^2 will be used for dinners with tables and chairs (occupant load factor 15 ft^2 per person, net). Find the occupant load, the total exit width needed at 0.2 in per person, and the number of 36 in exit doors.

Show solution

1. Occupant load: $N = 4200 \div 15 = 280$ people.
2. Exit width: $280 \times 0.2 = 56$ in.
3. Doors: $56 \div 36 = 1.6$, so at least 2 doors; and since the occupant load exceeds 49, the code requires at least 2 exits anyway, placed far apart.
4. (Codes also require that losing any one exit leaves at least half the capacity, so two 36 in doors, 72 in total, are a sound choice.)

Answer: 280 people; 56 in of exit width; at least two 36 in doors.

Example 2. An office floor of 18,000 ft^2 (gross) uses an occupant load factor of 150 ft^2 per person. The exits are two stairs. Find the occupant load and the width each stair needs at 0.3 in per person, if each must carry half.

Show solution

1. Occupant load: $18{,}000 \div 150 = 120$ people.
2. Total stair width: $120 \times 0.3 = 36$ in.
3. Each stair carries 60 people: $60 \times 0.3 = 18$ in, but codes set a minimum stair width (44 in for most buildings), which governs.

Answer: 120 people; the 44 in minimum stair width governs.

Example 3. A sprinkler system is designed for a density of 0.10 gallons per minute per square foot over the 1,500 ft^2 most remote area. Find the water flow, and the volume of water for 60 minutes.

Show solution

1. Flow: $0.10 \times 1500 = 150$ gpm.
2. Volume: $150 \times 60 = 9000$ gal.
3. That is about 34,000 liters, which the water supply must be able to provide at the needed pressure (plus hose streams for firefighters).

Answer: 150 gpm; 9,000 gallons for an hour.

PRACTICE

1. Easy What did the Rebuilding Act of 1667 require of new London buildings?

   Show answer

   Brick or stone construction instead of timber, with set wall thicknesses and story heights by class of house and street, and some widened streets.

2. Easy What are the three parts of a means of egress?

   Show answer

   Exit access, the exit, and the exit discharge.

3. Easy A 1,800 ft^2 room with chairs only (7 ft^2 per person). Find its occupant load.

   Show answer

   $1800/7 \approx 257$ people.

4. Medium For Problem 3, find the exit width at 0.2 in per person and the number of 36 in doors.

   Show answer

   $257 \times 0.2 \approx 51.4$ in; two doors.

5. Medium A sprinkler system needs 0.15 gpm/ft^2 over 2,500 ft^2. Find the flow.

   Show answer

   375 gpm.

6. Medium Why must exit doors from assembly rooms open outward?

Show answer

So that a crowd pressing toward the exit pushes the doors open instead of jamming them shut.

7. Medium Steel loses about half its strength near 550 °C. If a column works at 40% of its room-temperature strength, why is insulation still essential?

Show answer

Building fires reach 800–1,000 °C or more; steel at those temperatures retains far less than 40% of its strength, so without insulation the column would fail.

8. Hard An office building has 6 floors above ground of 15,000 ft^2 each at 150 ft^2 per person. The two stairs must carry the occupants of the largest floor. Find the stair width each needs at 0.3 in per person (each serving half). Compare with the 44 in minimum.

Show answer

100 people per floor; total stair width $100 \times 0.3 = 30$ in; each 15 in; the 44 in minimum governs.

9. Hard Heat needed to warm a 500 kg steel column by 500 °C ($c = 0.49\ \mathrm{kJ/kg{\cdot}^\circ C}$)? If fireproofing lets in heat at 10 kW, how long until it reaches that temperature?

Show answer

$Q = 500 \times 0.49 \times 500 = 122{,}500\ \mathrm{kJ}$; time $= 122{,}500/10 = 12{,}250\ \mathrm{s} \approx 3.4$ hours.

10. Hard Conceptual. Why do codes require two exits placed far apart rather than one wide exit?

Show answer

Model answer: a single fire or obstruction could block one exit; two exits far apart make it unlikely that both are lost, and they spread the crowd so no one door is overwhelmed.

11. **Challenge** A sanctuary seats 600 in chairs (7 ft^2 per person, so 4,200 ft^2). People can move through a door at about 60 persons per minute per 36 in of width. With two 72 in double doors, how long does it take to empty the room through the doors? What if one door is blocked?

Show answer

Each 72 in door passes about 120 persons/min; two doors 240/min: $600/240 = 2.5$ min. With one blocked: 5 min. Codes aim for evacuation in a few minutes, so a third exit or wider doors would be prudent.

COMMON MISTAKES

- Using gross area when the code calls for net area (or the reverse) in occupant load calculations.
- Counting a door that is locked, blocked or swings the wrong way as an exit.
- Thinking fireproofing makes steel fire-proof. It only slows heating for a rated time.

> **Make it a game · Exit Width Drill.** Beat the clock. Your parent names a room, such as a classroom with 30 people. You calculate occupant load and exit width, then sketch two exits. Name the fire-resistance rating of a pretend wall. Wrong calculation costs a point. Then recall from memory the key milestones from the 1667 London Rebuilding Act to modern codes.

UNIT 7

Tall Buildings

The skyscraper is the great American building type, and its story is a story of engineering. In this unit you will see how four inventions made tall buildings possible: the iron and steel frame (Jenney's Home Insurance Building), the safe elevator (Otis), structural systems that resist wind efficiently (Fazlur Khan's tubes), and modern concrete cores and wind engineering (the Burj Khalifa). Along the way you will meet the ironworkers of the Empire State Building, built in 410 days.

The unit ends with what keeps people safe in tall buildings and all buildings: fire-resistant construction, sprinklers and exits, and the building codes that grew out of hard lessons from the Great Fire of London onward. You will build and test model towers as walls and frames, elevators with counterweights and brakes, tubes in sideways load, tower shapes in front of a fan, and insulated columns in heat.

PREREQUISITES

- Units 4–5: loads, tributary areas, column loads, stress, bending and deflection
- Physics ideas: work, energy and power ($W = Fd$, $P = Fv$)
- Algebra: proportional reasoning with squares and powers

MASTERY CHECKLIST

- ☐ I can explain how the steel frame made the skyscraper possible and calculate column loads in a frame building.
- ☐ I can calculate elevator power and energy and explain how counterweights and safety brakes work.
- ☐ I can calculate wind force, overturning moment and drift for a tower and explain framed, trussed and bundled tubes.
- ☐ I can calculate construction rates, pumping pressure and vortex frequency, and explain the Burj Khalifa's buttressed core.
- ☐ I can compute occupant loads, exit widths and sprinkler demand, and explain how codes protect people from fire.

OPTIONAL KITS FOR THIS UNIT

- **LEGO Architecture**: Ages 12 and up for most sets. Skyline and landmark sets include famous towers; useful for studying massing and setbacks in Lesson 7.4. Never required.
- **KEVA planks**: Good for tall-tower stacking and wind tests in Lessons 7.3 and 7.4.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

7.1	**The Home Insurance Building and the Steel Frame**	3 sessions of 50 minutes
7.2	**Elevators and Otis's Safety Brake**	3 sessions of 50 minutes
7.3	**Wind and the Tube: Fazlur Khan**	3 sessions of 50 minutes
7.4	**From the Empire State Building to the Burj Khalifa**	3 sessions of 50 minutes
7.5	**Fire Safety and Building Codes**	3 sessions of 50 minutes

Unit 7 test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation. Answer short-answer questions in complete sentences. The test is worth 100 points.

1. **6 pts**
   What made the Home Insurance Building different from earlier tall buildings?

2. **6 pts**
   A 14-story frame building has columns on a 22 ft × 22 ft grid with 130 psf per floor. Find the ground-floor interior column load and the steel area at 20 ksi.

3. **5 pts**
   Why did early steel frames need fireproofing?

4. **5 pts**

   How did Otis's safety brake work?

5. **7 pts**

   A 1,400 kg car with a 1,200 kg load and a counterweight of 2,000 kg rises at 4 m/s. Find the motor power at 85% efficiency.

6. **6 pts**

   How much energy does it take to lift 1,000 kg to a height of 250 m?

7. **7 pts**

   A tower 50 m wide and 250 m tall has a uniform wind pressure of 1.6 kPa. Find the wind force, the overturning moment and the drift limit at $H/500$.

8. **6 pts**

   For the tower in Problem 7, if it is a tube 50 m deep, what force does each face carry?

9. **6 pts**

   Explain Fazlur Khan's tube idea and why it saves steel.

10. **6 pts**

    The Empire State Building rose 102 stories in 410 days. Find the average floors per week.

11. **6 pts**

    Find the minimum pressure to pump concrete (2,400 kg/m^3) to 450 m.

12. **6 pts**

    How does the Burj Khalifa's shape reduce wind effects?

13. **7 pts**

    A tower 40 m wide sways at 0.12 Hz. At what wind speed would vortex shedding match that sway (St = 0.15)?

14. **7 pts**

    A hall of 3,600 ft^2 is used for tables and chairs (15 ft^2/person). Find the occupant load, the exit width at 0.2 in per person and the minimum number of exits.

15. **6 pts**

A sprinkler system needs 0.20 gpm/ft^2 over 1,500 ft^2 for 90 minutes. Find the flow and the total water.

16. **8 pts**

Short essay. Explain how building codes grow out of disasters, using the Great Fire of London and at least one other example, and why codes are minimums rather than ideals.

ANSWER KEY

1. Its floors and much of its outer wall were carried by an internal frame of iron and (above the sixth floor) steel, so the walls no longer had to carry the building and grow thicker toward the bottom.
2. $14 \times 130 \times 484 \approx 880{,}900$ lb ≈ 881 kips; $A \approx 44$ in^2.
3. Steel loses much of its strength at fire temperatures, so it was insulated (with terra-cotta tile, later sprayed fireproofing or board) to stay cool enough to carry its loads during a fire.
4. The rope's tension kept a spring flexed; if the rope broke, the spring straightened and pushed pawls into ratchet teeth on the guide rails, stopping the car (it failed safe).
5. Imbalance 600 kg: $600 \times 9.81 \times 4 = 23{,}544$ W; $\div 0.85 \approx 27.7$ kW.
6. $1000 \times 9.81 \times 250 \approx 2.45$ MJ.
7. $F = 1.6 \times 50 \times 250 = 20{,}000$ kN; $M = 20{,}000 \times 125 = 2.5 \times 10^6$ kN·m; drift limit 0.50 m.
8. $2.5 \times 10^6/50 = 50{,}000$ kN.
9. Put closely spaced columns and deep beams (or diagonal braces) around the building's perimeter so the whole exterior acts as one hollow tube resisting wind like a cantilevered box beam; material at the perimeter is most effective against bending, so less steel per square foot is needed (the Hancock Center used about 30 lb/ft^2).
10. $102/(410/7) \approx 1.74$ floors per week.
11. $2400 \times 9.81 \times 450 \approx 10.6$ MPa.
12. Its Y-shaped plan and 26 spiraling setbacks change its width and shape with height, so wind vortices shed at different frequencies and cannot organize into a strong, rhythmic push; the tapering also reduces area high up.
13. $V = fB/\mathrm{St} = 0.12 \times 40/0.15 = 32$ m/s.
14. 240 people; 48 in; at least 2 exits (more than 49 occupants).
15. 300 gpm; 27,000 gallons.
16. Model answer: after the 1666 fire, the Rebuilding Act required brick and stone, set wall thicknesses and widened streets; after later fires (Chicago 1871, the Iroquois Theater 1903) codes added fireproofing, outward-swinging and unlocked exits, exit signs, sprinklers and egress widths. Each rule records a lesson. Codes are minimums because they must apply to all buildings and be

enforceable; good designers aim higher, and safety also depends on maintenance and on owners keeping exits clear and systems working.

PORTFOLIO PIECE: A TOWER DESIGN STUDY

Design a 30-story office or apartment tower for a site in a real city. Choose its structural system (frame, framed tube, braced tube or core with outriggers) and its shape, and justify the choice for wind. Compute the wind force, overturning moment and drift limit; the load on an interior ground-floor column; the elevators needed for morning traffic; and the stairs required for one floor's occupant load.

Build a 1:500 model (60 cm or so) of your tower, test it in front of a fan for sway, and write a three-page design report with plans, a section, your calculations and test results.

LOOK FOR	WHAT IT MEANS
Structural concept	System and shape chosen and justified with reference to wind and Khan's or the Burj Khalifa's ideas.
Calculations	Wind force, moment, drift limit, column load, elevator count and stair width computed correctly with units.
Model and test	Model built to scale and tested fairly; sway observations recorded and discussed.
Life safety	Exits, stairs and fire protection planned and checked against code numbers.
Report	Clear drawings, organized calculations and well-written conclusions.

UNIT 8

Foundations and Geotechnical Engineering

Every structure rests on the ground, and the ground is the least predictable material an engineer uses. In this unit you will meet the science of soils that Karl Terzaghi founded, learn how engineers investigate a site and calculate effective stress and settlement, and design spread footings and piles as Chicago's engineers learned to do on their swamp.

Then you will study three great geotechnical case studies: the Leaning Tower of Pisa, which leaned for eight centuries and was gently straightened in 2001; retaining walls from Coulomb's fortress to the Blue Ridge Parkway; and Hoover Dam, a mountain of concrete cooled by 582 miles of pipe that holds back a great river. You will build in sand and clay and water: jar tests, footings and piles, leaning towers, retaining walls and model dams.

PREREQUISITES

- Units 4–5: stress, pressure, moments, factor of safety
- Algebra and trigonometry: logarithms (base 10), sine and tangent, squares
- Physics: density, weight, pressure in fluids

MASTERY CHECKLIST

- ☐ I can classify soils, describe a site investigation, and calculate total stress, pore pressure and effective stress.
- ☐ I can estimate consolidation settlement and explain why clays settle slowly.
- ☐ I can size a spread footing and estimate a pile's capacity from skin friction and end bearing.
- ☐ I can calculate the eccentricity and bearing pressures of a leaning tower and explain how Pisa was stabilized.
- ☐ I can calculate active earth pressure and check a gravity wall, and explain why drainage matters.
- ☐ I can calculate water pressure and force on a dam and check a gravity dam against sliding, including uplift.

OPTIONAL KITS FOR THIS UNIT

- KEVA planks: Identical maple planks for building the leaning-tower model in Lesson 8.3 and block gravity walls in Lesson 8.4.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

8.1	**Soils and Site Investigation: Terzaghi's Science**	3 sessions of 50 minutes
8.2	**Shallow and Deep Foundations**	3 sessions of 50 minutes
8.3	**The Leaning Tower of Pisa**	3 sessions of 50 minutes

8.4	**Retaining Walls and Slopes**	3 sessions of 50 minutes
8.5	**Dams: Hoover Dam**	3 sessions of 50 minutes

Unit 8 test

Time allowed: 50 minutes. Calculators are allowed. Use $\gamma_w = 9.81\ \mathrm{kN/m^3}$. Show your work and units for every calculation. The test is worth 100 points.

1. **5 pts**
   State Terzaghi's principle of effective stress and explain why it matters.

2. **7 pts**
   Soil with $\gamma = 17\ \mathrm{kN/m^3}$ lies above a water table at 4 m; below it $\gamma_{\mathrm{sat}} = 19.5\ \mathrm{kN/m^3}$. Find σ, u and $\sigma\prime$ at 12 m.

3. **7 pts**
   A clay layer 5 m thick ($C_c = 0.35$, $e_0 = 1.0$) has $\sigma\prime_0 = 90$ kPa. A building adds 45 kPa. Estimate the settlement.

4. **6 pts**
   A column carries 3,000 kN on soil allowing 250 kPa. Size a square footing.

5. **8 pts**
   A pile 0.5 m in diameter and 18 m long has $f_s = 35$ kPa and $q_b = 2500$ kPa. Find its ultimate and allowable capacity (FS = 2.5).

6. **5 pts**
   Why is uneven settlement more harmful than uniform settlement?

7. **6 pts**
   Why did the Tower of Pisa lean, and why did the long pause in construction help it?

8. **8 pts**
   A 50,000 kN tower stands on a circular foundation 15 m in diameter. Its center of gravity is 18 m high and it leans 4°. Find e, $q_{\max}$ and $q_{\min}$.

9. **5 pts**

How did engineers reduce Pisa's lean without touching the tower?

10. **7 pts**

A wall 5 m high retains soil with $\gamma = 18$ and $\phi = 30^\circ$. Find K_a, P_a and its line of action.

11. **6 pts**

If water rises to the top behind the wall in Problem 10, what is the water force alone? Why do walls need drains?

12. **6 pts**

A dry slope of sand with $\phi = 34^\circ$ is cut at 28°. Find its factor of safety.

13. **7 pts**

Find the pressure at 120 m depth and the force per meter width on a dam holding 120 m of water.

14. **6 pts**

A triangular gravity dam 120 m high with a 100 m base (23.5 kN/m^3). Find its weight per meter and its sliding FS with $\mu = 0.7$ (no uplift), using Problem 13.

15. **3 pts**

Why was Hoover Dam's concrete cooled with pipes?

16. **8 pts**

Short essay. Using the St. Francis Dam (Lesson 1.4), Pisa and Hoover Dam, explain why site investigation and monitoring are the foundation of geotechnical engineering.

ANSWER KEY

1. $\sigma\prime = \sigma - u$: the stress carried grain to grain equals the total stress minus the pore water pressure. It controls soil strength and compression, so changes in water pressure change a soil's strength and cause settlement.
2. $\sigma = 68 + 156 = 224$ kPa; $u = 9.81 \times 8 \approx 78.5$ kPa; $\sigma\prime \approx 145.5$ kPa.
3. $S = (0.35 \times 5/2.0)\log_{10}(135/90) = 0.875 \times 0.176 \approx 0.154$ m.
4. $A = 12\ \mathrm{m}^2$; side ≈ 3.46, use 3.5 m.
5. Skin $35 \times \pi(0.5)(18) \approx 990$ kN; end $2500 \times \pi(0.25)^2 \approx 491$ kN; $Q_u \approx 1481$ kN; $Q_a \approx 592$ kN.
6. Uneven settlement distorts the structure, bending beams and walls and cracking finishes; uniform settlement moves the building as a whole.
7. It stands on soft, slightly uneven layers of silty sand and clay, softer on the south side, so it settled unevenly. The pause let the clay consolidate and gain strength under the early stories before more weight was added.
8. $e = 18\sin 4^\circ \approx 1.256$ m; $A \approx 176.7\ \mathrm{m}^2$, $S \approx 331.3\ \mathrm{m}^3$; $P/A \approx 283$; $M/S = 62{,}800/331.3 \approx 190$; $q_{\max} \approx 473$, $q_{\min} \approx 93$ kPa.
9. They extracted small amounts of soil from beneath the north (high) side through inclined drill holes; the ground there settled slightly and the tower rotated back about half a degree.
10. $K_a = 1/3$; $P_a = \frac{1}{2}(1/3)(18)(25) = 75$ kN/m, at 1.67 m above the base.
11. $\frac{1}{2}(9.81)(25) \approx 122.6$ kN/m, more than the dry soil's push; drains keep water from building up behind the wall.
12. $\tan 34^\circ / \tan 28^\circ \approx 0.675/0.532 \approx 1.27$.
13. $p \approx 1177$ kPa; $F = \frac{1}{2}(9.81)(14{,}400) \approx 70{,}600$ kN/m.
14. $W = \frac{1}{2}(100)(120)(23.5) = 141{,}000$ kN/m; $FS = 0.7 \times 141{,}000/70{,}600 \approx 1.40$.
15. To remove the heat of hydration quickly so the concrete would not crack as it slowly cooled and shrank.
16. Model answer: St. Francis failed on rock that was never properly investigated; Pisa leaned because it was built on soft, uneven soil no one understood, and it was saved only after decades of borings, testing and careful monitoring during the soil extraction; Hoover Dam's designers investigated and cleaned its foundation to sound rock, grouted and drained it, and it has been watched by instruments ever since. Because soil and rock vary and cannot be seen,

engineers must investigate before designing and keep observing during and after construction.

PORTFOLIO PIECE: A GEOTECHNICAL REPORT FOR A REAL SITE

Choose a site near you where something might be built (with the owner's permission), such as a backyard shed, a garden wall or a church pavilion. Describe its soil from a jar test and a shallow hand-dug hole (call 811 first), estimate its unit weight and bearing capacity from typical values, and note any water or slopes.

Write a short geotechnical report: the soil profile, the recommended foundation type and footing size for the structure's loads, a retaining-wall check if the site needs one, drainage recommendations, and what further investigation a professional engineer would require.

LOOK FOR	WHAT IT MEANS
Site description	Soil described from observation and a jar test; water, slopes and surroundings noted.
Calculations	Loads estimated; footing sized; effective stress or settlement or wall check calculated correctly.
Recommendations	Foundation type and drainage recommended with clear reasons.
Honesty about limits	States what is uncertain and what professional investigation would add.
Presentation	Organized report with a site sketch, soil profile and labeled calculations.

8.1 Soils and Site Investigation: Terzaghi's Science

OBJECTIVES

- Describe how soils are classified (gravel, sand, silt, clay) and how engineers investigate a site with borings, samples and tests.
- Calculate total stress, pore water pressure and effective stress at depth, and estimate consolidation settlement of a clay layer.
- Analyze a soil sample with a jar sedimentation test and compare the bearing of loose and compacted soil.

> **Warm-up.** Squeeze a handful of damp garden soil, a handful of damp sand and (if you can find it) a lump of damp clay. Which holds its shape? Which feels gritty, which smooth, which sticky? Rub a little between your fingers and describe each in one word.

WATCH

How Soil Destroys Buildings

Practical Engineering · 8:09

https://www.youtube.com/watch?v=SW-NoiM726U

> **WHEN TO WATCH**
> After the physics paragraph. Watch how a clay sample swells and shrinks with water, and why that cracks foundations.

BACKUP VIDEO

What is Geotechnical Investigation or Soil Investigation?

Civil Mentors · 6:00

https://www.youtube.com/watch?v=ClCUdEyqm4Q

With the engineering paragraph: the steps of a site investigation from desk study to laboratory testing.

KEY TERMS

geotechnical engineering

The branch of civil engineering that studies soil and rock and designs foundations, slopes, retaining walls, tunnels and earth dams.

grain size

Gravel (larger than about 4.75 mm), sand (0.075–4.75 mm), silt and clay (smaller than 0.075 mm); clay particles are tiny, flat and sticky (plastic).

boring and SPT

A drilled hole for taking samples; the Standard Penetration Test counts the blows of a 63.5 kg hammer falling 760 mm needed to drive a sampler 300 mm (the N-value).

pore water pressure (u)

The pressure of water in the spaces between soil grains, $u = \gamma_w z_w$ below the water table.

effective stress

The stress carried grain to grain, $\sigma' = \sigma - u$; it controls a soil's strength and compression (Terzaghi's principle).

consolidation

The slow settlement of a saturated clay as water is squeezed out of its pores under load.

CORE IDEAS

The story. For most of history, builders knew the ground only by experience: dig until it feels firm, and hope. Karl Terzaghi, born in Prague in 1883, trained as a mechanical engineer and worked on dams and foundations, where he saw that engineers had formulas for steel and concrete but almost none for the soil everything rested on. From 1919 to 1925, teaching at Robert College in Istanbul, he built his own testing apparatus, partly from odds and ends, and measured how clays compress and how water moves through them. In 1925 he published *Erdbaumechanik* (Soil Mechanics), and the science of geotechnical engineering was born. He later taught at MIT, Vienna and Harvard and advised on dams around the world.

How they figured it out. Terzaghi's key discovery was the principle of **effective stress**. In a saturated soil, part of the load is carried by the water in the pores and part by the grains pressing on one another. Only the grain-to-grain part, $\sigma' = \sigma - u$, gives the soil its strength and makes it compress. When a load is placed on a clay, the water at first carries it; as the water slowly seeps out, the load passes to the grains and the clay settles. This is **consolidation**, and it explained why buildings on clay

kept settling for years. Engineers then learned to investigate every site before designing: maps and records, borings with samples, field tests such as the Standard Penetration Test and the cone penetration test, and laboratory tests of strength and compressibility. At the great Grand Coulee Dam in the 1930s, inspectors were lowered into 26-inch holes drilled into the granite to see its seams for themselves.

The physics. At depth z, the total vertical stress is the weight of everything above: $\sigma = \sum \gamma_i h_i$, with unit weights of about 17 kN/m^3 for moist soil above the water table and 19–21 kN/m^3 for saturated soil below it. Below the water table, the water pressure is $u = \gamma_w z_w$, with $\gamma_w = 9.81\ \mathrm{kN/m^3}$. The effective stress is the difference. When a building adds stress $\Delta\sigma$ to a normally consolidated clay layer of thickness H, its settlement is about $S = \dfrac{C_c H}{1+e_0} \log_{10} \dfrac{\sigma'_0 + \Delta\sigma}{\sigma'_0}$, where C_c is the compression index from a lab test and e_0 is the void ratio (volume of voids ÷ volume of solids). Sands drain quickly and settle at once; clays drain slowly and settle over months or years. Some clays also swell when wet and shrink when dry, which cracks foundations, as the video shows.

The engineering. Every foundation design starts with a geotechnical report: the layers of soil and rock, the water table, the strength and compressibility of each layer, and recommendations for foundation type and bearing pressure. A few thousand dollars of borings can save millions, and lives: the St. Francis Dam (Lesson 1.4) failed on rock no one had properly investigated. Engineers also watch the water, because rising groundwater lowers effective stress and weakens soil, and because flowing water can carry fine soil away (piping) and undermine dams and levees.

Try it. In the build you will separate a soil sample into sand, silt and clay in a jar of water, and test how compaction changes what a soil can carry.

PICTURE STUDY

Looking Down a 26-Inch Drill Hole at Grand Coulee Dam, U.S. Bureau of Reclamation photographer, September 1935 · Photograph · National Archives

Image: Wikimedia Commons, public domain

Before building Grand Coulee Dam, one of the largest concrete structures ever made, the Bureau of Reclamation drilled holes 26 inches wide into the granite foundation so that inspectors could be lowered inside to look at the rock's seams directly. This is site investigation at its most thorough.

Look closely

- Find the hat. Use it to estimate the hole's diameter.
- Find the dark lines on the wall of the hole. What are they?
- Look at the surface around the hole. What is the rock like near the top?
- Imagine being lowered inside. What would you look for?

Talk about it

1. Why would seams in the rock matter for a dam?
2. What does this photograph have to do with the St. Francis Dam (Lesson 1.4)?
3. Most sites are investigated with much smaller borings. What can a small boring tell an engineer, and what might it miss?

> **Safety first.** Wash hands after handling soil. Do not dig on any property without permission, and never dig more than a hand trowel's depth without calling 811 first to locate buried utilities.

BUILD

Know Your Ground: A Jar Test and a Bearing Test

You will separate a soil sample from your yard into its sand, silt and clay fractions in a jar of water, then test how much load a loose and a compacted soil can carry under a small “footing.”

You will need

- A tall clear jar with a lid, water, a spoon of dish detergent (to separate clay particles), a ruler and a marker
- About two cups of soil from your yard (from below the topsoil if possible), sand for comparison
- A sturdy box or bucket, a wooden block about 5 cm square as a footing, weights (books or a bucket of water), a ruler held vertically
- A kitchen scale

Steps

1. **Jar test.** Fill the jar one-third with soil, add water to three-quarters full and a spoon of detergent. Shake hard for 3 minutes.
2. Set it down and mark the height of settled material after 1 minute (sand), after 2 hours (silt) and after 24–48 hours when the water clears (clay).
3. Measure each layer’s thickness and calculate the percentages of sand, silt and clay. Use a soil texture triangle (search the USDA soil texture triangle) to name your soil.
4. **Bearing test.** Fill the box with damp soil loosely to 15 cm. Set the block on top. **Predict** how much load it will carry before sinking 1 cm.
5. Add weight to the block in steps and measure the settlement after each. Stop at 2 cm or when it plunges.
6. Empty the box and refill it in 3 cm layers, compacting each layer firmly with a tamper. Repeat the test.
7. Convert the load at 1 cm settlement to a bearing pressure ($p = F/A$) for each case and compare. Repeat with sand if you have it.

8. **Improve.** Test a footing twice as wide on the compacted soil. Does the bearing pressure at 1 cm settlement stay the same, rise or fall?

Tips

- Use the same water content for loose and compacted tests so only compaction changes.
- Read settlement from a ruler fixed to the box, not to the block.
- Clay can take a long time to settle in the jar; be patient.

Check your work

☐ I measured my soil's sand, silt and clay fractions and named the soil.

☐ My bearing tests show the effect of compaction.

☐ I can calculate effective stress and consolidation settlement.

ENGINEER'S NOTEBOOK

1. A sketch of your jar with the layers measured and labeled, and your percentages.
2. A soil profile for a site (real or from your jar test): layers, water table, unit weights, and graphs of total stress, pore pressure and effective stress against depth.
3. Your bearing test data and graph of load against settlement for loose and compacted soil.

KEY FORMULAS

$$\sigma = \sum \gamma_i h_i, \quad u = \gamma_w z_w, \quad \sigma\prime = \sigma - u$$

$$e = \frac{V_v}{V_s}, \quad n = \frac{e}{1+e}$$

$$S = \frac{C_c H}{1+e_0} \log_{10} \frac{\sigma\prime_0 + \Delta\sigma}{\sigma\prime_0}$$

From history. Terzaghi kept detailed diaries of every project and insisted that engineers go and look at the ground themselves. He often stressed that judgment, built on careful observation of the ground, mattered as much as formulas. He also pioneered the observational method: watching a structure as it is built and adjusting the design to what is found.

Example 1. A site has moist sand ($\gamma = 17\ \mathrm{kN/m^3}$) to the water table at 3 m depth, then saturated clay ($\gamma = 20\ \mathrm{kN/m^3}$). Find the total stress, pore pressure and effective stress at 10 m depth.

Show solution

1. Total stress: $\sigma = 17(3) + 20(7) = 51 + 140 = 191\ \mathrm{kPa}$.
2. Pore pressure: the point is 7 m below the water table, so $u = 9.81 \times 7 \approx 68.7\ \mathrm{kPa}$.
3. Effective stress: $\sigma\prime = 191 - 68.7 \approx 122\ \mathrm{kPa}$.

Answer: $\sigma = 191$ kPa, $u \approx 69$ kPa, $\sigma\prime \approx 122$ kPa.

Example 2. A clay layer 4.0 m thick has $C_c = 0.30$, $e_0 = 0.90$ and an effective stress at its middle of 100 kPa. A new building adds 50 kPa. Estimate the consolidation settlement.

Show solution

1. $S = \dfrac{C_c H}{1 + e_0} \log_{10} \dfrac{\sigma\prime_0 + \Delta\sigma}{\sigma\prime_0} = \dfrac{0.30 \times 4.0}{1.90} \log_{10} \dfrac{150}{100}$.
2. $\dfrac{1.2}{1.9} = 0.632$; $\log_{10} 1.5 = 0.176$.
3. $S \approx 0.632 \times 0.176 \approx 0.111$ m, about 11 cm, taking place over months or years as water drains from the clay.

Answer: About 0.11 m (11 cm).

Example 3. A soil sample has a void ratio of 0.65. Find its porosity. If the soil is saturated, what fraction of its volume is water?

Show solution

1. Porosity: $n = e/(1 + e) = 0.65/1.65 \approx 0.39$.
2. Saturated means the voids are full of water, so water fills about 39% of the volume.

Answer: $n \approx 0.39$; about 39% water by volume.

PRACTICE

1. Easy List soil types from largest particles to smallest.

   Show answer

   Gravel, sand, silt, clay.

2. Easy What is effective stress?

   Show answer

   The stress carried grain to grain: total stress minus pore water pressure, $\sigma\prime = \sigma - u$.

3. Easy Find the water pressure 5 m below the water table.

   Show answer

   $9.81 \times 5 \approx 49$ kPa.

4. Medium Soil with $\gamma = 18\ \mathrm{kN/m^3}$ lies above a water table at 2 m, and soil with $\gamma_{\mathrm{sat}} = 20\ \mathrm{kN/m^3}$ below it. Find σ, u and $\sigma\prime$ at 8 m.

   Show answer

   $\sigma = 36 + 120 = 156$ kPa; $u = 9.81 \times 6 \approx 58.9$ kPa; $\sigma\prime \approx 97$ kPa.

5. Medium If the water table in Problem 4 rises to the ground surface (all soil saturated at 20 kN/m^3), what is $\sigma\prime$ at 8 m?

   Show answer

   $\sigma = 160$ kPa; $u = 78.5$ kPa; $\sigma\prime \approx 81.5$ kPa, lower: the soil is weaker.

6. Medium A sample has a void ratio of 1.2. Find its porosity.

   Show answer

   $1.2/2.2 \approx 0.55$.

7. Medium Why does a building on sand settle quickly but one on clay keeps settling for years?

   Show answer

   Sand drains easily, so water leaves its pores almost at once; clay has tiny pores, so water seeps out slowly and the load passes to the grains gradually (consolidation).

8. Hard A clay layer 6 m thick ($C_c = 0.40$, $e_0 = 1.10$) has $\sigma\prime_0 = 80$ kPa at its middle. A fill adds 60 kPa. Estimate the settlement.

Show answer

$$S = (0.40 \times 6/2.10)\log_{10}(140/80) \approx 1.143 \times 0.243 \approx 0.28 \text{ m}.$$

9. Hard In an SPT, a sampler is driven 150 mm to seat it, then 300 mm with 8 blows and then 7 blows for each 150 mm. What is the N-value, and is the sand loose or dense? (Roughly: N < 10 loose, 10–30 medium, > 30 dense.)

Show answer

N counts the last 300 mm: $8 + 7 = 15$ blows; medium dense.

10. Hard Conceptual. Why can a rise in groundwater cause a slope or foundation to fail even though nothing heavier has been added?

Show answer

Model answer: higher water raises the pore pressure and lowers the effective stress, which is what gives soil its friction and strength; the same load is now resisted by a weaker soil, and flowing water may also wash fine grains away.

11. Challenge Show that doubling the added stress does not double the consolidation settlement. Compare $\Delta\sigma = 50$ and 100 kPa for $\sigma\prime_0 = 100$ kPa.

Show answer

$\log_{10}(150/100) = 0.176$ and $\log_{10}(200/100) = 0.301$; ratio $0.301/0.176 \approx 1.71$, so the settlement rises by about 71%, not 100%, because clay stiffens as it is compressed.

COMMON MISTAKES

- Using the total unit weight below the water table without subtracting the pore pressure when computing effective stress.
- Assuming the soil at the surface is the same all the way down. Investigate the layers.
- Forgetting that clay settlement continues for a long time after construction.

> **Make it a game · Jar of Ground.** Fill a jar with soil, water and a pinch of dish soap, shake it, and let it settle. Predict the layers from bottom to top: gravel, sand, silt, clay. Sketch what you see in your notebook and label grain sizes. Then calculate effective stress at depth from total stress and pore pressure. Score for each correct layer.

> **Scripture.** “Where wast thou when I laid the foundations of the earth? declare, if thou hast understanding. Who hath laid the measures thereof, if thou knowest? or who hath stretched the line upon it? Whereupon are the foundations thereof fastened? or who laid the corner stone thereof;” (Job 38:4-6, KJV)
>
> God asks Job builder’s questions: measures, lines, foundations. Geotechnical engineers study, with humility, the ground God laid.

8.2 Shallow and Deep Foundations

OBJECTIVES

- Explain how spread footings, mats, piles and caissons carry loads into the ground, with Chicago's foundations as the case study.
- Size a spread footing for an allowable bearing pressure and estimate pile capacity from skin friction and end bearing.
- Test model footings and piles in sand and compare their capacities.

> **Warm-up.** Stand on soft ground or sand in sneakers, then on one foot, then on tiptoe. Which sinks most? Then lay a board down and stand on it. What did the board do for you?

