Physics

First edition | 10 units | 50 lessons | A video for every lesson

FIRST EDITION

Course guide

This is a full-year, college-preparatory physics course for students in grades 10–12. It covers the whole classical core: measurement and graphing, motion in one and two dimensions, Newton's laws, circular motion and gravitation, work and energy, momentum, rotation and statics, fluids and heat, waves, sound and light, and electricity and magnetism, ending with Maxwell's equations and a first look at the photon and the atom. Every lesson has real equations, two to four fully worked examples with units on every line, a practice set of 8 to 12 problems with answers, and a lab. Each unit ends with a 100-point test, and there is a midterm after Unit 5 and a final after Unit 10.

Physics was discovered by people, one experiment at a time, and the course is built around those experiments. Each lesson begins with **the story** (Kepler wrestling with Tycho's Mars data, Stapp riding a rocket sled, Cavendish weighing the Earth in a shed, Joule warming water with a falling weight, Buys Ballot's horn players on a train, Faraday's coil and magnet), then **how they figured it out**: what they measured, what went wrong, and how the idea was tested. Then comes **the physics**, with the equations and worked examples, and **the engineering**: where the idea is used today, from seat belts and roller coasters to cranes, hydraulics, power stations and fiber optics. Then the student repeats the discovery in a lab with a hypothesis, a procedure, a data table, a graph with a best-fit line, and an honest discussion of error.

Common toys make excellent lab apparatus, and the course uses them on purpose: Hot Wheels cars and track for kinematics, projectiles and the loop-the-loop, a pull-back car for motion graphs, a spring toy for Hooke's law and oscillation, a Newton's cradle and a Super Ball for collisions, a balancing-bird toy for center of mass, a yo-yo for rotational inertia, a top for angular momentum, a squeeze bottle or Super Soaker for pressure and Torricelli's law, a Slinky for waves, and a kaleidoscope or periscope for reflection. The rest of the equipment is a meter stick, a kitchen scale, a phone

stopwatch, string, marbles, a few batteries and magnets. God made an orderly world that can be measured, and many of the people in these stories, Kepler, Pascal, Boyle, Newton, Faraday, Joule, Kelvin and Maxwell among them, studied it as His handiwork. The old saying *Fabricando fit faber*, "by making, one becomes a maker," describes the method: you learn physics by doing it.

> Videos were chosen lesson by lesson from trusted teaching channels, most often Flipping Physics and The Organic Chemistry Tutor (clear, worked-problem lessons), with Khan Academy, Bozeman Science, Professor Matt Anderson, MIT OpenCourseWare, The Efficient Engineer, Veritasium, Steve Mould, the Royal Institution and NASA. Backup videos add history and live experiments from NASA, the Smithsonian, Practical Engineering, The B1M, Mark Rober, the National MagLab and others. Every video was checked in preview for accuracy and content. Some are 15 to 30 minutes long; the "when" note says when to watch, and a parent note flags the few with a sponsor message (skip it). Watch with the notebook open and pause to work each example before the presenter does.

Start with Lesson 1.1 →

Who it is for

Written for students in grades 10–12 who have completed Algebra I and geometry. Right-triangle trigonometry (sine, cosine, tangent) is taught in Lesson 1.5 where it is first needed, and no calculus is required. Students work fairly independently, reading the lesson and working the examples themselves; a parent or mentor checks the practice sets against the answers, discusses the essay questions and supervises the labs. No kits are required: every lab uses a scientific calculator, household materials and inexpensive hardware-store or craft-store items, and the free phyphox app, which reads a phone's sensors, makes many labs more precise. Optional kits are listed in each unit.

How to use this course

1. **The weekly rhythm.** Each lesson takes three sessions of about 50 minutes. Session 1: the warm-up, the video, the core ideas and the worked examples. Session 2: the lab and the lab notebook. Session 3: the practice set, checked against the answers, with every missed problem corrected in the notebook. Each unit ends with a test

day, and the midterm and final each have a review day. At five sessions a week the course fills a school year of about 33 weeks.

2. **Story, then physics, then engineering.** Read the core ideas in order. The story and "how they figured it out" explain why the equation exists; do not skip them. Then cover the solution of each worked example, try it yourself, and compare line by line.
3. **Units on every line.** Write the formula, substitute the numbers with their units, simplify the units along with the numbers, and box the answer with its unit and a sensible number of significant figures. Most physics errors are unit errors, and units are also your best check.
4. **Practice sets.** Problems are marked easy, medium, hard and challenge. Do all of the easy and medium problems and most of the hard ones; the challenge problem is for students who want more. Mark your work against the answers the same day, and rework every miss until you understand it.
5. **Picture study.** Each lesson has one historic engraving, drawing, photograph or painting of the experiment or the machine. Look at it quietly for two minutes, put it away and describe what you saw, then work through the "look for" items and the questions, which point to the physics.
6. **The lab notebook.** Use a bound graph-paper notebook for the whole year. For every lab write the date, the question, a hypothesis with a reason, a labeled sketch of the setup, a data table with units and uncertainties, a graph with a best-fit line, a comparison with theory (percent difference), and a paragraph on sources of error and what you would change. Record what actually happened, not what should have happened: "a just weight is his delight" (Proverbs 11:1).
7. **Tests and exams.** Each unit test is worth 100 points and mixes calculations, concept questions and one short essay. The midterm covers Units 1–5 and the final covers the whole course. Calculators are allowed; formulas are not given, so make a one-page formula sheet for each unit as you go and study from it.
8. **Safety.** Read the safety note before each lab. Electricity comes only from batteries, never from the outlets in the wall. Lasers are low-power pointers (5 mW or less) and are never pointed at anyone's eyes. Wear safety glasses for anything under tension, spinning or launched. Hot water and the water rocket need an adult nearby.

9. **Optional kits.** Some units list kits from PASCO, Arbor Scientific, Thames & Kosmos, Snap Circuits and others. They make a few labs more precise, but every lab can be done without them.

Scope and sequence

UNIT 1 · WEEKS 1–4

Measurement and the Scientific Method

How physics knows what it knows: Kepler and Galileo as data-driven scientists, SI units and conversions, significant figures and uncertainty, graphs and best-fit lines with Hooke's law, and vectors with right-triangle trigonometry.

1.1 Galileo, Kepler and Data-Driven Science

1.2 SI Units: The Meter, the Kilogram and the Lost Mars Orbiter

1.3 Uncertainty and Significant Figures: Michelson Measures Light

1.4 Graphs and Best-Fit Lines: Hooke's Law

1.5 Vectors and Right-Triangle Trigonometry: Stevin's Wreath of Spheres

UNIT 2 · WEEKS 4–7

Motion in One and Two Dimensions

Galileo's inclined plane, velocity and acceleration with the kinematic equations, free fall from Pisa to the Moon, motion graphs from Marey's chronophotographs, and projectile motion.

2.1 Galileo's Inclined Plane

2.2 Velocity and Acceleration: Stapp's Rocket Sled

2.3 Free Fall: From Pisa to the Hammer and the Feather

2.4 Motion Graphs: Marey's Chronophotographs

2.5 Projectiles: Galileo's Parabola

UNIT 3 · WEEKS 7–10

Newton's Laws

Newton's *Principia* and the three laws, inertia and mass, F = ma with free-body diagrams, friction from Leonardo to Coulomb, and inclines, tension and the Atwood machine.

3.1 The Principia and Newton's Three Laws

3.2 Inertia and Mass: Weighing Astronauts on Skylab

3.3 F = ma and Free-Body Diagrams: Otis and the Elevator

3.4 Friction: From Leonardo to Coulomb

3.5 Inclines, Tension and the Atwood Machine

UNIT 4 · WEEKS 10–13

Circular Motion and Gravitation

Centripetal acceleration and banked curves, Newton's Moon test and universal gravitation, Kepler's three laws, Cavendish's torsion balance, and orbits from Newton's mountain to Apollo.

4.1 Centripetal Acceleration: Huygens and the Curve

4.2 The Moon and the Apple: Universal Gravitation

4.3 Kepler's Laws of Planetary Motion

4.4 Cavendish Weighs the Earth

4.5 Orbits: From Newton's Mountain to Apollo

UNIT 5 · WEEKS 13–17

Work, Energy and Power

Joule's paddle wheel and the conservation of energy, work and kinetic energy, potential energy, power and Watt's horsepower, and roller coasters and efficiency.

5.1 Joule's Paddle Wheel: Energy Is Conserved

5.2 Work and Kinetic Energy: Balls Dropped into Clay

5.3 Potential Energy and Conservation: Galileo's Interrupted Pendulum

5.4 Power and Watt's Horsepower

5.5 Roller Coasters, Loops and Efficiency

UNIT 6 · WEEKS 17–20

Momentum

Impulse and the seat belt, conservation of momentum and Newton's cradle, elastic and inelastic collisions, rockets and Goddard, and collisions in two dimensions.

6.1 Impulse and the Seat Belt

6.2 Conservation of Momentum and Newton's Cradle

6.3 Elastic and Inelastic Collisions: The Super Ball

6.4 Rockets: Goddard and the Rocket Equation

6.5 Momentum in Two Dimensions: Billiards and Accident Reconstruction

UNIT 7 · WEEKS 20–23

Rotation and Statics

Torque and Archimedes' lever, center of mass and the Leaning Tower, static equilibrium in cranes and the Forth Bridge, rotational inertia and the yo-yo, and angular momentum in tops and gyroscopes.

7.1 Torque: Archimedes' Lever

7.2 Center of Mass and the Leaning Tower

7.3 Equilibrium: Cranes and the Forth Bridge

7.4 Rotational Inertia: The Yo-Yo and the Flywheel

7.5 Angular Momentum: Tops, Gyroscopes and Foucault

UNIT 8 · WEEKS 23–26

Fluids and Heat

Pressure and Pascal, buoyancy and Archimedes, Bernoulli and the lift on a wing, temperature, heat and calorimetry, and the laws of thermodynamics with Carnot and Kelvin.

8.1 Pressure: Pascal and the Puy de Dôme

8.2 Buoyancy: Archimedes' Principle

8.3 Moving Fluids: Bernoulli, Torricelli and the Wing

8.4 Temperature and Heat: Joseph Black and the Calorimeter

8.5 Thermodynamics and Engines: Carnot and Kelvin

UNIT 9 · WEEKS 27–30

Waves, Sound and Light

Simple harmonic motion and Huygens' pendulum clock, waves and sound, the Doppler effect, reflection, refraction and lenses, and Young's double slit.

9.1 Oscillations: Huygens and the Pendulum Clock

9.2 Waves and Sound: The Slinky and Chladni's Plates

9.3 The Doppler Effect: Horns on a Train

9.4 Reflection, Refraction and Lenses: Newton's Telescope

9.5 Light as a Wave: Young's Double Slit

UNIT 10 · WEEKS 30–33

Electricity and Magnetism

Charge and Coulomb's law, circuits and Ohm's law, magnetism and Oersted, Faraday's induction with generators and motors, and Maxwell's equations, light, the photon and the atom.

10.1 Charge and Coulomb's Law

10.2 Circuits and Ohm's Law

10.3 Magnetism: Gilbert, Oersted and the Electric Current

10.4 Faraday's Induction: Generators, Motors and Transformers

10.5 Maxwell, Light and the Photon

Weekly pacing calendar

WEEK	PLAN
1	• Day 1: Lesson 1.1 Galileo, Kepler and Data-Driven Science: video, teaching and worked examples • Day 2: Lesson 1.1: lab and notebook • Day 3: Lesson 1.1: practice set and corrections • Day 4: Lesson 1.2 SI Units: The Meter, the Kilogram and the Lost Mars Orbiter: video, teaching and worked examples • Day 5: Lesson 1.2: lab and notebook *Five sessions a week of about 50 minutes. Families on a four-day week can combine the practice day of one lesson with the teaching day of the next.*
2	• Day 1: Lesson 1.2: practice set and corrections • Day 2: Lesson 1.3 Uncertainty and Significant Figures: Michelson Measures Light: video, teaching and worked examples • Day 3: Lesson 1.3: lab and notebook • Day 4: Lesson 1.3: practice set and corrections • Day 5: Lesson 1.4 Graphs and Best-Fit Lines: Hooke's Law: video, teaching and worked examples
3	• Day 1: Lesson 1.4: lab and notebook • Day 2: Lesson 1.4: practice set and corrections • Day 3: Lesson 1.5 Vectors and Right-Triangle Trigonometry: Stevin's Wreath of Spheres: video, teaching and worked examples • Day 4: Lesson 1.5: lab and notebook • Day 5: Lesson 1.5: practice set and corrections
4	• Day 1: Unit 1 test • Day 2: Lesson 2.1 Galileo's Inclined Plane: video, teaching and worked examples • Day 3: Lesson 2.1: lab and notebook • Day 4: Lesson 2.1: practice set and corrections • Day 5: Lesson 2.2 Velocity and Acceleration: Stapp's Rocket Sled: video, teaching and worked examples
5	• Day 1: Lesson 2.2: lab and notebook • Day 2: Lesson 2.2: practice set and corrections • Day 3: Lesson 2.3 Free Fall: From Pisa to the Hammer and the Feather: video, teaching and worked examples

WEEK	PLAN
	• Day 4: Lesson 2.3: lab and notebook • Day 5: Lesson 2.3: practice set and corrections
6	• Day 1: Lesson 2.4 Motion Graphs: Marey's Chronophotographs: video, teaching and worked examples • Day 2: Lesson 2.4: lab and notebook • Day 3: Lesson 2.4: practice set and corrections • Day 4: Lesson 2.5 Projectiles: Galileo's Parabola: video, teaching and worked examples • Day 5: Lesson 2.5: lab and notebook
7	• Day 1: Lesson 2.5: practice set and corrections • Day 2: Unit 2 test • Day 3: Lesson 3.1 The Principia and Newton's Three Laws: video, teaching and worked examples • Day 4: Lesson 3.1: lab and notebook • Day 5: Lesson 3.1: practice set and corrections
8	• Day 1: Lesson 3.2 Inertia and Mass: Weighing Astronauts on Skylab: video, teaching and worked examples • Day 2: Lesson 3.2: lab and notebook • Day 3: Lesson 3.2: practice set and corrections • Day 4: Lesson 3.3 F = ma and Free-Body Diagrams: Otis and the Elevator: video, teaching and worked examples • Day 5: Lesson 3.3: lab and notebook
9	• Day 1: Lesson 3.3: practice set and corrections • Day 2: Lesson 3.4 Friction: From Leonardo to Coulomb: video, teaching and worked examples • Day 3: Lesson 3.4: lab and notebook • Day 4: Lesson 3.4: practice set and corrections • Day 5: Lesson 3.5 Inclines, Tension and the Atwood Machine: video, teaching and worked examples
10	• Day 1: Lesson 3.5: lab and notebook • Day 2: Lesson 3.5: practice set and corrections • Day 3: Unit 3 test • Day 4: Lesson 4.1 Centripetal Acceleration: Huygens and the Curve: video, teaching and worked examples

WEEK	PLAN
	• Day 5: Lesson 4.1: lab and notebook
11	• Day 1: Lesson 4.1: practice set and corrections • Day 2: Lesson 4.2 The Moon and the Apple: Universal Gravitation: video, teaching and worked examples • Day 3: Lesson 4.2: lab and notebook • Day 4: Lesson 4.2: practice set and corrections • Day 5: Lesson 4.3 Kepler's Laws of Planetary Motion: video, teaching and worked examples
12	• Day 1: Lesson 4.3: lab and notebook • Day 2: Lesson 4.3: practice set and corrections • Day 3: Lesson 4.4 Cavendish Weighs the Earth: video, teaching and worked examples • Day 4: Lesson 4.4: lab and notebook • Day 5: Lesson 4.4: practice set and corrections
13	• Day 1: Lesson 4.5 Orbits: From Newton's Mountain to Apollo: video, teaching and worked examples • Day 2: Lesson 4.5: lab and notebook • Day 3: Lesson 4.5: practice set and corrections • Day 4: Unit 4 test • Day 5: Lesson 5.1 Joule's Paddle Wheel: Energy Is Conserved: video, teaching and worked examples
14	• Day 1: Lesson 5.1: lab and notebook • Day 2: Lesson 5.1: practice set and corrections • Day 3: Lesson 5.2 Work and Kinetic Energy: Balls Dropped into Clay: video, teaching and worked examples • Day 4: Lesson 5.2: lab and notebook • Day 5: Lesson 5.2: practice set and corrections
15	• Day 1: Lesson 5.3 Potential Energy and Conservation: Galileo's Interrupted Pendulum: video, teaching and worked examples • Day 2: Lesson 5.3: lab and notebook • Day 3: Lesson 5.3: practice set and corrections • Day 4: Lesson 5.4 Power and Watt's Horsepower: video, teaching and worked examples • Day 5: Lesson 5.4: lab and notebook

WEEK	PLAN
16	• Day 1: Lesson 5.4: practice set and corrections • Day 2: Lesson 5.5 Roller Coasters, Loops and Efficiency: video, teaching and worked examples • Day 3: Lesson 5.5: lab and notebook • Day 4: Lesson 5.5: practice set and corrections • Day 5: Unit 5 test
17	• Day 1: Review day for the midterm • Day 2: Midterm exam • Day 3: Lesson 6.1 Impulse and the Seat Belt: video, teaching and worked examples • Day 4: Lesson 6.1: lab and notebook • Day 5: Lesson 6.1: practice set and corrections
18	• Day 1: Lesson 6.2 Conservation of Momentum and Newton's Cradle: video, teaching and worked examples • Day 2: Lesson 6.2: lab and notebook • Day 3: Lesson 6.2: practice set and corrections • Day 4: Lesson 6.3 Elastic and Inelastic Collisions: The Super Ball: video, teaching and worked examples • Day 5: Lesson 6.3: lab and notebook
19	• Day 1: Lesson 6.3: practice set and corrections • Day 2: Lesson 6.4 Rockets: Goddard and the Rocket Equation: video, teaching and worked examples • Day 3: Lesson 6.4: lab and notebook • Day 4: Lesson 6.4: practice set and corrections • Day 5: Lesson 6.5 Momentum in Two Dimensions: Billiards and Accident Reconstruction: video, teaching and worked examples
20	• Day 1: Lesson 6.5: lab and notebook • Day 2: Lesson 6.5: practice set and corrections • Day 3: Unit 6 test • Day 4: Lesson 7.1 Torque: Archimedes' Lever: video, teaching and worked examples • Day 5: Lesson 7.1: lab and notebook
21	• Day 1: Lesson 7.1: practice set and corrections • Day 2: Lesson 7.2 Center of Mass and the Leaning Tower: video, teaching and worked examples

WEEK	PLAN
	• Day 3: Lesson 7.2: lab and notebook • Day 4: Lesson 7.2: practice set and corrections • Day 5: Lesson 7.3 Equilibrium: Cranes and the Forth Bridge: video, teaching and worked examples
22	• Day 1: Lesson 7.3: lab and notebook • Day 2: Lesson 7.3: practice set and corrections • Day 3: Lesson 7.4 Rotational Inertia: The Yo-Yo and the Flywheel: video, teaching and worked examples • Day 4: Lesson 7.4: lab and notebook • Day 5: Lesson 7.4: practice set and corrections
23	• Day 1: Lesson 7.5 Angular Momentum: Tops, Gyroscopes and Foucault: video, teaching and worked examples • Day 2: Lesson 7.5: lab and notebook • Day 3: Lesson 7.5: practice set and corrections • Day 4: Unit 7 test • Day 5: Lesson 8.1 Pressure: Pascal and the Puy de Dôme: video, teaching and worked examples
24	• Day 1: Lesson 8.1: lab and notebook • Day 2: Lesson 8.1: practice set and corrections • Day 3: Lesson 8.2 Buoyancy: Archimedes' Principle: video, teaching and worked examples • Day 4: Lesson 8.2: lab and notebook • Day 5: Lesson 8.2: practice set and corrections
25	• Day 1: Lesson 8.3 Moving Fluids: Bernoulli, Torricelli and the Wing: video, teaching and worked examples • Day 2: Lesson 8.3: lab and notebook • Day 3: Lesson 8.3: practice set and corrections • Day 4: Lesson 8.4 Temperature and Heat: Joseph Black and the Calorimeter: video, teaching and worked examples • Day 5: Lesson 8.4: lab and notebook
26	• Day 1: Lesson 8.4: practice set and corrections • Day 2: Lesson 8.5 Thermodynamics and Engines: Carnot and Kelvin: video, teaching and worked examples

WEEK	PLAN
	• Day 3: Lesson 8.5: lab and notebook • Day 4: Lesson 8.5: practice set and corrections • Day 5: Unit 8 test
27	• Day 1: Lesson 9.1 Oscillations: Huygens and the Pendulum Clock: video, teaching and worked examples • Day 2: Lesson 9.1: lab and notebook • Day 3: Lesson 9.1: practice set and corrections • Day 4: Lesson 9.2 Waves and Sound: The Slinky and Chladni's Plates: video, teaching and worked examples • Day 5: Lesson 9.2: lab and notebook
28	• Day 1: Lesson 9.2: practice set and corrections • Day 2: Lesson 9.3 The Doppler Effect: Horns on a Train: video, teaching and worked examples • Day 3: Lesson 9.3: lab and notebook • Day 4: Lesson 9.3: practice set and corrections • Day 5: Lesson 9.4 Reflection, Refraction and Lenses: Newton's Telescope: video, teaching and worked examples
29	• Day 1: Lesson 9.4: lab and notebook • Day 2: Lesson 9.4: practice set and corrections • Day 3: Lesson 9.5 Light as a Wave: Young's Double Slit: video, teaching and worked examples • Day 4: Lesson 9.5: lab and notebook • Day 5: Lesson 9.5: practice set and corrections
30	• Day 1: Unit 9 test • Day 2: Lesson 10.1 Charge and Coulomb's Law: video, teaching and worked examples • Day 3: Lesson 10.1: lab and notebook • Day 4: Lesson 10.1: practice set and corrections • Day 5: Lesson 10.2 Circuits and Ohm's Law: video, teaching and worked examples
31	• Day 1: Lesson 10.2: lab and notebook • Day 2: Lesson 10.2: practice set and corrections • Day 3: Lesson 10.3 Magnetism: Gilbert, Oersted and the Electric Current: video, teaching and worked examples • Day 4: Lesson 10.3: lab and notebook

WEEK	PLAN
	• Day 5: Lesson 10.3: practice set and corrections
32	• Day 1: Lesson 10.4 Faraday's Induction: Generators, Motors and Transformers: video, teaching and worked examples • Day 2: Lesson 10.4: lab and notebook • Day 3: Lesson 10.4: practice set and corrections • Day 4: Lesson 10.5 Maxwell, Light and the Photon: video, teaching and worked examples • Day 5: Lesson 10.5: lab and notebook
33	• Day 1: Lesson 10.5: practice set and corrections • Day 2: Unit 10 test • Day 3: Review day for the final • Day 4: Final exam

Materials

Scientific calculator

With sine, cosine, tangent, square roots and scientific notation. A graphing calculator is fine but not needed.

Bound graph-paper lab notebook, pencil and ruler

For every lab, every graph and every corrected practice problem.

Meter stick, tape measure, protractor and a kitchen scale that reads to 1 g

The basic measuring tools for nearly every lab.

A smartphone with a stopwatch, slow-motion video and the free phyphox app

Phyphox (from RWTH Aachen University) reads the phone's accelerometer, gyroscope, microphone, light and magnetic sensors. Optional but excellent; every lab has a stopwatch or video method too.

String or fishing line, tape, paper clips, rubber bands, binder clips, washers or coins as standard masses, and zip-top bags

For pendulums, pulleys, hanging masses and force tables.

Marbles or steel balls, a smooth ramp board (about 1.5 m) and books to prop it

For Galileo's ramp, projectiles and rolling contests.

Toy apparatus: a Hot Wheels car with a few lengths of orange track and a loop, a pull-back car, a spring toy or small Slinky, a full-size metal Slinky, a yo-yo, a top, a balancing-bird toy, a bouncy rubber ball and a squeeze bottle or water blaster

Inexpensive toys used as lab equipment in about a dozen lessons. A Newton's cradle is optional (marbles in a groove work too).

Springs and spring scales (0–10 N or similar), two empty spools or a small pulley, and two large plastic syringes (no needles) with tubing

For Hooke's law, the Atwood machine and hydraulics.

A thermometer (digital kitchen probe), foam cups with lids, and a few steel washers or bolts

For calorimetry and the gas-law lab. An adult handles boiling water.

A flashlight, a small mirror, a magnifying glass, a clear glass or plastic box of water, an old CD or DVD, and a laser pointer of 5 mW or less

For the optics labs. Never point a laser at anyone's eyes or at a mirror toward a face.

AA batteries with holders, a 9 V battery, small bulbs or LEDs, a few resistors, alligator clip leads, magnet wire, an iron nail, strong magnets, a compass and an inexpensive digital multimeter

For Unit 10. Batteries only; never the outlets in the wall.

Balloons, aluminum foil, a wool sock and a clear jar

For the electrostatics lab.

Two 2-liter soda bottles, a rubber stopper, a ball-inflation needle and a bicycle pump with a gauge

For the water rocket in Lesson 6.4, flown outdoors with an adult.

Safety glasses

For anything under tension, spinning, bouncing hard or launched.

Optional: a PASCO or Arbor Scientific dynamics cart, a Newton's cradle, a gyroscope, a Snap Circuits set

Never required; see the kits listed in each unit.

Grading

Grades come from the practice sets, the labs, the lab notebook, the ten unit tests, the midterm and the final. A calculation earns full credit when the formula, the substitution with units and the answer with its unit and sensible significant figures are all shown. A lab earns full credit for honest data carefully analyzed, even when the result disagrees with theory, as long as the student explains why.

PART	%
Practice sets (completed and corrected)	**15**
Labs (procedure, data, graph, analysis)	**25**
Lab notebook (organization, honesty, error discussion)	**10**
Unit tests	**30**
Midterm exam	**10**
Final exam	**10**

Scale: A 90–100, B 80–89, C 70–79, D 60–69, F below 60.

- Practice sets are graded for completion and correction, not first-try accuracy: a problem done, checked and correctly reworked earns full credit.
- On tests, give partial credit for a correct formula and setup with units even if the arithmetic slips. A missing or wrong unit costs a point; an answer with no work earns at most half credit.
- A lab report earns full marks when it has a hypothesis with a reason, a data table with units, a graph with a best-fit line and its slope, a comparison with theory, and a specific paragraph on error and improvements.
- For a high-school transcript this course is one full credit of laboratory physics. Keep the lab notebook and tests as the record of the year.

A note to parents

You do not need to know physics to supervise this course well. Your jobs are to keep the rhythm (three sessions per lesson), check the practice sets against the answers, ask the student to explain one worked example aloud each week, read the lab notebook, and be present for the labs that need an adult: hot water, the water rocket, lasers and anything under tension. The best question you can ask is “How do you know?” A student who can answer it from the data is learning physics.

Expect the labs not to match theory exactly. Friction, air resistance, reaction time and imperfect equipment are always present, and finding and measuring them is half of what a physicist does. Praise careful measurement and honest records more than a perfect answer. The course often tells how the great experimenters got things wrong first, and how they found out: Kepler threw away years of work over 8 minutes of arc, and Joule repeated his paddle-wheel measurement for years to make it better.

Many of the founders of physics were believing Christians who saw their work as reading the book of nature that God wrote: Kepler, Boyle, Newton, Pascal, Faraday, Joule, Kelvin and Maxwell. The course says so plainly where it is true and leaves the rest for family discussion. It does not teach origins, and it ends with a short, honest look at the photon and the atom as the doorway to modern physics, for students going on to college science or engineering. *Fabricando fit faber*: by doing physics, a student becomes a physicist.

UNIT 1

Measurement and the Scientific Method

Physics is the science that measures. Before you can study motion, force or energy, you need the tools that every physicist uses every day: careful experiments with controlled variables, a common system of units, an honest statement of uncertainty, graphs that turn data into laws, and vectors for quantities that have direction. This unit teaches those tools through the people who forged them: Tycho and Kepler, who overturned two thousand years of circles with 8 minutes of arc; the surveyors who measured the meter from Dunkirk to Barcelona; Michelson, timing light with a spinning mirror; Hooke, hanging weights from springs; and Stevin, proving the law of the inclined plane with a necklace of balls.

You will run five labs: a fair test of Galileo's claim that heavy and light objects fall together, a density survey of household materials, a twenty-trial measurement of your own reaction time, the spring constant of a toy spring, and a three-scale force table. Right-triangle trigonometry is introduced in Lesson 1.5, and you will use it all year.

PREREQUISITES

- Algebra I: solving linear equations, working with exponents and scientific notation, graphing lines and finding slopes
- Geometry: the Pythagorean theorem, similar triangles and angle measure
- A scientific calculator, set to degree mode

MASTERY CHECKLIST

- ☐ I can design a fair test with independent, dependent and controlled variables and a testable hypothesis.
- ☐ I can convert units with conversion factors, including squared and cubed units, and check equations with dimensional analysis.
- ☐ I can report a measurement as a mean with an uncertainty and correct significant figures, and tell random from systematic error.
- ☐ I can graph data, draw a best-fit line, and interpret its slope and intercept with units, as in Hooke's law.
- ☐ I can resolve vectors into components with sine and cosine and add vectors to find a resultant.

OPTIONAL KITS FOR THIS UNIT

- **Arbor Scientific spring scales and force tables**: A classroom science supplier that sells to families. Inexpensive spring scales in several ranges make Lessons 1.4 and 1.5 easier and more precise; a tabletop force table with pulleys is a classic upgrade for Lesson 1.5.
- **phyphox (free app)**: Not a kit but a free app from RWTH Aachen University that turns a phone into a lab instrument. Its acoustic stopwatch and accelerometer tools are used in many labs this year, starting with the drop test in Lesson 1.1.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

1.1 Galileo, Kepler and Data-Driven Science — 3 sessions of 50 minutes

1.2 SI Units: The Meter, the Kilogram and the Lost Mars Orbiter

3 sessions of 50 minutes

1.3 Uncertainty and Significant Figures: Michelson Measures Light

3 sessions of 50 minutes

1.4 Graphs and Best-Fit Lines: Hooke's Law 3 sessions of 50 minutes

1.5 Vectors and Right-Triangle Trigonometry: Stevin's Wreath of Spheres

3 sessions of 50 minutes

Unit 1 test

Calculators are allowed. Show your work for every calculation: write the formula, substitute with units, and give the answer with its unit and sensible significant figures. Answer the concept questions in complete sentences. The test is worth 100 points.

1. **5 pts**
   Define independent, dependent and controlled variables, and identify each in this experiment: a student changes the length of a pendulum and times 10 swings, using the same bob and the same release angle.

2. **6 pts**
   Convert $72\ \mathrm{km/h}$ to meters per second.

3. **6 pts**
   Convert $2.5\ \mathrm{m}^2$ to square centimeters.

4. **6 pts**
   A stone has a mass of $81\ \mathrm{g}$ and displaces $30\ \mathrm{mL}$ of water. Find its density in $\mathrm{g/cm^3}$ and $\mathrm{kg/m^3}$.

5. **6 pts**
   Use dimensional analysis to decide whether $v^2 = 2ad$ can be correct (v in m/s, a in $\mathrm{m/s^2}$, d in m).

6. **6 pts**

   Explain in two or three sentences what caused the loss of the Mars Climate Orbiter and what rule would have prevented it.

7. **6 pts**

   State the number of significant figures in (a) 0.0207 (b) 4.500 (c) 3.0×10^8, and compute 6.21×0.30 with correct significant figures.

8. **7 pts**

   Four timings of a cart are 2.31, 2.28, 2.35 and 2.30 s. Find the mean and the standard deviation, and report the result.

9. **6 pts**

   A thermometer reads 2 °C too high every time. A second thermometer gives readings that scatter by ±1 °C around the true value. Which has a random error and which a systematic error? Which can be improved by averaging?

10. **7 pts**

    A ruler is caught after falling 16.0 cm. Find the reaction time.

11. **8 pts**

    A spring stretches 3.0, 6.1 and 8.9 cm under forces of 1.0, 2.0 and 3.0 N. Find the spring constant from the best-fit line through the origin, and predict the stretch for 2.5 N.

12. **5 pts**

    On a graph of position (m) against time (s), what are the units of the slope, and what does it represent?

13. **7 pts**

    Find the x- and y-components of a 40 N force at 35° above the horizontal.

14. **8 pts**

    A ship sails 12 km east, then 9.0 km north. Find its displacement (size and direction).

15. **7 pts**

A 50 N sign hangs from two cables, each 30° above the horizontal. Find the tension in each cable.

16. **4 pts**

Michelson reported $299{,}910 \pm 50$ km/s for the speed of light; the modern value is $299{,}792$ km/s. Find his percent error.

ANSWER KEY

1. Independent: changed on purpose (length). Dependent: measured (time for 10 swings). Controlled: kept the same (bob, release angle, timing method).
2. $72 \times 1000/3600 = 20$ m/s
3. 2.5×10^4 cm^2 $= 25{,}000$ cm^2
4. $81/30 = 2.7$ g/cm^3 $= 2700$ kg/m^3
5. Left: m^2/s^2. Right: (m/s^2)(m) $=$ m^2/s^2. The units match, so it can be correct.
6. One team's software reported thruster impulse in pound-force seconds and the navigation software assumed newton-seconds, so impulses were underestimated by a factor of 4.45 and the spacecraft flew too low at Mars. Requiring units on every number and checking them at every interface between teams would have caught it.
7. (a) 3 (b) 4 (c) 2; $6.21 \times 0.30 = 1.863 \to 1.9$
8. Mean 2.31 s (2.310); $s = 0.029$ s; report 2.31 ± 0.03 s.
9. The first has a systematic error; the second a random error. Only the second is improved by averaging many readings.
10. $t = \sqrt{2(0.160)/9.8} = 0.181$ s
11. The line through the origin and about (0.090 m, 3.0 N) has slope $k \approx 33$ N/m. Stretch for 2.5 N: $x = 2.5/33 = 0.075$ m $= 7.5$ cm.
12. m/s; the slope is the velocity.
13. $F_x = 40\cos 35^\circ = 32.8$ N; $F_y = 40\sin 35^\circ = 22.9$ N
14. $\sqrt{144 + 81} = 15$ km at $\tan^{-1}(9/12) = 36.9^\circ$ north of east
15. $2T\sin 30^\circ = 50$ N, so $T = 50$ N.
16. $118/299{,}792 \times 100\% = 0.039\%$

PORTFOLIO PIECE: A CALIBRATED RUBBER-BAND FORCE METER

Build a force meter from a rubber band (or a spring) mounted on a stick with a paper scale. Calibrate it with known masses from the kitchen scale, graph your calibration with uncertainties, and mark the scale in newtons. Then use your meter to measure three unknown forces (a drawer opening, a backpack lifted, a toy car pulled up a ramp) and repeat each measurement five times. Write a one-page report with your calibration graph, its best-fit line or curve, a table of measurements reported as mean ± uncertainty, and a discussion of how accurate your meter is and where it stops being reliable.

LOOK FOR	WHAT IT MEANS
Calibration and graph	At least 8 calibration points, axes labeled with units, best-fit line or smooth curve, and the range over which the meter is linear stated.
Measurements and uncertainty	Each unknown force measured five times and reported as mean ± uncertainty with correct significant figures.
Units and conversions	All masses converted to forces in newtons with $F = mg$; units shown on every number.
Error analysis	Identifies random and systematic errors specifically (such as zero drift, hysteresis, reading parallax) and suggests one improvement.
Honest record	Notebook pages are dated and show the actual readings, including any that were repeated.

1.1 Galileo, Kepler and Data-Driven Science

OBJECTIVES

- Describe how Kepler used Tycho Brahe's measurements to overturn a model he liked, and why that matters for science.
- Identify the independent, dependent and controlled variables in an experiment and write a testable hypothesis.
- Calculate a mean, a range and a percent error, and use them to judge whether data support a prediction.

> **Warm-up.** Hold a book and a flat sheet of paper at the same height and drop them together. Then crumple the paper into a tight ball and drop it with the book again. Before each drop, write a prediction in your notebook: which lands first, and why? After each drop, write what actually happened. Keep this page; you will come back to it in the lab.

WATCH

How the Bizarre Path of Mars Reshaped Astronomy [Kepler's Laws Part 1]

Welch Labs · 16:40

https://www.youtube.com/watch?v=Phscjl0u6TI

> **WHEN TO WATCH**
> After the story paragraph. It shows the Mars data Kepler was trying to fit, and why a circle would not do.

Note: About two minutes in, the video pauses for a sponsor message (a genealogy website); skip ahead to where the history resumes.

KEY TERMS

hypothesis
A testable prediction with a reason, such as "If the ramp is steeper, the marble will reach the bottom sooner, because more of its weight pulls it along the ramp."

independent variable
The one quantity the experimenter deliberately changes.

dependent variable

The quantity that is measured to see how it responds.

controlled variables

Everything else that could affect the result and is kept the same on purpose.

percent error

How far a measured value is from an accepted value, as a percentage of the accepted value.

law and theory

A scientific law describes a pattern (Kepler's laws say how planets move); a theory explains why (Newton's gravitation explains Kepler's laws).

CORE IDEAS

The story. For twenty years, from 1576 to 1597, the Danish nobleman Tycho Brahe ran an observatory on the island of Hven. There were no telescopes yet. Instead, Tycho built enormous sighting instruments, including a brass quadrant with a radius of about two meters fixed to a wall, and he and his assistants measured the positions of the stars and planets night after night, to an accuracy of about one or two minutes of arc (a minute is 1/60 of a degree). In 1600 a young German mathematician named Johannes Kepler came to work for him in Prague. Tycho died the next year, and Kepler inherited the best set of planetary measurements in the world.

How they figured it out. Kepler set out to find the orbit of Mars. For more than two thousand years astronomers had assumed that planets move in circles, and Kepler built the best circle-based model he could. He later wrote that he had carried out at least seventy trial calculations. His model matched most of Tycho's observations, but at some points it missed by 8 minutes of arc, about the width of a dime seen from 8 meters away. Many astronomers would have blamed the data. Kepler knew Tycho's measurements were better than that, so he threw his model away. After years more work he found that Mars moves on an *ellipse*, and he published the result in 1609 in *Astronomia nova* (*The New Astronomy*). In the same years, Galileo Galilei in Italy was rolling balls down ramps and timing them, refusing to accept an idea about motion until it had been measured.

The physics. What Kepler and Galileo did is what we now call the scientific method. Ask a clear question. Make a **hypothesis**: a prediction that could turn out wrong. Design a test that changes one **independent variable**, measures a **dependent variable**, and holds the **controlled variables** fixed. Repeat each measurement,

because every measurement scatters a little, and summarize with a mean. Then compare with the prediction. A useful number is the **percent error**: $\text{percent error} = \dfrac{|\text{measured} - \text{accepted}|}{\text{accepted}} \times 100\%$. If the difference is larger than the scatter in your data, the prediction (or your experiment) has a problem. Kepler's 8 minutes were larger than Tycho's scatter of 1 or 2 minutes, and that is how he knew.

The engineering. Engineers live by the same method. A new car design is crash-tested; a new wing is tested in a wind tunnel; a new bridge cable is pulled until it breaks. In each case the result is compared with the prediction, and when they disagree by more than the measurement uncertainty, the model is changed, not the data. Honest records matter: an engineer who fudges a test result can cost lives. Kepler, a devout Lutheran, saw astronomy as reading the book of nature that God had written, and he would not make the book say what it did not say.

Try it. In the lab you will test Galileo's claim that heavy and light objects fall together. You will need a careful plan, repeated trials and an honest comparison, exactly as Kepler did with Mars.

PICTURE STUDY

Tycho Brahe's Mural Quadrant, From Tycho Brahe, *Astronomiae instauratae mechanica*, 1598 · Hand-colored engraving · Published in Wandsbek (near Hamburg)

Image: Wikimedia Commons, public domain

Tycho published pictures of his instruments so other astronomers would trust his numbers. The great quadrant was fixed to a north–south wall; a star's height was read as it crossed the meridian, while an assistant noted the time.

Look closely

- Find the curved brass scale of the quadrant. Why would a larger radius allow more precise readings?
- Find the people reading the scale, watching the clocks and writing down the numbers. How many people does one measurement take?
- Find the small window in the wall at the top left. What is it for?
- Look at the painted background inside the arc. What does it tell you about the observatory?

Talk about it

1. Why did Tycho record the time of each observation as well as the angle?
2. Kepler trusted these measurements to within 2 minutes of arc. What features of the instrument make that believable?

3. What could make a measurement with this instrument wrong in the same direction every time?

Safety first. Drop objects only from a stable height (a stair landing or a sturdy step stool, never a chair on a table). Keep everyone's feet clear of the landing spot.

LAB

Do Heavier Things Fall Faster?

Aristotle taught that heavier objects fall faster. Galileo argued that, apart from air resistance, all objects fall together. You will design a fair test, repeat it, and decide from your data, not from authority.

You will need

- Pairs of objects of the same size and shape but different mass: a golf ball and a ping-pong ball, a full and an empty water bottle with caps, or a steel and a glass marble
- A flat sheet of paper and a coffee filter
- Tape measure
- A phone with slow-motion video (or the phyphox "Acoustic Stopwatch," which starts and stops on a clap or a click)
- A partner

Steps

1. **Question and hypothesis.** Write the question "Does mass change how long an object takes to fall 2.00 m?" and your hypothesis with a reason.
2. List your variables: independent (mass), dependent (fall time), controlled (height, shape and size, release method, timing method).
3. Measure and mark a drop height of 2.00 m above a hard floor. Weigh each object on the kitchen scale and record its mass.
4. Drop the two objects together from the same height, holding them at the same level and releasing at once (a ruler under both, pulled away quickly, works well). Film from the side in slow motion at floor level.
5. Repeat 5 times. In each video, note whether one object lands first and estimate the time gap from the frame count (frames ÷ frames per second).
6. Now time single drops of the heavy object: 5 trials. Find the mean, the range and the percent error against $t = \sqrt{2h/g}$.
7. **Second test.** Drop a flat sheet of paper and a crumpled sheet of the same mass. Then drop a coffee filter alone and two nested filters together. Record

what happens.

8. **Analyze.** Is any time gap between heavy and light objects larger than the scatter between trials? What does the paper test show about which variable really matters?
9. **Improve.** Name the largest source of error in your timing and change one thing to reduce it (a higher drop, a better release, a faster frame rate). Repeat one test and compare.

Tips

- Two objects released together are easier to judge than two separate timings: you only need to see which lands first.
- Most phones film slow motion at 120 or 240 frames per second, so one frame is $1/240$ s ≈ 0.004 s.

Check your work

- ☐ I wrote a testable hypothesis and named my independent, dependent and controlled variables.
- ☐ I computed a mean, a range and a percent error, and compared the difference with the scatter.
- ☐ I can explain, from my own data, why a sheet of paper falls slowly but a crumpled one does not.

LAB NOTEBOOK

1. Lab report: question, hypothesis, variables, a labeled sketch of the setup, a data table with units, and your conclusion in two or three sentences.
2. Return to your warm-up page. Next to each prediction, write what happened and what you now think explains it.
3. Write a paragraph comparing Kepler's 8 minutes of arc with your own data: when should a scientist change the model, and when should she suspect the measurement?

KEY FORMULAS

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

$$\text{percent error} = \frac{|\text{measured} - \text{accepted}|}{\text{accepted}} \times 100\%$$

$$\text{percent difference} = \frac{|A - B|}{(A + B)/2} \times 100\%$$

From history. Galileo and Kepler wrote to each other but never met. Kepler went on to discover two more laws of planetary motion (Unit 4), and he printed the *Rudolphine Tables* (1627), planetary tables based on Tycho's data that were far more accurate than any before them. Tycho's instruments did not survive, but his published drawings let modern historians judge how good they were.

WORKED EXAMPLES

Example 1. Kepler's circle model missed Tycho's Mars observations by 8 minutes of arc. Express this in degrees, and as a fraction of a full circle.

Show solution

1. There are 60 minutes in a degree: $8' \times \frac{1^\circ}{60'} = 0.133^\circ$.
2. A full circle is $360^\circ = 360 \times 60' = 21{,}600'$.
3. Fraction: $\frac{8'}{21{,}600'} = 3.70 \times 10^{-4}$, about 1 part in 2,700.

Answer: 0.133°, or about 3.7×10^{-4} of a circle.

Example 2. A student drops a golf ball from $h = 2.00$ m five times and records fall times of 0.62, 0.66, 0.63, 0.61 and 0.65 s. The time predicted for a fall with no air resistance is $t = \sqrt{2h/g}$ with $g = 9.8\ \mathrm{m/s^2}$. Find the mean, the range and the percent error.

Show solution

1. Mean: $\bar{t} = \dfrac{0.62 + 0.66 + 0.63 + 0.61 + 0.65}{5}\ \mathrm{s} = \dfrac{3.17}{5}\ \mathrm{s} = 0.634\ \mathrm{s}$.
2. Range: $0.66\ \mathrm{s} - 0.61\ \mathrm{s} = 0.05\ \mathrm{s}$, so the times scatter by about ± 0.025 s around the mean.
3. Prediction: $t = \sqrt{\dfrac{2(2.00\ \mathrm{m})}{9.8\ \mathrm{m/s^2}}} = \sqrt{0.408\ \mathrm{s^2}} = 0.639\ \mathrm{s}$.
4. Percent error: $\dfrac{|0.634 - 0.639|}{0.639} \times 100\% = 0.78\%$.

Answer: Mean 0.634 s, range 0.05 s, percent error about 0.8%. The difference (0.005 s) is much smaller than the scatter, so the data agree with the prediction.

Example 3. A student wants to know whether the steepness of a ramp changes how long a marble takes to roll 1.0 m. Name the independent, dependent and controlled variables, and write a hypothesis.

Show solution

1. Independent variable (what you change on purpose): the height of the raised end of the ramp, which sets its steepness.
2. Dependent variable (what you measure): the time to roll 1.0 m.
3. Controlled variables: the same marble, the same ramp surface, the same 1.0 m distance, release from rest, the same timing method.
4. Hypothesis: “If the ramp is raised higher, the roll time will decrease, because a larger part of the marble’s weight acts along the ramp.”

Answer: Independent: ramp height. Dependent: roll time. Controlled: marble, surface, distance, release, timing method. The hypothesis must make a prediction that could be wrong.

PRACTICE

1. Easy Convert 2 minutes of arc (Tycho's typical accuracy) to degrees.

   Show answer

   $2/60 = 0.033^\circ$

2. Easy In an experiment on how the length of a pendulum affects the time of one swing, name the independent and dependent variables.

   Show answer

   Independent: the length. Dependent: the time of one swing (the period).

3. Easy Which of these is a testable hypothesis? (a) Marbles are beautiful. (b) A steel marble will roll farther on a carpet than a glass marble of the same size. (c) Gravity is mysterious.

   Show answer

   (b). It makes a prediction that a measurement could prove wrong.

4. Medium Five measurements of a table's length are 152.3, 152.6, 152.1, 152.4 and 152.6 cm. Find the mean and the range.

   Show answer

   Mean $= 762.0/5 = 152.4$ cm; range $= 152.6 - 152.1 = 0.5$ cm.

5. Medium A student measures $g = 9.62\ \mathrm{m/s^2}$. The accepted value is $9.80\ \mathrm{m/s^2}$. Find the percent error.

   Show answer

   $|9.62 - 9.80|/9.80 \times 100\% = 1.8\%$

6. Medium Two lab groups measure the period of the same pendulum as 1.42 s and 1.48 s. Find the percent difference.

   Show answer

   $$\frac{0.06}{1.45} \times 100\% = 4.1\%$$

7. Medium Explain the difference between a scientific law and a scientific theory, using Kepler and Newton as examples.

Show answer

A law describes a pattern: Kepler's laws say planets move on ellipses and how their periods depend on distance. A theory explains the pattern: Newton's gravitation explains why the orbits are ellipses.

8. Medium A student's timing of a 2.00 m drop gives a mean of 0.70 s, while theory predicts 0.64 s, and all five trials were between 0.69 and 0.71 s. Is this random error or systematic error? Suggest a cause.

Show answer

The trials agree closely with each other but all are too long, so it is a systematic error: for example, the student starts the stopwatch on time but reacts late at the landing every time.

9. Hard Kepler's 8-arcminute disagreement was 4 to 8 times Tycho's measurement error. Why did that convince Kepler, and what should he have concluded if the disagreement had been 1 arcminute?

Show answer

A disagreement much larger than the measurement error means the model, not the data, is wrong. A 1-arcminute difference would be within Tycho's error, so the circle model could not have been rejected on that evidence.

10. Hard A ball falls 2.00 m in a mean time of 0.645 s. Using $h = \frac{1}{2}gt^2$, find the value of g this implies and its percent error from $9.80\ \mathrm{m/s^2}$.

Show answer

$g = 2h/t^2 = 2(2.00\ \mathrm{m})/(0.645\ \mathrm{s})^2 = 9.61\ \mathrm{m/s^2}$; percent error $= 1.9\%$.

11. **Challenge** Design an experiment to test whether the mass of a marble affects how long it takes to roll down a ramp. Give your hypothesis, variables, number of trials and how you will decide.

Show answer

Sample: use steel and glass marbles of the same diameter, a 1.0 m ramp at fixed height, release from rest with a gate, 5 timed trials each by slow-motion video. Decide that mass matters only if the difference in mean times is larger than the range of each set of trials.

COMMON MISTAKES

- Changing two things at once (a heavier and a larger ball), so that you cannot tell which change mattered.
- Throwing out a trial because it "looks wrong." Keep every measurement, and only discard one if you can name what went wrong.
- Calling a difference "error" when it is really a mistake. Error in science means the unavoidable scatter of measurement, which you estimate and report.

Make it a game · Variable Spotter. Your parent describes three experiments in one sentence each, such as watering plants with different amounts of light. In ten seconds each, name the independent variable, the dependent variable and two controlled variables, then phrase a testable hypothesis. Your parent plays Kepler and challenges one claim per round. Score a point for each clean setup.

Scripture. "It is the glory of God to conceal a thing: but the honour of kings is to search out a matter." (Proverbs 25:2, KJV)

Kepler searched out the orbit of Mars for years. Patient, honest searching is the honor of the scientist.

1.2 SI Units: The Meter, the Kilogram and the Lost Mars Orbiter

OBJECTIVES

- Name the SI base units for length, mass and time, and use the common metric prefixes.
- Convert units with conversion factors (the factor-label method), including squared and cubed units.
- Use dimensional analysis to check whether an equation can be correct.
- Explain how a unit mistake caused the loss of the Mars Climate Orbiter.

> **Warm-up.** Without measuring, estimate in your notebook: the length of your stride, the height of a door, the mass of an apple, and the mass of a full gallon of milk, each in SI units (meters and kilograms). Then measure each one. How close were you? Which estimate was furthest off?

WATCH

Dimensional Analysis

The Organic Chemistry Tutor · 15:47

https://www.youtube.com/watch?v=vWovdihteqs

> **WHEN TO WATCH**
> Before the worked examples. Pause after each example in the video and work it yourself before the presenter does.

BACKUP VIDEO

The Mars Metric Mistake

Smithsonian National Air and Space Museum · 2:29

https://www.youtube.com/watch?v=4DXFurrTM_g

With the story paragraph, before the physics.

KEY TERMS

SI

The International System of Units (Système international), used by scientists in every country: the meter, kilogram, second, ampere, kelvin, mole and candela, and units built from them.

base unit

One of the seven defined SI units. Length is measured in meters (m), mass in kilograms (kg), time in seconds (s).

derived unit

A unit built from base units, such as the newton, $1\ \mathrm{N} = 1\ \mathrm{kg} \cdot \mathrm{m/s^2}$, or the joule, $1\ \mathrm{J} = 1\ \mathrm{N} \cdot \mathrm{m}$.

conversion factor

A fraction equal to 1, such as $\frac{1000\ \mathrm{m}}{1\ \mathrm{km}}$, used to change units without changing the quantity.

dimensional analysis

Checking that both sides of an equation have the same units (dimensions).

CORE IDEAS

The story. In the 1700s every French town had its own units of length and weight, and there were hundreds of them. In 1791 the French Academy of Sciences proposed a unit based on the Earth itself: the **meter**, one ten-millionth of the distance from the North Pole to the equator. From 1792 to 1798 two astronomers, Jean-Baptiste Delambre and Pierre Méchain, measured the length of the meridian arc from Dunkirk to Barcelona by triangulation, sighting from church towers and hilltops. A platinum bar of the new length was made in 1799. In 1889 new standard bars and a cylinder of platinum-iridium called the International Prototype of the Kilogram were sent out to the nations that had signed the Metre Convention. Today all seven SI units are defined by fixed constants of nature: since 1983 the meter is the distance light travels in 1/299,792,458 of a second, and since 2019 the kilogram is defined through the Planck constant.

How they figured it out. Units matter only if everyone uses the same ones. On September 23, 1999, NASA's Mars Climate Orbiter fired its engine to go into orbit around Mars and was never heard from again. The investigation board found the

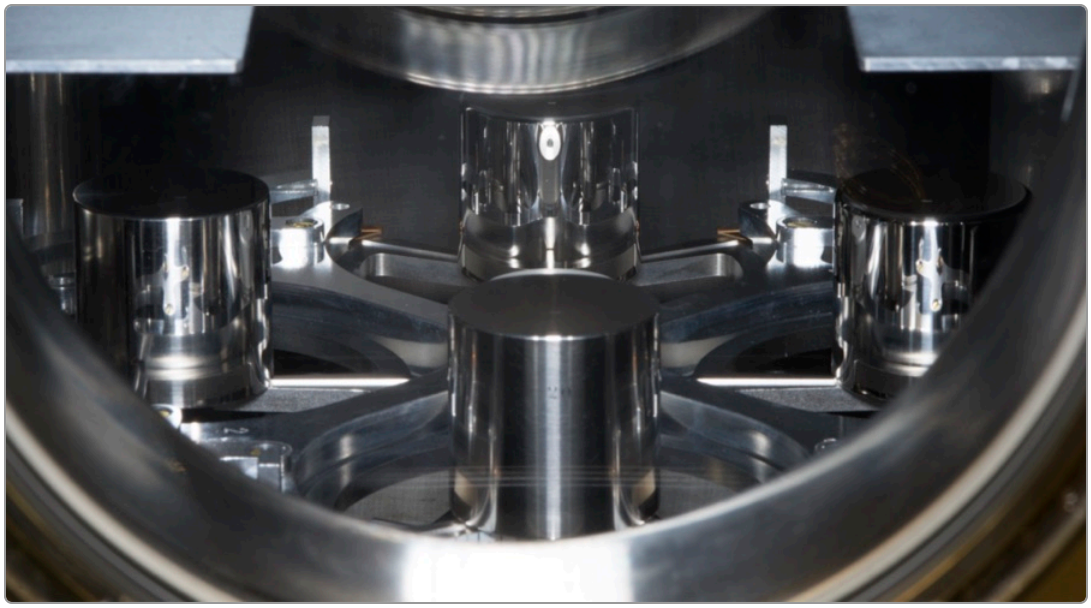
cause: software written by the spacecraft's builder reported the impulse of small thruster firings in pound-force seconds, while the navigation team's software expected newton-seconds. Every number was too small by a factor of 4.45. Over nine months of flight the small errors added up, and the spacecraft passed about 57 km above Mars instead of the planned 140 km or more, low enough to be destroyed in the atmosphere. The board's lesson was not only "use metric" but "check the units at every interface between teams."

The physics. Every physical quantity is a number *and* a unit, and the unit is part of the answer. To convert, multiply by conversion factors that equal one, arranged so the unwanted units cancel: $65\ \mathrm{mi/h} \times \dfrac{1609\ \mathrm{m}}{1\ \mathrm{mi}} \times \dfrac{1\ \mathrm{h}}{3600\ \mathrm{s}} = 29.1\ \mathrm{m/s}$. Squared and cubed units need squared and cubed factors: $1\ \mathrm{m}^2 = (100\ \mathrm{cm})^2 = 10^4\ \mathrm{cm}^2$. **Dimensional analysis** uses units to check equations: in $d = \frac{1}{2}at^2$, the right side has units $(\mathrm{m/s}^2)(\mathrm{s}^2) = \mathrm{m}$, matching the left side. An equation whose units do not match must be wrong.

The engineering. Engineers write units on every number and every column heading, and they check units where two teams' work meets. In 1983 an Air Canada Boeing 767 ran out of fuel at 41,000 feet because the fuel load had been worked out in pounds when the new airplane's system used kilograms. The pilots glided the airplane to a safe landing at a former air base in Gimli, Manitoba, and the "Gimli Glider" became a standard lesson in aviation training. Calibration laboratories such as the U.S. National Institute of Standards and Technology keep the measuring tools of industry tied to the SI definitions.

Try it. In the lab you will measure masses and volumes of everyday objects, convert between grams per cubic centimeter and kilograms per cubic meter, and identify materials by their density.

PICTURE STUDY



The United States' Kilogram Standards, National Institute of Standards and Technology, 2018 · Photograph · NIST, Gaithersburg, Maryland

Image: Wikimedia Commons, public domain

For 130 years the kilogram was a physical object. The United States received its national copies, K20 and K4, in 1890, and every scale in the country was traced back to them. Here four of the U.S. standards sit on the turntable of a mass comparator, which weighs them against one another inside a vacuum chamber. Since 2019 the kilogram has been defined by a constant of nature instead.

Look closely

- Find the four cylinders. Notice that they are about as tall as they are wide. Why might a designer choose that shape?
- Find the turntable that carries them, one at a time, onto the weighing position.
- Look at the round window and the thick steel walls of the chamber.

Talk about it

1. Why weigh standards in a vacuum? (Hint: air pushes up on everything, by Archimedes' principle, and the push depends on an object's volume.)
2. The official prototypes slowly drifted by about 50 micrograms compared with their copies. Why is that a problem for a world standard?

3. Why is it better to define a unit by a constant of nature than by an object kept in a vault?

LAB

Measuring in SI: The Density of Everyday Things

Every material has a characteristic density. You will measure the mass and volume of several objects, convert units carefully, and identify unknown materials by comparing with accepted densities.

You will need

- Kitchen scale (grams)
- Ruler with millimeters
- A clear measuring cup or a tall glass marked with a ruler (for displacement)
- Objects: a wooden block, a stack of coins of one kind, a steel bolt, an aluminum can tab or piece of aluminum, a rock, a candle or block of wax
- Water

Steps

1. Make a data table with columns: object, mass (g), volume method, volume (cm^3), density (g/cm^3), density (kg/m^3), accepted value, percent error.
2. For regular solids (the wooden block, the wax), measure length, width and height in centimeters to the nearest millimeter and calculate the volume.
3. For irregular objects (the bolt, the rock), use displacement: read the water level, lower the object in completely, read again. The difference in milliliters is the volume in cm^3 ($1\ \mathrm{mL} = 1\ \mathrm{cm}^3$).
4. Weigh each object. Calculate density in g/cm^3, then convert to kg/m^3 with the factor $1\ \mathrm{g/cm}^3 = 1000\ \mathrm{kg/m}^3$.
5. Compare with accepted values: water 1.00, pine about 0.4–0.6, paraffin wax 0.9, aluminum 2.70, granite about 2.7, steel 7.85, copper 8.96 g/cm^3. Compute percent errors.
6. **Improve.** For your largest percent error, find the measurement with the largest relative uncertainty (often a small volume). Measure it a better way (for example, displace ten coins at once instead of one) and recalculate.

7. Measure your stride: walk 10 steps, measure the distance in meters, and find one stride. Convert it to feet and inches with $1 \text{ in} = 2.54 \text{ cm}$.

Tips

- Displacement works best in a narrow container, where a small volume makes a big change in height.
- Shake off air bubbles that cling to an object underwater; they add false volume.