WATCH

Why Buildings Need Foundations

Practical Engineering · 14:51

https://www.youtube.com/watch?v=0_KhihMIOG8

> **WHEN TO WATCH**
> After the core ideas. Pause at each foundation type and sketch it.

Note: Includes a sponsor segment in the middle and a brief comedic movie clip.

BACKUP VIDEO

Why Bridges Don't Sink

Practical Engineering · 17:30

https://www.youtube.com/watch?v=XpTs1V2NQ24

For deep foundations: how piles carry load by friction and end bearing, with a load-test demonstration.

KEY TERMS

spread footing

A wide pad of concrete under a column or wall that spreads its load over enough soil to keep the bearing pressure safe.

mat (raft) foundation

A thick slab under a whole building, spreading its load over the entire footprint.

pile

A long column of timber, steel or concrete driven or drilled into the ground to carry load to deeper, stronger layers.

skin friction and end bearing

The two ways a pile carries load: friction along its sides and pressure on its tip.

caisson (drilled shaft)

A large-diameter shaft dug or drilled to firm soil or rock and filled with concrete.

allowable bearing pressure

The pressure a soil can safely carry, set by the geotechnical engineer to limit both failure and settlement.

CORE IDEAS

The story. Chicago was built on a swamp. Beneath the city lie tens of feet of soft, wet clay over a firmer layer called hardpan and, much deeper, limestone bedrock. Its first heavy buildings settled visibly: the Auditorium Building (1889) sank more than two feet, and its lobby floor still slopes. In 1882 John Root, designing the ten-story Montauk Block, invented the "floating raft": a grillage of crossed steel rails and beams embedded in concrete, spreading each column's load over a wide area of clay. In the 1890s Chicago engineers turned to caissons, hand-dug wells about 4–8 ft across, lined with timber and dug down to hardpan or rock, then filled with concrete. Today the city's skyscrapers stand on drilled shafts that reach bedrock.

How they figured it out. Foundations have always been learned by settlement. Venice stands on millions of wooden piles driven into the lagoon's clay and sand, which have lasted for centuries because they stay waterlogged and away from air. Chicago's engineers measured how much their buildings settled and adjusted, sometimes building with the first floor set high so it would sink to the right level. Terzaghi's soil mechanics (Lesson 8.1) finally let engineers predict bearing capacity

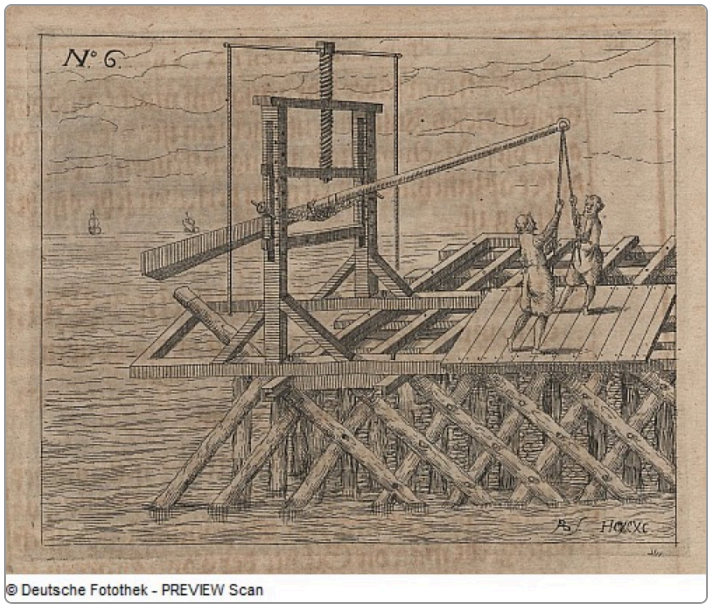
and settlement. Today they also load-test real piles on site, pushing or pulling them with jacks until they move, to confirm their capacity.

The physics. A **spread footing** must keep the soil pressure below the allowable bearing pressure: $q = \dfrac{P}{A} \le q_a$, so the area needed is $A = P/q_a$. A wider footing also stresses the soil deeper, so it may settle more on a clay layer below. A **pile** carries load by friction along its sides and pressure on its tip: $Q_u = f_s(\pi D L) + q_b\left(\dfrac{\pi D^2}{4}\right)$, where f_s is the unit skin friction and q_b the unit end bearing; engineers divide by a factor of safety of 2 to 3. Longer piles reach stronger soils and gain more friction. A **mat** spreads load over the whole footprint and can "float" a building by removing as much soil weight as the building adds.

The engineering. The geotechnical engineer chooses the foundation type: spread footings when good soil is near the surface, mats for heavy buildings on moderate soil, and piles or drilled shafts when the good layers are deep. Uneven settlement is often more dangerous than uniform settlement, because it bends and cracks the structure; so foundations are designed to settle evenly, and sensitive buildings are monitored. And every foundation must be protected from water: drains, waterproofing and careful grading keep the soil beneath dry and strong.

Try it. In the build you will press model footings into sand and pull model piles out of it, measuring their capacities.

PICTURE STUDY



A Pile Driver on a Timber Staging, Unknown woodcut artist, 1614 · Woodcut · Deutsche Fotothek, Saxon State and University Library, Dresden

Image: Wikimedia Commons, public domain

Pile drivers like this one, worked by men hauling ropes or turning winches, drove the timber piles that hold up bridges, quays and whole cities such as Venice and Amsterdam. This woodcut from a 1614 book of machines shows how it was done.

Look closely

- Find the heavy ram at the top of the frame. How is it lifted?
- Find the pile being driven. How do you think the workers keep it straight?
- Look at the trestle beneath the platform. How is it braced?
- Find the workers. How many people might it take to run this machine?

Talk about it

1. Why does each blow of the ram push the pile deeper, and why does each blow push it less as it goes deeper?
2. Why do timber piles last for centuries if they stay under water?
3. How would engineers today check how much load a pile can carry?

Safety first. Wear safety glasses when dropping weights onto model piles, and keep fingers clear of the drop.

BUILD

Footings and Piles in a Sandbox

You will test square footings of different sizes on sand to see how area affects capacity, and model piles of different lengths to see how skin friction adds up with depth.

You will need

- A bucket or box of damp sand at least 25 cm deep
- Square wooden or plastic plates 3 cm, 5 cm and 7 cm on a side as footings
- Wooden dowels or pencils as piles, with a small hook or eye screw in the top
- Weights (a bucket of water or books), a spring scale or luggage scale, a ruler fixed to the box

Steps

1. Prepare the sand the same way for every test: fill, tamp lightly in layers and level.
2. **Footings.** Place the 3 cm plate on the sand and load it until it sinks 1 cm. Record the load. Repeat with the 5 cm and 7 cm plates.
3. Compute the bearing pressure at 1 cm settlement for each. **Predict** first: will the pressure stay the same as the plates get bigger?
4. **Piles.** Push dowels into the sand to depths of 5, 10 and 15 cm. Pull each straight up with the spring scale and record the peak force (this measures skin friction).
5. Graph pull-out force against embedment depth. Is it a straight line, or does it curve upward (friction grows with depth because the soil pressure grows)?
6. Roughen a dowel with sandpaper or glue sand to it and repeat the 15 cm test. Compare.
7. **Pile group.** Join four dowels with a card pile cap, push them in 10 cm, and pull the group out. Is the group four times as strong as one pile?
8. **Improve.** Design the strongest foundation for a 2 kg load using no more than four dowels and one plate, and test it.

Tips

- Keep the sand's moisture the same for every test.
- Pull straight up, slowly and steadily, and read the peak.
- Mark depths on the dowels before pushing them in.

Check your work

☐ My footing tests show how bearing capacity changes with footing size.

☐ My pile tests show how skin friction grows with depth.

☐ I can size a footing and estimate a pile's capacity.

ENGINEER'S NOTEBOOK

1. Sketches of a spread footing, a mat, a pile group with cap and a drilled shaft, with arrows showing how each carries load.
2. Your footing and pile data tables and graphs.
3. A design calculation: footing size and number of piles for a column load you choose, with the soil values from Example 2.

KEY FORMULAS

$$A_{\text{footing}} = \frac{P}{q_a}$$

$$Q_u = f_s(\pi D L) + q_b \frac{\pi D^2}{4}$$

$$Q_a = \frac{Q_u}{FS}, \quad n_{\text{piles}} = \left\lceil \frac{P}{Q_a} \right\rceil$$

From history. When the Auditorium Building settled unevenly, the architects Adler and Sullivan had foreseen some of it: they had preloaded the foundations under one heavy part with pig iron and bricks during construction, to make it settle in step with the rest. It was only partly successful, and the building still shows its tilt, but the idea of preloading is used to this day.

Example 1. A column carries 1,800 kN (service load). The allowable bearing pressure is 150 kPa. Size a square spread footing (ignore its own weight).

Show solution

1. Area: $A = P/q_a = 1800 \div 150 = 12.0\ \text{m}^2$.
2. Side: $\sqrt{12.0} \approx 3.46$ m; use a 3.5 m square footing.
3. Check: $1800 \div 3.5^2 = 1800 \div 12.25 \approx 147\ \text{kPa} \leq 150\ \text{kPa}$. ✓

Answer: A 3.5 m × 3.5 m footing.

Example 2. A concrete pile 0.40 m in diameter and 15 m long is driven into clay with an average skin friction of 40 kPa and reaches sand with an end bearing of 2,000 kPa. Find its ultimate and allowable capacity (FS = 2.5) and the number of piles needed for the column in Example 1.

Show solution

1. Skin friction: $f_s(\pi DL) = 40 \times \pi(0.40)(15) \approx 754$ kN.
2. End bearing: $q_b(\pi D^2/4) = 2000 \times \pi(0.40)^2/4 \approx 251$ kN.
3. Ultimate: $Q_u \approx 1005$ kN; allowable: $1005 \div 2.5 \approx 402$ kN.
4. Piles: $1800 \div 402 \approx 4.5$, so 5 piles under a concrete pile cap.

Answer: About 1,005 kN ultimate, 402 kN allowable; 5 piles.

Example 3. A building weighs 120 kPa over its footprint. It has a two-level basement 7 m deep in soil weighing 18 kN/m^3. How much net pressure does the building add to the soil at the base of the basement?

Show solution

1. Soil removed: $18 \times 7 = 126$ kPa of overburden.
2. Net added pressure: $120 - 126 = -6$ kPa.
3. The building weighs slightly less than the soil dug out, so it “floats”: the clay below feels almost no new load and will hardly settle. This is a compensated (floating) mat foundation.

Answer: About −6 kPa: essentially no net added load.

PRACTICE

1. Easy When would an engineer choose piles instead of spread footings?

   Show answer

   When the soil near the surface is too weak or compressible and firm soil or rock lies deep.

2. Easy A wall footing carries 90 kN per meter on soil allowing 150 kPa. How wide must it be?

   Show answer

   $90/150 = 0.6$ m.

3. Easy What are the two ways a pile carries load?

   Show answer

   Skin friction along its sides and end bearing at its tip.

4. Medium A column carries 2,400 kN on soil allowing 200 kPa. Size a square footing.

   Show answer

   $A = 12\ \text{m}^2$; side ≈ 3.46, use 3.5 m.

5. Medium A steel pipe pile 0.30 m in diameter and 20 m long has $f_s = 30$ kPa and $q_b = 3000$ kPa. Find its ultimate capacity.

   Show answer

   Skin $30 \times \pi(0.30)(20) \approx 565$ kN; end $3000 \times \pi(0.3)^2/4 \approx 212$ kN; total ≈ 777 kN.

6. Medium For Problem 5, with FS = 3, how many piles carry 2,000 kN?

   Show answer

   Allowable ≈ 259 kN; $2000/259 \approx 7.7$, so 8 piles.

7. Medium Why are timber piles in Venice still sound after centuries?

   Show answer

   They are always submerged in waterlogged soil without oxygen, so wood-rotting fungi cannot grow.

8. Hard A building on a mat weighs 150 kPa. How deep a basement (soil 19 kN/m^3) would make it fully compensated?

Show answer

$150/19 \approx 7.9$ m.

9. Hard Two columns 6 m apart sit on separate footings. One settles 25 mm and the other 10 mm. Find the angular distortion (differential settlement ÷ spacing). A common limit is 1/300. Does it pass?

Show answer

$15 \text{ mm}/6000 \text{ mm} = 1/400$; it passes 1/300, but cracking of finishes is possible.

10. Hard Conceptual. Why is uneven settlement more harmful than even settlement?

Show answer

Model answer: if a building sinks evenly, it moves as a whole and its frame is not bent; uneven settlement distorts the frame, bending beams and walls, cracking masonry and glass, and tilting floors (as at Pisa).

11. Challenge A pile's skin friction in sand grows roughly with depth: $f_s = kz$. Show that the total friction on a pile of length L grows as L^2, and find how much more friction a 20 m pile has than a 10 m pile.

Show answer

Friction $= \int_0^L kz(\pi D)\,dz = \frac{1}{2}\pi D k L^2$, so it grows as L^2; a 20 m pile has 4 times the friction of a 10 m pile.

COMMON MISTAKES

- Forgetting that a bigger footing stresses the soil deeper and can increase settlement on a clay layer below.
- Adding up pile capacities in a closely spaced group without checking the group as a whole.
- Ignoring groundwater and drainage around foundations.

> **Make it a game · Footing Face-Off.** Sandbox duel. Press a flat lid and a narrow pencil into a bowl of sand with the same weight on top, and predict which sinks. Explain spread footings, piles, skin friction and end bearing. Then size a footing: load divided by allowable pressure. Closest estimate wins. Swap and reset the sand.

> **Scripture.** “He is like a man which built an house, and digged deep, and laid the foundation on a rock: and when the flood arose, the stream beat vehemently upon that house, and could not shake it: for it was founded upon a rock.” (Luke 6:48, KJV)
>
> In Luke’s telling the wise builder digged deep. Chicago’s engineers learned the same lesson: when the good ground is deep, the foundation must go down to it.

8.3 The Leaning Tower of Pisa

OBJECTIVES

- Explain why the Tower of Pisa began to lean, why it did not fall, and how engineers stabilized it in 1999–2001.
- Calculate the eccentricity of the tower's weight and the bearing pressure under its foundation, and the effect of reducing the tilt.
- Model a leaning tower on soft ground and test ways to reduce its lean.

> **Warm-up.** Stack coins or checkers into a tower on a soft surface (a folded towel or a cushion). Press gently on one side of the base. Now add coins one at a time. When does it begin to lean more? When does it fall?

WATCH

How Engineers Straightened the Leaning Tower of Pisa

Practical Engineering · 13:16

https://www.youtube.com/watch?v=0ZhHoyqQEhA

> **WHEN TO WATCH**
> After the story. Note why the counterweights helped and why soil extraction worked better than pushing on the tower.

BACKUP VIDEO

Civil engineering: An oral history part 2 - Propping up Pisa

Institution of Civil Engineers · 4:13

https://www.youtube.com/watch?v=04MbizZtHak

After the main video: Professor John Burland, who led the soil work, explains the idea with a model on his desk.

KEY TERMS

differential settlement

Settlement that is greater on one side of a structure than the other, causing it to tilt.

eccentricity

The horizontal distance between a structure's center of gravity and the center of its foundation; it creates a moment.

leaning instability

A condition in which a tall structure on soft ground leans more because its own lean shifts load to the soft side, which settles more, which increases the lean.

soil extraction (underexcavation)

Removing small amounts of soil from beneath the high side of a foundation so it settles slightly and the lean decreases.

kern (middle third)

The central zone of a foundation within which the resultant load must stay for the whole base to remain in compression.

CORE IDEAS

The story. In August 1173, builders in Pisa began a white marble bell tower beside their cathedral. By 1178, with three stories done, it had begun to lean. Work stopped for nearly a century, partly because of wars, and that pause probably saved it: the soft clay beneath had time to consolidate and grow stronger. When building resumed, the masons curved the upper stories slightly back toward vertical, which is why the tower is gently banana-shaped. The bell chamber was finished around 1372. Over the next six centuries the lean slowly increased, reaching about 5.5° by 1990, with the top overhanging the base by about 4.5 m. That year the tower was closed to visitors for fear it would fall.

How they figured it out. An international committee of engineers studied the tower for over a decade. Borings showed the reason for the lean: beneath the tower lies a layer of soft silty sand and clay over a deep layer of soft marine clay, a little softer on the south side. The tower was close to **leaning instability**: as it leaned, more of its weight shifted south, which made the south side settle more, which increased the lean. In 1993 the engineers placed 600 tonnes (later about 900 tonnes) of lead weights on the north side of the foundation as a temporary fix; the tower straightened slightly. Then, from 1999 to 2001, under the geotechnical engineer John Burland, they slowly drilled out small amounts of soil from beneath the north side with inclined drills. The ground there settled a little, and the tower rotated back about half a degree, to about 4°, the lean it had had in the early 1800s. It reopened in December 2001 and should be stable for at least two centuries.

The physics. When a tower of weight P leans by angle θ, its center of gravity at height h_g shifts sideways by $e = h_g \sin\theta$. The foundation then carries the vertical load plus a moment $M = Pe$. For a circular foundation of diameter D, the bearing pressure varies from $q_{\max} = \dfrac{P}{A} + \dfrac{M}{S}$ on the low side to $q_{\min} = \dfrac{P}{A} - \dfrac{M}{S}$ on the high side, where $A = \pi D^2/4$ and $S = \pi D^3/32$. As the lean grows, the pressure on the low side climbs and the soil there settles more: the feedback that drives leaning instability. If e ever reached $D/8$ (the edge of the kern for a circle), the high side would lift off the ground entirely.

The engineering. Pisa teaches several lessons: investigate the soil before building; tall, heavy, narrow structures on soft ground are dangerous; slow construction lets clay gain strength; and when repairing a fragile structure, the gentlest method that works is best. Pushing or pulling on the tower risked cracking its marble walls, which were already highly stressed; letting gravity do the work by removing soil from beneath the high side moved the tower without touching it. The engineers also monitored every millimeter of movement as they worked, adjusting as they went, the observational method that Terzaghi taught.

Try it. In the build you will set a model tower on soft foam or sand, make it lean, and try counterweights and soil extraction to straighten it.

PICTURE STUDY

The Campanile and Cathedral of Pisa, Unknown photographer, 1860s · Albumen photograph · Bibliothèque nationale de France

Image: Wikimedia Commons, public domain

This photograph was taken about 150 years ago, when the tower leaned a little less than it did in 1990. Look at the tower as a geotechnical engineer: the lean, the shape, the ground beneath it.

Look closely

- Use a ruler on the picture to estimate the angle of the lean.
- Look at the upper stories. Do they lean at the same angle as the lower ones, or curve back slightly?
- Find the base of the tower. Is it sunk below the level of the lawn?
- Compare the tower with the cathedral. Does the cathedral show any lean?

Talk about it

1. Why might the masons have curved the upper stories?
2. Why did a pause in construction of nearly a century help the tower survive?
3. If you could investigate the ground under the tower, what would you test?

Safety first. Wear safety glasses for the tower tests. Keep the model tower low (under 50 cm) and on the floor so a fall does no harm.

BUILD

Leaning Tower on Soft Ground

You will build a heavy model tower on a soft foundation, make it lean by uneven settlement, measure how the lean grows as you add weight, and then try counterweights and soil extraction to reduce it.

You will need

- A tube (a paper-towel roll or plastic bottle) filled partly with sand as the tower, with a round card base
- A shallow box of damp sand or a sheet of soft foam with one half softer (for example a thinner foam or looser sand)
- Coins or washers for added weight and counterweights; a protractor and a plumb line (thread and a nut)
- A drinking straw or thin tube for "soil extraction" and a spoon

Steps

1. Prepare the soft ground with one side slightly softer than the other (loosen the sand on one side).
2. Stand the tower on its base in the middle. Hang the plumb line from the top and measure the lean with the protractor.
3. **Predict.** As you add weight inside the tower, will the lean grow evenly or faster and faster?
4. Add weight in steps and measure the lean after each step. Graph lean against weight.
5. **Counterweight.** Place coins on the base on the high side. Measure the change in lean.
6. **Soil extraction.** Using the straw, gently remove small amounts of sand from beneath the high side of the base, a little at a time, measuring after each removal.
7. Compute the eccentricity of your tower's center of gravity at its greatest lean, and its ratio to the base diameter. How close was it to $D/8$?

8. **Improve.** Rebuild the tower on a wider base, or with a stiffer (compacted) foundation, and repeat the loading. Compare the final leans.

Tips

- Add weight gently; jolts make the tower lurch.
- Remove sand in tiny amounts; it is easy to overdo.
- Record everything, including what made the lean jump suddenly.

Check your work

- ☐ My graph shows how the lean grew with load.
- ☐ I tested counterweights and soil extraction and measured their effects.
- ☐ I can calculate eccentricity and bearing pressure for a leaning tower.

ENGINEER'S NOTEBOOK

1. A section through the Tower of Pisa and the soil layers beneath it, with the center of gravity, its eccentricity and the bearing pressure diagram.
2. Your lean-versus-weight data and graph, and the effects of counterweight and extraction.
3. A paragraph on leaning instability: why a lean can feed on itself.

KEY FORMULAS

$e = h_g \sin\theta, \quad M = Pe$

$q = \dfrac{P}{A} \pm \dfrac{M}{S}, \quad A = \dfrac{\pi D^2}{4}, \quad S = \dfrac{\pi D^3}{32}$ (circle)

Kern of a circular base: $e \le \dfrac{D}{8}$

From history. A famous story says that Galileo dropped balls of different weights from the tower to show they fall together. He may never have done it; the story comes from a biography written by his student Vincenzo Viviani years later. But the tower is a fitting place for the legend, because it is one of the world's great lessons in gravity and the ground.

Example 1. The Tower of Pisa weighs about 14,500 tonnes and stands on a ring foundation about 19.6 m in diameter (treat it as a full circle). Its center of gravity is about 22.6 m above the foundation. In 1990 it leaned 5.5°. Find the eccentricity, the moment and the bearing pressures on the low and high sides.

Show solution

1. Weight: $P = 14{,}500 \times 9.81 \approx 142{,}000$ kN.
2. Eccentricity: $e = 22.6 \sin 5.5^\circ \approx 2.17$ m; moment $M = Pe \approx 308{,}000$ kN·m.
3. $A = \pi(19.6)^2/4 \approx 302\ \mathrm{m}^2$; $S = \pi(19.6)^3/32 \approx 739\ \mathrm{m}^3$.
4. $P/A \approx 471$ kPa; $M/S \approx 417$ kPa.
5. $q_{\max} \approx 471 + 417 = 888$ kPa on the south (low) side; $q_{\min} \approx 471 - 417 = 55$ kPa on the north side.

Answer: $e \approx 2.2$ m; about 890 kPa on the low side and 55 kPa on the high side.

Example 2. After soil extraction, the lean was reduced to about 3.97°. Recompute the bearing pressures.

Show solution

1. $e = 22.6 \sin 3.97^\circ \approx 1.56$ m; $M \approx 142{,}000 \times 1.56 \approx 222{,}000$ kN·m.
2. $M/S \approx 222{,}000 \div 739 \approx 301$ kPa.
3. $q_{\max} \approx 471 + 301 = 772$ kPa; $q_{\min} \approx 471 - 301 = 170$ kPa.
4. The pressure on the soft south side fell by more than 100 kPa, moving the tower back from the edge of instability.

Answer: About 770 kPa and 170 kPa.

Example 3. How far could the center of gravity shift before the north edge of a 19.6 m circular base would carry no pressure at all? What lean is that for $h_g = 22.6$ m?

Show solution

1. The kern limit for a circle: $e = D/8 = 19.6/8 = 2.45$ m.
2. $\sin\theta = 2.45/22.6 \approx 0.108$, so $\theta \approx 6.2^\circ$.
3. By 1990 the tower, at 5.5°, was uncomfortably close (and the soil would likely have failed on the south side before that).

Answer: About 2.45 m, a lean of about 6.2°.

PRACTICE

1. Easy When did the Tower of Pisa start to lean, and why?

   Show answer

   About 1178, when three stories were built, because it stood on soft, uneven soil (silty sand and clay over soft marine clay), slightly softer on the south side.

2. Easy What did engineers do in 1999–2001 to reduce the lean?

   Show answer

   They carefully extracted soil from beneath the north (high) side with inclined drills, so it settled slightly and the tower rotated back about half a degree.

3. Easy Find the eccentricity of a center of gravity 30 m high in a tower leaning 3°.

   Show answer

   $30 \sin 3^\circ \approx 1.57$ m.

4. Medium Find A and S for a circular foundation 12 m in diameter.

   Show answer

   $A = \pi(12)^2/4 \approx 113.1\ \text{m}^2$; $S = \pi(12)^3/32 \approx 169.6\ \text{m}^3$.

5. Medium A 20,000 kN tower on the foundation of Problem 4 has an eccentricity of 0.8 m. Find $q_{\max}$ and $q_{\min}$.

   Show answer

   $P/A \approx 176.8$; $M/S = 16{,}000/169.6 \approx 94.3$; $q_{\max} \approx 271$, $q_{\min} \approx 82.5$ kPa.

6. Medium What is the kern limit for the foundation in Problem 4?

Show answer

$D/8 = 1.5$ m.

7. Medium Why did the long pause in construction help the tower?

Show answer

The clay had time to consolidate under the first stories' weight, gaining strength before more load was added.

8. Hard Pisa's lead counterweight of 900 tonnes was placed about 7 m north of the tower's center. What counter-moment did it provide? Compare it with the 1990 overturning moment of about 308,000 kN·m.

Show answer

$900 \times 9.81 \times 7 \approx 61{,}800$ kN·m, about 20% of the overturning moment: enough to start the tower moving back, though not to straighten it.

9. Hard Explain the feedback in leaning instability as a chain of causes.

Show answer

More lean → center of gravity moves toward the low side → more pressure on the low side → more settlement there → more lean; if the soil is soft enough, the chain does not stop until the tower falls.

10. Hard Conceptual. Why didn't the engineers simply push the tower upright with cables or jacks?

Show answer

Model answer: its marble walls were already highly stressed and cracked; forces applied to the tower could have caused a sudden collapse, and its soft foundation could have shifted unpredictably. Removing soil let gravity rotate the tower slowly and gently while they monitored it.

11. Challenge For a circular foundation, show that $q_{\min} = 0$ when $e = D/8$, using $q = P/A - Pe/S$.

Show answer

$P/A = Pe/S$ gives $e = S/A = (\pi D^3/32)/(\pi D^2/4) = D/8$.

COMMON MISTAKES

- Thinking the tower leans because it was built crooked. It was built straight; the soil settled unevenly.
- Forgetting that the lean increases the pressure on the low side, which increases the lean.
- Using the rectangular kern ($B/6$) for a circular foundation; for a circle it is $D/8$.

> **Make it a game · Lean and Stabilize.** Stack coins or blocks into a tower on a pillow, and watch it lean. Explain why Pisa leaned and why it did not fall. Predict when the weight leaves the kern. Then recall the 1999–2001 fix: underexcavation. Your parent plays a skeptical reporter and asks three questions. Answer them all for a point.

8.4 Retaining Walls and Slopes

OBJECTIVES

- Explain how soil pushes on a retaining wall (Coulomb and Rankine) and why water and drainage matter so much.
- Calculate active earth pressure and check a gravity wall for overturning and sliding.
- Measure the push of soil on a model wall and test how slope angle and water affect stability.

> **Warm-up.** Pour dry sand into a pile against a book standing on its edge. What does the sand do to the book? Now try damp sand, then very wet sand. Which pushes the book over most easily?

WATCH

Why Retaining Walls Collapse

Practical Engineering · 12:50

https://www.youtube.com/watch?v=--DKkzWVh-E

> **WHEN TO WATCH**
> After the physics paragraph. Note every way he shows a wall can fail, and the role of drainage.

Note: Includes a short sponsor message.

KEY TERMS

retaining wall
A wall that holds back soil where the ground level changes.

active earth pressure
The sideways pressure soil exerts when the wall moves slightly away from it; $p = K_a \gamma z$.

angle of internal friction (ϕ)
A measure of a soil's shear strength; for loose sand about 30°, dense sand up to about 40°.

gravity wall
A retaining wall that resists the soil's push by its own weight.

weep holes and drains

Openings and gravel drains that let water out from behind a wall so it cannot build up pressure.

factor of safety against sliding and overturning

Resisting force (or moment) divided by driving force (or moment); typically at least 1.5 for sliding and 2.0 for overturning.

CORE IDEAS

The story. In 1773 a young French army engineer, Charles-Augustin de Coulomb, read a paper to the French Academy of Sciences. For years he had been building the stone walls of a fort on the island of Martinique in the Caribbean, and he had wondered how thick such walls really needed to be. He imagined a wedge of soil behind the wall sliding down along a plane, held back by friction and by the wall, and he found the plane that would produce the greatest push. It was the first rational theory of earth pressure, and it still bears his name. Coulomb later became famous for his law of electric charges. In 1857 the Scottish engineer William Rankine, a professor at Glasgow, published a simpler theory based on the stresses within the soil, and engineers use both today.

How they figured it out. Coulomb's method was to look for the worst case: of all the wedges of soil that might slide, which one pushes hardest on the wall? That idea, testing every possible failure and designing for the worst, is still the heart of geotechnical design. Rankine showed that for a smooth vertical wall and level soil, the sideways pressure is a fixed fraction K_a of the vertical stress. Experience then taught engineers the rest: walls fail most often not because the soil is heavier than expected, but because water builds up behind them. Saturated soil pushes harder, and water itself pushes with its full weight sideways.

The physics. For dry soil of unit weight γ and friction angle ϕ, Rankine's active pressure coefficient is $K_a = \dfrac{1-\sin\phi}{1+\sin\phi} = \tan^2\!\left(45^\circ - \dfrac{\phi}{2}\right)$. For $\phi = 30^\circ$, $K_a = \frac{1}{3}$: the soil pushes sideways with one-third of its vertical stress. The pressure grows linearly with depth, so the total force on a wall of height H is $P_a = \frac{1}{2}K_a\gamma H^2$, acting at $H/3$ above the base. Water, by contrast, pushes sideways with its full pressure ($K = 1$): $P_w = \frac{1}{2}\gamma_w H^2$, which for the same height is larger than the dry soil's push. A slope of dry sand stands as long as its angle is less than about ϕ; its factor of safety is roughly $FS = \tan\phi / \tan\beta$ for a slope angle β.

The engineering. A gravity wall is checked for **overturning** (the moment of the soil's push about the toe against the moment of the wall's weight), **sliding** (the push against the friction under the base), and **bearing** on the soil beneath. Modern walls include cantilever walls of reinforced concrete, walls of soil reinforced with layers of geogrid (mechanically stabilized earth), sheet-pile and soldier-pile walls, and soil nails. Every one of them needs drainage: gravel behind the wall, a perforated drain pipe at its base and weep holes through it. A wall without drainage is a dam that was never meant to hold water.

Try it. In the build you will measure the push of dry and wet sand on a hinged model wall and test slopes to find their angle of failure.

PICTURE STUDY

The Great Retaining Wall on Grandfather Mountain, Blue Ridge Parkway, Historic American Engineering Record, date not recorded · Photograph · Library of Congress, Prints and Photographs Division

Image: Wikimedia Commons, public domain

Road builders on the Blue Ridge Parkway in North Carolina called this massive stone wall the "Great Wall of China." It holds up the roadway where it was cut into a steep mountainside. HAER photographed it as part of documenting the parkway's engineering.

Look closely

- Find the wall's face. Is it vertical or does it lean back into the hill (batter)?
- Look at the size and shape of the stones. How are they laid?
- Look for openings near the base that might be weep holes.
- Compare the height of the wall with the slope below it.

Talk about it

1. Why would a wall lean back into the soil it holds up?
2. What would happen to this wall if heavy rain could not drain out from behind it?
3. Why might the builders have chosen stone here instead of concrete?

Safety first. Wear safety glasses. Keep the sand box on the floor, and clean up wet sand promptly so no one slips.

BUILD

The Push of Soil

You will build a hinged model wall in a box, fill sand behind it, and measure the force needed to hold the wall upright, comparing dry, damp and saturated sand. Then you will test slopes of dry sand to find their failure angle.

You will need

- A clear plastic storage box or a wooden box with one removable side
- A stiff board as the wall, hinged at its base with tape, and a string tied to its top running horizontally over the box edge to a spring scale or a cup of pennies
- Dry sand, a watering can, a ruler and a protractor
- Gravel or aquarium stones and a strip of cloth for a drain

Steps

1. Stand the wall upright at one end of the box, hinged at the bottom, with the string pulled taut to hold it.
2. Fill sand behind it in 5 cm layers to a height H. After each layer, record the pull needed to keep the wall vertical (the string force).
3. **Predict** using $P_a = \frac{1}{2} K_a \gamma H^2$ and moments about the hinge: how should the string force grow with H?
4. Graph string force against H^2. Is it close to a straight line?
5. Wet the sand slowly from the top until water stands just below the surface, keeping the wall vertical. Record the new force.
6. Empty, then rebuild with a 3 cm gravel drain and a cloth filter against the wall and a gap at the bottom to let water out. Wet the sand again. Record the force.
7. **Slopes.** Pile dry sand into a long slope and steepen it until it slides. Measure the angle. Repeat with damp sand.
8. **Improve.** Design a small gravity wall of blocks that holds back 10 cm of damp sand without the string. Add a drain and test it with water.

Tips

- Fill the sand gently in layers; do not drop it from high up.
- Measure H from the hinge to the sand surface.
- Damp sand can stand steeper than dry sand for a while, because of surface tension; that strength vanishes when it dries or floods.

Check your work

☐ My data show the push of soil growing with the square of its height.

☐ I measured how water increased the push and how a drain reduced it.

☐ I can calculate

K_a,P_a

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ENGINEER'S NOTEBOOK

1. A section through a gravity wall and a cantilever wall showing the soil wedge, the pressure diagram, the resultant at $H/3$, the drain and weep holes.
2. Your force data and graph against H^2, with dry, wet and drained results.

3. Your slope-angle measurements and a comparison with typical values of ϕ.

KEY FORMULAS

$$K_a = \frac{1 - \sin\phi}{1 + \sin\phi}$$

$P_a = \frac{1}{2} K_a \gamma H^2$ (acting at $H/3$)

$$P_w = \frac{1}{2}\gamma_w H^2$$

$$FS_{\text{OT}} = \frac{\sum M_R}{\sum M_O}, \quad FS_{\text{slide}} = \frac{\mu W}{P_a}, \quad FS_{\text{slope}} \approx \frac{\tan\phi}{\tan\beta}$$

From history. Rankine also wrote one of the first engineering textbooks in English, *A Manual of Applied Mechanics* (1858), and he was one of the founders of thermodynamics; the Rankine temperature scale is named after him. He liked to write comic songs, too, and published a book of them.

WORKED EXAMPLES

Example 1. A wall 3.0 m high retains dry sand with $\gamma = 18\ \text{kN/m}^3$ and $\phi = 30^\circ$. Find K_a, the active force per meter of wall and where it acts.

Show solution

1. $K_a = (1 - \sin 30^\circ)/(1 + \sin 30^\circ) = 0.5/1.5 = 1/3$.
2. $P_a = \frac{1}{2}(1/3)(18)(3.0)^2 = \frac{1}{2}(1/3)(18)(9) = 27\ \text{kN/m}$.
3. It acts at $H/3 = 1.0$ m above the base.

Answer: $K_a = 1/3$; 27 kN per meter at 1.0 m above the base.

Example 2. The wall in Example 1 is a concrete gravity wall 3.0 m high and 1.2 m thick (rectangular, 24 kN/m^3). Check overturning about the toe and sliding with a base friction coefficient of 0.5.