Check your work

- ☐ I converted between g/cm^3 and kg/m^3 correctly and explained why the factor is 1000.
- ☐ I identified at least one unknown material by its density.
- ☐ I can name the measurement that limited my accuracy and how I improved it.

LAB NOTEBOOK

1. Data table for all objects with units in every column heading, and the conversion for one object written out with conversion factors.
2. Draw a "factor-label chain" for converting 1 mile per hour to meters per second, and for converting 1 cubic meter to liters.
3. Write a short incident report, as if for NASA, explaining in four sentences what went wrong with the Mars Climate Orbiter and what checking rule would have caught it.

KEY FORMULAS

$$1 \text{ km} = 10^3 \text{ m}, \quad 1 \text{ cm} = 10^{-2} \text{ m}, \quad 1 \text{ mm} = 10^{-3} \text{ m}, \quad 1\ \mu\text{m} = 10^{-6} \text{ m}$$

$$1 \text{ mi} = 1609 \text{ m}, \quad 1 \text{ in} = 2.54 \text{ cm (exact)}, \quad 1 \text{ lb (mass)} = 0.4536 \text{ kg}, \quad 1 \text{ lbf} = 4.448 \text{ N}$$

$$\rho = \frac{m}{V}, \qquad 1 \text{ g/cm}^3 = 1000 \text{ kg/m}^3$$

From history. Méchain found a small inconsistency in his own latitude measurements at Barcelona and worried over it for the rest of his life without telling anyone; the problem was later found to come from the limits of the instruments of the time. The meter they produced turned out to be about 0.2 mm short of a true ten-millionth of the quadrant, but by then it was the standard, and the standard is what matters.

Example 1. Convert a highway speed of 65 mi/h to meters per second.

Show solution

1. Write the quantity and multiply by factors that cancel miles and hours: $65\ \frac{\text{mi}}{\text{h}} \times \frac{1609\ \text{m}}{1\ \text{mi}} \times \frac{1\ \text{h}}{3600\ \text{s}}$.
2. Miles cancel with miles and hours with hours, leaving m/s.
3. Compute: $\frac{65 \times 1609}{3600}\ \text{m/s} = 29.1\ \text{m/s}$.

Answer: 29.1 m/s

Example 2. A thruster firing delivered an impulse that the builder's software reported as 100, meaning 100 lbf · s. The navigation software read it as 100 N · s. What was the true impulse in newton-seconds, and by what factor was it underestimated?

Show solution

1. Convert: $100\ \text{lbf} \cdot \text{s} \times \frac{4.448\ \text{N}}{1\ \text{lbf}} = 445\ \text{N} \cdot \text{s}$.
2. Ratio of true to assumed: $\frac{445\ \text{N} \cdot \text{s}}{100\ \text{N} \cdot \text{s}} = 4.45$.

Answer: The true impulse was 445 N · s; the navigation team's value was too small by a factor of 4.45.

Example 3. Iron has a density of $7.87\ \text{g/cm}^3$. Express this in kg/m^3, and find the mass of an iron block $10.0\ \text{cm} \times 5.0\ \text{cm} \times 2.0\ \text{cm}$.

Show solution

1. Convert: $7.87\ \frac{\text{g}}{\text{cm}^3} \times \frac{1\ \text{kg}}{1000\ \text{g}} \times \left(\frac{100\ \text{cm}}{1\ \text{m}}\right)^3 = 7.87 \times \frac{10^6}{10^3}\ \text{kg/m}^3 = 7870\ \text{kg/m}^3$.
2. Volume: $V = 10.0 \times 5.0 \times 2.0\ \text{cm}^3 = 100\ \text{cm}^3$.
3. Mass: $m = \rho V = 7.87\ \text{g/cm}^3 \times 100\ \text{cm}^3 = 787\ \text{g} = 0.787\ \text{kg}$.

Answer: $7870\ \text{kg/m}^3$; the block's mass is 787 g (0.787 kg).

Example 4. A student writes the speed of a falling object as $v = gt^2$. Use dimensional analysis to show this cannot be right.

Show solution

1. Units of the left side: v is in $\mathrm{m/s}$.
2. Units of the right side: $(\mathrm{m/s^2})(\mathrm{s^2}) = \mathrm{m}$.
3. $\mathrm{m/s} \neq \mathrm{m}$, so the equation is wrong. (The correct relation, $v = gt$, has units $(\mathrm{m/s^2})(\mathrm{s}) = \mathrm{m/s}$.)

Answer: The units do not match ($\mathrm{m/s}$ versus m), so $v = gt^2$ is impossible.

PRACTICE

1. Easy Convert $3.5\ \mathrm{km}$ to meters and to centimeters.

   Show answer

   $3500\ \mathrm{m}$; $3.5 \times 10^5\ \mathrm{cm}$

2. Easy Convert $450\ \mathrm{g}$ to kilograms.

   Show answer

   $0.450\ \mathrm{kg}$

3. Easy How many seconds are in one day?

   Show answer

   $24 \times 60 \times 60 = 86{,}400\ \mathrm{s}$

4. Medium A sprinter runs at $10.0\ \mathrm{m/s}$. What is this in kilometers per hour and in miles per hour?

   Show answer

   $36.0\ \mathrm{km/h}$; $10.0 \times 3600/1609 = 22.4\ \mathrm{mi/h}$

5. Medium Convert $1.0\ \mathrm{m^3}$ to cubic centimeters and to liters ($1\ \mathrm{L} = 1000\ \mathrm{cm^3}$).

   Show answer

   $10^6\ \mathrm{cm^3}$; $1000\ \mathrm{L}$

6. Medium The Gimli Glider's crew converted liters of fuel to mass using 1.77, which is pounds per liter, as if it were kilograms per liter; the correct factor was 0.80 kg/L. By what factor did they overestimate the mass of fuel on board, and roughly what fraction of the fuel they needed did they actually have?

Show answer

They overestimated by $1.77/0.80 = 2.2$, so they had only about $0.80/1.77 = 0.45$, less than half, of the fuel they needed.

7. Medium Aluminum has density 2.70 g/cm^3. What is the volume of a 54 g piece?

Show answer

$V = m/\rho = 54/2.70 = 20\ \text{cm}^3$

8. Medium Show with units that $\frac{1}{2}mv^2$ has the same units as force times distance.

Show answer

$\text{kg}(\text{m/s})^2 = \text{kg}\cdot\text{m}^2/\text{s}^2$, and $\text{N}\cdot\text{m} = (\text{kg}\cdot\text{m/s}^2)(\text{m}) = \text{kg}\cdot\text{m}^2/\text{s}^2$. They match (both are joules).

9. Hard A sheet of paper is 0.10 mm thick. A ream of 500 sheets stands how tall in centimeters? How many sheets would it take to make a stack 1.0 km high?

Show answer

$500 \times 0.10\ \text{mm} = 50\ \text{mm} = 5.0\ \text{cm}$. For 1.0 km: $\dfrac{1.0 \times 10^3\ \text{m}}{1.0 \times 10^{-4}\ \text{m}} = 1.0 \times 10^7$ sheets.

10. Hard The meter was meant to be one ten-millionth of the distance from the pole to the equator. Using that definition, what is the Earth's circumference in kilometers?

Show answer

Pole to equator is a quarter circle: 10^7 m. Full circle $= 4 \times 10^7\ \text{m} = 40{,}000$ km (the measured value is about 40,008 km).

11. Hard A student claims the period of a pendulum is $T = 2\pi\sqrt{g/L}$. Use units to decide whether this can be right.

Show answer

$\sqrt{(\text{m/s}^2)/\text{m}} = \sqrt{1/\text{s}^2} = 1/\text{s}$, not seconds, so it is wrong. The correct formula is $T = 2\pi\sqrt{L/g}$, which has units of seconds.

12. **Challenge** Light travels $299{,}792{,}458$ m in one second. How far does it travel in one nanosecond (10^{-9} s), in centimeters? Why does this matter for GPS, which measures distances by timing radio signals?

Show answer

$0.2998\ \mathrm{m} \approx 30\ \mathrm{cm}$ per nanosecond. A clock error of only 10 ns gives a position error of about 3 m, so GPS satellites need extremely accurate atomic clocks.

COMMON MISTAKES

- Converting squared or cubed units with a single factor. $1\ \mathrm{m}^3$ is $(100)^3 = 10^6\ \mathrm{cm}^3$, not $100\ \mathrm{cm}^3$.
- Writing a conversion factor upside down. Check that the unit you want to remove appears once on top and once on the bottom.
- Dropping units in the middle of a calculation and adding them back at the end by guesswork.

Make it a game · Unit Conversion Gauntlet. Your parent calls out a quantity, such as 72 km/h or 2.5 cubic centimeters. Convert it to SI using factor-label on paper, showing each cancelled unit. Beat the clock: five conversions in three minutes earns a pass. Include one squared or cubed unit. Then try to set a new personal best for speed with no errors.

Scripture. “A false balance is abomination to the LORD: but a just weight is his delight.” (Proverbs 11:1, KJV)

Honest weights and measures were a matter of honesty and fairness in Solomon’s day and still are. A unit is a promise that everyone means the same thing.

1.3 Uncertainty and Significant Figures: Michelson Measures Light

OBJECTIVES

- Distinguish accuracy from precision and random error from systematic error.
- Report a measurement with its uncertainty and with an appropriate number of significant figures.
- Apply the significant-figure rules for multiplying, dividing, adding and subtracting.
- Find the mean and standard deviation of repeated measurements.

> **Warm-up.** Have three people each measure the width of the same door with the same tape measure, to the nearest millimeter, without seeing each other's results. Write down all three. Do they agree exactly? Which digit is the first one where they disagree? That digit tells you something important about the measurement.

WATCH

Significant Figures - A Fast Review!

The Organic Chemistry Tutor · 15:09

https://www.youtube.com/watch?v=l2yuDvwYq5g

> **WHEN TO WATCH**
> After the physics paragraph, before the worked examples.

BACKUP VIDEO

Why are Significant Figures Important?

Tyler DeWitt · 7:44

https://www.youtube.com/watch?v=VAuslY-Uuf4

Optional, for a second explanation of why significant figures are about honesty in measurement.

KEY TERMS

accuracy
How close a measurement is to the true value.

precision
How close repeated measurements are to each other.

random error

Unpredictable scatter that makes repeated measurements differ, sometimes high and sometimes low; reduced by repeating and averaging.

systematic error

An error that pushes every measurement the same way, such as a miscalibrated scale or a consistent reaction delay; not reduced by averaging.

uncertainty

The range within which the true value probably lies, written as value $\pm$ uncertainty, for example 18.0 ± 0.4 cm.

significant figures

The digits in a reported number that carry real information: all the certain digits plus the first uncertain one.

CORE IDEAS

The story. In 1879 a 26-year-old instructor at the U.S. Naval Academy in Annapolis, Albert A. Michelson, set out to measure the speed of light better than anyone had. Improving a method of the Frenchman Léon Foucault, he bounced a beam of sunlight off a rapidly spinning mirror to a fixed mirror about 600 meters away along the bank of the Severn River and back. In the short time the light took for the round trip, the spinning mirror turned slightly, so the returning beam came back displaced by a tiny amount, about 13 centimeters at his eyepiece. From the displacement, the distance and the mirror's rotation rate, he calculated the speed.

How they figured it out. Michelson made about a hundred separate measurements, and they scattered. He reported the mean, 299,910 km/s, with an uncertainty of ±50 km/s. Today the speed of light is exactly 299,792.458 km/s, because since 1983 it has been used to define the meter. Michelson's result was 118 km/s too high, more than twice his stated uncertainty, which means some *systematic* error he did not find, perhaps in the temperature correction or the measured distance, pushed every result the same way. He spent much of his life refining the measurement, and in 1907 he became the first American to receive a Nobel Prize in the sciences, for his precision optical instruments.

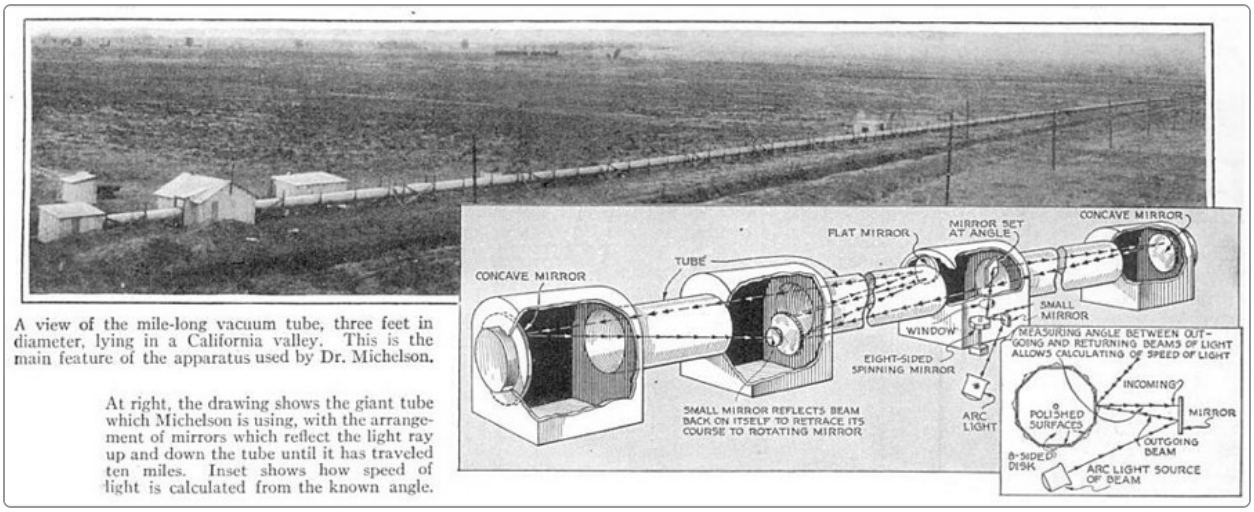
The physics. Every measurement has uncertainty. **Precision** is how tightly repeated measurements cluster; **accuracy** is how close they are to the truth. Random error makes the scatter and can be reduced by averaging many trials. Systematic error shifts everything and can only be found by checking the method. Report a result as

$\bar{x} \pm \delta x$, where the uncertainty may be estimated from the instrument (half the smallest division), from half the range of your trials, or from the standard deviation s. The digits you write should match the uncertainty: these are the **significant figures**. When you multiply or divide, keep as many significant figures as the least precise factor. When you add or subtract, keep as many decimal places as the least precise term.

The engineering. Engineers call uncertainty *tolerance*, and it is written on every drawing. A piston might be specified as 82.00 ± 0.01 mm and its cylinder as 82.05 ± 0.01 mm, so that every piston fits every cylinder. Interchangeable parts, from rifle locks in the early 1800s to the bolts in a modern airliner, depend on controlling and measuring uncertainty. A result written with too many digits is not more precise; it is dishonest, because it claims knowledge the measurement does not contain.

Try it. In the lab you will measure your own reaction time by catching a falling ruler, twenty times, and report it properly as a mean with an uncertainty.

PICTURE STUDY



Michelson's Mile-Long Vacuum Tube, Unknown photographer, 1930 · Newspaper photograph and diagram · Irvine Ranch, near Santa Ana, California

Image: Wikimedia Commons, public domain

Michelson spent fifty years improving his 1879 measurement. For his last attempt, begun in 1930, he and his colleagues built a pipe nearly a mile long, pumped most of the air out of it, and sent light back and forth inside it many times.

Look closely

- Follow the pipe into the distance. How straight does it have to be?
- Find the stands or supports. What would happen to the measurement if the ground settled under them?
- Look for any buildings or equipment at the end of the pipe.

Talk about it

1. Why would removing the air make the measurement more accurate? (Light travels slightly slower in air.)
2. Why does a longer light path reduce the percent uncertainty of the timing?
3. Name two systematic errors this design was meant to reduce.

LAB

Your Reaction Time, with Its Uncertainty

A falling ruler is a clock: the farther it falls before you catch it, the longer your reaction took. You will make twenty measurements, describe their scatter honestly, and look for systematic errors.

You will need

- A 30 cm ruler (or a meter stick)
- A partner
- Calculator or a free spreadsheet (Google Sheets or LibreOffice Calc)

Steps

1. **Hypothesis.** Predict your reaction time in seconds and write it down, with a reason.
2. Your partner holds the ruler vertically with the zero mark between your open thumb and finger, which do not touch it. Without warning, the partner lets go and you catch it as fast as you can. Read the distance at the top of your thumb, to the nearest 0.1 cm.
3. Repeat 20 times. Record every result, including the bad ones.
4. Find the mean, the range and the standard deviation s (the spreadsheet functions are AVERAGE and STDEV). Report $\bar{d} \pm s$ with sensible significant figures.
5. Convert the mean distance to a reaction time with $t = \sqrt{2d/g}$. Also find the reaction times for $\bar{d} - s$ and $\bar{d} + s$ to get the uncertainty in time.
6. Make a histogram: count how many catches fell in each 1 cm interval. Describe its shape.
7. **Look for systematic error.** Try 10 more catches when you can see your partner's hand, and 10 when a sound (your partner says "now") is the signal. Does the mean change by more than its uncertainty?
8. **Improve.** Did your results get better as you practiced? Compare the mean of your first 10 and last 10 trials. Decide whether "practice" is a variable you should have controlled.

Tips

- Keep your forearm resting on a table edge so only your fingers move.
- Your partner should vary the waiting time so you cannot anticipate the drop.

Check your work

- ☐ I reported my reaction time as a mean with an uncertainty and a sensible number of significant figures.
- ☐ I can explain the difference between the random scatter in my data and a systematic error.
- ☐ I can use the significant-figure rules for products and sums.

LAB NOTEBOOK

1. All 20 measurements in a table, the mean and standard deviation, and your histogram.
2. Draw four targets showing: accurate and precise, precise but not accurate, accurate on average but not precise, and neither. Label which kind of error each shows.
3. Write three sentences about Michelson: what his uncertainty claimed, what the true value shows, and what that teaches about systematic error.

KEY FORMULAS

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}}$$

$$\text{percent uncertainty} = \frac{\delta x}{x} \times 100\%$$

$$t = \sqrt{\frac{2d}{g}} \quad \text{(ruler-drop reaction time)}$$

From history. In 1887 Michelson and Edward Morley used a different optical instrument, the interferometer, to look for the motion of the Earth through a supposed “ether” that light was thought to travel through. They found none. That famous null result, measured with extraordinary precision, became one of the foundations of modern physics.

Example 1. Give each answer with the correct number of significant figures: (a) $12.11\ \text{cm} + 0.3\ \text{cm}$; (b) $4.56\ \text{m} \times 1.4\ \text{m}$; (c) $\dfrac{0.0360\ \text{m}}{1.2\ \text{s}}$.

Show solution

1. (a) Addition: keep the fewest decimal places. 0.3 has one decimal place: $12.11 + 0.3 = 12.41 \to 12.4\ \text{cm}$.
2. (b) Multiplication: keep the fewest significant figures. 1.4 has two: $4.56 \times 1.4 = 6.384 \to 6.4\ \text{m}^2$.
3. (c) 0.0360 has three significant figures (leading zeros do not count; the trailing zero does); 1.2 has two: $0.0360/1.2 = 0.0300 \to 0.030\ \text{m/s}$.

Answer: (a) 12.4 cm, (b) $6.4\ \text{m}^2$, (c) $0.030\ \text{m/s}$

Example 2. A student catches a dropped ruler five times and records catch distances of 18.2, 16.9, 19.5, 17.4 and 18.0 cm. Find the mean distance, the standard deviation, and the reaction time for the mean distance.

Show solution

1. Mean: $\bar{d} = \dfrac{18.2 + 16.9 + 19.5 + 17.4 + 18.0}{5}\ \text{cm} = \dfrac{90.0}{5}\ \text{cm} = 18.0\ \text{cm}$.
2. Deviations from the mean: $0.2, -1.1, 1.5, -0.6, 0.0$; squares: $0.04, 1.21, 2.25, 0.36, 0$; sum $= 3.86\ \text{cm}^2$.
3. Standard deviation: $s = \sqrt{\dfrac{3.86}{5-1}}\ \text{cm} = \sqrt{0.965}\ \text{cm} = 0.98\ \text{cm}$, so $d = 18.0 \pm 1.0\ \text{cm}$.
4. Reaction time: $t = \sqrt{\dfrac{2(0.180\ \text{m})}{9.8\ \text{m/s}^2}} = \sqrt{0.0367\ \text{s}^2} = 0.192\ \text{s}$.

Answer: $d = 18.0 \pm 1.0$ cm; reaction time about 0.19 s.

Example 3. Michelson reported $c = 299{,}910 \pm 50$ km/s. The modern value is 299,792 km/s. Find his percent uncertainty and his percent error. Was the true value inside his uncertainty range?

Show solution

1. Percent uncertainty: $\dfrac{50}{299{,}910} \times 100\% = 0.017\%$.
2. Percent error: $\dfrac{299{,}910 - 299{,}792}{299{,}792} \times 100\% = \dfrac{118}{299{,}792} \times 100\% = 0.039\%$.
3. His range was 299,860 to 299,960 km/s. The true value, 299,792 km/s, lies below it.

Answer: Uncertainty 0.017%, error 0.039%. The true value was outside his range, a sign of an undetected systematic error, even though the measurement was superb for its time.

PRACTICE

1. Easy How many significant figures are in each? (a) 0.00450 (b) 3.020 (c) 6.02×10^{23} (d) 1500 written as 1.5×10^3

   Show answer

   (a) 3 (b) 4 (c) 3 (d) 2

2. Easy Round 0.006734 to two significant figures.

   Show answer

   0.0067

3. Easy A bathroom scale always reads 1.5 kg too high. Is that a random or a systematic error? Will weighing yourself ten times and averaging fix it?

   Show answer

   Systematic. Averaging does not fix it, because every reading is high by the same amount.

4. Medium Compute and round correctly: 3.14159×2.0 m.

   Show answer

   6.3 m (two significant figures)

5. **Medium** Compute and round correctly: $105.3\ \text{g} - 2.47\ \text{g}$.

Show answer

$102.83 \to 102.8$ g (one decimal place)

6. **Medium** A ruler is caught after falling $22.5\ \text{cm}$. What was the reaction time?

Show answer

$t = \sqrt{2(0.225)/9.8} = 0.214\ \text{s}$

7. **Medium** Find the mean and standard deviation of 9.8, 9.6, 10.1 and 9.9 (m/s^2).

Show answer

Mean $9.85\ \text{m/s}^2$; $s = \sqrt{0.13/3} = 0.21\ \text{m/s}^2$. Report $9.85 \pm 0.21\ \text{m/s}^2$ (or 9.9 ± 0.2).

8. **Medium** A length is measured as $25.0 \pm 0.2\ \text{cm}$. What is the percent uncertainty?

Show answer

$0.2/25.0 \times 100\% = 0.8\%$

9. **Hard** A rectangle measures $12.0 \pm 0.1\ \text{cm}$ by $5.0 \pm 0.1\ \text{cm}$. Find its area and the uncertainty in the area, using the rule that percent uncertainties add when quantities are multiplied.

Show answer

Area $= 60\ \text{cm}^2$. Percent uncertainties: $0.83\% + 2.0\% = 2.8\%$, so $\delta A = 1.7\ \text{cm}^2$: $60 \pm 2\ \text{cm}^2$.

10. **Hard** Michelson's spinning mirror turned at about 257 turns per second, and the light's round trip was about 1210 m. How long did the round trip take, and through what angle (in degrees) did the mirror turn in that time? Use $c = 3.00 \times 10^8\ \text{m/s}$.

Show answer

$t = 1210/(3.00 \times 10^8) = 4.03 \times 10^{-6}$ s. Angle $= 257 \times 360^\circ \times 4.03 \times 10^{-6} = 0.373^\circ$.

11. **Challenge** Explain why reporting a calculator answer like 0.19166 s for a reaction time measured with a ruler read to 0.1 cm and a scatter of 1 cm is misleading, and how you should report it.

Show answer

The scatter of about 1 cm in 18 cm (about 5%) means the time is uncertain by about 3% (half the percent uncertainty, because of the square root), roughly ±0.005 s. Report about 0.192 ± 0.005 s; the extra digits claim a precision the measurement does not have.

COMMON MISTAKES

- Counting leading zeros as significant. In 0.0045, only the 4 and the 5 count.
- Rounding in the middle of a calculation. Keep extra digits until the final answer, then round once.
- Thinking more decimal places make an answer better. The number of digits should match the uncertainty.

> **Make it a game · Reaction Time Report.** Use the ruler-drop test: your parent drops a ruler and you catch it, then convert the distance to time with d = 1/2 g t squared. Do ten trials, record the average and report it with uncertainty and correct significant figures. Your parent challenges your last digit. Try to cut your spread next round.

1.4 Graphs and Best-Fit Lines: Hooke's Law

OBJECTIVES

- Plot data with labeled axes and units, and draw a best-fit straight line.
- Find the slope and intercept of a best-fit line, with units, and explain their physical meaning.
- State Hooke's law, $F = kx$, and find a spring constant from a graph.
- Recognize when data are not linear and where a law stops working.

> **Warm-up.** Hang a key ring from a rubber band and then from a spring (from a toy, a retractable pen or the hardware store). Add a second, then a third identical key ring. Each time, measure how much longer the rubber band or spring gets. Before you look closely at the numbers, predict: if one key ring stretches the spring 1 cm, how far will three stretch it? Then check.

WATCH

Two ways to find the spring constant - WITH GRAPHS

Rhett Allain · 12:11

https://www.youtube.com/watch?v=WviMC0DXaZM

> **WHEN TO WATCH**
> Before the lab. Watch how the presenter turns a table of masses and stretches into a graph and a slope.

BACKUP VIDEO

How to determine the spring constant

Physics with Professor Matt Anderson · 6:45

https://www.youtube.com/watch?v=tdy_wHZfdTo

Optional, if you want to see the same analysis done in a spreadsheet.

KEY TERMS

Hooke's law

For many springs, the stretching force is proportional to the extension: $F = kx$.

spring constant

The stiffness k of a spring, in newtons per meter (N/m): the force needed to stretch it one meter.

extension

How much a spring's length has increased from its relaxed length.

slope

Rise over run of a line, $\Delta y/\Delta x$, with the units of the y-axis divided by the units of the x-axis.

best-fit line

The straight line that passes as close as possible to all of the data points, with points scattered evenly above and below it.

elastic limit

The largest stretch from which a spring or material will spring back to its original length; beyond it, it is permanently bent.

CORE IDEAS

The story. Robert Hooke was the first Curator of Experiments of the Royal Society of London, appointed in 1662, and his job was to show the members a new experiment almost every week. Among hundreds of other projects he studied springs: coiled wire springs, watch springs, long straight wires and even pieces of wood. In 1676 he published his discovery as a Latin anagram, a jumble of letters, "ceiiinossttuv," so that he could claim it without revealing it. In 1678 he unscrambled it in a small book, *Lectures de Potentia Restitutiva, or Of Spring*: *ut tensio, sic vis*, "as the extension, so the force."

How they figured it out. Hooke hung a spring from a nail, added weights one at a time to a pan at the bottom, and measured how far the pan went down each time. With one weight the spring stretched a certain amount; with two weights, twice as much; with three, three times. He found the same pattern with every springy body he tried, as long as he did not stretch it too far. His drawings show the springs, the pans and the ruled scales. Hooke was also an architect who helped Christopher Wren rebuild London after the Great Fire of 1666, and he used his understanding of springs to design balance springs for pocket watches.

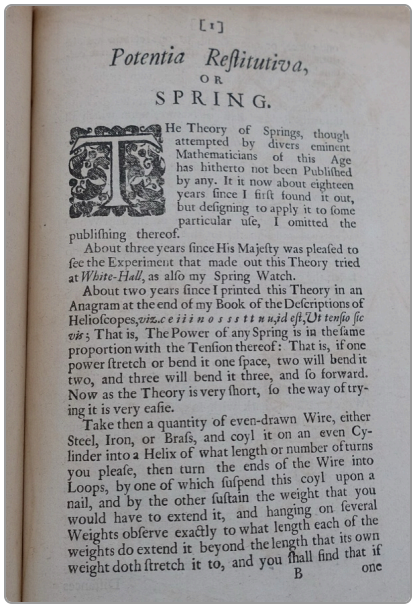
The physics. The best way to see a pattern in data is a graph. Put the variable you controlled on the horizontal axis and the one you measured on the vertical axis, label both with units, and draw the straight line that best fits the points. A straight line has the equation $y = mx + b$: the **slope** $m = \Delta y/\Delta x$ and the **intercept** b. Both have units and meanings. For a spring, plot force F against extension x: the points fall on a line through the origin, so $F = kx$, and the slope is the **spring constant** k in

N/m. A stiff car spring might have $k \approx 30{,}000\ \mathrm{N/m}$; a toy spring, about 10 N/m. Beyond the **elastic limit**, the line bends and the spring is permanently stretched.

The engineering. Springs that obey Hooke's law make it possible to measure force: a spring scale, a bathroom scale, and the strain gauges in the load cells that weigh trucks are all calibrated graphs of force against stretch. Engineers choose springs by their constant: a car suspension, a mattress, a garage-door spring and a pogo stick all need different values of k. They also design springs to stay well below their elastic limit, so the spring returns to its shape after millions of cycles.

Try it. In the lab you will measure the spring constant of a toy spring, check whether a rubber band obeys Hooke's law, and see what a graph tells you that a table cannot.

PICTURE STUDY



[1]

Potentia Restitutiva,
OR
SPRING.

THe Theory of Springs, though attempted by divers eminent Mathematicians of this Age has hitherto not been Published by any. It it now about eighteen years since I first found it out, but designing to apply it to some particular use, I omitted the publishing thereof.

About three years since His Majesty was pleased to see the Experiment that made out this Theory tried at *White-Hall*, as also my Spring Watch.

About two years since I printed this Theory in an Anagram at the end of my Book of the Descriptions of Heliostopes, *viz. c e i i i n o s s s t t u u, id est, Ut tensio sic vis*; That is, The Power of any Spring is in the same proportion with the Tension thereof: That is, if one power stretch or bend it one space, two will bend it two, and three will bend it three, and so forward. Now as the Theory is very short, so the way of trying it is very easie.

Take then a quantity of even-drawn Wire, either Steel, Iron, or Brass, and coyl it on an even Cylinder into a Helix of what length or number of turns you please, then turn the ends of the Wire into Loops, by one of which suspend this coyl upon a nail, and by the other sustain the weight that you would have to extend it, and hanging on several Weights observe exactly to what length each of the weights do extend it beyond the length that its own weight doth stretch it to, and you shall find that if

B one

Of Spring, Robert Hooke, 1678 · Printed page · *Lectures de Potentia Restitutiva, or Of Spring*, London (Niels Bohr Library & Archives copy)

Image: Wikimedia Commons, public domain

This is the page on which Hooke finally revealed the law he had hidden in an anagram two years before. Read it slowly; the English of 1678 is not hard.

Look closely

- Find the anagram, printed here as "c e i i i n o s s s t t u u" (in Latin type, u and v were the same letter), and its solution in Latin.
- Find Hooke's English translation: "The Power of any Spring is in the same proportion with the Tension thereof."
- Find his instructions beginning "Take then a quantity of even-drawn Wire." What apparatus is he describing?
- Find the mention of "His Majesty" and a "Spring Watch."

Talk about it

1. Hooke says if one power stretches a spring "one space," two will stretch it two. Sketch the graph this describes.
2. Why would Hooke publish his law first as an anagram?
3. Hooke's instructions are a recipe anyone can follow. Why does that matter in science?

> **Safety first.** Wear safety glasses when stretching springs and rubber bands; a band that slips or snaps can hit an eye. Do not stretch a spring so far that it stays bent.

LAB

The Spring Constant of a Toy Spring

You will repeat Hooke's experiment with a toy spring and a rubber band, graph force against extension, find the spring constant from the slope, and find out where the law stops working.

You will need

- A spring: a mini plastic or metal Slinky, a spring from a toy, or a hardware-store extension spring
- A large rubber band
- A small paper cup with a string or paper-clip handle (the "pan")
- Coins, washers or marbles as masses, weighed on the kitchen scale
- Meter stick or ruler taped vertically beside the spring
- A hook, door handle or clamp to hang the spring from
- Graph paper or a spreadsheet

Steps

1. **Hypothesis.** Predict the shape of the graph of force against extension for the spring and for the rubber band, with reasons.
2. Hang the spring with the cup on its end. Tape the ruler beside it and record the position of the cup's bottom with no added mass. That is your zero.
3. Add masses in equal steps (for example, 20 g at a time) up to at least 8 steps. After each, record the total added mass and the cup's position. Extension = position − zero.
4. Remove the masses one step at a time and record the positions again. Does the spring return along the same values?
5. Convert masses to forces ($F = mg$) and extensions to meters. Plot F against x and draw the best-fit line. Find the slope k in N/m, using two points on the line that are far apart (not two data points).
6. Repeat steps 2–5 with the rubber band. Plot it on the same axes.
7. **Analyze.** Over what range is each graph straight? Does the rubber band's graph curve, and does unloading follow the loading curve? Which one is a

good force-measuring spring?

8. **Test your graph.** Hang an object of unknown mass, measure the extension, use your k to predict its mass, then weigh it on the scale and find the percent error.
9. **Improve.** Identify the largest source of uncertainty (reading the ruler, the spring swinging) and change your method to reduce it. Repeat two points and see if they lie closer to the line.

Tips

- Read the ruler at eye level to avoid parallax.
- Let the spring stop bouncing before each reading; a slow, gentle release helps.
- A spreadsheet's SLOPE function, or "add trendline," gives the least-squares slope; compare it with your hand-drawn line.

Check your work

- ☐ I can find a slope with units from a best-fit line and explain that it is the spring constant.
- ☐ I can tell from a graph whether a spring obeys Hooke's law and over what range.
- ☐ I used my graph to predict an unknown mass and checked the prediction.

LAB NOTEBOOK

1. Data tables (loading and unloading) and one graph with both the spring and the rubber band, axes labeled with quantities and units, best-fit line drawn, slope calculation shown.
2. Sketch what the graph of a spring looks like beyond its elastic limit. Mark the limit.
3. Two springs are hung end to end. Predict whether the combination is stiffer or softer than one spring, and why. (You can test it.)

KEY FORMULAS

$$F = kx$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$y = mx + b$$

$$W = mg \quad (g = 9.8\ \mathrm{N/kg})$$

From history. Hooke and Christiaan Huygens argued bitterly over who first invented the spiral balance spring for watches; both made working versions in the 1670s. Hooke's many inventions include the universal joint used in car drive shafts and an improved compound microscope, with which he drew the cells of cork in *Micrographia* (1665) and gave the word "cell" its biological meaning.

WORKED EXAMPLES

Example 1. A student hangs masses of 0, 50, 100, 150 and 200 g from a spring and measures extensions of 0, 2.0, 4.1, 5.9 and 8.0 cm. Find the spring constant.

Show solution

1. Convert masses to forces with $F = mg$: $0.050\ \mathrm{kg} \times 9.8\ \mathrm{N/kg} = 0.49\ \mathrm{N}$, so the forces are $0, 0.49, 0.98, 1.47, 1.96\ \mathrm{N}$.
2. Convert extensions to meters: $0, 0.020, 0.041, 0.059, 0.080\ \mathrm{m}$.
3. Plot F (vertical) against x (horizontal). The points lie close to a straight line through the origin; the best-fit line passes through $(0, 0)$ and about $(0.080\ \mathrm{m}, 1.96\ \mathrm{N})$.
4. Slope: $k = \dfrac{\Delta F}{\Delta x} = \dfrac{1.96\ \mathrm{N} - 0}{0.080\ \mathrm{m} - 0} = 24.5\ \mathrm{N/m}$.

Answer: $k \approx 24.5\ \mathrm{N/m}$ (a least-squares fit of the same data gives $24.6\ \mathrm{N/m}$).

Example 2. The same spring stretches 5.3 cm when an unknown object hangs from it. What is the object's weight and mass?

Show solution

1. $F = kx = (24.5\ \mathrm{N/m})(0.053\ \mathrm{m}) = 1.30\ \mathrm{N}$.
2. $m = \dfrac{F}{g} = \dfrac{1.30\ \mathrm{N}}{9.8\ \mathrm{N/kg}} = 0.133\ \mathrm{kg} = 133\ \mathrm{g}$.

Answer: Weight 1.30 N; mass about 133 g.

Example 3. A spring's total length L is measured with different hanging forces: $L = 12.0$ cm at $F = 0$ and $L = 20.0$ cm at $F = 4.0$ N, with points in between on a straight line. Write the equation of the line of L against F and interpret the slope and intercept.

Show solution

1. Slope: $m = \dfrac{20.0\ \mathrm{cm} - 12.0\ \mathrm{cm}}{4.0\ \mathrm{N} - 0} = 2.0\ \mathrm{cm/N}$.
2. Intercept: $b = 12.0$ cm, the length with no force (the relaxed length).
3. Equation: $L = (2.0\ \mathrm{cm/N})F + 12.0\ \mathrm{cm}$.
4. The slope is the stretch per newton, $1/k$:
   $k = \dfrac{1}{2.0\ \mathrm{cm/N}} = 0.50\ \mathrm{N/cm} = 50\ \mathrm{N/m}$.

Answer: $L = (2.0\ \mathrm{cm/N})F + 12.0\ \mathrm{cm}$; the intercept is the relaxed length and the slope is $1/k$, so $k = 50\ \mathrm{N/m}$.

PRACTICE

1. Easy A spring with $k = 40\ \mathrm{N/m}$ is stretched 0.15 m. What force is needed?

   Show answer

   $F = kx = 40 \times 0.15 = 6.0\ \mathrm{N}$

2. Easy A 2.0 N force stretches a spring 5.0 cm. What is k in N/m?

   Show answer

   $k = 2.0\ \mathrm{N}/0.050\ \mathrm{m} = 40\ \mathrm{N/m}$

3. Easy What is the weight of a 250 g mass?

Show answer

$W = 0.250 \times 9.8 = 2.45\ \text{N}$

4. Medium Find the slope of the line through $(0.10\ \text{s}, 0.50\ \text{m})$ and $(0.60\ \text{s}, 3.00\ \text{m})$, with units. What quantity does it represent?

Show answer

$\dfrac{2.50\ \text{m}}{0.50\ \text{s}} = 5.0\ \text{m/s}$, a velocity.

5. Medium A car spring has $k = 3.0 \times 10^4\ \text{N/m}$. How far does it compress when it supports a quarter of a 1200 kg car?

Show answer

$F = \frac{1}{4}(1200)(9.8) = 2940\ \text{N};\ x = 2940/30{,}000 = 0.098\ \text{m} \approx 9.8\ \text{cm}$

6. Medium A graph of extension (cm) against force (N) has a slope of $0.80\ \text{cm/N}$. Find k in N/m.

Show answer

$k = 1/(0.80\ \text{cm/N}) = 1.25\ \text{N/cm} = 125\ \text{N/m}$

7. Medium A student's force–extension graph is straight for the first five points and then curves upward in extension. What has happened?

Show answer

The spring has passed its elastic limit (or the limit of proportionality); beyond it the extension grows faster than the force, and the spring may be permanently stretched.

8. Medium Suppose a spring like Hooke's stretches 1.5 cm for each 1 lb weight (4.45 N). What is its spring constant in N/m?

Show answer

$k = 4.45\ \text{N}/0.015\ \text{m} = 297\ \text{N/m} \approx 300\ \text{N/m}$

9. Hard A best-fit line of length L (cm) against hanging mass m (g) is $L = 0.050m + 15.0$. Find the relaxed length and the spring constant in N/m.

Show answer

Relaxed length 15.0 cm. Slope $0.050\ \text{cm/g} = 0.050\ \text{cm per } 0.0098\ \text{N}$, so $k = 0.0098\ \text{N}/0.00050\ \text{m} = 19.6\ \text{N/m}$.

10. Hard Explain why you should find a slope from two points on the best-fit line rather than from two of your data points.

Show answer

The best-fit line averages the scatter of all the points. Two data points each carry their own random error, so a slope from them can be noticeably off; points on the line use all the data.

11. Challenge Two identical springs, each with $k = 30\ \mathrm{N/m}$, are hung (a) end to end and (b) side by side, each carrying part of the load. Find the effective spring constant in each case.

Show answer

(a) Each spring feels the full force and stretches F/k, so the total stretch is $2F/k$: $k_{\text{eff}} = 15\ \mathrm{N/m}$. (b) Each carries half the force, so the stretch is $F/(2k)$: $k_{\text{eff}} = 60\ \mathrm{N/m}$.

COMMON MISTAKES

- Plotting mass in grams on the force axis. Convert mass to force in newtons first if you want k in N/m.
- Using total length instead of extension. Hooke's law uses the change in length from the relaxed spring.
- Forcing the line through every point, connecting the dots. A best-fit line passes between the points.

> **Make it a game · Slope Says What.** Your parent sketches a rough force-versus-extension scatter of six points on paper. You draw the best-fit line, compute the slope with units in 60 seconds, and say what it means physically. Then say what a nonzero intercept would suggest. Score a point per correct unit and meaning, and swap roles.

1.5 Vectors and Right-Triangle Trigonometry: Stevin's Wreath of Spheres

OBJECTIVES

- Distinguish scalars from vectors and give examples of each.
- Use sine, cosine and tangent to find the components of a vector and the angle of a resultant.
- Add vectors by components and by drawing them tip to tail.
- Explain Stevin's argument for the balance of forces on an inclined plane.

> **Warm-up.** Stand in a doorway. Walk 4 steps straight ahead, turn left 90°, and walk 3 steps. Now, without walking the path again, point straight at where you started. Guess how many steps it would take to walk straight back. Then do it and count. You have just added two vectors.

WATCH

Introduction to Tip-to-Tail Vector Addition, Vectors and Scalars

Flipping Physics · 10:26

https://www.youtube.com/watch?v=ZYl9-iz7nR8

> **WHEN TO WATCH**
> After the physics paragraph. Pause after each example and try it yourself first.

BACKUP VIDEO

Addition of Vectors By Means of Components - Physics

The Organic Chemistry Tutor · 15:21

https://www.youtube.com/watch?v=xS-gdFgZel0

Before worked example 2, for practice finding components with sine and cosine.

KEY TERMS

scalar

A quantity with size only: mass, time, temperature, distance, speed, energy.

vector

A quantity with size and direction: displacement, velocity, acceleration, force. Drawn as an arrow; written in bold or with an arrow, $\vec{F}$.

resultant

The single vector equal to the sum of two or more vectors.

components

The perpendicular parts of a vector, usually along x and y: $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

sine, cosine, tangent

In a right triangle,
$\sin\theta = \text{opposite}/\text{hypotenuse}$,
$\cos\theta = \text{adjacent}/\text{hypotenuse}$,
$\tan\theta = \text{opposite}/\text{adjacent}$.

CORE IDEAS

The story. Simon Stevin of Bruges (1548–1620) was a bookkeeper, a mathematician, and an engineer and adviser to Prince Maurice of Nassau in the Netherlands, for whom he designed windmills, sluices and drainage systems. He also popularized decimal fractions in a little book of 1585. In 1586 he published *The Elements of the Art of Weighing*, and on its title page he put a puzzle that became famous: a loop of equal balls, strung like a necklace, draped over a triangular block with one long sloping side and one short steep side. Stevin wrote above it, in Dutch, "Wonder en is gheen wonder": "A wonder, yet no wonder."

How they figured it out. The long slope holds more balls than the short slope. Will the heavier side pull the necklace around? If it did, the necklace would come back to the same position and keep going forever, a perpetual-motion machine, which Stevin knew was absurd. So it must stand still. The hanging loop below the block is symmetrical and pulls equally on both sides, so it can be cut away, and the balls on the two slopes must still balance. That means a few balls on a steep slope pull as hard along the slope as many balls on a gentle slope. Each ball pulls along its slope in proportion to the height divided by the length of the slope: what we now call the *sine* of the slope's angle. From this Stevin worked out the rule for combining forces that act in different directions.

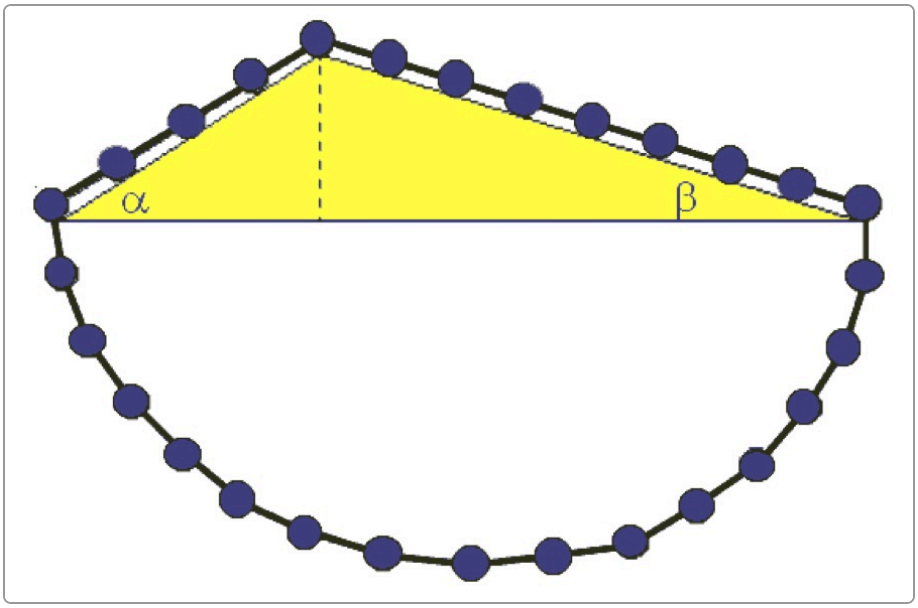
The physics. A **scalar** has only size (5 kg, 12 s); a **vector** has size and direction (5 m north, 12 N downward). Vectors add tip to tail, or by components. In a right triangle with hypotenuse A and angle θ from the x-axis, the components are $A_x = A\cos\theta$ and $A_y = A\sin\theta$. To add vectors, add their x-components and their y-components

separately; then the size of the resultant is $R = \sqrt{R_x^2 + R_y^2}$ (Pythagoras) and its direction is $\theta = \tan^{-1}(R_y/R_x)$. Stevin's balls are a vector problem: the part of a ball's weight W that pulls along a slope at angle θ is $W \sin\theta$. A slope twice as long with the same height has half the sine and twice the balls, so the pulls balance.

The engineering. Every structure and every journey is a vector problem. A pilot points the airplane into the wind so that the sum of the airplane's velocity through the air and the wind's velocity carries it to the runway. A surveyor breaks a property line into north and east components. A bridge engineer resolves the force in each diagonal member of a truss into horizontal and vertical parts, and a crane operator knows that a cable pulling at an angle lifts with only its vertical component. You will use components in almost every unit of this course.

Try it. In the lab you will pull on a ring with three spring scales until it balances, measure the forces and angles, and check whether their vector sum really is zero.

PICTURE STUDY



Stevin's Wreath of Spheres (redrawn), After Simon Stevin (1586), Modern redrawing · Diagram · Wikimedia Commons

Image: Wikimedia Commons, public domain

Stevin was so proud of this argument that he made it the emblem of his book on weighing, with the motto "Wonder en is gheen wonder" ("A wonder, yet no wonder"). This modern redrawing shows the idea plainly: a law of physics proved by reasoning about a machine that cannot work.

Look closely

- Count the balls on the long slope and on the short slope.
- Find the loop of balls hanging below the block. Why is it symmetrical?
- Compare the angles of the two slopes.

Talk about it

1. Why does the necklace not slide toward the side with more balls?
2. If you cut the hanging loop away, why must the balls on the slopes still balance?
3. How does this picture show that a force along a slope depends on the slope's angle?

LAB

A Force Table with Three Spring Scales

When three forces hold an object still, their vector sum must be zero. You will test that with spring scales pulling on a ring, measuring every force and angle and adding them by components.

You will need

- Three spring scales (or three rubber bands calibrated with your Hooke's-law graph)
- A key ring or loop of strong string
- Large sheet of paper taped to a table, protractor, ruler, pencil
- Three thumbtacks or partners to hold the scales

Steps

1. Tape the paper down and draw axes through its center. Hook the three scales to the ring.
2. Pull the scales in three different directions, not all 120° apart, until the ring sits still over the center. Hold or pin the scales in place.
3. Mark each string's direction on the paper and read each force in newtons.
4. **Predict.** Before calculating, draw the three forces tip to tail to scale (for example, 1 cm = 1 N). Does the drawing close into a triangle?
5. Measure each angle from the positive x-axis with the protractor. Calculate each force's x and y components and add them in a table.
6. Compare: are $\sum F_x$ and $\sum F_y$ zero within your uncertainty? Estimate the uncertainty of each scale reading and of each angle.
7. **Second test.** Hang a known weight from the ring with string and support it with two scales at angles. Predict the two scale readings from the weight and the angles before reading them.
8. **Improve.** Find the largest contributor to your imbalance (scale zero, friction on the paper, angle reading) and correct it. Repeat once.

Tips

- Zero each spring scale in the horizontal position before you begin; most are calibrated hanging vertically.
- Lift the ring slightly off the paper so friction does not hold it.

Check your work

- ☐ I can resolve a force into components using sine and cosine.
- ☐ I added three measured forces by components and found a sum close to zero.
- ☐ I predicted two scale readings from geometry before measuring them.

LAB NOTEBOOK

1. A scale drawing of the three forces tip to tail, and a component table with columns: force, angle, F_x, F_y.
2. Draw Stevin's necklace yourself on a block with slopes of 30° and 60°, and show with a short calculation why it balances.
3. Write the meaning of sine, cosine and tangent in your own words with a labeled right triangle; keep this page for the rest of the year.

KEY FORMULAS

$$A_x = A\cos\theta, \qquad A_y = A\sin\theta$$

$$R = \sqrt{R_x^2 + R_y^2}, \qquad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\sin\theta = \frac{\text{opp}}{\text{hyp}}, \quad \cos\theta = \frac{\text{adj}}{\text{hyp}}, \quad \tan\theta = \frac{\text{opp}}{\text{adj}}$$

From history. Stevin also showed, before Galileo, that a heavy and a light lead ball dropped together from about 30 feet hit a board below at the same moment; he and a friend did the experiment at Delft around 1586. He wrote his books in Dutch rather than Latin so that ordinary craftsmen could read them, and he coined many Dutch scientific words still in use, including *wiskunde* for mathematics.

Example 1. A surveyor walks 30 m east and then 40 m north. Find the size and direction of her displacement.

Show solution

1. Components: $R_x = 30\text{ m}$, $R_y = 40\text{ m}$.
2. Size: $R = \sqrt{(30\text{ m})^2 + (40\text{ m})^2} = \sqrt{2500\text{ m}^2} = 50\text{ m}$.
3. Direction: $\theta = \tan^{-1}\left(\dfrac{40\text{ m}}{30\text{ m}}\right) = 53.1^\circ$ north of east.

Answer: 50 m at 53.1° north of east.

Example 2. A rope pulls a sled with a force of 25 N at 30° above the horizontal. Find the horizontal and vertical components.

Show solution

1. Horizontal: $F_x = F\cos\theta = (25\text{ N})\cos 30^\circ = (25\text{ N})(0.866) = 21.7\text{ N}$.
2. Vertical: $F_y = F\sin\theta = (25\text{ N})\sin 30^\circ = (25\text{ N})(0.500) = 12.5\text{ N}$.
3. Check: $\sqrt{21.7^2 + 12.5^2}\text{ N} = 25.0\text{ N}$.

Answer: $F_x = 21.7$ N forward, $F_y = 12.5$ N upward.

Example 3. An airplane flies with a velocity of 200 km/h due north relative to the air. A wind blows due east at 50 km/h. Find the airplane's velocity relative to the ground.

Show solution

1. Add the velocities as vectors. North component: 200 km/h; east component: 50 km/h.
2. Size: $v = \sqrt{(200)^2 + (50)^2}\text{ km/h} = \sqrt{42{,}500}\text{ km/h} = 206\text{ km/h}$.
3. Direction: $\theta = \tan^{-1}\left(\dfrac{50}{200}\right) = 14.0^\circ$ east of north.

Answer: 206 km/h at 14.0° east of north. To fly due north, the pilot must point the airplane somewhat west of north.

Example 4. Stevin's block has a long slope at 30° and a short slope at $90^\circ - 30^\circ = 60^\circ$, both rising to the same peak. Each ball weighs W. If the long slope holds 4 balls, how many must the short slope hold to balance, according to $n_1 W \sin\theta_1 = n_2 W \sin\theta_2$?

Show solution

1. Pull along the long slope: $4W \sin 30^\circ = 4W(0.500) = 2.00W$.
2. Pull along the short slope with n balls: $nW \sin 60^\circ = nW(0.866)$.
3. Set equal: $n = \dfrac{2.00}{0.866} = 2.31$.
4. Check with lengths: the slopes have the same height h, so their lengths are $h/\sin 30^\circ = 2h$ and $h/\sin 60^\circ = 1.155h$, a ratio of 1.73, and $4/1.73 = 2.31$. The number of balls is proportional to the length, so the necklace balances.

Answer: $n = 2.31$ ball-weights (in a real necklace the balls fill each slope in proportion to its length, which is exactly this ratio).

PRACTICE

1. Easy Classify as scalar or vector: speed, velocity, mass, weight, temperature, displacement, time.

   Show answer

   Scalars: speed, mass, temperature, time. Vectors: velocity, weight (a force), displacement.

2. Easy Find $\sin 40^\circ$, $\cos 40^\circ$ and $\tan 40^\circ$ to three decimal places.

   Show answer

   0.643, 0.766, 0.839

3. Easy A hiker walks 5.0 km north and 12 km east. How far is she from her starting point?

   Show answer

   $\sqrt{25 + 144} = 13$ km

4. Medium Find the components of a 60 N force at 25° above the horizontal.

   Show answer

   $F_x = 60\cos 25^\circ = 54.4$ N; $F_y = 60\sin 25^\circ = 25.4$ N

5. Medium A ladder 5.0 m long leans against a wall at 70° to the ground. How high up the wall does it reach, and how far is its foot from the wall?

Show answer

Height $= 5.0 \sin 70^\circ = 4.70$ m; distance $= 5.0 \cos 70^\circ = 1.71$ m

6. Medium Add $\vec{A} = 8.0$ m east and $\vec{B} = 6.0$ m west. Then add $\vec{A}$ and $\vec{C} = 6.0$ m north.

Show answer

$\vec{A} + \vec{B} = 2.0$ m east. $\vec{A} + \vec{C} = 10.0$ m at 36.9° north of east.

7. Medium A boat heads straight across a river at $3.0\ \mathrm{m/s}$ while the current carries it downstream at $1.5\ \mathrm{m/s}$. What is its speed relative to the bank, and at what angle from straight across?

Show answer

$\sqrt{9.0 + 2.25} = 3.35\ \mathrm{m/s}$ at $\tan^{-1}(1.5/3.0) = 26.6^\circ$ downstream

8. Hard Add these displacements: 10.0 m at 30° and 15.0 m at 120° (angles measured counterclockwise from east).

Show answer

$R_x = 8.66 - 7.50 = 1.16$ m; $R_y = 5.00 + 12.99 = 17.99$ m; $R = 18.0$ m at 86.3°.

9. Hard A 20 N picture hangs from two wires, each at 40° above the horizontal. Find the tension in each wire.

Show answer

Vertical balance: $2T \sin 40^\circ = 20$ N, so $T = 20/(2 \times 0.643) = 15.6$ N.

10. Hard Explain Stevin's argument in your own words: why must a loop of balls draped over a triangular block stand still?

Show answer

If the loop moved, it would return to an identical arrangement and keep moving forever without any energy source, which is impossible. So it stands still; the hanging part pulls equally on both sides, so the balls on the two slopes must balance even though one slope holds more balls.

11. **Challenge** A pilot wants to fly due north at a ground speed of 200 km/h, with a 50 km/h wind blowing due east. In what direction must she point the airplane, and what airspeed does she need?

Show answer

The airplane's air velocity must have a west component of 50 km/h and a north component of 200 km/h: airspeed $= \sqrt{200^2 + 50^2} = 206$ km/h, heading $\tan^{-1}(50/200) = 14.0^\circ$ west of north.

COMMON MISTAKES

- Using sine where cosine belongs. The component next to (adjacent to) the angle uses cosine.
- Adding vector sizes directly: 3 m north plus 4 m east is 5 m, not 7 m.
- Leaving the calculator in radian mode. Check that $\sin 30^\circ$ gives 0.5.

> **Make it a game · Component Quick Draw.** Your parent calls out a vector, such as 40 newtons at 30 degrees. Sketch it, find its x and y components, and give the resultant angle for a second pair, all in 60 seconds. Check each answer with a calculator. Three correct in a row earns a hero point. Try for five in a row.

UNIT 1

Measurement and the Scientific Method

Physics is the science that measures. Before you can study motion, force or energy, you need the tools that every physicist uses every day: careful experiments with controlled variables, a common system of units, an honest statement of uncertainty, graphs that turn data into laws, and vectors for quantities that have direction. This unit teaches those tools through the people who forged them: Tycho and Kepler, who overturned two thousand years of circles with 8 minutes of arc; the surveyors who measured the meter from Dunkirk to Barcelona; Michelson, timing light with a spinning mirror; Hooke, hanging weights from springs; and Stevin, proving the law of the inclined plane with a necklace of balls.

You will run five labs: a fair test of Galileo's claim that heavy and light objects fall together, a density survey of household materials, a twenty-trial measurement of your own reaction time, the spring constant of a toy spring, and a three-scale force table. Right-triangle trigonometry is introduced in Lesson 1.5, and you will use it all year.

PREREQUISITES

- Algebra I: solving linear equations, working with exponents and scientific notation, graphing lines and finding slopes
- Geometry: the Pythagorean theorem, similar triangles and angle measure
- A scientific calculator, set to degree mode

MASTERY CHECKLIST

- ☐ I can design a fair test with independent, dependent and controlled variables and a testable hypothesis.
- ☐ I can convert units with conversion factors, including squared and cubed units, and check equations with dimensional analysis.
- ☐ I can report a measurement as a mean with an uncertainty and correct significant figures, and tell random from systematic error.
- ☐ I can graph data, draw a best-fit line, and interpret its slope and intercept with units, as in Hooke's law.
- ☐ I can resolve vectors into components with sine and cosine and add vectors to find a resultant.

OPTIONAL KITS FOR THIS UNIT

- **Arbor Scientific spring scales and force tables**: A classroom science supplier that sells to families. Inexpensive spring scales in several ranges make Lessons 1.4 and 1.5 easier and more precise; a tabletop force table with pulleys is a classic upgrade for Lesson 1.5.
- **phyphox (free app)**: Not a kit but a free app from RWTH Aachen University that turns a phone into a lab instrument. Its acoustic stopwatch and accelerometer tools are used in many labs this year, starting with the drop test in Lesson 1.1.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

1.1 Galileo, Kepler and Data-Driven Science — 3 sessions of 50 minutes

1.2 SI Units: The Meter, the Kilogram and the Lost Mars Orbiter

3 sessions of 50 minutes

1.3 Uncertainty and Significant Figures: Michelson Measures Light

3 sessions of 50 minutes

1.4 Graphs and Best-Fit Lines: Hooke's Law 3 sessions of 50 minutes

1.5 Vectors and Right-Triangle Trigonometry: Stevin's Wreath of Spheres

3 sessions of 50 minutes

Unit 1 test

Calculators are allowed. Show your work for every calculation: write the formula, substitute with units, and give the answer with its unit and sensible significant figures. Answer the concept questions in complete sentences. The test is worth 100 points.

1. **5 pts**
   Define independent, dependent and controlled variables, and identify each in this experiment: a student changes the length of a pendulum and times 10 swings, using the same bob and the same release angle.

2. **6 pts**
   Convert $72\ \mathrm{km/h}$ to meters per second.