Show solution

1. Weight: $W = 3.0 \times 1.2 \times 24 = 86.4$ kN/m, acting 0.6 m from the toe.
2. Overturning: $M_R = 86.4 \times 0.6 \approx 51.8$ kN·m; $M_O = 27 \times 1.0 = 27$ kN·m; $FS_{\text{OT}} \approx 1.92$, a little under 2.0.
3. Sliding: $FS = \mu W / P_a = 0.5 \times 86.4/27 = 1.6$, above 1.5.
4. The wall is close on overturning; widening its base or adding a heel that soil sits on would bring it above 2.0.

Answer: $FS_{\text{OT}} \approx 1.9$ (marginal); $FS_{\text{slide}} = 1.6$ (acceptable).

Example 3. Suppose the drains behind the wall clog and water rises to the top of the soil. Estimate the added water force, and explain what happens.

Show solution

1. Water force: $P_w = \frac{1}{2}(9.81)(3.0)^2 \approx 44.1$ kN/m, acting at 1.0 m.
2. The soil, now submerged, pushes with its buoyant weight (about $20 - 9.81 \approx 10$ kN/m^3), about $\frac{1}{2}(1/3)(10)(9) = 15$ kN/m.
3. Total push $\approx 44 + 15 = 59$ kN/m, more than double the dry 27 kN/m. The overturning factor falls to about $51.8/59 \approx 0.88$: the wall would tip over.

Answer: About 44 kN/m of water force; the total push more than doubles and the wall fails.

PRACTICE

1. Easy Find K_a for $\phi = 36^\circ$.

   Show answer

   $(1 - 0.588)/(1 + 0.588) \approx 0.26$.

2. Easy Where does the resultant of a triangular earth pressure act on a wall of height H?

   Show answer

   At $H/3$ above the base.

3. **Easy** Why do retaining walls need weep holes?

Show answer

To let water drain from behind them so it cannot build up and add its full pressure to the soil's.

4. **Medium** A wall 4 m high retains soil with $\gamma = 19\ \mathrm{kN/m^3}$ and $K_a = 0.30$. Find P_a.

Show answer

$\frac{1}{2}(0.30)(19)(16) = 45.6\ \mathrm{kN/m}$.

5. **Medium** Find the water force on the same 4 m wall if water stood to its top.

Show answer

$\frac{1}{2}(9.81)(16) \approx 78.5\ \mathrm{kN/m}$.

6. **Medium** A dry sand slope with $\phi = 32^\circ$ is cut at 25°. Find its factor of safety.

Show answer

$\tan 32^\circ / \tan 25^\circ \approx 0.625/0.466 \approx 1.34$.

7. **Medium** A gravity wall weighs 120 kN/m and is pushed by 40 kN/m. With $\mu = 0.55$, find the sliding factor of safety.

Show answer

$0.55 \times 120/40 = 1.65$.

8. **Hard** A concrete gravity wall 2.5 m high retains soil with $\gamma = 18$, $K_a = 1/3$. How thick must it be (rectangular, 24 kN/m³) for $FS_{\mathrm{OT}} = 2.0$?

Show answer

$P_a = \frac{1}{2}(1/3)(18)(6.25) = 18.75$ kN/m at 0.833 m: $M_O = 15.6$. Need $M_R = 31.25 = (2.5 \times t \times 24)(t/2) = 30t^2$, so $t \approx 1.02$ m.

9. **Hard** If the height of a wall doubles, by what factor does the overturning moment from earth pressure increase?

Show answer

Force goes as H^2 and its lever arm as H, so the moment goes as H^3: 8 times.

10. Hard Conceptual. Why did Coulomb look for the soil wedge that pushes hardest, instead of just one likely wedge?

Show answer

Model answer: the soil will fail along whichever plane is weakest, producing the greatest push; designing for that worst case ensures the wall is safe against every possible failure, a principle still used throughout geotechnical design.

11. Challenge Show that $\frac{1-\sin\phi}{1+\sin\phi} = \tan^2\left(45^\circ - \frac{\phi}{2}\right)$ for $\phi = 30^\circ$ by computing both sides.

Show answer

Left: $0.5/1.5 = 0.333$. Right: $\tan^2(30^\circ) = (0.5774)^2 = 0.333$. ✓

COMMON MISTAKES

- Placing the earth pressure resultant at mid-height instead of $H/3$.
- Forgetting water. Most retaining walls that fail do so because of water pressure behind them.
- Using K_a for water. Water pushes sideways with its full pressure ($K = 1$).

Make it a game · Overturn or Slide?. Place a book against a pile of dry rice, like soil against a retaining wall. Predict whether a heavier book resists overturning or sliding, then test. Calculate the factor of safety against each failure for a made-up wall. Your parent plays inspector and asks why drainage matters. Score a point for each right answer.

8.5 Dams: Hoover Dam

OBJECTIVES

- Tell how Hoover Dam was built (1931–1936) and explain how gravity, arch and arch-gravity dams hold back water.
- Calculate hydrostatic pressure and force on a dam and check a gravity dam against sliding.
- Build a model dam, measure water pressure with depth, and test the effect of shape and seepage.

> **Warm-up.** Push a cup upside down into a bowl of water. Feel the water push back. Push it deeper. Does the push increase? Hold your hand flat at the bottom of a full bathtub or sink and then near the top. Where is the pressure greater?

WATCH

Hoover Dam | All the Secrets of the Engineering Wonder

Sabin Civil Engineering · 9:47

https://www.youtube.com/watch?v=hopBzK6BOwo

> **WHEN TO WATCH**
> After the story. Watch how the arch shape and the gravity shape work together, and how the concrete was cooled.

KEY TERMS

hydrostatic pressure
Water pressure, which increases linearly with depth: $p = \gamma_w h$.

gravity dam
A dam that resists the water by its own weight, usually triangular in section.

arch dam
A dam curved upstream that carries the water's push sideways into the canyon walls, like an arch lying on its side.

arch-gravity dam
A dam that combines both actions, such as Hoover Dam.

uplift

Water pressure under a dam's base pushing it upward, reducing its effective weight; controlled with drains and grout curtains.

heat of hydration

Heat released as cement reacts with water; in a huge concrete mass it must be removed or the concrete cracks.

CORE IDEAS

The story. The Colorado River flooded farms in its lower valley in some years and ran nearly dry in others. In 1928 Congress authorized a great dam in Black Canyon on the Arizona–Nevada border, to control floods, store water for irrigation and cities, and make electricity. The U.S. Bureau of Reclamation designed it, and a group of contractors called Six Companies built it, starting in 1931. First, four tunnels 56 ft in diameter were blasted through the canyon walls to carry the river around the site, and cofferdams were built to keep the work dry. Then the canyon floor was dug down to bedrock, and concrete was placed day and night in a grid of blocks. The dam was finished in 1936, two years ahead of schedule. It stands 726 ft (221 m) high, 660 ft (201 m) thick at its base and 45 ft (14 m) thick at its crest.

How they figured it out. The Bureau's engineers designed Hoover Dam as an arch-gravity dam: a massive wedge curved upstream, so that its own weight and its arching into the canyon walls both resist the water. To check the design, they built and tested scale models and analyzed the dam with a method called trial-load analysis, dividing it into horizontal arches and vertical cantilevers and adjusting the share each carried until their movements matched. The biggest construction problem was heat. As cement hardens it gives off heat; the engineers calculated that a single monolithic pour of this size would take over a century to cool, shrinking and cracking as it did. So they built the dam as columns of separate blocks, each placed in 5 ft lifts, and ran 582 miles of 1-inch steel pipe through the concrete, carrying river water and then refrigerated water to draw the heat out. When the blocks had cooled and shrunk, the joints between them were filled with grout, making the dam one piece.

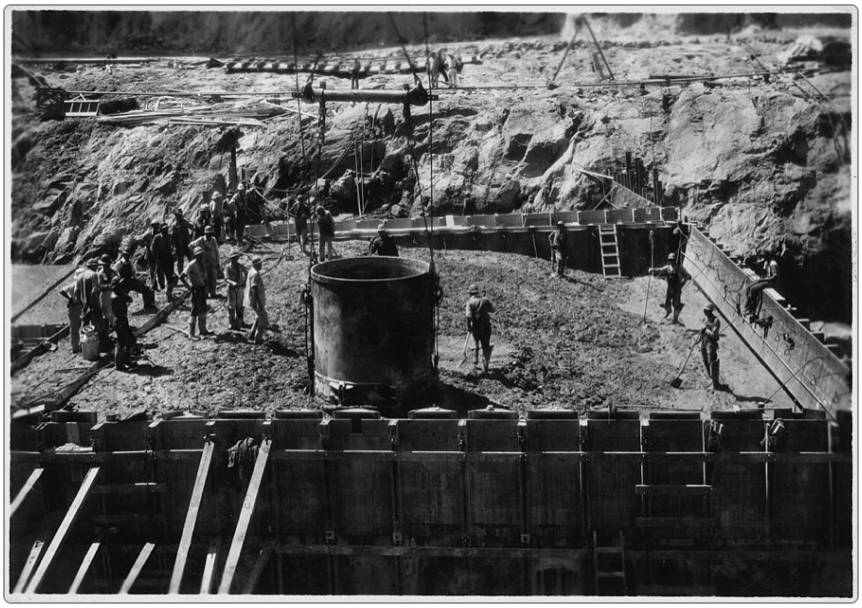
The physics. Water pressure increases with depth: $p = \gamma_w h$, with $\gamma_w = 9.81\ \mathrm{kN/m^3}$ (62.4 lb/ft^3). At 180 m depth that is about 1,770 kPa, roughly 17 times atmospheric pressure. The total horizontal force on each meter of a vertical dam face is the area of the triangular pressure diagram: $F = \frac{1}{2}\gamma_w H^2$, acting at $H/3$ above the base. The force grows with the square of the depth (Lesson 1.4). A **gravity dam** resists sliding

by friction on its base, μW, and overturning by the moment of its weight about the toe; uplift under the base reduces both, so drains and a curtain of grout injected into the rock are essential. An **arch dam** sends the water's push into the canyon walls, so it can be much thinner, if the rock walls are strong.

The engineering. Dams are among the most consequential structures engineers build: they store water for millions of people and generate electricity, and if they fail, the flood below can be devastating, as at St. Francis (Lesson 1.4). So dams are built on investigated foundations, with grout curtains and drains, monitored with instruments for the whole of their lives, and inspected regularly under dam-safety laws. Hoover Dam's concrete is still being watched, and it is still strong. The reservoir behind it stores the Colorado's water for farms and cities across the Southwest, and its power plant serves millions of people.

Try it. In the build you will measure water pressure with depth, build model dams of different shapes and test their stability and seepage.

PICTURE STUDY



Placing Concrete in a Block at Mid-Section of Hoover Dam, U.S. Bureau of Reclamation photographer, July 18, 1933 · Photograph · National Archives

Image: Wikimedia Commons, public domain

Hoover Dam was built as hundreds of separate blocks, each poured in 5 ft layers. Here an 8-cubic-yard bucket hangs from an overhead cableway over one block, and the crew spreads the concrete.

Look closely

- Find the bucket. How big is it compared with the workers?
- Find the timber forms around the block. Why build the dam in blocks instead of all at once?
- Look for pipes in the fresh concrete. What might they be for?
- Find the canyon wall behind. How does the dam connect to it?

Talk about it

1. Why does a huge mass of concrete need to be cooled?
2. What happens to the joints between blocks when the concrete cools and shrinks, and how were they closed?
3. Why must a dam's foundation be cleaned down to sound rock before concrete is placed?

> **Safety first.** Do the water tests outdoors or in a tub, with an adult present. Mop up spills promptly.

BUILD

Holding Back Water

You will measure how water pressure grows with depth using a bottle with holes, then build model dams of clay or sand in a trough and test their stability, seepage and the effect of a drain.

You will need

- A tall plastic bottle, a nail (adult heats or pushes holes), tape and a ruler
- A long plastic tub or a rain gutter with end caps as the river channel
- Modeling clay, sand, gravel and a strip of plastic sheet as a core
- A measuring cup, a stopwatch and food coloring

Steps

1. **Pressure.** Make holes at heights of 5, 10 and 15 cm in the bottle, tape them closed, and fill it with water. Remove the tape and measure how far each jet reaches. Which is strongest? Relate it to $p = \gamma_w h$.
2. **Gravity dam.** Build a clay dam across the trough, triangular in section, 8 cm high with a 10 cm base. Fill the upstream side slowly with water to 7 cm. Watch for sliding or leaks.
3. **Predict.** If the base is narrowed to 5 cm, at what water depth will it slide?
4. Build the narrower dam and test. Record the depth at failure.
5. **Earth dam.** Build a sand dam of the same height with gentle 1:2 slopes. Add food coloring upstream and watch for seepage through it. Time how long until colored water appears downstream.
6. Rebuild it with a clay or plastic “core” in the middle and a gravel drain at the downstream toe. Repeat the seepage test.
7. **Arch.** If your trough has firm sides, build a thin clay arch dam curving upstream and compare it with a straight thin wall of the same thickness.
8. **Improve.** Using the least clay, design a dam that holds 7 cm of water for 10 minutes without sliding or significant leaking.

Tips

- Press clay firmly into the bottom and sides of the trough; most model dams leak along the edges.
- Fill slowly to avoid waves.
- Seepage often shows as a damp patch on the downstream face before it flows.

Check your work

- ☐ My bottle test shows pressure increasing with depth.
- ☐ My dam tests compare base width, core and drain, and arch versus straight wall.
- ☐ I can calculate water pressure and force and check a gravity dam against sliding.

ENGINEER'S NOTEBOOK

1. A section through a gravity dam with the triangular water pressure diagram, the resultant at $H/3$, the dam's weight, the uplift and the drain.
2. Your jet distances and dam test results.
3. A plan sketch of an arch dam showing how the water's push is carried into the canyon walls.

KEY FORMULAS

$p = \gamma_w h$
$F = \frac{1}{2}\gamma_w H^2$ (per unit width, at $H/3$)
$FS_{\text{slide}} = \dfrac{\mu(W - U)}{F}$ (U = uplift force)
$Q = mc\Delta T$ (heat in concrete)

From history. The dam was first called Boulder Dam, then officially Hoover Dam after President Herbert Hoover, a mining engineer who had negotiated the agreement among the seven states that share the Colorado River. The Bureau of Reclamation's chief design engineer, John L. Savage, went on to advise on dams around the world.

Example 1. Find the water pressure at a depth of 180 m and the horizontal water force on each meter width of a vertical dam face holding 180 m of water.

Show solution

1. Pressure: $p = 9.81 \times 180 \approx 1766$ kPa, about 1.8 MPa.
2. Force: $F = \frac{1}{2}(9.81)(180)^2 = \frac{1}{2}(9.81)(32{,}400) \approx 158{,}900$ kN per meter of width.
3. It acts 60 m above the base ($H/3$).

Answer: About 1.77 MPa at the base; about 159,000 kN per meter, acting 60 m up.

Example 2. Model Hoover Dam's section as a trapezoid 221 m high, 14 m wide at the top and 201 m at the base, of concrete weighing 23.5 kN/m^3. Find its weight per meter, and its sliding factor of safety against the water force of Example 1 with a friction coefficient of 0.70, ignoring uplift and arch action.

Show solution

1. Area: $\frac{1}{2}(14 + 201)(221) \approx 23{,}760$ m^2.
2. Weight: $23{,}760 \times 23.5 \approx 558{,}300$ kN per meter.
3. $FS = \mu W/F = 0.70 \times 558{,}300 \div 158{,}900 \approx 2.46$.
4. Uplift would reduce this, which is why drains and grouting are vital; the arch action into the canyon walls adds a large reserve.

Answer: About 558,000 kN per meter; FS ≈ 2.5 against sliding (before uplift).

Example 3. In US units, find the pressure in psi at 590 ft of water depth, and the force per foot of dam width.

Show solution

1. $p = 62.4 \times 590 \approx 36{,}800$ lb/ft^2; divide by 144: about 256 psi.
2. $F = \frac{1}{2}(62.4)(590)^2 \approx 1.09 \times 10^7$ lb per foot of width, about 5,400 tons on each foot of dam.

Answer: About 256 psi; about 10.9 million lb per foot of width.

PRACTICE

1. Easy Find the water pressure at 50 m depth.

Show answer

$9.81 \times 50 \approx 491$ kPa.

2. Easy Why are dams thicker at the bottom than at the top?

Show answer

Water pressure, and so the force the dam must resist, increases with depth.

3. Easy What is uplift, and why is it dangerous?

Show answer

Water pressure under the base of a dam pushing upward; it reduces the dam's effective weight and its resistance to sliding and overturning.

4. Medium Find the horizontal force per meter width on a dam holding 40 m of water.

Show answer

$\frac{1}{2}(9.81)(1600) = 7848$ kN/m.

5. Medium A triangular gravity dam 40 m high with a 30 m base weighs (23.5 kN/m^3) how much per meter? With $\mu = 0.7$, find the sliding FS for Problem 4 (no uplift).

Show answer

$W = \frac{1}{2}(30)(40)(23.5) = 14{,}100$ kN/m; $FS = 0.7 \times 14{,}100/7848 \approx 1.26$, too low; a wider base is needed.

6. Medium Find the pressure at 100 ft depth in psi.

Show answer

$62.4 \times 100/144 \approx 43.3$ psi.

7. Medium Why was Hoover Dam built in separate blocks with cooling pipes?

Show answer

The cement's heat of hydration would have taken over a century to escape from a single mass, and cooling and shrinking would crack it; blocks with cooling pipes let it cool quickly, and the joints were then grouted.

8. Hard In Problem 5, uplift under the base acts as a triangle from $\gamma_w H$ at the heel to 0 at the toe, over the 30 m base. Find the uplift force and the new sliding FS.

Show answer

$U = \frac{1}{2}(9.81 \times 40)(30) = 5886$ kN/m; $FS = 0.7(14{,}100 - 5886)/7848 \approx 0.73$: it would slide. Drains that cut uplift are essential.

9. Hard Hoover Dam contains about 2.5 million m^3 of concrete. If each cubic meter released enough heat to warm it by 25 °C ($c \approx 0.9$ kJ/kg·°C, density 2,400 kg/m^3), how much heat had to be removed?

Show answer

$Q = mc\Delta T = (2.5 \times 10^6 \times 2400)(0.9)(25) \approx 1.35 \times 10^{11}$ kJ.

10. Hard Conceptual. Why can an arch dam be much thinner than a gravity dam of the same height?

Show answer

Model answer: the curved arch carries the water's push in compression sideways into the canyon walls, instead of resisting it by sheer weight; concrete is strong in compression, so a thin arch suffices, provided the rock abutments are strong.

11. Challenge For a triangular gravity dam of height H and base B (density 2,400 kg/m^3), with no uplift and $\mu = 0.7$, find the base-to-height ratio B/H needed for a sliding FS of 1.5.

Show answer

$W = \frac{1}{2}BH\gamma_c$, $F = \frac{1}{2}\gamma_w H^2$. $0.7 \cdot \frac{1}{2}BH(23.5) = 1.5 \cdot \frac{1}{2}(9.81)H^2$, so $B/H = 1.5 \times 9.81/(0.7 \times 23.5) \approx 0.89$.

COMMON MISTAKES

- Thinking the amount of water stored behind a dam determines the force on it. The force depends on the depth of water at the dam, not the length of the reservoir.
- Forgetting uplift under the base.
- Placing the water force at mid-height instead of $H/3$.

Make it a game · Dam Pressure Estimates. Fill a tall container with water. Predict the pressure at the bottom in pounds per square foot, then calculate it with $P = \rho gh$. Explain gravity, arch and arch-gravity dams from memory and say which suits Hoover Dam's canyon. Your parent covers one detail and you restate it. Closest prediction wins.

Scripture. “Thou hast set a bound that they may not pass over; that they turn not again to cover the earth.” (Psalm 104:9, KJV)

The psalmist praises God for setting bounds to the waters. A dam is a small human echo of that bound, built to protect and serve the people below it.

UNIT 8

Foundations and Geotechnical Engineering

Every structure rests on the ground, and the ground is the least predictable material an engineer uses. In this unit you will meet the science of soils that Karl Terzaghi founded, learn how engineers investigate a site and calculate effective stress and settlement, and design spread footings and piles as Chicago's engineers learned to do on their swamp.

Then you will study three great geotechnical case studies: the Leaning Tower of Pisa, which leaned for eight centuries and was gently straightened in 2001; retaining walls from Coulomb's fortress to the Blue Ridge Parkway; and Hoover Dam, a mountain of concrete cooled by 582 miles of pipe that holds back a great river. You will build in sand and clay and water: jar tests, footings and piles, leaning towers, retaining walls and model dams.

PREREQUISITES

- Units 4–5: stress, pressure, moments, factor of safety
- Algebra and trigonometry: logarithms (base 10), sine and tangent, squares
- Physics: density, weight, pressure in fluids

MASTERY CHECKLIST

- ☐ I can classify soils, describe a site investigation, and calculate total stress, pore pressure and effective stress.
- ☐ I can estimate consolidation settlement and explain why clays settle slowly.
- ☐ I can size a spread footing and estimate a pile's capacity from skin friction and end bearing.
- ☐ I can calculate the eccentricity and bearing pressures of a leaning tower and explain how Pisa was stabilized.
- ☐ I can calculate active earth pressure and check a gravity wall, and explain why drainage matters.
- ☐ I can calculate water pressure and force on a dam and check a gravity dam against sliding, including uplift.

OPTIONAL KITS FOR THIS UNIT

- **KEVA planks**: Identical maple planks for building the leaning-tower model in Lesson 8.3 and block gravity walls in Lesson 8.4.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

8.1	**Soils and Site Investigation: Terzaghi's Science**	3 sessions of 50 minutes
8.2	**Shallow and Deep Foundations**	3 sessions of 50 minutes
8.3	**The Leaning Tower of Pisa**	3 sessions of 50 minutes

8.4	**Retaining Walls and Slopes**	3 sessions of 50 minutes
8.5	**Dams: Hoover Dam**	3 sessions of 50 minutes

Unit 8 test

Time allowed: 50 minutes. Calculators are allowed. Use $\gamma_w = 9.81\ \mathrm{kN/m^3}$. Show your work and units for every calculation. The test is worth 100 points.

1. **5 pts**
   State Terzaghi's principle of effective stress and explain why it matters.

2. **7 pts**
   Soil with $\gamma = 17\ \mathrm{kN/m^3}$ lies above a water table at 4 m; below it $\gamma_{\mathrm{sat}} = 19.5\ \mathrm{kN/m^3}$. Find σ, u and σ' at 12 m.

3. **7 pts**
   A clay layer 5 m thick ($C_c = 0.35$, $e_0 = 1.0$) has $\sigma'_0 = 90$ kPa. A building adds 45 kPa. Estimate the settlement.

4. **6 pts**
   A column carries 3,000 kN on soil allowing 250 kPa. Size a square footing.

5. **8 pts**
   A pile 0.5 m in diameter and 18 m long has $f_s = 35$ kPa and $q_b = 2500$ kPa. Find its ultimate and allowable capacity (FS = 2.5).

6. **5 pts**
   Why is uneven settlement more harmful than uniform settlement?

7. **6 pts**
   Why did the Tower of Pisa lean, and why did the long pause in construction help it?

8. **8 pts**
   A 50,000 kN tower stands on a circular foundation 15 m in diameter. Its center of gravity is 18 m high and it leans 4°. Find e, $q_{\max}$ and $q_{\min}$.

9. **5 pts**

How did engineers reduce Pisa's lean without touching the tower?

10. **7 pts**

A wall 5 m high retains soil with $\gamma = 18$ and $\phi = 30^\circ$. Find K_a, P_a and its line of action.

11. **6 pts**

If water rises to the top behind the wall in Problem 10, what is the water force alone? Why do walls need drains?

12. **6 pts**

A dry slope of sand with $\phi = 34^\circ$ is cut at 28°. Find its factor of safety.

13. **7 pts**

Find the pressure at 120 m depth and the force per meter width on a dam holding 120 m of water.

14. **6 pts**

A triangular gravity dam 120 m high with a 100 m base (23.5 kN/m^3). Find its weight per meter and its sliding FS with $\mu = 0.7$ (no uplift), using Problem 13.

15. **3 pts**

Why was Hoover Dam's concrete cooled with pipes?

16. **8 pts**

Short essay. Using the St. Francis Dam (Lesson 1.4), Pisa and Hoover Dam, explain why site investigation and monitoring are the foundation of geotechnical engineering.

ANSWER KEY

1. $\sigma\prime = \sigma - u$: the stress carried grain to grain equals the total stress minus the pore water pressure. It controls soil strength and compression, so changes in water pressure change a soil's strength and cause settlement.
2. $\sigma = 68 + 156 = 224$ kPa; $u = 9.81 \times 8 \approx 78.5$ kPa; $\sigma\prime \approx 145.5$ kPa.
3. $S = (0.35 \times 5/2.0)\log_{10}(135/90) = 0.875 \times 0.176 \approx 0.154$ m.
4. $A = 12\ \text{m}^2$; side ≈ 3.46, use 3.5 m.
5. Skin $35 \times \pi(0.5)(18) \approx 990$ kN; end $2500 \times \pi(0.25)^2 \approx 491$ kN; $Q_u \approx 1481$ kN; $Q_a \approx 592$ kN.
6. Uneven settlement distorts the structure, bending beams and walls and cracking finishes; uniform settlement moves the building as a whole.
7. It stands on soft, slightly uneven layers of silty sand and clay, softer on the south side, so it settled unevenly. The pause let the clay consolidate and gain strength under the early stories before more weight was added.
8. $e = 18\sin 4^\circ \approx 1.256$ m; $A \approx 176.7\ \text{m}^2$, $S \approx 331.3\ \text{m}^3$; $P/A \approx 283$; $M/S = 62{,}800/331.3 \approx 190$; $q_{\max} \approx 473$, $q_{\min} \approx 93$ kPa.
9. They extracted small amounts of soil from beneath the north (high) side through inclined drill holes; the ground there settled slightly and the tower rotated back about half a degree.
10. $K_a = 1/3$; $P_a = \frac{1}{2}(1/3)(18)(25) = 75$ kN/m, at 1.67 m above the base.
11. $\frac{1}{2}(9.81)(25) \approx 122.6$ kN/m, more than the dry soil's push; drains keep water from building up behind the wall.
12. $\tan 34^\circ / \tan 28^\circ \approx 0.675/0.532 \approx 1.27$.
13. $p \approx 1177$ kPa; $F = \frac{1}{2}(9.81)(14{,}400) \approx 70{,}600$ kN/m.
14. $W = \frac{1}{2}(100)(120)(23.5) = 141{,}000$ kN/m; $FS = 0.7 \times 141{,}000/70{,}600 \approx 1.40$.
15. To remove the heat of hydration quickly so the concrete would not crack as it slowly cooled and shrank.
16. Model answer: St. Francis failed on rock that was never properly investigated; Pisa leaned because it was built on soft, uneven soil no one understood, and it was saved only after decades of borings, testing and careful monitoring during the soil extraction; Hoover Dam's designers investigated and cleaned its foundation to sound rock, grouted and drained it, and it has been watched by instruments ever since. Because soil and rock vary and cannot be seen,

engineers must investigate before designing and keep observing during and after construction.

PORTFOLIO PIECE: A GEOTECHNICAL REPORT FOR A REAL SITE

Choose a site near you where something might be built (with the owner's permission), such as a backyard shed, a garden wall or a church pavilion. Describe its soil from a jar test and a shallow hand-dug hole (call 811 first), estimate its unit weight and bearing capacity from typical values, and note any water or slopes.

Write a short geotechnical report: the soil profile, the recommended foundation type and footing size for the structure's loads, a retaining-wall check if the site needs one, drainage recommendations, and what further investigation a professional engineer would require.

LOOK FOR	WHAT IT MEANS
Site description	Soil described from observation and a jar test; water, slopes and surroundings noted.
Calculations	Loads estimated; footing sized; effective stress or settlement or wall check calculated correctly.
Recommendations	Foundation type and drainage recommended with clear reasons.
Honesty about limits	States what is uncertain and what professional investigation would add.
Presentation	Organized report with a site sketch, soil profile and labeled calculations.

UNIT 9

Water and Transportation

Civil engineering began as the work of the public: surveying the land, bringing water in, carrying wastewater away and building roads. In this unit you will follow those works from the Romans to today: the groma and the GPS; the slow sand filter, Croton's aqueduct and John Leal's chlorine; Bazalgette's sewers under London; the Appian Way, McAdam's roads and the AASHO Road Test; and Stephenson's Rocket and the science of traffic.

These are the systems most people never notice until they fail, and they serve more people than any single building. You will survey your yard with a homemade water level, clean muddy water, measure runoff, rut a model road, and count traffic, using the same calculations professional engineers use: slopes and closure errors, flows and doses, the rational method, the fourth-power law, banking and stopping distances.

PREREQUISITES

- Units 1–8: units, pressure, flow, effective stress and drainage
- Algebra: rates and proportions, powers, square roots
- Trigonometry: sine and cosine; physics: speed, acceleration and centripetal acceleration (v^2/R)

MASTERY CHECKLIST

- ☐ I can reduce leveling notes, compute slopes and closure errors, and explain how GPS finds position.
- ☐ I can describe water treatment and compute demand, pipe size, chemical dose and detention time.
- ☐ I can explain combined and separate sewers and compute runoff with

$Q = CiA$ and sewage flows.

- ☐ I can explain road layers and drainage and compute ESALs, crown height and pavement quantities.
- ☐ I can compute superelevation, stopping sight distance, yellow time and traffic flow.

OPTIONAL KITS FOR THIS UNIT

- Arduino (Uno and starter kits): Ages 12 and up with an adult. Optional for Lesson 9.5: program a model traffic signal with LEDs and set its yellow time from the formula in the lesson. Battery or USB power only.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

9.1	**Surveying: From the Roman Groma to GPS**	3 sessions of 50 minutes
9.2	**Water Supply and Treatment**	3 sessions of 50 minutes
9.3	**Wastewater and Stormwater: Bazalgette's London**	3 sessions of 50 minutes
9.4	**Roads and Pavement: The Romans and McAdam**	3 sessions of 50 minutes
9.5	**Railways and Traffic Engineering**	3 sessions of 50 minutes

Unit 9 test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation. Convert units carefully. The test is worth 100 points.

1. **6 pts**
   A benchmark is 245.30 ft. BS = 4.12 ft; FS on P = 6.85 ft. Find the HI and P's elevation.

2. **6 pts**
   A level loop has $\sum BS = 12.564$ m and $\sum FS = 12.531$ m. Find the closure error. Should the loop be accepted if the limit is 20 mm?

3. **5 pts**
   A sewer falls 0.62 m over 155 m. Find its slope in percent.

4. **5 pts**
   Why does a GPS receiver need signals from at least four satellites?

5. 7 pts

A town of 18,000 uses 110 gallons per person per day. Find the demand in MGD and ft^3/s, and the chlorine needed per day at 1.5 mg/L.

6. 6 pts

A basin of 300,000 gallons receives 1.8 MGD. Find the detention time.

7. 6 pts

Describe John Leal's contribution to safe water and why measurement was central to it.

8. 7 pts

Find the peak runoff from 8 acres of pavement ($C = 0.9$) and from the same land as woods ($C = 0.15$), for 2.2 in/h. How much storage would a 30-minute storm need to hold the difference (simplified)?

9. 6 pts

How did Bazalgette's intercepting sewers end the Great Stink?

10. 5 pts

What is self-cleansing velocity and why does it matter?

11. 6 pts

Find the ESAL of a 30,000 lb single axle and compare it with the standard axle.

12. 5 pts

Why did McAdam raise and crown his roads?

13. 6 pts

Find the asphalt tonnage for 2 miles of road 22 ft wide and 3 in thick (145 lb/ft^3).

14. 7 pts

A curve has $R = 350$ m, design speed 90 km/h, and $f = 0.12$. Find the superelevation needed.

15. 5 pts

Find the stopping sight distance at 80 km/h ($t = 2.5$ s, $a = 3.4$ m/s^2).

16. **4 pts**

A lane carries 1,500 veh/h at 50 km/h. Find its density.

17. **8 pts**

Short essay. Water mains, sewers and roads are “invisible until they fail.” Choose two of them and explain how engineers design and maintain them to serve the public, using one historical example.

ANSWER KEY

1. HI = 249.42 ft; P = 242.57 ft.
2. +0.033 m (33 mm); it exceeds 20 mm, so the loop should be rerun.
3. 0.40%.
4. To solve for four unknowns: its three coordinates and its own clock error.
5. 1.98 MGD; ≈ 3.06 ft^3/s; $8.34 \times 1.98 \times 1.5 \approx 24.8$ lb/day.
6. $0.3/1.8$ day = 4 h.
7. In 1908 he had Jersey City's water continuously chlorinated; when the practice was challenged in court, bacterial measurements showed the treated water was safe and much cleaner, and chlorination spread nationwide, cutting waterborne disease.
8. Pavement 15.84 ft^3/s; woods 2.64 ft^3/s; difference 13.2 ft^3/s × 1800 s ≈ 23,800 ft^3.
9. They ran along both banks of the Thames, catching the outflow of the city's sewers before it reached the river in London and carrying it east to outfalls far downstream, with pumping stations lifting it where needed.
10. The minimum flow speed, about 0.6 m/s (2 ft/s), that keeps solids moving; below it, solids settle and the sewer clogs.
11. $(30/18)^4 \approx 7.7$; it does about 7.7 times the damage.
12. To keep water out of the soil beneath, so the soil stayed dry and strong enough to carry the load, and to shed rain to the ditches.
13. $10{,}560 \times 22 \times 0.25 = 58{,}080$ ft^3; $\approx 8.42 \times 10^6$ lb ≈ 4,210 tons.
14. $v = 25$ m/s; $v^2/(gR) = 625/3434 \approx 0.182$; $e \approx 0.062$, about 6%.
15. $v = 22.2$; $55.6 + 72.6 \approx 128$ m.
16. 30 veh/km.
17. Model answer: e.g., sewers are designed for peak flows with slopes for self-cleansing velocity and sized for growth (Bazalgette doubled his diameters, and London still uses them), and are inspected and relined over time; water systems use multiple treatment barriers and constant testing (Leal's chlorination), with a residual in the pipes and regular main replacement. Good design anticipates growth and failure modes, and faithful maintenance keeps small problems from becoming disasters.

PORTFOLIO PIECE: A NEIGHBORHOOD INFRASTRUCTURE STUDY

Study the public works on your street: survey the slope of the street or a ditch with your water level; trace where rainwater goes and estimate the peak runoff from your block with $Q = CiA$; find out where your water comes from and how it is treated (water utilities publish annual reports); and count traffic at two times of day.

Present your findings in a short report with a map, data tables and calculations, and recommend one improvement (a rain garden, a crosswalk, a drainage fix) with a simple estimate of what it would do. Consider sending a polite letter with your best idea to your town's engineer or public works department, with a parent's help.

LOOK FOR	WHAT IT MEANS
Field data	Slope survey, runoff observations and traffic counts collected safely and recorded carefully.
Calculations	Slope, runoff, water demand and traffic flow calculated correctly with units.
Research	Water source and treatment described accurately from a reliable source (such as the utility's annual report).
Recommendation	A practical improvement proposed and supported with numbers.
Presentation	Clear map, tables and writing.

9.1 Surveying: From the Roman Groma to GPS

OBJECTIVES

- Describe how surveyors have measured land, from the Roman groma and chorobates to the theodolite, total station and GPS.
- Reduce a differential leveling survey, compute slopes and closure error, and use trigonometry to find distances and heights.
- Build a water level and a groma and run a closed level loop.

> **Warm-up.** Go outside and try to judge by eye which of two spots in your yard is higher. Then pour a little water on a flat board set between them, or roll a marble. Were you right? Why is it so hard to see small differences in height?

WATCH

How does land surveying work?