3. **6 pts**
   Convert $2.5\ \mathrm{m}^2$ to square centimeters.

4. **6 pts**
   A stone has a mass of $81\ \mathrm{g}$ and displaces $30\ \mathrm{mL}$ of water. Find its density in $\mathrm{g/cm^3}$ and $\mathrm{kg/m^3}$.

5. **6 pts**
   Use dimensional analysis to decide whether $v^2 = 2ad$ can be correct (v in m/s, a in $\mathrm{m/s^2}$, d in m).

6. **6 pts**

   Explain in two or three sentences what caused the loss of the Mars Climate Orbiter and what rule would have prevented it.

7. **6 pts**

   State the number of significant figures in (a) 0.0207 (b) 4.500 (c) 3.0×10^8, and compute 6.21×0.30 with correct significant figures.

8. **7 pts**

   Four timings of a cart are 2.31, 2.28, 2.35 and 2.30 s. Find the mean and the standard deviation, and report the result.

9. **6 pts**

   A thermometer reads 2 °C too high every time. A second thermometer gives readings that scatter by ±1 °C around the true value. Which has a random error and which a systematic error? Which can be improved by averaging?

10. **7 pts**

    A ruler is caught after falling 16.0 cm. Find the reaction time.

11. **8 pts**

    A spring stretches 3.0, 6.1 and 8.9 cm under forces of 1.0, 2.0 and 3.0 N. Find the spring constant from the best-fit line through the origin, and predict the stretch for 2.5 N.

12. **5 pts**

    On a graph of position (m) against time (s), what are the units of the slope, and what does it represent?

13. **7 pts**

    Find the x- and y-components of a 40 N force at 35° above the horizontal.

14. **8 pts**

    A ship sails 12 km east, then 9.0 km north. Find its displacement (size and direction).

15. **7 pts**

A 50 N sign hangs from two cables, each 30° above the horizontal. Find the tension in each cable.

16. **4 pts**

Michelson reported $299{,}910 \pm 50$ km/s for the speed of light; the modern value is $299{,}792$ km/s. Find his percent error.

ANSWER KEY

1. Independent: changed on purpose (length). Dependent: measured (time for 10 swings). Controlled: kept the same (bob, release angle, timing method).
2. $72 \times 1000/3600 = 20$ m/s
3. 2.5×10^4 cm^2 $= 25{,}000$ cm^2
4. $81/30 = 2.7$ g/cm^3 $= 2700$ kg/m^3
5. Left: $\mathrm{m}^2/\mathrm{s}^2$. Right: $(\mathrm{m}/\mathrm{s}^2)(\mathrm{m}) = \mathrm{m}^2/\mathrm{s}^2$. The units match, so it can be correct.
6. One team's software reported thruster impulse in pound-force seconds and the navigation software assumed newton-seconds, so impulses were underestimated by a factor of 4.45 and the spacecraft flew too low at Mars. Requiring units on every number and checking them at every interface between teams would have caught it.
7. (a) 3 (b) 4 (c) 2; $6.21 \times 0.30 = 1.863 \to 1.9$
8. Mean 2.31 s (2.310); $s = 0.029$ s; report 2.31 ± 0.03 s.
9. The first has a systematic error; the second a random error. Only the second is improved by averaging many readings.
10. $t = \sqrt{2(0.160)/9.8} = 0.181$ s
11. The line through the origin and about $(0.090\ \mathrm{m}, 3.0\ \mathrm{N})$ has slope $k \approx 33$ N/m. Stretch for 2.5 N: $x = 2.5/33 = 0.075$ m $= 7.5$ cm.
12. m/s; the slope is the velocity.
13. $F_x = 40\cos 35^\circ = 32.8$ N; $F_y = 40\sin 35^\circ = 22.9$ N
14. $\sqrt{144 + 81} = 15$ km at $\tan^{-1}(9/12) = 36.9^\circ$ north of east
15. $2T\sin 30^\circ = 50$ N, so $T = 50$ N.
16. $118/299{,}792 \times 100\% = 0.039\%$

PORTFOLIO PIECE: A CALIBRATED RUBBER-BAND FORCE METER

Build a force meter from a rubber band (or a spring) mounted on a stick with a paper scale. Calibrate it with known masses from the kitchen scale, graph your calibration with uncertainties, and mark the scale in newtons. Then use your meter to measure three unknown forces (a drawer opening, a backpack lifted, a toy car pulled up a ramp) and repeat each measurement five times. Write a one-page report with your calibration graph, its best-fit line or curve, a table of measurements reported as mean ± uncertainty, and a discussion of how accurate your meter is and where it stops being reliable.

LOOK FOR	WHAT IT MEANS
Calibration and graph	At least 8 calibration points, axes labeled with units, best-fit line or smooth curve, and the range over which the meter is linear stated.
Measurements and uncertainty	Each unknown force measured five times and reported as mean ± uncertainty with correct significant figures.
Units and conversions	All masses converted to forces in newtons with $F = mg$; units shown on every number.
Error analysis	Identifies random and systematic errors specifically (such as zero drift, hysteresis, reading parallax) and suggests one improvement.
Honest record	Notebook pages are dated and show the actual readings, including any that were repeated.

UNIT 2

Motion in One and Two Dimensions

Kinematics is the description of motion: where something is, how fast it is going, and how its speed is changing, without yet asking why. It began with Galileo, who slowed falling down on a ramp, timed it with a water clock and discovered that distance grows as the square of time. In this unit you will follow his ramp, ride with John Paul Stapp on a rocket sled that stopped in 1.4 seconds, drop a hammer and a feather on the Moon with Apollo 15, read motion from Marey's time photographs, and trace Galileo's parabola from a table edge to the floor.

The tools are the four constant-acceleration equations, motion graphs (slopes and areas), and the vectors from Unit 1, now used to split a projectile's motion into horizontal and vertical parts. Toys do much of the lab work: a Hot Wheels car on a ramp and off a table, and a pull-back car for motion graphs. You will also measure g yourself.

PREREQUISITES

- Unit 1: units, significant figures, graphing and slopes, vectors and components
- Algebra: solving quadratic equations with the quadratic formula (used in a few problems)

MASTERY CHECKLIST

☐ I can use

$d =$ for motion from rest and find

$\frac{1}{2}at^2$ acceleration from a linearized graph of

d against

t^2.

☐ I can choose and use the four constant-acceleration equations, with signs, including stopping-distance problems.

☐ I can solve free-fall problems for objects dropped and thrown upward, and explain terminal velocity.

☐ I can read and sketch

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- ☐ I can solve projectile problems by treating horizontal and vertical motion separately.

OPTIONAL KITS FOR THIS UNIT

- **Hot Wheels track sets**: Ages 4 and up. A basic set of straight orange track, a launcher and a loop is enough for Lessons 2.1, 2.5 and 5.5; extra straight track sections are sold separately and inexpensively.
- **PASCO dynamics carts and tracks**: A school laboratory supplier. A low-friction cart on an aluminum track gives very clean ramp and motion-graph data for Lessons 2.1, 2.4 and 3.3; useful for co-ops or families who want laboratory-grade equipment.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

2.1	**Galileo's Inclined Plane**	3 sessions of 50 minutes
2.2	**Velocity and Acceleration: Stapp's Rocket Sled**	3 sessions of 50 minutes

2.3	**Free Fall: From Pisa to the Hammer and the Feather**	3 sessions of 50 minutes
2.4	**Motion Graphs: Marey's Chronophotographs**	3 sessions of 50 minutes
2.5	**Projectiles: Galileo's Parabola**	3 sessions of 50 minutes

Unit 2 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$ and ignore air resistance unless a problem says otherwise. For every calculation, write the equation, substitute with units, and give the answer with its unit. Answer the essay question in a well-organized paragraph. The test is worth 100 points.

1. **5 pts**
   A cart starts from rest and accelerates uniformly. It covers $0.30\ \mathrm{m}$ in the first second. How far has it gone after $4.0\ \mathrm{s}$?

2. **6 pts**
   A ball rolls from rest down a ramp, covering $2.0\ \mathrm{m}$ in $2.5\ \mathrm{s}$. Find its acceleration and its speed at the bottom.

3. **6 pts**
   Find the acceleration of an object sliding without friction down a ramp inclined at 20°.

4. **7 pts**
   A car moving at $18\ \mathrm{m/s}$ brakes to a stop in $45\ \mathrm{m}$. Find its deceleration and the stopping time.

5. **7 pts**
   A driver traveling at $30\ \mathrm{m/s}$ has a reaction time of $1.5\ \mathrm{s}$ and brakes at $6.0\ \mathrm{m/s^2}$. Find the total stopping distance.

6. **6 pts**
   Stapp's sled slowed from $282\ \mathrm{m/s}$ to rest in $1.4\ \mathrm{s}$. Find the average deceleration in $\mathrm{m/s^2}$ and in g.

7. **6 pts**

A stone is dropped from rest. How far does it fall in 2.5 s and how fast is it moving then?

8. **7 pts**

A ball is thrown straight up at 12 m/s. Find its maximum height and its total time in the air (caught at the launch height).

9. **5 pts**

A ball is thrown straight up. Give its acceleration (size and direction) (a) on the way up, (b) at the top, (c) on the way down.

10. **5 pts**

Why did the hammer and feather fall together on the Moon but not on Earth?

11. **7 pts**

A velocity–time graph rises in a straight line from 0 to 8.0 m/s in 4.0 s, then stays at 8.0 m/s for 6.0 s. Find the acceleration during the first part and the total displacement.

12. **5 pts**

A position–time graph is a straight line from $(0, 2.0\ \mathrm{m})$ to $(5.0\ \mathrm{s}, 17.0\ \mathrm{m})$. What is the velocity?

13. **7 pts**

A marble leaves a 1.0 m high table horizontally at 2.0 m/s. Where does it land?

14. **8 pts**

A ball is kicked at 15 m/s at 35° above level ground. Find its time of flight, range and maximum height.

15. **4 pts**

Which launch angle gives the greatest range on level ground without air resistance? Name two other angles that give equal ranges.

16. 9 pts

Essay. Explain how Galileo used an inclined plane and a water clock to discover the law of falling bodies, and why this was an example of the scientific method at its best.

ANSWER KEY

1. $16 \times 0.30 = 4.8$ m
2. $a = 2(2.0)/6.25 = 0.64\ \mathrm{m/s^2}$; $v = at = 1.6$ m/s
3. $9.8 \sin 20^\circ = 3.35\ \mathrm{m/s^2}$
4. $a = 18^2/(2 \times 45) = 3.6\ \mathrm{m/s^2}$; $t = 18/3.6 = 5.0$ s
5. Reaction 45 m + braking $900/12 = 75$ m = 120 m
6. $201\ \mathrm{m/s^2}$; $20.6\,g$ (about 21 g)
7. $\frac{1}{2}(9.8)(6.25) = 30.6$ m; $v = 24.5$ m/s
8. $H = 144/19.6 = 7.3$ m; $t = 2(12)/9.8 = 2.4$ s
9. $9.8\ \mathrm{m/s^2}$ downward in all three cases.
10. The Moon has no air, so the only force on each is gravity, which gives every object the same acceleration. On Earth, air resistance slows the light, broad feather far more than the hammer.
11. $a = 2.0\ \mathrm{m/s^2}$; displacement $= 16 + 48 = 64$ m
12. $15/5.0 = 3.0$ m/s
13. $t = \sqrt{2/9.8} = 0.45$ s; $x = 0.90$ m from the table
14. $v_{0y} = 8.60$ m/s, $v_{0x} = 12.3$ m/s; $t = 1.76$ s; $R = 21.6$ m; $H = 3.8$ m
15. 45°. Any pair adding to 90°, such as 30° and 60°.
16. Model answer: A falling ball moves too fast to time, so Galileo rolled bronze balls down a long, smooth groove to slow the motion while keeping it uniformly accelerated. He timed rolls by weighing water that flowed from a vessel during each roll, repeated the trials many times over different fractions of the ramp, and found that distance was proportional to the square of time at every slope. He then reasoned that free fall is the limiting case of a steep ramp. It shows the method well: a clear question, a clever experiment that made the quantity measurable, repeated measurement, a mathematical law, and a prediction (for projectiles) later tested against data.

PORTFOLIO PIECE: THE TARGET CHALLENGE

Build a ramp-and-launch system (Hot Wheels track, a marble on a curtain-rod channel, or a ball on a gutter) that sends a projectile horizontally off a table. Calibrate it: measure the launch speed from at least four release points, and graph launch speed against release height. Then a parent places a cup on the floor at a distance you have not seen in advance. Measure the distance, use your calibration and projectile equations to choose a release point, and launch once. Write a two-page report with your calibration graph, your calculation, the result of the first launch, and an analysis of every source of error you can identify. If you miss, show how you corrected and how many launches it took.

LOOK FOR	WHAT IT MEANS
Calibration	At least four release points, five trials each, with launch speeds measured by video and graphed with a best-fit line or curve.
Prediction	Release point chosen by calculation before launching, with every step written with units.
Result and analysis	First-launch result recorded honestly; percent error found; specific sources of error identified (speed change at the edge, height measurement, drag, release consistency).
Physics explanation	Report explains independence of horizontal and vertical motion and why the fall time depends only on height.
Improvement	A documented correction with before-and-after data.

2.1 Galileo's Inclined Plane

OBJECTIVES

- Describe Galileo's inclined-plane experiment and his discovery that distance from rest grows with the square of the time.
- Use $d = \frac{1}{2}at^2$ and $v = at$ for motion that starts from rest with constant acceleration.
- Linearize data by graphing distance against time squared, and find the acceleration from the slope.
- Compare a measured acceleration on a ramp with $g \sin\theta$.

> **Warm-up.** Lay a ruler or a book under one end of a long board or a length of Hot Wheels track so it slopes gently. Let a car roll from the top and say "tick" aloud at a steady rhythm, about twice a second, while a partner marks where the car is at each tick. Look at the marks. Are they equally spaced? Write a sentence about the pattern before you read on.

WATCH

Galileo's Measure Of Gravity Explained By Jim Al-Khalili | The Amazing World Of Gravity

Spark · 5:01

https://www.youtube.com/watch?v=ZBr8Q2ROX9s

> **WHEN TO WATCH**
> After the story paragraph: a physicist rebuilds Galileo's ramp and water clock.

BACKUP VIDEO

Galileo's Inclined Plane Experiment

HPSC102 · 5:28

https://www.youtube.com/watch?v=ZUgYc6Bi46w

Optional, after the physics paragraph, for a second reconstruction of the experiment.

KEY TERMS

uniform acceleration

Acceleration that stays the same: the velocity changes by equal amounts in equal times.

odd-number rule

Galileo's discovery that an object starting from rest covers distances in the ratio 1 : 3 : 5 : 7 ... in successive equal time intervals.

linearize

To graph transformed variables (such as d against t^2) so that a curved relationship becomes a straight line whose slope can be measured.

inclined plane

A ramp. For an object sliding without friction, the acceleration along the ramp is $g \sin \theta$.

CORE IDEAS

The story. Galileo Galilei wanted to know how a falling body gains speed. Does it fall at a steady pace, or faster and faster, and by what rule? A dropped ball falls 5 meters in the first second, far too fast to time with any clock of 1600. So Galileo slowed gravity down. In his *Two New Sciences* (1638) he described a wooden beam about 12 cubits long (about 7 meters), with a straight groove cut along it a little more than a finger wide and lined with smooth, polished parchment. He raised one end a cubit or two, and let a hard, smooth bronze ball roll down the groove.

How they figured it out. For a clock, Galileo used water. A large vessel of water had a thin pipe in its bottom; he opened the pipe when the ball was released and closed it when the ball reached the mark, then weighed the water that had run out on a sensitive balance. The weight of water measured the time. He wrote that he repeated the trials "a full hundred times," rolling the ball the full length, half, a quarter, and other fractions. The result was always the same: the distances were to each other as the squares of the times. A ball that rolled 1 unit in the first interval had rolled 4 units in two intervals, 9 in three and 16 in four, so in each successive interval it covered 1, 3, 5, 7 units: the odd numbers. Changing the ramp's slope changed how fast the ball gained speed, but never the pattern. His manuscripts, studied in the 1970s, include a page of real measurements from about 1604 that show this rule.

The physics. When an object starts from rest and its acceleration a is constant, its velocity grows in proportion to time, $v = at$, and its distance grows with the square of time, $d = \frac{1}{2}at^2$. A graph of d against t is a curve (half a parabola). A graph of d

against t^2 is a straight line through the origin with slope $\frac{1}{2}a$; this trick of choosing axes to get a straight line is called **linearizing**. On a frictionless ramp at angle θ, only the part of gravity along the ramp, $g \sin\theta$, accelerates the object. A rolling ball accelerates less, $a = \frac{5}{7} g \sin\theta$ for a solid ball, because some of the energy goes into spinning (Lesson 7.4). Galileo saw that as the ramp approaches vertical, the motion approaches free fall, so his ramp law was a law of falling.

The engineering. Engineers use the ramp law wherever gravity does the work. A wheelchair ramp is limited by building codes to a slope of 1 in 12 (about 4.8°) so that a person can control it. Runaway-truck ramps beside steep mountain highways are built uphill, filled with loose gravel, to stop trucks whose brakes have failed. Package conveyors in warehouses use gentle roller slopes so boxes speed up just enough. The same kinematics, $d = \frac{1}{2}at^2$, sets the length of an airport runway and the spacing of railway signals.

Try it. In the lab you will repeat Galileo's experiment with a Hot Wheels car on orange track and a phone camera instead of a water clock.

PICTURE STUDY

Galileo Shows the Law of Falling Bodies, Giuseppe Bezzuoli, 1841 · Fresco · Tribuna di Galileo, Museo La Specola, Florence

Image: Wikimedia Commons, public domain

Two hundred years after Galileo, the city of Florence built a hall in his honor and painted scenes from his life on its walls. This one imagines him showing his law of falling bodies to a prince of the Medici family and a crowd of doubtful professors.

Look closely

- Find Galileo. How has the painter made him stand out?
- Find any apparatus he is using to make his point.
- Look at the faces of the onlookers. Who seems convinced, and who seems doubtful?

Talk about it

1. Why would an experiment that anyone could watch be more convincing than an argument from an old book?
2. The painter was not there; this is an imagined scene. What does it tell us about how people in 1841 remembered Galileo?
3. If you were Galileo, what measurement would you show the doubters to convince them that distance grows as the square of time?

LAB

Galileo's Ramp with a Hot Wheels Car

You will measure how far a toy car rolls in a given time on a straight ramp, test whether distance grows with the square of time, find the acceleration from a linearized graph, and run a fair test of the ramp's slope.

You will need

- Hot Wheels car and at least 1.5 m of straight orange track (or a smooth board with a groove, or a marble on a curtain-rod channel)
- Books to raise one end; tape measure; masking tape
- A phone that records slow-motion video
- Graph paper or a spreadsheet

Steps

1. Raise one end of the track 5.0 cm per meter of length. Put tape marks at 20, 40, 60, 80, 100 and 120 cm from the start line. Record the length and height.
2. **Hypothesis.** If the car takes time T to reach 20 cm, predict how long it takes to reach 80 cm, and why.
3. Release the car from rest at the start line (hold it behind a ruler and lift the ruler). Film from the side in slow motion with the marks in view. Do 3 trials.
4. Step through the video and record the time at each mark (frame number ÷ frame rate, counting from the release frame). Average the 3 trials.
5. Make a table: d (m), t (s), t^2 (s^2). Graph d against t, then d against t^2. Draw the best-fit line on the second graph and find its slope; $a = 2 \times$ slope.
6. Check the odd-number rule: are the distances in equal time intervals roughly in the ratio 1 : 3 : 5?
7. **Fair test.** Change only the height: 2.5 cm, 5.0 cm and 10.0 cm per meter. Find a for each and graph a against $\sin\theta$. Is it a straight line? What should its slope be?
8. Compare each measured a with $g \sin\theta$ and compute the percent difference.
9. **Improve.** Identify the largest source of error (release, frame timing, a joint in the track) and change one thing. Repeat the 5.0 cm test.

Tips

- Tape the track joints flat; a bump at a joint shows up as a jump in your graph.
- Start the clock on the frame where the car first moves, not when your hand moves.

Check your work

☐ My graph of

d against

t^2 is a straight line through the origin, and I found

a from its slope.

☐ I can explain why a rolling car accelerates a little less than

$g \sin\theta$.

☐ My graph of

a against

$\sin\theta$ tests whether acceleration is proportional to the slope.

LAB NOTEBOOK

1. Data table, the curved d–t graph and the straight d–t^2 graph, with the slope worked out.
2. Graph of a against $\sin\theta$ for your three heights, with the best-fit line and its slope compared with g.
3. Draw Galileo's water clock and explain in three sentences how weighing water measures time, and what its sources of error would be.

KEY FORMULAS

$$v = at \quad \text{(from rest)}$$

$$d = \tfrac{1}{2}at^2 \quad \text{(from rest)}$$

$$a = g\sin\theta \quad \text{(sliding, no friction)}, \qquad a = \tfrac{5}{7}g\sin\theta \quad \text{(rolling solid ball)}$$

From history. Galileo was 74 and nearly blind when *Two New Sciences* was published, and he was living under house arrest after his trial of 1633. The manuscript was carried out of Italy and printed in Leiden. His idea of "diluting" gravity on a ramp was used again by George Atwood in 1784, who slowed falling with a pulley instead (Lesson 3.5).

Example 1. A ball released from rest on a ramp rolls 0.20 m in the first second. How far has it rolled after 3.0 s, how far does it roll during the third second alone, and what is its acceleration?

Show solution

1. Distance grows as t^2: after 3.0 s it has gone $3^2 = 9$ times as far: $9 \times 0.20\ \text{m} = 1.8\ \text{m}$.
2. By the odd-number rule, the third second's distance is 5 units: $5 \times 0.20\ \text{m} = 1.0\ \text{m}$. (Check: after 2 s it had gone $4 \times 0.20 = 0.80$ m, and $1.8 - 0.80 = 1.0$ m.)
3. From $d = \frac{1}{2}at^2$: $a = \dfrac{2d}{t^2} = \dfrac{2(0.20\ \text{m})}{(1.0\ \text{s})^2} = 0.40\ \text{m/s}^2$.

Answer: 1.8 m after 3 s; 1.0 m during the third second; $a = 0.40\ \text{m/s}^2$.

Example 2. A ramp 2.00 m long has one end raised 0.10 m. Find the acceleration of an object sliding down it without friction, and of a solid ball rolling down it. How long does the ball take to roll the full length?

Show solution

1. $\sin\theta = \dfrac{\text{height}}{\text{length}} = \dfrac{0.10\ \text{m}}{2.00\ \text{m}} = 0.050$.
2. Sliding: $a = g\sin\theta = (9.8\ \text{m/s}^2)(0.050) = 0.49\ \text{m/s}^2$.
3. Rolling: $a = \frac{5}{7}(0.49\ \text{m/s}^2) = 0.35\ \text{m/s}^2$.
4. Time: $t = \sqrt{\dfrac{2d}{a}} = \sqrt{\dfrac{2(2.00\ \text{m})}{0.35\ \text{m/s}^2}} = \sqrt{11.4\ \text{s}^2} = 3.4\ \text{s}$.

Answer: Sliding $0.49\ \text{m/s}^2$; rolling $0.35\ \text{m/s}^2$; about 3.4 s to roll 2.00 m.

Example 3. A student times a Hot Wheels car at several marks and plots d (m) against t^2 (s^2). The best-fit line through the origin has a slope of $0.21\ \mathrm{m/s^2}$. What is the car's acceleration? If the track is raised $0.050\ \mathrm{m}$ over $1.00\ \mathrm{m}$, how does this compare with $g \sin\theta$?

Show solution

1. For $d = \frac{1}{2}at^2$, the slope of d against t^2 is $\frac{1}{2}a$. So $a = 2 \times 0.21\ \mathrm{m/s^2} = 0.42\ \mathrm{m/s^2}$.
2. $g \sin\theta = (9.8\ \mathrm{m/s^2})\left(\dfrac{0.050}{1.00}\right) = 0.49\ \mathrm{m/s^2}$.
3. Percent difference from $g \sin\theta$: $\dfrac{0.49 - 0.42}{0.49} \times 100\% = 14\%$ lower.

Answer: $a = 0.42\ \mathrm{m/s^2}$, about 14% less than $g \sin\theta$. Friction in the wheels and the energy of the spinning wheels account for the difference.

PRACTICE

1. Easy A cart starts from rest and accelerates at $0.50\ \mathrm{m/s^2}$. How fast is it moving after $4.0\ \mathrm{s}$?

   Show answer

   $v = at = 2.0\ \mathrm{m/s}$

2. Easy How far does the same cart go in $4.0\ \mathrm{s}$?

   Show answer

   $d = \frac{1}{2}(0.50)(4.0)^2 = 4.0\ \mathrm{m}$

3. Easy A ball rolls 3 cm in the first time interval. By the odd-number rule, how far does it roll in the second and third intervals, and how far in total after three intervals?

   Show answer

   9 cm and 15 cm; total 27 cm (which is $3^2 \times 3$ cm).

4. Medium A ball rolls $1.20\ \mathrm{m}$ from rest in $2.0\ \mathrm{s}$. Find its acceleration and its speed at $2.0\ \mathrm{s}$.

   Show answer

   $a = 2d/t^2 = 0.60\ \mathrm{m/s^2}$; $v = at = 1.2\ \mathrm{m/s}$

5. Medium Find $g \sin\theta$ for a ramp at 10°.

Show answer

$9.8 \sin 10^\circ = 1.70 \text{ m/s}^2$

6. Medium How long does an object take to slide 3.0 m from rest down a frictionless 10° ramp?

Show answer

$t = \sqrt{2(3.0)/1.70} = 1.88 \text{ s}$

7. Medium Why did Galileo use a ramp instead of timing a dropped ball, and why was it reasonable to apply the ramp result to free fall?

Show answer

A dropped ball falls too fast to time with a water clock; the ramp slows the motion while keeping it uniformly accelerated. Galileo saw that the same pattern held at every slope and that a ramp turned toward vertical becomes free fall.

8. Medium A wheelchair ramp may rise at most 1 in 12. If it were frictionless, what would the acceleration of a rolling chair be? (Use $\sin\theta \approx 1/12$.)

Show answer

$a \approx 9.8/12 = 0.82 \text{ m/s}^2$

9. Hard Galileo's ramp was about 7 m long. If a ball took 10 s to roll the whole length from rest, what was its acceleration, and if it was a solid rolling ball, what was the angle of the ramp?

Show answer

$a = 2(7)/100 = 0.14 \text{ m/s}^2$. $\sin\theta = a/(\frac{5}{7}g) = 0.14/7.0 = 0.020$, so $\theta \approx 1.1^\circ$, a very gentle slope.

10. Hard A student plots d against t^2 and gets a straight line with slope 0.35 m/s^2 that does not pass through the origin but crosses the d-axis at 0.04 m. What does the slope tell you, and what might the intercept mean?

Show answer

$a = 0.70 \text{ m/s}^2$. The intercept suggests the distances were measured from a point 4 cm behind the actual release point (or the clock started late), a systematic error.

11. **Challenge** Show that the odd-number rule follows from $d = \frac{1}{2}at^2$: find the distance covered during the nth interval of length T.

Show answer

$d_n - d_{n-1} = \frac{1}{2}aT^2[n^2 - (n-1)^2] = \frac{1}{2}aT^2(2n-1)$, and $2n-1$ is the nth odd number: 1, 3, 5, 7, ...

COMMON MISTAKES

- Using $d = \frac{1}{2}at^2$ for an object that was already moving at the start. It holds only from rest.
- Graphing d against t, seeing a curve, and drawing a straight line through it anyway. Linearize first.
- Measuring the ramp's angle with the length along the floor instead of along the ramp in $\sin\theta = h/L$.

Make it a game · Odd Number Rule Test. Roll a toy car or marble down a slightly tilted board and mark where it is at equal time beats, counted aloud by your parent. Predict whether the gaps follow Galileo's odd numbers, 1, 3, 5, 7. Measure the gaps, compare to prediction and give percent difference. Retry to improve the match.

Scripture. "I gave my heart to seek and search out by wisdom concerning all things that are done under heaven" (Ecclesiastes 1:13, KJV)

Galileo searched out the motion of a rolling ball with a hundred careful trials. Seeking and searching out is a calling, and sometimes a long labor.

2.2 Velocity and Acceleration: Stapp's Rocket Sled

OBJECTIVES

- Distinguish distance from displacement and speed from velocity, and calculate average velocity and average acceleration.
- Choose and use the four constant-acceleration equations to solve one-dimensional motion problems.
- Calculate stopping distances, including reaction distance, and relate them to road design.
- Express accelerations in multiples of g.

> **Warm-up.** Ride in a car (or imagine one) as it stops at a red light. Hold a small object on a string from the rearview mirror, or just notice which way your body leans. Which way does the hanging object swing as the car speeds up? As it slows? As it cruises? Write your observations; you will explain them with acceleration.

WATCH

Introduction to Uniformly Accelerated Motion with Examples of Objects in UAM

Flipping Physics · 6:41

https://www.youtube.com/watch?v=WCR2Ki6hFf4

> **WHEN TO WATCH**
> After the physics paragraph. Pause before each worked example and predict the answer.

BACKUP VIDEO

The Rocket Sled Trials of Colonel John Stapp

The History Guy: History Deserves to Be Remembered · 14:05

https://www.youtube.com/watch?v=JHGJ_y4aJII

With the story paragraph: the history of Stapp's sled tests and what they changed.

KEY TERMS

displacement

The change in position, $\Delta x = x_f - x_i$, a vector (it can be negative). Distance is the total path length, a scalar.

average velocity

$\bar{v} = \Delta x / \Delta t$. Average speed is total distance divided by time.

acceleration

The rate of change of velocity, $a = \Delta v / \Delta t$, in m/s^2. Slowing down is acceleration opposite to the velocity.

g-force

An acceleration expressed as a multiple of $g = 9.8\ \mathrm{m/s^2}$. An acceleration of $49\ \mathrm{m/s^2}$ is 5 g.

reaction distance

The distance a vehicle travels at constant speed between the moment a driver sees a hazard and the moment the brakes begin to act.

CORE IDEAS

The story. After the Second World War, aircraft were becoming fast enough that engineers wondered whether a pilot could survive an emergency ejection or a crash at all. The accepted wisdom was that a human body could not survive much more than 18 g. Colonel John Paul Stapp, an Air Force flight surgeon, decided to find out by measuring it on himself. Beginning in 1947 at Muroc (now Edwards Air Force Base) in California, he rode a rocket-powered sled down a long track that was stopped abruptly by water brakes. On December 10, 1954, at Holloman Air Force Base in New Mexico, he rode the sled *Sonic Wind No. 1* to 632 miles per hour (282 m/s) in about 5 seconds, then stopped in about 1.4 seconds. For a moment he experienced 46.2 g. Newspapers called him “the fastest man on Earth.”

How they figured it out. Stapp’s team mounted accelerometers on the sled and on Stapp, filmed every run with high-speed cameras, and worked up from mild stops to severe ones, run by run, recording the effects on the body. They tested harness designs, seat shapes and strap widths. The results showed that a properly restrained person, facing forward or backward, could survive far larger decelerations than anyone had believed, if the force was spread over the body by good restraints. Stapp then spent years arguing for seat belts in automobiles, since he realized that far

more people died on highways than in aircraft, and he helped start an annual conference on car-crash research that still meets. A side note: an engineer on his team, Edward Murphy, is the source of the saying now called Murphy's law.

The physics. Velocity is the rate of change of position, with direction; **acceleration** is the rate of change of velocity. Average acceleration is $a = \Delta v / \Delta t$. When acceleration is constant, four equations connect the five quantities v_0 (initial velocity), v (final velocity), a, t and Δx. Each equation leaves out one quantity, so choose the one that leaves out the quantity you neither know nor need. Stapp's sled: speeding up, $a = \dfrac{282\ \mathrm{m/s} - 0}{5.0\ \mathrm{s}} = 56\ \mathrm{m/s^2} \approx 5.8\,g$ on average; stopping, $a = \dfrac{0 - 282\ \mathrm{m/s}}{1.4\ \mathrm{s}} = -200\ \mathrm{m/s^2}$, about $-20\,g$ on average, with a short peak of 46 g. Signs matter: pick a positive direction and keep it.

The engineering. Highway engineers design for stopping. A driver needs about 1.5 seconds to see a hazard and press the brake, and during that time the car does not slow at all. Then the brakes give a deceleration of about 7 m/s² on dry pavement, less on wet. The total stopping distance, reaction plus braking, sets the "stopping sight distance" that determines how far ahead a driver must be able to see over a hill or around a curve, the length of yellow lights, and the spacing of warning signs. Because braking distance grows with v^2, doubling your speed quadruples it.

Try it. In the lab you will measure the deceleration of a ball or toy car rolling to a stop on carpet, using two different equations, and see whether they agree.

PICTURE STUDY

Stapp on the Rocket Sled, U.S. Air Force (NASA Glenn archive), 1956 · Photograph · NASA Glenn Research Center archive

Image: Wikimedia Commons, public domain

Stapp visited NASA's Lewis (now Glenn) laboratory in 1956 to show how the sled and its restraints worked. Look at how the harness is designed to spread the force over the body.

Look closely

- Find the shoulder straps, lap belt and any straps across the legs. Where do they press on the body?
- Find the seat and the structure that holds it. How strong must it be?
- Look for any instruments or cameras attached to the sled.

Talk about it

1. Why does a wide strap make a large deceleration more survivable than a narrow one?
2. If the sled stops in 1.4 s instead of 0.7 s, how does the average deceleration change?
3. What connection do you see between this harness and the seat belt in a car?

Safety first. Roll the ball or car only on the floor; keep the area clear so no one steps on it.

LAB

Rolling to a Stop: Measuring Deceleration Two Ways

A ball rolling across carpet slows down at a nearly constant rate. You will measure its starting speed, its stopping distance and its stopping time, and find the deceleration from two different equations.

You will need

- A ball (tennis or rubber) or a toy car
- A short ramp (a book propped on another book) to launch it
- Carpet or a rug at least 3 m long
- Tape measure, masking tape
- Phone with slow-motion video

Steps

1. Set the ramp at the edge of the carpet. Put two tape marks 20 cm apart on the carpet just past the bottom of the ramp, and a meter stick along the path for the video.
2. **Hypothesis.** If you double the launch speed, predict how the stopping distance changes, and why.
3. Release the ball from a fixed point on the ramp and film the whole roll from the side. Repeat 5 times.
4. From the video: find the launch speed v_0 = 0.20 m ÷ (time between the two marks); the stopping distance d measured from the second mark; and the stopping time t from the second mark to the stop.
5. Find the deceleration two ways: $a_1 = v_0^2/(2d)$ and $a_2 = v_0/t$. Compare them; what assumption must be true for them to agree?
6. **Fair test.** Release from two other heights on the ramp to change v_0. Graph stopping distance d against v_0^2. Is it a straight line? Its slope is $1/(2a)$.
7. Compare surfaces: carpet, a rug, a bare floor (if it stops within the room). Which has the largest deceleration?
8. **Improve.** If a_1 and a_2 disagree by more than 10%, find out why (the ball still speeding up at the first mark, a slope in the floor, timing). Change one thing

and retest.

Tips

- The ball should roll, not bounce, onto the carpet; a gentle ramp works best.
- Use the same release point for each set of trials.

Check your work

- ☐ I found the deceleration by two methods and explained why they agree or disagree.
- ☐ My graph of d against v_0^2 shows how stopping distance depends on speed.
- ☐ I can use the four constant-acceleration equations and choose the right one.

LAB NOTEBOOK

1. Data table for all trials, with both calculated decelerations and their percent difference.
2. Graph of stopping distance against v_0^2 with a best-fit line, and the deceleration found from its slope.
3. A velocity–time sketch of a car stopping: flat during the reaction time, then a straight line down to zero. Shade the area under each part and label them "reaction distance" and "braking distance."

KEY FORMULAS

$$v = v_0 + at$$

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a\Delta x$$

$$\Delta x = \tfrac{1}{2}(v_0 + v)t$$

From history. Stapp's research helped lead to the 1966 U.S. law requiring seat belts in new cars, and he attended its signing. He received the National Medal of Science in 1991. His sled tests also improved the design of aircraft seats and harnesses that have saved many pilots.

Example 1. *Sonic Wind No. 1* went from rest to 282 m/s in 5.0 s, then stopped from 282 m/s in 1.4 s. Assuming constant acceleration in each phase, find each acceleration (in m/s^2 and in g) and the distance covered while stopping.

Show solution

1. Speeding up: $a = \dfrac{\Delta v}{\Delta t} = \dfrac{282\ \mathrm{m/s} - 0}{5.0\ \mathrm{s}} = 56\ \mathrm{m/s^2}$; in g: $\dfrac{56}{9.8} = 5.8\,g$.
2. Stopping: $a = \dfrac{0 - 282\ \mathrm{m/s}}{1.4\ \mathrm{s}} = -2.0 \times 10^2\ \mathrm{m/s^2}$; in g: $\dfrac{201}{9.8} = 21\,g$ (average; the peak was higher).
3. Stopping distance: $\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(282\ \mathrm{m/s} + 0)(1.4\ \mathrm{s}) = 197\ \mathrm{m}$.

Answer: $56\ \mathrm{m/s^2}$ ($5.8\,g$) speeding up; $-2.0 \times 10^2\ \mathrm{m/s^2}$ (about $21\,g$ average) stopping, over about 200 m.

Example 2. A car travels at 25 m/s (about 56 mi/h). The driver's reaction time is 1.5 s, and the brakes then decelerate the car at $7.0\ \mathrm{m/s^2}$. Find the total stopping distance.

Show solution

1. Reaction distance (constant speed): $d_1 = vt = (25\ \mathrm{m/s})(1.5\ \mathrm{s}) = 37.5\ \mathrm{m}$.
2. Braking distance: use $v^2 = v_0^2 + 2a\Delta x$ with $v = 0$, $a = -7.0\ \mathrm{m/s^2}$:
   $\Delta x = \dfrac{-v_0^2}{2a} = \dfrac{-(25\ \mathrm{m/s})^2}{2(-7.0\ \mathrm{m/s^2})} = 44.6\ \mathrm{m}$.
3. Total: $37.5\ \mathrm{m} + 44.6\ \mathrm{m} = 82\ \mathrm{m}$.

Answer: About 82 m, nearly half of it before the brakes even act.

Example 3. A runner accelerates from $2.0\ \mathrm{m/s}$ to $8.0\ \mathrm{m/s}$ over $15\ \mathrm{m}$. Find her acceleration and the time taken.

Show solution

1. Known: $v_0 = 2.0\ \mathrm{m/s}$, $v = 8.0\ \mathrm{m/s}$, $\Delta x = 15\ \mathrm{m}$; time is unknown, so use $v^2 = v_0^2 + 2a\Delta x$.
2. $a = \dfrac{v^2 - v_0^2}{2\Delta x} = \dfrac{(8.0)^2 - (2.0)^2}{2(15)}\ \mathrm{m/s^2} = \dfrac{60}{30}\ \mathrm{m/s^2} = 2.0\ \mathrm{m/s^2}$.
3. Time: $t = \dfrac{v - v_0}{a} = \dfrac{6.0\ \mathrm{m/s}}{2.0\ \mathrm{m/s^2}} = 3.0\ \mathrm{s}$.

Answer: $a = 2.0\ \mathrm{m/s^2}$, $t = 3.0\ \mathrm{s}$.

PRACTICE

1. Easy A car goes from 0 to $27\ \mathrm{m/s}$ in $9.0\ \mathrm{s}$. What is its average acceleration?

   Show answer

   $3.0\ \mathrm{m/s^2}$

2. Easy A runner goes once around a $400\ \mathrm{m}$ track in $80\ \mathrm{s}$ and ends where she started. Find her average speed and her average velocity.

   Show answer

   Average speed $5.0\ \mathrm{m/s}$; average velocity 0 (zero displacement).

3. Easy Express an acceleration of $29.4\ \mathrm{m/s^2}$ in g.

   Show answer

   $3.0\,g$

4. Medium A train moving at $30\ \mathrm{m/s}$ brakes at $1.0\ \mathrm{m/s^2}$. How long does it take to stop, and how far does it go?

   Show answer

   $t = 30\ \mathrm{s}$; $\Delta x = 30^2/2 = 450\ \mathrm{m}$

5. Medium A bicycle at $4.0\ \mathrm{m/s}$ accelerates at $0.50\ \mathrm{m/s^2}$ for $6.0\ \mathrm{s}$. Find its final speed and the distance covered.

   Show answer

   $v = 7.0\ \mathrm{m/s}$; $\Delta x = 4.0(6.0) + \frac{1}{2}(0.50)(36) = 33\ \mathrm{m}$

6. Medium An airliner needs a speed of $75\ \mathrm{m/s}$ to take off and accelerates at $2.5\ \mathrm{m/s^2}$. What is the minimum runway length?

Show answer

$$\Delta x = 75^2/(2 \times 2.5) = 1125\ \mathrm{m} \approx 1.1\ \mathrm{km}$$

7. Medium Using the car in Worked Example 2, find the total stopping distance at $35\ \mathrm{m/s}$. By what factor did the braking distance grow compared with $25\ \mathrm{m/s}$?

Show answer

Reaction $52.5\ \mathrm{m}$; braking $35^2/14 = 87.5\ \mathrm{m}$; total $140\ \mathrm{m}$. Braking distance grew by $(35/25)^2 = 1.96$.

8. Medium Can an object have zero velocity and nonzero acceleration at the same moment? Give an example.

Show answer

Yes. A ball thrown straight up has zero velocity at the top of its path, but its acceleration is still $9.8\ \mathrm{m/s^2}$ downward.

9. Hard Stapp's 1954 sled stopped from $282\ \mathrm{m/s}$ with a peak of $46.2\,g$. If the whole stop had been at $46.2\,g$, how long would it have taken and how far would the sled have gone?

Show answer

$$a = 46.2 \times 9.8 = 453\ \mathrm{m/s^2};\ t = 282/453 = 0.62\ \mathrm{s};\ \Delta x = 282^2/(2 \times 453) = 88\ \mathrm{m}$$

10. Hard A driver at $20\ \mathrm{m/s}$ sees a deer $50\ \mathrm{m}$ ahead. Her reaction time is $1.2\ \mathrm{s}$ and her brakes give $6.5\ \mathrm{m/s^2}$. Does she stop in time? By how much?

Show answer

Reaction $24\ \mathrm{m}$; braking $400/13 = 30.8\ \mathrm{m}$; total $54.8\ \mathrm{m}$. She does not stop in time: she reaches the deer's position still moving, about $4.8\ \mathrm{m}$ short of a full stop.

11. Challenge A car accelerates from rest at $3.0\ \mathrm{m/s^2}$. At the same moment, a truck passes it moving at a steady $15\ \mathrm{m/s}$. When and where does the car catch the truck, and how fast is the car going then?

Show answer

Set $\frac{1}{2}(3.0)t^2 = 15t$: $t = 10\ \mathrm{s}$, at $150\ \mathrm{m}$; the car is going $30\ \mathrm{m/s}$.

COMMON MISTAKES

- Forgetting that “stops” means final velocity zero, and that a slowing object has acceleration opposite its velocity (a negative sign if the velocity is positive).
- Leaving out the reaction distance in stopping problems.
- Mixing km/h or mi/h with m/s^2 in the same equation. Convert speeds to m/s first.

> **Make it a game · Pick the Equation.** Your parent reads a one-sentence motion problem, with three knowns and one unknown. In 20 seconds, name which of the four constant-acceleration equations to use and why, before solving. A wrong choice costs a point. Race through six problems, then find your personal best score.

2.3 Free Fall: From Pisa to the Hammer and the Feather

OBJECTIVES

- State that, without air resistance, all objects near the Earth's surface fall with the same acceleration, $g = 9.8\ \mathrm{m/s^2}$ downward.
- Solve free-fall problems for objects dropped and thrown upward, using a consistent sign convention.
- Describe the effect of air resistance and the idea of terminal velocity.
- Measure g from a graph of drop height against time squared.

> **Warm-up.** Toss a ball straight up and catch it. At the very top, for an instant, it is not moving. Write down your answer to this question before reading: at that instant, is the ball accelerating? If so, which way? If not, why does it start to come down?

WATCH

Introduction to Free-Fall and the Acceleration due to Gravity

Flipping Physics · 12:11

https://www.youtube.com/watch?v=vyvDzI22sOE

> **WHEN TO WATCH**
> After the physics paragraph, before the worked examples.

BACKUP VIDEO

Apollo 15 Hammer-Feather Drop

NASA Solar System · 1:23

https://www.youtube.com/watch?v=oYEgdZ3iEKA

With the story paragraph: watch the actual 1971 film from the Moon.

KEY TERMS

free fall

Motion under the influence of gravity alone, with no significant air resistance. An object in free fall accelerates at g downward whether it is moving up, down or sideways.

acceleration due to gravity

$g = 9.80\ \mathrm{m/s^2}$ near the Earth's surface (it varies slightly with location); on the Moon it is $1.62\ \mathrm{m/s^2}$.

air resistance (drag)

The force of air pushing against a moving object; it grows with speed and with the object's area.

terminal velocity

The steady speed a falling object reaches when air resistance has grown to equal its weight, so it no longer accelerates.

CORE IDEAS

The story. Aristotle taught that heavier things fall faster, in proportion to their weight, and for nearly two thousand years most scholars agreed. A few doubted it: John Philoponus in the sixth century wrote that two very different weights dropped together land at nearly the same time, and in 1586 Simon Stevin and a friend dropped two lead balls, one ten times heavier than the other, from about 30 feet onto a board in Delft and heard them land together. Galileo's student Vincenzo Viviani later wrote that Galileo had dropped balls from the Leaning Tower of Pisa in front of professors and students. Historians are not sure that public test happened, but Galileo's ramp experiments, and his argument in *Two New Sciences*, settled the question.

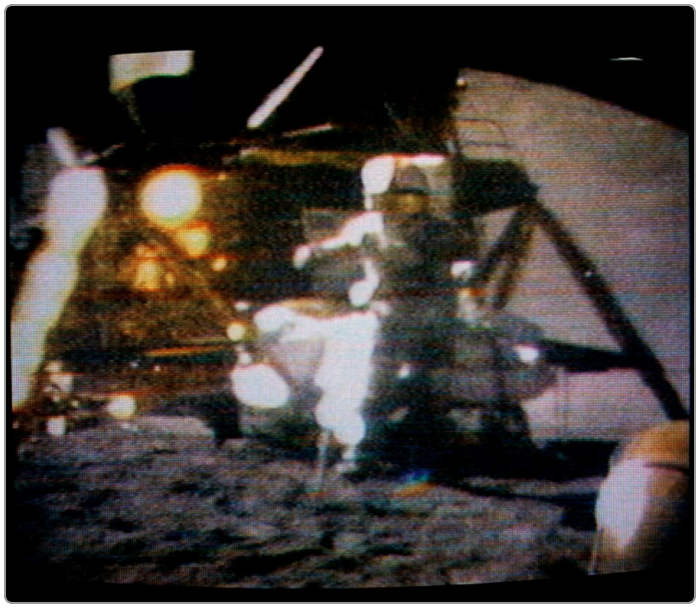
How they figured it out. Galileo argued with a thought experiment. Suppose a heavy stone falls faster than a light one. Tie them together. The light stone should slow the heavy one down, so the pair falls slower than the heavy stone alone; but the pair is heavier than the heavy stone, so it should fall faster. Both cannot be true, so the assumption is wrong: weight does not set the rate of fall. What does make a feather fall slowly is the air. On August 2, 1971, at the end of the third moonwalk of Apollo 15, Commander Dave Scott stood in front of the television camera holding a 1.32 kg geologic hammer in one hand and a falcon feather in the other. With no air on the Moon, he dropped them together. They hit the dust at the same instant. "How about that," he said; "Mr. Galileo was correct."

The physics. Near the Earth's surface, every object in free fall has the same acceleration, $g = 9.8\ \mathrm{m/s^2}$, directed downward. Free fall is just constant-acceleration motion with $a = -g$ if you call up positive. An object thrown upward slows by 9.8 m/s every second, stops for an instant at the top, and speeds up by 9.8 m/s every second on the way down; on level ground the trip up and the trip down take equal times. At the top, the velocity is zero but the acceleration is still g downward. In air, a falling object also feels drag, which grows with speed. When drag equals weight, the acceleration is zero and the object falls at its **terminal velocity**: about 55 m/s for a skydiver spread out, a few meters per second for a raindrop, and less than 1 m/s for a feather.

The engineering. NASA tests how flames, fluids and spacecraft parts behave without gravity by letting them fall. At the Zero Gravity Research Facility at NASA's Glenn Research Center in Ohio, an experiment package is dropped 132 meters down a shaft inside a vacuum chamber, giving 5.2 seconds of free fall, then caught in a container of polystyrene beads. Engineers also design parachutes and drogue chutes by controlling terminal velocity, and they calculate the speed of falling objects to design safety nets, catch fences on construction sites and the energy absorbers at the bottom of elevator shafts.

Try it. In the lab you will measure g by dropping a ball from several heights and linearizing your data, just as you linearized Galileo's ramp.

PICTURE STUDY



The Hammer and the Feather, NASA, August 2, 1971 · Photograph · NASA Johnson Space Center

Image: Wikimedia Commons, public domain

This picture is a frame from the live television broadcast from the Moon. It shows one of the simplest and most famous physics experiments ever performed.

Look closely

- Find the hammer and the feather. Are they at the same height?
- Look at the ground. What do you notice about the lunar surface and the lack of anything that would show wind?
- Find the lunar module or other equipment in the background, if visible.

Talk about it

1. Why would this experiment fail on Earth, even though Galileo was right?
2. If Scott had thrown the hammer up instead of dropping it, would the feather still land at the same time if thrown up at the same speed? Why?
3. What other experiments would be easy on the Moon and hard on Earth?

Safety first. Drop balls only from a stable height, such as a stair landing with an adult present. Never lean over a railing.

LAB

Measuring g with a Falling Ball

You will drop a ball from several heights, time each fall, and find g from the slope of a graph of height against time squared.

You will need

- A small, dense ball (a golf ball, a steel ball, a marble wrapped in tape)
- Tape measure
- A phone with slow-motion video, or the phyphox app's "Acoustic Stopwatch" (it starts on one sound and stops on the next: a click at release, the thump at landing)
- A hard floor; a stair landing or sturdy step stool for the higher drops

Steps

1. **Hypothesis.** Predict the shape of a graph of height h against time t, and of h against t^2.
2. Mark drop heights of 0.50, 1.00, 1.50, 2.00 and (if safe) 2.50 m, measured to the bottom of the ball.
3. For each height, drop the ball from rest 5 times. Time the fall from the video (frames ÷ frame rate) or with the acoustic stopwatch (snap your fingers at release, or let the ball fall from between two fingers you click together).
4. Average the times and compute t^2 for each height.
5. Graph h against t^2. Draw the best-fit line; its slope is $g/2$, so $g = 2 \times$ slope. Find the percent error from $9.80\ \mathrm{m/s^2}$.
6. Also graph h against t, and describe why it curves.
7. **Air resistance test.** Repeat the 2.00 m drop with a ping-pong ball and a crumpled paper ball. Do they fit your line? Calculate the "g" each one seems to have.
8. **Improve.** Identify the main source of error (the moment of release, the frame rate, the measured height). If your line does not pass through the origin, what systematic error does the intercept suggest? Fix it and repeat one height.

Tips

- At 240 frames per second, each frame is about 0.004 s; a 2 m fall is about 0.64 s, about 150 frames.
- A systematic delay at release shows up as a line that crosses the t^2 axis instead of passing through the origin.

Check your work

☐ I measured

g from the slope of

h against

t^2 and found its percent error.

☐ I can explain why the ping-pong ball gave a smaller apparent

g.

☐ I can solve free-fall problems for objects thrown up or dropped.

LAB NOTEBOOK

1. Data table (height, five times, mean, t^2) and the h–t^2 graph with the slope and g worked out.
2. Sketch position, velocity and acceleration against time for a ball thrown straight up and caught. Mark the top of the path on each graph.
3. Write a paragraph explaining Galileo's tied-stones argument in your own words, and why the hammer and feather fell together on the Moon.

KEY FORMULAS

$$v = v_0 - gt$$

$$y = y_0 + v_0 t - \tfrac{1}{2} g t^2$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

$$t = \sqrt{\frac{2h}{g}} \quad \text{(dropped from rest through height } h\text{)}$$

From history. The geologic hammer stayed in the Apollo 15 equipment, but the feather was left on the Moon. It came from a falcon, a nod to the lunar module's name, *Falcon*, and to the falcon mascot of the U.S. Air Force Academy, where Scott and two of his crewmates had ties. Today's best measurements of g use atom interferometers that drop clouds of atoms in a vacuum and are precise to better than one part in a billion.

WORKED EXAMPLES

Example 1. Scott released the hammer and feather from about 1.6 m above the lunar surface, where $g_{\text{Moon}} = 1.62\ \text{m/s}^2$. How long did they take to fall? How long would the hammer take on Earth?

Show solution

1. From rest, $h = \frac{1}{2}gt^2$, so $t = \sqrt{\dfrac{2h}{g}}$.
2. Moon: $t = \sqrt{\dfrac{2(1.6\ \text{m})}{1.62\ \text{m/s}^2}} = \sqrt{1.98\ \text{s}^2} = 1.4\ \text{s}$.
3. Earth: $t = \sqrt{\dfrac{2(1.6\ \text{m})}{9.8\ \text{m/s}^2}} = \sqrt{0.327\ \text{s}^2} = 0.57\ \text{s}$.

Answer: About 1.4 s on the Moon, compared with 0.57 s on Earth. (That is why the film looks like slow motion.)

Example 2. A ball is thrown straight up at 15 m/s. Find (a) the time to reach the top, (b) the maximum height, and (c) the total time until it returns to the hand.

Show solution

1. Take up as positive, $a = -9.8\ \text{m/s}^2$. (a) At the top $v = 0$:
   $t = \dfrac{v_0}{g} = \dfrac{15\ \text{m/s}}{9.8\ \text{m/s}^2} = 1.53\ \text{s}$.
2. (b) $v^2 = v_0^2 - 2gh$ with $v = 0$: $h = \dfrac{v_0^2}{2g} = \dfrac{(15\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 11.5\ \text{m}$.
3. (c) The trip down takes as long as the trip up: $2 \times 1.53\ \text{s} = 3.06\ \text{s}$. Check with $0 = v_0 t - \frac{1}{2}gt^2$: $t = \dfrac{2v_0}{g} = 3.06\ \text{s}$.

Answer: (a) 1.53 s, (b) 11.5 m, (c) 3.06 s.

Example 3. NASA Glenn's drop tower lets an experiment fall 132 m in a vacuum. Find the free-fall time and the speed at the bottom.

Show solution

1. $t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2(132\text{ m})}{9.8\text{ m/s}^2}} = \sqrt{26.9\text{ s}^2} = 5.19\text{ s}.$
2. $v = gt = (9.8\text{ m/s}^2)(5.19\text{ s}) = 50.9\text{ m/s}$, or
   $v = \sqrt{2gh} = \sqrt{2(9.8)(132)}\text{ m/s} = 50.9\text{ m/s}.$

Answer: About 5.2 s of free fall, ending at about 51 m/s (about 114 mi/h).

Example 4. A stone dropped from a bridge hits the water 2.4 s later. How high is the bridge? (Ignore air resistance and the time for the sound to return.)

Show solution

1. $h = \frac{1}{2}gt^2 = \frac{1}{2}(9.8\text{ m/s}^2)(2.4\text{ s})^2 = (4.9\text{ m/s}^2)(5.76\text{ s}^2) = 28\text{ m}.$

Answer: About 28 m.

PRACTICE

1. Easy How fast is a dropped object moving after 3.0 s of free fall?

   Show answer

   $v = gt = 29\text{ m/s}$

2. Easy How far does it fall in those 3.0 s?

   Show answer

   $\frac{1}{2}(9.8)(9.0) = 44\text{ m}$

3. Easy A ball is thrown straight up. What are its velocity and acceleration at the highest point?

   Show answer

   Velocity zero; acceleration 9.8 m/s^2 downward.

4. Medium How long does it take an object to fall 20 m from rest?

   Show answer

   $t = \sqrt{40/9.8} = 2.0\text{ s}$

5. **Medium** A ball is thrown up at $9.8\ \mathrm{m/s}$. How high does it go and when does it return to the starting height?

Show answer

$h = 9.8^2/19.6 = 4.9\ \mathrm{m}$; returns at $2.0\ \mathrm{s}$.

6. **Medium** On the Moon, a ball is thrown up at $9.8\ \mathrm{m/s}$. How high does it go?

Show answer

$h = 9.8^2/(2 \times 1.62) = 29.6\ \mathrm{m}$, six times higher than on Earth.

7. **Medium** A stone is thrown downward from a $30\ \mathrm{m}$ cliff at $5.0\ \mathrm{m/s}$. How fast is it moving when it hits the ground?

Show answer

$v = \sqrt{25 + 2(9.8)(30)} = \sqrt{613} = 24.8\ \mathrm{m/s}$

8. **Medium** Explain why a skydiver stops accelerating after about 12 seconds of fall, even though gravity still acts.

Show answer

Air resistance grows with speed. When it equals the skydiver's weight, the net force is zero, so the acceleration is zero and the skydiver falls at constant terminal velocity.

9. **Hard** A ball is thrown up at $12\ \mathrm{m/s}$ from a $20\ \mathrm{m}$ balcony. How long until it hits the ground below?

Show answer

$-20 = 12t - 4.9t^2$, so $4.9t^2 - 12t - 20 = 0$: $t = \dfrac{12 + \sqrt{144 + 392}}{9.8} = 3.6\ \mathrm{s}$.

10. **Hard** A student's graph of drop height (m) against t^2 (s^2) has a slope of $4.82\ \mathrm{m/s^2}$. What value of g does she find, and what is her percent error?

Show answer

$g = 9.64\ \mathrm{m/s^2}$; percent error 1.6%.

11. Hard A rock dropped into a well is heard to splash 2.0 s after release. Ignoring the sound's travel time, how deep is the well? Is your answer too large or too small, and why?

Show answer

$h = 4.9(4.0) = 19.6$ m. It is slightly too large: part of the 2.0 s was the time for the sound to come back up (about 0.06 s), so the true fall time and depth are a little less.

12. Challenge Two balls are released from the same height, the second 0.50 s after the first. Show that the distance between them grows steadily with time, and find it 1.0 s after the second release.

Show answer

$\Delta y = \frac{1}{2}g(t+0.5)^2 - \frac{1}{2}gt^2 = g(0.5t + 0.125)$, which grows linearly. At $t = 1.0$ s: $9.8(0.625) = 6.1$ m.

COMMON MISTAKES

- Thinking the acceleration is zero at the top of a throw. Only the velocity is zero.
- Mixing signs: using $+9.8$ for g with up positive. Choose a direction and give every vector the right sign.
- Assuming heavier objects fall faster. In air, it is the ratio of weight to drag, not the weight alone, that matters.

Make it a game · Drop Time Estimation Duel. Pick three heights in your home, such as a table, a counter and a stairway railing. Both of you predict the fall time of a small ball using g = 9.8 m/s2. Measure the height, calculate the time, then check with a phone stopwatch if available. The closest estimate scores a point.

Scripture. "He stretcheth out the north over the empty place, and hangeth the earth upon nothing." (Job 26:7, KJV)

The same gravity that pulls a stone to the ground holds the Earth and the Moon in their places, hung upon nothing that we can see.

2.4 Motion Graphs: Marey's Chronophotographs

OBJECTIVES

- Read and sketch position–time, velocity–time and acceleration–time graphs for the same motion.
- Find velocity from the slope of a position–time graph (including the tangent slope for curved graphs) and displacement from the area under a velocity–time graph.
- Analyze the motion of a pull-back toy car from video data.

> **Warm-up.** Look at a ruler-straight line of footprints in snow or sand (or imagine one): close together where someone walked slowly, far apart where they ran. Sketch a row of dots that shows an object starting slowly, speeding up, then slowing to a stop. That row of dots is the idea behind this whole lesson.

WATCH

Position vs. time graphs | One-dimensional motion | Physics | Khan Academy

Khan Academy Physics · 15:19

https://www.youtube.com/watch?v=GtoamALPOP0

> **WHEN TO WATCH**
> After the physics paragraph. Work the graph examples with the video paused.

BACKUP VIDEO

How to Match Motion Graphs in Physics

How To Physics · 12:46

https://www.youtube.com/watch?v=lGlzeytpiJ0

For extra practice turning one kind of motion graph into another.

KEY TERMS

position–time graph

A graph of position against time. Its slope at any point is the velocity.

velocity–time graph

A graph of velocity against time. Its slope is the acceleration, and the area between the line and the time axis is the displacement.

instantaneous velocity

The velocity at a single instant: the slope of the tangent line to the position–time curve at that time.

tangent line

A straight line that just touches a curve at one point and has the same steepness as the curve there.

chronophotograph

Marey's term for a photograph that records a moving object several times on the same plate at equal time intervals.

CORE IDEAS

The story. Étienne-Jules Marey (1830–1904) was a French physician and physiologist who wanted to understand motion: how a heart beats, how a horse trots, how a bird beats its wings. He built recording instruments that traced motion onto smoked paper, and then in 1882 he turned to photography. Marey designed a camera that opened its shutter many times a second, recording a moving subject again and again on a single glass plate against a black background. He called the results *chronophotographs*, "time photographs." He photographed a man walking and running, a pelican in flight, a jumping cat, and even a bouncing ball, its successive images strung out along its arching path.

How they figured it out. Marey realized that a chronophotograph is a graph. Because the images are taken at equal time intervals, the spacing between them shows the speed: wide gaps mean fast motion, crowded images mean slow motion. To make the data even clearer, he dressed his runners in black suits with thin white stripes and metal buttons along the arms and legs, so that only lines and dots showed up on the plate, and then he measured the positions and drew them as curves. His method of measuring motion frame by frame is the ancestor of motion capture, the sports-science and engineering method used today with high-speed cameras and markers.

The physics. The three motion graphs tell one story. On a **position–time graph**, the slope is the velocity: a straight line means constant velocity, a curve getting steeper means speeding up, a flat line means at rest. For a curved graph, the velocity at an instant is the slope of the tangent line there. On a **velocity–time graph**, the slope is the acceleration, and the area under the graph is the displacement (area below the time axis counts as negative displacement). On an **acceleration–time graph**,

constant acceleration is a flat line, and the area under it is the change in velocity. For constant acceleration, the x–t graph is a parabola, the v–t graph is a straight line, and the a–t graph is flat.

The engineering. Every car built today records motion graphs. The event data recorder in an air-bag module saves velocity against time for the few seconds around a crash, and accident investigators read the slope and area to find braking and impact speeds. Elevator and roller-coaster designers shape their velocity–time graphs to limit acceleration and jerk for comfort. The accelerometer in your phone produces an acceleration–time graph, and phyphox can show it to you live.

Try it. A pull-back toy car makes a wonderful motion-graph lab: it speeds up while its spring unwinds and then coasts to a stop. You will film it, measure its position frame by frame like Marey, and build all three graphs.

PICTURE STUDY

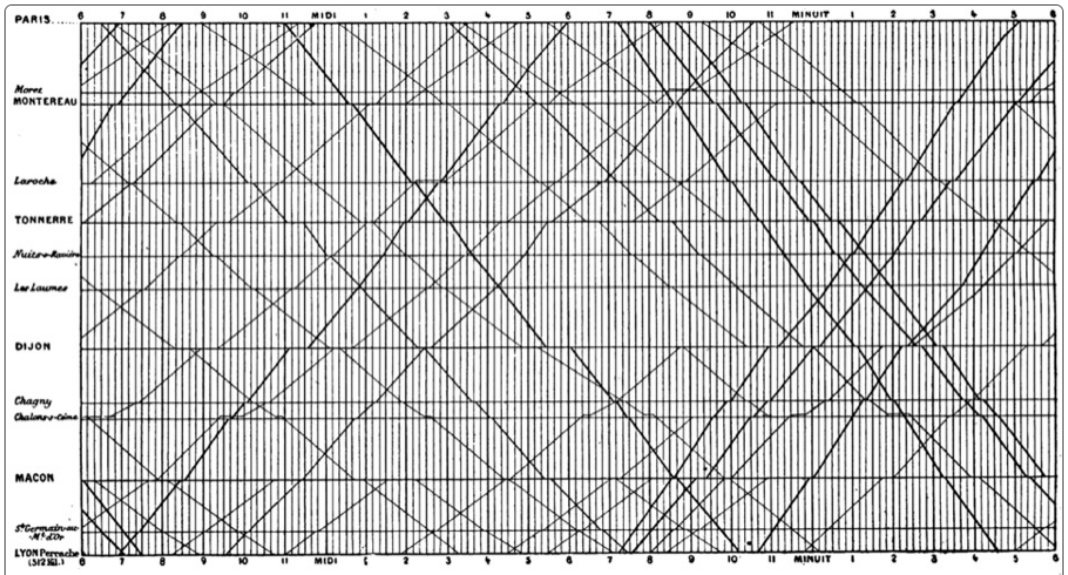


Fig. 7. Graphique de la marche des trains sur un chemin de fer, d'après la méthode de Ibry.

Ibry's Train Schedule, Paris to Lyon, Charles Ibry (reproduced by Étienne-Jules Marey), 1885 · Printed graph · Marey, *La méthode graphique*, Paris

Image: Wikimedia Commons, public domain

Marey reproduced this railway engineer's chart in his book on graphical methods as an example of how a graph can show motion at a glance. Every line is a position–time graph of one train.

Look closely

- Find Paris at the top and Lyon at the bottom. Where is time shown?
- Follow one slanting line. Which way is that train going?
- Find a line with short flat sections. What is the train doing there?
- Find two lines that cross. What happens to the two trains at that point?

Talk about it

1. How can you tell from the chart which trains are fastest?
2. What does the slope of a line represent, and what are its units?
3. Why would railway dispatchers find this chart more useful than a table of times?

LAB

Motion Graphs of a Pull-Back Car

You will film a pull-back toy car beside a meter stick, measure its position at equal time steps, and build its position–time, velocity–time and acceleration–time graphs.

You will need

- A pull-back toy car
- A smooth floor or table at least 2 m long
- Meter stick or tape measure laid along the path
- Phone with slow-motion video (or a free video-analysis app such as Tracker or phyphox)
- Graph paper or a spreadsheet

Steps

1. Lay the tape measure flat beside the path. Mark a start line. Set the phone on a stack of books, 1–2 m away, looking straight across at the path.
2. **Hypothesis.** Sketch what you expect the x–t and v–t graphs to look like.
3. Pull the car back a fixed distance (count the clicks or measure), set it at the start line, release it and film until it stops. Repeat 3 times with the same pull-back.
4. Step through the best video and record the car's front position every 0.10 s (every 24 frames at 240 fps; every 12 at 120 fps) until it stops.
5. Graph x against t. Then find the average velocity in each interval, $\Delta x/\Delta t$, and graph it at the middle of each interval to get v against t.
6. From the slopes of the v–t graph, find the acceleration during the push phase and the deceleration during the coasting phase. Sketch the a–t graph.
7. Check: find the area under your v–t graph and compare it with the total distance the car actually traveled.
8. **Fair test.** Change only the pull-back distance (for example, 5 cm and 10 cm). How do the peak velocity and the stopping distance change?

9. **Improve.** Identify the largest uncertainty in your positions (parallax, blur) and reduce it, for example by putting a bright tape mark on the car.

Tips

- The camera must not move or zoom during the video.
- Parallax: the tape measure should be as close as possible to the car's path so their positions line up in the image.

Check your work

☐ I made all three motion graphs from my own video data.

☐ I found acceleration from the slope of my

v –

t graph and distance from its area.

☐ I can explain how the shape of the

x –

t graph changes when the car speeds up, coasts and stops.

LAB NOTEBOOK

1. Position table and all three graphs, aligned one above another with the same time axis.
2. Mark on your x–t graph the moment of greatest velocity, and draw the tangent line there; compare its slope with your v–t peak.
3. Sketch the three graphs for an elevator ride from the ground floor up to the fifth floor, starting and ending at rest.

KEY FORMULAS

$$v = \text{slope of } x\text{–}t = \frac{\Delta x}{\Delta t}$$

$$a = \text{slope of } v\text{–}t = \frac{\Delta v}{\Delta t}$$

$$\Delta x = \text{area under } v\text{–}t$$

From history. Marey's work in Paris and Eadweard Muybridge's in California inspired each other; Muybridge visited Marey in 1881. Marey's camera designs led directly to the motion-picture cameras of the 1890s. He also studied air flow by photographing lines of smoke streaming past objects, one of the first smoke wind tunnels.

WORKED EXAMPLES

Example 1. A car's velocity–time graph rises in a straight line from 0 to 12 m/s during the first 4.0 s, stays flat at 12 m/s until $t = 10.0$ s, then falls in a straight line to 0 at $t = 13.0$ s. Find the acceleration in each phase and the total distance.

Show solution

1. Phase 1: $a = \dfrac{12\ \text{m/s}}{4.0\ \text{s}} = 3.0\ \text{m/s}^2$; area $= \frac{1}{2}(4.0\ \text{s})(12\ \text{m/s}) = 24\ \text{m}$.
2. Phase 2: $a = 0$; area $= (6.0\ \text{s})(12\ \text{m/s}) = 72\ \text{m}$.
3. Phase 3: $a = \dfrac{0 - 12\ \text{m/s}}{3.0\ \text{s}} = -4.0\ \text{m/s}^2$; area $= \frac{1}{2}(3.0\ \text{s})(12\ \text{m/s}) = 18\ \text{m}$.
4. Total distance: $24 + 72 + 18 = 114\ \text{m}$.

Answer: 3.0, 0 and $-4.0\ \text{m/s}^2$; total distance 114 m.

Example 2. Video of a pull-back car gives these positions: $t = 0$: 0 cm; 0.20 s: 4 cm; 0.40 s: 16 cm; 0.60 s: 36 cm; 0.80 s: 58 cm; 1.00 s: 76 cm. Find the average velocity in each interval and describe the motion.

Show solution

1. Average velocity for each 0.20 s interval: $\dfrac{4}{0.20} = 20$, $\dfrac{12}{0.20} = 60$, $\dfrac{20}{0.20} = 100$, $\dfrac{22}{0.20} = 110$, $\dfrac{18}{0.20} = 90$ cm/s.
2. Place each value at the middle of its interval ($t = 0.10, 0.30, 0.50, 0.70, 0.90$ s) to build a velocity–time graph.
3. From 0.10 to 0.50 s the velocity rises by 80 cm/s in 0.40 s: $a = 200\ \text{cm/s}^2 = 2.0\ \text{m/s}^2$ while the spring pushes. After about 0.6 s the velocity peaks and then begins to fall as the car coasts.

Answer: The car speeds up at about $2.0\ \text{m/s}^2$ for roughly the first 0.6 s, reaches about 1.1 m/s, and then begins to slow down.

Example 3. A ball is thrown straight up at 10 m/s and caught at the same height. Describe its x–t, v–t and a–t graphs, with key numbers.

Show solution

1. x–t: a downward-opening parabola, rising to a peak of $\frac{v_0^2}{2g} = \frac{100}{19.6} = 5.1$ m at $t = \frac{10}{9.8} = 1.02$ s, returning to zero at 2.04 s.
2. v–t: a straight line from $+10$ m/s to -10 m/s, crossing zero at 1.02 s; its slope is $-9.8\ \mathrm{m/s^2}$.
3. a–t: a flat line at $-9.8\ \mathrm{m/s^2}$ for the whole flight.
4. Check: the area under the v–t line from 0 to 1.02 s is $\frac{1}{2}(1.02)(10) = 5.1$ m, the height of the peak.

Answer: Parabola peaking at 5.1 m; straight line of slope $-9.8\ \mathrm{m/s^2}$ crossing zero at 1.02 s; constant $a = -9.8\ \mathrm{m/s^2}$.

PRACTICE

1. Easy On a position–time graph, what does a horizontal line mean? A straight line sloping upward?

   Show answer

   Horizontal: at rest. Sloping straight up: moving at constant positive velocity.

2. Easy A v–t graph is a horizontal line at 6.0 m/s from $t = 0$ to $t = 5.0$ s. Find the displacement.

   Show answer

   30 m

3. Easy A v–t graph rises from 2 m/s to 10 m/s in 4 s. Find the acceleration.

   Show answer

   $2\ \mathrm{m/s^2}$

4. Medium Find the displacement for the graph in the previous problem.

   Show answer

   Trapezoid: $\frac{1}{2}(2 + 10)(4) = 24$ m

5. Medium A position–time graph passes through $(1.0\ \text{s}, 3.0\ \text{m})$ and $(4.0\ \text{s}, 12.0\ \text{m})$ in a straight line. What is the velocity?

Show answer

$9.0/3.0 = 3.0\ \text{m/s}$

6. Medium Sketch (describe) the x–t graph of a car that moves forward at constant speed, stops for a while, then backs up at a slower constant speed.

Show answer

A straight line sloping up, then a horizontal segment, then a straight line sloping down less steeply than the first.

7. Medium A velocity–time graph goes from $+8\ \text{m/s}$ to $-4\ \text{m/s}$ in a straight line over $6\ \text{s}$. Find the acceleration, the time when the object is momentarily at rest, and the displacement.

Show answer

$a = -2\ \text{m/s}^2$; at rest at $t = 4\ \text{s}$; displacement
$= \frac{1}{2}(4)(8) - \frac{1}{2}(2)(4) = 16 - 4 = 12\ \text{m}$.

8. Medium Marey's bouncing-ball chronophotograph shows images crowded together at the tops of the bounces and spread apart near the floor. Explain why.

Show answer

The images are equally spaced in time. Near the tops the ball is moving slowly, so it moves only a short distance between exposures; near the floor it is moving fastest, so the images are far apart.

9. Hard An elevator accelerates upward at $1.0\ \text{m/s}^2$ for $2.0\ \text{s}$, moves at constant speed for $8.0\ \text{s}$, and slows at $1.0\ \text{m/s}^2$ for $2.0\ \text{s}$. How far does it rise?

Show answer

Top speed $2.0\ \text{m/s}$. Areas: $2.0 + 16 + 2.0 = 20\ \text{m}$.

10. Hard On a curved position–time graph, the tangent line at $t = 2.0\ \text{s}$ passes through $(1.0\ \text{s}, 0.5\ \text{m})$ and $(3.0\ \text{s}, 4.5\ \text{m})$. What is the instantaneous velocity at $2.0\ \text{s}$?

Show answer

$$\frac{4.0\ \text{m}}{2.0\ \text{s}} = 2.0\ \text{m/s}$$

11. **Challenge** A car's a–t graph shows $+2.0\ \mathrm{m/s^2}$ for 5 s, then 0 for 5 s, then $-1.0\ \mathrm{m/s^2}$ for 10 s. If it starts from rest, sketch the v–t graph and find the total distance.

Show answer

v rises to $10\ \mathrm{m/s}$ at 5 s, stays at 10 until 10 s, falls to 0 at 20 s. Distance $= 25 + 50 + 50 = 125\ \mathrm{m}$.

COMMON MISTAKES

- Reading a position–time graph as a picture of the path (thinking a line going up means climbing a hill).
- Using the height of a v–t graph for the distance instead of the area under it.
- Placing an interval's average velocity at the end of the interval instead of its middle.

> **Make it a game · Graph Charades.** Your parent walks across the room in a pattern: steady, speeding up, stopping, reversing. You sketch position-time and velocity-time graphs from watching, in 30 seconds. Then swap: you walk a hidden pattern and your parent graphs it. Score one point for each graph that matches the real motion.

2.5 Projectiles: Galileo's Parabola

OBJECTIVES

- Explain why the horizontal and vertical motions of a projectile are independent.
- Solve problems for horizontally launched projectiles and for projectiles launched at an angle.
- Use the range equation for level ground and explain why 45° gives the greatest range without air resistance.
- Predict where a toy car launched off a table will land, and test the prediction.

> **Warm-up.** Hold one coin at the edge of a table and a second coin flat on the table behind it. Flick the second coin so that it knocks the first straight off the edge and flies off itself. Listen. Do the two coins hit the floor at the same moment? Predict before you try it, then try it five times.

WATCH

Introduction to Projectile Motion

Flipping Physics · 6:57

https://www.youtube.com/watch?v=GiiWsXtt5GE

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Horizontally launched projectile | Two-dimensional motion | Physics | Khan Academy

Khan Academy Physics · 12:26

https://www.youtube.com/watch?v=jmSWImPs6fQ

Before the lab, for horizontally launched projectiles step by step.

KEY TERMS

projectile

An object moving through the air under gravity alone after it has been launched.

trajectory

The path of a projectile. Without air resistance it is a parabola.

independence of motion

The horizontal motion (constant velocity) and the vertical motion (constant acceleration g downward) of a projectile happen at the same time without affecting each other.

launch angle

The angle above the horizontal at which a projectile is launched.

range

The horizontal distance a projectile travels before landing.

CORE IDEAS

The story. Before Galileo, writers on motion pictured a thrown object traveling in a straight line until its "impetus" ran out and then falling straight down, joined by a curve. Around 1608 Galileo tested this on a table. He let a ball roll down an inclined ramp from different heights, so that it left the table edge horizontally at different speeds, and he measured where it landed on the floor. A page of his notes, folio 116v, preserved in Florence, shows the curved paths he drew and the distances he measured, in his units called *punti*.

How they figured it out. Galileo combined two things he already knew. A ball rolling on a level table keeps moving at a steady speed (his idea of inertia); a falling ball covers distances that grow as the square of the time. If the ball does both at once, after 1, 2, 3 time units it has moved 1, 2, 3 units sideways and 1, 4, 9 units down. Those points lie on a **parabola**, the curve that the Greek geometer Apollonius had studied two thousand years earlier. In *Two New Sciences* Galileo proved that a projectile's path is a parabola, and that, on level ground, a launch at 45° gives the farthest range, with angles equally above and below 45° (say 30° and 60°) giving equal ranges.

The physics. Treat the horizontal and vertical motions separately; the only thing they share is the time. Horizontally there is no acceleration (ignoring air): $x = v_{0x}t$. Vertically the acceleration is g downward: $y = v_{0y}t - \frac{1}{2}gt^2$. For a launch at speed v_0 and angle θ, the components are $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$. For a horizontal launch from height h, the fall time is $t = \sqrt{2h/g}$, the same as if the object were

simply dropped, and the landing distance is $x = v_0 t$. On level ground the range is $R = \dfrac{v_0^2 \sin 2\theta}{g}$, largest when $2\theta = 90^\circ$.

The engineering. Fountain designers calculate the angle and speed of each water jet so the arcs land exactly in the basin. Fire engineers know that a hose stream reaches farthest at about 30–40° rather than 45°, because air resistance shortens the far end of the arc. Agricultural sprinklers are set to low angles so wind blows the water less. Ski-jump hills are shaped so that the landing slope matches the jumper's parabola, keeping the landing gentle. And every basketball shot is a projectile problem solved by practice.

Try it. In the lab you will launch a Hot Wheels car horizontally off a table, measure its launch speed, predict where it will land, place a target there and see how close you come.

PICTURE STUDY

The Fountains of Peterhof, Detroit Publishing Company, c. 1890–1900 · Photochrom print · Library of Congress

Image: Wikimedia Commons, public domain

The fountains of Peterhof, the summer palace built by Tsar Peter the Great near St. Petersburg in the early 1700s, run by gravity alone: water flows from ponds on higher ground through pipes to the nozzles. Every jet is a stream of projectiles.

Look closely

- Find the tallest jets. Where does the water slow down most?
- Look at how the jets spread or bend at the top. What is pushing them?
- Find the fountains at the far end near the palace. Are they shaped differently?

Talk about it

1. If a vertical jet rises 10 m, how fast must the water leave the nozzle? (Use $v = \sqrt{2gh}$.)
2. If a nozzle were tilted to 45°, what shape would its jet make, and where would it land?
3. Why does the water at the top of a jet look like it hangs in the air?

> **Safety first.** Launch the car only toward a clear floor area; wear safety glasses if anyone is close to the landing zone.

LAB

Hit the Target: A Hot Wheels Car off the Table

You will measure a toy car's launch speed, predict its landing point using projectile physics, put a target there, and launch. Then you will find out why your prediction missed, if it did, and improve it.

You will need

- Hot Wheels car and orange track with a ramp down to a flat section that ends at the table edge
- A table at least 0.7 m high
- Tape measure, masking tape
- A sheet of paper with a target drawn on it (or a paper cup laid on its side), and carbon paper or a little flour to mark the landing
- Phone with slow-motion video

Steps

1. Set up the track so the car runs down a ramp and then along at least 30 cm of flat track to the table edge. Clamp or tape the track firmly.
2. Measure the table height h from the track surface to the floor.
3. Put two tape marks on the flat track 20.0 cm apart, ending at the edge. Release the car from a fixed point and film it crossing the marks. Do 5 trials and find the mean launch speed $v_0 = 0.200\ \mathrm{m}/\Delta t$.
4. **Predict.** Calculate the fall time $t = \sqrt{2h/g}$ and the landing distance $x = v_0 t$, measured from the point on the floor directly below the table edge (use a plumb line: a string with a weight).
5. Tape the target at your predicted distance. Launch the car 5 times from the same release point. Mark and measure each landing.
6. Find the mean landing distance and its spread, and the percent difference from your prediction.
7. **Fair test.** Release from a higher point on the ramp to increase v_0. Measure the new v_0, predict, and test again. Graph landing distance against v_0. Is it a straight line through the origin? What should the slope be?

8. **Improve.** If your landings are consistently long or short, find the cause (the car slowing on the flat track after the marks, the measured height, air drag) and fix it. Then let a parent choose a target distance; set the release point by calculation and try to hit it on the first launch.

Tips

- The car must leave the edge flat, not tipped up or down; a short piece of flat track at the edge matters.
- Measure the height to the car's center when it leaves the edge, not to the tabletop.

Check your work

- ☐ I predicted the landing point from the launch speed and table height before launching.
- ☐ My graph of landing distance against launch speed is a straight line, and I can explain its slope.
- ☐ I can explain why the fall time does not depend on the launch speed.

LAB NOTEBOOK

1. Labeled sketch of the setup with h, v_0 and the predicted and measured x; data table of 5 launches.
2. Graph of landing distance against launch speed with the predicted line (slope $\sqrt{2h/g}$) drawn on the same axes.
3. Draw a parabola made of dots at equal time intervals for a horizontal launch, using horizontal steps of 1, 2, 3, 4 units and vertical drops of 1, 4, 9, 16. Explain how it combines Galileo's two discoveries.

KEY FORMULAS

$$x = v_0 \cos\theta\, t, \qquad y = v_0 \sin\theta\, t - \tfrac{1}{2} g t^2$$

$$t_{\text{fall}} = \sqrt{\frac{2h}{g}} \quad \text{(horizontal launch)}$$

$$R = \frac{v_0^2 \sin 2\theta}{g} \quad \text{(level ground)}$$

$$H = \frac{(v_0 \sin\theta)^2}{2g}$$

From history. Galileo's folio 116v was not recognized as a record of a real experiment until the historian Stillman Drake studied it in the 1970s. Drake found that Galileo's measured landing distances agreed with the predicted parabola to within a few percent, which showed that Galileo tested his theory by measurement and did not reason only from first principles.

WORKED EXAMPLES

Example 1. A marble rolls off a table 0.90 m high at 1.5 m/s. How long is it in the air, and how far from the table does it land?

Show solution

1. Vertical: it starts with zero vertical velocity, so
$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(0.90\ \mathrm{m})}{9.8\ \mathrm{m/s^2}}} = \sqrt{0.184\ \mathrm{s^2}} = 0.43\ \mathrm{s}.$$
2. Horizontal: $x = v_0 t = (1.5\ \mathrm{m/s})(0.429\ \mathrm{s}) = 0.64\ \mathrm{m}$.

Answer: 0.43 s in the air; it lands 0.64 m from the table.

Example 2. A soccer ball is kicked at 20 m/s at 30° above level ground. Find its time of flight, range and maximum height.

Show solution

1. Components: $v_{0x} = 20\cos 30^\circ = 17.3\ \mathrm{m/s}$; $v_{0y} = 20\sin 30^\circ = 10.0\ \mathrm{m/s}$.
2. Time of flight (returns to $y = 0$): $t = \dfrac{2v_{0y}}{g} = \dfrac{2(10.0\ \mathrm{m/s})}{9.8\ \mathrm{m/s^2}} = 2.04\ \mathrm{s}$.
3. Range: $R = v_{0x}t = (17.3\ \mathrm{m/s})(2.04\ \mathrm{s}) = 35.3\ \mathrm{m}$. Check:
$$\frac{v_0^2 \sin 60^\circ}{g} = \frac{400(0.866)}{9.8} = 35.3\ \mathrm{m}.$$
4. Maximum height: $H = \dfrac{v_{0y}^2}{2g} = \dfrac{(10.0)^2}{19.6}\ \mathrm{m} = 5.1\ \mathrm{m}$.

Answer: 2.04 s; range 35.3 m; maximum height 5.1 m.

Example 3. A fountain jet must land in a pool 3.0 m away at the same height as the nozzle. If the nozzle points at 60°, what speed is needed? What other angle would work with the same speed?

Show solution

1. $R = \dfrac{v_0^2 \sin 2\theta}{g}$, so
   $$v_0 = \sqrt{\frac{gR}{\sin 2\theta}} = \sqrt{\frac{(9.8\ \mathrm{m/s^2})(3.0\ \mathrm{m})}{\sin 120^\circ}} = \sqrt{\frac{29.4}{0.866}}\ \mathrm{m/s} = 5.8\ \mathrm{m/s}.$$
2. $\sin 2\theta$ is the same for $\theta = 60^\circ$ and $\theta = 30^\circ$ ($\sin 120^\circ = \sin 60^\circ$), so a 30° jet at the same speed lands at the same place.

Answer: 5.8 m/s; a 30° jet would land in the same spot, but on a lower, faster arc.

PRACTICE

1. Easy A ball rolls off a 1.25 m high table. How long does it take to reach the floor?

   Show answer

   $t = \sqrt{2.5/9.8} = 0.51\ \mathrm{s}$

2. Easy In the warm-up, does the coin knocked off with a sideways speed land before, after or at the same time as the coin that simply drops? Why?

   Show answer

   At the same time (from the same height). The sideways motion does not change the vertical motion.

3. Easy Find the horizontal and vertical components of a launch at 15 m/s and 40°.

   Show answer

   11.5 m/s and 9.6 m/s

4. Medium A stone is thrown horizontally at 8.0 m/s from a 20 m cliff. How far from the base does it land, and how fast is it moving vertically when it lands?

   Show answer

   $t = 2.02$ s; $x = 16.2$ m; $v_y = 19.8$ m/s downward

5. Medium Find the range of a ball kicked at 25 m/s at 45° on level ground.

Show answer

$R = 625/9.8 = 63.8$ m

6. Medium A ball is launched at 18 m/s at 60°. How high does it go?

Show answer

$H = (18 \sin 60^\circ)^2/19.6 = 12.4$ m

7. Medium Two projectiles are launched at the same speed, one at 25° and one at 65°. Which goes farther on level ground? Which stays in the air longer?

Show answer

They have the same range (25° + 65° = 90°). The 65° projectile stays up longer and goes higher.

8. Medium In Galileo's notes, a ball left the table with one speed and landed a certain distance away. If the speed is doubled and the table height is the same, how does the landing distance change? If instead the height is quadrupled?

Show answer

Doubling the speed doubles the distance. Quadrupling the height doubles the fall time, so it also doubles the distance.

9. Hard A Hot Wheels car leaves a 0.80 m table and lands 0.96 m from the edge. What was its launch speed?

Show answer

$t = \sqrt{1.6/9.8} = 0.404$ s; $v_0 = 0.96/0.404 = 2.4$ m/s

10. Hard A basketball is shot from 2.0 m high at 8.0 m/s and 50°. Is it above or below the 3.05 m rim when it has traveled 4.0 m horizontally? By how much?

Show answer

$t = 4.0/(8.0 \cos 50^\circ) = 0.778$ s;
$y = 2.0 + 8.0 \sin 50^\circ (0.778) - 4.9(0.778)^2 = 3.80$ m, about 0.75 m above the rim.

11. **Challenge** Show that the range on level ground, $R = \dfrac{v_0^2 \sin 2\theta}{g}$, follows from the separate horizontal and vertical motions. (Use $\sin 2\theta = 2 \sin\theta \cos\theta$.)

Show answer

Time of flight $t = 2v_0 \sin\theta / g$. Then
$R = v_0 \cos\theta \cdot t = 2v_0^2 \sin\theta \cos\theta / g = v_0^2 \sin 2\theta / g$.

COMMON MISTAKES

- Using the launch speed v_0 in the vertical equations instead of its vertical component $v_0 \sin\theta$.
- Thinking a faster horizontal launch makes the object stay up longer. The fall time depends only on the height.
- Using the range formula when the landing point is higher or lower than the launch point. It works only on level ground.

> **Make it a game · Table Edge Target.** Roll a small ball off a table edge. Measure the table height and horizontal speed with a ruler and timed roll, then predict the landing spot using independent motions. Place a paper target there and test. Score by distance from the target in centimeters, and try to tighten your miss over three rounds.

UNIT 2

Motion in One and Two Dimensions

Kinematics is the description of motion: where something is, how fast it is going, and how its speed is changing, without yet asking why. It began with Galileo, who slowed falling down on a ramp, timed it with a water clock and discovered that distance grows as the square of time. In this unit you will follow his ramp, ride with John Paul Stapp on a rocket sled that stopped in 1.4 seconds, drop a hammer and a feather on the Moon with Apollo 15, read motion from Marey's time photographs, and trace Galileo's parabola from a table edge to the floor.

The tools are the four constant-acceleration equations, motion graphs (slopes and areas), and the vectors from Unit 1, now used to split a projectile's motion into horizontal and vertical parts. Toys do much of the lab work: a Hot Wheels car on a ramp and off a table, and a pull-back car for motion graphs. You will also measure g yourself.

PREREQUISITES

- Unit 1: units, significant figures, graphing and slopes, vectors and components
- Algebra: solving quadratic equations with the quadratic formula (used in a few problems)

MASTERY CHECKLIST

☐ I can use

$d =$ for motion from rest and find

$\frac{1}{2}at^2$ acceleration from a linearized graph of

d against

t^2.

☐ I can choose and use the four constant-acceleration equations, with signs, including stopping-distance problems.

☐ I can solve free-fall problems for objects dropped and thrown upward, and explain terminal velocity.

☐ I can read and sketch

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graphs and find velocity from slo

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- ☐ I can solve projectile problems by treating horizontal and vertical motion separately.

OPTIONAL KITS FOR THIS UNIT

- **Hot Wheels track sets**: Ages 4 and up. A basic set of straight orange track, a launcher and a loop is enough for Lessons 2.1, 2.5 and 5.5; extra straight track sections are sold separately and inexpensively.
- **PASCO dynamics carts and tracks**: A school laboratory supplier. A low-friction cart on an aluminum track gives very clean ramp and motion-graph data for Lessons 2.1, 2.4 and 3.3; useful for co-ops or families who want laboratory-grade equipment.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

2.1	**Galileo's Inclined Plane**	3 sessions of 50 minutes
2.2	**Velocity and Acceleration: Stapp's Rocket Sled**	3 sessions of 50 minutes

2.3	**Free Fall: From Pisa to the Hammer and the Feather**	3 sessions of 50 minutes
2.4	**Motion Graphs: Marey's Chronophotographs**	3 sessions of 50 minutes
2.5	**Projectiles: Galileo's Parabola**	3 sessions of 50 minutes

Unit 2 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$ and ignore air resistance unless a problem says otherwise. For every calculation, write the equation, substitute with units, and give the answer with its unit. Answer the essay question in a well-organized paragraph. The test is worth 100 points.

1. **5 pts**
   A cart starts from rest and accelerates uniformly. It covers $0.30\ \mathrm{m}$ in the first second. How far has it gone after $4.0\ \mathrm{s}$?

2. **6 pts**
   A ball rolls from rest down a ramp, covering $2.0\ \mathrm{m}$ in $2.5\ \mathrm{s}$. Find its acceleration and its speed at the bottom.

3. **6 pts**
   Find the acceleration of an object sliding without friction down a ramp inclined at 20°.

4. **7 pts**
   A car moving at $18\ \mathrm{m/s}$ brakes to a stop in $45\ \mathrm{m}$. Find its deceleration and the stopping time.

5. **7 pts**
   A driver traveling at $30\ \mathrm{m/s}$ has a reaction time of $1.5\ \mathrm{s}$ and brakes at $6.0\ \mathrm{m/s^2}$. Find the total stopping distance.

6. **6 pts**
   Stapp's sled slowed from $282\ \mathrm{m/s}$ to rest in $1.4\ \mathrm{s}$. Find the average deceleration in $\mathrm{m/s^2}$ and in g.

7. **6 pts**

A stone is dropped from rest. How far does it fall in 2.5 s and how fast is it moving then?

8. **7 pts**

A ball is thrown straight up at 12 m/s. Find its maximum height and its total time in the air (caught at the launch height).

9. **5 pts**

A ball is thrown straight up. Give its acceleration (size and direction) (a) on the way up, (b) at the top, (c) on the way down.

10. **5 pts**

Why did the hammer and feather fall together on the Moon but not on Earth?

11. **7 pts**

A velocity–time graph rises in a straight line from 0 to 8.0 m/s in 4.0 s, then stays at 8.0 m/s for 6.0 s. Find the acceleration during the first part and the total displacement.

12. **5 pts**

A position–time graph is a straight line from $(0, 2.0\ \text{m})$ to $(5.0\ \text{s}, 17.0\ \text{m})$. What is the velocity?

13. **7 pts**

A marble leaves a 1.0 m high table horizontally at 2.0 m/s. Where does it land?

14. **8 pts**

A ball is kicked at 15 m/s at 35° above level ground. Find its time of flight, range and maximum height.

15. **4 pts**

Which launch angle gives the greatest range on level ground without air resistance? Name two other angles that give equal ranges.

16. 9 pts

Essay. Explain how Galileo used an inclined plane and a water clock to discover the law of falling bodies, and why this was an example of the scientific method at its best.

ANSWER KEY

1. $16 \times 0.30 = 4.8$ m
2. $a = 2(2.0)/6.25 = 0.64\ \mathrm{m/s^2}$; $v = at = 1.6$ m/s
3. $9.8 \sin 20^\circ = 3.35\ \mathrm{m/s^2}$
4. $a = 18^2/(2 \times 45) = 3.6\ \mathrm{m/s^2}$; $t = 18/3.6 = 5.0$ s
5. Reaction 45 m + braking $900/12 = 75$ m = 120 m
6. $201\ \mathrm{m/s^2}$; $20.6\,g$ (about 21 g)
7. $\frac{1}{2}(9.8)(6.25) = 30.6$ m; $v = 24.5$ m/s
8. $H = 144/19.6 = 7.3$ m; $t = 2(12)/9.8 = 2.4$ s
9. $9.8\ \mathrm{m/s^2}$ downward in all three cases.
10. The Moon has no air, so the only force on each is gravity, which gives every object the same acceleration. On Earth, air resistance slows the light, broad feather far more than the hammer.
11. $a = 2.0\ \mathrm{m/s^2}$; displacement $= 16 + 48 = 64$ m
12. $15/5.0 = 3.0$ m/s
13. $t = \sqrt{2/9.8} = 0.45$ s; $x = 0.90$ m from the table
14. $v_{0y} = 8.60$ m/s, $v_{0x} = 12.3$ m/s; $t = 1.76$ s; $R = 21.6$ m; $H = 3.8$ m
15. 45°. Any pair adding to 90°, such as 30° and 60°.
16. Model answer: A falling ball moves too fast to time, so Galileo rolled bronze balls down a long, smooth groove to slow the motion while keeping it uniformly accelerated. He timed rolls by weighing water that flowed from a vessel during each roll, repeated the trials many times over different fractions of the ramp, and found that distance was proportional to the square of time at every slope. He then reasoned that free fall is the limiting case of a steep ramp. It shows the method well: a clear question, a clever experiment that made the quantity measurable, repeated measurement, a mathematical law, and a prediction (for projectiles) later tested against data.

PORTFOLIO PIECE: THE TARGET CHALLENGE

Build a ramp-and-launch system (Hot Wheels track, a marble on a curtain-rod channel, or a ball on a gutter) that sends a projectile horizontally off a table. Calibrate it: measure the launch speed from at least four release points, and graph launch speed against release height. Then a parent places a cup on the floor at a distance you have not seen in advance. Measure the distance, use your calibration and projectile equations to choose a release point, and launch once. Write a two-page report with your calibration graph, your calculation, the result of the first launch, and an analysis of every source of error you can identify. If you miss, show how you corrected and how many launches it took.

LOOK FOR	WHAT IT MEANS
Calibration	At least four release points, five trials each, with launch speeds measured by video and graphed with a best-fit line or curve.
Prediction	Release point chosen by calculation before launching, with every step written with units.
Result and analysis	First-launch result recorded honestly; percent error found; specific sources of error identified (speed change at the edge, height measurement, drag, release consistency).
Physics explanation	Report explains independence of horizontal and vertical motion and why the fall time depends only on height.
Improvement	A documented correction with before-and-after data.

UNIT 3

Newton's Laws

Kinematics described motion; dynamics explains it. In 1687 Isaac Newton published three laws that, together with his law of gravity, explained motion from falling apples to orbiting planets. This unit teaches those laws and the method that goes with them: draw a free-body diagram of one object, add the forces as vectors, and set the net force equal to mass times acceleration.

The stories run from Halley's visit to Cambridge, through the Skylab astronauts who were weighed by their inertia and Elisha Otis cutting his own elevator rope, to Leonardo's friction sketches, Coulomb's prize essay and Atwood's classroom machine. The labs measure the thrust of a balloon rocket, build an inertial balance, show that acceleration is proportional to force with a toy car, test the laws of friction, and measure g with an Atwood machine built from a thread spool.

PREREQUISITES

- Units 1–2: vectors and components, kinematics equations, graphing and linearizing
- Right-triangle trigonometry (Lesson 1.5)

MASTERY CHECKLIST

- ☐ I can state Newton's three laws and identify third-law force pairs correctly.
- ☐ I can distinguish mass from weight and explain how mass is measured by inertia.
- ☐ I can draw a complete free-body diagram and apply

$\sum F$ along chosen axes, including

ma elevators and ropes at angles.

- ☐ I can use static and kinetic friction and find

μ_s from a slipping angle.

- ☐ I can solve incline, pulley and Atwood machine problems for acceleration and tension.

OPTIONAL KITS FOR THIS UNIT

- **Arbor Scientific (Atwood machine, low-friction pulleys, carts)**: Classroom physics supplier. A low-friction pulley or a ready-made Atwood machine makes Lesson 3.5 much more precise; their spring scales fit Lessons 3.1 and 3.4.
- **Vernier Go Direct sensors**: School-grade wireless sensors (force, motion, acceleration) that pair with a phone or computer. A force sensor and motion detector turn Lessons 3.3–3.5 into real-time graphs; optional and more expensive than the household methods.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

3.1	**The Principia and Newton's Three Laws**	3 sessions of 50 minutes
3.2	**Inertia and Mass: Weighing Astronauts on Skylab**	3 sessions of 50 minutes
3.3	**F = ma and Free-Body Diagrams: Otis and the Elevator**	3 sessions of 50 minutes
3.4	**Friction: From Leonardo to Coulomb**	3 sessions of 50 minutes
3.5	**Inclines, Tension and the Atwood Machine**	3 sessions of 50 minutes

Unit 3 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$. Draw a free-body diagram for every force problem; it earns points even if the arithmetic slips. Show the equation, the substitution with units and the answer with its unit. The test is worth 100 points.

1. **6 pts**
   State Newton's three laws of motion in your own words.

2. **5 pts**
   A horse pulls a cart forward. By the third law, the cart pulls back on the horse equally. Explain why they can still accelerate.

3. 6 pts

A 1200 kg car accelerates from 0 to 18 m/s in 6.0 s. Find the net force on it.

4. 5 pts

A 90 kg astronaut is on the Moon ($g = 1.62$ N/kg). Give her mass and her weight there.

5. 6 pts

An inertial balance with $k = 40$ N/m oscillates with a period of 0.50 s. Find the oscillating mass.

6. 7 pts

A 70 kg person stands on a scale in an elevator. What does the scale read (in newtons) when the elevator (a) moves up at constant speed, (b) accelerates upward at 2.0 $\mathrm{m/s^2}$, (c) accelerates downward at 2.0 $\mathrm{m/s^2}$?

7. 7 pts

A 900 kg elevator car slows to a stop while moving downward, with an acceleration of 1.5 $\mathrm{m/s^2}$ upward. Find the cable tension.

8. 7 pts

A 15 kg box is pulled across a floor by a rope at 35° above the horizontal with a 60 N force. $\mu_k = 0.30$. Find the normal force, the friction force and the acceleration.

9. 5 pts

A block begins to slide down a board tilted to 28°. Find μ_s.

10. 6 pts

A car skids to a stop from 20 m/s on a road with $\mu_k = 0.60$. How far does it skid?

11. 5 pts

Explain why the friction between two surfaces does not depend (approximately) on the area of contact.

12. 6 pts

A 6.0 kg box slides down a 40° ramp with $\mu_k = 0.25$. Find its acceleration.

13. **8 pts**

An Atwood machine has masses of 2.0 kg and 2.5 kg. Find the acceleration and the tension.

14. **8 pts**

A 4.0 kg cart on a frictionless table is tied over a pulley to a 1.0 kg hanging mass. Find the acceleration and the tension. How long does the hanging mass take to fall 0.80 m from rest?

15. **5 pts**

Explain how Elisha Otis's safety brake worked, in terms of the forces on the platform and the spring.

16. **8 pts**

Essay. Explain, using the story of the *Principia*, how Newton's laws were tested. Why did agreement between theory and measurement in many different situations (falling bodies, the Moon, the planets, tides) make the laws convincing?

ANSWER KEY

1. 1: An object keeps its velocity (at rest or moving straight at constant speed) unless a net force acts. 2: Net force equals mass times acceleration, in the same direction. 3: Forces come in equal and opposite pairs acting on two different objects.
2. The two forces act on different objects. The horse accelerates because the ground pushes it forward (friction on its hooves) more than the cart pulls back; the cart accelerates because the horse's pull exceeds the friction on the cart.
3. $a = 3.0\ \mathrm{m/s^2}$; $F = 3600\ \mathrm{N}$
4. Mass 90 kg; weight 146 N
5. $m = kT^2/4\pi^2 = 40(0.25)/39.5 = 0.25\ \mathrm{kg}$
6. (a) 686 N (b) 826 N (c) 546 N
7. $T = 900(9.8 + 1.5) = 10{,}170\ \mathrm{N}$
8. $N = 147 - 34.4 = 112.6\ \mathrm{N}$; $f = 33.8\ \mathrm{N}$; $a = (49.1 - 33.8)/15 = 1.02\ \mathrm{m/s^2}$
9. $\tan 28^\circ = 0.53$
10. $a = 5.88\ \mathrm{m/s^2}$; $d = 400/11.76 = 34\ \mathrm{m}$
11. Surfaces touch only at microscopic high points. The true contact area is set by how hard the surfaces are pressed together, so it is proportional to the load, not to the apparent area. Spreading the same load over a bigger area just makes each contact point carry less.
12. $9.8(\sin 40^\circ - 0.25\cos 40^\circ) = 9.8(0.643 - 0.192) = 4.42\ \mathrm{m/s^2}$
13. $a = 0.5(9.8)/4.5 = 1.09\ \mathrm{m/s^2}$; $T = 2.0(10.89) = 21.8\ \mathrm{N}$
14. $a = 9.8/5.0 = 1.96\ \mathrm{m/s^2}$; $T = 4.0 \times 1.96 = 7.84\ \mathrm{N}$; $t = \sqrt{1.6/1.96} = 0.90\ \mathrm{s}$
15. The rope's tension held a wagon spring bent above the platform. If the rope broke, the tension disappeared, the spring snapped straight and pushed pawls into toothed rails beside the shaft, so the rails supported the platform instead of the rope.
16. Model answer: Halley asked what orbit an inverse-square force would produce, and Newton showed that it gives an ellipse, matching Kepler's laws drawn from Tycho's data. In the *Principia* Newton then used the same three laws and one law of gravity to calculate falling bodies, pendulums, the Moon's orbit, the planets, comets and tides, and the results matched measurements in every case. A theory that fits one set of data might be fitted to it by chance; a few simple laws that predict many unrelated measurements, including new ones, are very

unlikely to be coincidences. That is why scientists accepted them, and also why they were later ready to test them where they might fail.

PORTFOLIO PIECE: A COUNTERWEIGHTED LIFT

Design and build a small model elevator: a lightweight car (a paper cup or small box) that rides up and down guided by two strings or straws, lifted by a thread over a spool pulley and balanced by a counterweight. Choose the counterweight using an Atwood-machine calculation so that the loaded car (with a specified number of coins as "passengers") rises slowly when you give it a small extra pull or add a small rider. Measure the acceleration for at least three loads, compare with your predictions, and estimate pulley friction. Add a simple safety catch in the spirit of Otis (for example, a springy stick that jams against the guides if the thread goes slack) and test it by releasing the thread. Write a report with free-body diagrams, calculations, data and a photo or sketch of the catch working.

LOOK FOR	WHAT IT MEANS
Design calculation	Counterweight chosen by calculation; free-body diagrams of car and counterweight; predicted accelerations for each load.
Measurements	Acceleration measured for at least three loads, three trials each, compared with predictions and percent differences.
Friction analysis	Pulley friction estimated from the data and discussed.
Safety catch	A working catch that stops the car when the thread goes slack, explained with forces.
Report	Clear, honest, with units on every number and a paragraph on what you would change.

3.1 The Principia and Newton's Three Laws

OBJECTIVES

- Tell how Halley's question led Newton to write the *Principia* (1687).
- State Newton's three laws of motion and apply each to everyday situations.
- Identify third-law force pairs and explain why they never cancel each other.
- Calculate the acceleration produced by a net force, $\vec{F}_{\text{net}} = m\vec{a}$.

> **Warm-up.** Stand on a skateboard (or sit on a rolling office chair) facing a wall and push gently off the wall with both hands. Which way do you move? You pushed on the wall; what pushed on you? Write your answer in one sentence, then try pushing harder and see what changes.

WATCH

The Most Famous Physics Textbook

Tibees · 17:00

https://www.youtube.com/watch?v=1KA9z4J0d4E

> **WHEN TO WATCH**
> After the story paragraph. Notice how the presenter reads Newton's own statements of the laws.

BACKUP VIDEO

Introduction to Newton's Third Law of Motion

Flipping Physics · 5:57

https://www.youtube.com/watch?v=bAevUhFUhV4

After the physics paragraph, for a careful look at the third law with live examples.

KEY TERMS

force

A push or pull on an object by another object, measured in newtons: $1\ \text{N} = 1\ \text{kg} \cdot \text{m/s}^2$.

net force

The vector sum of all the forces acting on an object.

Newton's first law

An object stays at rest or moves in a straight line at constant speed unless a net force acts on it.

Newton's second law

The acceleration of an object is the net force on it divided by its mass: $\vec{a} = \vec{F}_{\text{net}}/m$.

Newton's third law

When object A exerts a force on object B, B exerts a force of equal size and opposite direction on A.

force pair

The two forces of a third-law interaction. They act on different objects, are the same kind of force and are equal and opposite.

CORE IDEAS

The story. In August 1684 the astronomer Edmond Halley rode to Cambridge to ask Isaac Newton a question that he, Christopher Wren and Robert Hooke had been arguing over in a London coffeehouse: if the Sun pulled on a planet with a force that weakened as the square of the distance, what path would the planet follow? Newton answered at once: an ellipse. He had calculated it years before. He could not find the paper, so he promised to work it out again. Three months later Halley received a short treatise, *On the Motion of Bodies in Orbit*, and urged Newton to expand it. For the next eighteen months Newton did little else. The result, *Philosophiæ Naturalis Principia Mathematica* (*The Mathematical Principles of Natural Philosophy*), was published in London in July 1687. The Royal Society had spent its printing money on a lavish book about fish, so Halley paid for the printing himself.

How they figured it out. Newton began the *Principia* with three "axioms, or laws of motion," built on the work of Galileo, Descartes and Huygens, and then derived from them, step by step, the motion of falling bodies, pendulums, projectiles, the Moon, the planets, comets and the tides. The test of the laws was that the same three rules, applied to every one of these, gave answers that matched the measurements. In the second edition (1713) Newton added a closing essay, the General Scholium, in which he wrote that the solar system could only come from "the counsel and dominion of an intelligent and powerful Being." For Newton, discovering the laws of motion was discovering how the Creator governs His world.

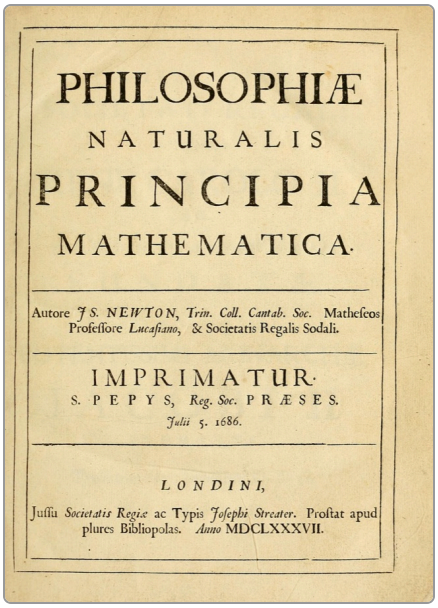
The physics. The **first law**: with no net force, an object keeps its velocity (zero, or constant in a straight line). The **second law**: a net force causes an acceleration in the same direction, $\vec{F}_{\text{net}} = m\vec{a}$; a newton is the force that accelerates 1 kg at 1 m/s^2. The

third law: forces come in pairs. When you push on a wall, the wall pushes on you with an equal and opposite force. The two forces of a pair act on *different* objects, so they never cancel each other; whether an object accelerates depends only on the forces acting *on it*. A book resting on a table has two forces on it, its weight (the Earth pulls it down) and the normal force (the table pushes it up); these balance, but they are not a third-law pair. The pair of the book's weight is the book pulling up on the Earth.

The engineering. Every propulsion system is a third-law machine. A rocket pushes exhaust gas backward and the gas pushes the rocket forward. A propeller or a jet engine pushes air backward; a ship's screw pushes water backward; a car's tires push backward on the road through friction, and the road pushes the car forward. Firefighters lean into a hose because the water jet pushes back on the nozzle. Engineers size motors and engines with the second law: to give a 1,500 kg car an acceleration of 3 m/s^2 takes a net force of 4,500 N.

Try it. In the lab you will check that two spring scales pulled against each other always read the same, and then measure the thrust of a balloon rocket from its acceleration.

PICTURE STUDY



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore JS. NEWTON, Trin. Coll. Cantab. Soc. Matheseos Professore Lucasiano, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,
Jussu Societatis Regiæ ac Typis Josephi Streater. Prostat apud plures Bibliopolas. Anno MDCLXXXVII.

The Title Page of the Principia, Isaac Newton (printed by Joseph Streater), 1687 · Printed title page · London

Image: Wikimedia Commons, public domain

This page opened the most influential book in the history of physics. Read it like a historian: every line tells you something about how science was published in 1687.

Look closely

- Find the title words "Philosophiæ Naturalis Principia Mathematica." Why "mathematical" principles?
- Find "Autore Js. Newton" and the description of his post at Cambridge.
- Find "Imprimatur" and the name S. Pepys. Samuel Pepys, the famous diarist, was President of the Royal Society that year.
- Find the date, written in Roman numerals.

Talk about it

1. Why would a scientific society give its official approval to a book?
2. Newton wrote in Latin. Who could read the book in 1687, and who could not?
3. What does it tell you that Halley, not the Society, paid for the printing?

Safety first. Inflate balloons with your own breath or a pump; never with any gas from a tank. Wear safety glasses in case a balloon pops near your face.

LAB

Action and Reaction: Spring Scales and a Balloon Rocket

You will test the third law with two spring scales, then use the second law to measure the thrust of a balloon rocket from its mass and acceleration.

You will need

- Two spring scales
- A long balloon, a drinking straw, tape
- 5–6 m of smooth fishing line or kite string stretched tight between two chairs
- Kitchen scale, tape measure, phone with slow-motion video
- Binder clip (to hold the balloon closed while you set up)

Steps

1. **Spring scales.** Hook the two scales together. You pull one, your partner pulls the other, harder and softer, steady and jerky. Record both readings at five moments. Then hook one scale to a doorknob and pull. What does the doorknob "read"?
2. **Hypothesis.** Predict whether the two scales can ever read differently, and why.
3. **Rocket.** Thread the straw onto the line and stretch the line tight and level. Put tape marks every 0.50 m along the first 2.0 m.
4. Blow up the balloon, clip its neck, and tape it to the straw. Weigh the balloon-and-straw on the kitchen scale with the same amount of air (an inflated balloon weighs about the same as an empty one, since the air inside is only slightly compressed).
5. Release the clip and film the rocket's first 2.0 m. Record the time at each mark. Do 3 trials with the balloon inflated the same amount (measure its length).
6. Plot d against t^2 for the first meter, where the thrust is nearly steady; $a = 2 \times$ slope. Estimate the net force $F = ma$, which is the thrust minus friction on the line.

7. **Fair test.** Change one variable: the balloon's inflation, or the nozzle (tape a short straw into the neck to narrow it). Measure the new acceleration.
8. **Improve.** Friction on the line reduces your thrust estimate. Measure it: tilt the line until the uninflated straw-and-balloon just slides at steady speed, and use that angle to estimate the friction force. Add it back to your thrust.

Tips

- The mass is small (a few grams), so record it to 0.1 g if your scale can.
- Tape the balloon so it pushes straight along the line, not at an angle.

Check your work

☐ I showed that two scales pulling on each other always read the same.

☐ I found the rocket's acceleration from a linearized graph and calculated the net force with

$F = ma$.

☐ I can name the force pair that moves the rocket.

LAB NOTEBOOK

1. Table of paired spring-scale readings and one sentence of conclusion.
2. Free-body sketch of the balloon rocket in flight, with every force labeled, and a separate sketch of the air being pushed out, showing the third-law partner of the thrust.
3. For a horse pulling a cart, draw both objects separately with all forces, and explain why the cart moves even though the cart pulls back on the horse as hard as the horse pulls forward.

KEY FORMULAS

$$\vec{F}_{\text{net}} = \sum \vec{F} = m\vec{a}$$

$$\vec{F}_{A \text{ on } B} = -\vec{F}_{B \text{ on } A}$$

$$W = mg$$

From history. Newton wrote the *Principia* in Latin; Andrew Motte's English translation appeared in 1729. The third edition (1726) was the last Newton saw. Halley used the new physics to predict that a comet seen in 1531, 1607 and 1682 would return about 1758. It did, after both men had died, and it is now called Halley's Comet.

Example 1. A 1500 kg car's tires push it forward with 4000 N while air resistance and rolling friction total 1000 N backward. Find its acceleration.

Show solution

1. Net force (forward positive): $F_{\text{net}} = 4000\ \text{N} - 1000\ \text{N} = 3000\ \text{N}$.
2. $a = \dfrac{F_{\text{net}}}{m} = \dfrac{3000\ \text{N}}{1500\ \text{kg}} = 2.0\ \text{m/s}^2$ forward.

Answer: $2.0\ \text{m/s}^2$ forward.

Example 2. Two skaters stand on smooth ice. Anna (40 kg) pushes Ben (60 kg) with a force of 120 N for a moment. Find the force on Anna and each skater's acceleration during the push.

Show solution

1. Third law: Ben pushes back on Anna with 120 N in the opposite direction.
2. Ben: $a = \dfrac{120\ \text{N}}{60\ \text{kg}} = 2.0\ \text{m/s}^2$ away from Anna.
3. Anna: $a = \dfrac{120\ \text{N}}{40\ \text{kg}} = 3.0\ \text{m/s}^2$ away from Ben.

Answer: Equal forces of 120 N on each; Ben accelerates at $2.0\ \text{m/s}^2$ and Anna at $3.0\ \text{m/s}^2$, in opposite directions. The forces are equal, the accelerations are not.

Example 3. A book rests on a table. List every force on the book, and give the third-law partner of each.

Show solution

1. Forces on the book: its weight $\vec{W}$ (the Earth pulls it down) and the normal force $\vec{N}$ (the table pushes it up).
2. Partner of the weight: the book pulls up on the Earth with an equal force.
3. Partner of the normal force: the book pushes down on the table with an equal force.
4. $\vec{W}$ and $\vec{N}$ are equal and opposite because the book is not accelerating (second law), not because of the third law: they act on the same object and are different kinds of force.

Answer: Weight (partner: book pulls Earth up) and normal force (partner: book pushes table down). Weight and normal force balance by the second law but are not a pair.

PRACTICE

1. Easy What net force gives a 3.0 kg object an acceleration of $4.0\ \mathrm{m/s^2}$?

   Show answer

   12 N

2. Easy A 20 N net force acts on a 5.0 kg cart. Find its acceleration.

   Show answer

   $4.0\ \mathrm{m/s^2}$

3. Easy A hockey puck slides across smooth ice at constant velocity. What is the net force on it?

   Show answer

   Zero (first law).

4. Medium A 70 kg sprinter accelerates at $3.5\ \mathrm{m/s^2}$. What horizontal force does the track exert on her? What force does she exert on the track?

   Show answer

   245 N forward on her; 245 N backward on the track.

5. Medium A 1200 kg car slows from 20 m/s to 0 in 5.0 s. Find the size of the net force on it.

Show answer

$a = 4.0\ \mathrm{m/s^2}$; $F = 4800\ \mathrm{N}$

6. Medium A truck and a small car collide head-on. Which feels the larger force? Which has the larger acceleration?

Show answer

The forces are equal (third law). The car, having less mass, has the larger acceleration.

7. Medium A student says, "If every force has an equal and opposite partner, they always cancel, so nothing can ever accelerate." What is wrong with this reasoning?

Show answer

The two forces of a pair act on different objects. Only the forces acting on one object add to give its net force.

8. Medium An elevator of mass 500 kg hangs at rest from its cable. What is the tension in the cable? What is the third-law partner of the elevator's weight?

Show answer

$T = mg = 4900\ \mathrm{N}$. The elevator pulls up on the Earth with 4900 N.

9. Hard A 0.50 kg model rocket's engine gives 8.0 N of thrust straight up. Find the net force and acceleration at liftoff.

Show answer

$F_{\mathrm{net}} = 8.0 - 4.9 = 3.1\ \mathrm{N}$ up; $a = 6.2\ \mathrm{m/s^2}$ up.

10. Hard Halley's question concerned a force that weakens as the square of the distance. If a planet's distance from the Sun doubles, by what factor does the Sun's pull change? If it triples?

Show answer

To 1/4; to 1/9.

11. **Challenge** Two blocks, 2.0 kg and 3.0 kg, sit side by side on a frictionless floor. A 15 N push acts on the 2.0 kg block. Find the acceleration and the force each block exerts on the other.