Practical Engineering · 6:26

https://www.youtube.com/watch?v=SPCewaAfqPA

> **WHEN TO WATCH**
> After the story. Watch closely when he demonstrates leveling with a rod and level; you will do the same in the build.

KEY TERMS

groma

A Roman surveying instrument: a cross on a staff with four plumb lines, used to sight straight lines and right angles.

chorobates

A Roman leveling table about 20 ft long with plumb lines and a water channel on top, described by Vitruvius.

benchmark

A permanent point of known elevation from which other elevations are measured.

differential leveling

Finding differences in elevation by reading a graduated rod through a level: elevation = known elevation + backsight − foresight.

total station

An electronic theodolite that measures horizontal and vertical angles and distances by laser.

GPS (GNSS)

Satellite positioning: a receiver finds its position from the travel times of signals from at least four satellites.

CORE IDEAS

The story. Roman surveyors, the *agrimensores*, laid out roads, camps, cities and farmland across an empire with remarkably simple tools. With the **groma**, a cross on a pole with plumb lines at the four arms, they sighted straight lines and right angles. With the **chorobates** they leveled. The aqueduct that supplied Nîmes in France, crossing the Pont du Gard, drops only about 12.6 m over about 50 km, an average of 25 cm per kilometer, a slope so gentle that it shows extraordinary care. Seventeen centuries later, in 1749, a seventeen-year-old named George Washington was appointed surveyor of Culpeper County, Virginia; he had already spent a season surveying the western lands of Lord Fairfax. From 1763 to 1767 the English astronomer Charles Mason and surveyor Jeremiah Dixon ran a boundary line of over 230 miles between Pennsylvania and Maryland, checking their direction by the stars.

How they figured it out. Surveying is geometry carried out on the ground with honest care. Surveyors learned to check everything: to close every traverse back on its starting point, to run a level loop back to the benchmark, and to compare the result with zero. The theodolite (a telescope that measures angles) and the steel tape replaced the Roman tools; triangulation networks of measured angles covered whole nations; and in the 1990s the Global Positioning System made it possible to fix a point to within centimeters using signals from satellites. But every new tool still relies on the old discipline: measure twice, check by a different route, and record faithfully. The Bible counts it a sin to move a neighbor's boundary stone; surveyors are trusted keepers of boundaries.

The physics. A **level** defines a horizontal line of sight; a **rod** held upright on a point is read through it. Starting from a benchmark of known elevation, the reading on it (the backsight, BS) gives the height of the instrument: $HI = \text{elev}_{BM} + BS$. The reading on a new point (the foresight, FS) gives its elevation: $\text{elev} = HI - FS$. Slope is rise over run, often in percent. A total station measures a slope distance s and a vertical angle θ, so the horizontal distance is $s\cos\theta$ and the height difference $s\sin\theta$. GPS works by timing: each satellite sends the time; the receiver measures how long

each signal took and multiplies by the speed of light, $d = ct$. With distances to four satellites it can solve for its three coordinates and its own clock error. Because light travels about 300 m in a microsecond, the clocks must be extraordinarily accurate.

The engineering. Every civil project begins and ends with surveying: boundary and topographic surveys before design, construction staking to place every footing and curb, and as-built surveys afterward. Surveyors set grades for roads, pipes that must flow downhill, and the foundations of buildings. A small slope error in a sewer can mean a pipe that never drains; an error in a boundary can cost a family its land. That is why surveyors are licensed professionals, like engineers and architects.

Try it. In the build you will make a water-tube level and a groma, then run a level loop around your yard and see how close you come to closing.

PICTURE STUDY

Measuring a Road with a Wagon, after Vitruvius, Unknown woodcut artist, 1575 · Woodcut · Deutsche Fotothek, Saxon State and University Library, Dresden

Image: Wikimedia Commons, public domain

The German heading reads, roughly, “Figure of how one should measure the road from a wagon by an artful machine, according to Vitruvius.” Vitruvius described an odometer: gears driven by a wheel of known size that dropped a pebble into a box at every mile. Surveyors have always looked for clever, honest ways to measure distance.

Look closely

- Find the large toothed wheel inside the carriage. What turns it?
- Find the carriage wheels. Why must their size be known exactly?
- Look for the drum or box where the count would be recorded.
- Compare this method with pacing, a measuring chain and GPS.

Talk about it

1. Vitruvius’s wheels were 4 Roman feet across, and he reckoned about 400 turns to the Roman mile of 5,000 feet. Check his arithmetic using $C = \pi d$.
2. What errors could make a wagon odometer read too long or too short?
3. Why do surveyors still check one measurement with another method?

> **Safety first.** Work only in your own yard or with permission, away from streets. Never drive stakes deeper than a few inches without calling 811 to locate buried utilities.

BUILD

A Water Level, a Groma and a Level Loop

You will build two old tools that still work: a water-tube level (water finds its own level) and a groma. With them you will run a closed level loop around your yard and lay out a right angle, then check your accuracy.

You will need

- About 6 m of clear vinyl tubing (10 mm), water with a drop of food coloring, two yardsticks or 2 m rods taped to stakes
- For the groma: two sticks crossed and nailed at the middle, four strings with washers as plumb bobs, a broomstick as the staff
- A tape measure, stakes or flags, a notebook

Steps

1. Fill the tube with colored water, without bubbles. Tape each end to a rod so the water shows near mid-height on both.
2. Test it: hold both rods side by side on a flat floor. The water levels must read the same. Record any offset.
3. Choose a benchmark (a step or a stake) and call its elevation 10.000 m. Choose three or four points around the yard.
4. Set one rod on the benchmark and the other on point A. Read the water level on each rod (from the ground up). The difference in readings is the difference in elevation. Record them as you would a BS and FS.
5. Move around the loop point by point and finish back on the benchmark. Compute every elevation and the closure error.
6. **Predict** before you compute: how many millimeters of closure error do you expect?
7. Use the groma to lay out a right angle at a corner (sight along two arms through the hanging strings). Check it with the 3-4-5 rule: 3 m along one line, 4 m along the other, and the diagonal should be 5 m. Record the error.
8. **Improve.** Rerun the loop more carefully (shorter sights, steady rods, waiting for the water to settle) and compare the new closure error.

Tips

- Keep the tube free of kinks and bubbles.
- Hold the rods plumb; a leaning rod reads too high.
- Wait for the water to stop moving before reading.

Check your work

- ☐ I built a water level and ran a closed level loop with a computed closure error.
- ☐ I laid out and checked a right angle with a groma and the 3-4-5 rule.
- ☐ I can reduce leveling notes and compute slopes and GPS distances.

ENGINEER'S NOTEBOOK

1. A sketch map of your yard with your points, the benchmark and the elevations you found.
2. Your level book: a table of BS, HI, FS and elevation for each setup, with the check $\sum BS - \sum FS$.
3. A labeled drawing of the groma and chorobates, and one sentence on how each worked.

KEY FORMULAS

$HI = \text{elev}_{\text{known}} + BS, \quad \text{elev}_{\text{new}} = HI - FS$

$\text{closure error} = \text{elev}_{\text{end}} - \text{elev}_{\text{start}}$ (for a closed loop)

$\text{slope} = \dfrac{\text{rise}}{\text{run}} \times 100\%$

$d_h = s\cos\theta, \quad \Delta h = s\sin\theta, \quad d = ct$

From history. Mason and Dixon marked their line every mile with stones carved with a P on one side and an M on the other, and every fifth mile with a crownstone bearing the arms of the Penn and Calvert families. Many still stand. Their line later became famous as a symbol of the division between North and South, but to them it was simply the most accurate long boundary of their age.

Example 1. A benchmark has an elevation of 100.00 m. The backsight on it reads 1.45 m and the foresight on point A reads 2.10 m. Find the height of instrument and the elevation of A.

Show solution

1. $HI = 100.00 + 1.45 = 101.45$ m.
2. $\text{elev}_A = 101.45 - 2.10 = 99.35$ m.
3. A is 0.65 m lower than the benchmark.

Answer: HI = 101.45 m; elevation of A = 99.35 m.

Example 2. A level loop starts at BM (100.00 m), runs to A and B, and returns to BM. Readings: setup 1, BS on BM 1.20, FS on A 0.85; setup 2, BS on A 1.62, FS on B 2.48; setup 3, BS on B 1.95, FS on BM 1.47. Find the elevations of A and B and the closure error.

Show solution

1. Setup 1: $HI = 101.20$; A $= 101.20 - 0.85 = 100.35$.
2. Setup 2: $HI = 100.35 + 1.62 = 101.97$; B $= 101.97 - 2.48 = 99.49$.
3. Setup 3: $HI = 99.49 + 1.95 = 101.44$; BM (computed) $= 101.44 - 1.47 = 99.97$.
4. Closure error: $99.97 - 100.00 = -0.03$ m. Check: $\sum BS - \sum FS = 4.77 - 4.80 = -0.03$. ✓ The 3 cm error would be spread among the points or the loop rerun, depending on the required accuracy.

Answer: A = 100.35 m, B = 99.49 m; closure error −0.03 m.

Example 3. The Nîmes aqueduct falls about 12.6 m over about 50 km. Find its average slope in m/km and in percent. A modern sewer must fall at least 0.5%. How many times steeper is that?

Show solution

1. $12.6 \div 50 \approx 0.25\ \mathrm{m/km}$.
2. Percent: $0.25\ \mathrm{m} \div 1000\ \mathrm{m} \times 100\% = 0.025\%$.
3. $0.5\% \div 0.025\% = 20$. A typical small sewer is twenty times steeper than the Roman aqueduct's average slope.

Answer: About 0.25 m/km (0.025%); a 0.5% sewer is 20 times steeper.

PRACTICE

1. Easy A benchmark is 50.00 m. BS = 1.30 m; FS on point P = 0.90 m. Find P's elevation.

   Show answer

   $50.00 + 1.30 - 0.90 = 50.40\ \mathrm{m}$.

2. Easy A driveway rises 1.8 m over 30 m. What is its slope in percent?

   Show answer

   6%.

3. Easy What did the Roman groma do?

   Show answer

   It let surveyors sight straight lines and lay out right angles using plumb lines hung from a cross.

4. Medium A total station measures a slope distance of 120.0 m at a vertical angle of 4.0°. Find the horizontal distance and height difference.

   Show answer

   $120 \cos 4^\circ \approx 119.71\ \mathrm{m}$; $120 \sin 4^\circ \approx 8.37\ \mathrm{m}$.

5. Medium A sewer must drop at 1.0% over 85 m. How much lower is its downstream end?

   Show answer

   0.85 m.

6. Medium A level loop has $\sum BS = 8.415$ m and $\sum FS = 8.402$ m. What is the closure error?

Show answer

$+0.013$ m (13 mm).

7. Medium A GPS signal takes 0.0680 s to arrive. How far away is the satellite? ($c = 3.00 \times 10^8$ m/s)

Show answer

2.04×10^7 m, about 20,400 km.

8. Hard A receiver's clock is off by one microsecond. How large a distance error does that cause? Why does GPS need a fourth satellite?

Show answer

$3 \times 10^8 \times 10^{-6} = 300$ m. The receiver's cheap clock is not exact, so its time offset is a fourth unknown along with x, y and z; four satellites give four equations to solve for all four.

9. Hard Check a right angle by the 3-4-5 rule: the legs are 6.00 m and 8.00 m. What should the diagonal be? If it measures 10.06 m, is the angle larger or smaller than 90°?

Show answer

10.00 m. A longer diagonal means the angle is a little larger than 90° (by the law of cosines, about 0.7°).

10. Hard Conceptual. Why do surveyors always close a loop back to the starting point?

Show answer

Model answer: closing the loop gives a check; if the computed elevation (or position) of the start does not match its known value, the difference reveals the size of the errors, and a blunder can be caught before it reaches construction.

11. **Challenge** Two points 400 m apart are leveled with a single setup halfway between them. Earth's curvature makes a level line of sight rise above the surface by about $0.0785d^2$ meters (d in km). Why does setting up halfway cancel this effect?

Show answer

Curvature raises both rod readings by the same amount ($0.0785 \times 0.2^2 \approx 3.1$ mm each at 200 m), so in BS − FS they cancel. Balanced sight distances cancel curvature and many instrument errors.

COMMON MISTAKES

- Adding the foresight instead of subtracting it.
- Reading a rod that is not plumb, which always reads too high.
- Not closing a loop, so errors go undetected.

Make it a game · Level Loop Race. Use a clear tube or hose with water as a water level. Mark equal heights at two points across the room and compare. Record the readings, then compute the closure error of your loop. Explain the groma and the benchmark in your own words. Smallest error wins; try to beat it next time.

Scripture. "Go and walk through the land, and describe it, and come again to me" (Joshua 18:8, KJV)

Joshua sent men to survey and describe the land before it was divided. Every fair division of land still begins with an honest survey.

9.2 Water Supply and Treatment

OBJECTIVES

- Trace how cities learned to deliver clean water: aqueducts, slow sand filters (1829), and chlorination (Jersey City, 1908), and describe a modern treatment plant.
- Estimate water demand, size a pipe from flow and velocity, and compute chemical doses and detention times.
- Test coagulation and filtration of muddy water and measure the improvement in clarity.

> **Warm-up.** Turn on a tap and fill a glass. Write down every step you think the water went through between a river or well and your glass. Then compare your list with the plant in this lesson.

WATCH

How City Water Purification Works: Drinking and Wastewater

Animagraffs · 12:26

https://www.youtube.com/watch?v=KsVfshmK0Ak

> **WHEN TO WATCH**
> After the core ideas. Watch the first half (drinking water) now; save the second half (wastewater) for Lesson 9.3.

KEY TERMS

coagulation and flocculation

Adding a chemical (such as alum) that makes tiny particles clump together, then gently stirring so the clumps (floc) grow large enough to settle.

sedimentation

Letting water stand slowly in a basin so floc and particles sink to the bottom.

filtration

Passing water through sand, gravel or membranes to remove remaining particles; slow sand filters also grow a living layer that removes germs.

disinfection

Destroying remaining harmful microbes, usually with chlorine, ultraviolet light or ozone.

MGD

Million gallons per day, a common unit of water flow (1 MGD ≈ 1.55 cubic feet per second).

detention time

How long water stays in a basin: volume ÷ flow.

CORE IDEAS

The story. For most of history, city water came from wells, rivers and aqueducts, and it often carried disease. In 1829 James Simpson, engineer of the Chelsea Waterworks Company in London, built the first large slow sand filter: water trickled slowly down through beds of sand and gravel, coming out clear. In 1842 New York opened the Croton Aqueduct, carrying clean upland water 41 miles to the city. But many cities still suffered from typhoid fever, spread by contaminated water. In 1908 Dr. John L. Leal, a physician and public-health officer, had the water supplying Jersey City, New Jersey, treated continuously with a small dose of chlorine. The water company was taken to court over it; Leal's data showed the chlorinated water was safe and far cleaner. Within a few years chlorination spread across America, and deaths from waterborne disease fell dramatically.

How they figured it out. Engineers and scientists learned water treatment step by step. Slow sand filters worked well, and later researchers found why: a thin living layer on the sand surface traps and consumes bacteria. Rapid sand filters, which work much faster, needed chemical coagulation first. Bacteriologists learned to test water for indicator organisms. Leal and the engineer George Warren Fuller worked out how much chlorine was enough to disinfect without harming taste or health. Every step was proved by measurement: samples taken, counted and compared.

The physics. A city's demand is roughly its population times a per-person rate, about 100 gallons (about 380 liters) per person per day in many American towns, with peaks two to three times the average. Flow, area and velocity are linked by $Q = Av$; pipes are usually sized so water moves at about 1–2 m/s (3–6 ft/s). Chemical doses are given in milligrams per liter (mg/L); in US practice, $\text{lb/day} = 8.34 \times Q_{\text{MGD}} \times \text{dose}_{\text{mg/L}}$. Sedimentation basins and chlorine contact tanks need enough **detention time**, $t = V/Q$, for particles to settle and for chlorine to act. Particles settle faster when they are bigger, which is why coagulation and flocculation come first.

The engineering. A modern plant takes water from a river, lake or wells; screens it; adds a coagulant; flocculates and settles it; filters it through sand, anthracite or membranes; disinfects it; adjusts its chemistry to protect pipes; and pumps it to water towers and mains (Grade 6). Operators test it around the clock. It is one of the great quiet achievements of civil engineering that a family can drink from any tap in town without a thought. Clean water is a gift to neighbors and a matter of trust.

Try it. In the build you will clear muddy water with alum and a sand filter, and measure its clarity with a homemade turbidity tube.

PICTURE STUDY

Inside a Covered Slow Sand Filter, Lake Whitney Filtration Plant, Historic American Engineering Record, late 20th century · Photograph · Library of Congress, Prints and Photographs Division

Image: Wikimedia Commons, public domain

Early twentieth-century cities in cold climates built their slow sand filters under concrete roofs, covered with earth, so the water would not freeze. HAER photographed this one in New Haven, Connecticut, drained so you can see its sand bed.

Look closely

- Find the sand floor. This is the filter. How deep do you think the water stood above it?
- Find the columns and the roof slab. Why cover a filter?
- Look for pipes and valves. Where does the water come in and go out?
- Find the marks or tools on the sand. Why would workers scrape the sand surface from time to time?

Talk about it

1. How does slow water through sand clean it?
2. Why would a filter this size be needed for only a medium-sized city?
3. What step after filtration made city water truly safe from germs?

> **Safety first.** Never drink water from your homemade filter, however clear it looks. Alum is safe to handle but keep it away from eyes and wash hands after use. An adult should supervise.

BUILD

Clearing Muddy Water: Coagulation and Filtration

You will make muddy water, treat samples with different doses of alum, let them settle, and pass them through a sand filter, measuring clarity at each step with a turbidity tube.

You will need

- Clear jars or cups, garden soil or clay, water, measuring spoons
- Potassium alum (from the spice aisle) dissolved in water to make a known solution (for example 1 g in 100 mL)
- A 2-liter bottle with the bottom cut off, cotton or a coffee filter, fine sand, coarse sand and gravel
- A tall clear tube or a 2-liter bottle with a black-and-white "Secchi" disk taped inside at the bottom, a ruler, a stopwatch

Steps

1. Make a big batch of muddy water and stir it well. Measure its clarity: pour into the tube until you can no longer see the disk; record the depth.
2. **Jar test.** Fill five jars with 500 mL each. Add alum solution to give doses of 0, 10, 20, 40 and 80 mg/L. Stir quickly for 1 minute, then slowly for 10 minutes, then let settle for 30 minutes.
3. **Predict** which dose will give the clearest water.
4. Measure the clarity of the top water in each jar. Plot clarity against dose. Find the best dose.
5. Build the filter: cotton at the neck, then gravel, coarse sand and 10 cm of fine sand. Rinse it with clean water until it runs clear.
6. Pour settled water from the best jar through the filter. Time how long 500 mL takes and compute the filtration rate (cm per minute through the sand area).
7. Measure the clarity of the filtered water and compare it with the untreated, settled and filtered results.
8. **Improve.** Filter untreated muddy water through the same filter and compare. Then try a deeper sand layer. Which change helps most?

Tips

- Use the same mud batch for every test.
- Too much alum can make water worse; there is a best dose.
- Pour gently onto the filter so you do not dig a hole in the sand.

Check your work

- ☐ My jar test found a best alum dose.
- ☐ I measured clarity before and after each treatment step.
- ☐ I can compute water demand, pipe size, chemical dose and detention time.

ENGINEER'S NOTEBOOK

1. A flow diagram of a water treatment plant from intake to tap, with a sentence on each step.
2. Your jar-test table and graph of clarity against dose.
3. A short paragraph on John Leal and why measurement mattered in convincing the court.

KEY FORMULAS

$$Q = Av$$

$$\text{chemical (lb/day)} = 8.34 \times Q\ \text{(MGD)} \times \text{dose (mg/L)}$$

$$t = \frac{V}{Q}$$

$$1\ \text{MGD} \approx 1.547\ \text{ft}^3/\text{s} \approx 3785\ \text{m}^3/\text{day}$$

From history. The Croton Aqueduct of 1842 crossed the Harlem River on High Bridge, a stone arch bridge that still stands as the oldest bridge in New York City. When the aqueduct opened, New Yorkers celebrated with a parade and fountains, and the city soon had more water than it had ever known.

Example 1. A town of 25,000 people uses 100 gallons per person per day. Find its average demand in MGD and ft^3/s, and the diameter of a main carrying it at 4 ft/s.

Show solution

1. Demand: $25{,}000 \times 100 = 2{,}500{,}000 \text{ gal/day} = 2.5 \text{ MGD}$.
2. In ft^3/s: $2.5 \times 1.547 \approx 3.87 \text{ ft}^3/\text{s}$.
3. Area: $A = Q/v = 3.87 \div 4 \approx 0.967 \text{ ft}^2$; diameter $= \sqrt{4A/\pi} \approx 1.11 \text{ ft} \approx 13.3 \text{ in}$.
4. Choose a 14 in main (and check it for peak demand and fire flow, which may require larger).

Answer: 2.5 MGD (3.87 ft^3/s); about a 14 in main.

Example 2. The plant in Example 1 doses chlorine at 2.0 mg/L. How many pounds of chlorine are used per day? If the chlorine contact tank must give 30 minutes of detention at peak flow (2.5 times average), what volume must it have?

Show solution

1. Chlorine: $8.34 \times 2.5 \times 2.0 \approx 41.7 \text{ lb/day}$.
2. Peak flow: $2.5 \times 3.87 \approx 9.67 \text{ ft}^3/\text{s}$.
3. Volume: $V = Qt = 9.67 \times 1800 \text{ s} \approx 17{,}400 \text{ ft}^3$ (about 130,000 gallons).

Answer: About 42 lb/day; about 17,400 ft^3.

Example 3. A slow sand filter is operated at a rate of 0.20 m/h (meters of water passing down through the sand per hour). What area of filter is needed for 10,000 m^3/day?

Show solution

1. Hourly flow: $10{,}000 \div 24 \approx 417 \text{ m}^3/\text{h}$.
2. Area: $417 \div 0.20 \approx 2083 \text{ m}^2$, about half a hectare.
3. That large area is why cities later switched to rapid filters running 20–50 times faster with chemical pretreatment.

Answer: About 2,100 m^2.

PRACTICE

1. Easy List the main steps of conventional drinking-water treatment in order.

Show answer

Screening, coagulation, flocculation, sedimentation, filtration, disinfection (and chemical adjustment).

2. Easy What did John Leal do in 1908?

Show answer

He had Jersey City's water supply continuously disinfected with chlorine, the first large U.S. public supply treated this way.

3. Easy A town of 8,000 uses 120 gallons per person per day. Find its demand in MGD.

Show answer

0.96 MGD.

4. Medium Convert 0.96 MGD to ft^3/s.

Show answer

$0.96 \times 1.547 \approx 1.49\ \mathrm{ft}^3/\mathrm{s}$.

5. Medium A pipe 0.30 m in diameter carries water at 1.5 m/s. Find the flow in m^3/s and m^3/day.

Show answer

$A \approx 0.0707\ \mathrm{m}^2$; $Q \approx 0.106\ \mathrm{m}^3/\mathrm{s} \approx 9160\ \mathrm{m}^3/\mathrm{day}$.

6. Medium A plant treats 5 MGD with alum at 25 mg/L. How many pounds of alum a day?

Show answer

$8.34 \times 5 \times 25 \approx 1043\ \mathrm{lb/day}$.

7. Medium A settling basin holds 400,000 gallons and receives 2.4 MGD. Find its detention time in hours.

Show answer

$0.4/2.4 = 0.167\ \mathrm{day} = 4\ \mathrm{h}$.

8. Hard A fire requires 1,500 gallons per minute on top of a peak demand of 3 MGD. What total flow in ft^3/s must the main carry? What diameter keeps the velocity at 6 ft/s?

Show answer

1,500 gpm $= 2.16$ MGD; total 5.16 MGD ≈ 7.98 ft^3/s; $A = 1.33$ ft^2, $D \approx 1.30$ ft ≈ 15.6 in, so a 16 in main.

9. Hard Why does coagulation make settling so much faster?

Show answer

Settling speed grows rapidly with particle size (roughly with the square of the diameter for small particles); coagulation and flocculation turn tiny particles that would take days to settle into large flocs that settle in hours.

10. Hard Conceptual. Why must water still be disinfected after filtration, and why is a small chlorine residual kept in the pipes?

Show answer

Model answer: filters remove most particles and many microbes but not all; disinfection destroys the rest. A small residual protects the water from contamination on its way through miles of pipes to the tap.

11. Challenge A slow sand filter at 0.15 m/h and a rapid filter at 7.5 m/h each treat 20,000 m^3/day. Compare their areas.

Show answer

Hourly flow 833 m^3/h: slow filter $833/0.15 \approx 5560$ m^2; rapid filter $833/7.5 \approx 111$ m^2, 50 times smaller.

COMMON MISTAKES

- Mixing units of flow: gallons per day, MGD, gallons per minute and ft^3/s. Convert carefully.
- Assuming clear water is safe. Germs are invisible; disinfection and testing are essential.
- Using more coagulant than needed. There is an optimum dose.

> **Make it a game · Clear the Water.** Stir soil into a jar of water, then add a spoonful of alum or let it settle through a coffee filter. Predict which step clears it best: coagulation, sedimentation or filtration. Tell the story of slow sand filters and chlorination at Jersey City. Calculate detention time for a made-up tank. Point for each correct claim.

9.3 Wastewater and Stormwater: Bazalgette's London

OBJECTIVES

- Tell how Joseph Bazalgette's intercepting sewers (1859–1875) cleaned up London, and how modern wastewater and stormwater systems work.
- Estimate sewage flow, check self-cleansing velocity and calculate storm runoff with the rational method $Q = CiA$.
- Measure runoff from different surfaces in a model and design a simple detention or infiltration feature.

> **Warm-up.** During the next rain, look at where water goes in your neighborhood: gutters, downspouts, storm drains, ditches, puddles, lawns. Sketch the path from your roof to the nearest stream or drain.

WATCH

Where Does Stormwater Go?

Practical Engineering · 11:47

https://www.youtube.com/watch?v=wdcXmerZWDc

> **WHEN TO WATCH**
> After the core ideas. Note the difference between combined and separate sewers and how detention ponds work.

Note: Includes a short sponsor segment.

KEY TERMS

sanitary sewer
A pipe system that carries wastewater from homes and businesses to a treatment plant.

storm sewer
A system of inlets and pipes that carries rainwater runoff to streams or ponds.

combined sewer
An older system carrying both sewage and stormwater in the same pipes, as in Bazalgette's London.

intercepting sewer
A large sewer running across the ends of smaller sewers to catch their flow and carry it away.

runoff coefficient (C)

The fraction of rainfall that runs off a surface: about 0.9 for pavement and roofs, 0.1–0.3 for lawns.

self-cleansing velocity

The minimum flow speed, about 0.6 m/s (2 ft/s), that keeps solids from settling in a sewer.

CORE IDEAS

The story. In the hot summer of 1858, the River Thames in London was an open sewer. The stench was so bad that the windows of the Houses of Parliament were hung with curtains soaked in lime, and the newspapers called it the Great Stink. Parliament finally acted, and the Metropolitan Board of Works' chief engineer, Joseph Bazalgette, was given the task he had already planned. From 1859 to 1875 his men built about 82 miles of great brick **intercepting sewers** running east along both banks of the river, catching the flow of more than 1,100 miles of new street sewers and carrying it far downstream to outfalls at Beckton and Crossness. Pumping stations lifted it where gravity was not enough. The work used some 318 million bricks and new, stronger Portland cement, and along the river it built the Victoria, Albert and Chelsea Embankments, with roads and gardens on top. Outbreaks of waterborne disease in central London ended.

How they figured it out. Bazalgette calculated the flow each sewer would carry from the population it served and the rain that would fall on its district, and then, it is said, doubled the diameter, reasoning that the job would only be done once and the city would grow. He was right: London's population more than doubled and his sewers still serve it. He tested cement samples for strength, a new practice, and specified it carefully. His system was combined: in heavy rain, overflows discharged into the Thames to keep sewage from backing up into homes, a problem modern London has addressed with a new "super sewer" tunnel beneath the river, completed in the 2020s.

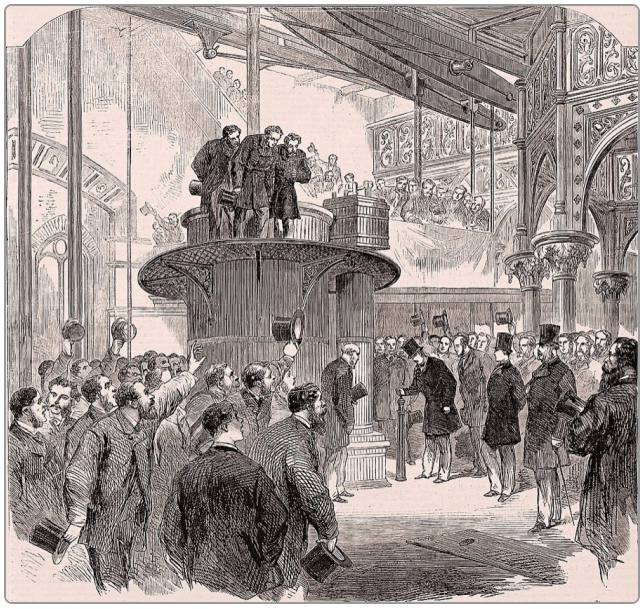
The physics. Sewers are designed for gravity flow, sloping steadily downhill (Lesson 9.1). Wastewater flow is roughly the population times about 100 gallons per person per day, with peaks several times the average. The pipe must be large enough for the peak flow ($Q = Av$) and steep enough that the velocity at ordinary flow stays above about 0.6 m/s (2 ft/s), the **self-cleansing velocity**, so solids do not settle. Storm runoff is estimated with the **rational method**: $Q = CiA$, where C is the runoff coefficient, i the rainfall intensity and A the area. In US units, with i in inches per hour and A in

acres, Q comes out in ft^3/s almost exactly. Paving a field raises C from perhaps 0.2 to 0.9, more than quadrupling the peak runoff.

The engineering. Today most new developments have separate sanitary and storm sewers. Wastewater goes to treatment plants that screen it, settle solids, let bacteria digest the dissolved waste, and disinfect the effluent before returning it to a river (the second half of the Animagraffs video). Stormwater is managed with inlets and pipes, but also with detention ponds that store runoff and release it slowly, and with rain gardens, swales and permeable pavement that let it soak into the ground. These systems protect homes from flooding and streams from erosion. Like the water supply, they are invisible until they fail, and they are among the most important things engineers build for the health of their neighbors.

Try it. In the build you will measure how much water runs off different surfaces and design a rain garden that holds back a storm.

PICTURE STUDY



Opening of the Main Drainage Works at Crossness, Engraving in the *Illustrated London News*, April 15, 1865 · Wood engraving · *Illustrated London News*, London

Image: Wikimedia Commons, public domain

In April 1865 the Prince of Wales started the great steam engines of Bazalgette's southern outfall pumping station at Crossness. Even a sewage works was built with beautiful ornamental ironwork, a sign of how proud the Victorians were of this achievement.

Look closely

- Find the great beam engines. What were they pumping, and why did the sewage need lifting?
- Look at the decorated iron columns and screens. Why might engineers decorate a pumping station?
- Find the dignitaries on the platform. Why would a royal visit matter for such a project?
- Look for clues to the size of the engines compared with the people.

Talk about it

1. Why did London need pumping stations if sewers flow by gravity?
2. How did intercepting sewers along the river solve the Great Stink?
3. What is a sewage works like today? How has it changed?

> **Safety first.** Use clean water and soil only. Do not go near real storm drains, culverts or ditches during storms; moving water is dangerous.

BUILD

Runoff and a Rain Garden

You will measure how much of a “rainfall” runs off different surfaces in trays, compute their runoff coefficients, and design a small rain garden or detention box that holds back the runoff from a model parking lot.

You will need

- Three or four identical baking trays or boxes lined with plastic, each with a spout at one end
- Surfaces: a smooth board or tile (pavement), packed soil, loose soil with grass sod, gravel
- A watering can with a sprinkler head, a measuring cup, a stopwatch
- Containers to catch runoff; a small box with holes and gravel for the detention model

Steps

1. Tilt each tray at the same slope (about 5%). Place a different surface in each.
2. **Predict** the order of runoff from most to least.
3. Sprinkle exactly 1 liter evenly over each tray in 1 minute. Collect the runoff for 5 minutes. Measure its volume and record when it began and when it peaked.
4. Compute each surface’s runoff coefficient: $C =$ runoff $\div$ rainfall.
5. Use your C values to compute $Q = CiA$ for a real 1-acre lot of each surface in a 2 in/h storm.
6. Build a detention model: run the pavement tray’s outflow into a box with a small outlet hole and gravel storage. Measure the peak outflow (by timing how fast a cup fills) with and without the box.
7. Build a mini rain garden: a basin of loose soil and gravel at the outlet of the pavement tray. Measure how much water soaks in.
8. **Improve.** Change the outlet hole size or the storage volume to cut the peak outflow in half. Record your design.

Tips

- Sprinkle evenly and at the same rate for every test.
- Measure the timing as well as the volume: a peak that comes later is easier to handle.
- Dry soil and wet soil behave differently; test each surface the same way.

Check your work

- ☐ I measured runoff coefficients for at least three surfaces.
- ☐ My detention or rain-garden model reduced the peak flow, and I measured how much.
- ☐ I can explain Bazalgette's system and compute storm runoff and sewage flows.

ENGINEER'S NOTEBOOK

1. A map-style sketch of Bazalgette's system: the Thames, the intercepting sewers north and south, the pumping stations and the outfalls.
2. Your runoff data, C values and the rational-method calculations for a real lot.
3. A sketch of your detention or rain-garden design with dimensions and results.

KEY FORMULAS

$Q = CiA$ (US: ft³/s with i in in/h and A in acres; SI: $Q = CiA/360$ in m³/s with i in mm/h and A in hectares)

$Q = Av, \quad v_{\text{min}} \approx 0.6 \text{ m/s } (2 \text{ ft/s})$

$V_{\text{storage}} \approx (Q_{\text{in}} - Q_{\text{out}}) \times t$

From history. Bazalgette was knighted in 1875. His great-great-grandson, a television producer, has helped tell the story. Bazalgette is buried in Wimbledon, and a monument to him on the Victoria Embankment, which his work created, carries the Latin motto *Flumini vincula posuit*, "He placed the river in chains."

Example 1. A 10-acre shopping center is almost all pavement and roofs ($C = 0.9$). A design storm has an intensity of 2.0 in/h. Find the peak runoff. What was it before development, when the land was grassland ($C = 0.25$)?

Show solution

1. Developed: $Q = CiA = 0.9 \times 2.0 \times 10 = 18 \text{ ft}^3/\text{s}$.
2. Grassland: $Q = 0.25 \times 2.0 \times 10 = 5 \text{ ft}^3/\text{s}$.
3. Development multiplied the peak by 3.6. A detention pond would be required to hold the difference and release it at no more than 5 ft^3/s.

Answer: 18 ft^3/s developed; 5 ft^3/s as grassland.

Example 2. For Example 1, if the storm lasts 30 minutes and the pond releases 5 ft^3/s while 18 ft^3/s comes in, roughly how much storage is needed?

Show solution

1. Difference: $18 - 5 = 13 \text{ ft}^3/\text{s}$.
2. Over 30 minutes: $13 \times 1800 = 23{,}400 \text{ ft}^3$.
3. A pond 100 ft × 80 ft would need to be about $23{,}400 \div 8000 \approx 2.9$ ft deep (a simplified estimate; real ponds are designed with full storm hydrographs).

Answer: About 23,400 ft^3 (roughly 3 ft deep over 8,000 ft^2).