Show answer

$a = 15/5.0 = 3.0\ \mathrm{m/s^2}$. The 3.0 kg block needs 9.0 N, so the blocks push on each other with 9.0 N (equal and opposite).

COMMON MISTAKES

- Calling weight and normal force a third-law pair. A pair acts on two different objects.
- Thinking a moving object needs a force to keep it moving. It needs a force only to change its velocity.
- Using a single force instead of the net force in $F = ma$.

Make it a game · Three Laws, Three Examples. In 60 seconds, state Newton's three laws from memory and give a fresh everyday example for each, with no repeats from the lesson. Your parent scores one point per law and one per good example. Then your parent describes a situation and you identify the force pair. Beat your score in round two.

Scripture. “And he is before all things, and by him all things consist.” (Colossians 1:17, KJV)

Newton’s three laws describe how all things hold together in motion. Scripture says that they hold together because of Christ.

3.2 Inertia and Mass: Weighing Astronauts on Skylab

OBJECTIVES

- Explain inertia and Newton's first law using Galileo's two-ramp thought experiment.
- Distinguish mass (a measure of inertia, in kilograms) from weight (a force, in newtons), and calculate weight with $W = mg$.
- Explain how mass can be measured without gravity, as on Skylab and the International Space Station.
- Build and calibrate an inertial balance.

> **Warm-up.** Put a coin on an index card on top of an empty glass. Flick the card sharply sideways. Where does the coin go? Now try pulling the card slowly. Write down the difference and your best explanation.

WATCH

Newton's first law | Physics | Khan Academy

Khan Academy · 8:57

https://www.youtube.com/watch?v=rjkQcfw5fkM

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

STEMonstrations: Newton's First Law of Motion

NASA Johnson · 4:19

https://www.youtube.com/watch?v=-luKN6mad5w

With the engineering paragraph: astronauts show inertia and mass aboard the International Space Station.

KEY TERMS

inertia
The tendency of an object to keep its state of motion; the resistance of an object to a change in velocity.

mass
The amount of inertia an object has, measured in kilograms. It is the same everywhere.

weight

The gravitational force on an object, $W = mg$, measured in newtons. It changes from place to place (a 70 kg person weighs 686 N on Earth and 113 N on the Moon).

inertial reference frame

A point of view (such as the ground, for most purposes) from which Newton's first law holds: objects with no net force move at constant velocity.

inertial balance

A device that measures mass by how fast an object oscillates on a spring, without using its weight.

CORE IDEAS

The story. On Skylab, America's first space station (1973–1974), doctors wanted to know whether the astronauts were losing mass during months in orbit. But a bathroom scale reads zero in orbit: the astronaut and the scale are both in free fall around the Earth. Engineers built a Body Mass Measuring Device: a seat mounted on springs. The astronaut strapped in, tucked up tight, released the seat and let it rock back and forth, and an electronic timer measured the period of the oscillation. The heavier the astronaut, the slower the rocking. Alan Bean, commander of the second Skylab crew, was photographed riding it, and the measurements showed that astronauts lost mass in the first days of a mission. Today the International Space Station uses a device that gives the astronaut a known push and measures the acceleration.

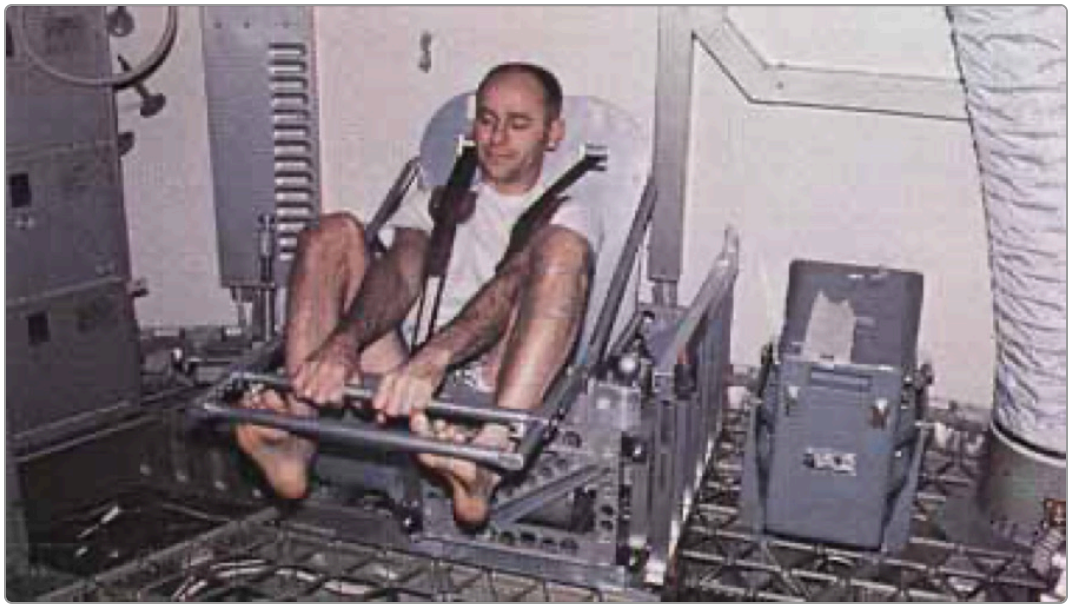
How they figured it out. The idea that mass is resistance to acceleration came from Galileo. In his *Dialogue* (1632) and later work he imagined a ball rolling down one smooth ramp and up another: it rises to nearly its starting height. Make the second ramp less steep and the ball rolls farther to reach that height. Make it level, and the ball should roll forever. So motion does not need a cause; only *changes* in motion do. Descartes stated the straight-line version, and Newton made it his first law. Newton then defined mass, "quantity of matter," as what resists a force, and showed by pendulum experiments with different materials (wood, gold, silver, lead, sand, salt, water, wheat) that the mass that resists acceleration is the same mass that gravity pulls on, to about one part in a thousand.

The physics. Inertia is resistance to changes in velocity, and **mass** is its measure. Mass is in kilograms and does not change from place to place. **Weight** is a force, $W = mg$, in newtons. On Earth $g = 9.8\ \mathrm{N/kg}$ (the same number as $9.8\ \mathrm{m/s^2}$); on the Moon $g = 1.62\ \mathrm{N/kg}$, so you would weigh one-sixth as much but have the same mass, and it would be just as hard to start or stop you sideways. A mass on a spring oscillates with period $T = 2\pi\sqrt{m/k}$, which depends on mass and spring stiffness but not on gravity. That is why the Skylab device worked, and why you can build one at home.

The engineering. Engineers design for inertia every day. Cargo straps, load bars in trucks and the nets in the backs of cars keep loads from continuing forward when a vehicle stops. Head restraints catch your head when a car is hit from behind and your body is pushed forward. Spacecraft engineers count every kilogram, because every kilogram must be accelerated. Inertial navigation systems in aircraft and submarines measure the tiny forces needed to accelerate a known test mass and calculate position from them without any outside signal.

Try it. In the lab you will build an inertial balance from a ruler or a hacksaw blade, calibrate it with known masses, and use it to measure an unknown mass, without ever using its weight.

PICTURE STUDY



Alan Bean on the Body Mass Measuring Device, NASA, 1973 · Photograph · Skylab, in Earth orbit

Image: Wikimedia Commons, public domain

This photograph shows a solution to a strange problem: how do you weigh someone when nothing has weight? The answer was to measure inertia instead.

Look closely

- Find the seat and how Bean is holding his body. Why must he hold perfectly still and tucked in?
- Find the frame or rails the seat moves on.
- Look at the background. What tells you this is in orbit?

Talk about it

1. Why does a bathroom scale read zero on a space station?
2. If Bean had lost 2 kg, would the seat rock faster or slower?
3. Why would doctors care about an astronaut's mass during a long mission?

Safety first. Clamp the ruler or blade firmly to the table and keep your face away from the end while it vibrates. Hacksaw blades have sharp teeth: wrap them in tape or use a ruler instead.

LAB

An Inertial Balance

You will measure mass the way Skylab did: by timing oscillations. Because the balance moves sideways, gravity plays no part.

You will need

- A flexible 30 cm plastic or thin metal ruler (or a hacksaw blade with the teeth taped)
- A C-clamp or heavy books to fix one end flat on a table edge
- A small paper cup or bottle cap taped near the free end
- Coins or washers of known mass (weigh them on the kitchen scale)
- Stopwatch or phone
- Graph paper or spreadsheet

Steps

1. Clamp the ruler flat (horizontally) at the table edge so 20–25 cm sticks out sideways, with the flat face vertical so it vibrates side to side, not up and down. Tape the cup at the end.
2. **Hypothesis.** Predict how the period will change if the mass in the cup doubles.
3. Pull the end 2–3 cm sideways and release. Time 10 complete oscillations and divide by 10. Repeat 3 times with the empty cup.
4. Add known masses in steps (for example 5, 10, 15, 20, 25 g of coins) and time 10 oscillations for each, 3 trials each.
5. Graph T^2 against added mass m. Theory: $T^2 = \frac{4\pi^2}{k}(m + m_0)$, a straight line whose slope gives k and whose intercept gives m_0, the effective mass of the cup and ruler.
6. **Measure an unknown.** Put an unknown object (an eraser, a key) in the cup, find its period, and read its mass from your graph. Then weigh it on the kitchen scale and find the percent difference.

7. **Test the gravity claim.** Hold a strong magnet below the cup (if the masses are steel washers) to add a downward pull. Does the period change? Why or why not?
8. **Improve.** Identify the largest source of error (counting oscillations, damping, the mass sliding in the cup). Change one thing and repeat the unknown.

Tips

- Timing 10 oscillations divides your reaction-time error by 10.
- Keep the amplitude small and the same each time.

Check your work

☐ My graph of

T^2 against mass is a straight line, and I used it to measure an unknown mass.

☐ I can explain the difference between mass and weight and why the balance measures mass.

☐ I can explain how Skylab astronauts were weighed.

LAB NOTEBOOK

1. Calibration table and the T^2–m graph with best-fit line; the unknown mass read from the graph and compared with the kitchen scale.
2. Draw Galileo's two-ramp thought experiment in three stages and explain how it leads to the first law.
3. Explain in a paragraph why it is just as hard to push a stalled car into motion on the Moon as on Earth, even though it weighs one-sixth as much.

KEY FORMULAS

$$W = mg$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \Longrightarrow \quad m = \frac{kT^2}{4\pi^2}$$

$$m = \frac{F}{a}$$

From history. Skylab's three crews spent 28, 59 and 84 days aboard, and the Body Mass Measuring Device was used almost daily, along with a smaller version for weighing food and waste. In 1889 the Hungarian physicist Loránd Eötvös began torsion-balance experiments that showed inertial and gravitational mass to be equal to about one part in a hundred million; modern experiments have pushed that past one part in 10^{15}.

WORKED EXAMPLES

Example 1. An astronaut has a mass of 70 kg. Find her weight on Earth and on the Moon ($g_{\text{Moon}} = 1.62\ \text{N/kg}$). What is her mass on the Moon?

Show solution

1. Earth: $W = mg = (70\ \text{kg})(9.8\ \text{N/kg}) = 686\ \text{N}$.
2. Moon: $W = (70\ \text{kg})(1.62\ \text{N/kg}) = 113\ \text{N}$.
3. Mass does not change: 70 kg.

Answer: 686 N on Earth, 113 N on the Moon; mass 70 kg in both places.

Example 2. On the space station, a mass-measuring device pushes an astronaut with a steady 40 N and measures an acceleration of $0.50\ \text{m/s}^2$. What is the astronaut's mass (including a 3 kg moving arm, which must be subtracted)?

Show solution

1. $m_{\text{total}} = \dfrac{F}{a} = \dfrac{40\ \text{N}}{0.50\ \text{m/s}^2} = 80\ \text{kg}$.
2. Astronaut: $80\ \text{kg} - 3\ \text{kg} = 77\ \text{kg}$.

Answer: 77 kg.

Example 3. A home inertial balance has an effective spring constant $k = 60\ \mathrm{N/m}$. With an object in its cup, it oscillates with a period of $0.80\ \mathrm{s}$. What is the total oscillating mass?

Show solution

1. $m = \dfrac{kT^2}{4\pi^2} = \dfrac{(60\ \mathrm{N/m})(0.80\ \mathrm{s})^2}{4\pi^2}$.
2. $= \dfrac{(60)(0.64)}{39.5}\ \mathrm{kg} = 0.97\ \mathrm{kg}$.
3. Check units: $\mathrm{N/m} \cdot \mathrm{s}^2 = \dfrac{\mathrm{kg{\cdot}m/s^2}}{\mathrm{m}}\mathrm{s}^2 = \mathrm{kg}$.

Answer: About $0.97\ \mathrm{kg}$ (including the cup and the moving part of the balance).

PRACTICE

1. Easy Find the weight of a $12\ \mathrm{kg}$ dog on Earth.

   Show answer

   $118\ \mathrm{N}$

2. Easy A bag of rice weighs $49\ \mathrm{N}$ on Earth. What is its mass?

   Show answer

   $5.0\ \mathrm{kg}$

3. Easy Which has more inertia: a bowling ball or a beach ball of the same size? How could you tell in a space station?

   Show answer

   The bowling ball. In orbit, it would be much harder to start or stop moving, or to shake back and forth.

4. Medium An astronaut weighs $735\ \mathrm{N}$ on Earth. What does she weigh on Mars, where $g = 3.71\ \mathrm{N/kg}$?

   Show answer

   $m = 75\ \mathrm{kg}$; $W = 278\ \mathrm{N}$

5. Medium Explain, using the first law, why passengers lurch forward when a bus brakes suddenly.

Show answer

Their bodies tend to keep moving forward at the bus's old speed; the bus slows beneath them until the seat, belt or their feet supply a backward force.

6. Medium An inertial balance has $k = 25\ \mathrm{N/m}$. Find the period for a total mass of $0.10\ \mathrm{kg}$.

Show answer

$T = 2\pi\sqrt{0.10/25} = 0.40\ \mathrm{s}$

7. Medium If the mass on an inertial balance is quadrupled, what happens to the period?

Show answer

It doubles, since $T \propto \sqrt{m}$.

8. Medium A force of $150\ \mathrm{N}$ gives an astronaut (with the device's $3.0\ \mathrm{kg}$ moving arm) an acceleration of $1.8\ \mathrm{m/s^2}$. What is the astronaut's mass?

Show answer

$150/1.8 = 83.3\ \mathrm{kg}$; astronaut $80.3\ \mathrm{kg}$

9. Hard An inertial balance oscillates with period $0.50\ \mathrm{s}$ when empty and $0.70\ \mathrm{s}$ with a $40\ \mathrm{g}$ mass added. Find the effective empty mass m_0 and the spring constant k.

Show answer

$\dfrac{0.49}{0.25} = \dfrac{m_0 + 0.040}{m_0}$, so $m_0 = 0.040/0.96 = 0.042\ \mathrm{kg}$; $k = 4\pi^2 m_0/T^2 = 6.6\ \mathrm{N/m}$

10. Hard A performer pulls a tablecloth out from under a full table setting. Explain with inertia and friction why it works best when pulled quickly.

Show answer

Friction acts on the dishes only while the cloth slides under them. A fast pull makes that time very short, so the friction force gives the dishes only a tiny change in velocity; their inertia keeps them nearly at rest.

11. **Challenge** Newton tested whether the mass that resists acceleration equals the mass that gravity pulls on by using pendulums of different materials with the same length. Explain why identical periods for, say, gold and wood show that the two kinds of mass are proportional.

Show answer

The pull of gravity on the bob is proportional to its gravitational mass, while its resistance to acceleration is its inertial mass. A pendulum's period depends on their ratio; if every material gives the same period, the ratio is the same for every material, so the two masses are proportional (and can be made equal by choosing units).

COMMON MISTAKES

- Giving weight in kilograms. Weight is a force, in newtons; kilograms measure mass.
- Thinking astronauts are weightless because there is no gravity in orbit. Gravity at the station's height is about 90% as strong as on the ground; they float because they are in free fall.
- Thinking a force is needed to keep something moving. A force is needed only to change the motion.

> **Make it a game · Mass or Weight.** Your parent names ten quantities such as 'my 60 kg', 'a 500 newton force' and 'the moon's pull on me'. Call each mass or weight in three seconds, then convert one with W = mg. Include the moon at g near 1.6. Parent slips in one trick answer per round, and you win a bonus point for catching it.

3.3 F = ma and Free-Body Diagrams: Otis and the Elevator

OBJECTIVES

- Draw a correct free-body diagram for an object, showing every force acting on it.
- Apply $\sum F = ma$ along chosen axes to find accelerations, tensions and normal forces.
- Calculate apparent weight in an accelerating elevator.
- Show experimentally that acceleration is proportional to net force for a fixed mass.

> **Warm-up.** Stand on a bathroom scale in an elevator, or, if you don't have one handy, stand on the scale and bend your knees quickly downward and then push up. Watch the reading. When does it read more than your weight? Less? Write down what your body was doing (speeding up, slowing down, which direction) each time.

WATCH

Newton's Second Law of Motion - Force, Mass, & Acceleration

The Organic Chemistry Tutor · 19:06

https://www.youtube.com/watch?v=Ee6CHn0MRKE

> **WHEN TO WATCH**
> After the physics paragraph. Work each free-body diagram with the presenter.

BACKUP VIDEO

Do You Feel Your Weight?

Flipping Physics · 6:10

https://www.youtube.com/watch?v=Li5c9RVItmI

With the elevator worked example: what you feel when an elevator accelerates.

KEY TERMS

free-body diagram

A sketch of one object alone, with an arrow for every force acting on it, drawn from the object's center and labeled.

normal force

The push of a surface on an object, perpendicular to the surface.

tension

The pull of a rope, cable or string, directed along it, away from the object.

apparent weight

The force a scale or floor pushes up on you, which is what you feel as your weight; it equals $m(g + a)$ in an elevator accelerating upward at a.

safety factor

The ratio of the load a part can carry before failing to the largest load it is designed to carry.

CORE IDEAS

The story. Hoists had lifted goods for centuries, but people would not ride them, because when a rope broke the platform fell. In 1852 Elisha Otis, a master mechanic at a bedstead factory in Yonkers, New York, designed a hoist with a safety brake: a wagon spring at the top of the platform, held bent by the tension of the rope. If the rope went slack, the spring snapped straight and pushed two pawls into toothed ratchet rails on each side of the shaft, locking the platform in place. In 1854, at the Exhibition of the Industry of All Nations in New York's Crystal Palace, Otis rode his platform up high above the crowd and had the only rope cut with an axe. The platform dropped a few inches and stopped. He is said to have called out, "All safe, gentlemen, all safe."

How they figured it out. Otis's brake works because of forces on the platform. While the rope holds, its tension keeps the spring bent; the moment the tension vanishes, the forces on the spring change and it acts. In 1857 the first Otis passenger elevator went into service in a store on Broadway, and in 1861 he patented the improved design. Together with steel frames, the safe elevator made tall buildings practical: no one will climb twenty flights of stairs. Every elevator engineer since has begun the same way, by drawing the car alone with every force on it and applying Newton's second law.

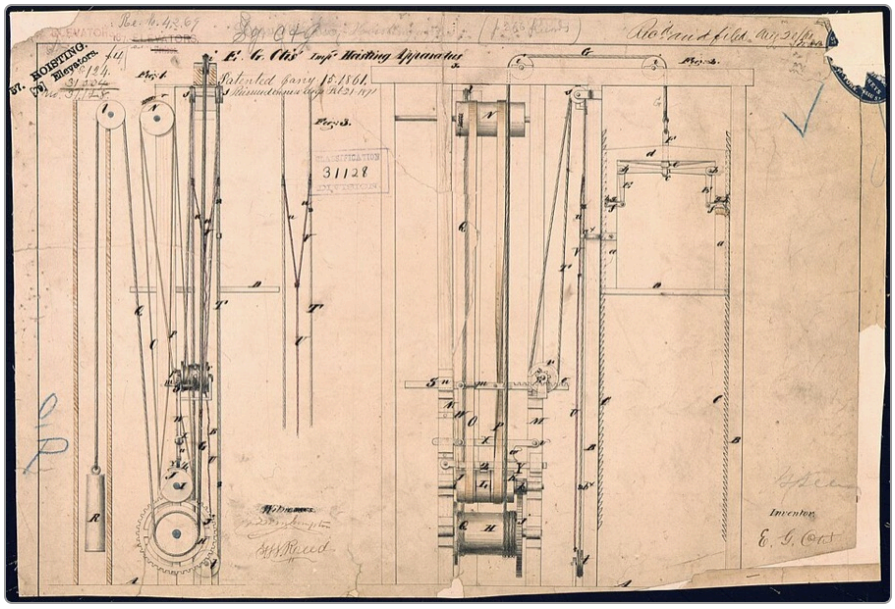
The physics. A **free-body diagram** shows one object and every force acting on it, nothing else. Then choose axes, write $\sum F_x = ma_x$ and $\sum F_y = ma_y$, and solve. For an elevator car of mass m hanging from a cable with tension T: $T - mg = ma$ (up positive), so $T = m(g + a)$. Speeding up going up, or slowing down going down, a is upward and the cable must pull harder than the car's weight. For a person standing

on a scale in the car, the scale's normal force is $N = m(g + a)$: that is your **apparent weight**, the weight you feel. If the cable broke and the car fell freely, $a = -g$ and $N = 0$.

The engineering. Elevators are designed to accelerate gently, about 1 m/s^2 in most buildings, so riders feel at most about a 10% change in weight. The hoist ropes are many times stronger than the heaviest load they ever carry (a large safety factor), there are several ropes where one would do, and modern cars still have a descendant of Otis's brake: a governor that grips the rails if the car ever moves too fast. Engineers also use apparent weight in reverse: the accelerometer in your phone measures the normal force on a tiny test mass and reports $g + a$.

Try it. In the lab you will pull a toy car with a hanging weight and show that, for a fixed total mass, the acceleration is proportional to the net force.

PICTURE STUDY



Otis's Elevator Patent, Elisha Graves Otis, January 15, 1861 · Patent drawing · U.S. National Archives

Image: Wikimedia Commons, public domain

Otis's patent, "Improvement in Hoisting Apparatus," shows the safety device that made passenger elevators possible. A patent drawing must show the idea clearly enough for a skilled mechanic to build it.

Look closely

- Find the toothed rails on each side of the shaft.
- Find the spring above the platform and the pawls (catches) at its ends.
- Find where the hoisting rope attaches. Trace what holds the spring bent.

Talk about it

1. Draw a free-body diagram of the platform while the rope holds it. What changes the instant the rope breaks?
2. Why does the brake need no electricity or operator?
3. Why would two rails and two pawls be better than one?

LAB

Force and Acceleration with a Toy Car

You will pull a toy car across a table with a string over a pulley and a hanging weight. By moving washers from the car to the hanger, you change the force while keeping the total mass the same, and you test whether $a \propto F$.

You will need

- A toy car that rolls freely in a straight line (a Hot Wheels car on straight track, or a larger toy truck)
- Thread or fishing line about 1.5 m long
- A pulley: an empty thread spool on a pencil clamped at the table edge, or a smooth round dowel
- A small paper cup or binder clip as the hanger
- 10 identical washers or coins (weigh them)
- Kitchen scale, tape measure, phone with slow-motion video

Steps

1. Weigh the car, the hanger and one washer. Tape 8 washers on the car and put 2 on the hanger.
2. Tie the thread from the car over the pulley to the hanger. Mark a start line and a 1.00 m finish on the table (or as much as your table allows).
3. **Hypothesis.** Predict how the acceleration will change if you double the number of washers on the hanger (moving them from the car).
4. Release the car from rest and time it over 1.00 m with video. Do 3 trials. Find $a = 2d/t^2$.
5. Move one washer from the car to the hanger (so the total mass stays the same) and repeat. Continue until 6 washers hang.
6. Graph a against the hanging weight $m_h g$. Theory: $a = \dfrac{m_h g - f}{M_{\text{total}}}$, a straight line with slope $1/M_{\text{total}}$ and an intercept set by friction f.
7. Find your slope and compare $1/\text{slope}$ with the total mass you weighed. Find friction from the intercept.

8. **Second test.** Keep 3 washers hanging and add mass to the car (extra washers from outside the set). Graph a against $1/M_{\text{total}}$.
9. **Improve.** Reduce the largest error (string rubbing, the car steering into the edge, release). Repeat one point.

Tips

- Pad the floor under the hanger with a towel.
- If the car is too fast to time, use a lighter hanger or a heavier car.

Check your work

- ☐ My graph of acceleration against force is a straight line for a fixed total mass.
- ☐ I can draw free-body diagrams of the car and the hanger.
- ☐ I can calculate apparent weight in an elevator.

LAB NOTEBOOK

1. Free-body diagrams of the car and of the hanging mass, then the data table and the a-against-force graph with the slope interpreted.
2. Free-body diagrams of a person in an elevator in five moments: at rest, starting up, cruising up, slowing to a stop, and (if the cable broke) falling freely. Write N for each.
3. Use phyphox's "Elevator" or "Acceleration (without g)" experiment on an elevator ride, if you can, and sketch the graph you see.

KEY FORMULAS

$$\sum F_x = ma_x, \qquad \sum F_y = ma_y$$

$$T = m(g + a) \quad \text{(elevator, up positive)}$$

$$N = m(g + a) \quad \text{(apparent weight)}$$

From history. Otis died in 1861, the year of his patent, and his sons Charles and Norton built the Otis Elevator Company. In 1889 the company installed the elevators of the Eiffel Tower, whose curved legs required cars that climbed at a changing angle. The Haughwout Building, home of the first passenger elevator, still stands at Broadway and Broome Street in New York.

Example 1. An 800 kg elevator car starts upward with an acceleration of $1.2\ \mathrm{m/s^2}$. Find the tension in its cable. What is the tension when it moves up at constant speed?

Show solution

1. Free-body diagram: tension T up, weight mg down. Up is positive.
2. $\sum F = ma$: $T - mg = ma$, so
   $T = m(g + a) = (800\ \mathrm{kg})(9.8 + 1.2)\ \mathrm{m/s^2} = 8800\ \mathrm{N}$.
3. Constant speed: $a = 0$, so $T = mg = (800\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) = 7840\ \mathrm{N}$.

Answer: 8800 N while accelerating up; 7840 N at constant speed.

Example 2. A 60 kg student stands on a scale in an elevator that is moving up but slowing down at $1.5\ \mathrm{m/s^2}$. What does the scale read in newtons, and what "mass" would a scale marked in kilograms show?

Show solution

1. Slowing down while moving up means the acceleration is downward:
   $a = -1.5\ \mathrm{m/s^2}$.
2. $N - mg = ma$, so $N = m(g + a) = (60\ \mathrm{kg})(9.8 - 1.5)\ \mathrm{m/s^2} = 498\ \mathrm{N}$.
3. A kilogram scale divides by g: $\dfrac{498\ \mathrm{N}}{9.8\ \mathrm{N/kg}} = 50.8\ \mathrm{kg}$.

Answer: 498 N; the scale would show about 51 kg, though the student's mass is still 60 kg.

Example 3. A 20 kg sled is pulled across snow by a rope at 30° above the horizontal with a tension of 50 N. Friction is 10 N. Find the normal force and the acceleration.

Show solution

1. Forces: tension 50 N at 30°, weight $mg = 196$ N down, normal force N up, friction 10 N backward.
2. Vertical ($a_y = 0$): $N + 50 \sin 30^\circ - 196 = 0$, so $N = 196 - 25 = 171$ N.
3. Horizontal: $50 \cos 30^\circ - 10 = 20a$, so $43.3 - 10 = 20a$ and $a = \dfrac{33.3 \text{ N}}{20 \text{ kg}} = 1.67 \text{ m/s}^2$.

Answer: $N = 171$ N (less than the weight, because the rope lifts a little); $a = 1.67 \text{ m/s}^2$.

PRACTICE

1. Easy Draw (describe) the free-body diagram of a lamp hanging from a ceiling cord.

   Show answer

   Two forces: tension up along the cord, weight down. They are equal because the lamp is at rest.

2. Easy A 50 kg box is pushed across a frictionless floor with 100 N. Find its acceleration.

   Show answer

   2.0 m/s^2

3. Easy A 1000 kg elevator moves down at constant speed. What is the cable tension?

   Show answer

   9800 N

4. Medium The same elevator accelerates downward at 2.0 m/s^2. Find the tension.

   Show answer

   $1000(9.8 - 2.0) = 7800$ N

5. Medium A 65 kg person stands on a scale in an elevator accelerating upward at $1.0\ \mathrm{m/s^2}$. What does the scale read?

Show answer

$65(10.8) = 702\ \mathrm{N}$

6. Medium A 2.0 kg lamp hangs from a cord in an accelerating car. The cord tension is 21 N and makes an angle of 21° with the vertical. Find the car's acceleration.

Show answer

Vertical: $T\cos 21^\circ = 19.6\ \mathrm{N}$; horizontal: $T\sin 21^\circ = 7.5\ \mathrm{N} = 2.0a$, so $a = 3.8\ \mathrm{m/s^2}$ (equivalently $a = g\tan 21^\circ$).

7. Medium A 5.0 kg box is pulled by a rope at 40° above horizontal with 30 N on a frictionless floor. Find its acceleration and the normal force.

Show answer

$a = 30\cos 40^\circ/5.0 = 4.6\ \mathrm{m/s^2}$; $N = 49 - 30\sin 40^\circ = 29.7\ \mathrm{N}$

8. Medium Why does a bathroom scale read zero if you jump off a table while standing on it?

Show answer

You and the scale fall freely together with acceleration g, so the scale does not need to push on you: $N = m(g - g) = 0$.

9. Hard An elevator's cable can safely carry 2.0×10^4 N. The car and passengers have a mass of 1500 kg. What is the largest upward acceleration allowed?

Show answer

$a = T/m - g = 13.3 - 9.8 = 3.5\ \mathrm{m/s^2}$

10. Hard In the lab, the total mass is 0.30 kg, the hanging mass is 0.020 kg, and friction is 0.050 N. Predict the acceleration and the time to roll 1.00 m.

Show answer

$a = (0.196 - 0.050)/0.30 = 0.49\ \mathrm{m/s^2}$; $t = \sqrt{2/0.49} = 2.0\ \mathrm{s}$

11. **Challenge** A 70 kg window washer pulls himself up in a bosun's chair (10 kg) with a rope that runs from the chair, over a pulley at the roof and back down to his hands. How hard must he pull to rise at constant speed? To accelerate up at $0.50\ \mathrm{m/s^2}$?

Show answer

Two rope segments pull up on the person-plus-chair system: $2T = (80)(9.8)$, so $T = 392$ N at constant speed. Accelerating: $2T = 80(10.3)$, $T = 412$ N.

COMMON MISTAKES

- Putting forces on the free-body diagram that the object exerts on other things. Only forces acting on the object belong there.
- Adding a separate "force of motion" or "force of acceleration." ma is the result of the forces, not one of them.
- Assuming the normal force always equals mg. It does not when there is vertical acceleration or another vertical force.

> **Make it a game · Free-Body Face-Off.** Your parent names a scenario: a book on a table, an elevator accelerating up, a hanging lamp. Sketch the free-body diagram in 30 seconds, label every force and give the net force direction. Swap and critique each other's diagrams. A missing or invented force costs a point. Highest score after six rounds wins.

3.4 Friction: From Leonardo to Coulomb

OBJECTIVES

- Distinguish static from kinetic friction and use $f_s \leq \mu_s N$ and $f_k = \mu_k N$.
- State the classical laws of friction (Leonardo, Amontons, Coulomb) and explain them with the idea of microscopic contact.
- Find μ_s from the angle at which an object begins to slide, $\mu_s = \tan\theta$.
- Apply friction to braking and to the design of tires and brakes.

> **Warm-up.** Put a book on a board or a large hardcover book and slowly tilt it until the book starts to slide. Measure (or estimate) the angle. Now try the same book lying on its spine instead of its cover, and with a second book on top. Predict before each trial whether the angle will change.

WATCH

Understanding the Force of Friction Equation

Flipping Physics · 6:57

https://www.youtube.com/watch?v=quBTyhdVqQE

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Introduction to Static and Kinetic Friction by Bobby

Flipping Physics · 4:03

https://www.youtube.com/watch?v=i-DdiI7gFoM

Before the lab: a short look at the difference between static and kinetic friction.

KEY TERMS

static friction
The force that keeps two surfaces from starting to slide; it adjusts to whatever is needed, up to a maximum $\mu_s N$.

kinetic friction
The force that opposes sliding once it has started: $f_k = \mu_k N$, nearly independent of speed.

coefficient of friction

The ratio μ of the friction force to the normal force for a pair of surfaces; usually $\mu_k < \mu_s$.

angle of repose

The steepest angle at which an object (or a pile of material) stays put; $\tan\theta = \mu_s$.

tribology

The science and engineering of friction, wear and lubrication.

CORE IDEAS

The story. Around 1493 Leonardo da Vinci filled pages of his notebooks with small sketches of blocks on tables being dragged by strings that ran over pulleys to hanging weights. He tried blocks lying on their wide faces and on their narrow sides, and wrote that friction does not depend on the area of contact and that it is about one quarter of the weight. His notebooks were not published, and the discovery was lost. In 1699 the French physicist Guillaume Amontons found the same laws again and announced them to the Paris Academy of Sciences, where they were received with disbelief: surely a wider block rubs more? In 1781 Charles-Augustin de Coulomb, a military engineer who would later measure electric force (Lesson 10.1), won a prize from the Academy for a long study of friction in machines.

How they figured it out. Coulomb tested oak, fir, elm, iron and brass under heavy loads, dry and greased, at different speeds and after different times at rest. He confirmed Amontons' laws: friction is proportional to the load pressing the surfaces together, and nearly independent of the apparent area of contact. He also found that the force needed to *start* sliding is larger than the force needed to keep sliding, and that for kinetic friction the speed matters little. The modern explanation came in the twentieth century: even polished surfaces touch only at microscopic high points. Press harder and the points flatten and the true contact area grows in proportion to the load, whatever the apparent area.

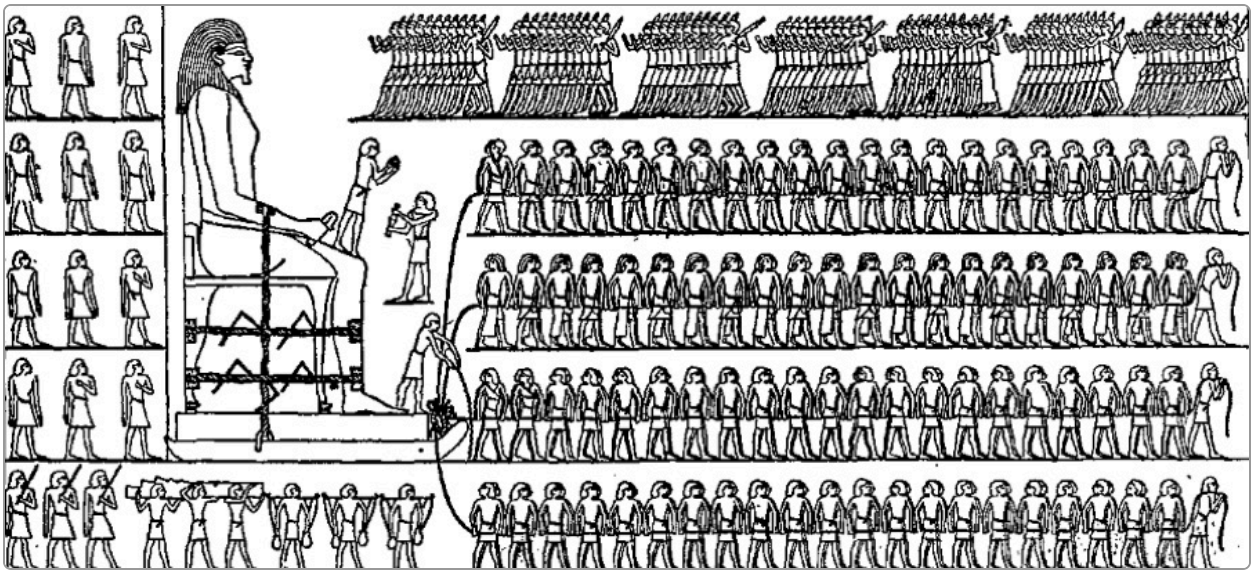
The physics. Friction acts along the surface, opposing sliding (or the tendency to slide). **Static friction** is as large as it needs to be, up to a maximum: $f_s \le \mu_s N$. **Kinetic friction** is $f_k = \mu_k N$. The coefficients depend on the pair of materials: about 0.7 for rubber on dry asphalt, 0.5 for wood on wood, 0.15 for steel on steel with oil, and 0.03 for steel on ice. On a ramp tilted until an object just begins to slide, the

component of gravity along the ramp, $mg\sin\theta$, equals the maximum static friction, $\mu_s mg\cos\theta$, so $\mu_s = \tan\theta$.

The engineering. Friction is both wasted energy and the reason anything works. A car accelerates, steers and stops only because of friction between tires and road. A skidding tire uses kinetic friction, which is smaller than static, so antilock brakes (ABS) pulse the brakes many times a second to keep the tires rolling at the edge of skidding, using static friction and keeping steering control. Engineers fight friction elsewhere with ball bearings, oil and smooth surfaces: studies estimate that about a third of a car's fuel energy goes to overcoming friction in the engine, transmission, tires and brakes.

Try it. In the lab you will test Amontons' laws with a toy car (wheels taped so it slides), a ramp and a spring scale: does friction depend on weight? On area? Is static friction larger than kinetic?

PICTURE STUDY



Moving the Colossus of Djehutihotep, After a tomb painting at Deir el-Bersha (c. 1900 BC), drawn for a 1903 history, 1903 drawing of an ancient painting · Line drawing · Tomb of Djehutihotep, Deir el-Bersha, Egypt

Image: Wikimedia Commons, public domain

This ancient Egyptian painting shows a statue about 7 meters tall being dragged on a sledge. Long before Leonardo, engineers were fighting friction.

Look closely

- Count the rows of men pulling. Estimate how many there are.
- Find the man at the front of the sledge pouring something onto the ground.
- Find the ropes. Are they parallel to the ground or angled?

Talk about it

1. Why would pouring water on sand make the sledge easier to pull? (In 2014, physicists at the University of Amsterdam measured that the right amount of water roughly halves the friction of a sledge on desert sand, because damp sand is stiffer and does not pile up in front of the runners.)
2. If the statue and sledge weighed about 600,000 N and μ_k on wet sand were about 0.3, how much force would the team need? About how much per person?
3. What other ways could the engineers have reduced friction?

LAB

Testing the Laws of Friction

You will test the three classical laws of friction: that friction is proportional to the normal force, that it does not depend on the area of contact, and that static friction is greater than kinetic friction.

You will need

- A toy car with its wheels taped so it cannot roll (or a small wooden block)
- A smooth board or a length of track to use as a ramp; protractor (or tape measure to find height and length)
- A spring scale (0–5 N or 0–10 N)
- A shoe, a brick-shaped block with different-sized faces, and washers or coins for added load
- Kitchen scale

Steps

1. **Hypotheses.** Write a prediction for each law, with a reason.
2. **Slip angle.** Put the taped car on the board and slowly raise one end until the car just starts to slide. Measure the angle (or $\tan\theta = \text{height}/\text{horizontal run}$). Repeat 5 times. Find μ_s.
3. Add washers to the car (tape them on) to double and triple its mass. Find the slip angle again. Does μ_s change?
4. **Kinetic friction.** Tilt the board to a lower angle, give the car a nudge, and find the angle at which it slides down at a steady speed once started. Then $\mu_k = \tan\theta$. Compare with μ_s.
5. **Spring-scale method.** Weigh a shoe. Pull it across a table with the spring scale held level. Record the peak reading just before it starts moving (static) and the steady reading while it slides at constant speed (kinetic). Repeat with 1, 2 and 3 added masses inside the shoe.
6. Graph friction force against normal force for both static and kinetic friction. Find each slope: these are μ_s and μ_k.

7. **Area test.** Pull a rectangular block lying on its largest face, then on its smallest face. Compare the kinetic friction readings.
8. **Improve.** Identify your largest uncertainty (pulling at an angle, uneven speed, reading the jumpy scale). Change one thing and repeat.

Tips

- Pull slowly and steadily; a jerky pull makes the scale bounce.
- Clean, dry surfaces give more repeatable results; dust acts like a lubricant.

Check your work

☐ My graph of friction against normal force is a straight line through the origin, and I found

μ_s and

μ_k from its slopes.

☐ I can explain with microscopic contact points why friction does not depend on apparent area.

☐ I can explain how ABS shortens stopping distances.

LAB NOTEBOOK

1. Data tables for the ramp and spring-scale tests; the friction-against-normal-force graph with two best-fit lines.
2. Free-body diagram of the car at the slipping angle, with weight split into components, showing why $\mu_s = \tan\theta$.
3. Sketch, greatly magnified, two surfaces touching at a few high points. Show how pressing harder increases the true contact area.

KEY FORMULAS

$$f_s \le \mu_s N, \qquad f_k = \mu_k N$$

$$\mu_s = \tan\theta_{\text{slip}}$$

$$a = \mu_k g \quad \text{(skidding on level ground)}$$

From history. Amontons was deaf from childhood and also invented an early air thermometer and a hygrometer. Coulomb's friction study, *Théorie des machines simples*, was written while he was supervising construction work as an army engineer; his friction law $f = \mu N$ is still called Coulomb friction by engineers.

Example 1. A 2.0 kg block rests on a table with $\mu_s = 0.45$ and $\mu_k = 0.30$. What horizontal force starts it moving? What force keeps it moving at constant speed? What is the friction force if you push with only 7.0 N?

Show solution

1. Normal force: $N = mg = (2.0\text{ kg})(9.8\text{ m/s}^2) = 19.6\text{ N}$.
2. To start: $f_{s,\max} = \mu_s N = (0.45)(19.6\text{ N}) = 8.8\text{ N}$.
3. To keep moving: $f_k = \mu_k N = (0.30)(19.6\text{ N}) = 5.9\text{ N}$.
4. With a 7.0 N push the block stays still ($7.0 < 8.8$), so static friction is exactly 7.0 N, opposite the push.

Answer: 8.8 N to start; 5.9 N to keep sliding; with a 7.0 N push the friction is 7.0 N.

Example 2. A toy car with its wheels taped begins to slide down a board when the board is tilted to 24°. Find μ_s.

Show solution

1. At the slipping angle, $mg\sin\theta = \mu_s mg\cos\theta$.
2. $\mu_s = \tan\theta = \tan 24^\circ = 0.45$.

Answer: $\mu_s = 0.45$ (the mass cancels).

Example 3. A car moving at 25 m/s brakes on dry asphalt. Compare the stopping distance if the wheels lock and skid ($\mu_k = 0.70$) with ABS keeping the tires at the edge of static friction ($\mu_s = 0.90$).

Show solution

1. On level ground the friction force is the only horizontal force: $\mu mg = ma$, so $a = \mu g$.
2. Skidding: $a = (0.70)(9.8) = 6.9\ \mathrm{m/s^2}$; $d = \dfrac{v^2}{2a} = \dfrac{(25\ \mathrm{m/s})^2}{2(6.9\ \mathrm{m/s^2})} = 46\ \mathrm{m}$.
3. ABS: $a = (0.90)(9.8) = 8.8\ \mathrm{m/s^2}$; $d = \dfrac{625}{2(8.8)}\ \mathrm{m} = 35\ \mathrm{m}$.

Answer: About 46 m skidding versus 35 m with ABS, and the driver can still steer with ABS.

PRACTICE

1. Easy A 10 kg crate on a floor has $\mu_k = 0.40$. What is the kinetic friction force?

   Show answer

   39 N

2. Easy A box just starts to slide down a board at 30°. Find μ_s.

   Show answer

   $\tan 30^\circ = 0.58$

3. Easy Which is usually larger for the same surfaces, μ_s or μ_k? What do you feel that shows it?

   Show answer

   μ_s. It takes a harder push to start a heavy box moving than to keep it moving.

4. Medium A 25 kg box is pushed with 120 N across a floor with $\mu_k = 0.35$. Find its acceleration.

   Show answer

   $f = 85.8\ \mathrm{N}$; $a = 34.2/25 = 1.37\ \mathrm{m/s^2}$

5. Medium A hockey puck slides 40 m on ice before stopping, starting at 8.0 m/s. Find μ_k.

Show answer

$a = 64/80 = 0.80\ \mathrm{m/s^2}$; $\mu_k = 0.80/9.8 = 0.082$

6. Medium A skid mark at an accident is 32 m long. If $\mu_k = 0.70$, how fast was the car going when it began to skid?

Show answer

$v = \sqrt{2(0.70)(9.8)(32)} = 21\ \mathrm{m/s}$ (about 47 mi/h)

7. Medium Leonardo found friction to be about one quarter of the weight. What coefficient is that, and at what ramp angle would a block begin to slide?

Show answer

$\mu = 0.25$; $\theta = \tan^{-1}(0.25) = 14^\circ$

8. Medium Why do racing cars have wide tires if friction does not depend on area?

Show answer

The simple law is approximate for rubber. Wide tires spread the load, run cooler and wear less, and soft rubber on a wider patch grips somewhat better; they also give more stable handling.

9. Hard A 5.0 kg block slides down a 35° ramp with $\mu_k = 0.25$. Find its acceleration.

Show answer

$a = g(\sin 35^\circ - 0.25\cos 35^\circ) = 9.8(0.574 - 0.205) = 3.6\ \mathrm{m/s^2}$

10. Hard In the picture study, estimate the force to pull a 6.0×10^5 N statue on its sledge with $\mu_k = 0.3$, and the force per person if 172 men pulled.

Show answer

1.8×10^5 N; about 1050 N each, more than the weight of a man, which is why so many were needed and why reducing μ mattered.

11. **Challenge** You push a 20 kg box with force F directed 30° below the horizontal (pushing down and forward). If $\mu_s = 0.50$, what is the smallest F that will start the box moving?

Show answer

$F \cos 30^\circ = 0.50(196 + F \sin 30^\circ)$, so $0.866F - 0.25F = 98$ and $F = 159$ N (pushing down increases the normal force and the friction).

COMMON MISTAKES

- Using $f_s = \mu_s N$ when the object is not on the verge of slipping. Static friction is only as large as needed.
- Assuming the normal force is always mg. On a ramp it is $mg \cos \theta$, and a rope pulling at an angle changes it too.
- Thinking friction always opposes motion. Static friction pushes you forward when you walk and pushes a car forward when it accelerates.

> **Make it a game · Friction Estimation.** Slide a few household objects, such as a mug, a shoe and a book, across a table. Predict which has the highest coefficient of friction, then tilt the surface until each begins to slip and measure the angle. Calculate mu as tan of the angle. Score points for correct ranking and for estimates close to the measured value.

3.5 Inclines, Tension and the Atwood Machine

OBJECTIVES

- Resolve weight into components parallel and perpendicular to an incline.
- Solve problems with several objects connected by strings over pulleys, finding the acceleration and the tension.
- Derive and use the Atwood machine equation, $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2}$.
- Build an Atwood machine and use it to measure g.

> **Warm-up.** Hang two equal bags of coins (or two identical water bottles) over a smooth broom handle with a string. Do they move? Add one coin to one bag. What happens? Predict how fast it will move compared with simply dropping the bag, and write your prediction down.

WATCH

31.4 Worked Example - Atwood Machine

MIT OpenCourseWare · 11:02

https://www.youtube.com/watch?v=FlHKTvUjD6g

> **WHEN TO WATCH**
> After the physics paragraph. Pause at the free-body diagrams and draw your own first.

BACKUP VIDEO

How to Analyze an Atwood's Machine

The Physics Classroom · 8:46

https://www.youtube.com/watch?v=Mmy8C_vvQ8A

Optional: a second worked analysis of the Atwood machine.

KEY TERMS

Atwood machine

Two masses hung from a string over a light, nearly frictionless pulley; the difference in their weights accelerates both.

tension

The pulling force in a string. For a light string over a frictionless pulley, the tension is the same all along it.

system

The group of objects you choose to analyze together. Internal forces (like tension between connected masses) cancel when you treat the system as a whole.

counterweight

A weight that balances most of a load, so that a motor needs to supply only the difference, as in elevators and funiculars.

CORE IDEAS

The story. George Atwood was a tutor and lecturer at Trinity College, Cambridge, the college where Newton had worked a century earlier. He wanted to show his students Newton's laws in action, but falling bodies move too fast to watch. In 1784 he published *A Treatise on the Rectilinear Motion and Rotation of Bodies*, describing a machine he had designed: a tall stand with a light brass pulley at the top, whose axle rested on the rims of four other wheels ("friction wheels") to make it turn almost freely, and two nearly equal weights hanging from a fine silk thread. A seconds pendulum ticked beside a long scale so the class could see the weights pass each mark.

How they figured it out. With two equal weights the machine balances. Add a small rider to one side and the system accelerates, but slowly, because the small difference in weight must accelerate the whole mass of both weights. Atwood could make the acceleration as small as he liked, and show that distances grew as the square of time, just as Galileo had shown with his ramp. A clever ring could lift the rider off partway down; after that the weights moved at constant speed, a beautiful display of the first law. Atwood machines became standard equipment in physics classrooms for two hundred years.

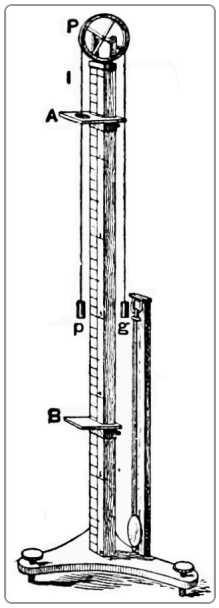
The physics. For connected objects, draw a free-body diagram for each, choose a positive direction that follows the motion around the pulley, and write $\sum F = ma$ for each. In an Atwood machine with $m_2 > m_1$: $m_2 g - T = m_2 a$ and $T - m_1 g = m_1 a$. Adding them eliminates T: $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2}$. On an incline of angle θ, split weight into $mg \sin\theta$ down the slope and $mg \cos\theta$ into the slope; the normal force balances the second (if nothing else pushes), and friction, if any, is $\mu_k mg \cos\theta$. A block on a ramp tied to a hanging weight is a "modified Atwood machine," solved the same way.

The engineering. Every traction elevator is an Atwood machine. The car hangs on one end of the ropes and a counterweight, typically equal to the car plus about 40 to

50% of its rated load, hangs on the other. The motor needs to supply only the difference in weight plus the force to accelerate, which saves a great deal of energy. Funicular railways on steep hills use the same idea: two cars are tied to one cable over a pulley at the top, so the descending car helps lift the ascending one. Some, like the Lynton and Lynmouth Cliff Railway in England (1890), still run by filling the upper car's water tank until it is heavier than the lower one.

Try it. In the lab you will build an Atwood machine with a spool, thread and cups of washers, measure its acceleration, and use it to find g.

PICTURE STUDY



Atwood's Machine, Unknown illustrator, 1905 · Engraving · *The New International Encyclopædia* (1905)
Image: Wikimedia Commons, public domain

For more than a century, nearly every school physics room had a machine like this. It made Newton's second law visible.

Look closely

- Find the pulley at the top and the small wheels around its axle.
- Find the two weights and the graduated scale on the post.
- Look for the ring or platform partway down. What is it for?

Talk about it

1. Why are the two weights nearly equal?
2. Why did Atwood work so hard to reduce friction in the pulley?
3. How would you use this machine to show both Newton's first law and his second law?

Safety first. Pad the floor under the falling cup. Keep fingers away from the spool while it spins.

LAB

An Atwood Machine at the Kitchen Table

You will build an Atwood machine from a thread spool, measure its acceleration for several mass differences, and use your data to measure g and to estimate the effects of friction.

You will need

- An empty thread spool and a smooth pencil or dowel for its axle (or a small pulley from a hardware store)
- A way to hold the axle high: clamped between two stacks of books on a high shelf, or tied above a doorway
- Strong thread or fishing line
- Two identical small paper cups with string handles
- 20 or more identical washers or coins, weighed
- Tape measure, phone with slow-motion video, towel

Steps

1. Mount the pulley at least 1.5 m above the floor. Hang the cups from the two ends of a thread over it.
2. Put 10 washers in each cup and check that they balance. Then move 1 washer from cup 1 to cup 2: the total mass stays the same while the difference grows.
3. **Hypothesis.** Predict the acceleration from $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2}$ for differences of 2, 4 and 6 washers.
4. Hold the heavier cup at a mark, release from rest, and film it falling 1.00 m (or as far as possible). Find $a = 2d/t^2$. 3 trials.
5. Repeat with differences of 4, 6 and 8 washers (moving washers from one cup to the other).
6. Graph a against $\dfrac{m_2 - m_1}{m_1 + m_2}$. Theory says a straight line through the origin with slope g. Find your slope.
7. If the line has a negative intercept on the a-axis, friction in the pulley is at work. Use the intercept to estimate the friction force.

8. **Improve.** Lubricate the axle (a little graphite from a pencil or soap), or use a lighter spool, and repeat two points. Did your g move closer to 9.8 m/s^2?

Tips

- The cups must not swing; release them gently.
- Use the heaviest total mass you can while keeping the motion slow enough to time; friction then matters less.

Check your work

☐ I derived the Atwood equation from free-body diagrams of both masses.

☐ My graph gives a value of

g, and I found its percent error.

☐ I can explain how an elevator counterweight is an Atwood machine.

LAB NOTEBOOK

1. Free-body diagrams of both cups and the derivation of the Atwood equation; data table and graph with slope.
2. Free-body diagram of a block on a ramp tied over a pulley to a hanging weight. Write the two equations and show how to find which way it moves.
3. Paragraph: how much work does an elevator motor save with a counterweight? Use numbers for a 1,000 kg car and a 1,300 kg counterweight carrying 600 kg of passengers.

KEY FORMULAS

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}, \qquad T = \frac{2m_1 m_2 g}{m_1 + m_2}$$

$$F_{\parallel} = mg\sin\theta, \qquad F_{\perp} = mg\cos\theta$$

$$a = g(\sin\theta - \mu_k \cos\theta) \quad \text{(sliding down a ramp)}$$

From history. Atwood also worked as a financial adviser to Prime Minister William Pitt, calculating the costs of public loans. His machine is still made for physics classrooms, now often with a light plastic pulley and an electronic timer or a rotary motion sensor that records the pulley's turning directly.

Example 1. An Atwood machine holds $m_1 = 0.48$ kg and $m_2 = 0.52$ kg. Find the acceleration and the tension.

Show solution

1. $a = \dfrac{(m_2 - m_1)g}{m_1 + m_2} = \dfrac{(0.04\text{ kg})(9.8\text{ m/s}^2)}{1.00\text{ kg}} = 0.392\text{ m/s}^2$.
2. From the lighter mass: $T - m_1 g = m_1 a$, so
   $T = m_1(g + a) = (0.48\text{ kg})(10.19\text{ m/s}^2) = 4.89\text{ N}$.
3. Check with the heavier mass: $m_2(g - a) = (0.52)(9.41)\text{ N} = 4.89\text{ N}$.

Answer: $a = 0.39\text{ m/s}^2$ (one twenty-fifth of g); $T = 4.89$ N.

Example 2. A 5.0 kg box slides down a frictionless ramp inclined at 30°. Find its acceleration and the normal force.

Show solution

1. Along the ramp: $ma = mg\sin\theta$, so $a = (9.8\text{ m/s}^2)\sin 30^\circ = 4.9\text{ m/s}^2$.
2. Perpendicular: $N = mg\cos\theta = (5.0\text{ kg})(9.8\text{ m/s}^2)(0.866) = 42\text{ N}$.

Answer: $a = 4.9\text{ m/s}^2$; $N = 42$ N (less than the weight of 49 N).

Example 3. A 2.0 kg cart on a level table ($\mu_k = 0.10$) is pulled by a string over a pulley at the table's edge to a hanging 0.50 kg mass. Find the acceleration and the tension.

Show solution

1. Cart: $T - \mu_k m_c g = m_c a$, with friction $(0.10)(2.0)(9.8) = 1.96$ N.
2. Hanging mass: $m_h g - T = m_h a$, with weight $(0.50)(9.8) = 4.9$ N.
3. Add: $4.9 - 1.96 = (2.0 + 0.50)a$, so $a = \dfrac{2.94\text{ N}}{2.5\text{ kg}} = 1.18\text{ m/s}^2$.
4. $T = m_h(g - a) = (0.50)(9.8 - 1.18)\text{ N} = 4.31\text{ N}$.

Answer: $a = 1.18\text{ m/s}^2$; $T = 4.31$ N.

Example 4. In an Atwood machine with $m_1 = 0.480$ kg and $m_2 = 0.520$ kg, the heavier mass falls 1.00 m from rest in 2.30 s. What value of g does this give?

Show solution

1. $a = \dfrac{2d}{t^2} = \dfrac{2(1.00\text{ m})}{(2.30\text{ s})^2} = 0.378\text{ m/s}^2$.
2. $g = a\dfrac{m_1 + m_2}{m_2 - m_1} = (0.378\text{ m/s}^2)\dfrac{1.000}{0.040} = 9.45\text{ m/s}^2$.
3. Percent error: $\dfrac{9.8 - 9.45}{9.8} \times 100\% = 3.6\%$, low, as expected from friction and the pulley's own inertia.

Answer: $g \approx 9.5\text{ m/s}^2$, about 4% low.

PRACTICE

1. Easy Find the components of the weight of a 10 kg box on a 25° incline.

   Show answer

   Along: 41.4 N; perpendicular: 88.8 N

2. Easy An Atwood machine has 1.0 kg and 1.2 kg masses. Find the acceleration.

   Show answer

   $a = 0.2(9.8)/2.2 = 0.89\text{ m/s}^2$

3. Easy In an Atwood machine, if both masses are equal, what is the acceleration? Can they still be moving?

   Show answer

   Zero acceleration. They can still be moving at constant velocity if they were given a push.

4. Medium Find the tension in the string for the masses in the previous problem.

   Show answer

   $T = 1.0(9.8 + 0.89) = 10.7\text{ N}$

5. Medium A sled slides down a 20° snowy hill with $\mu_k = 0.10$. Find its acceleration.

   Show answer

   $9.8(\sin 20^\circ - 0.10\cos 20^\circ) = 2.43\text{ m/s}^2$

6. **Medium** A 3.0 kg block on a frictionless table is tied over a pulley to a 1.0 kg hanging block. Find the acceleration and the tension.

Show answer

$a = 9.8/4.0 = 2.45\ \mathrm{m/s^2}$; $T = 3.0 \times 2.45 = 7.35\ \mathrm{N}$

7. **Medium** What force, parallel to a frictionless 15° ramp, will push a 40 kg cart up it at constant speed?

Show answer

$40(9.8)\sin 15^\circ = 101\ \mathrm{N}$

8. **Medium** An elevator car of 1000 kg carries 400 kg of people, with a counterweight of 1200 kg. What force must the motor supply (through friction on the drive sheave) to move the car up at constant speed?

Show answer

Difference in weights: $(1400 - 1200)(9.8) = 1960\ \mathrm{N}$

9. **Hard** A 4.0 kg block sits on a 30° ramp ($\mu_k = 0.20$) tied over a pulley at the top to a hanging 3.0 kg mass. Which way does it move, and with what acceleration?

Show answer

Hanging weight 29.4 N exceeds $mg\sin 30^\circ = 19.6\ \mathrm{N}$, so the block moves up the ramp. Friction $= 0.20(4.0)(9.8)\cos 30^\circ = 6.79\ \mathrm{N}$.
$a = (29.4 - 19.6 - 6.79)/7.0 = 0.43\ \mathrm{m/s^2}$.

10. **Hard** Atwood wanted an acceleration of exactly $g/50$. If the total mass is 0.500 kg, what must the two masses be?

Show answer

$(m_2 - m_1)/0.500 = 1/50$, so $m_2 - m_1 = 0.010$ kg: $m_2 = 0.255$ kg, $m_1 = 0.245$ kg.