Example 3. A sewer serves 4,000 people at 100 gallons per person per day, with a peaking factor of 3. Find the peak flow in ft^3/s. If an 8 in pipe flowing full at 2.5 ft/s carries 0.87 ft^3/s, is it big enough?

Show solution

1. Average: $4000 \times 100 = 400{,}000 \text{ gal/day} = 0.4 \text{ MGD} \approx 0.62 \text{ ft}^3/\text{s}$.
2. Peak: $3 \times 0.62 \approx 1.86 \text{ ft}^3/\text{s}$.
3. The 8 in pipe's 0.87 ft^3/s is too small; a 12 in pipe at a similar slope (about 2.6 ft^3/s) would be chosen.

Answer: Peak about 1.9 ft^3/s; the 8 in pipe is too small.

PRACTICE

1. Easy What was the Great Stink, and what did it lead to?

   Show answer

   The overwhelming smell of the sewage-filled Thames in the hot summer of 1858; it led Parliament to fund Bazalgette's intercepting sewer system.

2. Easy What is the difference between a combined and a separate sewer system?

   Show answer

   A combined system carries sewage and stormwater in the same pipes; a separate system has one set of pipes for sewage and another for stormwater.

3. Easy Find the peak runoff from 5 acres of lawn ($C = 0.2$) in a 3 in/h storm.

   Show answer

   $0.2 \times 3 \times 5 = 3\ \mathrm{ft}^3/\mathrm{s}$.

4. Medium The same 5 acres is paved ($C = 0.9$). Find the peak runoff and the ratio to lawn.

   Show answer

   13.5 ft^3/s; 4.5 times.

5. Medium In SI, find Q for a 2-hectare parking lot ($C = 0.9$) with a rainfall intensity of 50 mm/h.

   Show answer

   $Q = 0.9 \times 50 \times 2/360 = 0.25\ \mathrm{m}^3/\mathrm{s}$.

6. Medium A sewer 0.30 m in diameter flows full at 0.8 m/s. Find its flow. Is it self-cleansing?

   Show answer

   $A \approx 0.0707$; $Q \approx 0.057\ \mathrm{m}^3/\mathrm{s}$; yes, 0.8 m/s exceeds 0.6 m/s.

7. Medium A town of 12,000 people produces 100 gallons of wastewater per person per day. Find the average flow in MGD and the peak at a factor of 2.5.

   Show answer

   1.2 MGD; peak 3.0 MGD.

8. Hard A developer must keep the peak runoff from a 6-acre site at its grassland value ($C = 0.3$) for a 2.5 in/h storm. After development, $C = 0.75$. Find both peaks and the storage needed for a 40-minute storm (simplified).

Show answer

Grassland $0.3 \times 2.5 \times 6 = 4.5$; developed $11.25\ \text{ft}^3/\text{s}$; storage $\approx 6.75 \times 2400 = 16{,}200\ \text{ft}^3$.

9. Hard Why did Bazalgette's decision to enlarge the sewers prove so wise?

Show answer

London's population and paved area grew far beyond what he planned for; larger sewers had spare capacity, so the system still works over 150 years later instead of needing early replacement.

10. Hard Conceptual. How do rain gardens and permeable pavement reduce flooding, and why is that sometimes better than bigger pipes?

Show answer

Model answer: they let water soak into the ground where it falls, reducing both the volume and the peak of runoff; bigger pipes only move the flood faster downstream, while infiltration protects streams from erosion and recharges groundwater.

11. Challenge The rational method assumes the whole area contributes at once, which happens only when the storm lasts at least as long as the time of concentration t_c (the time for water to flow from the farthest point). If t_c for a large site is 30 min and intensity for a 30-min storm is 2.4 in/h, but for a 10-min storm is 4.0 in/h, which intensity should be used, and why?

Show answer

2.4 in/h, the intensity for a duration equal to t_c: a shorter, more intense storm ends before water from the far parts of the site arrives, so not all the area contributes at its peak.

COMMON MISTAKES

- Using the wrong units in $Q = CiA$. In US units, i in in/h and A in acres give ft^3/s; in SI, divide by 360.
- Sizing a sewer for average flow instead of peak flow.
- Designing a pipe with too little slope, so solids settle and it clogs.

Make it a game · Runoff Rationale. Pour a cup of water over sponge, bare soil and a plate, and predict which sheds the most runoff. Calculate Q = CiA with a coefficient for each. Then explain why Bazalgette built intercepting sewers. Your parent plays city council and asks one hard question. Answer from memory to earn the point.

9.4 Roads and Pavement: The Romans and McAdam

OBJECTIVES

- Trace road building from Roman layered roads to Telford's and McAdam's methods and modern asphalt and concrete pavements.
- Calculate pavement quantities, cross slopes and the damage done by heavy axles with the fourth-power law.
- Test how base thickness and drainage affect a model road on soft ground.

> **Warm-up.** Look closely at a road surface. Find cracks, potholes or ruts. Where are they worst: in the wheel paths, at the edges, where water stands? Write down your guess for why roads fail.

WATCH

How Are Highways Designed?

Practical Engineering · 12:21

https://www.youtube.com/watch?v=9XIjqdk69O4

> **WHEN TO WATCH**
> After the core ideas. Listen for how design speed sets curves, slopes and sight distances; you will use this again in Lesson 9.5.

KEY TERMS

subgrade

The natural soil beneath a road, which ultimately carries every load.

base and subbase

Layers of compacted crushed stone or gravel that spread wheel loads and drain water.

macadam

John McAdam's road of small, angular broken stones compacted in layers on a well-drained, crowned subgrade (1810s–1820s).

flexible and rigid pavements

Asphalt pavements flex and spread load through their layers; concrete pavements are stiff slabs that bridge over weak spots.

cross slope (crown)

The sideways slope of a road surface, about 2%, that sheds water to the edges.

ESAL

Equivalent single-axle load: the number of standard 18,000 lb axle passes that would do the same damage as a given traffic mix.

CORE IDEAS

The story. In 312 BC the Roman censor Appius Claudius began a road from Rome southward to Capua: the Appian Way, the "queen of roads." Roman engineers dug down to firm ground and built in layers: large stones, then rubble in mortar, then gravel and lime, and on top fitted paving stones, crowned so water ran off into ditches. Their roads lasted for centuries. After Rome fell, road building in Europe declined for over a thousand years. In the early 1800s two Scots revived it. Thomas Telford (Lesson 6.2) built roads on a foundation of large, hand-set stones. John Loudon McAdam, surveyor of roads for Bristol from 1816, found a cheaper way: keep the soil beneath dry by raising and crowning it, then build up layers of small, angular broken stone, none larger than about 2 inches, which traffic compacted into a solid crust. In 1823 the Boonsborough Turnpike in Maryland became the first macadam road in America.

How they figured it out. McAdam's great insight was that the natural soil carries the load, provided it is kept dry; the stone layer's job is to spread the load and keep the water out. Angular stones lock together under traffic, while round stones roll. When cars arrived, their rubber tires sucked the dust out of macadam roads, so builders sprayed tar or bound the stones with asphalt: in 1902 the English engineer Edgar Purnell Hooley patented "tarmac." Then, from 1958 to 1960, the American Association of State Highway Officials built test loops near Ottawa, Illinois, and ran loaded trucks over them for two years, day and night, measuring how each kind of pavement wore out. The AASHO Road Test showed that pavement damage grows roughly with the **fourth power** of the axle load.

The physics. A wheel load spreads out as it passes down through the pavement layers, roughly like a cone, so the pressure on the subgrade is much smaller than the tire pressure. Water is the enemy: saturated subgrade soils lose strength (Lesson 8.1), and water that freezes in cracks expands and breaks the pavement apart. So roads are crowned with a cross slope of about 2% and drained with ditches and underdrains. The fourth-power law means that one axle twice as heavy does about

$2^4 = 16$ times the damage: a single 18,000 lb truck axle does as much damage as thousands of car axles.

The engineering. Pavement engineers design the thickness of each layer from the traffic expected over the road's life (in ESALs), the strength of the subgrade, the climate and the materials. They choose flexible (asphalt) or rigid (concrete) pavement, specify compaction, and design drainage. Highway engineers also design the road's alignment: grades, curves, sight distances and lane widths for the design speed. Roads are some of the most used structures engineers make, and maintaining them faithfully, sealing cracks and fixing drainage before small problems become potholes, saves enormous cost.

Try it. In the build you will press a model wheel load on roads built with different base thicknesses over soft wet soil, and test how a crown sheds water.

PICTURE STUDY

The First American Macadam Road, 1823, Carl Rakeman, painted 1920s · Oil painting (for the U.S. Bureau of Public Roads) · Federal Highway Administration

Image: Wikimedia Commons, public domain

In the 1920s the Bureau of Public Roads commissioned the artist Carl Rakeman to paint scenes from the history of American roads. This one imagines the building of the Boonsborough Turnpike in Maryland in 1823, the first macadam road in the United States.

Look closely

- Find the workers breaking stone. How big are the pieces they are making?
- Find the road surface. Is it flat or crowned?
- Find the man in the top hat. What might he be checking?
- Look at the ground beside the road. Is there a ditch for drainage?

Talk about it

1. Why did McAdam insist on small, angular stones?
2. Why does keeping the soil beneath the road dry matter so much?
3. Which modern machines do the jobs these men are doing by hand?

Safety first. Wear safety glasses when pressing loads on the model road, and keep fingers out from under the weight.

BUILD

A Road on Soft Ground

You will build model roads with different thicknesses of gravel base over soft, wet soil, press a "wheel" into them with a measured load, and measure the rut. Then you will test how a crowned surface sheds water compared with a flat one.

You will need

- A long plastic tub, wet clay-rich soil or modeling sand for the subgrade
- Gravel or aquarium stones (angular if possible) and some round pebbles or marbles for comparison
- A short length of dowel or a toy wheel on a block as the wheel, weights or a bucket of water, a kitchen scale or bathroom scale
- A ruler and a watering can

Steps

1. Fill the tub with 8 cm of wet, soft soil and smooth it. Divide it into four lanes with strips of cardboard.
2. Leave lane 1 bare. Cover lanes 2, 3 and 4 with 1, 2 and 4 cm of compacted angular gravel. In a fifth test, use 2 cm of round pebbles.
3. **Predict** how the rut depth will change with base thickness, and whether round stones will do as well as angular.
4. Press the wheel straight down on each lane with the same load (for example 5 kg) for 10 seconds. Measure the rut depth.
5. Roll the loaded wheel back and forth 20 times over each lane and measure the rut again.
6. Graph rut depth against base thickness.
7. **Drainage.** Build two small gravel "roads" on soil, one flat and one crowned at about 4%. Sprinkle water on each and observe where it goes. Then test the wheel on each.
8. **Improve.** With the same amount of gravel as lane 3, design a better road (add a cloth separator between soil and gravel, a crown, or side ditches) and test it.

Tips

- Make the soil equally wet and soft in every lane.
- Compact the gravel firmly; loose gravel ruts easily.
- Read rut depth from a straightedge laid across the lane.

Check your work

- ☐ My data show how base thickness reduces rutting on soft soil.
- ☐ I compared angular and round stones and crowned and flat roads.
- ☐ I can compute ESALs, crown height and pavement quantities.

ENGINEER'S NOTEBOOK

1. Cross sections of a Roman road, a macadam road and a modern asphalt highway, with each layer labeled.
2. Your rut-depth table and graph.
3. A paragraph on the AASHO Road Test and the fourth-power law.

KEY FORMULAS

$$\text{ESAL} = \left(\frac{P}{18{,}000\ \text{lb}}\right)^4 \text{ (single axle, approximate)}$$

$$\text{crown rise} = \text{cross slope} \times \text{half-width}$$

$$V = L \times W \times t, \quad m = \rho V$$

From history. McAdam spent much of his own money on his road experiments and was later repaid by Parliament. His method gave English the verb "to macadamize," and the word tarmac, from tar-macadam, is still used for airport aprons and runways.

Example 1. Using the fourth-power law, compare the pavement damage of a 24,000 lb single axle and a 2,000 lb car axle with the standard 18,000 lb axle. How many car axles equal one standard axle?

Show solution

1. 24,000 lb: $(24/18)^4 = (1.333)^4 \approx 3.16$ standard axles.
2. 2,000 lb: $(2/18)^4 = (0.111)^4 \approx 0.000152$.
3. Car axles per standard axle: $1 \div 0.000152 \approx 6560$.
4. A single heavy axle does as much damage as thousands of cars, which is why axle weights are limited by law and enforced at weigh stations.

Answer: 3.16 ESAL for the 24,000 lb axle; about 6,560 car axles equal one 18,000 lb axle.

Example 2. A two-lane road is 24 ft wide with a 2% cross slope from the center line. How much higher is the crown than the edges?

Show solution

1. Half-width: 12 ft.
2. Rise: $0.02 \times 12 = 0.24$ ft ≈ 2.9 in.

Answer: About 2.9 in.

Example 3. How many tons of asphalt are needed to pave 1 mile of a 24 ft road 4 in thick? (Compacted asphalt weighs about 145 lb/ft^3.)

Show solution

1. Volume: $5280 \times 24 \times (4/12) = 42{,}240$ ft^3.
2. Weight: $42{,}240 \times 145 \approx 6.12 \times 10^6$ lb.
3. Tons: $6.12 \times 10^6 \div 2000 \approx 3060$ tons, about 120 truckloads of 25 tons.

Answer: About 3,060 tons.

PRACTICE

1. Easy What was McAdam's key idea?

   Show answer

   Keep the natural soil dry (raised and crowned) so it carries the load, and cover it with layers of small, angular broken stone that lock together into a hard surface.

2. Easy Find the crown rise of a 36 ft road (center crown) at a 2% cross slope.

   Show answer

   $0.02 \times 18 = 0.36 \text{ ft} \approx 4.3 \text{ in.}$

3. Easy How many times more damage does an axle twice the standard weight do?

   Show answer

   $2^4 = 16$ times.

4. Medium Find the ESAL of a 12,000 lb single axle.

   Show answer

   $(12/18)^4 \approx 0.198.$

5. Medium A truck has a 10,000 lb steer axle and two 18,000 lb single axles (simplified). Find its total ESAL.

   Show answer

   $(10/18)^4 + 1 + 1 \approx 0.095 + 2 = 2.10.$

6. Medium A gravel base 8 in thick is placed on a road 30 ft wide and 2,000 ft long. Find its volume in cubic yards.

   Show answer

   $2000 \times 30 \times (8/12) = 40{,}000 \text{ ft}^3 \approx 1481 \text{ yd}^3.$

7. Medium In metric, find the mass of asphalt for a road 1 km long, 7.3 m wide and 0.10 m thick at 2,350 kg/m^3.

   Show answer

   730 m^3; $1.72 \times 10^6 \text{ kg} \approx 1720$ tonnes.

8. Hard A road sees 500 trucks a day each of 2.1 ESAL. How many ESALs over a 20-year design life (365 days/yr)? Why are cars usually ignored in this calculation?

Show answer

$500 \times 2.1 \times 365 \times 20 \approx 7.67 \times 10^6$ ESAL. Cars, at about 0.0003 ESAL each, add almost nothing by comparison.

9. Hard Why do potholes appear mostly in late winter and early spring?

Show answer

Water seeps into cracks and the base, freezes and expands, then thaws, leaving voids and a weak, wet base; traffic then breaks the weakened pavement into holes.

10. Hard Conceptual. Why are angular stones better than round stones for a road base?

Show answer

Model answer: angular stones interlock and resist sliding past each other, so the layer is stiff and spreads load; round stones roll and shift, so the base ruts and deforms.

11. Challenge Two states allow 20,000 lb and 22,400 lb single axles. By what percent does the higher limit increase pavement damage per axle?

Show answer

$(22.4/20)^4 = 1.12^4 \approx 1.574$, about 57% more damage per axle.

COMMON MISTAKES

- Thinking the pavement surface carries the load. The layers spread it to the subgrade, which ultimately carries it.
- Ignoring drainage. Most road failures begin with water.
- Treating damage as proportional to weight; it grows about as the fourth power of axle load.

> **Make it a game · Road Layers Layered.** Build a road in a loaf pan with dirt, gravel and sand. Press a toy car through it and predict which layer fails first. Name each layer from subgrade to surface. Then use the fourth-power law: if the axle load doubles, how much more damage? Sixteen times. Closest prediction scores. Beat your own streak.

> **Scripture.** “The voice of him that crieth in the wilderness, Prepare ye the way of the LORD, make straight in the desert a highway for our God. Every valley shall be exalted, and every mountain and hill shall be made low: and the crooked shall be made straight, and the rough places plain:” (Isaiah 40:3-4, KJV)
>
> Isaiah’s picture of preparing a highway, filling valleys, cutting hills and straightening curves, is exactly what road builders do. The prophet uses it to describe preparing hearts for the coming of the Lord.

9.5 Railways and Traffic Engineering

OBJECTIVES

- Tell how Stephenson's Rocket and the first railways set the pattern for gentle grades and curves, and how traffic engineering grew with the automobile.
- Calculate superelevation on curves, stopping sight distance, signal yellow time, and traffic flow from density and speed.
- Conduct a safe traffic count and speed study and compute flow, speed and density.

> **Warm-up.** On a car ride, notice what happens on a highway curve: does the road tilt? Notice how far ahead you can see before a hill or curve. Write down where you felt the car lean and why.

WATCH

How Do Traffic Signals Work?

Practical Engineering · 12:37

https://www.youtube.com/watch?v=DP62ogEZgkI

> **WHEN TO WATCH**
> After the traffic-engineering paragraph. Note how detectors and timing plans adapt to traffic.

KEY TERMS

grade

The slope of a road or railway along its length, in percent; mainline railways are usually kept below about 1–2%.

superelevation

The banking of a curve, tilting the road or track toward the inside so part of the needed sideways force comes from the slope.

stopping sight distance

The distance a driver needs to see ahead to perceive a hazard, react and brake to a stop.

flow, density and speed

Flow q (vehicles per hour), density k (vehicles per kilometer per lane), and speed v, linked by $q = kv$.

capacity

The greatest flow a lane or road can carry, roughly 2,000 vehicles per hour per freeway lane.

yellow change interval

The duration of the yellow light, set so a driver can either stop safely or clear the crossing.

CORE IDEAS

The story. In October 1829, the directors of the new Liverpool and Manchester Railway held trials at Rainhill to choose a locomotive. George Stephenson and his son Robert entered the *Rocket*, with a multi-tube boiler and a blast pipe that drew the fire hard. It was the only engine to complete the trials and reached about 29 mph, faster than any person had traveled on land. The railway opened in 1830, the first intercity line run entirely by steam, and within a generation railways crossed every continent. A century later, the automobile brought a new problem: crowded streets. In 1914 Cleveland, Ohio, installed one of the first electric traffic signals; in 1923 the inventor Garrett Morgan patented a signal with a warning position between stop and go; and in the 1930s Bruce Greenshields photographed traffic from above and measured the relation between how crowded a road is and how fast it moves.

How they figured it out. Steel wheels on steel rails roll with very little friction, so a locomotive can pull a long, heavy train, but only up gentle grades: the same low friction means little grip. Railway engineers therefore built long cuttings, embankments, tunnels and viaducts to keep grades low, and laid out curves of large radius, banked so trains could take them at speed. Greenshields found that as traffic density rises, speed falls, and flow rises to a maximum and then falls, the familiar experience of a jam forming. Traffic engineers have used such measurements ever since to time signals, design ramps and set speed limits.

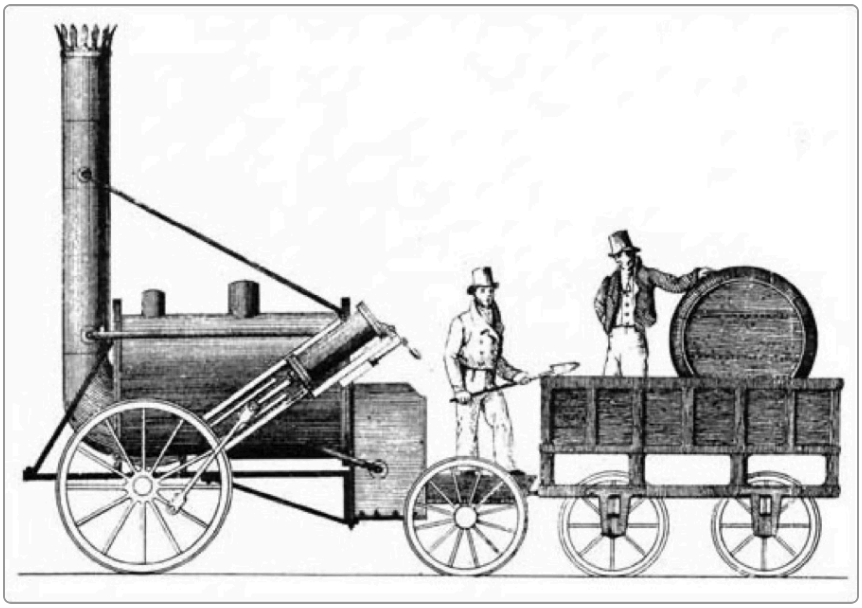
The physics. A vehicle going around a curve of radius R at speed v needs a sideways (centripetal) acceleration v^2/R. On a banked road, part of it comes from the bank and part from tire friction: $e + f = \dfrac{v^2}{gR}$, where e is the superelevation (a slope, such as 0.08) and f the side-friction factor drivers can use comfortably (about 0.10–0.17, lower at high speeds). To stop, a driver needs time to react ($t \approx 2.5$ s for design) and distance to brake: $d = vt + \dfrac{v^2}{2a}$, with a comfortable deceleration $a \approx 3.4\ \mathrm{m/s^2}$. Yellow lights follow similar physics: $y = t + \dfrac{v}{2a}$, about 1 s of reaction plus braking time. Traffic flow follows $q = kv$; in Greenshields's model, speed falls linearly with

density, and the greatest flow happens at half the jam density and half the free speed: $q_{\max} = \dfrac{v_f k_j}{4}$.

The engineering. Transportation engineers design roads and railways for safety first: sight distances, curve radii, banking, lane widths, signals and signs, all chosen for a design speed. They study crashes to find dangerous spots and fix them; modern roundabouts, for example, greatly reduce serious crashes at busy crossroads. They time signals in coordinated plans so that platoons of cars meet green lights, and they plan transit and rail. Every rule of the road is an engineering judgment meant to bring people home safely.

Try it. In the build you will count and time traffic from a safe place and compute flow, speed and density.

PICTURE STUDY



Stephenson's Rocket, Unknown engraver, 19th century · Engraving · Wikimedia Commons

Image: Wikimedia Commons, public domain

This side view shows the *Rocket* much as it ran at Rainhill in 1829. Look at it as an engineer and find the parts that made it fast.

Look closely

- Find the large driving wheels and the cylinder and rod that turn them.
- Find the tall chimney. The blast pipe inside it used exhaust steam to draw air through the fire. Why would that help?
- Find the tender with its barrel. What does it carry?
- Compare the size of the wheels with the men. How fast must the wheels turn at 29 mph?

Talk about it

1. Why can a locomotive pull a heavy train on level track but struggle on a steep grade?
2. Why did railway builders spend so much on cuttings, tunnels and viaducts?
3. How is a railway curve designed like a highway curve?

Safety first. Count traffic only from a safe place well back from the road, such as a front yard, a sidewalk set far back, or a window, with an adult. Never stand in or at the edge of a road, and never try to stop or signal drivers.

BUILD

A Traffic Study

You will count vehicles on a nearby street from a safe place, time them over a measured distance to find their speeds, and compute flow, average speed and density. If you have a toy car track, you will also test how banking lets cars take a curve faster.

You will need

- A tally counter or notebook, a stopwatch or phone, a tape measure
- Two landmarks along the road a known distance apart (for example two mailboxes or utility poles 50 m apart, measured along the sidewalk)
- Optional: a toy car track with a curve you can bank, books to tilt it, and a ramp for a starting speed

Steps

1. With an adult, choose a safe viewing spot. Measure the distance between two landmarks along the sidewalk.
2. Count all vehicles passing in one direction for 15 minutes. Multiply by 4 for the hourly flow q.
3. Time 20 vehicles between the two landmarks. Compute each speed ($v = d/t$) and the average speed in km/h.
4. Compute the density: $k = q/v$.
5. **Predict** how the numbers will change at a busier time. Repeat the study at a busy hour and a quiet hour.
6. Compare your flow with the roughly 2,000 vehicles per hour per lane capacity of a freeway lane. How close is your street to its capacity?
7. **Optional banking test.** Release a toy car from increasing heights on a ramp into a flat curve until it flies off. Then bank the curve and repeat. Compute the speeds from $v = \sqrt{2gh}$ (ignoring friction) and compare.
8. **Improve.** Using your data, suggest one change (a signal timing, a crosswalk, a turning lane) and explain how it would help safety or flow.

Tips

- Count in short blocks (5 minutes) so a mistake does not ruin the whole count.
- Start and stop the stopwatch as the front of the vehicle passes each landmark.
- Note the weather and time of day; they affect traffic.

Check your work

- ☐ I collected flow and speed data safely and computed density.
- ☐ I can calculate superelevation, stopping sight distance and yellow time.
- ☐ I can explain why railways need gentle grades and why roads are banked on curves.

ENGINEER'S NOTEBOOK

1. A diagram of a car on a banked curve with forces: weight, normal force, friction, and the centripetal direction.
2. Your traffic data table: counts, times, speeds, flow, density, at two times of day.
3. A sketch of a stopping-sight-distance problem: a driver approaching a hill crest, with the distance needed.

KEY FORMULAS

$e + f = \dfrac{v^2}{gR}$ (SI); $\quad e + f = \dfrac{V^2}{15R}$ (US, V in mph, R in ft)

$d_{\text{stop}} = vt + \dfrac{v^2}{2a}$

$y = t + \dfrac{v}{2a}$

$q = kv, \quad q_{\max} = \dfrac{v_f k_j}{4}$

From history. George Stephenson, the son of a colliery fireman, could not read until he was eighteen. He set the gauge of his railways at 4 ft 8½ in, the width of the colliery wagonways he knew, and that "standard gauge" is still used on most of the world's railways today.

Example 1. A highway curve has a radius of 400 m and a design speed of 100 km/h. With a side-friction factor of 0.12, what superelevation is needed?

Show solution

1. Speed: $100 \div 3.6 \approx 27.8$ m/s.
2. $v^2/(gR) = 27.8^2 \div (9.81 \times 400) = 772 \div 3924 \approx 0.197$.
3. $e = 0.197 - 0.12 \approx 0.077$, about 7.7% (designers would use 8%, a common maximum).

Answer: About 7.7%, rounded up to 8%.

Example 2. Find the stopping sight distance at 100 km/h with a reaction time of 2.5 s and a deceleration of 3.4 m/s^2.

Show solution

1. Reaction distance: $27.8 \times 2.5 \approx 69.4$ m.
2. Braking distance: $27.8^2 \div (2 \times 3.4) = 772 \div 6.8 \approx 113.5$ m.
3. Total: about 183 m. A driver must be able to see that far ahead over hills and around curves.

Answer: About 183 m.

Example 3. Greenshields's model for a lane has a free speed of 70 km/h and a jam density of 120 vehicles per km. Find the maximum flow and the speed and density at which it occurs.

Show solution

1. $q_{\max} = v_f k_j / 4 = 70 \times 120 \div 4 = 2100$ vehicles per hour.
2. It occurs at half the free speed, 35 km/h, and half the jam density, 60 vehicles/km.
3. Check: $q = kv = 60 \times 35 = 2100$. ✓ If more cars crowd in, speed falls faster than density rises, and flow drops: a jam.

Answer: 2,100 veh/h at 35 km/h and 60 veh/km.

Example 4. Find the yellow interval for a 45 mph approach (66 ft/s) with a 1.0 s reaction time and a deceleration of 10 ft/s^2.

Show solution

1. $y = t + v/(2a) = 1.0 + 66 \div 20 = 1.0 + 3.3 = 4.3$ s.

Answer: About 4.3 s.

PRACTICE

1. Easy Why are mainline railway grades kept low?

   Show answer

   Steel wheels on steel rails have little grip, so locomotives cannot pull heavy trains up steep slopes.

2. Easy A street carries 150 vehicles in 10 minutes in one lane. Find the hourly flow.

   Show answer

   900 vehicles per hour.

3. Easy If the flow is 900 veh/h and the average speed 45 km/h, find the density.

   Show answer

   20 vehicles per km.

4. Medium A curve of radius 250 m has a design speed of 80 km/h and $e = 0.08$. Find the side friction needed.

   Show answer

   $v = 22.2$ m/s; $v^2/(gR) = 494/2453 \approx 0.201$; $f \approx 0.12$.

5. Medium In US units, find the minimum radius for 60 mph with $e = 0.08$ and $f = 0.12$.

   Show answer

   $R = V^2/(15(e + f)) = 3600/3.0 = 1200$ ft.

6. Medium Find the stopping sight distance at 60 km/h ($t = 2.5$ s, $a = 3.4$ m/s^2).

   Show answer

   $v = 16.7$ m/s; $41.7 + 40.9 \approx 82.6$ m.

7. Medium Find the yellow interval at 35 mph (51.3 ft/s) with $t = 1.0$ s and $a = 10$ ft/s^2.

Show answer

$1.0 + 51.3/20 \approx 3.6$ s.

8. Hard A Greenshields lane has $v_f = 100$ km/h and $k_j = 125$ veh/km. Find the capacity. At a density of 90 veh/km, what are the speed and flow?

Show answer

$q_{\max} = 100 \times 125/4 = 3125$ veh/h (the model overestimates real capacity). At $k = 90$: $v = 100(1 - 90/125) = 28$ km/h; $q = 90 \times 28 = 2520$ veh/h.

9. Hard At what speed does a curve of radius 300 m with $e = 0.06$ need no friction at all?

Show answer

$v = \sqrt{egR} = \sqrt{0.06 \times 9.81 \times 300} \approx 13.3$ m/s, about 48 km/h.

10. Hard Conceptual. Why does adding one more car to a crowded freeway sometimes reduce the total flow?

Show answer

Model answer: beyond the critical density, speed drops faster than density rises, so $q = kv$ decreases; drivers brake and space out unevenly, and a jam forms that moves fewer cars per hour.

11. Challenge Show from $d = vt + v^2/(2a)$ that doubling speed more than doubles stopping distance. Compute both at 15 m/s and 30 m/s with $t = 2.5$ s and $a = 3.4$ m/s^2.

Show answer

At 15 m/s: $37.5 + 33.1 = 70.6$ m. At 30 m/s: $75 + 132.4 = 207.4$ m, about 2.9 times as far, because the braking part grows with v^2.

COMMON MISTAKES

- Using speed in km/h in formulas that need m/s. Divide km/h by 3.6.
- Forgetting reaction distance in stopping sight distance.
- Thinking more lanes always cure congestion. Demand often grows to fill new capacity, and bottlenecks move.

> **Make it a game · Stopping Sight Drill.** Quick-fire drill. Your parent calls a speed in miles per hour and a road condition. You estimate stopping sight distance, yellow-light time or superelevation in ten seconds, writing the formula. Wrong answers cost a point. Finish with a two-minute story of Stephenson's Rocket and why railways wanted gentle grades. Personal record to beat.

UNIT 9

Water and Transportation

Civil engineering began as the work of the public: surveying the land, bringing water in, carrying wastewater away and building roads. In this unit you will follow those works from the Romans to today: the groma and the GPS; the slow sand filter, Croton's aqueduct and John Leal's chlorine; Bazalgette's sewers under London; the Appian Way, McAdam's roads and the AASHO Road Test; and Stephenson's Rocket and the science of traffic.

These are the systems most people never notice until they fail, and they serve more people than any single building. You will survey your yard with a homemade water level, clean muddy water, measure runoff, rut a model road, and count traffic, using the same calculations professional engineers use: slopes and closure errors, flows and doses, the rational method, the fourth-power law, banking and stopping distances.

PREREQUISITES

- Units 1–8: units, pressure, flow, effective stress and drainage
- Algebra: rates and proportions, powers, square roots
- Trigonometry: sine and cosine; physics: speed, acceleration and centripetal acceleration (v^2/R)

MASTERY CHECKLIST

- ☐ I can reduce leveling notes, compute slopes and closure errors, and explain how GPS finds position.
- ☐ I can describe water treatment and compute demand, pipe size, chemical dose and detention time.
- ☐ I can explain combined and separate sewers and compute runoff with

$Q = CiA$ and sewage flows.

- ☐ I can explain road layers and drainage and compute ESALs, crown height and pavement quantities.
- ☐ I can compute superelevation, stopping sight distance, yellow time and traffic flow.

OPTIONAL KITS FOR THIS UNIT

- Arduino (Uno and starter kits): Ages 12 and up with an adult. Optional for Lesson 9.5: program a model traffic signal with LEDs and set its yellow time from the formula in the lesson. Battery or USB power only.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

9.1	**Surveying: From the Roman Groma to GPS**	3 sessions of 50 minutes
9.2	**Water Supply and Treatment**	3 sessions of 50 minutes
9.3	**Wastewater and Stormwater: Bazalgette's London**	3 sessions of 50 minutes
9.4	**Roads and Pavement: The Romans and McAdam**	3 sessions of 50 minutes
9.5	**Railways and Traffic Engineering**	3 sessions of 50 minutes

Unit 9 test

Time allowed: 50 minutes. Calculators are allowed. Show your work and units for every calculation. Convert units carefully. The test is worth 100 points.

1. **6 pts**
   A benchmark is 245.30 ft. BS = 4.12 ft; FS on P = 6.85 ft. Find the HI and P's elevation.

2. **6 pts**
   A level loop has $\sum BS = 12.564$ m and $\sum FS = 12.531$ m. Find the closure error. Should the loop be accepted if the limit is 20 mm?

3. **5 pts**
   A sewer falls 0.62 m over 155 m. Find its slope in percent.

4. **5 pts**
   Why does a GPS receiver need signals from at least four satellites?

5. 7 pts

A town of 18,000 uses 110 gallons per person per day. Find the demand in MGD and ft^3/s, and the chlorine needed per day at 1.5 mg/L.

6. 6 pts

A basin of 300,000 gallons receives 1.8 MGD. Find the detention time.

7. 6 pts

Describe John Leal's contribution to safe water and why measurement was central to it.

8. 7 pts

Find the peak runoff from 8 acres of pavement ($C = 0.9$) and from the same land as woods ($C = 0.15$), for 2.2 in/h. How much storage would a 30-minute storm need to hold the difference (simplified)?

9. 6 pts

How did Bazalgette's intercepting sewers end the Great Stink?

10. 5 pts

What is self-cleansing velocity and why does it matter?

11. 6 pts

Find the ESAL of a 30,000 lb single axle and compare it with the standard axle.

12. 5 pts

Why did McAdam raise and crown his roads?

13. 6 pts

Find the asphalt tonnage for 2 miles of road 22 ft wide and 3 in thick (145 lb/ft^3).

14. 7 pts

A curve has $R = 350$ m, design speed 90 km/h, and $f = 0.12$. Find the superelevation needed.

15. 5 pts

Find the stopping sight distance at 80 km/h ($t = 2.5$ s, $a = 3.4$ m/s^2).

16. **4 pts**

A lane carries 1,500 veh/h at 50 km/h. Find its density.

17. **8 pts**

Short essay. Water mains, sewers and roads are “invisible until they fail.” Choose two of them and explain how engineers design and maintain them to serve the public, using one historical example.