11. **Challenge** Show that the tension in an Atwood machine is $T = \dfrac{2m_1 m_2 g}{m_1 + m_2}$, and that it lies between $m_1 g$ and $m_2 g$.

Show answer

$T = m_1(g + a) = m_1 g\left(1 + \dfrac{m_2 - m_1}{m_1 + m_2}\right) = \dfrac{2m_1 m_2 g}{m_1 + m_2}$. Since the lighter mass accelerates up, $T > m_1 g$; since the heavier accelerates down, $T < m_2 g$.

COMMON MISTAKES

- Using mg instead of $mg \sin\theta$ for the pull along a ramp.
- Assuming the tension equals the hanging weight. It does only if nothing accelerates.
- Choosing "up" as positive for both masses of an Atwood machine; when one goes up the other goes down. Choose a direction that follows the string.

> **Make it a game · Incline Component Drill.** Your parent names an incline angle and a mass, for example 5 kg at 25 degrees. Calculate the weight component along and perpendicular to the slope in 60 seconds, then the acceleration if frictionless. Check with a calculator. Add an Atwood twist for a bonus: acceleration of 3 kg and 2 kg masses on a pulley.

UNIT 3

Newton's Laws

Kinematics described motion; dynamics explains it. In 1687 Isaac Newton published three laws that, together with his law of gravity, explained motion from falling apples to orbiting planets. This unit teaches those laws and the method that goes with them: draw a free-body diagram of one object, add the forces as vectors, and set the net force equal to mass times acceleration.

The stories run from Halley's visit to Cambridge, through the Skylab astronauts who were weighed by their inertia and Elisha Otis cutting his own elevator rope, to Leonardo's friction sketches, Coulomb's prize essay and Atwood's classroom machine. The labs measure the thrust of a balloon rocket, build an inertial balance, show that acceleration is proportional to force with a toy car, test the laws of friction, and measure g with an Atwood machine built from a thread spool.

PREREQUISITES

- Units 1–2: vectors and components, kinematics equations, graphing and linearizing
- Right-triangle trigonometry (Lesson 1.5)

MASTERY CHECKLIST

- ☐ I can state Newton's three laws and identify third-law force pairs correctly.
- ☐ I can distinguish mass from weight and explain how mass is measured by inertia.
- ☐ I can draw a complete free-body diagram and apply

$\sum F$ along chosen axes, including

ma elevators and ropes at angles.

- ☐ I can use static and kinetic friction and find

μ_s from a slipping angle.

- ☐ I can solve incline, pulley and Atwood machine problems for acceleration and tension.

OPTIONAL KITS FOR THIS UNIT

- **Arbor Scientific (Atwood machine, low-friction pulleys, carts)**: Classroom physics supplier. A low-friction pulley or a ready-made Atwood machine makes Lesson 3.5 much more precise; their spring scales fit Lessons 3.1 and 3.4.
- **Vernier Go Direct sensors**: School-grade wireless sensors (force, motion, acceleration) that pair with a phone or computer. A force sensor and motion detector turn Lessons 3.3–3.5 into real-time graphs; optional and more expensive than the household methods.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

3.1	**The Principia and Newton's Three Laws**	3 sessions of 50 minutes
3.2	**Inertia and Mass: Weighing Astronauts on Skylab**	3 sessions of 50 minutes
3.3	**F = ma and Free-Body Diagrams: Otis and the Elevator**	3 sessions of 50 minutes
3.4	**Friction: From Leonardo to Coulomb**	3 sessions of 50 minutes
3.5	**Inclines, Tension and the Atwood Machine**	3 sessions of 50 minutes

Unit 3 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$. Draw a free-body diagram for every force problem; it earns points even if the arithmetic slips. Show the equation, the substitution with units and the answer with its unit. The test is worth 100 points.

1. **6 pts**
   State Newton's three laws of motion in your own words.

2. **5 pts**
   A horse pulls a cart forward. By the third law, the cart pulls back on the horse equally. Explain why they can still accelerate.

3. 6 pts

A 1200 kg car accelerates from 0 to 18 m/s in 6.0 s. Find the net force on it.

4. 5 pts

A 90 kg astronaut is on the Moon ($g = 1.62$ N/kg). Give her mass and her weight there.

5. 6 pts

An inertial balance with $k = 40$ N/m oscillates with a period of 0.50 s. Find the oscillating mass.

6. 7 pts

A 70 kg person stands on a scale in an elevator. What does the scale read (in newtons) when the elevator (a) moves up at constant speed, (b) accelerates upward at 2.0 $\mathrm{m/s^2}$, (c) accelerates downward at 2.0 $\mathrm{m/s^2}$?

7. 7 pts

A 900 kg elevator car slows to a stop while moving downward, with an acceleration of 1.5 $\mathrm{m/s^2}$ upward. Find the cable tension.

8. 7 pts

A 15 kg box is pulled across a floor by a rope at 35° above the horizontal with a 60 N force. $\mu_k = 0.30$. Find the normal force, the friction force and the acceleration.

9. 5 pts

A block begins to slide down a board tilted to 28°. Find μ_s.

10. 6 pts

A car skids to a stop from 20 m/s on a road with $\mu_k = 0.60$. How far does it skid?

11. 5 pts

Explain why the friction between two surfaces does not depend (approximately) on the area of contact.

12. 6 pts

A 6.0 kg box slides down a 40° ramp with $\mu_k = 0.25$. Find its acceleration.

13. **8 pts**

An Atwood machine has masses of 2.0 kg and 2.5 kg. Find the acceleration and the tension.

14. **8 pts**

A 4.0 kg cart on a frictionless table is tied over a pulley to a 1.0 kg hanging mass. Find the acceleration and the tension. How long does the hanging mass take to fall 0.80 m from rest?

15. **5 pts**

Explain how Elisha Otis's safety brake worked, in terms of the forces on the platform and the spring.

16. **8 pts**

Essay. Explain, using the story of the *Principia*, how Newton's laws were tested. Why did agreement between theory and measurement in many different situations (falling bodies, the Moon, the planets, tides) make the laws convincing?

ANSWER KEY

1. 1: An object keeps its velocity (at rest or moving straight at constant speed) unless a net force acts. 2: Net force equals mass times acceleration, in the same direction. 3: Forces come in equal and opposite pairs acting on two different objects.
2. The two forces act on different objects. The horse accelerates because the ground pushes it forward (friction on its hooves) more than the cart pulls back; the cart accelerates because the horse's pull exceeds the friction on the cart.
3. $a = 3.0\ \mathrm{m/s^2}$; $F = 3600\ \mathrm{N}$
4. Mass 90 kg; weight 146 N
5. $m = kT^2/4\pi^2 = 40(0.25)/39.5 = 0.25\ \mathrm{kg}$
6. (a) 686 N (b) 826 N (c) 546 N
7. $T = 900(9.8 + 1.5) = 10{,}170\ \mathrm{N}$
8. $N = 147 - 34.4 = 112.6\ \mathrm{N}$; $f = 33.8\ \mathrm{N}$; $a = (49.1 - 33.8)/15 = 1.02\ \mathrm{m/s^2}$
9. $\tan 28^\circ = 0.53$
10. $a = 5.88\ \mathrm{m/s^2}$; $d = 400/11.76 = 34\ \mathrm{m}$
11. Surfaces touch only at microscopic high points. The true contact area is set by how hard the surfaces are pressed together, so it is proportional to the load, not to the apparent area. Spreading the same load over a bigger area just makes each contact point carry less.
12. $9.8(\sin 40^\circ - 0.25\cos 40^\circ) = 9.8(0.643 - 0.192) = 4.42\ \mathrm{m/s^2}$
13. $a = 0.5(9.8)/4.5 = 1.09\ \mathrm{m/s^2}$; $T = 2.0(10.89) = 21.8\ \mathrm{N}$
14. $a = 9.8/5.0 = 1.96\ \mathrm{m/s^2}$; $T = 4.0 \times 1.96 = 7.84\ \mathrm{N}$; $t = \sqrt{1.6/1.96} = 0.90\ \mathrm{s}$
15. The rope's tension held a wagon spring bent above the platform. If the rope broke, the tension disappeared, the spring snapped straight and pushed pawls into toothed rails beside the shaft, so the rails supported the platform instead of the rope.
16. Model answer: Halley asked what orbit an inverse-square force would produce, and Newton showed that it gives an ellipse, matching Kepler's laws drawn from Tycho's data. In the *Principia* Newton then used the same three laws and one law of gravity to calculate falling bodies, pendulums, the Moon's orbit, the planets, comets and tides, and the results matched measurements in every case. A theory that fits one set of data might be fitted to it by chance; a few simple laws that predict many unrelated measurements, including new ones, are very

unlikely to be coincidences. That is why scientists accepted them, and also why they were later ready to test them where they might fail.

PORTFOLIO PIECE: A COUNTERWEIGHTED LIFT

Design and build a small model elevator: a lightweight car (a paper cup or small box) that rides up and down guided by two strings or straws, lifted by a thread over a spool pulley and balanced by a counterweight. Choose the counterweight using an Atwood-machine calculation so that the loaded car (with a specified number of coins as "passengers") rises slowly when you give it a small extra pull or add a small rider. Measure the acceleration for at least three loads, compare with your predictions, and estimate pulley friction. Add a simple safety catch in the spirit of Otis (for example, a springy stick that jams against the guides if the thread goes slack) and test it by releasing the thread. Write a report with free-body diagrams, calculations, data and a photo or sketch of the catch working.

LOOK FOR	WHAT IT MEANS
Design calculation	Counterweight chosen by calculation; free-body diagrams of car and counterweight; predicted accelerations for each load.
Measurements	Acceleration measured for at least three loads, three trials each, compared with predictions and percent differences.
Friction analysis	Pulley friction estimated from the data and discussed.
Safety catch	A working catch that stops the car when the thread goes slack, explained with forces.
Report	Clear, honest, with units on every number and a paragraph on what you would change.

UNIT 4

Circular Motion and Gravitation

Why does the Moon go around the Earth instead of flying away or falling down? Answering that question was Newton's greatest achievement, and it required two ideas: that moving in a circle means accelerating toward the center, and that the same gravity that pulls an apple reaches across space, weakening as the square of the distance. This unit follows Huygens working out the force of circular motion for his clocks, Newton testing his idea on the Moon, Kepler finding the three laws of the planets in Tycho's data, Cavendish measuring gravity between lead balls in a sealed room, and the Apollo 8 crew orbiting the Moon on Christmas Eve, 1968.

You will whirl a stopper on a string, test an inverse-square law with light, draw ellipses and graph planetary data, build a torsion pendulum like Cavendish's, and simulate Newton's mountain in a spreadsheet. By the end you will be able to calculate the speed of the space station, the height of a geostationary satellite and the mass of the Earth.

PREREQUISITES

- Unit 3: Newton's laws and free-body diagrams
- Unit 2: free fall and projectiles
- Scientific notation and square and cube roots on a calculator

MASTERY CHECKLIST

- ☐ I can calculate centripetal acceleration and force and identify the real force that supplies it, including on flat and banked curves.
- ☐ I can explain Newton's Moon test and use

$F = Gm_1m_2/r^2$ and

$g = GM/r^2$.

- ☐ I can state Kepler's three laws and use

$T^2 = a^3$ (years and AU) and Newton's form of the third law.

- ☐ I can explain how Cavendish's torsion balance measured gravity between laboratory masses and use it to find the Earth's mass.
- ☐ I can calculate orbital speed, period, geostationary radius and escape speed, and explain weightlessness as free fall.

OPTIONAL KITS FOR THIS UNIT

- **Celestron binoculars and small telescopes**: Any steadily held 10×50 binoculars or a small telescope shows Jupiter's four large moons, which you can track over several nights for the unit portfolio, as Galileo did in 1610. Optional; published moon data work too.
- **PASCO centripetal force and gravitational torsion balance apparatus**: Laboratory versions of Lessons 4.1 and 4.4 for co-ops and schools. The torsion balance is expensive and delicate; the household labs need neither.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

4.1	**Centripetal Acceleration: Huygens and the Curve**	3 sessions of 50 minutes
4.2	**The Moon and the Apple: Universal Gravitation**	3 sessions of 50 minutes
4.3	**Kepler's Laws of Planetary Motion**	3 sessions of 50 minutes
4.4	**Cavendish Weighs the Earth**	3 sessions of 50 minutes
4.5	**Orbits: From Newton's Mountain to Apollo**	3 sessions of 50 minutes

Unit 4 Test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$, $G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$, $M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m and $GM_E = 3.99 \times 10^{14}\ \mathrm{m^3/s^2}$. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   An object moves in a circle at constant speed. Is it accelerating? Explain, and give the direction of the acceleration.

2. **6 pts**
   A 0.15 kg ball whirls in a horizontal circle of radius 0.50 m once every 0.40 s. Find its speed, its centripetal acceleration and the string tension.

3. **7 pts**
   A car rounds a flat curve of radius 80 m. If $\mu_s = 0.60$, what is the maximum safe speed? What bank angle would let a car take the curve at 20 m/s with no friction?

4. **5 pts**
   When a car turns sharply left, a passenger slides toward the right door. Explain what is really happening, without using "centrifugal force."

5. 7 pts

Explain Newton's Moon test and show the calculation: the Moon is 60 Earth radii away and has a period of 27.3 days at radius 3.84×10^8 m.

6. 6 pts

Find the gravitational force between a 1000 kg satellite and the Earth when the satellite is 7.0×10^6 m from the Earth's center.

7. 5 pts

If the distance between two objects is halved, by what factor does the gravitational force between them change?

8. 6 pts

Find g at an altitude of 1000 km.

9. 6 pts

State Kepler's three laws.

10. 6 pts

Saturn's semi-major axis is 9.54 AU. Find its period. An asteroid has a period of 5.2 years; find its semi-major axis.

11. 5 pts

A comet's distance from the Sun is 0.5 AU at perihelion and 20 AU at aphelion. How many times faster does it move at perihelion?

12. 6 pts

Describe the precautions Cavendish took in his torsion-balance experiment and explain why each was needed.

13. 6 pts

Use g, R_E and G to find the Earth's mass.

14. 7 pts

A satellite orbits 800 km above the Earth's surface. Find its orbital speed and period.

15. **5 pts**

Find the escape speed from the surface of Mars ($M = 6.42 \times 10^{23}$ kg, $R = 3.39 \times 10^{6}$ m).

16. **4 pts**

Why are astronauts in the space station weightless, even though gravity there is about 90% as strong as on the ground?

17. **8 pts**

Essay. Kepler abandoned his beautiful regular-solid model, and later his circle model of Mars, because they did not fit the data. Newton's gravity was accepted because it explained Kepler's laws, the Moon and falling apples together. Using these examples, explain what makes a scientific theory trustworthy.

ANSWER KEY

1. Yes. Its velocity changes direction continuously, so it accelerates; the acceleration points toward the center of the circle.
2. $v = 7.85\ \mathrm{m/s}$; $a = 123\ \mathrm{m/s^2}$; $T = 18.5\ \mathrm{N}$
3. $v = \sqrt{0.60 \times 9.8 \times 80} = 21.7\ \mathrm{m/s}$; $\tan\theta = 400/784 = 0.510$, $\theta = 27^\circ$
4. The passenger's body tends to continue in a straight line (first law) while the car turns left beneath it; the right door then pushes the passenger leftward, supplying the centripetal force.
5. If gravity weakens as $1/r^2$, the Moon's acceleration toward Earth should be $g/3600 = 2.72 \times 10^{-3}\ \mathrm{m/s^2}$. Its centripetal acceleration is $4\pi^2 r/T^2 = 4\pi^2(3.84 \times 10^8)/(2.36 \times 10^6)^2 = 2.72 \times 10^{-3}\ \mathrm{m/s^2}$. They agree, so the same gravity pulls apples and holds the Moon.
6. $F = GMm/r^2 = 8.1 \times 10^3\ \mathrm{N}$
7. It becomes 4 times larger.
8. $9.8(6370/7370)^2 = 7.3\ \mathrm{m/s^2}$
9. 1: Planets move on ellipses with the Sun at one focus. 2: The line from the Sun to a planet sweeps out equal areas in equal times. 3: The square of the period is proportional to the cube of the semi-major axis.
10. $T = 9.54^{1.5} = 29.5$ years; $a = 5.2^{2/3} = 3.0\ \mathrm{AU}$
11. 40 times
12. He sealed the apparatus in a closed room and enclosed the rod in a case to stop air currents; he observed from outside through telescopes so his body heat would not create drafts; he moved the large balls by ropes from outside; he timed the rod's oscillation to calibrate the wire's stiffness; and he repeated the measurement many times and averaged. The gravitational force was only about 10^{-7} N, so any disturbance would have hidden it.
13. $M = gR^2/G = 9.8(6.37 \times 10^6)^2/6.674 \times 10^{-11} = 5.96 \times 10^{24}\ \mathrm{kg}$
14. $r = 7.17 \times 10^6\ \mathrm{m}$; $v = \sqrt{3.99 \times 10^{14}/7.17 \times 10^6} = 7.46\ \mathrm{km/s}$; $T = 2\pi r/v = 6040\ \mathrm{s} = 101\ \mathrm{min}$
15. $\sqrt{2GM/R} = 5.03\ \mathrm{km/s}$
16. They and the station are in continuous free fall around the Earth, so the floor exerts no force on them.
17. Model answer: A trustworthy theory must agree with careful measurements within their uncertainty; Kepler rejected appealing models when they missed

Tycho's data by more than Tycho's errors. It must make definite, testable predictions: Newton's inverse-square law predicted the Moon's acceleration from the apple's, a number no one had adjusted to fit. And it gains strength when one simple principle explains many different phenomena (orbits, falling bodies, tides, comets). Beauty alone is not enough, though the true laws turned out to be beautiful as well. A good scientist holds a theory humbly, ready to test it further.

PORTFOLIO PIECE: WEIGHING JUPITER

Use Kepler's third law in Newton's form to measure the mass of Jupiter from the orbits of its four large moons, which Galileo discovered in 1610. Either observe the moons with binoculars or a small telescope on at least five nights, sketching their positions and working out Callisto's or Ganymede's period from your sketches, or use the published values (Io: 421,700 km, 1.769 days; Europa: 671,000 km, 3.551 days; Ganymede: 1,070,400 km, 7.155 days; Callisto: 1,882,700 km, 16.69 days). Graph T^2 against a^3, find the slope, and calculate Jupiter's mass from $M = 4\pi^2/(G \times \text{slope})$. Compare with the accepted value, 1.898×10^{27} kg, and compare Jupiter's mass with the Earth's. Write a two-page report explaining Kepler's third law, Newton's derivation of it, your graph and your result.

LOOK FOR	WHAT IT MEANS
Data	Observations sketched and dated, or published data tabulated with units converted to meters and seconds.
Graph	T^2 against a^3 with labeled axes, units, best-fit line and slope.
Calculation	Jupiter's mass computed correctly with every step and unit shown; percent difference from the accepted value.
Explanation	Clear derivation of $T^2 = 4\pi^2 a^3/GM$ from gravity supplying the centripetal force.
History and reflection	Connects the result to Galileo, Kepler and Newton, and discusses the main sources of error.

4.1 Centripetal Acceleration: Huygens and the Curve

OBJECTIVES

- Explain why an object moving in a circle at constant speed is accelerating, and toward where.
- Calculate centripetal acceleration and force with $a_c = v^2/r$, $v = 2\pi r/T$ and $F_c = mv^2/r$.
- Identify which real force (tension, friction, gravity, normal force) supplies the centripetal force in a given situation.
- Calculate the friction needed on a flat curve and the bank angle for a design speed.

> **Warm-up.** Tie a soft object (a rolled-up sock with a washer inside) to a string and whirl it in a horizontal circle over your head, outdoors and away from people. Feel the pull in your hand. Whirl faster: what happens to the pull? Shorten the string at the same rotation rate: what happens? Then let go at one point in the circle and notice which way the sock flies. Write all three observations.

WATCH

Centripetal Acceleration Introduction

Flipping Physics · 6:19

https://www.youtube.com/watch?v=RHmrcxQCbwg

> **WHEN TO WATCH**
> After the physics paragraph, before the worked examples.

BACKUP VIDEO

Centripetal force | Physics | Khan Academy

Khan Academy · 13:39

https://www.youtube.com/watch?v=4bMawIIWi7w

For more practice identifying which force is the centripetal force.

KEY TERMS

uniform circular motion

Motion in a circle at constant speed. The velocity keeps changing direction, so the object is accelerating.

centripetal acceleration

The acceleration of an object in circular motion, directed toward the center: $a_c = v^2/r$.

centripetal force

The net force toward the center that produces circular motion, $F_c = mv^2/r$. It is not a new kind of force; it is supplied by tension, friction, gravity, a normal force, or a combination.

period

The time T for one complete trip around the circle; $v = 2\pi r/T$.

banked curve

A road or track tilted toward the inside of a curve, so that part of the normal force points toward the center.

CORE IDEAS

The story. The Dutch scientist Christiaan Huygens (1629–1695) was the finest mathematician-engineer of his generation: he discovered Saturn's ring and its moon Titan with telescopes he ground himself, and he built the first accurate pendulum clocks (Lesson 9.1). While designing clocks with conical pendulums, weights swinging in horizontal circles, he needed to know how hard a weight pulls on its string when it moves in a circle. In a manuscript of 1659 he worked out the answer: the pull grows as the square of the speed and inversely with the radius. He published the result in 1673 at the end of his book on clocks, and the full proof, *De vi centrifuga*, appeared after his death. Newton, working alone in the 1660s, found the same rule and used it to study the Moon.

How they figured it out. Huygens imagined himself riding on the edge of a turning wheel and asked how far a stone, if released, would move away from the wheel in a short time. Since the released stone travels in a straight line along the tangent, while the wheel's rim curves away, the gap grows as the square of the time, like falling. Comparing this "falling away" with ordinary falling, he found the stone's tendency to fly outward is proportional to v^2/r. Newton turned the idea around: the stone

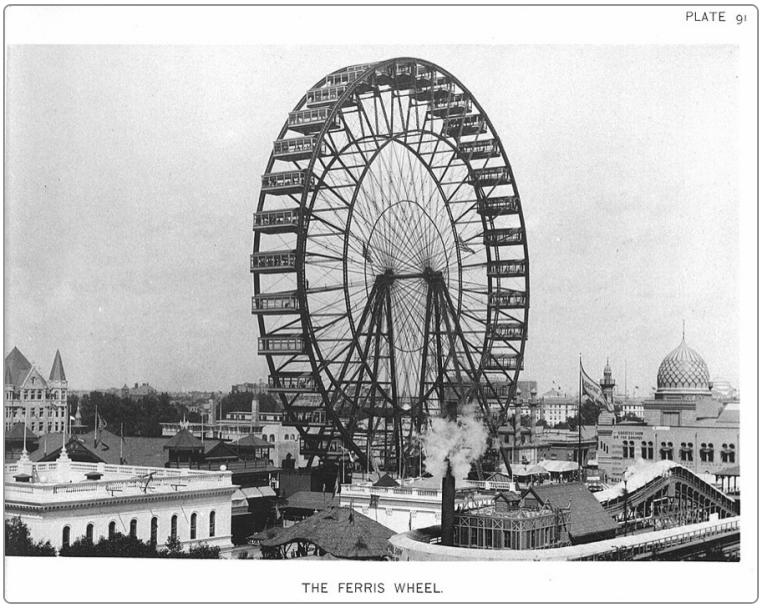
moves in a circle only because something pulls it *inward*, continually bending its straight-line path toward the center. The acceleration of circular motion points toward the center, and we call it **centripetal**, "center-seeking."

The physics. In uniform circular motion the speed is constant but the direction of the velocity changes, so there is an acceleration, $a_c = v^2/r$, always toward the center. By the second law there must be a net inward force, $F_c = mv^2/r$. Ask what supplies it. For a ball on a string, tension. For a car on a flat curve, static friction between tires and road; the car can stay on the curve only if $mv^2/r \leq \mu_s mg$. For the Moon, gravity. On a banked curve, the normal force tilts toward the center and can supply the whole centripetal force at the design speed, where $\tan\theta = \dfrac{v^2}{rg}$. If the inward force disappears, the object does not fly straight outward; it continues along the tangent, as the first law says.

The engineering. Highway engineers set the radius and the banking (called superelevation) of every curve for a design speed, and post lower speed limits where the radius is small. Railway tracks are banked ("canted") the same way. Velodromes for bicycle racing bank as steeply as 45° at the ends. Centrifuges spin samples so fast that the needed centripetal force is thousands of times their weight, separating blood cells from plasma. The first Ferris wheel, built by the engineer George Washington Gale Ferris Jr. for the 1893 World's Columbian Exposition in Chicago, turned slowly enough that its riders felt almost no centripetal acceleration at all.

Try it. In the lab you will whirl a rubber stopper on a string through a tube, with hanging washers supplying the force, and test whether $F = mv^2/r$.

PICTURE STUDY



The Ferris Wheel at the World's Columbian Exposition, Charles Dudley Arnold, 1893 · Photograph · Chicago

Image: Wikimedia Commons, public domain

George Ferris designed his wheel as America's answer to the Eiffel Tower. Its steel spokes are slender rods in tension, like a bicycle wheel's, hanging the rim from the hub.

Look closely

- Find the hub and the two great towers that hold its axle.
- Find the thin spokes. Are they thick enough to push, or only to pull?
- Count the cars you can see and estimate the size of one car from the people near the base.

Talk about it

1. A rider at the top and a rider at the bottom move at the same speed. Which way does the centripetal acceleration point for each?
2. Why does a slowly turning wheel need almost no extra force to keep riders on a circular path?
3. Why would the engineers worry about the wind pushing on such a large wheel?

> **Safety first.** Use a soft rubber stopper and swing it only outdoors or in a clear space, above head height and away from people and windows. Wear safety glasses. Use strong string and check the knot.

LAB

The Whirling Stopper

A rubber stopper whirls in a horizontal circle on a string that runs down through a tube to a set of hanging washers. The washers' weight supplies the centripetal force. You will measure the radius and period and test whether $F = 4\pi^2 mr/T^2$.

You will need

- A rubber stopper or a large eraser with a hole (10–20 g)
- 1.5 m of strong string or fishing line
- A smooth tube about 15 cm long: an empty ballpoint pen barrel or a piece of PVC pipe with taped ends
- Washers and a paper clip to hang them on
- A small binder clip or tape flag as a radius marker
- Kitchen scale, meter stick, stopwatch
- Safety glasses

Steps

1. Weigh the stopper and a set of washers. Tie the stopper to one end of the string, thread the string down through the tube, and hang washers on a paper clip at the bottom.
2. Put a binder clip on the string just below the tube, set so the radius from the tube's top to the stopper's center is 0.60 m. Keeping the clip just below the tube (not touching it) means the radius stays constant.
3. **Hypothesis.** Predict how the period changes if you double the hanging weight at the same radius.
4. Whirl the stopper in a horizontal circle above your head, adjusting the speed until the clip stays just below the tube. A partner times 20 turns; divide by 20.
5. Calculate the predicted force $4\pi^2 mr/T^2$ and compare it with the washers' weight $m_h g$. Find the percent difference.
6. **Fair test.** Keep the radius at 0.60 m and use 4 different numbers of washers. Graph the hanging weight against $1/T^2$. Theory says a straight line through

the origin with slope $4\pi^2 mr$.

7. Now keep the washers fixed and change the radius (0.40, 0.60, 0.80, 1.00 m). Graph T^2 against r.
8. **Improve.** Your measured force is probably a little off, partly because the string is not perfectly horizontal and partly because of friction in the tube. Estimate the angle of the string below horizontal and correct the radius, or smooth the tube; repeat one point.

Tips

- Whirl from the wrist with small circles of the tube; big arm motions make the radius wander.
- Count "zero" at the start of the first turn, not "one."

Check your work

☐ My graph of force against

$1/T^2$ is a straight line, and its slope agrees with

$4\pi^2 mr$.

☐ I can name the force that provides the centripetal force in my lab and in three other situations.

☐ I can calculate the bank angle for a curve.

LAB NOTEBOOK

1. Labeled sketch of the apparatus with free-body diagrams of the stopper and of the washers; data table and the two graphs.
2. Draw a car on a banked curve with its weight and normal force, and show how the normal force splits into a vertical part and a part toward the center.
3. Draw a circle with velocity arrows at four points and acceleration arrows at the same points. Then draw the path of a ball released at one point.

KEY FORMULAS

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$F_c = \frac{mv^2}{r}$$

$$v = \frac{2\pi r}{T}$$

$$\tan\theta = \frac{v^2}{rg} \quad \text{(banked curve, no friction needed)}$$

From history. Ferris's wheel had 36 cars, each able to carry 60 people, and its huge steel axle was made in one piece by a Pittsburgh forge. Huygens' centrifugal-force results also explained why the Earth, spinning, bulges at the equator, a bulge Newton calculated and French expeditions to Lapland and Peru confirmed in the 1730s.

WORKED EXAMPLES

Example 1. A 1200 kg car rounds a flat curve of radius 50 m at 15 m/s. Find its centripetal acceleration, the friction force required, and the smallest coefficient of static friction that will hold it on the road.

Show solution

1. $a_c = \dfrac{v^2}{r} = \dfrac{(15\ \text{m/s})^2}{50\ \text{m}} = 4.5\ \text{m/s}^2$.
2. $F_c = ma_c = (1200\ \text{kg})(4.5\ \text{m/s}^2) = 5400\ \text{N}$, supplied by static friction.
3. $\mu_s mg \geq mv^2/r$, so $\mu_s \geq \dfrac{a_c}{g} = \dfrac{4.5}{9.8} = 0.46$.

Answer: $4.5\ \text{m/s}^2$; 5400 N; $\mu_s \geq 0.46$ (dry pavement is fine; ice is not).

Example 2. A highway curve of radius 200 m is to be banked so that cars at 25 m/s (about 56 mi/h) need no friction at all. What bank angle is required?

Show solution

1. Forces: weight down and normal force perpendicular to the road. Vertical: $N\cos\theta = mg$. Horizontal (toward center): $N\sin\theta = mv^2/r$.
2. Divide: $\tan\theta = \dfrac{v^2}{rg} = \dfrac{(25\text{ m/s})^2}{(200\text{ m})(9.8\text{ m/s}^2)} = 0.319$.
3. $\theta = \tan^{-1}(0.319) = 17.7^\circ$.

Answer: About 18°. (Real highways bank much less, a few degrees, and rely partly on friction, so that a stopped car will not slide down on ice.)

Example 3. The first Ferris wheel was about 76 m across. If it made one turn in 9.0 min, find a rider's speed and centripetal acceleration.

Show solution

1. $r = 38\text{ m}$, $T = 9.0 \times 60\text{ s} = 540\text{ s}$.
2. $v = \dfrac{2\pi r}{T} = \dfrac{2\pi(38\text{ m})}{540\text{ s}} = 0.44\text{ m/s}$.
3. $a_c = \dfrac{v^2}{r} = \dfrac{(0.442\text{ m/s})^2}{38\text{ m}} = 0.0051\text{ m/s}^2$, about 1/2000 of g.

Answer: 0.44 m/s; $5.1 \times 10^{-3}\text{ m/s}^2$, far too small to feel.

Example 4. In the stopper lab, a 12 g stopper whirls on a radius of 0.60 m with a period of 0.75 s. What tension is needed, and what hanging mass supplies it?

Show solution

1. $F_c = \dfrac{4\pi^2 mr}{T^2} = \dfrac{4\pi^2(0.012\text{ kg})(0.60\text{ m})}{(0.75\text{ s})^2} = 0.51\text{ N}$.
2. The hanging washers' weight equals the tension: $m_h = \dfrac{0.51\text{ N}}{9.8\text{ m/s}^2} = 0.052\text{ kg} = 52\text{ g}$.

Answer: 0.51 N, about 52 g of washers.

PRACTICE

1. Easy A ball moves in a circle of radius 0.50 m at 3.0 m/s. Find its centripetal acceleration.

   Show answer

   $18\ \mathrm{m/s^2}$

2. Easy A merry-go-round horse is 4.0 m from the center and goes around once every 8.0 s. Find its speed.

   Show answer

   $v = 2\pi(4.0)/8.0 = 3.1\ \mathrm{m/s}$

3. Easy When you whirl a ball on a string and the string breaks, does the ball fly straight outward, along the tangent, or keep circling? Why?

   Show answer

   Along the tangent, in the direction it was moving at that instant, because with no force it moves in a straight line (first law).

4. Medium Find the tension in a string whirling a 0.20 kg ball in a horizontal circle of radius 0.80 m at 4.0 m/s (ignore gravity's effect on the string angle).

   Show answer

   $0.20(16)/0.80 = 4.0\ \mathrm{N}$

5. Medium What is the maximum speed at which a car can round a flat curve of radius 40 m if $\mu_s = 0.70$?

   Show answer

   $v = \sqrt{\mu_s g r} = \sqrt{0.70 \times 9.8 \times 40} = 16.6\ \mathrm{m/s}$

6. Medium A centrifuge spins a sample at radius 0.10 m at 3000 turns per minute. Find the centripetal acceleration in g.

   Show answer

   $T = 0.020\ \mathrm{s};\ a = 4\pi^2(0.10)/(0.020)^2 = 9870\ \mathrm{m/s^2} \approx 1000\,g$

7. Medium Name the force that supplies the centripetal force for: (a) the Moon orbiting the Earth, (b) a car on a flat curve, (c) clothes in a washing machine's spin cycle, (d) a skater whirling a partner by the hands.

Show answer

(a) gravity (b) static friction (c) the normal force of the drum wall (d) tension in the arms.

8. Medium A railway curve has radius 500 m and is designed for trains at 30 m/s. What bank angle is needed for no sideways force on the rails?

Show answer

$\tan\theta = 900/4900 = 0.184$; $\theta = 10.4^\circ$

9. Hard A 60 kg rider is at the bottom of a vertical loop of radius 10 m, moving at 15 m/s. What force does the seat exert on her? How many times her weight is that?

Show answer

$N - mg = mv^2/r$: $N = 60(9.8 + 22.5) = 1938$ N, about 3.3 times her weight.

10. Hard A conical pendulum (Huygens' clock idea): a 0.50 kg bob on a 1.0 m string swings in a horizontal circle with the string at 30° from vertical. Find the tension and the speed.

Show answer

$T\cos 30^\circ = mg \Rightarrow T = 5.66$ N. $r = 0.50$ m;
$T\sin 30^\circ = mv^2/r \Rightarrow v = \sqrt{gr\tan 30^\circ} = 1.68$ m/s.

11. Challenge Show that for an object in uniform circular motion, $a_c = 4\pi^2 r/T^2$, and use it to find the centripetal acceleration of a point on the Earth's equator due to the Earth's daily rotation ($R = 6.37 \times 10^6$ m, $T = 86{,}164$ s). What fraction of g is it?

Show answer

$a = v^2/r$ with $v = 2\pi r/T$ gives $4\pi^2 r/T^2$. Equator: $a = 0.0339$ m/s^2, about 0.35% of g, which is why you weigh very slightly less at the equator.

COMMON MISTAKES

- Drawing a separate "centripetal force" on the free-body diagram in addition to the real forces. The centripetal force is the net inward force from the real ones.

- Thinking an outward “centrifugal force” throws you against the car door on a curve. Your body tends to go straight; the door pushes you inward.
- Using the diameter instead of the radius.

> **Make it a game · Always Accelerating.** Whirl a keychain on a string in a clear space away from people and objects, or just trace a circle with your finger. Your parent asks 'speed constant, so no acceleration?' and you defend the right answer in 30 seconds, naming direction. Then calculate a_c for a 0.5 m radius and 3 m/s speed.

4.2 The Moon and the Apple: Universal Gravitation

OBJECTIVES

- Retell Newton's "Moon test" and calculate the Moon's centripetal acceleration.
- Explain why gravity follows an inverse-square law, using the spreading of influence over a sphere.
- Use $F = Gm_1m_2/r^2$ to calculate gravitational forces, and $g = GM/R^2$ to find g at different heights.
- Test an inverse-square law with light from a small source.

> **Warm-up.** Shine a small flashlight straight at a wall from 0.5 m away and trace the edge of the bright spot. Step back to 1.0 m and trace it again. How much bigger is the spot's area? If the same light is spread over a bigger area, how much dimmer is each part of it? Write a prediction for 1.5 m.

WATCH

Newton's law of gravitation | Physics | Khan Academy

Khan Academy · 13:04

https://www.youtube.com/watch?v=PIpnGilqefE

> **WHEN TO WATCH**
> After the story paragraph and before the Moon-test worked example.

BACKUP VIDEO

How Newton derived his law of universal gravitation

UNSW Physics · 4:26

https://www.youtube.com/watch?v=Ki5g0CpRj1I

Optional, for a short look at how Newton connected the falling apple and the orbiting Moon.

KEY TERMS

law of universal gravitation

Every two masses attract each other with a force $F = Gm_1m_2/r^2$, along the line between their centers.

gravitational constant

$G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$, the same everywhere in the universe.

inverse-square law

A quantity that falls off as $1/r^2$: double the distance and it becomes one quarter as large.

gravitational field strength

The gravitational force per kilogram at a place, $g = GM/r^2$; at the Earth's surface, 9.8 N/kg.

CORE IDEAS

The story. In 1665 and 1666 Cambridge University closed, and the young Isaac Newton went home to his family's farm at Woolsthorpe in Lincolnshire. There, as he told his friend William Stukeley sixty years later, he saw an apple fall in the orchard and wondered: why does the apple always fall straight down, toward the center of the Earth? And does the same pull reach as far as the Moon? If it does, it might be what keeps the Moon from flying off in a straight line, bending its path into an orbit. The story of the apple is told by Stukeley and others who heard it from Newton himself; it was the question, not the apple, that mattered.

How they figured it out. Newton tested the idea with numbers. The Moon is about 60 Earth radii from the Earth's center. If gravity weakens as the square of the distance, the Moon should fall toward the Earth with an acceleration $60^2 = 3600$ times smaller than an apple's: $9.8/3600 = 0.0027\ \mathrm{m/s^2}$. He then computed the Moon's actual centripetal acceleration from its distance and its period of 27.3 days. In 1666, using a poor value for the Earth's radius, he found them "pretty nearly" in agreement. When better measurements of the Earth became available, the agreement was excellent, and in the *Principia* he announced that the same force that makes an apple fall holds the Moon in its orbit and the planets around the Sun: gravitation is *universal*.

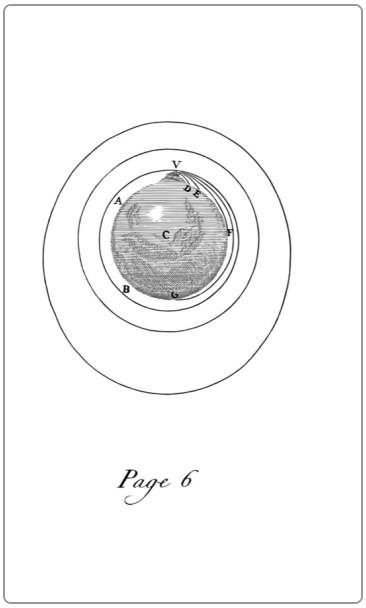
The physics. Newton's law of universal gravitation: $F = \dfrac{Gm_1m_2}{r^2}$. The force is proportional to each mass and falls off as the inverse square of the distance between centers. A simple picture explains the inverse square: whatever spreads out from a point in all directions (light, sound, or gravitational influence) is spread over the surface of a sphere of area $4\pi r^2$, so its strength per unit area falls as $1/r^2$. The constant $G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$ is tiny, so gravity between everyday objects is too small to notice; it matters only when one mass is enormous. At the Earth's surface, $g = GM_E/R_E^2 = 9.8$ N/kg; at height h, $g = GM_E/(R_E + h)^2$.

The engineering. Engineers and geologists measure tiny changes in g from place to place with gravimeters to find dense ore bodies or underground caverns before drilling. The twin GRACE satellites (2002–2017), followed by GRACE Follow-On,

measured how the gravitational pull of the Earth beneath them changes from month to month, mapping groundwater and ice. Every satellite orbit, every trajectory to the Moon and every GPS position depends on calculating gravity accurately, starting from Newton's law.

Try it. In the lab you will measure how the brightness of a small light falls off with distance and test whether it follows an inverse-square law, as Newton's gravity does.

PICTURE STUDY



Newton's Mountain, From Isaac Newton, *A Treatise of the System of the World*, 1728 · Engraving (redrawn) · London

Image: Wikimedia Commons, public domain

Newton imagined throwing an object horizontally from a very high mountain, faster and faster. The figure connects the falling apple to the orbiting Moon.

Look closely

- Find the mountain and the first short path that lands nearby.
- Follow the longer paths. Where does each land?
- Find the path that never lands.

Talk about it

1. Why does a faster throw land farther away, and why does the Earth's curve matter?
2. The object on the outermost path is still falling. Toward what? Why doesn't it hit the ground?
3. How does this picture explain why the Moon does not fall onto the Earth?

LAB

An Inverse-Square Law with Light

Light spreading from a small source is the easiest inverse-square law to measure at home. You will measure brightness against distance and test whether it falls as $1/r^2$, as Newton's gravity does.

You will need

- A small, bright LED (a single-LED flashlight or a phone's flash)
- A phone with the phyphox app ("Light" sensor) or a free light-meter app; or a second phone's camera brightness reading
- Meter stick or tape measure
- A dark room

Steps

1. In a dark room, fix the light so it points at the phone's light sensor (usually near the front camera). Find the sensor's exact spot by covering parts of the phone.
2. **Hypothesis.** Predict the brightness at 40 cm if it is 1000 lux at 20 cm, and explain your reason.
3. Measure the brightness at distances of 20, 30, 40, 50, 60, 80 and 100 cm, measured from the LED to the sensor. Take 3 readings at each.
4. Measure the background reading with the LED off and subtract it from each reading.
5. Graph brightness against distance; then graph brightness against $1/r^2$. Is the second a straight line through the origin?
6. For a stronger test, graph $\log(\text{brightness})$ against $\log(r)$. An inverse-square law gives a slope of -2. Find your slope.
7. **Fair test.** Repeat with a broad source (a lampshade or a flashlight with a wide reflector). Does it still follow $1/r^2$ at short distances? At long ones?
8. **Improve.** Reflections from walls and ceiling add light at large distances. Hang dark cloth or work in the middle of a large room, and repeat the farthest points.

Tips

- Keep the LED and sensor at the same height and pointed straight at each other.
- Measure distance to the LED's chip, not to the flashlight's front.

Check your work

☐ My graph of brightness against

$1/r^2$ (or my log–log slope near

-2) shows an inverse-square law.

☐ I can explain the inverse square by the spreading of light over a sphere.

☐ I can repeat Newton's Moon test with numbers.

LAB NOTEBOOK

1. Data table, the brightness–$1/r^2$ graph and the log–log graph with slope.
2. Draw a point source with two spheres around it at r and $2r$. Show why the same light covers four times the area.
3. Write out Newton's Moon test as a short argument, as if explaining it to Halley: the hypothesis, the prediction, the measurement and the conclusion.

KEY FORMULAS

$$F = \frac{Gm_1m_2}{r^2}, \qquad G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

$$g = \frac{GM}{r^2}$$

$$a_{\mathrm{Moon}} = \frac{4\pi^2 r}{T^2}$$

From history. Newton's rival Robert Hooke had suggested an inverse-square attraction in letters of 1679–1680 and felt he deserved credit; Newton's contribution was to prove what such a force would do, including that a uniform sphere attracts as if all its mass were at its center. Newton's home at Woolsthorpe Manor is now owned by the National Trust, and an apple tree there is said to be descended from the original.

Example 1. Newton's Moon test: the Moon's orbit has radius 3.84×10^8 m (about 60 Earth radii) and period 27.32 days. Find its centripetal acceleration and compare it with $g/60^2$.

Show solution

1. $T = 27.32 \times 86{,}400 \text{ s} = 2.36 \times 10^6 \text{ s}$.
2. $a = \dfrac{4\pi^2 r}{T^2} = \dfrac{4\pi^2(3.84 \times 10^8 \text{ m})}{(2.36 \times 10^6 \text{ s})^2} = 2.72 \times 10^{-3} \text{ m/s}^2$.
3. Prediction from the inverse square: $\dfrac{9.8 \text{ m/s}^2}{3600} = 2.72 \times 10^{-3} \text{ m/s}^2$.

Answer: Both are 2.72×10^{-3} m/s^2. The Moon falls toward the Earth exactly as an inverse-square gravity predicts.

Example 2. Find the gravitational force between the Earth (5.97×10^{24} kg) and the Moon (7.35×10^{22} kg), 3.84×10^8 m apart.

Show solution

1. $F = \dfrac{Gm_1m_2}{r^2} = \dfrac{(6.674 \times 10^{-11})(5.97 \times 10^{24})(7.35 \times 10^{22})}{(3.84 \times 10^8)^2}$ N.
2. Numerator: 2.93×10^{37}; denominator: 1.47×10^{17}.
3. $F = 1.99 \times 10^{20}$ N.

Answer: About 2.0×10^{20} N, and by the third law the Moon pulls the Earth equally hard.

Example 3. The International Space Station orbits about 400 km above the surface ($R_E = 6371$ km). What is g there?

Show solution

1. g falls as $1/r^2$: $g_{\text{ISS}} = g\left(\dfrac{R_E}{R_E + h}\right)^2 = (9.8 \text{ m/s}^2)\left(\dfrac{6371}{6771}\right)^2$.
2. $= (9.8)(0.885) \text{ m/s}^2 = 8.7 \text{ m/s}^2$.

Answer: 8.7 m/s^2, about 89% of its surface value. Astronauts float because they are falling around the Earth, not because gravity is absent.

PRACTICE

1. Easy If the distance between two masses triples, by what factor does the gravitational force change?

   Show answer

   1/9

2. Easy If one mass doubles and the distance stays the same, what happens to the force?

   Show answer

   It doubles.

3. Easy Find the gravitational force between two 70 kg people 1.0 m apart.

   Show answer

   3.3×10^{-7} N, far too small to feel.

4. Medium What is g at a height equal to one Earth radius above the surface?

   Show answer

   $9.8/4 = 2.45\ \mathrm{m/s^2}$

5. Medium Find g on the Moon from its mass 7.35×10^{22} kg and radius 1.737×10^{6} m.

   Show answer

   $g = GM/R^2 = 1.62\ \mathrm{m/s^2}$

6. Medium Mars has 0.107 times the Earth's mass and 0.532 times its radius. Find g on Mars.

   Show answer

   $9.8 \times 0.107/0.532^2 = 3.7\ \mathrm{m/s^2}$

7. Medium Explain why the Moon, which is being pulled toward the Earth, does not fall onto it.

   Show answer

   It is falling, continually, but it is also moving sideways so fast that as it falls the Earth's surface curves away beneath it. Its path bends into an orbit instead of a straight line.

8. Medium Find the Sun's gravitational force on the Earth ($M_S = 1.99 \times 10^{30}$ kg, $r = 1.50 \times 10^{11}$ m).

Show answer

$F = 3.5 \times 10^{22}$ N

9. Hard In 1666 Newton used an Earth radius that was about 15% too small. If he computed the Moon's distance as 60 of those radii, by roughly what fraction would his calculated Moon acceleration be off?

Show answer

The Moon's distance in meters is proportional to the radius he used, and $a = 4\pi^2 r/T^2$ is proportional to that distance, so his computed acceleration was about 15% too small, which is why the agreement was only "pretty nearly."

10. Hard At what height above the Earth's surface is g half its surface value?

Show answer

$(R+h)^2 = 2R^2$, so $h = (\sqrt{2}-1)R = 0.414 \times 6371 = 2640$ km

11. Challenge Where between the Earth and the Moon would a spacecraft feel equal and opposite pulls from each? (Earth 5.97×10^{24} kg, Moon 7.35×10^{22} kg, distance 3.84×10^8 m.)

Show answer

$\dfrac{M_E}{x^2} = \dfrac{M_M}{(d-x)^2}$ gives $\dfrac{d-x}{x} = \sqrt{M_M/M_E} = 0.111$, so $x = d/1.111 = 3.46 \times 10^8$ m from the Earth's center, about 90% of the way to the Moon.

COMMON MISTAKES

- Measuring r from the surface instead of from the center of the Earth.
- Forgetting to square the distance, or squaring the masses.
- Thinking there is no gravity in space. Gravity reaches everywhere; it only weakens with distance.

Make it a game · Moon Test Retell. Retell Newton's Moon test as if you were explaining it to a friend, in 90 seconds, hitting four key facts: the Moon's distance, its orbit time, the centripetal acceleration and the inverse-square comparison. Your parent counts the facts you hit. Then predict how weight changes at twice Earth's radius. Beat your count in round two.

4.3 Kepler's Laws of Planetary Motion

OBJECTIVES

- State Kepler's three laws and explain what each means physically.
- Use $T^2 \propto a^3$ (with T in years and a in astronomical units, $T^2 = a^3$) for objects orbiting the Sun.
- Use the law of equal areas to compare a planet's speeds at perihelion and aphelion.
- Draw ellipses and measure their eccentricity.

> **Warm-up.** Push two thumbtacks into a piece of cardboard 8 cm apart. Tie a loop of string about 24 cm around, put it around the tacks, and pull it tight with a pencil. Draw the curve all the way around. Then move the tacks to 4 cm apart and draw again, and to 0 cm (one tack). What happens to the shape as the tacks come together?

WATCH

Solar System Dynamics: Orbits and Kepler's Laws

NASA Solar System · 6:54

https://www.youtube.com/watch?v=wjOOrr2uPuU

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Kepler's Laws

Physics with Professor Matt Anderson · 12:42

https://www.youtube.com/watch?v=HJ1fHPzhkZM

After the worked examples, for more practice with the third law.

KEY TERMS

ellipse

A closed curve for which the sum of the distances from any point to two fixed points (the foci) is the same.

semi-major axis

Half the longest diameter of an ellipse, a; for an orbit it is the average of the closest and farthest distances.

eccentricity

How stretched an ellipse is: $e = c/a$, where c is the distance from the center to a focus. A circle has $e = 0$.

perihelion and aphelion

The points of an orbit closest to and farthest from the Sun.

astronomical unit (AU)

The average Earth–Sun distance, 1.496×10^{11} m.

CORE IDEAS

The story. In Lesson 1.1 we left Kepler at war with Mars. When he gave up circles, it took him years to find the right curve. He first found that Mars sweeps out equal areas in equal times, moving faster when near the Sun; then, around 1605, that its orbit is an ellipse with the Sun at one focus. He published both laws in *Astronomia nova* (1609). But Kepler wanted more: a single pattern linking all the planets. For years he searched, and on May 15, 1618, he found it: the square of a planet's period is proportional to the cube of its average distance from the Sun. He published this third law in *Harmonices mundi* (*The Harmony of the World*, 1619), a book full of praise to God for the order of the heavens.

How they figured it out. Kepler tried many relationships between the planets' periods and distances, and he wrote that the third law first came into his head on March 8, 1618, was rejected because of an arithmetic error, and then returned and was confirmed with the numbers. Earlier, in *Mysterium cosmographicum* (1596), he had tried to explain the spacing of the six known planets by nesting the five regular solids of geometry between their spheres, a beautiful idea that turned out to be wrong. The lesson is one Kepler himself taught: a theory, however lovely, must yield to the measurements. Seventy years later, Newton showed that all three laws follow from his laws of motion and an inverse-square gravity.

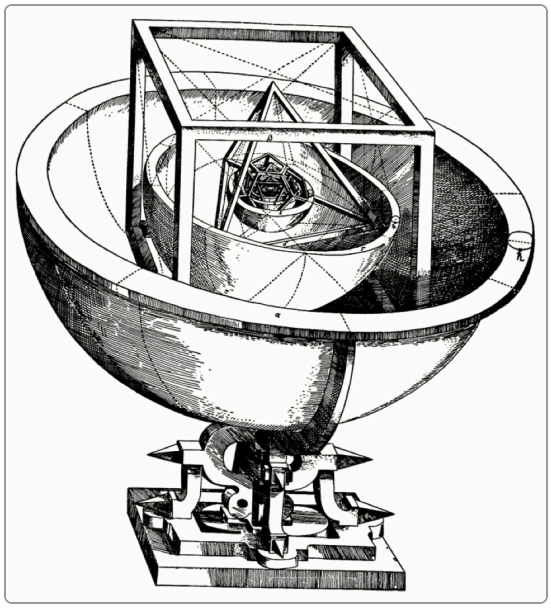
The physics. First law: each planet moves on an ellipse with the Sun at one focus. Most planetary orbits are nearly circular: Earth's eccentricity is 0.017, Mars's 0.093. **Second law:** the line from the Sun to a planet sweeps out equal areas in equal times, so the planet moves fastest at perihelion and slowest at aphelion; for small eccentricity, $\frac{v_p}{v_a} = \frac{r_a}{r_p}$. **Third law:** $T^2 \propto a^3$. For anything orbiting the Sun, with T in years and a in AU, $T^2 = a^3$. Newton's version for any central body of mass M is

$T^2 = \frac{4\pi^2}{GM} a^3$, which lets us find the mass of the Sun, or of any planet with a moon, from an orbit.

The engineering. Spacecraft follow Kepler's laws. The cheapest way to go from Earth to Mars, the Hohmann transfer orbit (Walter Hohmann, 1925), is half of an ellipse that touches Earth's orbit at perihelion and Mars's at aphelion; Kepler's third law gives its travel time, about 8.5 months. Launch windows to Mars open about every 26 months, when the planets are in the right positions. Communications satellites use the third law to sit in geostationary orbit, where the period is exactly one day.

Try it. In the lab you will draw ellipses, measure their eccentricities, and use real planetary data to test the third law with a graph.

PICTURE STUDY



Kepler's Model of the Solar System, Johannes Kepler, 1596 · Engraving · *Mysterium cosmographicum*, Tübingen

Image: Wikimedia Commons, public domain

Kepler's first book proposed that the five regular solids, nested inside one another, set the spacing of the planets. It was wrong, but it started his search for a true pattern.

Look closely

- Find the outermost sphere (Saturn) and the cube just inside it.
- Find the bands drawn on each sphere. What might they represent?
- Count the solids. How many are there, and why did that number matter to Kepler?

Talk about it

1. Why did a model with exactly five solids seem to explain why there were exactly six planets?
2. What kind of evidence finally showed Kepler that this model could not be right?
3. Compare this picture with the third law. Which is more beautiful to you, and which is true?

LAB

Ellipses and the Third Law

You will draw ellipses with string and tacks, measure their eccentricities, and then use real planetary data to test Kepler's third law with a graph.

You will need

- Cardboard, paper, two thumbtacks, string, pencil, ruler
- Graph paper or spreadsheet
- Planet table: Mercury 0.387 AU, 0.241 yr; the second planet 0.723 AU, 0.615 yr; Earth 1.000, 1.000; Mars 1.524, 1.881; Jupiter 5.203, 11.86; Saturn 9.537, 29.46; Uranus 19.19, 84.01; Neptune 30.07, 164.8

Steps

1. Draw four ellipses with the same string loop and the tacks 0, 3, 6 and 9 cm apart. For each, measure the major axis and the focus separation and calculate e.
2. **Hypothesis.** Predict the shape of a graph of T against a for the planets, and of T^2 against a^3.
3. Draw an orbit with $e = 0.093$ (Mars) and one with $e = 0.017$ (Earth) at the same size. Can you tell them from circles by eye?
4. On one of your eccentric ellipses, mark the focus as the Sun. Draw two thin "pie slices" from the focus, one near perihelion and one near aphelion, each with the same area (count graph-paper squares). Compare the lengths of the arcs: these are the distances traveled in equal times.
5. Graph T against a for the eight planets. Then graph T^2 against a^3 (use the first five planets on one graph so the points are not crowded). Find the slope.
6. For a strong test, graph $\log T$ against $\log a$ for all eight planets. Kepler's third law predicts a straight line of slope 1.5.
7. **Use it.** The dwarf planet Ceres has $a = 2.77$ AU. Predict its period from your graph, then compare with the published 4.60 years.
8. **Improve.** What limited the accuracy of your hand-drawn equal-area test? Redraw with thinner slices and check again.

Tips

- Keep the string taut and the pencil upright.
- A spreadsheet's LOG function and trendline make the log–log graph quick.

Check your work

☐ I can state Kepler's three laws and say what each means physically.

☐ My log–log graph gives a slope close to 1.5.

☐ I can use the third law to predict the period of an orbit.

LAB NOTEBOOK

1. Your four ellipses with measurements and eccentricities, and the equal-area slices with arc lengths compared.
2. The T^2–a^3 graph and the log–log graph with slopes.
3. Draw the Hohmann transfer from Earth to Mars to scale, marking where Mars must be at launch for the spacecraft to meet it.

KEY FORMULAS

$$T^2 = a^3 \quad (T \text{ in years}, a \text{ in AU, around the Sun})$$

$$T^2 = \frac{4\pi^2}{GM} a^3$$

$$e = \frac{c}{a}, \qquad r_p = a(1-e), \quad r_a = a(1+e)$$

$$\frac{v_p}{v_a} = \frac{r_a}{r_p}$$

From history. Kepler also wrote the first scientific account of how lenses form images (1604, 1611) and designed the Keplerian telescope, with two convex lenses, which became the standard astronomical telescope. In *Harmonices mundi* he wrote a prayer of thanksgiving to the Creator for letting him see the beauty of His works.

Example 1. Mars's orbit has a semi-major axis of 1.524 AU. Find its period.

Show solution

1. $T^2 = a^3 = (1.524)^3 = 3.540$.
2. $T = \sqrt{3.540} = 1.881$ years (687 days).

Answer: 1.88 years.

Example 2. Earth's orbit has $e = 0.017$. By what factor is Earth faster at perihelion (early January) than at aphelion (early July)?

Show solution

1. $r_p = a(1-e)$ and $r_a = a(1+e)$.
2. $\dfrac{v_p}{v_a} = \dfrac{r_a}{r_p} = \dfrac{1+e}{1-e} = \dfrac{1.017}{0.983} = 1.035$.

Answer: About 3.5% faster in January, which is one reason the northern winter is a few days shorter than the northern summer.

Example 3. A Hohmann transfer orbit from Earth (1.000 AU) to Mars (1.524 AU) is half an ellipse with perihelion at Earth and aphelion at Mars. How long is the trip?

Show solution

1. Semi-major axis: $a = \dfrac{1.000 + 1.524}{2}$ AU $= 1.262$ AU.
2. Full period: $T = a^{3/2} = (1.262)^{1.5} = 1.418$ years.
3. The trip is half an orbit: 0.709 years $\times$ 365.25 days/year $= 259$ days.

Answer: About 259 days, roughly 8.5 months.

Example 4. An ellipse is drawn with a string loop around two tacks. Its longest diameter is 20.0 cm and the tacks (foci) are 6.0 cm apart. Find its eccentricity.

Show solution

1. $a = 10.0$ cm; $c = 3.0$ cm (half the distance between the foci).
2. $e = \dfrac{c}{a} = \dfrac{3.0}{10.0} = 0.30$.

Answer: $e = 0.30$, far more eccentric than any planet's orbit except Mercury's (0.21) comes close.

PRACTICE

1. Easy State Kepler's first law.

   Show answer

   Each planet moves on an ellipse with the Sun at one focus.

2. Easy Jupiter's semi-major axis is 5.20 AU. Find its period.

   Show answer

   $T = 5.20^{1.5} = 11.9$ years

3. Easy Where in its orbit does a planet move fastest?

   Show answer

   At perihelion, closest to the Sun.