ANSWER KEY

1. HI = 249.42 ft; P = 242.57 ft.
2. +0.033 m (33 mm); it exceeds 20 mm, so the loop should be rerun.
3. 0.40%.
4. To solve for four unknowns: its three coordinates and its own clock error.
5. 1.98 MGD; ≈ 3.06 ft^3/s; $8.34 \times 1.98 \times 1.5 \approx 24.8$ lb/day.
6. $0.3/1.8$ day = 4 h.
7. In 1908 he had Jersey City's water continuously chlorinated; when the practice was challenged in court, bacterial measurements showed the treated water was safe and much cleaner, and chlorination spread nationwide, cutting waterborne disease.
8. Pavement 15.84 ft^3/s; woods 2.64 ft^3/s; difference 13.2 ft^3/s × 1800 s ≈ 23,800 ft^3.
9. They ran along both banks of the Thames, catching the outflow of the city's sewers before it reached the river in London and carrying it east to outfalls far downstream, with pumping stations lifting it where needed.
10. The minimum flow speed, about 0.6 m/s (2 ft/s), that keeps solids moving; below it, solids settle and the sewer clogs.
11. $(30/18)^4 \approx 7.7$; it does about 7.7 times the damage.
12. To keep water out of the soil beneath, so the soil stayed dry and strong enough to carry the load, and to shed rain to the ditches.
13. $10{,}560 \times 22 \times 0.25 = 58{,}080$ ft^3; $\approx 8.42 \times 10^6$ lb ≈ 4,210 tons.
14. $v = 25$ m/s; $v^2/(gR) = 625/3434 \approx 0.182$; $e \approx 0.062$, about 6%.
15. $v = 22.2$; $55.6 + 72.6 \approx 128$ m.
16. 30 veh/km.
17. Model answer: e.g., sewers are designed for peak flows with slopes for self-cleansing velocity and sized for growth (Bazalgette doubled his diameters, and London still uses them), and are inspected and relined over time; water systems use multiple treatment barriers and constant testing (Leal's chlorination), with a residual in the pipes and regular main replacement. Good design anticipates growth and failure modes, and faithful maintenance keeps small problems from becoming disasters.

PORTFOLIO PIECE: A NEIGHBORHOOD INFRASTRUCTURE STUDY

Study the public works on your street: survey the slope of the street or a ditch with your water level; trace where rainwater goes and estimate the peak runoff from your block with $Q = CiA$; find out where your water comes from and how it is treated (water utilities publish annual reports); and count traffic at two times of day.

Present your findings in a short report with a map, data tables and calculations, and recommend one improvement (a rain garden, a crosswalk, a drainage fix) with a simple estimate of what it would do. Consider sending a polite letter with your best idea to your town's engineer or public works department, with a parent's help.

LOOK FOR	WHAT IT MEANS
Field data	Slope survey, runoff observations and traffic counts collected safely and recorded carefully.
Calculations	Slope, runoff, water demand and traffic flow calculated correctly with units.
Research	Water source and treatment described accurately from a reliable source (such as the utility's annual report).
Recommendation	A practical improvement proposed and supported with numbers.
Presentation	Clear map, tables and writing.

UNIT 10

Design Studio

In this final unit you will put the whole course to work. You will design a small house for a family or a small chapel for a congregation, from the first conversation with the client to a finished presentation of drawings and a model. Each lesson is a studio step: the client and the program (with Jefferson's academical village as a model), site planning (with Olmsted and Vaux's Central Park), schematic design (with Asher Benjamin's pattern books), codes, cost and schedule (with the long pause of the Washington Monument), and the presentation (with Thornton's design for the Capitol).

Along the way you will use everything you have learned: scale drawings, plans, elevations and sections; proportion and precedent; loads, beams and foundations; daylight, sun angles and drainage; exits and fire safety; estimates and schedules. You will finish the year as both architect and engineer, with a design you can show and explain. *Fabricando fit faber*.

PREREQUISITES

- Units 1–9 of this course: drawing, history, structures, materials, foundations and site systems
- A client (real or imagined) willing to be interviewed and to review the design
- Drawing tools, model materials and optionally SketchUp Free or Tinkercad

MASTERY CHECKLIST

- ☐ I can interview a client, write a program, compute net and gross areas and a first budget, and draw a bubble diagram.
- ☐ I can analyze a site and compute sun angles, overhangs, setbacks, FAR and grading slopes.
- ☐ I can develop a schematic design with plans, elevations, a section and a study model, and check it with calculations.
- ☐ I can prepare square-foot and quantity estimates with contingency and escalation, a schedule with a critical path, and a code checklist.
- ☐ I can present a complete design clearly and honestly with drawings and a model, and reflect on the process.

OPTIONAL KITS FOR THIS UNIT

- **SketchUp Free (web)**: Free browser-based 3D modeling for the studio design, its perspectives and sun studies; a parent creates the account.
- **KEVA planks**: Handy for quick massing studies of the studio building before the card study model.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

10.1 The Client and the Program — 3 sessions of 50 minutes

10.2 Site Planning — 3 sessions of 50 minutes

10.3	**Designing a Small House or Chapel**	3 sessions of 50 minutes
10.4	**Codes, Cost and Schedule: Counting the Cost**	3 sessions of 50 minutes
10.5	**Presenting the Design: Drawings and a Model**	3 sessions of 50 minutes

Unit 10 test

Time allowed: 50 minutes. Calculators and an architect's scale are allowed. Show your work and units for every calculation. Answer short-answer questions in complete sentences. The test is worth 100 points.

1. **5 pts**
   What is an architectural program, and why is it written before design begins?

2. **6 pts**
   A chapel program has a sanctuary for 100 at 10 ft^2 per seat, a narthex of 200 ft^2, a fellowship hall for 60 at 15 ft^2 per person, a kitchen of 200 ft^2, two classrooms of 250 ft^2, two restrooms of 180 ft^2 and storage of 120 ft^2. Find the net area, and the gross area at 1.30.

3. **6 pts**
   For Problem 2 at \$240 per ft^2, with 8% fees and 10% contingency, find the budget.

4. **6 pts**
   Find the noon solar altitude at 38° N on the summer and winter solstices.

5. **6 pts**
   At 38° N, how deep must an overhang be to shade a window whose sill is 4 ft below the overhang at summer noon?

6. **6 pts**
   A lot is 70 ft × 120 ft with setbacks 25 ft front, 7 ft sides, 20 ft rear, and FAR 0.45. Find the buildable footprint and the maximum floor area.

7. 5 pts

Why did Olmsted and Vaux sink the cross-town roads in Central Park?

8. 6 pts

A chapel seats 108 in pews 9 ft long at 18 in per person, two pews per row, 36 in apart. Find the number of rows and the seating depth.

9. 6 pts

A room is 13 ft × 16 ft. Find the minimum glass and openable window areas.

10. 6 pts

A building 30 ft wide has an 8:12 roof. Find the rafter length with no overhang.

11. 6 pts

Escalate a $1,200,000 estimate at 5% a year for 2 years and add a 7% contingency.

12. 6 pts

A slab 40 ft × 60 ft × 4 in. How many cubic yards (round up)? At $160 per yd^3, find the concrete cost.

13. 6 pts

Tasks: A site work 2 wk; B foundation 3 wk after A; C framing 4 wk after B; D roof 2 wk after C; E windows 1 wk after C; F interior 6 wk after D and E. Find the critical path, duration and float of E.

14. 6 pts

What does the Washington Monument's two-colored marble teach about project planning?

15. 6 pts

A chapel 36 ft × 56 ft: what size is its plan at ¼″ = 1′-0″, and will it fit with an elevation of the same length (24 ft tall) side by side on a 24 in × 36 in board with 2 in margins?

16. 6 pts

Why was Thornton's Capitol design later changed by professional architects?

17. **6 pts**

Short essay. Explain how a small building can meet all three of Vitruvius's tests, using examples from your own studio design.

ANSWER KEY

1. A statement of the spaces, sizes, relationships, uses, budget and goals of a building; it is written first so the design answers the client's real needs and budget, avoiding costly changes later.
2. Net $1000 + 200 + 900 + 200 + 500 + 360 + 120 = 3280\ \text{ft}^2$; gross $\approx 4264\ \text{ft}^2$.
3. $4264 \times 240 \times 1.18 \approx \$1{,}207{,}600$.
4. $90 - 38 + 23.4 = 75.4^\circ$; $90 - 38 - 23.4 = 28.6^\circ$.
5. $4/\tan 75.4^\circ \approx 4/3.83 \approx 1.04$ ft.
6. Footprint $(70 - 14)(120 - 45) = 56 \times 75 = 4200\ \text{ft}^2$; floor area $0.45 \times 8400 = 3780\ \text{ft}^2$.
7. To carry city traffic across the park out of sight and without crossing the park's paths, keeping the park safe, quiet and unbroken.
8. 6 per pew, 12 per row; 9 rows; depth 27 ft.
9. Glass $0.08 \times 208 = 16.6\ \text{ft}^2$; open $8.3\ \text{ft}^2$.
10. Run 15, rise 10: $\sqrt{225 + 100} \approx 18.0$ ft.
11. $1{,}200{,}000 \times 1.1025 \times 1.07 \approx \$1{,}415{,}600$.
12. $800\ \text{ft}^3 = 29.6\ \text{yd}^3$, order 30: $4800.
13. A–B–C–D–F = 17 weeks; E can finish at week 10 but F starts at week 11, so E has 1 week of float.
14. Work stopped for over twenty years when funds ran out at about 150 ft; when it resumed, stone from a different source was used, leaving a visible line. It shows the cost of not securing funding (counting the cost) and the difficulty of resuming a halted project.
15. Plan 9×14 in; elevation 14×6 in; side by side need 28 in of the 32 in usable width and 9 in of the 20 in height. Yes.
16. His presentation drawings conveyed an appealing idea but left practical problems unsolved; Latrobe and others had to fit real rooms, structure and systems into it.
17. Model answer: firmness through a clear load path, adequate joists and rafters, a foundation suited to the soil, bracing for wind; commodity through a program confirmed by the client, good adjacencies, daylight, exits and accessibility, and an affordable budget; delight through proportion, orientation to sun and view,

honest materials and a fitting place on the site. A good design resolves conflicts among the three rather than sacrificing one.

PORTFOLIO PIECE: THE STUDIO DESIGN: A SMALL HOUSE OR CHAPEL

Assemble the complete design project from the five studio steps: the client interview and program, the site analysis and site plan, the schematic design and study model, the estimate, schedule and code check, and the final presentation set and model.

Bind or file everything in order with a cover sheet, and include photographs of your presentation, the questions asked and your one-page reflection. This is the capstone of the course and a strong piece for a high-school portfolio or college application.

LOOK FOR	WHAT IT MEANS
Program and client	A confirmed program with areas, adjacencies and budget, reflecting a real interview.
Site and design	A site analysis and plan, and a design whose plan, elevations, section and model agree and respond to the site.
Technical checks	Structure, daylight, stairs, roof, seating or rooms, exits, code and zoning checked with correct calculations.
Cost and schedule	Two estimates with contingency and escalation, a schedule with a critical path, and value-engineering choices explained.
Presentation and reflection	Clear, honest presentation with consistent scales and a well-made model, and a thoughtful reflection on the process.

10.1 The Client and the Program

OBJECTIVES

- Interview a client and write an architectural program: the spaces, sizes, relationships and goals a building must meet.
- Compute net and gross areas, apply a grossing factor and make a first budget from cost per square foot.
- Draw adjacency (bubble) diagrams that turn the program into the beginnings of a plan.

> **Warm-up.** Imagine your family were building a new house or your church a new hall. Write down five things the building must do, three things it should do if possible, and one thing it must never do. Who should you ask to be sure your list is right?

WATCH

Architecture Short Course: How to Develop a Design Concept

30X40 Design Workshop · 19:24

https://www.youtube.com/watch?v=k4dVgbuxBAw

> **WHEN TO WATCH**
> After the core ideas. Write down how the architect moves from the client's program and site to a design concept.

Note: A practicing residential architect explains his process at his desk. Calm and practical.

KEY TERMS

client
The person or group who will own and use the building and who pays for its design and construction.

program
A written statement of what a building must contain and accomplish: the spaces and their sizes, relationships, uses, budget and goals.

adjacency
Which spaces should be next to each other, near, or kept apart.

bubble diagram
A quick drawing of spaces as circles sized roughly to their area and linked by lines showing adjacencies.

grossing factor

The ratio of gross building area to net program area, covering walls, halls, stairs and mechanical spaces; often 1.25–1.40.

design concept

The central idea that organizes a design and answers the program and site.

CORE IDEAS

The story. In his old age Thomas Jefferson set out to found a university in Virginia. He did not begin by drawing a building. He began by asking what a university was for. His answer was an "academical village": a long lawn terraced down a hill, lined on each side by ten pavilions, each a home for a professor with a classroom below, joined by colonnades in front of the students' rooms, with outer ranges and dining halls behind, and at the head of the lawn a domed Rotunda holding the library, about half the size of Rome's Pantheon. Students and teachers would live and learn together, and the library, not a chapel or a ruler's palace, would stand at the center. Classes began in 1825. Jefferson was so proud of it that his tombstone names him "Father of the University of Virginia," along with only two other accomplishments.

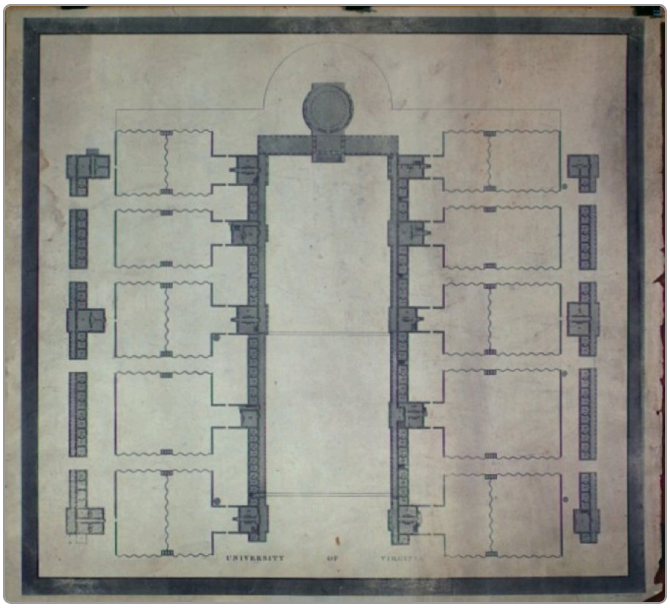
How they figured it out. Jefferson wrote letters to scholars and architects, including Benjamin Latrobe and William Thornton, asking their ideas, and he revised his scheme many times. He set out the needs (lodging, teaching rooms, a library, dining, gardens), sized them, arranged them by how they related to each other, and then gave them form, using Palladio's proportions (Lesson 3.3). That is the architect's method still: listen first, write the **program**, diagram it, and only then design. A good program records not just rooms and square feet, but how the client wants to live, work or worship, and what they can afford.

The physics. A program begins as a table of spaces with their **net** areas, often found from the number of people and a standard area per person: about 7–10 ft^2 per seat in a church or auditorium, 15 ft^2 per person at tables, about 150 ft^2 per office worker. Adding the net areas and multiplying by a **grossing factor** (often 1.25 to 1.40) estimates the gross area, which includes walls, corridors, stairs, restrooms and mechanical rooms. Multiplying the gross area by a cost per square foot for that type of building gives a first budget, to which designers add fees and contingency (Lesson 1.5). Early numbers like these let a client see whether their wishes and their budget agree before anyone draws a line.

The engineering. The program then becomes diagrams. A **bubble diagram** places each space as a circle of roughly the right size, linked to those it must be near. Some adjacencies are about use (the kitchen beside the fellowship hall), some about sound (the nursery away from the sanctuary), some about light and view, and some about structure and services (stack the restrooms, keep the mechanical room central). From diagrams come the first plans. Architects revisit the program at every stage and confirm it with the client, because a beautiful building that does not serve its people has failed the first test of commodity.

Try it. In this lesson you will choose the client and project for your Unit 10 design studio, a small house or a small chapel, interview the client, write the program and draw the bubble diagram.

PICTURE STUDY



Plan of the University of Virginia, Peter Maverick, after Thomas Jefferson, 1825 · Engraving · University of Virginia Library

Image: Wikimedia Commons, public domain

This engraved plan, made for Jefferson in 1825, shows his academical village as built. Read it as a program made visible: each kind of space is placed where its purpose belongs.

Look closely

- Find the Rotunda. Why is it at the head of the lawn?
- Count the pavilions. How are they spaced along each side?
- Find the rows of small rooms between the pavilions. Who lived there?
- Find the outer rows and the gardens between them. What might they be for?

Talk about it

1. What does this plan tell you about how Jefferson thought students and teachers should live?
2. Which spaces are next to each other, and why?
3. If you wrote the program for this university, what would be on it?

BUILD

Studio Step 1: Client, Program and Bubble Diagram

This unit is a design studio. You will design a small house for a family or a small chapel for a congregation of about 80–120 people. In this first step you choose your client, interview them and write the program.

You will need

- A notebook, graph paper, colored pencils
- A real or imagined client: a family member, your family, your pastor or a church committee
- A calculator or spreadsheet

Steps

1. Choose your project: a house (about 1,200–2,000 ft^2) or a chapel (80–120 seats).
2. Prepare ten interview questions: who will use it, what they will do there, what they love and dislike about their current building, special needs (accessibility, storage, music, hospitality), budget and timing.
3. Interview your client for 20–30 minutes and take careful notes. Read the notes back to them to confirm.
4. Write a program table: each space, the number of people, its use, its net area and any special needs (light, sound, view, plumbing).
5. Total the net area and apply a grossing factor (about 1.25 for a house, 1.30 for a chapel) to estimate the gross area.
6. Estimate a first budget with a cost per square foot your client agrees is realistic (look up local costs or ask a builder), plus fees and contingency.
7. Draw a bubble diagram: circles sized to scale for each space, with heavy lines for strong adjacencies and dashed lines for "nearby." Mark entrances and outdoor connections.
8. **Review.** Show the program, budget and diagram to your client. Record their changes and revise.

Tips

- Ask open questions ("Tell me about a typical Sunday") rather than yes-or-no questions.
- Do not draw a plan yet. Programs and diagrams first.
- Keep every version; architects learn as much from what changed as from the final answer.

Check your work

- ☐ My program lists every space with its area, use and needs, and has been confirmed by the client.
- ☐ I computed net and gross areas and a first budget.
- ☐ My bubble diagram shows the important adjacencies.

ENGINEER'S NOTEBOOK

1. Your interview questions and notes.
2. The program table with net areas, the grossing factor, gross area and first budget.
3. Your bubble diagram, and a second version after the client review.

KEY FORMULAS

$A_{\text{net}} = \sum(\text{people} \times \text{area per person})$ or $\sum A_{\text{room}}$
$A_{\text{gross}} = A_{\text{net}} \times \text{grossing factor}$
$\text{budget} = A_{\text{gross}} \times \text{cost per ft}^2 \times (1 + \text{fees} + \text{contingency})$

From history. Jefferson called the University of Virginia the last service of his life. His tombstone lists, by his own instruction, three things: author of the Declaration of Independence, author of the Virginia Statute for Religious Freedom, and father of the University of Virginia. He did not list being president.

Example 1. A small church wants: a sanctuary for 120 at 10 ft^2 per seat; a chancel of 300 ft^2; a narthex (entry) of 250 ft^2; a fellowship hall for 80 at 15 ft^2 per person; a kitchen of 250 ft^2; three classrooms of 300 ft^2 each; an office of 150 ft^2; two restrooms of 200 ft^2 each; storage of 150 ft^2; and a mechanical room of 150 ft^2. Find the net area and the gross area with a grossing factor of 1.30.

Show solution

1. Sanctuary: $120 \times 10 = 1200$; fellowship hall: $80 \times 15 = 1200$; classrooms: $3 \times 300 = 900$; restrooms: $2 \times 200 = 400$.
2. Net: $1200 + 300 + 250 + 1200 + 250 + 900 + 150 + 400 + 150 + 150 = 4950\ \text{ft}^2$.
3. Gross: $4950 \times 1.30 = 6435\ \text{ft}^2$.

Answer: Net 4,950 ft^2; gross about 6,435 ft^2.

Example 2. For the church in Example 1, construction costs about \$250 per ft^2. Add 8% for design fees and 10% for contingency. What budget should the building committee plan for?

Show solution

1. Construction: $6435 \times \$250 = \$1{,}608{,}750$.
2. Fees and contingency: $\times 1.18$.
3. Total: $\$1{,}608{,}750 \times 1.18 \approx \$1{,}898{,}000$.

Answer: About \$1.9 million.

Example 3. The committee has only \$1.5 million. Using the same costs, by how much must the gross area shrink? Suggest one program change that would do it.

Show solution

1. Allowed construction cost: $\$1{,}500{,}000 \div 1.18 \approx \$1{,}271{,}000$.
2. Allowed gross area: $\$1{,}271{,}000 \div \$250 \approx 5085\ \text{ft}^2$.
3. Reduction: $6435 - 5085 = 1350\ \text{ft}^2$ gross, or about $1350 \div 1.30 \approx 1040\ \text{ft}^2$ net.
4. One option: use the fellowship hall for Sunday-school classes, dropping two classrooms (600 ft^2 net), and reduce the sanctuary to 100 seats (200 ft^2) with room for later expansion, plus trimming storage and the narthex.

Answer: About 1,350 ft^2 less gross area (about 1,040 ft^2 of program), for example by combining uses.

PRACTICE

1. Easy What is an architectural program?

   Show answer

   A written statement of what a building must contain and do: its spaces, sizes, relationships, uses, budget and goals.

2. Easy A sanctuary seats 90 at 9 ft^2 per seat. Find its net area.

   Show answer

   810 ft^2.

3. Easy Net area is 2,000 ft^2 and the grossing factor 1.25. Find the gross area.

   Show answer

   2,500 ft^2.

4. Medium A house program: living 350, kitchen 200, dining 160, three bedrooms of 150, 140 and 130, two baths of 60, laundry 50, storage 80 (all ft^2). Find the net and the gross at 1.25.

   Show answer

   Net $= 350 + 200 + 160 + (150 + 140 + 130) + 120 + 50 + 80 = 1380$; gross $= 1725\ \text{ft}^2$.

5. Medium For Problem 4 at \$210 per ft^2, with 7% fees and 10% contingency, find the budget.

Show answer

$1725 \times 210 \times 1.17 \approx \$423{,}800.$

6. Medium Name two spaces in a chapel that should be adjacent and two that should be kept apart, with reasons.

Show answer

Adjacent: narthex and sanctuary (people gather, then enter); kitchen and fellowship hall (serving meals). Apart: nursery and sanctuary (sound), mechanical room and sanctuary (noise), or restrooms opening directly into the sanctuary.

7. Medium Why did Jefferson place the library at the head of the University of Virginia's lawn?

Show answer

Because he believed learning, represented by the library, should be the center of the university; the plan expressed his idea of what the institution was for.

8. Hard A client's budget is \$350,000 for construction only, at \$200 per ft^2. If the program's net area is 1,500 ft^2 and the grossing factor 1.25, can it be built? If not, how much net area must be cut?

Show answer

Gross needed 1875 ft^2 costs \$375,000; allowed gross $= 1750$, net $= 1400$; cut 100 ft^2 of net program.

9. Hard Why might a grossing factor for an office building be lower than one for a hospital?

Show answer

Offices have simple open floors with few corridors and small service areas; hospitals need wide corridors, many shafts and large mechanical spaces, so more gross area is needed for each square foot of usable room.

10. Hard Conceptual. Why do architects confirm the program with the client before designing?

Show answer

Model answer: design decisions build on the program; an error or misunderstanding there (missing rooms, wrong sizes, unaffordable scope) leads to wasted work or a building that does not serve its users. Confirming early is cheap; changing later is expensive.

11. Challenge A church expects to grow from 100 to 160 members over 15 years. Design the sanctuary for 100 seats now with room to add 60 later. If seats need 9 ft^2 each and the shell can be built 20% larger than needed now for an extra 6% in cost, compare that with adding a separate 60-seat wing later at 140% of the per-square-foot cost.

Show answer

Now: 900 ft^2; 20% larger shell = 1,080 ft^2 (120 seats) for 6% more, not quite enough for 160. A future 60-seat wing (540 ft^2) at 140% cost is expensive and disruptive; a better plan may be a shell sized for 160 (1,440 ft^2) with movable seating, or a planned expansion wall, chosen by comparing actual costs. (Model answer; the point is to plan for growth in the program.)

COMMON MISTAKES

- Designing before writing the program.
- Forgetting circulation, walls and mechanical space (the grossing factor) when working out size and cost.
- Listening to only one person on a committee. Programs should reflect all the people who will use the building.

> **Make it a game · Client Interview.** Role-play. Your parent plays a client who wants a small chapel or house and answers your questions honestly. Within five minutes, list the rooms, sizes and adjacencies on paper as a program. Draw a bubble diagram of how rooms connect. Compute net area, then gross with a grossing factor. Client approves or sends you back.

10.2 Site Planning

OBJECTIVES

- Analyze a site for sun, wind, slope, drainage, views, access, utilities and zoning, as Olmsted and Vaux did for Central Park.
- Calculate solar altitude at noon, size an overhang, check grading slopes, setbacks and floor area ratio.
- Draw a site analysis and place your studio building on its site with reasons.

> **Warm-up.** Stand at the place where your studio building might go (or a similar spot). Where does the sun rise and set? Where does water go when it rains? Where are the best and worst views? Which way does the wind usually blow? Note four things on a quick sketch.

WATCH

Architectural Design Process | Form, Orientation and Sunlight

30X40 Design Workshop · 9:54

https://www.youtube.com/watch?v=ly8orBNiNQM

> **WHEN TO WATCH**
> After the physics paragraph. Watch how he uses the sun's path to shape overhangs, windows and the building's orientation.

BACKUP VIDEO

How a Site Can Shape a Home

30X40 Design Workshop · 5:08

https://www.youtube.com/watch?v=hweVW3ujfbA

Before you draw your own site analysis.

KEY TERMS

site analysis

A study of a site's features: topography, drainage, soils, sun, wind, vegetation, views, access, utilities, neighbors and legal limits.

solar altitude

The sun's angle above the horizon; at noon it is about $90^\circ - \text{latitude} + \delta$, where δ ranges from about +23.4° (summer solstice) to −23.4° (winter solstice).

setback

The minimum distance a building must be from a property line, set by zoning.

floor area ratio (FAR)

Gross floor area divided by lot area; a zoning limit on how much building a lot may hold.

grading

Shaping the ground so water flows away from buildings, typically falling at least 6 in in the first 10 ft (5%).

orientation

How a building faces the sun, wind, views and street.

CORE IDEAS

The story. In 1857 New York City held a design competition for a great park on 778 acres in the middle of Manhattan, a strip of rocky outcrops, swamps and scattered shanties. Frederick Law Olmsted, the park's new superintendent, and the English-born architect Calvert Vaux won with their "Greensward Plan" in 1858. They studied every feature of the land: they kept the great rock outcrops, shaped lawns, lakes and woodlands, and, most cleverly, sank four cross-town roads below the level of the park so that city traffic could cross it unseen, with bridges carrying the park's paths over them. Building Central Park took nearly twenty years, moved millions of cartloads of earth and rock, and laid miles of drain pipe under the lawns. What looks like nature is a carefully engineered site plan.

How they figured it out. Olmsted and Vaux began with the site: where water collected, where rock was close to the surface, where views opened, how people would enter and move. They separated different kinds of traffic (carriages, riders, walkers and through traffic) on their own paths. Engineers, including the drainage expert George Waring, laid tile drains to dry the swampy ground. Site planning today follows the same order: understand the land before placing the building, and let the building and the site improve each other.

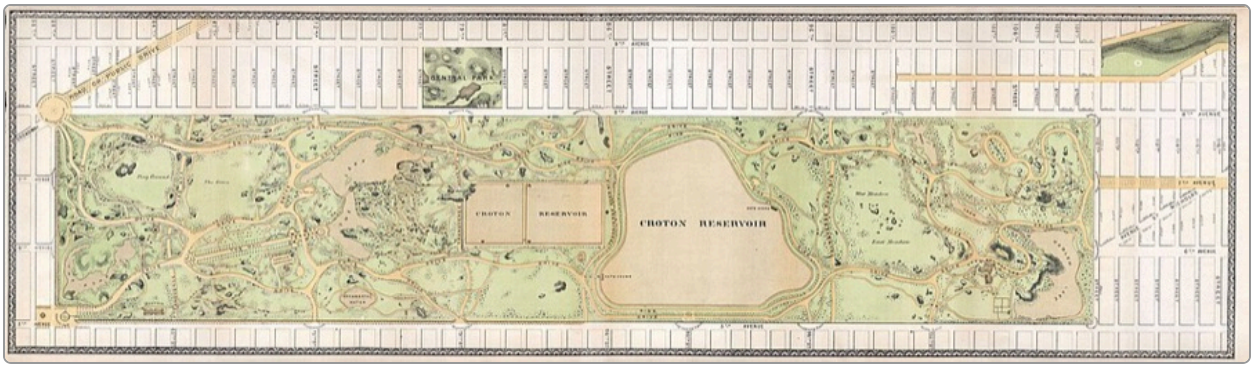
The physics. The sun's path shapes good buildings. At solar noon the sun's altitude is about $\alpha = 90^\circ - \phi + \delta$, where ϕ is the latitude and δ the sun's declination: +23.4° at the summer solstice, 0° at the equinoxes, −23.4° at the winter solstice. At 40° north, the noon sun stands about 73° high in June and only about 27° in December. A roof overhang of projection P above a south window casts a shadow down the wall a distance $P \tan \alpha$: deep enough to shade the window from the high summer sun, but shallow enough to let the low winter sun shine in. Water flows downhill, so the

ground must slope away from the building, and roofs, paving and lawns must send water to drains, swales or rain gardens (Lesson 9.3). Wind matters too: buildings can be sheltered by trees or other buildings from cold winter winds and opened to cooling summer breezes.

The engineering. A site plan also obeys laws and practical needs. Zoning sets setbacks from property lines, height limits, lot coverage and floor area ratio. Utilities (water, sewer, power, gas, communications) must reach the building. Driveways and parking must fit with safe sight lines. Soil and slope affect the foundation (Unit 8), and grading must balance cut and fill if possible. A good site plan places the building where it uses the land's gifts (sun, view, firm ground) and protects it from its dangers (flooding, unstable slopes), and it treats neighbors well.

Try it. In this studio step you will analyze your site, compute sun angles and overhangs, check zoning limits, and place your building.

PICTURE STUDY



Map of Central Park by Olmsted and Vaux, Frederick Law Olmsted and Calvert Vaux (published map), 1868 · Printed map · Geographicus Rare Antique Maps (via Wikimedia Commons)

Image: Wikimedia Commons, public domain

This map, published ten years after the Greensward Plan won the competition, shows Central Park much as Olmsted and Vaux shaped it. Look for how they planned the site.

Look closely

- Find the straight cross-town roads. How do they differ from the park's curving drives?
- Find the two large reservoirs. How did the designers work around them?
- Find the lakes and ponds. Where would water naturally collect on this land?
- Compare the city's straight grid outside the park with the park's curves inside.

Talk about it

1. Why did Olmsted and Vaux sink the cross-town roads?
2. How does the plan separate different kinds of traffic?
3. What would a site analysis of this land have shown before the park was built?

Safety first. Visit sites only with permission and an adult. Stay away from steep drop-offs, water and busy roads.

BUILD

Studio Step 2: Site Analysis and Site Plan

You will analyze a real site for your studio building (your yard, a church lot, or an empty lot in town, with permission) and produce a site analysis drawing and a site plan placing your building.

You will need

- Tape measure, compass (or a phone compass app), your water level from Lesson 9.1, a camera
- Graph paper or a free online map or aerial photo of the site printed to scale
- Your zoning rules (from the town's website or planning office) or the values in Worked Example 3

Steps

1. Measure the site boundaries (or take them from a map) and draw the site at a scale such as 1″ = 20′.
2. Mark north. Record the sun's rising and setting directions now, and compute the noon altitude for your latitude at both solstices.
3. Survey the slope with your water level and mark spot elevations. Draw arrows showing which way water flows.
4. Mark trees, views (good and bad), neighbors, noise, the usual wind direction, the street and where utilities enter.
5. Draw the setback lines and compute the buildable area and the FAR limit. Is your program's gross area allowed?
6. Place your building's footprint (from your bubble diagram) in two or three different positions. Score each against sun, view, drainage, access and neighbors.
7. Choose the best position, then draw the site plan: building, entrance, path or drive, parking if needed, grading arrows away from the building (5% for the first 10 ft), and a place for runoff to go.
8. **Review.** Show the site plan to your client and record their response.

Tips

- Visit the site at different times of day if you can.
- Use the sun angles to decide which side gets the main windows.
- Keep the building off the lowest, wettest part of the site.

Check your work

- ☐ My site analysis shows sun, slope, drainage, views, access, utilities and setbacks.
- ☐ I computed solar altitudes, an overhang and the zoning limits.
- ☐ My site plan places the building with clear reasons.

ENGINEER'S NOTEBOOK

1. Your site analysis drawing with all the observations labeled.
2. Sun-angle calculations and an overhang section for your main south windows.
3. The site plan with grading arrows and a short paragraph explaining the placement.

KEY FORMULAS

$\alpha_{\text{noon}} = 90^\circ - \phi + \delta, \quad \delta = +23.4^\circ$ (June), 0° (March, September), -23.4° (December)

shadow depth $= P \tan \alpha$ (below an overhang of projection P)

$FAR = \dfrac{A_{\text{gross floor}}}{A_{\text{lot}}}, \quad \text{slope} = \dfrac{\Delta h}{L}$

From history. Olmsted went on to design Prospect Park in Brooklyn, the grounds of the U.S. Capitol, the campus of Stanford University and parks in many cities. He believed that parks were for everyone and that beautiful public landscapes did people good. Vaux designed many of Central Park's bridges, each one different.

Example 1. Find the noon solar altitude at 40° N latitude at the summer and winter solstices and the equinoxes.

Show solution

1. Summer: $90 - 40 + 23.4 = 73.4^\circ$.
2. Equinox: $90 - 40 + 0 = 50^\circ$.
3. Winter: $90 - 40 - 23.4 = 26.6^\circ$.

Answer: 73.4°, 50° and 26.6°.

Example 2. A south window runs from 3.0 ft to 7.0 ft above the floor, and an overhang projects at 7.5 ft. How far must it project to shade the whole window at noon on the summer solstice at 40° N? How much of the window does that overhang shade at noon in midwinter?

Show solution

1. To shade down to the sill, the shadow must reach $7.5 - 3.0 = 4.5$ ft below the overhang.
2. $P = 4.5/\tan 73.4^\circ = 4.5/3.36 \approx 1.34$ ft, about 16 in.
3. Winter: shadow depth $= 1.34 \tan 26.6^\circ \approx 1.34 \times 0.50 \approx 0.67$ ft, only 8 in below the overhang, which is above the window head (0.5 ft below). So nearly the whole window gets winter sun.

Answer: About 1.34 ft (16 in); in winter the window is almost fully sunlit.

Example 3. A lot is 80 ft × 125 ft in a zone with FAR 0.40, front setback 25 ft, side setbacks 8 ft and rear setback 20 ft. Find the largest allowed gross floor area and the buildable footprint area inside the setbacks. Will a one-story chapel of 3,600 ft^2 fit?

Show solution

1. Lot area: $80 \times 125 = 10{,}000$ ft^2; allowed floor area: $0.40 \times 10{,}000 = 4000$ ft^2.
2. Buildable width: $80 - 2(8) = 64$ ft; buildable depth: $125 - 25 - 20 = 80$ ft; area $64 \times 80 = 5120$ ft^2.
3. A 3,600 ft^2 one-story chapel is under the FAR limit (4,000) and fits in the 5,120 ft^2 envelope, though parking may also need room.

Answer: 4,000 ft^2 by FAR; 5,120 ft^2 buildable footprint; yes, it fits.

PRACTICE

1. Easy Find the noon sun altitude at the equinox at 35° N.

   Show answer

   55°.

2. Easy What is a setback?

   Show answer

   The minimum distance a building must be from a property line, set by zoning.