4. Medium An asteroid has a period of 8.0 years. What is its semi-major axis?

   Show answer

   $a = 8.0^{2/3} = 4.0$ AU

5. Medium A comet has perihelion 0.60 AU and aphelion 35 AU. Find its semi-major axis, eccentricity and period.

   Show answer

   $a = 17.8$ AU; $e = (35 - 0.6)/(35 + 0.6) = 0.966$; $T = 17.8^{1.5} = 75$ years (Halley's Comet).

6. Medium For the comet in the previous problem, how many times faster is it at perihelion than at aphelion?

Show answer

$35/0.60 = 58$ times

7. Medium Explain the second law in words, and why it means planets speed up near the Sun.

Show answer

Equal areas in equal times: near the Sun the line to the planet is short, so to sweep the same area the planet must cover a longer arc in the same time, that is, move faster.

8. Medium Mercury has $e = 0.206$ and $a = 0.387$ AU. Find its perihelion and aphelion distances.

Show answer

$r_p = 0.387(0.794) = 0.307$ AU; $r_a = 0.387(1.206) = 0.467$ AU

9. Hard Use $T^2 = \dfrac{4\pi^2}{GM}a^3$ with Earth's orbit ($a = 1.496 \times 10^{11}$ m, $T = 3.156 \times 10^7$ s) to find the mass of the Sun.

Show answer

$$M = \frac{4\pi^2 a^3}{GT^2} = 1.99 \times 10^{30} \text{ kg}$$

10. Hard Kepler's regular-solid model gave planet spacings within about 10% for several planets. Why was that not good enough, and what does this teach about testing theories?

Show answer

Tycho's data were accurate to about a minute of arc, much better than 10%, so the model disagreed with the measurements by far more than their uncertainty. A theory must fit within the measurement error, not merely look roughly right.

11. Challenge A Hohmann transfer from Earth to Jupiter (5.20 AU): find the travel time.

Show answer

$a = 3.10$ AU; $T = 3.10^{1.5} = 5.46$ years; half is 2.73 years.

COMMON MISTAKES

- Putting the Sun at the center of the ellipse instead of at a focus.
- Using $T^2 = a^3$ with meters and seconds. That simple form works only in years and AU, and only for orbits around the Sun.
- Thinking the seasons are caused by the changing distance from the Sun. They are caused by the tilt of the Earth's axis; Earth is closest to the Sun in January.

Make it a game · Kepler Quick Check. Your parent names a planet or imagined object with its orbit size in AU. You calculate its year with T squared equals a cubed, in 45 seconds, then say which Kepler law you used. Add the second law: where does a planet move fastest? Five correct answers makes a perfect round.

Scripture. "The heavens declare the glory of God; and the firmament sheweth his handywork." (Psalm 19:1, KJV)

Kepler believed that the order he found in the planets' motions was part of how the heavens declare God's glory.

4.4 Cavendish Weighs the Earth

OBJECTIVES

- Describe how Henry Cavendish measured the gravitational attraction between lead balls with a torsion balance.
- Calculate tiny gravitational forces, and use $g = GM/R^2$ to find the mass and density of the Earth.
- Explain how a torsion pendulum's period measures the stiffness of a wire, $\kappa = 4\pi^2 I/T^2$.
- Build a torsion pendulum and see why Cavendish needed such a sensitive one.

> **Warm-up.** Hang a pencil horizontally from its middle by a long thread (tape the thread on so it balances). Twist it gently a quarter turn and let go. Count how long one slow back-and-forth twist takes. Now try a shorter thread, or a thicker one. Which twists more slowly? Which would be more sensitive to a tiny push?

WATCH

Watch gravity pull two metal balls together

Steve Mould · 12:46

https://www.youtube.com/watch?v=70-_GBymrck

> **WHEN TO WATCH**
> After the story paragraph. Watch closely as the two balls drift together.

Note: Includes a short sponsor message for an online learning site; skip it.

BACKUP VIDEO

Weighing the Earth: The Cavendish Experiment

Physics Almanac · 10:54

https://www.youtube.com/watch?v=5q6LpmxZPdE

With the physics paragraph: a step-by-step derivation of how Cavendish's measurements give G and the mass of the Earth.

KEY TERMS

torsion balance

A horizontal rod hung from a thin fiber; a tiny sideways force at the ends twists the fiber, and the twist angle measures the force.

torsion constant

The stiffness of a fiber against twisting, κ, in N·m per radian: the torque needed to twist it one radian.

torsion pendulum

A torsion balance set twisting back and forth; its period is $T = 2\pi\sqrt{I/\kappa}$, where I is the rod's rotational inertia.

density

Mass per volume. The Earth's average density is about 5.5 $\mathrm{g/cm^3}$, twice that of surface rock.

CORE IDEAS

The story. Newton's law of gravitation contains a constant, G, that Newton could not measure: the attraction between any objects small enough to handle is far too weak to detect. Henry Cavendish, a shy and wealthy English scientist, took up an apparatus designed by his friend the Reverend John Michell, who died in 1793 before he could use it. In 1797 and 1798, in an outbuilding at his house in Clapham, near London, Cavendish hung a wooden rod 1.8 m long from a thin wire, with a 0.73 kg lead ball at each end. Beside them he could swing two large lead balls of 158 kg each, so that each large ball came within 23 cm of a small one. The large balls' gravity should pull the small balls and twist the rod, ever so slightly.

How they figured it out. The force was about one fifty-millionth of the small balls' weight, so the slightest draft or temperature difference would swamp it. Cavendish sealed the apparatus in its room and watched from outside through telescopes, reading a vernier scale at each end of the rod, lit by lamps shining in through the walls. He moved the large balls from one side to the other with ropes from outside and measured the shift of the rod's resting position: a few millimeters. To turn that into a force he needed the wire's stiffness, which he found by timing the rod's slow natural swing: about 7.5 minutes per oscillation with his stiffer wire. From his measurements he reported that the Earth's mean density was 5.48 times that of water (a small arithmetic slip, found later, makes his data give 5.45). The modern value is 5.51.

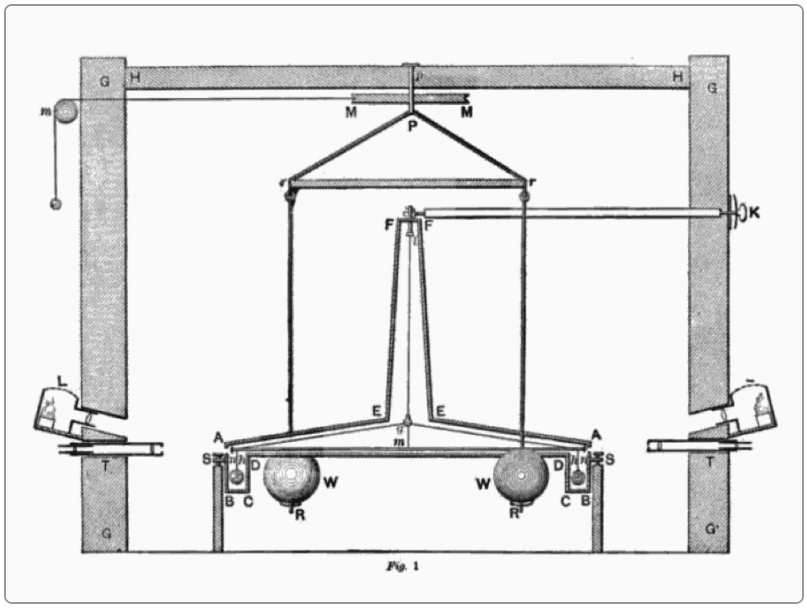
The physics. Cavendish compared two forces on the same small ball: the Earth's pull, its weight mg, and a large lead ball's pull, GMm/r^2. Their ratio gave the Earth's

mass relative to the lead ball's, and so the Earth's density. In today's language his experiment measured $G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$, and then $M_E = gR_E^2/G = 5.97 \times 10^{24}$ kg. The torsion pendulum works because a twisted fiber exerts a restoring torque $\tau = \kappa\theta$; its period is $T = 2\pi\sqrt{I/\kappa}$, so $\kappa = 4\pi^2 I/T^2$. A long period means a weak fiber and a sensitive balance. Cavendish's 7.5-minute period meant that a force of a ten-millionth of a newton could visibly turn his rod.

The engineering. Cavendish's real achievement was isolating a tiny signal from noise: sealing the room, observing remotely, averaging many trials, measuring the instrument's own calibration. Precision engineers do the same today. Torsion balances measure tiny forces in physics laboratories; spring gravimeters, descendants of the idea, detect density differences underground for mining and civil engineering; and G itself is still one of the hardest constants to measure, known only to about two parts in 100,000.

Try it. In the lab you will build a torsion pendulum, measure its torsion constant from its period, and calculate how much a pair of heavy objects would twist it, so you can see the scale of Cavendish's problem.

PICTURE STUDY



Cavendish's Apparatus, Henry Cavendish, 1798 · Engraving · *Philosophical Transactions of the Royal Society*

Image: Wikimedia Commons, public domain

This is Cavendish's own drawing, published with his paper "Experiments to Determine the Density of the Earth." It shows as much about controlling the environment as about measuring gravity.

Look closely

- Find the long rod and the thin wire from which it hangs.
- Find the large balls and the mechanism used to swing them into place.
- Find the telescopes and where they pass through the walls.
- Find the lamps.

Talk about it

1. Why did Cavendish observe from outside the room?
2. Why did he enclose the rod in a narrow case?
3. What would happen to the measurement if one wall of the room were warmer than the other?

LAB

A Torsion Pendulum

You will build a torsion pendulum like Cavendish's, measure its torsion constant from its period, and calculate the twist that gravity from heavy objects would produce.

You will need

- A wooden ruler or dowel about 30 cm long
- Two equal lumps of modeling clay or two equal stacks of coins (weighed)
- Fine thread, about 1 m (sewing thread or thin fishing line)
- A hook or a pencil taped across a doorway or shelf
- Tape, stopwatch, protractor
- A cardboard box with one side cut open, as a draft shield

Steps

1. Attach equal masses at the two ends of the ruler, the same distance from the center. Measure that distance r and the masses m.
2. Tie the thread to the ruler's exact balance point and hang it so the ruler is level. Put the draft-shield box around it.
3. **Hypothesis.** Predict how the period will change if you move the masses halfway to the center.
4. Twist the ruler about 30° and release. Time 5 complete oscillations, 3 times. Find the period T.
5. Calculate $I = 2mr^2$ and $\kappa = 4\pi^2 I/T^2$.
6. **Fair test.** Move the masses to half the distance and measure the period again. By theory, $T \propto r$ when the rod's own inertia is small. Check it.
7. Use your κ to calculate the twist angle if two 5 kg objects (full water jugs) were placed 5 cm from your clay balls: $F = Gm_1m_2/d^2$, $\theta = 2Fr/\kappa$. Could you see it?
8. **Improve.** How long would your thread need to be, or how thin, to give a 7.5-minute period like Cavendish's? Try a longer or finer fiber and measure the new period.

Tips

- Let the pendulum come to rest before each trial; small side-to-side swinging adds error.
- Air currents matter even at this scale; keep the draft shield on and stand still.

Check your work

☐ I measured a torsion constant from the period of oscillation.

☐ I calculated the twist that gravity between lab objects would cause, and can explain why Cavendish needed such a weak fiber.

☐ I can find the Earth's mass from

g, R_E

an G.

d

LAB NOTEBOOK

1. Labeled drawing of your pendulum with dimensions; data table; the calculated κ and the predicted gravitational twist.
2. Top view of Cavendish's apparatus with the four balls and arrows for the attractive forces and the resulting torque.
3. A paragraph: list every precaution Cavendish took and the error each one prevented.

KEY FORMULAS

$$M_E = \frac{gR_E^2}{G}$$

$$\tau = \kappa\theta, \qquad T = 2\pi\sqrt{\frac{I}{\kappa}} \quad \Longrightarrow \quad \kappa = \frac{4\pi^2 I}{T^2}$$

$$I = 2m\left(\tfrac{L}{2}\right)^2 \quad \text{(two small balls on a light rod)}$$

From history. Cavendish was famously shy and published little, but his unpublished notebooks, read decades later by James Clerk Maxwell, showed he had discovered many results in electricity before others did, including Coulomb's law and Ohm's law. Cambridge University's physics laboratory, founded in 1874 with Maxwell as its first director, is named the Cavendish Laboratory after the family.

Example 1. Find the gravitational force between one of Cavendish's large balls (158 kg) and a small ball (0.73 kg) with centers 0.225 m apart. Compare it with the small ball's weight.

Show solution

1. $F = \dfrac{GMm}{r^2} = \dfrac{(6.674 \times 10^{-11})(158)(0.73)}{(0.225)^2}\ \mathrm{N} = 1.52 \times 10^{-7}\ \mathrm{N}.$
2. Weight: $mg = (0.73\ \mathrm{kg})(9.8\ \mathrm{m/s^2}) = 7.15\ \mathrm{N}.$
3. Ratio: $\dfrac{1.52 \times 10^{-7}}{7.15} = 2.1 \times 10^{-8}$, about one fifty-millionth.

Answer: 1.5×10^{-7} N, about 1/47,000,000 of the ball's weight.

Example 2. From $g = 9.80\ \mathrm{m/s^2}$, $R_E = 6.371 \times 10^6$ m and G, find the mass and average density of the Earth.

Show solution

1. $M_E = \dfrac{gR_E^2}{G} = \dfrac{(9.80)(6.371 \times 10^6)^2}{6.674 \times 10^{-11}}\ \mathrm{kg} = 5.96 \times 10^{24}\ \mathrm{kg}.$
2. Volume: $V = \frac{4}{3}\pi R_E^3 = \frac{4}{3}\pi(6.371 \times 10^6\ \mathrm{m})^3 = 1.083 \times 10^{21}\ \mathrm{m}^3.$
3. Density: $\rho = \dfrac{M}{V} = \dfrac{5.96 \times 10^{24}}{1.083 \times 10^{21}}\ \mathrm{kg/m^3} = 5500\ \mathrm{kg/m^3} = 5.5\ \mathrm{g/cm^3}.$

Answer: $M_E \approx 6.0 \times 10^{24}$ kg; density 5.5 $\mathrm{g/cm^3}$, about twice that of surface rock, so the Earth's interior must be much denser.

Example 3. Cavendish's rod carried two 0.73 kg balls 0.93 m from the wire, and oscillated with a period of 7.5 min. Find the torsion constant and the angle through which the two attracting forces in Worked Example 1 would twist the rod.

Show solution

1. $I = 2mr^2 = 2(0.73\text{ kg})(0.93\text{ m})^2 = 1.26\text{ kg}\cdot\text{m}^2$ (ignoring the light rod).
2. $T = 450$ s: $\kappa = \dfrac{4\pi^2 I}{T^2} = \dfrac{4\pi^2(1.26)}{(450)^2}\text{ N}\cdot\text{m/rad} = 2.46\times10^{-4}\text{ N}\cdot\text{m/rad}$.
3. Torque from two forces:
   $\tau = 2Fr = 2(1.52\times10^{-7}\text{ N})(0.93\text{ m}) = 2.83\times10^{-7}\text{ N}\cdot\text{m}$.
4. $\theta = \dfrac{\tau}{\kappa} = \dfrac{2.83\times10^{-7}}{2.46\times10^{-4}}\text{ rad} = 1.15\times10^{-3}\text{ rad}$, which moves the ends of the rod $(0.93\text{ m})(1.15\times10^{-3}) = 1.1\text{ mm}$.

Answer: $\kappa = 2.5\times10^{-4}\text{ N}\cdot\text{m/rad}$; a twist of about 0.0012 rad (0.066°), about a millimeter at the ends. Moving the large balls to the other side doubled the shift.

PRACTICE

1. Easy Find the gravitational force between two 1.0 kg masses 0.10 m apart.

   Show answer

   6.7×10^{-9} N

2. Easy What did Cavendish actually report as his result?

   Show answer

   The Earth's mean density, about 5.48 times that of water (his data correctly give 5.45).

3. Easy Why could Newton not measure G?

   Show answer

   The gravitational force between objects small enough to handle in a laboratory is far too tiny to detect without an extremely sensitive instrument like the torsion balance.

4. **Medium** Use $g = 9.80\ \mathrm{m/s^2}$ at the surface and M_E to find the Earth's radius, as a check.

Show answer

$$R = \sqrt{GM/g} = \sqrt{(6.674 \times 10^{-11})(5.97 \times 10^{24})/9.80} = 6.38 \times 10^6\ \mathrm{m}$$

5. **Medium** A torsion pendulum with $I = 0.0020\ \mathrm{kg \cdot m^2}$ has a period of 30 s. Find κ.

Show answer

$$4\pi^2(0.0020)/900 = 8.8 \times 10^{-5}\ \mathrm{N \cdot m/rad}$$

6. **Medium** If a torsion fiber is replaced by one that is four times stiffer, how does the period change?

Show answer

It halves, since $T \propto 1/\sqrt{\kappa}$.

7. **Medium** Find the Moon's mass from $g_{\mathrm{Moon}} = 1.62\ \mathrm{m/s^2}$ and $R = 1.737 \times 10^6\ \mathrm{m}$.

Show answer

$$M = gR^2/G = 7.33 \times 10^{22}\ \mathrm{kg}$$

8. **Medium** The Earth's crust has density about $2.7\ \mathrm{g/cm^3}$. Cavendish found an average of $5.5\ \mathrm{g/cm^3}$. What does that tell us about the interior?

Show answer

The interior must be much denser than the crust, more than twice as dense on average; today we know the Earth has a dense iron-nickel core.

9. **Hard** Cavendish's period with his first wire was about 15 min, and with the stiffer wire 7.5 min. By what factor was the second wire stiffer?

Show answer

$\kappa \propto 1/T^2$: $(15/7.5)^2 = 4$ times stiffer.

10. Hard With the stiffer wire, a twist of 1.15×10^{-3} rad was produced. What twist would the same force have given with the first wire? Why might Cavendish still have preferred the stiffer one?

Show answer

Four times as much, 4.6×10^{-3} rad. But a longer period (15 min) makes each measurement slower and more affected by slow drifts in temperature and air, so a quicker, stiffer system can give better results overall.

11. Challenge Show that the Earth's density can be written $\rho = \dfrac{3g}{4\pi G R_E}$, and use it to find ρ.

Show answer

$M = gR^2/G$ and $V = \frac{4}{3}\pi R^3$, so $\rho = \dfrac{gR^2/G}{\frac{4}{3}\pi R^3} = \dfrac{3g}{4\pi G R}$. Numerically

$$\rho = \frac{3(9.80)}{4\pi(6.674 \times 10^{-11})(6.371 \times 10^6)} = 5500 \text{ kg/m}^3.$$

COMMON MISTAKES

- Thinking Cavendish measured G directly. He measured the Earth's density; G follows from his data.
- Forgetting that a torsion constant is a torque per angle (N·m/rad), not a force per length like a spring constant.
- Using the diameter of the Earth for R_E.

Make it a game · Weigh the Earth Walkthrough. Teach back Cavendish's experiment: your parent plays a skeptical student, asking five questions such as 'why a torsion balance?' and 'how do you get G?' You answer aloud without notes. Then use g = GM/R squared with R about 6.4 million meters to estimate Earth's mass on paper. Score by percent from the true value.

4.5 Orbits: From Newton's Mountain to Apollo

OBJECTIVES

- Derive and use the speed and period of a circular orbit, $v = \sqrt{GM/r}$ and $T = 2\pi\sqrt{r^3/GM}$.
- Explain weightlessness in orbit as free fall.
- Calculate the radius of a geostationary orbit and the escape speed from the Earth.
- Model an orbit step by step in a spreadsheet, as Newton imagined with his mountain.

> **Warm-up.** Poke two holes near the bottom of a paper cup, cover them with your fingers, and fill the cup with water. Over a sink or outdoors, uncover the holes and watch the water stream out. Now drop the cup while the water is streaming. What happens to the streams while the cup falls? Write down your explanation.

WATCH

Speed of a Satellite in Circular Orbit, Orbital Velocity, Period, Centripetal Force, Physics Problem

The Organic Chemistry Tutor · 17:18

https://www.youtube.com/watch?v=QIaAleG0Eb4

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Apollo 8's Christmas Eve 1968 Message

NASA Video · 2:01

https://www.youtube.com/watch?v=ToHhQUhdyBY

With the story paragraph: the Apollo 8 mission and its Christmas Eve broadcast.

KEY TERMS

orbit

The path of an object falling around another under gravity alone, never reaching the surface because it moves sideways fast enough.

orbital speed

The speed needed for a circular orbit of radius r: $v = \sqrt{GM/r}$, about 7.7 km/s just above the Earth's atmosphere.

geostationary orbit

A circular orbit above the equator with a period of one sidereal day, so a satellite stays over the same spot on the ground.

escape speed

The launch speed needed to leave a planet's gravity entirely (ignoring air): $v_{\text{esc}} = \sqrt{2GM/r}$, 11.2 km/s from the Earth's surface.

weightlessness

The sensation of having no weight, which happens when you and everything around you are in free fall together.

CORE IDEAS

The story. On December 21, 1968, Frank Borman, Jim Lovell and Bill Anders lifted off on Apollo 8, the first time human beings rode the giant Saturn V rocket and the first time anyone left Earth orbit. Three days later they fired their engine behind the Moon, where no radio could reach them, to slow down into lunar orbit; when they came around and radio contact returned, Mission Control knew the burn had worked. They circled the Moon ten times, about two hours per orbit. On the fourth orbit Anders photographed the Earth rising over the lunar horizon, the picture called *Earthrise*. On Christmas Eve, in a broadcast watched by one of the largest audiences in history, the three astronauts took turns reading the first ten verses of Genesis.

How they figured it out. The mathematics of the flight came straight from Newton. In his *Treatise of the System of the World*, Newton imagined a cannon on a mountain so high that it stood above the air, firing horizontally. A slow shot falls nearby; a faster one lands farther around the curved Earth; and at a certain speed the shot falls all the way around and returns to the mountain: an orbit. The Apollo flight dynamics

engineers in Houston used the same physics, solved step by step on computers, to plan a path from Earth to the Moon and a “free-return” trajectory that would swing the spacecraft around the Moon and back to Earth if the engine failed. On the far side, the burn to enter lunar orbit had to slow the spacecraft by about 0.9 km/s, exactly enough for the Moon’s gravity to capture it.

The physics. In a circular orbit, gravity supplies the centripetal force: $\frac{GMm}{r^2} = \frac{mv^2}{r}$, so $v = \sqrt{GM/r}$, independent of the satellite’s mass. The period is $T = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r^3}{GM}}$, which is Kepler’s third law. Astronauts in orbit float not because gravity is absent but because they, their spacecraft and everything in it are falling together around the Earth, just like the cup of water you dropped. To escape a planet entirely, an object needs a speed $\sqrt{2}$ times the circular orbital speed at that distance: $v_{\text{esc}} = \sqrt{2GM/r}$.

The engineering. Orbits are chosen for their jobs. Low Earth orbits (300–2,000 km) circle in about 90–120 minutes and are used for the space station and imaging satellites. GPS satellites orbit about 20,200 km up, twice a day. Weather and communications satellites sit in geostationary orbit, 35,786 km above the equator, where the period equals one rotation of the Earth. Engineers must also plan to remove satellites at the end of their lives, either lowering them to burn up in the atmosphere or raising them to a “graveyard” orbit, so that space stays usable.

Try it. In the lab you will build Newton’s mountain in a spreadsheet: a projectile launched horizontally above a round Earth, calculated in small time steps, and see it fall, land, orbit or escape depending on its speed.

PICTURE STUDY

Earthrise, William Anders, NASA, December 24, 1968 · Color photograph · Apollo 8, in lunar orbit

Image: Wikimedia Commons, public domain

Anders took this picture through a window of the command module on the fourth orbit. It shows the Earth from about 380,000 km away, the first time anyone had seen it whole.

Look closely

- Find the line between day and night on the Earth.
- Look at the lunar horizon. How far away do you think it is?
- Notice how black the sky is. Why are there no stars?

Talk about it

1. The spacecraft was moving about 1.6 km/s around the Moon. Why did the astronauts feel weightless?
2. Why would a photograph from orbit make the Earth look like it is rising?
3. Why do you think the Apollo 8 crew chose to read the opening of Genesis when they saw this view?

Safety first. Do the water-cup drop outdoors or over a sink or tub.

LAB

Newton's Mountain in a Spreadsheet

You will simulate Newton's thought experiment: a projectile launched horizontally from a high mountain above a round Earth. A spreadsheet will compute gravity, velocity and position in small time steps, the way the Apollo engineers' computers did.

You will need

- A computer with a free spreadsheet (Google Sheets or LibreOffice Calc)
- A paper cup, water, a sink (for the opening experiment)

Steps

1. **Free-fall demo.** Poke holes in a paper cup, fill it, and drop it while it streams. Record what happens to the streams and explain it.
2. Set up columns: t, x, y, v_x, v_y, $r = \sqrt{x^2 + y^2}$, $a_x = -GMx/r^3$, $a_y = -GMy/r^3$. Use SI units and $GM = 3.986 \times 10^{14}\ \mathrm{m^3/s^2}$.
3. Start at $x = 0$, $y = 6.771 \times 10^6$ m (a "mountain" 400 km high), $v_y = 0$, and v_x = your launch speed. Use a time step $\Delta t = 10$ s.
4. Each new row: $v_x \leftarrow v_x + a_x \Delta t$, $v_y \leftarrow v_y + a_y \Delta t$, then $x \leftarrow x + v_x \Delta t$, $y \leftarrow y + v_y \Delta t$ (update the velocity first). Fill down 1,000 rows.
5. **Hypothesis.** Predict the launch speed that gives a circular orbit, using $v = \sqrt{GM/r}$.
6. Make a scatter chart of y against x, and draw the Earth as a circle of radius 6.371×10^6 m for reference. Try launch speeds of 4000, 6000, 7672, 9000 and 11,000 m/s. Describe each path.
7. For the circular orbit, find the period from your table (when x returns to zero going positive) and compare with $T = 2\pi\sqrt{r^3/GM}$.
8. **Test the method.** Repeat the circular orbit with $\Delta t = 60$ s and $\Delta t = 1$ s. How does the step size affect whether the orbit closes? This is the engineer's check on a numerical model.
9. **Extend.** Find, by trial, the smallest launch speed that never returns. Compare with $\sqrt{2}$ times the circular speed.

Tips

- Updating velocity before position (the Euler–Cromer method) keeps orbits from spiraling outward.
- Set the chart's axes to the same scale so circles look like circles.

Check your work

- ☐ My spreadsheet shows Newton's mountain: short falls, an orbit and an escape, depending on speed.
- ☐ I found the period of my simulated circular orbit and compared it with the formula.
- ☐ I can explain weightlessness as free fall.

LAB NOTEBOOK

1. Printouts or sketches of your simulated paths for five launch speeds, labeled.
2. Derivation of $v = \sqrt{GM/r}$ and $T = 2\pi\sqrt{r^3/GM}$ from $F = ma$ and Newton's law of gravitation.
3. Explain the water-cup experiment and connect it to astronauts floating on the space station.

KEY FORMULAS

$$v = \sqrt{\frac{GM}{r}}$$

$$T = 2\pi\sqrt{\frac{r^3}{GM}}$$

$$v_{\text{esc}} = \sqrt{\frac{2GM}{r}}$$

$$GM_E = 3.986 \times 10^{14}\ \text{m}^3/\text{s}^2$$

From history. Apollo 8 splashed down in the Pacific Ocean on December 27, 1968. Its success cleared the way for the first landing seven months later. Jim Lovell later commanded Apollo 13, and Frank Borman read the Genesis passage's closing verses and then wished a merry Christmas to "all of you on the good Earth."

Example 1. The International Space Station orbits about 410 km above the surface ($R_E = 6371$ km). Find its speed and period.

Show solution

1. $r = 6371 \text{ km} + 410 \text{ km} = 6.781 \times 10^6 \text{ m}$.
2. $v = \sqrt{\dfrac{GM}{r}} = \sqrt{\dfrac{3.986 \times 10^{14} \text{ m}^3/\text{s}^2}{6.781 \times 10^6 \text{ m}}} = 7670 \text{ m/s}$.
3. $T = \dfrac{2\pi r}{v} = \dfrac{2\pi(6.781 \times 10^6 \text{ m})}{7670 \text{ m/s}} = 5560 \text{ s} = 92.6 \text{ min}$.

Answer: About 7.67 km/s (27,600 km/h), once around every 93 min, about 15 or 16 times a day.

Example 2. Find the radius and altitude of a geostationary orbit, whose period is one sidereal day, 86,164 s.

Show solution

1. From $T = 2\pi\sqrt{r^3/GM}$: $r = \left(\dfrac{GMT^2}{4\pi^2}\right)^{1/3}$.
2. $r = \left(\dfrac{(3.986 \times 10^{14})(86{,}164)^2}{4\pi^2}\right)^{1/3} \text{ m} = 4.216 \times 10^7 \text{ m}$.
3. Altitude: $42{,}164 \text{ km} - 6371 \text{ km} = 35{,}790 \text{ km}$.

Answer: $r = 42{,}160$ km; altitude about 35,800 km.

Example 3. Apollo 8 orbited about 110 km above the Moon ($R = 1737$ km, $GM = 4.905 \times 10^{12} \text{ m}^3/\text{s}^2$). Find the period.

Show solution

1. $r = 1847 \text{ km} = 1.847 \times 10^6 \text{ m}$.
2. $T = 2\pi\sqrt{\dfrac{r^3}{GM}} = 2\pi\sqrt{\dfrac{(1.847 \times 10^6)^3}{4.905 \times 10^{12}}} \text{ s} = 2\pi(1133 \text{ s}) = 7120 \text{ s}$.
3. $7120 \text{ s} = 1.98 \text{ h}$.

Answer: About 2 hours per orbit, matching the roughly two-hour orbits the crew flew.

Example 4. Find the escape speed from the Earth's surface.

Show solution

1. $v_{\text{esc}} = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2(3.986 \times 10^{14})}{6.371 \times 10^{6}}}\ \text{m/s} = 11{,}200\ \text{m/s}.$

Answer: 11.2 km/s. Apollo 8's third-stage engine brought the spacecraft close to this speed for its trip to the Moon.

PRACTICE

1. Easy Does the orbital speed of a satellite depend on its mass?

   Show answer

   No: $v = \sqrt{GM/r}$ depends only on the planet's mass and the orbit radius.

2. Easy Find the speed of a satellite in a circular orbit of radius 7.0×10^{6} m around the Earth.

   Show answer

   $v = \sqrt{3.986 \times 10^{14}/7.0 \times 10^{6}} = 7550\ \text{m/s}$

3. Easy Why do astronauts on the space station float?

   Show answer

   They and the station are in free fall together around the Earth, so nothing presses on them; gravity still acts, at about 89% of its surface strength.

4. Medium Find the period of the orbit in the previous problem.

   Show answer

   $T = 2\pi r/v = 5825\ \text{s} = 97\ \text{min}$

5. Medium GPS satellites have a period of half a sidereal day (43,082 s). Find their orbital radius and altitude.

   Show answer

   $r = (GMT^2/4\pi^2)^{1/3} = 2.656 \times 10^{7}$ m; altitude 20,200 km

6. Medium Find the escape speed from the Moon ($GM = 4.905 \times 10^{12}\ \text{m}^3/\text{s}^2$, $R = 1.737 \times 10^{6}$ m).

   Show answer

   2.38 km/s, which is why the small lunar module could return to orbit.

7. Medium If a satellite's orbit radius is made four times larger, by what factors do its speed and period change?

Show answer

Speed halves ($v \propto r^{-1/2}$); the period grows 8 times ($T \propto r^{3/2}$).

8. Medium In Newton's mountain picture, why must the mountain be above the atmosphere?

Show answer

Air resistance would slow the projectile, so it would spiral down and land instead of orbiting.

9. Hard The Moon orbits the Earth at 3.84×10^8 m. Use $T = 2\pi\sqrt{r^3/GM}$ to find its period in days.

Show answer

$T = 2.37 \times 10^6$ s $= 27.4$ days

10. Hard A spacecraft in a 200 km circular parking orbit must reach escape speed at that height. How much must it speed up?

Show answer

$r = 6.571 \times 10^6$ m: $v_{\text{circ}} = 7.79$ km/s, $v_{\text{esc}} = 11.01$ km/s; it must gain about 3.2 km/s.

11. Challenge Show that for any circular orbit the kinetic energy per kilogram is $\frac{1}{2}v^2 = \dfrac{GM}{2r}$, and use the escape-speed formula to show that escape requires exactly twice the kinetic energy of a circular orbit at the same radius.

Show answer

$v^2 = GM/r$ gives $\frac{1}{2}v^2 = GM/2r$. Escape: $\frac{1}{2}v_{\text{esc}}^2 = GM/r$, which is twice $GM/2r$.

COMMON MISTAKES

- Using the altitude instead of the distance from the planet's center for r.
- Saying there is no gravity in orbit.
- Thinking a heavier satellite needs to go faster to stay in the same orbit.

Make it a game · Orbit or Fall. Your parent names an orbital height, such as low Earth orbit or geostationary. Estimate, before calculating, whether the speed will be bigger or smaller than the Space Station's 7.7 km/s, and the period bigger or smaller than 90 minutes. Then compute. Score a point per correct direction and extra for percent within ten.

Scripture. “In the beginning God created the heaven and the earth.” (Genesis 1:1, KJV)

These were the first words the Apollo 8 crew read to the world from lunar orbit on Christmas Eve, 1968.

UNIT 4

Circular Motion and Gravitation

Why does the Moon go around the Earth instead of flying away or falling down? Answering that question was Newton's greatest achievement, and it required two ideas: that moving in a circle means accelerating toward the center, and that the same gravity that pulls an apple reaches across space, weakening as the square of the distance. This unit follows Huygens working out the force of circular motion for his clocks, Newton testing his idea on the Moon, Kepler finding the three laws of the planets in Tycho's data, Cavendish measuring gravity between lead balls in a sealed room, and the Apollo 8 crew orbiting the Moon on Christmas Eve, 1968.

You will whirl a stopper on a string, test an inverse-square law with light, draw ellipses and graph planetary data, build a torsion pendulum like Cavendish's, and simulate Newton's mountain in a spreadsheet. By the end you will be able to calculate the speed of the space station, the height of a geostationary satellite and the mass of the Earth.

PREREQUISITES

- Unit 3: Newton's laws and free-body diagrams
- Unit 2: free fall and projectiles
- Scientific notation and square and cube roots on a calculator

MASTERY CHECKLIST

- ☐ I can calculate centripetal acceleration and force and identify the real force that supplies it, including on flat and banked curves.
- ☐ I can explain Newton's Moon test and use

$F = Gm_1m_2/r^2$ and

$g = GM/r^2$.

- ☐ I can state Kepler's three laws and use

$T^2 = a^3$ (years and AU) and Newton's form of the third law.

- ☐ I can explain how Cavendish's torsion balance measured gravity between laboratory masses and use it to find the Earth's mass.
- ☐ I can calculate orbital speed, period, geostationary radius and escape speed, and explain weightlessness as free fall.

OPTIONAL KITS FOR THIS UNIT

- **Celestron binoculars and small telescopes**: Any steadily held 10×50 binoculars or a small telescope shows Jupiter's four large moons, which you can track over several nights for the unit portfolio, as Galileo did in 1610. Optional; published moon data work too.
- **PASCO centripetal force and gravitational torsion balance apparatus**: Laboratory versions of Lessons 4.1 and 4.4 for co-ops and schools. The torsion balance is expensive and delicate; the household labs need neither.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

4.1	Centripetal Acceleration: Huygens and the Curve	3 sessions of 50 minutes
4.2	The Moon and the Apple: Universal Gravitation	3 sessions of 50 minutes
4.3	Kepler's Laws of Planetary Motion	3 sessions of 50 minutes
4.4	Cavendish Weighs the Earth	3 sessions of 50 minutes
4.5	Orbits: From Newton's Mountain to Apollo	3 sessions of 50 minutes

Unit 4 Test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$, $G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$, $M_E = 5.97 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m and $GM_E = 3.99 \times 10^{14}\ \mathrm{m^3/s^2}$. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   An object moves in a circle at constant speed. Is it accelerating? Explain, and give the direction of the acceleration.

2. **6 pts**
   A 0.15 kg ball whirls in a horizontal circle of radius 0.50 m once every 0.40 s. Find its speed, its centripetal acceleration and the string tension.

3. **7 pts**
   A car rounds a flat curve of radius 80 m. If $\mu_s = 0.60$, what is the maximum safe speed? What bank angle would let a car take the curve at 20 m/s with no friction?

4. **5 pts**
   When a car turns sharply left, a passenger slides toward the right door. Explain what is really happening, without using "centrifugal force."

5. 7 pts

Explain Newton's Moon test and show the calculation: the Moon is 60 Earth radii away and has a period of 27.3 days at radius 3.84×10^8 m.

6. 6 pts

Find the gravitational force between a 1000 kg satellite and the Earth when the satellite is 7.0×10^6 m from the Earth's center.

7. 5 pts

If the distance between two objects is halved, by what factor does the gravitational force between them change?

8. 6 pts

Find g at an altitude of 1000 km.

9. 6 pts

State Kepler's three laws.

10. 6 pts

Saturn's semi-major axis is 9.54 AU. Find its period. An asteroid has a period of 5.2 years; find its semi-major axis.

11. 5 pts

A comet's distance from the Sun is 0.5 AU at perihelion and 20 AU at aphelion. How many times faster does it move at perihelion?

12. 6 pts

Describe the precautions Cavendish took in his torsion-balance experiment and explain why each was needed.

13. 6 pts

Use g, R_E and G to find the Earth's mass.

14. 7 pts

A satellite orbits 800 km above the Earth's surface. Find its orbital speed and period.

15. **5 pts**

Find the escape speed from the surface of Mars ($M = 6.42 \times 10^{23}$ kg, $R = 3.39 \times 10^{6}$ m).

16. **4 pts**

Why are astronauts in the space station weightless, even though gravity there is about 90% as strong as on the ground?

17. **8 pts**

Essay. Kepler abandoned his beautiful regular-solid model, and later his circle model of Mars, because they did not fit the data. Newton's gravity was accepted because it explained Kepler's laws, the Moon and falling apples together. Using these examples, explain what makes a scientific theory trustworthy.

ANSWER KEY

1. Yes. Its velocity changes direction continuously, so it accelerates; the acceleration points toward the center of the circle.
2. $v = 7.85\ \mathrm{m/s}$; $a = 123\ \mathrm{m/s^2}$; $T = 18.5\ \mathrm{N}$
3. $v = \sqrt{0.60 \times 9.8 \times 80} = 21.7\ \mathrm{m/s}$; $\tan\theta = 400/784 = 0.510$, $\theta = 27^\circ$
4. The passenger's body tends to continue in a straight line (first law) while the car turns left beneath it; the right door then pushes the passenger leftward, supplying the centripetal force.
5. If gravity weakens as $1/r^2$, the Moon's acceleration toward Earth should be $g/3600 = 2.72 \times 10^{-3}\ \mathrm{m/s^2}$. Its centripetal acceleration is $4\pi^2 r/T^2 = 4\pi^2(3.84 \times 10^8)/(2.36 \times 10^6)^2 = 2.72 \times 10^{-3}\ \mathrm{m/s^2}$. They agree, so the same gravity pulls apples and holds the Moon.
6. $F = GMm/r^2 = 8.1 \times 10^3\ \mathrm{N}$
7. It becomes 4 times larger.
8. $9.8(6370/7370)^2 = 7.3\ \mathrm{m/s^2}$
9. 1: Planets move on ellipses with the Sun at one focus. 2: The line from the Sun to a planet sweeps out equal areas in equal times. 3: The square of the period is proportional to the cube of the semi-major axis.
10. $T = 9.54^{1.5} = 29.5$ years; $a = 5.2^{2/3} = 3.0\ \mathrm{AU}$
11. 40 times
12. He sealed the apparatus in a closed room and enclosed the rod in a case to stop air currents; he observed from outside through telescopes so his body heat would not create drafts; he moved the large balls by ropes from outside; he timed the rod's oscillation to calibrate the wire's stiffness; and he repeated the measurement many times and averaged. The gravitational force was only about 10^{-7} N, so any disturbance would have hidden it.
13. $M = gR^2/G = 9.8(6.37 \times 10^6)^2/6.674 \times 10^{-11} = 5.96 \times 10^{24}\ \mathrm{kg}$
14. $r = 7.17 \times 10^6\ \mathrm{m}$; $v = \sqrt{3.99 \times 10^{14}/7.17 \times 10^6} = 7.46\ \mathrm{km/s}$; $T = 2\pi r/v = 6040\ \mathrm{s} = 101\ \mathrm{min}$
15. $\sqrt{2GM/R} = 5.03\ \mathrm{km/s}$
16. They and the station are in continuous free fall around the Earth, so the floor exerts no force on them.
17. Model answer: A trustworthy theory must agree with careful measurements within their uncertainty; Kepler rejected appealing models when they missed

Tycho's data by more than Tycho's errors. It must make definite, testable predictions: Newton's inverse-square law predicted the Moon's acceleration from the apple's, a number no one had adjusted to fit. And it gains strength when one simple principle explains many different phenomena (orbits, falling bodies, tides, comets). Beauty alone is not enough, though the true laws turned out to be beautiful as well. A good scientist holds a theory humbly, ready to test it further.

PORTFOLIO PIECE: WEIGHING JUPITER

Use Kepler's third law in Newton's form to measure the mass of Jupiter from the orbits of its four large moons, which Galileo discovered in 1610. Either observe the moons with binoculars or a small telescope on at least five nights, sketching their positions and working out Callisto's or Ganymede's period from your sketches, or use the published values (Io: 421,700 km, 1.769 days; Europa: 671,000 km, 3.551 days; Ganymede: 1,070,400 km, 7.155 days; Callisto: 1,882,700 km, 16.69 days). Graph T^2 against a^3, find the slope, and calculate Jupiter's mass from $M = 4\pi^2/(G \times \text{slope})$. Compare with the accepted value, 1.898×10^{27} kg, and compare Jupiter's mass with the Earth's. Write a two-page report explaining Kepler's third law, Newton's derivation of it, your graph and your result.

LOOK FOR	WHAT IT MEANS
Data	Observations sketched and dated, or published data tabulated with units converted to meters and seconds.
Graph	T^2 against a^3 with labeled axes, units, best-fit line and slope.
Calculation	Jupiter's mass computed correctly with every step and unit shown; percent difference from the accepted value.
Explanation	Clear derivation of $T^2 = 4\pi^2 a^3/GM$ from gravity supplying the centripetal force.
History and reflection	Connects the result to Galileo, Kepler and Newton, and discusses the main sources of error.

UNIT 5

Work, Energy and Power

Energy is the most useful idea in physics, because it is conserved: it changes form but never appears or disappears. This unit follows the discovery of that law, from 's Gravesande's balls dropped into clay, which showed that the effect of motion grows as v^2, and Galileo's pendulum, which always returns to its starting height, to Joule's paddle wheel, which turned falling weights into warm water at a fixed exchange rate. You will learn to calculate work, kinetic energy, gravitational and elastic potential energy, and power, and to use them to solve problems that would be hard with forces alone.

The engineering runs from James Watt's steam engine and his horsepower to pumped-storage power stations, brakes, elevators and roller coasters. The labs warm sand by flipping it, make craters in flour, launch a ball with a spring, measure your own horsepower on the stairs, and find the lowest start that sends a Hot Wheels car around a loop. The midterm exam follows this unit.

PREREQUISITES

- Units 2–4: kinematics, Newton's laws, free-body diagrams and centripetal force
- Hooke's law (Lesson 1.4)

MASTERY CHECKLIST

- ☐ I can calculate work with $W = Fd\cos\theta$ and kinetic energy with $\frac{1}{2}mv^2$, and use the work–energy theorem.
- ☐ I can calculate gravitational and elastic potential energy and solve problems with conservation of mechanical energy.
- ☐ I can explain Joule's experiment and convert mechanical energy to a temperature rise with $Q = mc\Delta T$.
- ☐ I can calculate power with $P = W/t$ and $P = Fv$, and convert between watts, horsepower and kilowatt-hours.
- ☐ I can find the minimum height for a loop-the-loop, the g-forces in a loop, and the efficiency of a real machine.

OPTIONAL KITS FOR THIS UNIT

- **K'NEX roller coaster building sets**: Ages 9 and up. Larger coaster sets have a motorized lift and adjustable track, useful for measuring drop heights, loops and efficiency in Lesson 5.5 and the portfolio.
- **Hot Wheels track sets with a loop**: Ages 4 and up. A basic loop set is enough for Lesson 5.5; extra straight track lengthens the ramp.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

5.1	Joule's Paddle Wheel: Energy Is Conserved	3 sessions of 50 minutes
5.2	Work and Kinetic Energy: Balls Dropped into Clay	3 sessions of 50 minutes
5.3	Potential Energy and Conservation: Galileo's Interrupted Pendulum 3 sessions of 50 minutes	
5.4	Power and Watt's Horsepower	3 sessions of 50 minutes
5.5	Roller Coasters, Loops and Efficiency	3 sessions of 50 minutes

Unit 5 Test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$, $c_{\text{water}} = 4186\ \mathrm{J/(kg \cdot K)}$ and $1\ \mathrm{hp} = 746\ \mathrm{W}$. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   State the law of conservation of energy and give an example of energy changing form three times.

2. **6 pts**
   A $5.0\ \mathrm{kg}$ mass falls $2.0\ \mathrm{m}$ ten times, turning a paddle wheel in $0.50\ \mathrm{kg}$ of water. If all the energy heats the water, find the temperature rise.

3. **5 pts**
   Explain why Joule's paddle-wheel result showed that heat is not a fluid.

4. **6 pts**
   A rope pulls a crate $15\ \mathrm{m}$ across a floor with a force of $120\ \mathrm{N}$ at 20° above the horizontal. How much work does the rope do?

5. **6 pts**
   A $0.060\ \mathrm{kg}$ tennis ball is served at $50\ \mathrm{m/s}$. Find its kinetic energy. How much work did the racket do on it (starting from rest)?

6. 7 pts

A 1500 kg car at 24 m/s brakes to a stop in 48 m. Find the average braking force. If the speed had been 12 m/s, what stopping distance would the same force give?

7. 5 pts

’s Gravesande dropped balls into clay. If a ball dropped from h makes a dent 1 cm deep, how deep is the dent from $4h$? Why did this support Leibniz rather than Descartes?

8. 6 pts

A pendulum bob is released 0.45 m above its lowest point. Find its speed at the bottom. If a nail catches the string, how high does it rise on the other side?

9. 7 pts

A spring ($k = 800$ N/m) is compressed 6.0 cm and launches a 40 g ball upward. Predict the maximum height. The ball actually rises 2.9 m; find the efficiency.

10. 6 pts

A roller-coaster car starts from rest at 45 m and passes over a 20 m hill. Find its speed on the hill (no friction).

11. 6 pts

A 70 kg student climbs 4.5 m of stairs in 5.0 s. Find her power in watts and horsepower.

12. 6 pts

A truck engine delivers 150 kW to its wheels at 25 m/s. What is the total resisting force?

13. 5 pts

A 2.0 kW heater runs for 3.0 h. How much energy does it use in kWh and in joules? At 15 cents per kWh, what does it cost?

14. 6 pts

Find the minimum release height and the minimum speed at the top for a frictionless loop of radius 0.20 m.

15. **6 pts**

A car enters a circular loop of radius 6.0 m at the bottom at 17 m/s. What is the rider's apparent weight in g there?

16. **4 pts**

Explain why modern roller-coaster loops are teardrop-shaped (clothoids).

17. **8 pts**

Essay. Using the stories of 's Gravesande, Galileo, Watt and Joule, explain how the idea of energy grew from an argument about moving balls into a law that engineers use to design machines.

ANSWER KEY

1. Energy is never created or destroyed; it only changes form or moves. Example: chemical energy in food → kinetic energy of a cyclist → thermal energy in the brakes when she stops (or gravitational potential energy on a hill).
2. $W = 10(5.0)(9.8)(2.0) = 980$ J; $\Delta T = 980/(0.50 \times 4186) = 0.47$ K
3. A fixed amount of mechanical work always produced the same amount of heating, and the heat was produced from nothing but motion; a conserved fluid could not be created that way. Heat must be a form of energy.
4. $120(15)\cos 20^\circ = 1690$ J
5. 75 J; 75 J
6. $KE = 432{,}000$ J; $F = 9000$ N; at half speed, a quarter of the distance: 12 m
7. 4 cm. From $4h$ the speed is only doubled, so the effect grew as v^2 (mv^2), not as v (mv).
8. $v = \sqrt{2(9.8)(0.45)} = 2.97$ m/s; it rises to 0.45 m (ignoring friction).
9. $PE_s = 1.44$ J; $h = 1.44/(0.040 \times 9.8) = 3.7$ m; efficiency $= 2.9/3.7 = 79\%$
10. $v = \sqrt{2(9.8)(25)} = 22$ m/s
11. $P = 70(9.8)(4.5)/5.0 = 617$ W $= 0.83$ hp
12. $F = P/v = 6000$ N
13. 6.0 kWh $= 2.16 \times 10^7$ J; 90 cents
14. $h = 0.50$ m; $v = \sqrt{9.8 \times 0.20} = 1.40$ m/s
15. $1 + 289/58.8 = 5.9\,g$
16. A clothoid has a large radius near the bottom and a small radius near the top. The train can enter fast on a gentle curve (low g-force) and pass over the top slowly in a tight curve (still enough centripetal force to stay on the track), so the g-forces stay moderate everywhere, unlike a circular loop, which needs about 6 g at the bottom.
17. Model answer: 's Gravesande's clay showed that the effect of a moving body grows with mv^2, the quantity now called kinetic energy. Galileo's interrupted pendulum showed that a falling body gains exactly enough speed to rise back to its starting height, the idea behind potential energy and its conservation. Watt, building engines to replace horses, defined power and showed that doing the same work with less fuel was possible, the beginning of efficiency. Joule proved that work and heat are two forms of one conserved quantity, completing the law. Engineers now use energy accounting to design brakes, elevators, power

stations and roller coasters, choosing materials and sizes so that every joule is accounted for.

PORTFOLIO PIECE: AN ENERGY-ACCOUNTED MARBLE COASTER

Design and build a marble roller coaster from foam pipe insulation (cut lengthwise), card stock or Hot Wheels track, with a first drop, at least one loop, a hill and a final stop in a catch cup. Before building, draw a scaled side view with every height labeled, predict the marble's speed at four points from energy conservation, and size the loop with $h_{\text{min}} = 2.5r$. After building, measure the marble's speed at the same four points with slow-motion video, calculate the efficiency from the start to each point, and adjust your predictions using the measured efficiency. Make the coaster run reliably 8 times out of 10. Write a two-page engineering report with your drawing, an energy table (predicted and measured), efficiency calculations and the changes you made.

LOOK FOR	WHAT IT MEANS
Design and predictions	Scaled drawing with heights; predicted speeds from energy conservation; loop sized by calculation.
Working coaster	Includes a drop, a loop, a hill and a safe stop; works at least 8 times out of 10.
Measurements	Speeds measured by video at four points, three trials each, with a table comparing predicted and measured values.
Energy accounting	Efficiency calculated at each point; losses explained (friction, rolling, air); corrected predictions.
Iteration and report	At least one documented design change with before-and-after data; clear, honest report with units.

5.1 Joule's Paddle Wheel: Energy Is Conserved

OBJECTIVES

- Describe Joule's paddle-wheel experiment and what the "mechanical equivalent of heat" means.
- State the law of conservation of energy and trace energy through a chain of transformations.
- Use $Q = mc\Delta T$ and $E = mgh$ to relate mechanical work to a temperature rise.
- Measure the warming of sand by repeated falls.

> **Warm-up.** Rub your palms together hard for ten seconds. Then bend a paper clip back and forth ten times and touch the bend (carefully). Finally, hammer a nail into a scrap of wood a few times and touch the nail head. In each case, where did the warmth come from? Write one sentence for each.

WATCH

What is the First Law of Thermodynamics?

The Royal Institution · 4:09

https://www.youtube.com/watch?v=1OFlW8OXN64

> **WHEN TO WATCH**
> After the story paragraph.

BACKUP VIDEO

Historical Didactical Video on Joule's Paddlewheel Experiment

HistoLab_EUF · 4:41

https://www.youtube.com/watch?v=oP5ET5uxPow

With the "how they figured it out" paragraph: a reconstruction of Joule's apparatus.

KEY TERMS

energy

The capacity to do work or to heat something, measured in joules (J). It comes in many forms: kinetic, gravitational, elastic, thermal, chemical, electrical, light.

conservation of energy

Energy is never created or destroyed; it changes form or moves from one object to another, and the total stays the same.

mechanical equivalent of heat

The amount of work that produces a given amount of heating: 4.186 J of work raises 1 g of water by 1 °C (one calorie).

specific heat capacity

The energy needed to raise 1 kg of a substance by 1 °C (or 1 K): $c = 4186\ \mathrm{J/(kg \cdot K)}$ for water, about 830 for sand, 450 for steel.

thermal energy

The energy of the random motion of a substance's molecules; a higher temperature means more of it.

CORE IDEAS

The story. In the 1840s most scientists thought heat was an invisible fluid, called caloric, that flowed from hot objects to cold ones and could be neither made nor destroyed. James Prescott Joule (1818–1889), a young man from a family business near Manchester who did experiments in a laboratory at home, suspected otherwise. He found that electric currents, the compression of air and the friction of water all produced heat, and always in the same proportion to the work done. If work could be turned into heat at a fixed rate, then heat was not a substance at all; it was a form of energy.

How they figured it out. Joule's most famous apparatus, refined from 1845 to 1850, was a brass paddle wheel inside an insulated copper vessel of water, with fixed vanes to stop the water from simply swirling. Two lead weights, hung on cords wound around a drum, fell slowly through about a meter and a half, turning the paddles and churning the water. Joule then wound the weights back up and let them fall again, twenty times, and measured the temperature rise of the water: a fraction of a degree, read on thermometers he had made to show one two-hundredth of a degree Fahrenheit. He corrected for the friction of the pulleys and for heat leaking in or out. In 1850 he reported that 772 foot-pounds of work raise one pound of water by one degree Fahrenheit, within 1% of the modern value. Joule, a devout Christian, said in an 1847 lecture that the great agents of nature are "by the Creator's fiat, indestructible."

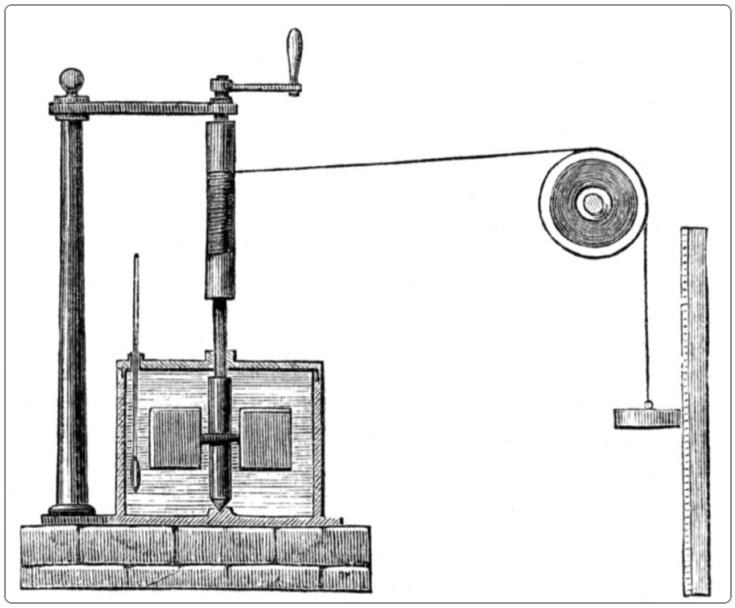
The physics. Joule's result is the **conservation of energy**: the energy of the falling weights, mgh, did not disappear; it became the thermal energy of the water. Heating a mass m of a substance with specific heat c through ΔT takes $Q = mc\Delta T$. For

water, $c = 4186\ \mathrm{J/(kg \cdot K)}$, which is very large: lifting 1 kg of water 427 m and dropping it would warm it by only 1 °C. The unit of energy is named the **joule** in his honor: 1 J is the work done by a 1 N force over 1 m. Energy can be followed through any chain of changes, like money through accounts, and the books always balance.

The engineering. Engineers do energy accounting constantly. A car's brakes turn its kinetic energy into heat in the brake discs, which must be massive enough not to overheat on a long mountain descent. Electric and hybrid cars use regenerative braking to turn some of that energy back into electrical energy in the battery instead. Power-plant engineers track every joule from fuel to steam to turbine to wire, and the lost energy appears as heat carried away by cooling water or towers. Every machine's efficiency is a statement of where its energy goes.

Try it. In the lab you will repeat Joule's idea with a tube of sand: turn it over a hundred times so the sand falls the length of the tube each time, and measure how much warmer it gets.

PICTURE STUDY



Joule's Apparatus, Unknown engraver, for *Harper's New Monthly Magazine*, 1869 · Wood engraving · *Harper's New Monthly Magazine*, New York

Image: Wikimedia Commons, public domain

Twenty years after Joule's experiments, this engraving explained them to ordinary American readers. It shows the falling weights, the pulleys and the vessel of water in which the paddles turned.

Look closely

- Find the weights and trace their cords to the drum on the spindle.
- Find the vessel that holds the water and the paddles.
- Find any handle or crank used to wind the weights back up.

Talk about it

1. Why did Joule make the weights fall slowly instead of dropping them freely?
2. Why must the vessel be insulated, and what would happen to the result if heat leaked out?
3. Where else in the apparatus does some of the falling weights' energy go?

Safety first. Seal the tube's ends firmly with tape so no sand escapes. Do the lab over a tray.

LAB

Joule's Experiment in a Tube of Sand

You will turn the potential energy of falling sand into thermal energy, as Joule turned falling weights into warm water, and compare the temperature rise with $\Delta T = Ngh/c$.

You will need

- A cardboard mailing tube or a stiff plastic tube about 1 m long with end caps (or two 2-liter bottles taped mouth to mouth)
- About 0.5 kg of dry sand
- A digital probe thermometer reading to 0.1 °C
- Duct tape, meter stick, an oven mitt or insulated glove
- Foam pipe insulation or a towel to wrap the tube (optional)

Steps

1. Let the sand sit at room temperature for an hour. Measure the inside length of the tube; the sand falls about this far each flip.
2. **Hypothesis.** Predict the temperature rise after 100 flips, using $\Delta T = Ngh/c$.
3. Pour in the sand, measure its starting temperature by pushing the probe into it through one end, then cap and tape the tube.
4. Holding the tube by the middle with gloves (so your hands do not warm it), flip it end over end 100 times, pausing each time for the sand to fall all the way.
5. Quickly open one end and measure the temperature again. Record the rise.
6. **Fair test.** Repeat with 200 flips (starting again from room temperature). Is the rise doubled? Repeat with half as much sand. Does the rise stay the same, as the equation predicts?
7. Compare each measured rise with the prediction and calculate the percent of the energy that stayed in the sand.
8. **Improve.** Heat leaks into the tube walls and the air. Wrap the tube in insulation and repeat 100 flips. Did more of the energy stay in the sand?

Tips

- Flip at a steady pace; count aloud in tens.
- Sand that is even slightly damp sticks and falls less than the full length.

Check your work

- ☐ I predicted and measured the warming of sand from repeated falls.
- ☐ I can explain why the amount of sand does not matter in the prediction.
- ☐ I can trace energy from my arms to the sand's height to its motion to heat.

LAB NOTEBOOK

1. Data table for each trial: flips, mass, starting and final temperature, measured and predicted rise, percent retained.
2. An energy-flow diagram (boxes and arrows) for one flip: chemical energy in your muscles → potential energy of the sand → kinetic energy → thermal energy, with losses marked.
3. Write a short paragraph explaining why Joule's result showed that heat is not a fluid.