3. Easy A lot of 12,000 ft^2 has an FAR of 0.5. What is the maximum gross floor area?

   Show answer

   6,000 ft^2.

4. Medium Find the winter-solstice noon altitude at 45° N.

   Show answer

   $90 - 45 - 23.4 = 21.6^\circ$.

5. Medium At 45° N in summer (68.4°), how far must an overhang project to shade a window whose sill is 5 ft below the overhang?

   Show answer

   $5/\tan 68.4^\circ \approx 5/2.53 \approx 1.98$ ft.

6. Medium Ground must fall 6 in in the first 10 ft from a house. What percent slope is that?

Show answer

5%.

7. Medium A lot is 60 ft × 100 ft with setbacks of 20 ft front, 5 ft sides and 15 ft rear. Find the buildable area.

Show answer

$(60 - 10)(100 - 35) = 50 \times 65 = 3250 \text{ ft}^2$.

8. Hard Why do Northern Hemisphere architects often put the largest windows on the south side with overhangs, and fewer on the west?

Show answer

South windows receive low winter sun (welcome heat and light) and high summer sun that a modest overhang can block; west windows get low, hot afternoon summer sun that overhangs cannot block, causing glare and overheating.

9. Hard An overhang of 2.0 ft projects at 8.0 ft above the floor of a south room at 40° N. At winter noon (26.6°), how far into the room does sunlight reach on the floor, if the window extends down to the floor and nothing else blocks it?

Show answer

The sun enters below the overhang edge; the edge is 8 ft up and 2 ft out, so the sunbeam passing the edge reaches the floor at $8/\tan 26.6^\circ \approx 16.0$ ft from the overhang edge, about 14 ft into the room.

10. Hard Conceptual. Why did Olmsted and Vaux's sunken transverse roads make Central Park better?

Show answer

Model answer: they let cross-town carriages and later cars pass through without crossing the park's paths at grade, keeping park visitors safe and quiet and the views unbroken; separating kinds of traffic is a basic principle of site planning.

11. **Challenge** A building's south wall faces 20° east of due south. At noon the sun is due south. Does a horizontal overhang shade as well as it would on a true south wall? Explain qualitatively.

Show answer

Not quite: the sun strikes the wall at an angle, so its rays travel farther under the overhang to reach the glass and the effective shading geometry changes; at noon the shade is somewhat less effective, and in the afternoon the wall turns away from the sun. Designers adjust overhang depth or add side fins.

COMMON MISTAKES

- Forgetting to check zoning limits before designing.
- Placing the building at the low point of the site where water collects.
- Using magnetic north for sun angles without correcting for magnetic declination where it is large.

Make it a game · Sun Path Estimate. Go outside or look through a window. Predict where the sun will be at noon and how high. Calculate the noon solar altitude for your latitude and the shade of an overhang. Sketch a site with orientation, slope and views. Your parent plays zoning officer and challenges one setback. Closest estimate scores a point.

Scripture. "Prepare thy work without, and make it fit for thyself in the field; and afterwards build thine house." (Proverbs 24:27, KJV)

Prepare the ground first, then build: that is the order of good site planning.

10.3 Designing a Small House or Chapel

OBJECTIVES

- Develop a design from program and site to plan, elevations, section and structure, using pattern books such as Asher Benjamin's as precedent.
- Check the design with calculations: room daylight, stair, roof framing, joist spans and seating.
- Produce schematic drawings and a study model and revise them after a review.

> **Warm-up.** Find a building you admire (a house, a church, a library) and sketch it from memory. Then go look at it, or a photo, and correct the sketch. What did you remember well? What did you get wrong about its proportions?

WATCH

Floor Plan Design TUTORIAL

30X40 Design Workshop · 8:37

https://www.youtube.com/watch?v=R7YxG4nsqeg

> **WHEN TO WATCH**
> Before you begin your plan. Copy his steps: diagram, rough sketch, then more formal plan.

BACKUP VIDEO

30 Minute Study Model - Here's How

30X40 Design Workshop · 24:29

https://www.youtube.com/watch?v=t7GpYvyQoEs

Before you build your study model.

KEY TERMS

pattern book

A published book of building designs, details and rules that builders followed; common in the 1700s and 1800s.

precedent

An earlier building studied for ideas, as Asher Benjamin's plates were for New England builders.

schematic design

The first design phase: the overall plan, form and structure, drawn simply and studied with models.

structural grid

The regular spacing of columns or bearing walls that organizes a plan and keeps spans reasonable.

natural light and ventilation

Codes require habitable rooms to have glazing of at least about 8% of the floor area, and openable windows of about 4%.

study model

A quick, rough model used to test massing, light and proportion during design.

CORE IDEAS

The story. In 1797 a young Massachusetts carpenter, Asher Benjamin, published *The Country Builder's Assistant*, the first architectural book written by an American. It gave country carpenters plans, elevations and details for houses and meetinghouses, with the dimensions written in, and rules for orders, stairs and roofs, so that a builder in a village far from any architect could build well. Benjamin published several more books, and in town after town across New England and the Midwest, white churches with columned porches and slender steeples were built from his plates. A good small building, like a good sentence, follows clear rules and serves its purpose with grace.

How they figured it out. Benjamin and his readers worked by precedent and proportion. They started from a type that worked (a rectangular meetinghouse with a porch and a tower, a center-hall house), adjusted it to the program and site, and kept the structure simple: walls on a regular rhythm, roof trusses spanning the width, windows set in a steady pattern. Architects still design this way. They develop the plan from the bubble diagram, test it in section and elevation, check it with numbers, build a study model, and revise, again and again.

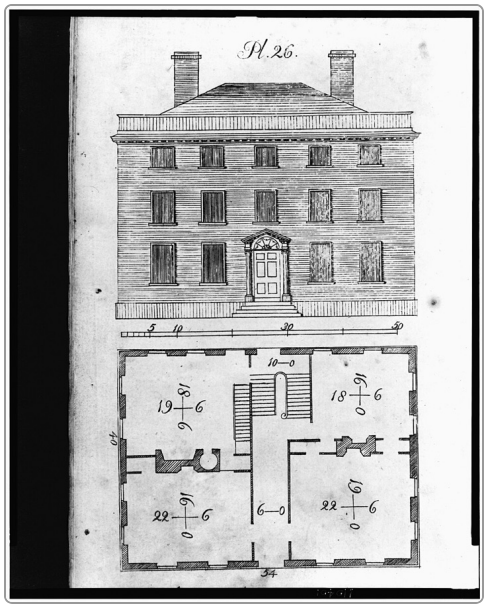
The physics. Many design checks are simple calculations you already know. Daylight: glass of at least 8% of each room's floor area, and openable area of 4% for fresh air. Stairs: risers no more than 7¾ in and treads at least 10 in (Lesson 2.3). Roof: a pitch chosen for the climate and look, with rafter length from the Pythagorean theorem. Structure: joists and rafters sized for their spans (Lesson 5.5), and a clear load path to the foundation (Lesson 4.5). Seating: about 18–20 in of pew per person

and about 36 in from pew to pew. Each check is quick, and together they turn a sketch into something that can be built.

The engineering. A good small house or chapel integrates everything in this course: firmness (a clear structure on a sound foundation, resisting wind, snow and earthquakes), commodity (a plan that works, with light, air, warmth, accessibility and safe exits) and delight (proportion, light, materials and a fitting place on its site). Integration is the hard part: the window that delights must also meet the code, the roof that looks right must also shed snow, and the beam that carries the roof must fit in the ceiling. Designers resolve these conflicts by working back and forth between drawings, models and calculations.

Try it. In this studio step you will design your building in schematic form: plan, elevations, section, structure and a study model, with the checks above.

PICTURE STUDY



Front Façade and Floor Plan, from The Country Builder's Assistant, Asher Benjamin, 1797 · Engraving · Library of Congress

Image: Wikimedia Commons, public domain

Asher Benjamin's plates gave country builders exactly what they needed: an elevation, a plan, and the dimensions. This house is a classic center-hall plan, a type built thousands of times across New England.

Look closely

- Find the central door in the elevation and the hall behind it in the plan. How do they line up?
- Read the dimensions in the rooms. What proportions do the rooms have?
- Count the windows on each floor. How are they spaced?
- Find the chimneys in the elevation and the fireplaces in the plan.

Talk about it

1. Why would a builder far from any architect value a book like this?
2. How does the plan's symmetry make the structure simple?
3. What would you change in this plan for a family today?

Safety first. An adult handles any craft knife used for the study model; use a cutting mat and a metal ruler.

BUILD

Studio Step 3: Schematic Design and Study Model

You will turn your program, bubble diagram and site plan into a schematic design: floor plan, two elevations, one section, a structural sketch, the design checks, and a study model, then revise after a review.

You will need

- Graph paper, tracing paper, an architect's scale, pencils
- Card stock, foam board or chipboard, glue, a craft knife (adult) and cutting mat
- SketchUp Free or Tinkercad (optional)
- Your program, budget, bubble diagram and site plan

Steps

1. Lay tracing paper over your bubble diagram and draw a first plan at ⅛″ = 1′-0″, with walls, doors and windows. Choose a structural grid or bearing walls with spans of 12–16 ft for joists (or trusses for a chapel roof).
2. Draw at least two more plan alternatives on fresh tracing paper. Compare them against your program and pick one.
3. Draw the chosen plan at ¼″ = 1′-0″, with room names and areas. Check that the total gross area matches your program within 10%.
4. Draw two elevations and a section through the main room, showing floor, ceiling and roof heights and the roof pitch.
5. Run the checks: glass and openable area in every habitable room; stair risers and treads; rafter length; seating (for a chapel); two exits if more than 49 occupants (Lesson 7.5); accessibility of the main entrance.
6. Build a study model at ⅛″ = 1′-0″ from card in an hour or two. Photograph it in sunlight from the south at morning and noon.
7. **Review.** Present the drawings and model to your client and one other person. Record comments.
8. **Revise.** Make at least two improvements and redraw the affected sheets.

Tips

- Draw quickly at first; save care for the final drawings.
- Keep the structure simple: stack walls over walls and keep spans regular.
- Look at precedents (a church or house you admire) and borrow their good ideas honestly.

Check your work

- ☐ My schematic design includes plan, two elevations, a section and a study model.
- ☐ My design checks (light, stairs, roof, seating, exits) are calculated and pass.
- ☐ I revised the design after a review and recorded what changed and why.

ENGINEER'S NOTEBOOK

1. Your plan alternatives on tracing paper and the reason you chose one.
2. The design-check calculations in a table.
3. Photographs of your study model with notes on light and proportion, and your revision list.

KEY FORMULAS

$$A_{\text{glass}} \geq 0.08\, A_{\text{floor}}, \quad A_{\text{open}} \geq 0.04\, A_{\text{floor}}$$

$$L_{\text{rafter}} = \sqrt{\text{run}^2 + \text{rise}^2}$$

$$\text{pew capacity} = \frac{\text{pew length}}{18\text{–}20 \text{ in per person}}$$

$$n_{\text{risers}} = \left\lceil \frac{\text{total rise}}{7.75 \text{ in}} \right\rceil$$

From history. Asher Benjamin later lived in Boston and designed churches and houses there; his Old West Church (1806) still stands. His books were so widely used that historians can trace his influence by finding his doorways, mantels and steeples in towns he never visited.

Example 1. A chapel must seat 120 in two banks of pews with a 5 ft center aisle and 3.5 ft side aisles. Each pew is 10 ft long, at 20 in per person, with pews 36 in apart. Find the nave's width and the length of the seating area.

Show solution

1. People per pew: $120 \text{ in} \div 20 \text{ in} = 6$; per row (two pews): 12.
2. Rows: $120 \div 12 = 10$ rows; seating depth $= 10 \times 3 \text{ ft} = 30 \text{ ft}$.
3. Width: $10 + 10 + 5 + 2(3.5) = 32 \text{ ft}$.

Answer: 32 ft wide; 30 ft of seating depth (plus space for the chancel and narthex).

Example 2. The 32 ft wide chapel has a 10:12 roof. Find the rafter length (no overhang), and the minimum glass for a 32 ft × 50 ft nave.

Show solution

1. Run: 16 ft; rise: $16 \times 10/12 \approx 13.33 \text{ ft}$.
2. Rafter: $\sqrt{16^2 + 13.33^2} = \sqrt{256 + 177.8} \approx 20.8 \text{ ft}$.
3. Glass: $0.08 \times (32 \times 50) = 0.08 \times 1600 = 128 \text{ ft}^2$, for example eight windows 4 ft × 4 ft.

Answer: Rafters about 20.8 ft; at least 128 ft^2 of glass.

Example 3. A house living room is 14 ft × 18 ft. Find the minimum glass and openable area. Will two windows 3 ft × 5 ft, each opening half its area, be enough?

Show solution

1. Floor: $14 \times 18 = 252 \text{ ft}^2$.
2. Glass: $0.08 \times 252 \approx 20.2 \text{ ft}^2$; open: $0.04 \times 252 \approx 10.1 \text{ ft}^2$.
3. Two windows: glass $2 \times 15 = 30 \text{ ft}^2$ ✓; openable $2 \times 7.5 = 15 \text{ ft}^2$ ✓.

Answer: Yes: 30 ft^2 of glass (needs 20.2) and 15 ft^2 openable (needs 10.1).

PRACTICE

1. Easy Find the minimum glass area for a 12 ft × 13 ft bedroom.

   Show answer

   $0.08 \times 156 = 12.5\ \text{ft}^2$.

2. Easy A pew is 12 ft long. How many people does it seat at 18 in each?

   Show answer

   8.

3. Easy Who was Asher Benjamin?

   Show answer

   A Massachusetts carpenter and architect who in 1797 published The Country Builder's Assistant, the first architectural book by an American, giving builders plans and details.

4. Medium A house is 28 ft wide with a 6:12 roof. Find the rafter length with a 1.5 ft overhang (measured horizontally).

   Show answer

   Run $14 + 1.5 = 15.5$ ft, rise 7.75 ft; $L = \sqrt{240.25 + 60.06} \approx 17.3$ ft.

5. Medium A chapel seats 90 in pews 9 ft long at 18 in per person, two pews per row, 36 in apart. How many rows, and how deep is the seating?

   Show answer

   6 per pew, 12 per row, $90/12 = 7.5$, so 8 rows; depth 24 ft.

6. Medium A stair rises 9′-0″. How many risers at most 7¾ in, and what is each?

   Show answer

   $108/7.75 = 13.9$, so 14 risers of about 7.71 in.

7. Medium A schematic plan totals 2,180 ft^2 gross against a program target of 1,950 ft^2. By what percent is it over? What might you do?

   Show answer

   About 11.8% over; tighten circulation, combine spaces or reduce some room sizes.

8. Hard Floor joists 2×10 at 16 in on center can span about 14 ft for a typical house floor. A living room is 16 ft × 20 ft. How could you frame it?

Show answer

Run the joists the 16 ft way only with deeper joists (2×12) or closer spacing; or add a beam at mid-span (8 ft joists); or use engineered I-joists; choose by checking span tables or calculations (Lesson 5.5).

9. Hard Why is a symmetrical center-hall plan structurally efficient?

Show answer

Its bearing walls line up on each floor, its spans are equal and moderate, the central stair and chimneys stack, and loads go straight down, so framing and foundations are simple and economical.

10. Hard Conceptual. Give one example of a conflict between delight and commodity or firmness in a small building, and how a designer might resolve it.

Show answer

Model answer: a tall wall of glass at the chancel delights but overheats and glares in afternoon sun and weakens the wall's bracing; resolve with a north- or east-facing window, deep reveals or overhangs, and a braced frame or shear walls elsewhere.

11. Challenge A chapel's nave is 32 ft wide and 54 ft long with a 10:12 roof starting 14 ft above the floor. Find the volume of the nave (box plus triangular prism). If heating needs about 0.5 Btu per hour per cubic foot per °F of difference for a building of this type in cold weather, estimate the heating needed for a 50 °F difference.

Show answer

Box $32 \times 54 \times 14 = 24{,}192$; roof $\frac{1}{2}(32)(13.33)(54) \approx 11{,}520$; total $\approx 35{,}700$ ft^3. Heat $\approx 0.5 \times 35{,}700 \times 50 \approx 893{,}000$ Btu/h (a rough rule; an engineer would calculate losses through each surface).

COMMON MISTAKES

- Drawing one plan and falling in love with it. Draw alternatives and compare.
- Ignoring structure until late. Spans and load paths shape the plan.
- Skipping the code checks until the end, then having to redesign rooms.

Make it a game · Daylight and Stairs. Walk your home with a tape measure. Estimate the daylight area of a window as a fraction of floor area, check a stair's rise and run, and measure a joist span. Then your parent hides one number from a design check, and you reconstruct it. Whoever reconstructs more checks wins the round.

Scripture. “Through wisdom is an house builded; and by understanding it is established: And by knowledge shall the chambers be filled with all precious and pleasant riches.” (Proverbs 24:3-4, KJV)

Wisdom, understanding and knowledge: the program, the structure and the details that fill a house with good things.

10.4 Codes, Cost and Schedule: Counting the Cost

OBJECTIVES

- Explain how building codes, zoning and permits shape a design, and how the Washington Monument's long pause shows the cost of not counting the cost.
- Prepare a quantity-based and a square-foot cost estimate with contingency and escalation, and a simple construction schedule with a critical path.
- Check the studio design against key code and zoning requirements.

> **Warm-up.** Read Luke 14:28-30. Then list everything you think it would cost to build your studio building: not only materials and labor, but design, permits, utilities, furniture and the unexpected. Which items might people forget?

WATCH

Easily Estimate Your Cost to Build

Armchair Builder · 7:35

https://www.youtube.com/watch?v=TyPGPCXsAwQ

> **WHEN TO WATCH**
> Before the worked examples. Note how he separates land, labor and materials, and the builder's costs.

KEY TERMS

building permit

Official permission to build, granted after the building department reviews the drawings for code compliance; inspections follow during construction.

zoning

Local law controlling land use, building size, height, setbacks and parking.

quantity takeoff

Measuring the quantities of materials from the drawings (cubic yards of concrete, square feet of roofing) to price them.

unit cost

The cost per unit of work, such as dollars per square foot of wall or per cubic yard of concrete.

escalation

The expected rise in construction prices between the estimate and the time of building.

life-cycle cost

The total cost of a building over its life: construction plus energy, maintenance and repairs.

CORE IDEAS

The story. On July 4, 1848, the cornerstone was laid in Washington, D.C., for a monument to George Washington, designed by the architect Robert Mills as a tall obelisk rising from a circular colonnade. The project was paid for by public subscription through the Washington National Monument Society. By 1854 the shaft stood about 152 ft high, and the money was gone. For more than twenty years, through troubled times and the Civil War, the stump stood unfinished. In 1876 Congress took over, and in 1879 the Army Corps of Engineers, under Lieutenant Colonel Thomas Lincoln Casey, resumed work. Casey found that the old foundation was too small for the finished height and enlarged it with a broad concrete slab beneath and around it, while the monument stood. The capstone was set in December 1884; at 555 ft it was briefly the tallest structure in the world. You can still see where the work stopped: about 150 ft up, the marble from a different quarry is a slightly different shade.

How they figured it out. The Monument's long pause shows what Jesus described in Luke 14: a builder who did not count the cost and could not finish. Modern projects avoid this with estimates at every phase, from square-foot estimates during schematic design to detailed quantity takeoffs before bidding; with contingencies for the unknown; with funding secured before construction; and with schedules that sequence the work. Casey's foundation work also shows that changes in design (here, a taller, plainer obelisk without Mills's colonnade) must be checked against everything already built.

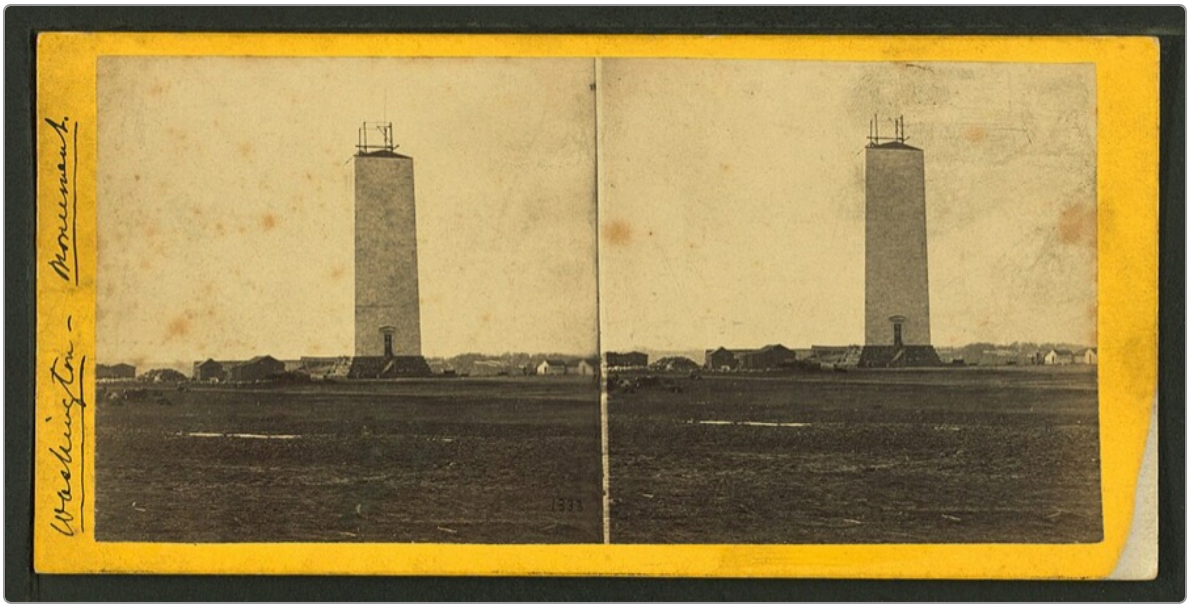
The physics. A cost estimate is arithmetic done carefully. A **square-foot estimate** multiplies gross area by a unit cost for the building type and place. A **quantity estimate** measures each material from the drawings and multiplies by unit prices: concrete by the cubic yard, framing by the board foot or piece, roofing by the square (100 ft^2), and so on, plus labor, equipment, overhead and profit. Prices rise over time, so estimates add **escalation**: $C_{\text{future}} = C_{\text{now}}(1+r)^n$. Schedules use the critical path method you learned in Lesson 1.5. And codes and zoning set limits that the design

must meet: construction type and fire ratings, exits, accessibility, energy use, structural loads, height, setbacks and parking.

The engineering. A design is not finished until it is buildable, affordable and legal. Architects and engineers check the code at every phase, meet with the building department, and revise the design when estimates come in high, a process called value engineering, which should keep what matters most and cut what matters least. They also consider life-cycle cost: a better roof or more insulation may cost more now and much less over fifty years. Counting the cost is not only prudence; it is honesty toward the client and the people who give money to build.

Try it. In this studio step you will estimate the cost of your design two ways, schedule its construction, and check it against the code and zoning.

PICTURE STUDY



The Unfinished Washington Monument, E. & H. T. Anthony (photographers and publishers), 1860s–1870s · Stereograph · Wikimedia Commons

Image: Wikimedia Commons, public domain

For more than twenty years the Washington Monument stood like this, about 152 ft of a planned 555 ft, because the money had run out. Stereographs like this one were sold as souvenirs and viewed in a stereoscope to appear three-dimensional.

Look closely

- Find the top of the stump. What is the small structure on it?
- Look at the ground around the base. What was the site like?
- Compare the shaft's width with its height. How tall would it be when finished?
- Look for any sign of the colonnade Mills had planned.

Talk about it

1. What lessons about cost and funding does this photograph teach?
2. Why did the engineers who finished it have to check the foundation first?
3. How does Luke 14:28-30 describe this picture?

BUILD

Studio Step 4: Estimate, Schedule and Code Check

You will estimate the cost of your studio design two ways, make a construction schedule, and check the design against key code and zoning requirements, then make any changes needed.

You will need

- Your schematic drawings and program
- A spreadsheet or calculator
- Local cost information (a builder, a cost guide, or the figures in this lesson), and your town's zoning and building code basics (online)

Steps

1. **Square-foot estimate.** Multiply your gross area by a realistic cost per square foot for your building type and region.
2. **Quantity estimate.** Take off at least six major items from your drawings: foundation concrete (yd^3), slab or floor framing (ft^2), exterior walls (ft^2), roofing (squares), windows and doors (count), interior finishes (ft^2). Price them with unit costs. Add 15% for items not counted, plus overhead and profit of 15%.
3. Compare the two estimates. If they differ by more than 15%, find out why.
4. Add design fees (8%), contingency (10%) and escalation to your planned start date. Compare with your client's budget.
5. **Schedule.** List the main construction tasks (site work, foundation, framing, roof, windows, rough-in of plumbing and electrical, insulation, drywall, finishes, landscaping), estimate durations, draw the network and find the critical path and total time.
6. **Code and zoning check.** Check setbacks, height, FAR, parking (if any), exits and egress widths, accessibility of the entrance and restroom, stair dimensions, glazing and smoke alarms. Record each requirement and whether your design passes.

7. **Value engineering.** If the cost exceeds the budget, propose three changes that save money while keeping what the client values most. Estimate the savings of each.
8. Review the results with your client and revise the design as needed.

Tips

- Write down every assumption with its source.
- Round quantities up; materials come in whole units and some are wasted.
- The critical path is usually foundation, framing, roof and interior finishes; utility connections often surprise people.

Check your work

- ☐ I prepared two estimates and explained their difference.
- ☐ My schedule shows a critical path and total duration.
- ☐ My code and zoning checklist is complete, and the design passes or has been revised.

ENGINEER'S NOTEBOOK

1. Your estimate tables: square-foot and quantity-based, with fees, contingency and escalation.
2. Your construction network diagram and Gantt chart.
3. Your code and zoning checklist and the value-engineering proposals.

KEY FORMULAS

$C = A_{\text{gross}} \times (\$/\text{ft}^2)$ or $C = \sum(\text{quantity} \times \text{unit cost})$

$C_{\text{total}} = C \times (1 + \text{contingency}), \quad C_{\text{future}} = C_{\text{now}}(1 + r)^n$

Project duration = critical path length

$q = \dfrac{W}{A}$ (foundation bearing pressure)

From history. The Washington Monument's capstone is a pyramid of aluminum, then a rare and costly metal, inscribed on one face with the Latin words *Laus Deo*, "Praise be to God." When it was set in 1884 it also served as the tip of the monument's lightning rod.

Example 1. A chapel estimate is \$1,900,000 in today's prices. Construction will start in 3 years, with prices rising 4% a year. What should the budget be? Add a 5% construction contingency on top.

Show solution

1. Escalated: $1{,}900{,}000 \times 1.04^3 = 1{,}900{,}000 \times 1.1249 \approx \$2{,}137{,}000$.
2. With contingency: $2{,}137{,}000 \times 1.05 \approx \$2{,}244{,}000$.

Answer: About \$2.24 million.

Example 2. Quantity takeoff for a slab: the chapel's floor is 32 ft × 70 ft and 5 in thick, with a thickened edge adding 6 yd^3. Concrete costs \$165 per cubic yard delivered, and labor and finishing \$4.50 per ft^2. Find the cost of the slab.

Show solution

1. Slab volume: $32 \times 70 \times (5/12) \approx 933\ \text{ft}^3 = 34.6\ \text{yd}^3$; plus 6 $yd^3 = 40.6\ yd^3$; order 42 yd^3 (allowing waste).
2. Concrete: $42 \times \$165 = \$6930$.
3. Labor and finishing: $2240\ \text{ft}^2 \times \$4.50 = \$10{,}080$.
4. Total: about \$17,010 for the slab.

Answer: About \$17,000.

Example 3. The finished Washington Monument weighs about 81,000 tons. Its original foundation was about 80 ft square. Casey enlarged it to about 126.5 ft square. Estimate the average bearing pressure on the soil before and after (ignoring the foundation's own weight).

Show solution

1. Weight: 81,000 tons $\times$ 2000 $= 1.62 \times 10^8$ lb.
2. Before: $1.62 \times 10^8 \div (80^2) = 1.62 \times 10^8 \div 6400 \approx 25{,}300$ psf, about 12.7 tons per ft^2.
3. After: $1.62 \times 10^8 \div (126.5^2) \approx 1.62 \times 10^8 \div 16{,}000 \approx 10{,}100$ psf, about 5.1 tons per ft^2.
4. Enlarging the base cut the pressure by more than half, a decision that let the monument be finished safely.

Answer: About 12.7 tons/ft^2 before and 5.1 tons/ft^2 after.

PRACTICE

1. Easy A 2,400 ft^2 building costs \$185 per ft^2. Find the construction cost.

   Show answer

   \$444,000.

2. Easy Why did work on the Washington Monument stop in 1854?

   Show answer

   The money raised by public subscription ran out (and troubled times and the Civil War followed).

3. Easy Add a 12% contingency to \$300,000.

   Show answer

   \$336,000.

4. Medium Escalate \$800,000 at 3.5% a year for 4 years.

   Show answer

   $800{,}000 \times 1.035^4 \approx \$918{,}000$.

5. Medium A roof has 2,600 ft^2 of area. Shingles cost $140 per square (100 ft^2) installed. Add 10% waste. Find the cost.

Show answer

$2860/100 = 28.6$ squares, so 29 squares × $140 = $4060.

6. Medium Footings: 180 ft of wall footing 2 ft wide and 1 ft deep. How many cubic yards? At $170/$yd^3$, what does the concrete cost?

Show answer

$360\ ft^3 = 13.3\ yd^3$, order 14 yd^3: $2380.

7. Medium Name three things a building permit review checks.

Show answer

Any three: structural design, fire and life safety (exits, ratings, alarms, sprinklers), accessibility, energy code, plumbing, mechanical and electrical, zoning compliance.

8. Hard Option A roofing costs $12,000 and lasts 20 years; option B costs $20,000 and lasts 50 years. Over 50 years (ignoring interest), which costs less?

Show answer

A needs replacement at 20 and 40 years: $3 \times 12{,}000 = \$36{,}000$ (prorating the third to year 50 gives about $30,000); B costs $20,000. B costs less over the life.

9. Hard Tasks: site work 2 wk; foundation 3 wk (after site); framing 5 wk (after foundation); roofing 2 wk (after framing); windows 1 wk (after framing); rough-in 3 wk (after roofing and windows); insulation and drywall 3 wk (after rough-in); finishes 4 wk (after drywall); landscaping 2 wk (after site work). Find the critical path and duration, and the float of landscaping.

Show answer

Site–foundation–framing–roofing–rough-in–drywall–finishes: $2 + 3 + 5 + 2 + 3 + 3 + 4 = 22$ weeks. Landscaping can start at week 2 and must end by week 22: float $= 22 - 4 = 18$ weeks (in practice done near the end).

10. Hard Conceptual. What is value engineering, and what should it not do?

Show answer

Model answer: reviewing a design to lower its cost while keeping its essential functions and quality; it should not cut safety, durability or the things the client values most, or simply move costs into later maintenance.

11. Challenge A design comes in 14% over a $1.6 million budget. A change to a simpler roof saves $65,000, reducing the sanctuary length saves $110,000, and choosing different finishes saves $48,000. Which combination brings it within budget with the least change to the sanctuary?

Show answer

Over by $0.14 \times 1.6\ \mathrm{M} = \$224{,}000$. Roof + finishes save $113,000 (not enough); roof + sanctuary $175,000 (not enough); all three save $223,000, about $1,000 short, so all three plus a small further saving are needed, or the budget must grow. A good designer would look for other savings before shortening the sanctuary.

COMMON MISTAKES

- Forgetting soft costs: design fees, permits, testing, furniture and utility connections.
- Leaving out contingency and escalation.
- Treating codes as a final check instead of a guide from the start.

Make it a game · Count the Cost. Pretend budget duel. Invent a 20 by 30 foot structure and price it by the square foot, then by a quantity list of studs and sheets. Compare, add contingency and escalation. Your parent plays the Washington Monument committee and asks where the money stopped. Closest to a sensible final figure scores.

Scripture. “For which of you, intending to build a tower, sitteth not down first, and counteth the cost, whether he have sufficient to finish it? Lest haply, after he hath laid the foundation, and is not able to finish it, all that behold it begin to mock him, Saying, This man began to build, and was not able to finish.” (Luke 14:28-30, KJV)

Jesus uses a builder’s budget to teach about following Him wholeheartedly. Engineers and architects take the lesson literally as well: count the cost before you begin.

10.5 Presenting the Design: Drawings and a Model

OBJECTIVES

- Tell how William Thornton won the 1793 competition for the U.S. Capitol, and explain what a complete design presentation includes.
- Prepare a presentation set of drawings at consistent scales and a finished model, and calculate their sizes and layout.
- Present the studio design to a client and an audience, answer questions and reflect on the whole design process.

> **Warm-up.** Explain your studio design in exactly one minute to someone who has never seen it, using only words and your hands. What was hardest to explain? What drawing or model would have helped most?

WATCH

Architecture Client Presentation

30X40 Design Workshop · 12:48

https://www.youtube.com/watch?v=XYFcwofisPo

> **WHEN TO WATCH**
> Before you prepare your own presentation. Notice the order in which he shows drawings, models and materials, and how he tells the story of the design.

KEY TERMS

presentation drawings
Clear, finished drawings made to communicate a design to clients and the public: site plan, plans, elevations, sections and perspectives.

presentation model
A careful scale model showing the finished design in its setting.

rendering
A drawing or computer image showing a design realistically, with light, color and materials.

design narrative
The short story of a design: the client's needs, the site, the main idea and how the building answers them.

construction documents

The detailed drawings and specifications (after presentation and approval) from which contractors build.

CORE IDEAS

The story. In March 1792 Secretary of State Thomas Jefferson announced a public competition, with a prize of $500 and a city lot, for the design of a Capitol for the new nation in the federal city on the Potomac. About a dozen designs arrived, many by amateurs. Months after the deadline, a physician and inventor from the Virgin Islands named William Thornton, who had taught himself architecture from books, asked to submit a design. His drawings showed a long building with a central dome, a classical portico and two wings for the two houses of Congress. President George Washington praised it for its grandeur, simplicity and convenience, and it won. The cornerstone was laid on September 18, 1793. Thornton's design was later carried forward and changed by professional architects, including Benjamin Latrobe and Charles Bulfinch, and the great dome came later, but the Capitol began with a set of drawings that convinced the people who would decide.

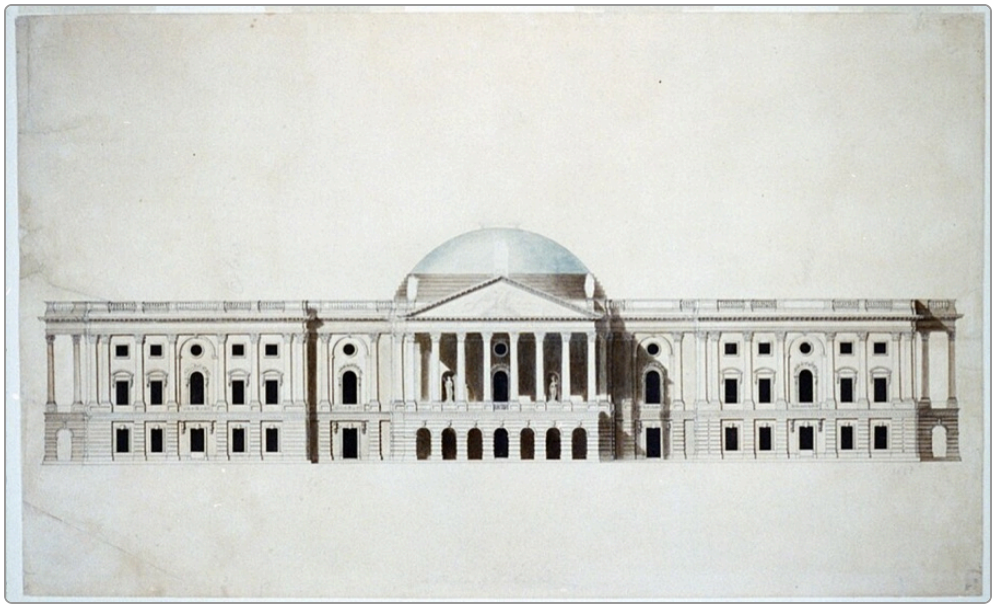
How they figured it out. Thornton won not because his drawings were the most technical, but because they communicated a clear, dignified idea that fit what the nation wanted. The professionals who followed, especially Latrobe, then had to make it work: fitting real rooms into the plan, designing vaults and stairs, and solving problems that the presentation drawings had glossed over. That is the lesson of presentation: it must be honest. Good presentations tell a clear story, show the design truly, and acknowledge open questions, so that approval rests on what will really be built.

The physics. Presentation is also measurement. Each drawing is at a stated scale (Lesson 2.1), and the scales are chosen so the drawings fit the sheets: a 32 ft × 50 ft chapel at ¼″ = 1′-0″ is 8 in × 12½ in on paper, comfortably on a 24 in × 36 in board with its elevations. A model at ⅛″ = 1′-0″ is 4 in × 6¼ in. Perspectives follow the geometry of Lesson 2.4, and shadows on elevations follow the sun angles of Lesson 10.2. Every dimension in the presentation must agree with every other, because clients, builders and officials will rely on them.

The engineering. A complete schematic presentation for a small building includes: a title and design narrative; a site plan; floor plans; at least two elevations; a section; a perspective or rendering; a model; a summary of the structure and systems; the code and zoning summary; and the cost estimate and schedule. The presenter tells the story in order: the client and program, the site, the main idea, the plan, the form and structure, the cost and schedule, and the questions still to be answered. After approval come the construction documents and, at last, the building. *Fabricando fit faber*: by making, one becomes a maker.

Try it. In this final studio step you will prepare and deliver the presentation of your design.

PICTURE STUDY



William Thornton's Winning Design for the U.S. Capitol, William Thornton, about 1793–1797 · Ink and watercolor on paper · Library of Congress

Image: Wikimedia Commons, public domain

Thornton, a physician and self-taught architect, drew this elevation of his design for the Capitol. It was the drawing that convinced Washington and Jefferson.

Look closely

- Find the central dome and portico. What do they tell the viewer about the building's importance?
- Compare the two wings. Why might they be identical?
- Look at the rhythm of windows and columns. How does it create order?
- Look at the shading. How does it make a flat elevation look three-dimensional?

Talk about it

1. Why might this drawing have persuaded the decision-makers more than a technical plan?
2. What would a builder still need to know that this drawing does not show?
3. How would you present your own design so that it is both convincing and honest?

> **Safety first.** An adult handles craft knives and hot glue for the final model; use a cutting mat and low-temperature glue.

BUILD

Studio Step 5: The Final Presentation

You will prepare a presentation set and a finished model of your studio design and present it to your client and an audience, as Thornton presented the Capitol.

You will need

- Your revised drawings, estimate, schedule and code check
- Drawing paper or presentation boards (24 in × 36 in or 11 in × 17 in sheets), pens, colored pencils or watercolor
- Foam board, basswood, card, clear plastic for windows, glue, a craft knife (adult)
- SketchUp Free or Tinkercad for a perspective rendering (optional)

Steps

1. Write a one-paragraph design narrative: client, program, site, main idea.
2. Prepare the drawings at consistent scales: site plan (1″ = 20′ or 1″ = 10′), floor plan and two elevations (¼″ = 1′-0″), a section, and a perspective (by hand or from your computer model).
3. Add a summary sheet: structure (load path and main members), systems (heating, water, light), code and zoning summary, cost estimate and schedule.
4. Lay out the sheets with titles, scales and north arrows. Check that every dimension agrees across drawings.
5. Build a presentation model at ⅛″ = 1′-0″ on a base showing 20 ft of site around the building, with the correct orientation.
6. Rehearse a 10-minute presentation, timing each part. Prepare answers to likely questions (cost, maintenance, accessibility, future growth).
7. Present to your client and an audience (family, church members, friends). Invite questions and record them.
8. **Reflect.** Write a page on what you would change if you did the project again, and what you learned about being both architect and engineer.

Tips

- Tell a story: problem, idea, solution.
- Let the drawings and model do the talking; point to them as you speak.
- Be honest about open questions; it builds trust.

Check your work

- ☐ My presentation set includes a narrative, site plan, plans, elevations, section, perspective and a summary sheet.
- ☐ My model is built to scale and agrees with the drawings.
- ☐ I presented the design, answered questions and wrote a reflection.

ENGINEER'S NOTEBOOK

1. A layout sketch of each presentation sheet before you draw it.
2. Your presentation outline with times for each part.
3. The questions asked at your presentation and your answers, and your reflection.

KEY FORMULAS

$$\text{drawing size} = \frac{\text{real size}}{k} \quad (k = 48 \text{ at } ¼'' = 1'\text{-}0''; \; k = 96 \text{ at } ⅛'' = 1'\text{-}0'')$$

$$A_{\text{model}} = A_{\text{real}}/k^2$$

$$\text{time per sheet} = \frac{\text{total presentation time}}{\text{number of sheets}}$$

From history. The Capitol's cornerstone was laid by George Washington himself on September 18, 1793. Thornton went on to become the first superintendent of the U.S. Patent Office, and when British troops burned Washington in 1814, he is said to have persuaded them to spare the Patent Office building, arguing that its models belonged to all mankind.

Example 1. A chapel is 32 ft wide, 50 ft long and 34 ft to the ridge. Find its size on a ¼″ = 1′-0″ elevation and as an ⅛″ = 1′-0″ model. Will the plan and two elevations at ¼″ fit on one 24 in × 36 in board with 2 in margins?

Show solution

1. At ¼″ = 1′-0″ (4 ft per inch): plan 8×12.5 in; long elevation 12.5 in wide × 8.5 in tall; end elevation 8×8.5 in.
2. At ⅛″ = 1′-0″: model base 4×6.25 in, height 4.25 in.
3. Board usable area: 20×32 in. Plan (12.5 in) and long elevation (12.5 in) side by side need 25 in of width plus a gap; the end elevation fits below or beside. Heights: $8 + 8.5 = 16.5$ in, under 20 in. They fit with room for titles.

Answer: Elevation about 12.5 in × 8.5 in; model 4 in × 6.25 in × 4.25 in; yes, they fit.

Example 2. You have 12 minutes to present 8 sheets and a model, and you must leave 4 minutes for questions. How much time per item?

Show solution

1. Talking time: $12 - 4 = 8$ minutes for 9 items.
2. $8 \div 9 \approx 0.9$ minutes, a little under a minute each.
3. That means one clear point per sheet; combine or drop sheets that do not tell part of the story.

Answer: Under a minute per item; plan one point per sheet.

Example 3. A model base is to show the building plus 20 ft of site on every side at ⅛″ = 1′-0″. For the 32 ft × 50 ft chapel, how large is the base?

Show solution

1. Real site area shown: $(32 + 40) \times (50 + 40) = 72 \times 90$ ft.
2. At 8 ft per inch: $72 \div 8 = 9$ in and $90 \div 8 = 11.25$ in.

Answer: A base 9 in × 11¼ in.

PRACTICE

1. **Easy** Who won the 1793 competition for the U.S. Capitol?

   Show answer

   William Thornton, a physician and self-taught architect.

2. **Easy** How long is a 40 ft wall on a ¼″ = 1′-0″ drawing?

   Show answer

   10 in.

3. **Easy** List five drawings in a schematic presentation set.

   Show answer

   Any five: site plan, floor plan, elevations, section, perspective or rendering (plus model, narrative, summary).

4. **Medium** A house is 44 ft × 30 ft. What size is its plan at ⅛″ = 1′-0″ and at ¼″ = 1′-0″?

   Show answer

   5.5 in × 3.75 in; 11 in × 7.5 in.

5. **Medium** A 1:100 model of a chapel 10 m × 16 m: what is its size in mm?

   Show answer

   100 mm × 160 mm.

6. **Medium** A site plan at 1″ = 20′ must show a lot 80 ft × 125 ft. Will it fit on an 11 in × 17 in sheet?

   Show answer

   4 in × 6.25 in; yes, easily.

7. **Medium** Why must presentation drawings agree with each other in every dimension?

   Show answer

   Clients, officials and builders rely on them; disagreements cause confusion, mistrust and errors in later documents and construction.

8. Hard A 24 in × 36 in board with 1.5 in margins must hold a plan at ¼″ = 1′-0″ of a building 60 ft × 40 ft and its long elevation (60 ft long, 28 ft tall). Will both fit side by side? If not, what scale works?

Show answer

At ¼″: plan 15 in × 10 in, elevation 15 in × 7 in. Usable 33 in × 21 in: side by side 30 in wide fits, heights 10 in and 7 in fit. Yes.

9. Hard Explain why Latrobe and others had to change Thornton's design.

Show answer

Thornton's presentation drawings showed the idea and appearance but not workable details; the professionals had to fit real rooms, structure, stairs and services into the plan and solve construction problems, which required changes.

10. Hard Conceptual. What makes a presentation both persuasive and honest?

Show answer

Model answer: a clear story linking the client's needs and the site to the design idea; accurate drawings and models at true scales; realistic cost and schedule; and openness about unresolved questions and trade-offs, so that approval rests on what will really be built.

11. Challenge Your model at ⅛″ = 1′-0″ represents a building of 6,400 ft^2 of floor area. What area of model floor is that, in square inches? If you build the same model at ¼″ = 1′-0″, how many times more foam board does the floor take?

Show answer

$6400 \div 96^2 \text{ ft}^2 = 0.694 \text{ ft}^2 = 100 \text{ in}^2$; at ¼″ it is $(2)^2 = 4$ times as much, 400 in^2.

COMMON MISTAKES

- Using different scales for the plan and elevations on the same board without labeling them.
- Showing too many drawings and talking too long; one clear point per sheet.
- Presenting only the attractive views and hiding the problems.

Make it a game · Present in Sixty. Pitch drill. Pin up your drawings in order and give a sixty-second presentation to your parent: concept, plan, section, model, cost. Your parent timer-calls overtime. Then answer three questions about Thornton's 1793 Capitol entry. Record your time and clarity score, and beat it on a second try.

Scripture. “And the LORD answered me, and said, Write the vision, and make it plain upon tables, that he may run that readeth it.” (Habakkuk 2:2, KJV)

A design presentation writes the vision plainly so that others can understand it and act on it.

UNIT 10

Design Studio

In this final unit you will put the whole course to work. You will design a small house for a family or a small chapel for a congregation, from the first conversation with the client to a finished presentation of drawings and a model. Each lesson is a studio step: the client and the program (with Jefferson's academical village as a model), site planning (with Olmsted and Vaux's Central Park), schematic design (with Asher Benjamin's pattern books), codes, cost and schedule (with the long pause of the Washington Monument), and the presentation (with Thornton's design for the Capitol).

Along the way you will use everything you have learned: scale drawings, plans, elevations and sections; proportion and precedent; loads, beams and foundations; daylight, sun angles and drainage; exits and fire safety; estimates and schedules. You will finish the year as both architect and engineer, with a design you can show and explain. *Fabricando fit faber*.

PREREQUISITES

- Units 1–9 of this course: drawing, history, structures, materials, foundations and site systems
- A client (real or imagined) willing to be interviewed and to review the design
- Drawing tools, model materials and optionally SketchUp Free or Tinkercad

MASTERY CHECKLIST

- ☐ I can interview a client, write a program, compute net and gross areas and a first budget, and draw a bubble diagram.
- ☐ I can analyze a site and compute sun angles, overhangs, setbacks, FAR and grading slopes.
- ☐ I can develop a schematic design with plans, elevations, a section and a study model, and check it with calculations.
- ☐ I can prepare square-foot and quantity estimates with contingency and escalation, a schedule with a critical path, and a code checklist.
- ☐ I can present a complete design clearly and honestly with drawings and a model, and reflect on the process.

OPTIONAL KITS FOR THIS UNIT

- **SketchUp Free (web)**: Free browser-based 3D modeling for the studio design, its perspectives and sun studies; a parent creates the account.
- **KEVA planks**: Handy for quick massing studies of the studio building before the card study model.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

10.1 The Client and the Program — 3 sessions of 50 minutes

10.2 Site Planning — 3 sessions of 50 minutes

10.3	**Designing a Small House or Chapel**	3 sessions of 50 minutes
10.4	**Codes, Cost and Schedule: Counting the Cost**	3 sessions of 50 minutes
10.5	**Presenting the Design: Drawings and a Model**	3 sessions of 50 minutes

Unit 10 test

Time allowed: 50 minutes. Calculators and an architect's scale are allowed. Show your work and units for every calculation. Answer short-answer questions in complete sentences. The test is worth 100 points.

1. **5 pts**
   What is an architectural program, and why is it written before design begins?

2. **6 pts**
   A chapel program has a sanctuary for 100 at 10 ft^2 per seat, a narthex of 200 ft^2, a fellowship hall for 60 at 15 ft^2 per person, a kitchen of 200 ft^2, two classrooms of 250 ft^2, two restrooms of 180 ft^2 and storage of 120 ft^2. Find the net area, and the gross area at 1.30.

3. **6 pts**
   For Problem 2 at \$240 per ft^2, with 8% fees and 10% contingency, find the budget.

4. **6 pts**
   Find the noon solar altitude at 38° N on the summer and winter solstices.

5. **6 pts**
   At 38° N, how deep must an overhang be to shade a window whose sill is 4 ft below the overhang at summer noon?

6. **6 pts**
   A lot is 70 ft × 120 ft with setbacks 25 ft front, 7 ft sides, 20 ft rear, and FAR 0.45. Find the buildable footprint and the maximum floor area.

7. **5 pts**

   Why did Olmsted and Vaux sink the cross-town roads in Central Park?

8. **6 pts**

   A chapel seats 108 in pews 9 ft long at 18 in per person, two pews per row, 36 in apart. Find the number of rows and the seating depth.

9. **6 pts**

   A room is 13 ft × 16 ft. Find the minimum glass and openable window areas.

10. **6 pts**

    A building 30 ft wide has an 8:12 roof. Find the rafter length with no overhang.

11. **6 pts**

    Escalate a $1,200,000 estimate at 5% a year for 2 years and add a 7% contingency.

12. **6 pts**

    A slab 40 ft × 60 ft × 4 in. How many cubic yards (round up)? At $160 per yd^3, find the concrete cost.

13. **6 pts**

    Tasks: A site work 2 wk; B foundation 3 wk after A; C framing 4 wk after B; D roof 2 wk after C; E windows 1 wk after C; F interior 6 wk after D and E. Find the critical path, duration and float of E.

14. **6 pts**

    What does the Washington Monument's two-colored marble teach about project planning?

15. **6 pts**

    A chapel 36 ft × 56 ft: what size is its plan at ¼″ = 1′-0″, and will it fit with an elevation of the same length (24 ft tall) side by side on a 24 in × 36 in board with 2 in margins?

16. **6 pts**

    Why was Thornton's Capitol design later changed by professional architects?

17. **6 pts**

Short essay. Explain how a small building can meet all three of Vitruvius's tests, using examples from your own studio design.

ANSWER KEY

1. A statement of the spaces, sizes, relationships, uses, budget and goals of a building; it is written first so the design answers the client's real needs and budget, avoiding costly changes later.
2. Net $1000 + 200 + 900 + 200 + 500 + 360 + 120 = 3280\ \text{ft}^2$; gross $\approx 4264\ \text{ft}^2$.
3. $4264 \times 240 \times 1.18 \approx \$1{,}207{,}600$.
4. $90 - 38 + 23.4 = 75.4^\circ$; $90 - 38 - 23.4 = 28.6^\circ$.
5. $4/\tan 75.4^\circ \approx 4/3.83 \approx 1.04$ ft.
6. Footprint $(70 - 14)(120 - 45) = 56 \times 75 = 4200\ \text{ft}^2$; floor area $0.45 \times 8400 = 3780\ \text{ft}^2$.
7. To carry city traffic across the park out of sight and without crossing the park's paths, keeping the park safe, quiet and unbroken.
8. 6 per pew, 12 per row; 9 rows; depth 27 ft.
9. Glass $0.08 \times 208 = 16.6\ \text{ft}^2$; open $8.3\ \text{ft}^2$.
10. Run 15, rise 10: $\sqrt{225 + 100} \approx 18.0$ ft.
11. $1{,}200{,}000 \times 1.1025 \times 1.07 \approx \$1{,}415{,}600$.
12. $800\ \text{ft}^3 = 29.6\ \text{yd}^3$, order 30: $4800.
13. A–B–C–D–F = 17 weeks; E can finish at week 10 but F starts at week 11, so E has 1 week of float.
14. Work stopped for over twenty years when funds ran out at about 150 ft; when it resumed, stone from a different source was used, leaving a visible line. It shows the cost of not securing funding (counting the cost) and the difficulty of resuming a halted project.
15. Plan 9×14 in; elevation 14×6 in; side by side need 28 in of the 32 in usable width and 9 in of the 20 in height. Yes.
16. His presentation drawings conveyed an appealing idea but left practical problems unsolved; Latrobe and others had to fit real rooms, structure and systems into it.
17. Model answer: firmness through a clear load path, adequate joists and rafters, a foundation suited to the soil, bracing for wind; commodity through a program confirmed by the client, good adjacencies, daylight, exits and accessibility, and an affordable budget; delight through proportion, orientation to sun and view,

honest materials and a fitting place on the site. A good design resolves conflicts among the three rather than sacrificing one.

PORTFOLIO PIECE: THE STUDIO DESIGN: A SMALL HOUSE OR CHAPEL

Assemble the complete design project from the five studio steps: the client interview and program, the site analysis and site plan, the schematic design and study model, the estimate, schedule and code check, and the final presentation set and model.

Bind or file everything in order with a cover sheet, and include photographs of your presentation, the questions asked and your one-page reflection. This is the capstone of the course and a strong piece for a high-school portfolio or college application.

LOOK FOR	WHAT IT MEANS
Program and client	A confirmed program with areas, adjacencies and budget, reflecting a real interview.
Site and design	A site analysis and plan, and a design whose plan, elevations, section and model agree and respond to the site.
Technical checks	Structure, daylight, stairs, roof, seating or rooms, exits, code and zoning checked with correct calculations.
Cost and schedule	Two estimates with contingency and escalation, a schedule with a critical path, and value-engineering choices explained.
Presentation and reflection	Clear, honest presentation with consistent scales and a well-made model, and a thoughtful reflection on the process.

COURSE EXAM

Midterm Exam

Covers Units 1–5. Time: 100 minutes.

You have 100 minutes. A scientific calculator and an architect's scale are allowed. Show your work for every calculation, with the formula, the numbers with units and the answer with its unit; correct setup earns partial credit even if the arithmetic slips. Draw free-body diagrams where they help. Use $g = 9.81\ \mathrm{m/s^2}$. Answer the essay in complete sentences. The exam has 28 problems worth a total of 100 points.

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1. **3 pts** **Lesson 1.1**

   Name Vitruvius's three tests of a good building and give one example of how a building can fail each test.

2. **3 pts** **Lesson 1.1**

   A granite block measures 0.80 m × 0.60 m × 0.50 m (2,700 kg/m^3). Find its mass and weight.

3. **4 pts** **Lesson 1.1**

   A column carries 220 kN to soil that safely carries 160 kPa. Size a square footing (round up to 0.1 m).

4. **4 pts** **Lesson 1.2**

   A pyramid has a square base 50 m on a side and is 32 m tall. Find its volume and the slope of its faces.

5. **3 pts** **Lesson 1.2**

   Convert a seked of 5¼ palms to a slope angle.

6. **3 pts** **Lesson 1.4**

   A hanger can carry 75 kN. Find its factor of safety for a 30 kN load, and for 45 kN after a design change.

7. **3 pts** **Lesson 1.4**

   The water behind a dam rises from 25 m to 30 m. By what factor does the horizontal water force grow?

8. 4 pts **Lesson 1.5**

Tasks: A 2 weeks; B 6 weeks after A; C 3 weeks after A; D 4 weeks after C; E 2 weeks after B and D. Find the critical path, the duration and the float of C and D.

9. 4 pts **Lesson 2.1**

A plan at ¼″ = 1′-0″ shows a wall 4⅜ in long. How long is it? How long is a 6.3 m wall at 1:50?

10. 3 pts **Lesson 2.1**

A 1:100 model has a floor area of 80 cm^2. What is the real floor area?

11. 4 pts **Lesson 2.3**

A gable roof on a building 30 ft wide has a 10:12 pitch. Find the rise, rafter length (no overhang) and angle.

12. 4 pts **Lesson 2.3**

A stair rises 9′-4″. With risers no more than 7¾ in and 10 in treads, find the number of risers, their height and the total run.

13. 3 pts **Lesson 2.4**

A tower 12 m tall is 60 m away; the picture plane is 0.50 m from the eye. How tall is its image?

14. 4 pts **Lesson 3.2**

An arch carries 20 kN/m over 18 m with a rise of 6 m. Find its horizontal thrust. How would the thrust change if the rise were 9 m?

15. 4 pts **Lesson 3.2**

A buttress pier weighs 800 kN and is 2.0 m wide. A thrust of 60 kN acts 10 m above its base. Find the factor of safety against overturning.

16. 4 pts **Lesson 3.3**

Find the arithmetic, geometric and harmonic means of 18 ft and 32 ft.

17. 3 pts **Lesson 3.4**

A 200 m line of iron girders warms by 30 °C ($\alpha = 12 \times 10^{-6}/^{\circ}\mathrm{C}$). How much does it grow?

18. 3 pts **Lesson 3.5**

A uniformly loaded cantilever is lengthened by 20%. By what factor does its tip deflection grow?

19. 4 pts **Lesson 4.1**

A room 20 ft × 25 ft has $D = 15$ psf and $L = 50$ psf. Find the factored pressure ($1.2D + 1.6L$) and total load.

20. 3 pts **Lesson 4.1**

Find the wind pressure for a 100 mph wind.

21. 3 pts **Lesson 4.2**

An 8 m beam carries 16 kN at 3 m from the left support. Find the reactions.

22. 4 pts **Lesson 4.3**

A triangular truss spans 8 m with its apex 2 m high and 10 kN at the apex. Find the force in each rafter and the tie (T or C).

23. 4 pts **Lesson 4.4**

A chain hangs with $a = 12$ cm between supports 12 cm apart. Find its sag with the catenary and the parabola approximation.

24. 4 pts **Lesson 4.5**

A six-story building has interior columns on a 7 m × 9 m grid, each floor (and roof) 9 kPa factored. Find the ground-floor column load.

25. 4 pts **Lesson 5.3**

A steel rod 30 mm in diameter and 4.0 m long carries 80 kN. Find its stress and stretch ($E = 200$ GPa).

26. 3 pts **Lesson 5.5**

A 2×10 joist (1½ in × 9¼ in) carries a moment of 20,000 in·lb. Find the bending stress.

27. 4 pts **Lesson 5.4**

A concrete beam is 300 mm wide, $d = 450$ mm, with $A_s = 800$ mm^2, $f_y = 415$ MPa, $f\prime_c = 25$ MPa. Find a and M_n.

28. 4 pts **Lesson 3.1–3.2**

Short essay. The Pantheon's dome and a Gothic cathedral's vaults both push outward. Explain how the Roman and Gothic builders each handled that thrust, and what their solutions made possible.

ANSWER KEY

1. Firmness (it stands safely: a roof that sags or a wall that cracks fails it); commodity (it serves its use: a church with poor sight lines or no exits fails it); delight (it gives pleasure: a dreary, ugly building fails it).
2. $V = 0.24\ \mathrm{m}^3$; $m = 648$ kg; $W \approx 6.36$ kN.
3. $A = 1.375\ \mathrm{m}^2$; side ≈ 1.17, use 1.2 m.
4. $V = \frac{1}{3}(2500)(32) \approx 26{,}700\ \mathrm{m}^3$; $\theta = \tan^{-1}(32/25) \approx 52.0^\circ$.
5. $\tan\theta = 7/5.25 \approx 1.333$, $\theta \approx 53.1^\circ$.
6. 2.5; 1.67.
7. $(30/25)^2 = 1.44$.
8. A–B–E = 10 weeks; A–C–D–E = 11 weeks, critical. Duration 11 weeks; C and D are on the critical path (0 float); B has 1 week of float.
9. $4.375 \times 4 = 17.5$ ft $= 17'$-6″; $6300/50 = 126$ mm.
10. $80 \times 10^4\ \mathrm{cm}^2 = 80\ \mathrm{m}^2$.
11. Rise 12.5 ft; rafter $\sqrt{15^2 + 12.5^2} \approx 19.5$ ft; angle $\approx 39.8^\circ$.
12. $112/7.75 = 14.5$, so 15 risers of about 7.47 in; 14 treads; run 140 in (11′-8″).
13. $0.50 \times 12/60 = 0.10$ m.
14. $H = 20 \times 324/48 = 135$ kN; at 9 m, 90 kN (two-thirds).
15. $M_r = 800$ kN·m; $M_o = 600$; $FS \approx 1.33$.
16. $A = 25$; $G = \sqrt{576} = 24$; $H = 2(18)(32)/50 = 23.04$ ft.
17. $0.072\ \mathrm{m} = 72$ mm.
18. $1.2^4 \approx 2.07$.
19. 98 psf; 49,000 lb.
20. $0.00256 \times 10{,}000 = 25.6$ psf.
21. $R_R = 16 \times 3/8 = 6$ kN; $R_L = 10$ kN.
22. Rafter length $\sqrt{20} \approx 4.47$ m, $\sin\theta \approx 0.447$: rafters $5/0.447 \approx 11.2$ kN C; tie $11.2 \times (4/4.47) = 10.0$ kN T.
23. Catenary $12(\cosh 0.5 - 1) \approx 1.53$ cm; parabola $36/24 = 1.5$ cm.
24. $6 \times 63 \times 9 = 3402$ kN.
25. $A \approx 707\ \mathrm{mm}^2$; $\sigma \approx 113$ MPa; $\delta \approx 2.3$ mm.
26. $S = 1.5 \times 9.25^2/6 \approx 21.4\ \mathrm{in}^3$; $\sigma \approx 935$ psi.

27. $a = 332{,}000/6375 \approx 52.1$ mm;
    $M_n = 332{,}000(450 - 26.0) \approx 1.41 \times 10^8$ N·mm $= 141$ kN·m.
28. Model answer: the Romans made the Pantheon's dome thick at the base with stepped rings and heavy drum walls, and light at the top with pumice concrete and coffers, so the drum could absorb the thrust and the dome could span 43 m without columns. Gothic builders used pointed arches to reduce thrust and ribbed vaults to gather it onto piers, then carried it outside with flying buttresses weighted by pinnacles, so the walls between could be glass. Each turned an understanding of thrust into a new kind of space.

COURSE EXAM

Final Exam

Covers Units 1–10. Time: 120 minutes.

You have 120 minutes. A scientific calculator and an architect's scale are allowed. Show your work for every calculation, with units; correct setup earns partial credit. Use $g = 9.81\ \mathrm{m/s^2}$ and $\gamma_w = 9.81\ \mathrm{kN/m^3}$. Answer short-answer questions and the essay in complete sentences. The exam has 30 problems worth a total of 100 points.

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1. **3 pts** **Lesson 1.4**

   Distinguish the work and licensing of an architect and a civil engineer.

2. **3 pts** **Lesson 2.1**

   A plan at ⅛″ = 1′-0″ shows a wall 6¼ in long. How long is the wall?

3. **3 pts** **Lesson 2.2**

   A building is 36 ft × 48 ft outside, with 1,400 ft² of usable rooms. Find its gross area and efficiency.

4. **3 pts** **Lesson 3.2**

   A pointed vault carries 35 kN/m over 14 m with a rise of 10 m. Find its thrust.

5. **3 pts** **Lesson 3.4**

   Why did the Crystal Palace need diagonal tie rods, and why could it be built so fast?

6. **3 pts** **Lesson 4.1**

   A town's ground snow load is 35 psf. Estimate the flat-roof snow load and the total snow on a 40 ft × 60 ft roof.

7. **3 pts** **Lesson 4.2**

   A 10 m beam carries 5 kN/m over its length and a 20 kN load 4 m from the left support. Find the reactions.

8. **4 pts** **Lesson 4.3**

   A triangular truss spans 12 m with its apex 4.5 m high and 18 kN at the apex. Find the rafter and tie forces.

9. 3 pts **Lesson 5.5**

A 4×10 beam (3½ in × 9¼ in) carries a moment of 60,000 in·lb. Find its bending stress.

10. 3 pts **Lesson 5.5**

A beam deflects 12 mm. What would it deflect if (a) its depth were 1.5 times larger, (b) its span were 25% longer?

11. 3 pts **Lesson 5.2**

A masonry wall 50 m tall ($\gamma = 19\ \mathrm{kN/m^3}$) carries 375 kN/m from its floors. With an allowable stress of 1.6 MPa, find its minimum thickness.

12. 4 pts **Lesson 6.3**

A suspension span of 800 m with a sag of 80 m carries 150 kN/m. Find H, V and $T_{\max}$.

13. 3 pts **Lesson 6.4**

Find the Euler buckling load of a pinned steel strut 40 mm square and 3.0 m long ($E = 200$ GPa).

14. 3 pts **Lesson 6.4**

Contrast the causes of the Quebec Bridge (1907) and Tacoma Narrows (1940) failures.

15. 3 pts **Lesson 7.2**

An elevator car of 1,000 kg with an 800 kg load and a 1,400 kg counterweight rises at 3 m/s. Find the motor power at 80% efficiency.

16. 4 pts **Lesson 7.3**

A tower 45 m wide and 300 m tall has an average wind pressure of 2.0 kPa. Find the wind force, the base moment and the drift limit at $H/500$.

17. 3 pts **Lesson 7.5**

A fellowship hall of 2,700 ft^2 is used with tables and chairs (15 ft^2/person). Find the occupant load, the exit width at 0.2 in per person, and the minimum number of exits.

18. (4 pts) **Lesson 8.1**

Soil weighs 18 kN/m^3 above a water table at 2 m and 20 kN/m^3 below. Find σ, u and $\sigma\prime$ at 9 m.

19. (3 pts) **Lesson 8.2**

A column carries 2,800 kN on soil allowing 200 kPa. Size a square footing.

20. (4 pts) **Lesson 8.2**

A pile 0.45 m in diameter and 20 m long has $f_s = 30$ kPa and $q_b = 2200$ kPa. Find its ultimate and allowable capacity (FS = 3).

21. (4 pts) **Lesson 8.4**

A wall 4 m high retains dry soil with $\gamma = 18$ and $\phi = 30^\circ$. Find P_a and where it acts. What would water alone add if the drains clogged?

22. (3 pts) **Lesson 8.5**

Find the horizontal water force per meter on a dam holding 60 m of water, and where it acts.

23. (3 pts) **Lesson 9.1**

A benchmark is 75.00 m. BS = 1.85 m; FS on point Q = 0.62 m. Find Q's elevation.

24. (3 pts) **Lesson 9.3**

Find the peak runoff from 6 acres ($C = 0.85$) in a 2.4 in/h storm.

25. (3 pts) **Lesson 9.2**

A town of 30,000 uses 100 gallons per person per day. Find the demand in MGD and the chlorine needed at 1.8 mg/L.

26. (3 pts) **Lesson 9.4**

Find the ESAL of a 27,000 lb single axle.

27. (3 pts) **Lesson 9.5**

A curve of radius 500 m has a design speed of 110 km/h. With $f = 0.11$, what superelevation is needed?

28. **4 pts** **Lesson 10.2**

At 42° N, find the noon sun altitude at both solstices, and the overhang projection that shades a window whose sill is 4 ft below the overhang at summer noon.

29. **3 pts** **Lesson 10.4**

A 2,000 ft^2 house costs $225 per ft^2 with 18% for fees and contingency. Find the budget today and escalated 3 years at 4% a year.

30. **6 pts** **Lesson whole course**

Essay. Choose three case studies from this course (for example the St. Francis Dam, the Quebec Bridge, Ronan Point, Hartford, Fallingwater or Pisa) and explain what each taught engineers about their responsibility to the public. End with how you would apply those lessons in your own work.

ANSWER KEY

1. The architect designs the building's plan, form and spaces and coordinates the team; licensed after an accredited degree, experience and the ARE. The civil engineer designs structures, foundations and public works; licensed as a PE after an accredited degree, the FE exam, about four years' experience and the PE exam.
2. 50 ft.
3. 1,728 ft^2; about 81%.
4. $35 \times 196/80 \approx 85.8$ kN.
5. Diagonals kept the column-and-beam frame from racking in wind; it was built quickly because it was made of a few standard, factory-made parts on a 24 ft module, delivered by rail and bolted together.
6. $0.7 \times 35 = 24.5$ psf; $24.5 \times 2400 = 58{,}800$ lb.
7. Total 70 kN; $R_R = (50 \times 5 + 20 \times 4)/10 = 33$ kN; $R_L = 37$ kN.
8. Rafter length 7.5 m, $\sin\theta = 0.6$, $\cos\theta = 0.8$: rafters $9/0.6 = 15$ kN C; tie 12 kN T.
9. $S \approx 49.9\ \text{in}^3$; $\sigma \approx 1200$ psi.
10. (a) $12/3.375 \approx 3.6$ mm; (b) $12 \times 2.44 \approx 29$ mm.
11. $t = 375/(1600 - 950) \approx 0.58$ m.
12. $H = 150 \times 640{,}000/640 = 150{,}000$ kN; $V = 60{,}000$ kN; $T \approx 161{,}600$ kN.
13. $I \approx 213{,}300\ \text{mm}^4$; $P \approx 46.8$ kN.
14. Quebec: a compression chord buckled because its lacing was too weak and the dead load had been underestimated after the span was lengthened, with no independent check. Tacoma: the slender, shallow solid-girder deck was aerodynamically unstable and twisted apart by aeroelastic flutter in moderate wind.
15. Imbalance 400 kg: 11,772 W; $\div 0.8 \approx 14.7$ kW.
16. $F = 27{,}000$ kN; $M = 4.05 \times 10^6$ kN·m; 0.60 m.
17. 180 people; 36 in; 2 exits.
18. $\sigma = 176$ kPa; $u \approx 68.7$ kPa; $\sigma\prime \approx 107$ kPa.
19. $A = 14\ \text{m}^2$; side ≈ 3.74, use 3.8 m.
20. Skin ≈ 848 kN; end ≈ 350 kN; $Q_u \approx 1198$ kN; $Q_a \approx 399$ kN.
21. $P_a = \frac{1}{2}(1/3)(18)(16) = 48$ kN/m at 1.33 m; water about 78.5 kN/m.

22. $\frac{1}{2}(9.81)(3600) \approx 17{,}660$ kN/m, at 20 m above the base.
23. 76.23 m.
24. ≈ 12.2 ft^3/s.
25. 3 MGD; $8.34 \times 3 \times 1.8 \approx 45$ lb/day.
26. $(27/18)^4 = 1.5^4 \approx 5.06$.
27. $v = 30.6$ m/s; $v^2/(gR) \approx 0.190$; $e \approx 0.08$ (8%).
28. 71.4° and 24.6°; $4/\tan 71.4^\circ \approx 1.35$ ft.
29. \$531,000; $\times 1.04^3 \approx$ \$597,300.
30. Model answer: e.g., St. Francis: investigate the ground and require independent review, leading to licensure; Quebec: verify every assumption, recalculate after changes and stop work when warnings appear (the iron ring); Ronan Point: design for robustness so that local damage cannot become progressive collapse. Engineers must hold the public's safety paramount, check their work and others', listen to warnings, and be honest about what they do not know. A thoughtful conclusion applies these to the student's own design studio project.