KEY FORMULAS

$$E_{\text{grav}} = mgh$$

$$Q = mc\Delta T$$

$$1 \text{ cal} = 4.186 \text{ J}$$

$$\Delta T = \frac{Ngh}{c} \quad (\text{sand falling } h, N \text{ times})$$

From history. Joule's work was at first ignored, but the young William Thomson (later Lord Kelvin) heard him speak in 1847, became his friend and collaborator, and helped make energy conservation the foundation of thermodynamics (Lesson 8.5). Hermann von Helmholtz in Germany and Julius Robert Mayer reached the same principle in the same years from different directions.

Example 1. Joule found that 772 ft · lbf of work raises 1 lb of water by $1^\circ\mathrm{F}$. Convert this to joules per calorie, using $1\ \mathrm{ft}\cdot\mathrm{lbf} = 1.356\ \mathrm{J}$, $1\ \mathrm{lb} = 453.6\ \mathrm{g}$ and $1^\circ\mathrm{F} = \frac{5}{9}{}^\circ\mathrm{C}$.

Show solution

1. Work: $772 \times 1.356\ \mathrm{J} = 1047\ \mathrm{J}$.
2. Heat: one pound of water raised $1^\circ\mathrm{F}$ is $453.6\ \mathrm{g} \times \frac{5}{9}{}^\circ\mathrm{C} = 252\ \mathrm{g}\cdot{}^\circ\mathrm{C} = 252\ \mathrm{cal}$.
3. Ratio: $\dfrac{1047\ \mathrm{J}}{252\ \mathrm{cal}} = 4.15\ \mathrm{J/cal}$, compared with the modern $4.186\ \mathrm{J/cal}$, a difference of less than 1%.

Answer: $4.15\ \mathrm{J/cal}$, within 1% of today's value.

Example 2. In an apparatus like Joule's, two 13 kg weights each fall 1.6 m, twenty times, and all the energy goes into 6.0 kg of water. Find the temperature rise.

Show solution

1. Work: $W = N(2m)gh = 20(26\ \mathrm{kg})(9.8\ \mathrm{m/s^2})(1.6\ \mathrm{m}) = 8150\ \mathrm{J}$.
2. $\Delta T = \dfrac{W}{m_w c} = \dfrac{8150\ \mathrm{J}}{(6.0\ \mathrm{kg})(4186\ \mathrm{J/(kg\cdot K)})} = 0.32\ \mathrm{K}$.

Answer: Only about $0.32^\circ\mathrm{C}$, which is why Joule needed such precise thermometers.

Example 3. A 1500 kg car brakes from 25 m/s to rest. If all its kinetic energy goes into four steel brake discs with a total mass of 32 kg ($c = 450\ \mathrm{J/(kg\cdot K)}$), how much do they warm?

Show solution

1. $KE = \frac{1}{2}mv^2 = \frac{1}{2}(1500\ \mathrm{kg})(25\ \mathrm{m/s})^2 = 4.69 \times 10^5\ \mathrm{J}$.
2. $\Delta T = \dfrac{Q}{mc} = \dfrac{4.69 \times 10^5\ \mathrm{J}}{(32\ \mathrm{kg})(450\ \mathrm{J/(kg\cdot K)})} = 33\ \mathrm{K}$.

Answer: About $33^\circ\mathrm{C}$ per stop, which is why repeated hard stops make brakes very hot.

Example 4. Sand in a closed tube falls 1.0 m each time the tube is flipped. Show that the temperature rise does not depend on the amount of sand, and find it for 100 flips ($c_{\text{sand}} = 830\ \text{J/(kg}\cdot\text{K)}$).

Show solution

1. Energy per flip: mgh; after N flips: $Nmgh$.
2. $\Delta T = \dfrac{Nmgh}{mc} = \dfrac{Ngh}{c}$; the mass cancels.
3. $\Delta T = \dfrac{100(9.8\ \text{m/s}^2)(1.0\ \text{m})}{830\ \text{J/(kg}\cdot\text{K)}} = 1.2\ \text{K}$.

Answer: About 1.2°C for 100 flips, whatever the mass of sand (some heat will leak to the tube and air, so expect less).

PRACTICE

1. Easy How much energy is needed to warm 2.0 kg of water by 10°C?

   Show answer

   $Q = 2.0(4186)(10) = 8.4 \times 10^4\ \text{J}$

2. Easy Convert 500 cal to joules.

   Show answer

   $2093\ \text{J} \approx 2.1\ \text{kJ}$

3. Easy A ball is dropped and bounces lower each time until it stops. Where did its energy go?

   Show answer

   Into thermal energy of the ball and floor (they warm slightly) and into sound.

4. Medium A 50 kg load is lifted 2.0 m. How much energy is that? If that energy could all go into 1.0 kg of water, how much warmer would the water get?

   Show answer

   $980\ \text{J};\ \Delta T = 0.23\ \text{K}$

5. Medium How high would you have to lift 1 kg of water and drop it to warm it by 1°C?

   Show answer

   $h = c/g = 4186/9.8 = 427\ \text{m}$

6. Medium Water at the top of a 50 m waterfall falls to the bottom. If all its potential energy became thermal energy in the water, how much warmer would it be at the bottom?

Show answer

$\Delta T = gh/c = 9.8(50)/4186 = 0.12$ K (Joule hoped to measure this at a waterfall in Switzerland in 1847).

7. Medium A 0.20 kg lead weight falls and hits a concrete floor at 15 m/s without bouncing. If half its kinetic energy heats the lead ($c = 128$ J/(kg · K)), how much does it warm?

Show answer

$KE = 22.5$ J; half is 11.25 J; $\Delta T = 11.25/(0.20 \times 128) = 0.44$ K

8. Medium Describe the energy changes in a hydroelectric power plant, from water behind the dam to a light bulb.

Show answer

Gravitational potential energy of the water → kinetic energy of falling water → kinetic energy of the spinning turbine → electrical energy in the generator → light and heat in the bulb, with some energy lost as heat at every stage.

9. Hard In Joule's 1850 data, the work done was about 4.15 J for every calorie of heating. If his thermometers read 0.1°F too high at the end of a run in which the water warmed 0.56°F, by what percent would his value be off, and in which direction?

Show answer

The measured rise would be 0.66 instead of 0.56°F, 18% too large, so his J/cal would be 15% too small (about 3.5 J/cal). This is why he worked so hard on thermometer accuracy.

10. Hard A 70 kg hiker climbs 1000 m. The human body is about 20% efficient. How much food energy does she use, in joules and in food Calories (1 Cal = 4186 J)?

Show answer

$mgh = 6.86 \times 10^5$ J; food energy $= 3.43 \times 10^6$ J $= 820$ Cal

11. **Challenge** An electric kettle heats 1.5 kg of water from $20^\circ\mathrm{C}$ to $100^\circ\mathrm{C}$. From what height would the same water have to fall to gain that much energy? What does this tell you about the size of thermal energy compared with everyday mechanical energy?

Show answer

$Q = 1.5(4186)(80) = 5.0 \times 10^5$ J; $h = Q/(mg) = 34{,}000$ m. Heating takes enormous amounts of energy compared with lifting, which is why Joule's temperature rises were so small.

COMMON MISTAKES

- Treating heat as a substance contained in objects. Objects contain thermal energy; heat is energy moving because of a temperature difference.
- Forgetting that the mass can cancel, as in the sand tube.
- Mixing calories and joules, or food Calories (kilocalories) and calories.

Make it a game · Energy Chain Story. Choose a start, such as sunlight, and build a chain of energy transformations aloud with your parent, taking turns adding links: plant, food, muscle, a paddle wheel, heat. A link is invalid if energy is not accounted for. Keep it going to twelve links without a repeat, and then do Joule's experiment in reverse.

Scripture. "whatsoever God doeth, it shall be for ever: nothing can be put to it, nor any thing taken from it" (Ecclesiastes 3:14, KJV)

Joule saw the conservation of energy as a sign of the Creator's faithfulness: in His world, nothing is added and nothing is lost.

5.2 Work and Kinetic Energy: Balls Dropped into Clay

OBJECTIVES

- Calculate the work done by a force, $W = Fd\cos\theta$, and say when work is positive, negative or zero.
- Calculate kinetic energy, $KE = \frac{1}{2}mv^2$, and explain why it depends on the square of speed.
- Use the work–energy theorem, $W_{\text{net}} = \Delta KE$, to find speeds, forces and distances.
- Test the dependence of impact energy on drop height with craters in flour or sand.

> **Warm-up.** Hold a book out at arm's length without moving it until your arm is tired. Then carry it across the room at a steady height. Then lift it from the floor to the table. In which of the three did you do work on the book, in the physicist's sense? Write your guess and a reason before reading on.

WATCH

Work and Kinetic Energy - Physics

The Organic Chemistry Tutor · 13:04

https://www.youtube.com/watch?v=MR8jQZLcHKM

> **WHEN TO WATCH**
> After the physics paragraph, before the worked examples.

BACKUP VIDEO

What Is the Work-Energy Theorem? | Physics in Motion

GPB Education · 6:03

https://www.youtube.com/watch?v=K-RtNL8abKk

Optional, for a short worked derivation of the work–energy theorem.

KEY TERMS

work

Energy transferred by a force acting through a displacement: $W = Fd\cos\theta$, where θ is the angle between the force and the displacement. Unit: the joule.

kinetic energy

The energy of motion, $KE = \frac{1}{2}mv^2$.

work–energy theorem

The net work done on an object equals its change in kinetic energy: $W_{\text{net}} = \Delta KE$.

vis viva

"Living force," Leibniz's name for the quantity mv^2, the forerunner of kinetic energy.

negative work

Work done by a force opposite the motion (such as friction or braking), which removes kinetic energy.

CORE IDEAS

The story. For fifty years, Europe's natural philosophers argued about the true measure of a moving body's "force." Descartes and his followers said it was the quantity of motion, mv. In 1686 Gottfried Leibniz argued it was mv^2, which he called *vis viva*, living force. In 1722 the Dutch professor Willem 's Gravesande, a careful experimenter and one of the first to teach Newton's physics in Europe, settled the question with an experiment of beautiful simplicity: he dropped brass balls into a tray of soft clay and measured the dents.

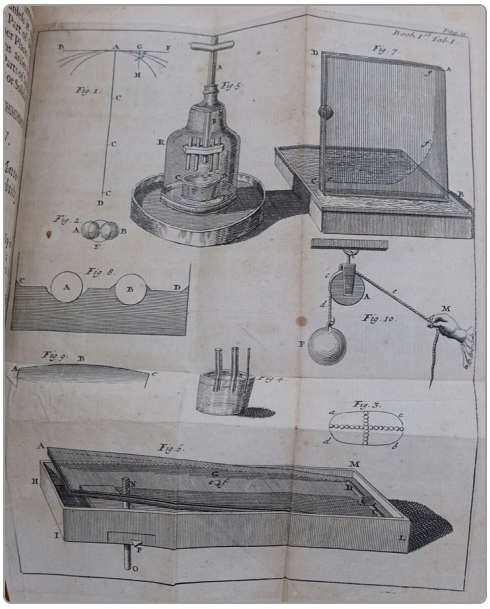
How they figured it out. 's Gravesande found that a ball dropped from four times the height made a dent four times as deep, and a ball of twice the mass dropped from the same height made a dent twice as deep. But from four times the height the ball is moving only twice as fast (because $v^2 \propto h$ in free fall). So the effect grows as mv^2, not mv. The French scholar Émilie du Châtelet took up the result in her *Institutions de physique* (1740), explained it clearly, and argued for Leibniz's measure; she later translated Newton's *Principia* into French. In 1829 the engineer Gaspard-Gustave Coriolis gave the quantity the factor of one half and gave us the word *work* for force times distance.

The physics. Work is done when a force acts through a distance: $W = Fd\cos\theta$. A force along the motion does positive work; a force against the motion (friction, brakes) does negative work; a force perpendicular to the motion (the normal force on a sliding box, or holding a book still) does no work at all. **Kinetic energy** is $KE = \frac{1}{2}mv^2$. The **work–energy theorem** ties them together: the net work on an object equals its change in kinetic energy. Because kinetic energy grows with v^2, doubling a car's speed gives it four times the kinetic energy, so the brakes must do four times the work, and at the same braking force the car needs four times the distance to stop.

The engineering. The work–energy theorem is the reason for speed limits near schools: a car at 40 mi/h carries about 1.8 times the kinetic energy of one at 30 mi/h, and a pedestrian's injuries depend on that energy. Pile drivers, which drive bridge and building foundations, are rated by the energy of each blow, mgh of the falling ram. Crash-barrier and guardrail engineers design barriers to do a known amount of negative work over a known distance. Even a hammer is a kinetic-energy tool: you give it energy over a long swing and it delivers that energy over the short distance the nail moves.

Try it. In the lab you will repeat 's Gravesande's experiment, dropping a heavy ball into flour or damp sand from different heights and measuring the craters.

PICTURE STUDY



Experiments from 's Gravesande's Physics, Willem Jacob 's Gravesande, 1720 · Engraved plate · *Mathematical Elements of Physicks, Prov'd by Experiments* (English edition), Niels Bohr Library & Archives

Image: Wikimedia Commons, public domain

's Gravesande filled his textbook with engraved plates of apparatus so that teachers could build them and show every law by experiment. His book's title says it all: physics "prov'd by experiments."

Look closely

- Count the numbered figures on the plate.
- Find any balls or pendulums. What experiments might they be used for?
- Find the long trough or box. What could it hold?

Talk about it

1. Why would a professor in 1720 want every law shown by an experiment rather than just stated?
2. If you dropped a ball into a trough of clay, what would you measure, and how would you keep the test fair?
3. Why is clay a good material for recording an impact?

Safety first. Use a tray and drop only onto it. Keep flour dust away from flames.

LAB

’s Gravesande’s Craters

You will drop a heavy ball into a tray of flour (or damp sand) from different heights and measure the craters, testing whether the effect of an impact depends on v or on v^2.

You will need

- A baking tray filled 4 cm deep with flour, leveled with a ruler, with a thin dusting of cocoa powder on top to show the crater
- Two balls of the same size and different mass (a steel ball and a glass marble, or a golf ball and a ping-pong ball weighted with clay)
- Meter stick, ruler, kitchen scale
- A pencil or skewer to measure crater depth

Steps

1. Weigh both balls. Level the flour and dust it with cocoa.
2. **Hypothesis.** If the drop height is quadrupled, predict whether the crater depth doubles or quadruples, and why.
3. Drop the heavier ball from 0.25 m, measured to the bottom of the ball, onto a fresh spot. Measure the crater’s diameter and its depth (push a skewer to the bottom and mark the surface level). Lift the ball out gently.
4. Repeat from 0.50, 1.00 and 1.50 m. Do 3 drops at each height; relevel between drops.
5. Calculate the impact speed $v = \sqrt{2gh}$ and the kinetic energy $\frac{1}{2}mv^2$ for each height.
6. Graph crater depth against v and against v^2 (or against h). Which is closer to a straight line?
7. **Fair test.** Drop the lighter ball from the same heights. Graph crater depth against kinetic energy for both balls on one graph. Do they fall on one line?
8. **Improve.** Crater diameter is easier to measure but may not be proportional to energy. Decide which measurement is more reliable and why; repeat your worst point.

Tips

- The cocoa shows the crater rim and the ejected flour clearly.
- Drop the ball by releasing it from your fingertips, not by throwing.

Check your work

☐ I calculated the kinetic energy at impact for each drop.

☐ My graph shows whether crater depth tracks

v or

v^2.

☐ I can use the work–energy theorem to explain braking distance.

LAB NOTEBOOK

1. Data table with height, speed, kinetic energy, crater depth and diameter; graphs of depth against v and v^2.
2. Draw three situations in which a force does positive work, negative work and zero work, with arrows for force and displacement.
3. A short dialogue between Descartes and Leibniz, with 's Gravesande stepping in with his data.

KEY FORMULAS

$$W = Fd\cos\theta$$

$$KE = \tfrac{1}{2}mv^2$$

$$W_{\text{net}} = \Delta KE = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

From history. Du Châtelet worked on her translation of the *Principia*, with her own commentary, until shortly before her death in 1749; it was published in 1759 and is still the standard French translation. Coriolis is better remembered today for the Coriolis effect, the apparent sideways deflection of motion on a rotating Earth.

Example 1. A gardener pushes a lawn mower 20 m with a force of 50 N directed along the handle, 30° below the horizontal. How much work does she do?

Show solution

1. Only the component of the force along the motion does work: $W = Fd\cos\theta$.
2. $W = (50\text{ N})(20\text{ m})\cos 30^\circ = (1000\text{ J})(0.866) = 866\text{ J}$.

Answer: 870 J (the downward part of her push does no work on the mower's horizontal motion).

Example 2. Compare the kinetic energy of a 1200 kg car at 15 m/s and at 30 m/s. If brakes supply a constant 6000 N, how far does the car take to stop from each speed?

Show solution

1. $KE_1 = \frac{1}{2}(1200\text{ kg})(15\text{ m/s})^2 = 1.35 \times 10^5\text{ J}$;
   $KE_2 = \frac{1}{2}(1200)(30)^2\text{ J} = 5.40 \times 10^5\text{ J}$, four times as much.
2. Work–energy theorem: $-Fd = 0 - KE$, so $d = KE/F$.
3. $d_1 = \dfrac{1.35 \times 10^5\text{ J}}{6000\text{ N}} = 22.5\text{ m}$; $d_2 = \dfrac{5.40 \times 10^5\text{ J}}{6000\text{ N}} = 90\text{ m}$.

Answer: Double the speed, four times the kinetic energy and four times the braking distance: 22.5 m versus 90 m.

Example 3. 's Gravesande drops a 0.20 kg ball from 0.50 m and from 2.0 m. Find its speed and kinetic energy at impact each time, and the ratio of dent depths if depth is proportional to kinetic energy.

Show solution

1. Speeds: $v = \sqrt{2gh}$: $\sqrt{2(9.8)(0.50)} = 3.13$ m/s and $\sqrt{2(9.8)(2.0)} = 6.26$ m/s, twice as fast.
2. Kinetic energies: $\frac{1}{2}(0.20)(3.13)^2 = 0.98$ J and $\frac{1}{2}(0.20)(6.26)^2 = 3.92$ J, four times as much.
3. Dent depth ratio: $3.92/0.98 = 4$.

Answer: Speeds 3.13 and 6.26 m/s; energies 0.98 and 3.92 J; the higher drop makes a dent 4 times as deep, matching mv^2, not mv (which would give 2 times).

PRACTICE

1. Easy How much work is done lifting a 15 kg box 1.2 m at constant speed?

   Show answer

   $W = mgh = 176$ J

2. Easy Find the kinetic energy of a 0.145 kg baseball at 40 m/s.

   Show answer

   116 J

3. Easy A waiter carries a tray at constant height across a room. How much work does his upward force do on the tray? Why?

   Show answer

   None: the force is vertical and the displacement is horizontal, so $\cos 90^\circ = 0$.

4. Medium A sled is pulled 30 m by a rope at 25° above horizontal with a tension of 80 N. How much work does the rope do?

   Show answer

   $80(30)\cos 25^\circ = 2175$ J

5. Medium A 2.0 kg cart moving at 3.0 m/s is pushed forward with 10 N for 4.0 m. Find its final speed (no friction).

Show answer

$\Delta KE = 40$ J; $KE_f = 9 + 40 = 49$ J; $v = 7.0$ m/s

6. Medium A 1000 kg car at 20 m/s skids 35 m to a stop. Find the average friction force.

Show answer

$F = KE/d = 2.0 \times 10^5/35 = 5700$ N

7. Medium By what factor does kinetic energy increase when speed rises from 30 to 40 mi/h?

Show answer

$(40/30)^2 = 1.78$

8. Medium Why does a falling object's kinetic energy grow in proportion to the distance fallen, while its speed does not?

Show answer

Gravity does work mgh, so $KE = mgh$ grows linearly with h. Speed is $\sqrt{2gh}$, which grows only as the square root of the distance.

9. Hard A 0.50 kg hammer moving at 8.0 m/s drives a nail 1.5 cm into wood. Find the average force on the nail (assume all the hammer's kinetic energy does work on the nail).

Show answer

$KE = 16$ J; $F = 16/0.015 = 1070$ N

10. Hard A pile driver's 2000 kg ram falls 3.0 m onto a pile, which sinks 5.0 cm. Find the energy per blow and the average force on the pile.

Show answer

$E = 58{,}800$ J; $F = 58{,}800/0.050 = 1.18 \times 10^6$ N

11. Challenge Derive the work–energy theorem for a constant net force from $v^2 = v_0^2 + 2ad$ and $F = ma$.

Show answer

Multiply $v^2 - v_0^2 = 2ad$ by $\frac{1}{2}m$: $\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mad = Fd = W_{\text{net}}$.

COMMON MISTAKES

- Using the whole force instead of its component along the displacement.
- Forgetting to square the speed, or squaring the mass.
- Thinking that holding something heavy still is work. It tires your muscles, but no energy is transferred to the object.

> **Make it a game · Work Positive Negative Zero.** Your parent names ten actions, such as carrying a bag across a room, lowering a box, pushing a wall. Call work as positive, negative or zero within three seconds, and justify using the angle between force and motion. Then compare KE at 10 and 20 m/s. Parent adds one trick question per round.

5.3 Potential Energy and Conservation: Galileo's Interrupted Pendulum

OBJECTIVES

- Calculate gravitational potential energy, mgh, and elastic potential energy, $\frac{1}{2}kx^2$.
- Apply conservation of mechanical energy to pendulums, ramps, springs and falling objects.
- Explain why the speed at the bottom of a frictionless path depends only on the height of the drop.
- Measure how much of a toy spring's stored energy becomes height.

> **Warm-up.** Hang a weight (a key ring or a washer) on a string about 1 m long from a doorframe. Pull it to one side until it is level with a mark on the wall (a sticky note), release it, and watch how high it rises on the other side. Then, with a partner, hold a pencil against the string partway down so the string catches on it as it swings. Predict before you try it: how high will the weight rise now?

WATCH

Kinetic Energy and Potential Energy

The Organic Chemistry Tutor · 13:18

https://www.youtube.com/watch?v=DyaVgHGssos

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Introduction to Elastic Potential Energy with Examples

Flipping Physics · 7:17

https://www.youtube.com/watch?v=7ZnpsCV2GSY

After the worked examples, for springs and elastic potential energy.

KEY TERMS

gravitational potential energy

Energy stored by height in a gravitational field: $PE_g = mgh$, measured from a chosen reference level.

elastic potential energy

Energy stored in a stretched or compressed spring: $PE_s = \frac{1}{2}kx^2$.

mechanical energy

The sum of kinetic and potential energy, $KE + PE$.

conservation of mechanical energy

When only gravity and spring forces do work (no friction or air drag), $KE + PE$ stays constant.

path independence

The work done by gravity (or a spring) depends only on the starting and ending positions, not on the path between them.

CORE IDEAS

The story. In the third day of his *Two New Sciences* (1638), Galileo described an experiment anyone could do. Hang a lead ball on a fine thread from a nail in a wall, pull it aside to a height marked by a horizontal line, and let it go. It swings down and rises on the other side almost to the same line. Now drive a second nail into the wall below the first, so that as the ball swings through the bottom the thread catches on it and the ball swings up a tighter arc. Where does it rise to? Galileo reported that it still rises to the same horizontal line, and if the nail is so low that it cannot, the ball swings right over the nail and wraps around it.

How they figured it out. Galileo used the interrupted pendulum to argue that a ball rolling down a ramp gains the same speed whatever the ramp's slope, as long as the height is the same: the speed it gains in falling is just enough to carry it back to the height it came from, by any path. Christiaan Huygens turned this into a principle in 1673: the center of gravity of a group of falling bodies can never rise above the height from which it started. A century and a half later, with the ideas of work and energy in place, the principle became the conservation of mechanical energy.

The physics. Lifting an object of mass m through a height h stores **gravitational potential energy** mgh. Compressing or stretching a spring by x stores **elastic potential energy** $\frac{1}{2}kx^2$ (the area under Hooke's straight-line graph). When only gravity and springs do work, mechanical energy is conserved: $KE_i + PE_i = KE_f + PE_f$. Then the speed at the bottom of a drop is $v = \sqrt{2gh}$ by any path: a free fall, a ramp, a pendulum, a roller coaster. Only the height matters, because gravity's work depends only on the change of height. When friction acts, some mechanical energy becomes thermal energy, and the total energy, including the heat, is still conserved, as Joule showed.

The engineering. Pumped-storage hydroelectric stations store energy as gravitational potential energy on a huge scale. When electricity is plentiful, pumps lift water from a lower reservoir to an upper one; when it is needed, the water runs back down through turbines. The Bath County station in Virginia, completed in 1985, lifts water about 380 m and can deliver about 3,000 megawatts. At a smaller scale, the springs in a car's suspension, a pogo stick, a pull-back car and a pinball plunger all store elastic potential energy and give it back.

Try it. In the lab you will repeat Galileo's interrupted pendulum, then measure how high a spring launcher toy sends a small ball, and find the fraction of its elastic energy that becomes height.

PICTURE STUDY

Penstock Pipe at Hoover Dam, U.S. Bureau of Reclamation, 1934 · Photograph · Hoover Dam, Nevada–Arizona

Image: Wikimedia Commons, public domain

The giant pipes called penstocks carry water from the reservoir down to the turbines in the power plant at the foot of Hoover Dam. Each is a channel through which gravitational potential energy becomes kinetic energy and then electricity.

Look closely

- Find the pipe section and the cables holding it.
- Compare the pipe's size with any people or machines you can see.
- Look for the canyon walls. How far below must the turbines be?

Talk about it

1. Why must the pipe carry water down to the bottom of the dam before it reaches the turbines?
2. If the water falls about 150 m, what speed could it reach with no losses?
3. Why does the power a dam can produce depend on both the height of the water and how much water flows each second?

Safety first. Wear safety glasses when using a spring launcher. Launch straight up, away from faces and light fixtures.

LAB

Galileo's Interrupted Pendulum and a Spring Launcher

First you will repeat Galileo's experiment to see that a pendulum returns to its starting height by any path. Then you will measure the energy stored in a spring and find out how much of it a toy launcher turns into height.

You will need

- String about 1.2 m, a heavy bob (a large nut or a fishing sinker), a doorframe or hook
- A pencil or dowel held as the "nail," a sheet of paper taped to the wall behind the pendulum
- A spring launcher: a spring of known k (from Lesson 1.4) in a cardboard or PVC tube with a stopper, or a toy pop-up launcher
- A light ball (ping-pong ball), meter stick, phone with slow-motion video
- Safety glasses

Steps

1. **Pendulum.** Hang the pendulum in front of the paper. Draw a horizontal line on the paper 20 cm above the bob's lowest point.
2. Release the bob from the line and mark how high it rises on the other side. Repeat with a partner holding the pencil firmly against the wall so the string catches on it at three different heights. Mark the rise each time.
3. **Hypothesis.** Predict what happens when the nail is placed so low that the shortened pendulum cannot reach the line. Test it.
4. **Spring launcher.** Measure k for your spring (hang known masses, as in Lesson 1.4) and the compression x you will use. Weigh the ball.
5. Calculate the stored energy $\frac{1}{2}kx^2$ and the predicted maximum height $h = kx^2/(2mg)$.
6. Launch the ball straight up 5 times beside a meter stick taped to the wall (or film it) and record the peak heights. Find the mean.
7. Calculate the efficiency: $\dfrac{mgh_{\text{measured}}}{\frac{1}{2}kx^2}$.

8. **Fair test.** Change only the compression (for example, half and full). Predict and measure. Does the height grow as x^2?
9. **Improve.** Identify where the missing energy went (the spring's own motion, friction in the tube, air drag, sound). Change one thing, such as a heavier ball, and see whether the efficiency changes.

Tips

- Film the launch from a few meters away so the peak height can be read against the meter stick.
- Release the pendulum by letting go, not pushing.

Check your work

- ☐ I showed that the pendulum returns to the same height whatever path the string takes.
- ☐ I calculated the energy stored in a spring and the predicted launch height.
- ☐ I measured the launcher's efficiency and explained where the lost energy went.

LAB NOTEBOOK

1. Sketch of the interrupted pendulum with the height marks, and a sentence on what happened when the nail was too low.
2. Energy bar charts (KE, PE, thermal) for the launched ball at three moments: compressed, leaving the spring, at the top.
3. Data and efficiency calculations for the launcher, and a graph of height against x^2.

KEY FORMULAS

$$PE_g = mgh, \qquad PE_s = \tfrac{1}{2}kx^2$$

$$KE_i + PE_i = KE_f + PE_f$$

$$v = \sqrt{2gh}$$

From history. Bath County Pumped Storage Station, in the Allegheny Mountains of Virginia, has been called the world's largest battery; its upper and lower reservoirs are dammed valleys. Pumped storage is about 75–80% efficient over a full cycle of pumping and generating, and it still provides most of the grid-scale energy storage in the world.

Example 1. A pendulum bob is released from rest 0.20 m above its lowest point. How fast is it moving at the bottom? With the string caught on a nail, how high does it rise on the other side?

Show solution

1. Energy: $mgh = \frac{1}{2}mv^2$, so $v = \sqrt{2gh} = \sqrt{2(9.8\ \mathrm{m/s^2})(0.20\ \mathrm{m})} = 1.98\ \mathrm{m/s}$; the mass cancels.
2. On the far side, the kinetic energy turns back into potential energy. With no friction it rises until $v = 0$: $h = \dfrac{v^2}{2g} = 0.20\ \mathrm{m}$, the same height, nail or no nail.

Answer: 1.98 m/s at the bottom; it rises back to 0.20 m, as Galileo saw.

Example 2. A toy launcher's spring ($k = 400\ \mathrm{N/m}$) is compressed 5.0 cm and launches a 20 g ball straight up. How high can it go, ignoring friction and air?

Show solution

1. Stored energy: $PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(400\ \mathrm{N/m})(0.050\ \mathrm{m})^2 = 0.50\ \mathrm{J}$.
2. All of it becomes mgh at the top: $h = \dfrac{0.50\ \mathrm{J}}{(0.020\ \mathrm{kg})(9.8\ \mathrm{m/s^2})} = 2.6\ \mathrm{m}$.

Answer: About 2.6 m (in practice less, because the spring and launcher also move and air resists the ball).

Example 3. A skier starts from rest at the top of a 40 m high hill and glides down a winding trail with negligible friction. How fast is she going at the bottom? Does the shape of the trail matter?

Show solution

1. $v = \sqrt{2gh} = \sqrt{2(9.8\ \mathrm{m/s^2})(40\ \mathrm{m})} = 28\ \mathrm{m/s}$.
2. With no friction, only the height matters: a steep straight run and a long winding trail give the same final speed (though they take different times).

Answer: 28 m/s (about 63 mi/h), whatever the path.

Example 4. A pumped-storage station raises 1.0×10^6 m^3 of water (1.0×10^9 kg) through 300 m. How much energy is stored, in joules and in megawatt-hours?

Show solution

1. $PE = mgh = (1.0 \times 10^9 \text{ kg})(9.8 \text{ m/s}^2)(300 \text{ m}) = 2.94 \times 10^{12}$ J.
2. $1 \text{ MWh} = 10^6 \text{ W} \times 3600 \text{ s} = 3.6 \times 10^9$ J, so $\dfrac{2.94 \times 10^{12}}{3.6 \times 10^9} = 817$ MWh.

Answer: 2.9×10^{12} J, about 820 MWh, enough to supply roughly 800 megawatts for an hour (less the losses in pumps and turbines).

PRACTICE

1. Easy Find the potential energy gained by lifting a 3.0 kg bag 1.5 m.

   Show answer

   44 J

2. Easy How much energy is stored in a spring with $k = 200$ N/m stretched 0.10 m?

   Show answer

   1.0 J

3. Easy A ball is dropped from 5.0 m. Find its speed just before it lands.

   Show answer

   9.9 m/s

4. Medium A pendulum swings through its lowest point at 2.5 m/s. How high above the lowest point does it rise?

   Show answer

   $h = v^2/2g = 0.32$ m

5. Medium A roller-coaster car starts at rest 30 m high and goes over a hill 12 m high. How fast is it moving at the top of the hill (no friction)?

   Show answer

   $v = \sqrt{2(9.8)(18)} = 18.8$ m/s

6. Medium A 0.050 kg dart is shot from a spring gun ($k = 250$ N/m, compressed 4.0 cm) horizontally. Find its launch speed.

Show answer

$\frac{1}{2}kx^2 = 0.20$ J; $v = \sqrt{2(0.20)/0.050} = 2.8$ m/s

7. Medium Two identical balls start from the same height and roll without friction down two ramps, one steep and one gentle, to the same floor. Which is faster at the bottom? Which gets there first?

Show answer

They have the same speed at the bottom (same height dropped). The ball on the steep ramp arrives first because it accelerates more.

8. Medium A 60 kg pole vaulter running at 9.0 m/s converts all her kinetic energy into height. How high does her center of mass rise?

Show answer

$h = 81/19.6 = 4.1$ m (plus her starting center-of-mass height of about 1 m, close to real vaults).

9. Hard A 2.0 kg block slides from rest down a frictionless ramp 0.80 m high and then compresses a spring ($k = 500$ N/m) on the level floor. How far is the spring compressed?

Show answer

$mgh = 15.7$ J $= \frac{1}{2}(500)x^2$, so $x = 0.25$ m

10. Hard A toy launcher stores 0.40 J and sends a 25 g ball 1.1 m high. What is its efficiency?

Show answer

$mgh = 0.27$ J; efficiency $= 67\%$

11. **Challenge** Galileo's pendulum is 1.0 m long and is released from horizontal. A nail is placed a distance d directly below the pivot. What is the smallest d for which the bob swings all the way around the nail? (At the top of the small circle, the bob needs $v^2 \geq gr$, where $r = 1.0 - d$.)

Show answer

Energy: at the top of the small circle the bob is $2r$ above the bottom and has fallen $1.0 - 2r$ from the start, so $v^2 = 2g(1.0 - 2r)$. Require $2g(1 - 2r) \geq gr$: $r \leq 0.40$ m, so $d \geq 0.60$ m.

COMMON MISTAKES

- Using the length of a ramp or path instead of the change in height in mgh.
- Forgetting to square the compression in $\frac{1}{2}kx^2$, or using the spring's total length instead of x.
- Applying conservation of mechanical energy when friction is large. Then account for the energy turned into heat.

> **Make it a game · Energy Prediction Duel.** Hold a ball at a chosen height and predict its speed at the floor using mgh equals half m v squared. Write your guess before the calculator. Closest to the computed value wins the round. Then try a spring or rubber band launch and predict height. Play five rounds with different heights.

5.4 Power and Watt's Horsepower

OBJECTIVES

- Define power as the rate of doing work, $P = W/t = Fv$, and use watts and horsepower.
- Explain how James Watt defined the horsepower and why he needed it.
- Calculate the power needed to lift loads and to drive vehicles, and the energy used in kilowatt-hours.
- Measure your own power output running up stairs.

> **Warm-up.** Walk up a flight of stairs slowly. Then run up the same stairs (safely, holding the rail). Did you do more work running? Did you feel more tired? Write down what was the same and what was different between the two climbs.

WATCH

What Is Horsepower? | Earth Science

BBC Earth Science · 7:49

https://www.youtube.com/watch?v=Xym2XNezZ-w

> **WHEN TO WATCH**
> With the story paragraph.

BACKUP VIDEO

Horsepower: James Watt and the Transition from Horse to Steam

The History Guy: History Deserves to Be Remembered · 12:05

https://www.youtube.com/watch?v=3apcSyFUeOs

Optional, for more on how Watt's engine and the horsepower came about.

KEY TERMS

power

The rate of doing work or transferring energy: $P = W/t$. Unit: the watt, $1\ \mathrm{W} = 1\ \mathrm{J/s}$.

horsepower

James Watt's unit of power, 33,000 foot-pounds per minute, equal to about 746 W.

kilowatt-hour

A unit of energy used for electricity: the energy delivered by 1 kW for 1 hour, 3.6×10^6 J.

efficiency

The fraction of the input energy or power that becomes useful output: $\eta = P_{\text{out}}/P_{\text{in}}$.

separate condenser

Watt's 1765 invention: condensing the steam in a separate chamber so the main cylinder could stay hot, which cut fuel use by about three quarters.

CORE IDEAS

The story. In 1763 James Watt, an instrument maker at the University of Glasgow, was asked to repair a small model of a Newcomen steam engine, the kind used since 1712 to pump water from mines. Watt noticed that it wasted most of its steam: each stroke, cold water was sprayed into the cylinder to condense the steam, chilling the cylinder, which then had to be reheated by fresh steam. In 1765, walking on Glasgow Green on a Sunday afternoon, he thought of condensing the steam in a separate vessel, so the cylinder could stay hot. With the Birmingham manufacturer Matthew Boulton he built engines that used about a quarter of the coal of the old ones.

How they figured it out. To sell an engine, Boulton and Watt had to tell customers how many horses it would replace. Watt measured how fast mill horses could work, walking in a circle turning a mill wheel, and settled on a round figure: a horse could raise 33,000 pounds one foot in one minute. He called this one **horsepower**. It was a generous figure, more than most horses could keep up all day, so buyers who replaced ten horses with a "ten-horsepower" engine were never disappointed. In 1882 the British Association for the Advancement of Science named the metric unit of power the **watt** in his honor.

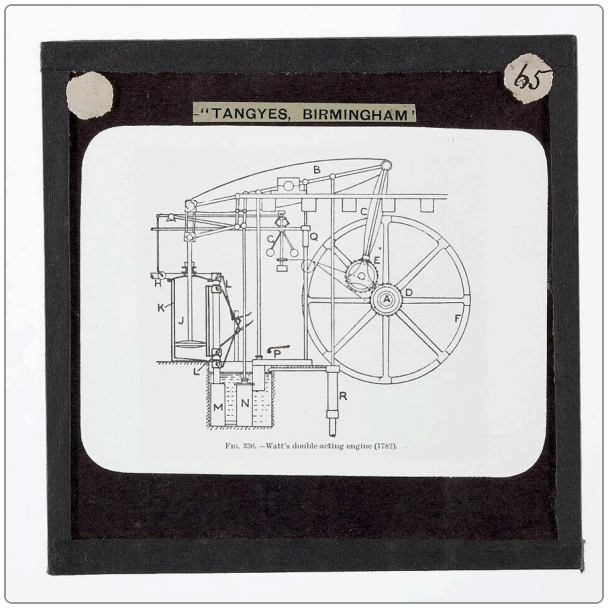
The physics. Power is the rate of doing work: $P = \dfrac{W}{t}$, measured in watts (joules per second). Walking and running up stairs do the same work, mgh, but running does it in less time, so it takes more power. For a force pushing an object at a steady speed, $P = Fv$. One horsepower is $33{,}000\ \text{ft} \cdot \text{lbf/min} = 746\ \text{W}$. A healthy person can produce about 100 W for hours and well over 1000 W for a few seconds.

Energy is power times time, $E = Pt$; electric companies sell energy in kilowatt-hours, $1\ \mathrm{kWh} = 3.6 \times 10^6\ \mathrm{J}$.

The engineering. Engineers choose motors and engines by power. An elevator motor must deliver at least mgv to lift its unbalanced load at speed v; a car's engine must supply the drag and rolling resistance times the speed, which grows rapidly with speed because air drag itself grows as v^2, so the power to overcome it grows as v^3. Every motor's nameplate lists its power, and every appliance's label tells you, in watts, how fast it uses energy. Watt's improved engine, by doing the same work with less fuel, also introduced the idea of efficiency that engineers have chased ever since.

Try it. In the lab you will measure your own power climbing stairs, and find out how many horsepower you are.

PICTURE STUDY



Watt's Rotative Steam Engine, Unknown illustrator (lantern slide), Engine of about 1782; slide c. 1900 · Lantern slide of a sectional drawing · Museums Victoria

Image: Wikimedia Commons, public domain

Watt's later engines did not just pump water; they turned wheels, so they could drive mills and factories. This cross-section shows the parts that made it efficient.

Look closely

- Find the cylinder and the piston rod.
- Find the separate condenser. How is it connected to the cylinder?
- Find the beam and the flywheel. How is the up-and-down motion turned into rotation?

Talk about it

1. Why does keeping the cylinder hot save fuel?
2. What job does the heavy flywheel do?
3. How would Watt measure this engine's power in horsepower?

Safety first. Run stairs only if you are healthy, wearing good shoes, holding the rail, with the stairs clear. Walk down.

LAB

How Many Horsepower Are You?

You will measure the power you produce running up a flight of stairs and lifting a load, and compare it with Watt's horse.

You will need

- A flight of stairs (at least 10 steps)
- Tape measure
- Bathroom scale (for your mass) and a backpack with books
- Stopwatch or phone, a partner

Steps

1. Measure the height of one step and count the steps to find the total height h. Find your mass from the bathroom scale (kg).
2. **Hypothesis.** Predict your power in watts running up the stairs, and whether it will be more or less than 1 hp.
3. Walk up at a normal pace while your partner times you. Then run up (safely) as fast as you can, timed. Do 3 trials of each, resting between.
4. Calculate the work mgh and the power mgh/t for each trial, in watts and horsepower.
5. **Fair test.** Carry a backpack of known mass and run again. Does your power change, or only your time?
6. **Endurance.** Climb the stairs at a steady pace 5 times in a row, timing the whole set. Compare your sustained power with your sprint power.
7. Compare your results with Watt's horse (746 W) and with a typical sustained human output (about 100 W).
8. **Improve.** Identify the largest uncertainty (timing start and stop, measuring height). Change your method (start the clock on a foot on the first step, stop on the last) and repeat.

Tips

- Only the vertical height matters, not the length of the staircase.

- Your body is only about 20–25% efficient, so you burn about four times as much food energy as the work you do.

Check your work

- ☐ I calculated my power climbing stairs in watts and horsepower.
- ☐ I can explain why walking and running up stairs take the same work but different power.
- ☐ I can calculate the power needed to lift a load at a given speed.

LAB NOTEBOOK

1. Data table: trial, mass, height, time, work, power (W and hp).
2. Bar chart comparing your sprint power, your sustained power, Watt's horse and a 100 W light bulb of the old kind.
3. Read the power ratings on five appliances in your home and calculate the energy each uses in a day of typical use, in kWh.

KEY FORMULAS

$$P = \frac{W}{t} = \frac{\Delta E}{t}$$

$$P = Fv$$

$$1\ \text{hp} = 746\ \text{W}, \qquad 1\ \text{kWh} = 3.6 \times 10^6\ \text{J}$$

From history. Watt also invented the centrifugal governor for engines, a pair of spinning balls that automatically adjusted the steam supply, one of the first feedback control systems, and a copying press for letters. His partner Boulton told the visiting James Boswell in 1776, "I sell here, sir, what all the world desires to have: Power."

WORKED EXAMPLES

Example 1. Show that Watt's 33,000 ft · lbf/min equals about 746 W ($1\ \text{ft} \cdot \text{lbf} = 1.356\ \text{J}$).

Show solution

1. $33{,}000\ \text{ft} \cdot \text{lbf} \times \dfrac{1.356\ \text{J}}{1\ \text{ft} \cdot \text{lbf}} = 44{,}740\ \text{J}.$
2. Per second: $\dfrac{44{,}740\ \text{J}}{60\ \text{s}} = 746\ \text{W}.$

Answer: $1\ \text{hp} = 746\ \text{W}.$

Example 2. A 60 kg student runs up stairs 3.0 m high in 4.0 s. Find her power output in watts and horsepower.

Show solution

1. Work against gravity: $W = mgh = (60\text{ kg})(9.8\text{ m/s}^2)(3.0\text{ m}) = 1764\text{ J}$.
2. $P = \dfrac{W}{t} = \dfrac{1764\text{ J}}{4.0\text{ s}} = 441\text{ W}$.
3. $\dfrac{441\text{ W}}{746\text{ W/hp}} = 0.59\text{ hp}$.

Answer: 441 W, about 0.6 hp, briefly.

Example 3. A car moving at a steady 25 m/s must overcome a total drag and rolling resistance of 600 N. What power must the engine deliver to the wheels? How much energy does it use in one hour, in kWh, if the engine and drivetrain are 25% efficient?

Show solution

1. $P = Fv = (600\text{ N})(25\text{ m/s}) = 15{,}000\text{ W} = 15\text{ kW}$ (about 20 hp).
2. Input power: $\dfrac{15\text{ kW}}{0.25} = 60\text{ kW}$.
3. Energy in one hour: $(60\text{ kW})(1\text{ h}) = 60\text{ kWh}$ of fuel energy (about 2 gallons of gasoline).

Answer: 15 kW at the wheels; about 60 kWh of fuel energy per hour.

Example 4. An elevator motor lifts an unbalanced load of 1000 kg at 2.0 m/s. What power is needed? If electricity costs 15 cents per kWh, what does a 30-second ride cost?

Show solution

1. $P = Fv = mgv = (1000\text{ kg})(9.8\text{ m/s}^2)(2.0\text{ m/s}) = 19{,}600\text{ W} = 19.6\text{ kW}$.
2. Energy: $(19.6\text{ kW})\left(\dfrac{30}{3600}\text{ h}\right) = 0.163\text{ kWh}$.
3. Cost: $0.163 \times 15 = 2.5$ cents.

Answer: About 20 kW; a 30-second ride costs about 2.5 cents.

PRACTICE

1. Easy A motor does 3000 J of work in 10 s. What is its power?

   Show answer

   300 W

2. Easy Convert 150 hp to kilowatts.

   Show answer

   112 kW

3. Easy How much energy does a 1500 W space heater use in 2.0 h, in kWh and in joules?

   Show answer

   $3.0\ \text{kWh} = 1.08 \times 10^7\ \text{J}$

4. Medium A crane lifts a 500 kg beam 20 m in 25 s. Find its power output.

   Show answer

   $500(9.8)(20)/25 = 3920\ \text{W}$

5. Medium A cyclist produces 250 W against a total resisting force of 25 N. How fast does she ride?

   Show answer

   $v = P/F = 10\ \text{m/s}$

6. Medium A pump lifts 50 kg of water per minute through 12 m. What power does it deliver? If it is 60% efficient, what power does it use?

   Show answer

   98 W; 163 W

7. Medium Two students do the same work climbing the same stairs, but one takes 6 s and the other 3 s. Compare their power.

   Show answer

   The faster student produces twice the power.

8. Medium In Watt's day, a mine pump had to lift 600 kg of water per minute from 50 m down. How many horsepower was that?

Show answer

$600(9.8)(50)/60 = 4900\ \mathrm{W} = 6.6\ \mathrm{hp}$

9. Hard Air drag on a car is about $F = 0.40v^2$ (in newtons, v in m/s). Find the power to overcome drag at 20 m/s and at 30 m/s.

Show answer

$P = 0.40v^3$: 3.2 kW at 20 m/s, 10.8 kW at 30 m/s, 3.4 times as much for 1.5 times the speed.

10. Hard A Newcomen engine burned about four times the coal of a Watt engine doing the same work. If a Watt engine was 2% efficient, what was the Newcomen engine's efficiency? What does this say about early steam engines?

Show answer

About 0.5%. Almost all the energy of the coal was lost as heat; even Watt's engine wasted 98%.

11. Challenge An elevator car of 1200 kg with a counterweight of 1500 kg carries 800 kg of passengers up at 1.5 m/s. What power must the motor supply (ignoring friction)? What if the car went up empty?

Show answer

Loaded: unbalanced mass $2000 - 1500 = 500$ kg;
$P = 500(9.8)(1.5) = 7.35$ kW. Empty: the counterweight is heavier by 300 kg, so the motor must hold the car back; gravity supplies 4.4 kW, which some modern elevators recover as electricity.

COMMON MISTAKES

- Confusing power with energy. A kilowatt is a rate; a kilowatt-hour is an amount.
- Using the length of the staircase instead of its height.
- Forgetting to convert minutes or hours to seconds when calculating watts.

> **Make it a game · Stair Power Challenge.** Run up a flight of stairs at a steady, safe pace while your parent times you. Measure the vertical height with a ruler or tape, then calculate your power in watts using your mass, g and the time. Convert to horsepower. Rest, then try to beat your own personal record, taking the stairs at a walk, never a sprint.

5.5 Roller Coasters, Loops and Efficiency

OBJECTIVES

- Use energy conservation and centripetal force to find the minimum height for a loop-the-loop.
- Calculate the forces riders feel at the top and bottom of a loop, and explain why modern loops are not circles.
- Calculate efficiency and use it to account for friction in real machines.
- Test the minimum release height for a Hot Wheels loop.

> **Warm-up.** If you have a Hot Wheels track with a loop, start a car at different heights on the ramp and find the lowest start that gets it all the way around without falling. If not, swing a cup of water on a string in a vertical circle outdoors (go fast!) and then slower until the water starts to spill. In either case, write down what you think decides whether it makes it around the top.

WATCH

The Real Physics of Roller Coaster Loops

Art of Engineering · 18:02

https://www.youtube.com/watch?v=4q2W5SJc5j4

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Minimum Height for a Roller Coaster to Loop the Loop | Physics with Professor Matt Anderson | M11-06

Physics with Professor Matt Anderson · 11:56

https://www.youtube.com/watch?v=D63ru7ygpQc

Before the lab, for the minimum-height derivation step by step.

KEY TERMS

loop-the-loop

A vertical loop on a track. To stay on the track at the top, an object needs a speed of at least $\sqrt{gr}$.

apparent weight (g-force)

The normal force from the seat divided by the rider's weight; what a rider feels, in multiples of g.

clothoid loop

A teardrop-shaped loop whose curvature increases gradually toward the top, keeping riders' g-forces lower than in a circular loop.

efficiency

Useful energy out divided by energy in, $\eta = E_{\text{out}}/E_{\text{in}}$, always less than 100% in real machines.

lift hill

The first climb of a roller coaster, where a chain or cable gives the train the gravitational potential energy for the whole ride.

CORE IDEAS

The story. In 1884 LaMarcus Thompson opened the Switchback Railway at Coney Island in Brooklyn: riders climbed a tower, rolled down a gentle track at about 6 miles per hour to a second tower, and climbed out to ride back. It cost a nickel and was an instant success. Inventors soon wanted more excitement. In 1895 Lina Beecher's Flip Flap Railway opened at Coney Island's Sea Lion Park with the first vertical loop in America, a perfect circle about 25 feet across. It worked, but riders complained of sore necks; many people came to watch, but few came back to ride.

How they figured it out. The trouble was physics. To get around a circular loop, the car had to enter it fast, and at the bottom of a tight circle a fast car needs a huge centripetal force: the seat pushed riders with about six times their weight. In 1901 Edward Prescott's Loop the Loop at Coney Island used a taller, oval-shaped loop that was gentler at the top and bottom. But the real solution came much later. In 1976 a coaster in California, designed with the German engineer Werner Stengel, used a loop shaped like a teardrop, a *clothoid*, whose curve tightens gradually toward the top. The car can enter with a gentle curve at high speed and go over the top slowly in a tight curve, keeping the g-forces comfortable everywhere. Nearly every looping coaster since uses Stengel's shape.

The physics. At the top of a loop of radius r, the track and gravity both push the car toward the center. The car stays on the track only if gravity alone is not more than enough: $mg \le mv^2/r$, so $v_{\text{top}} \ge \sqrt{gr}$. Energy conservation then says the release height, measured from the bottom, must be at least $h = 2r + \dfrac{v_{\text{top}}^2}{2g} = 2.5r$ for a sliding

object with no friction. At the bottom, $N - mg = mv^2/r$; released from $2.5r$, $v^2 = 5gr$ and $N = 6mg$: six g. Real cars and balls need more height, because friction and air drag remove energy and a rolling object stores some energy in spinning. The **efficiency** of a ride, $\eta = \dfrac{\text{energy remaining}}{\text{energy at the start}}$, tells the designer how tall each later hill can be.

The engineering. Coaster engineers work within limits on g-forces set by safety standards: sustained positive g-forces are generally kept below about 5 g and negative (lifting out of the seat) forces small and brief. They shape every curve with clothoids and banking so the forces change smoothly. The lift hill sets the energy budget, and every later hill must be lower than the first by enough to cover friction losses. At the end, magnetic or friction brakes remove the remaining kinetic energy as heat.

Try it. In the lab you will find the lowest release height that sends a Hot Wheels car around a loop, compare it with $2.5r$, and measure the efficiency of the track.

PICTURE STUDY

The Flip Flap Railway, Detroit Publishing Company, 1901 · Photograph · Sea Lion Park, Coney Island, New York (Library of Congress)

Image: Wikimedia Commons, public domain

America's first looping roller coaster looked thrilling, but its perfectly circular loop was hard on riders. This photograph lets us see why.

Look closely

- Find the loop. Is it a circle or an oval?
- Find the ramp that leads into the loop. How high is its top compared with the top of the loop?
- Find the person standing near the loop's base, and use him to estimate the loop's size.

Talk about it

1. If the loop's radius is about 3.8 m, what is the lowest starting height that would get a car around it?
2. Why does a circular loop give riders a much stronger push at the bottom than at the top?
3. How would you reshape this loop to make it gentler?

Safety first. Swing a cup of water in a vertical circle only outdoors, away from people. Keep fingers clear of the track's launcher.

LAB

The Hot Wheels Loop: Minimum Height and Efficiency

You will find, by careful testing, the lowest release height that gets a Hot Wheels car around a vertical loop, compare it with the frictionless prediction $2.5r$, and use the difference to measure the energy lost to friction.

You will need

- Hot Wheels orange track with a loop (or bend a length of track into a loop and clamp it), a car
- A tall support for the ramp (a stack of books or a chair), tape, binder clips
- Meter stick, phone with slow-motion video
- Graph paper

Steps

1. Build a ramp leading straight into the loop. Measure the loop's radius r to the track's running surface (half the inside height of the loop).
2. **Hypothesis.** Predict the minimum release height from $2.5r$, and predict whether the real car will need more or less, and why.
3. Mark release points on the ramp every 2 cm in height. Starting high, release the car from rest at each mark, working downward, 3 trials each. Record whether it completes the loop.
4. Find the lowest height at which the car completes the loop 3 times out of 3. That is your measured $h_{\min}$.
5. Compare with $2.5r$: find the ratio $h_{\min}/(2.5r)$.
6. **Efficiency.** Remove the loop. Release the car from a measured height and film it on the flat track just after the ramp; find its speed from two marks 10 cm apart. Calculate $\eta = \frac{1}{2}v^2/(gh)$.
7. Use your efficiency to correct the prediction: if only a fraction η of the energy remains at the loop, $h_{\min} \approx 2.5r/\eta$. Does this explain your result?
8. **Fair test.** Try a heavier car or a car with different wheels from the same height. Which completes the loop from lower down?

9. **Improve.** Tighten joints, clean the wheels, or straighten the track and repeat the minimum-height test. How much lower can you get?

Tips

- The loop must be clamped firmly; a loop that sways absorbs energy.
- Release by lifting your finger, not by pushing the car.

Check your work

☐ I found the minimum release height for my loop and compared it with $2.5r$.

☐ I measured the track's efficiency and used it to explain the extra height needed.

☐ I can explain why modern loops are clothoids, not circles.

LAB NOTEBOOK

1. Labeled side view of the ramp and loop with r, h_{min} and $2.5r$ marked; data table of trials.
2. Free-body diagrams of the car at the top and at the bottom of the loop.
3. Sketch a circular loop and a clothoid loop of the same height and explain in two sentences why the clothoid is gentler.

KEY FORMULAS

$$v_{\text{top}} \geq \sqrt{gr}$$

$$h_{\text{min}} = 2.5r \quad \text{(sliding, no friction)}$$

$$N_{\text{bottom}} = mg + \frac{mv^2}{r}$$

$$\eta = \frac{E_{\text{out}}}{E_{\text{in}}}$$

From history. LaMarcus Thompson patented his "Roller Coasting Structure" in 1885 and built dozens of scenic railways. Edward Prescott's Loop the Loop of 1901 carried only four riders per car and was soon closed for lack of riders; looping coasters disappeared for seventy years until steel tracks and clothoid loops made them both safe and comfortable.

Example 1. A Hot Wheels loop has radius $r = 0.15$ m. Find the minimum speed at the top and the minimum release height (no friction, ignoring wheel spin).

Show solution

1. $v_{\text{top}} = \sqrt{gr} = \sqrt{(9.8\ \text{m/s}^2)(0.15\ \text{m})} = 1.21\ \text{m/s}$.
2. Energy from release (height h, at rest) to the top of the loop (height $2r$): $mgh = mg(2r) + \frac{1}{2}mv_{\text{top}}^2$.
3. $h = 2r + \dfrac{gr}{2g} = 2.5r = 2.5(0.15\ \text{m}) = 0.375\ \text{m}$.

Answer: $1.21\ \text{m/s}$ at the top; release at least $0.38\ \text{m}$ above the bottom of the loop.

Example 2. A circular loop like the Flip Flap Railway's has radius $3.8\ \text{m}$. A car released from the minimum height $2.5r$ reaches the bottom. How fast is it moving, and what force does the seat exert on a $60\ \text{kg}$ rider there?

Show solution

1. $h = 2.5(3.8\ \text{m}) = 9.5\ \text{m}$; $v = \sqrt{2gh} = \sqrt{2(9.8)(9.5)}\ \text{m/s} = 13.6\ \text{m/s}$.
2. At the bottom: $N - mg = \dfrac{mv^2}{r}$, so
$$N = (60\ \text{kg})\left(9.8 + \frac{(13.6)^2}{3.8}\right)\ \text{m/s}^2 = (60)(9.8 + 48.9)\ \text{N} = 3530\ \text{N}.$$
3. In multiples of weight: $\dfrac{3530}{588} = 6.0\,g$.

Answer: $13.6\ \text{m/s}$; about $3500\ \text{N}$, six times the rider's weight, which explains the sore necks.

Example 3. A Hot Wheels car released from $0.50\ \mathrm{m}$ is measured at $2.6\ \mathrm{m/s}$ at the bottom of the ramp. Find the track's efficiency to that point.

Show solution

1. Energy at start (per kg): $gh = (9.8)(0.50) = 4.90\ \mathrm{J/kg}$.
2. Energy at bottom (per kg): $\frac{1}{2}v^2 = \frac{1}{2}(2.6)^2 = 3.38\ \mathrm{J/kg}$.
3. $\eta = \dfrac{3.38}{4.90} = 0.69$, or 69%.

Answer: About 69%: almost a third of the energy went into friction, air drag and spinning the wheels.

Example 4. A coaster's first drop is $60\ \mathrm{m}$. If 10% of the energy is lost by the bottom, how fast is the train going there?

Show solution

1. Kinetic energy at the bottom $= 0.90\,mgh$: $\frac{1}{2}mv^2 = 0.90\,mgh$.
2. $v = \sqrt{2(0.90)(9.8\ \mathrm{m/s^2})(60\ \mathrm{m})} = 32.5\ \mathrm{m/s}$.

Answer: About $32.5\ \mathrm{m/s}$ (73 mi/h), compared with $34.3\ \mathrm{m/s}$ with no losses.

PRACTICE

1. Easy What is the minimum speed at the top of a loop of radius $10\ \mathrm{m}$?

   Show answer

   $\sqrt{98} = 9.9\ \mathrm{m/s}$

2. Easy What is the minimum release height for a frictionless loop of radius $10\ \mathrm{m}$?

   Show answer

   $25\ \mathrm{m}$

3. Easy A machine takes in $500\ \mathrm{J}$ and delivers $350\ \mathrm{J}$ of useful work. Find its efficiency.

   Show answer

   70%

4. Medium A coaster car is released from 30 m with no friction. How fast is it at the bottom of a valley 5 m above the ground?

Show answer

$v = \sqrt{2(9.8)(25)} = 22 \text{ m/s}$

5. Medium A 70 kg rider is at the top of a loop of radius 8.0 m, moving at 12 m/s. Find the force of the seat on the rider.

Show answer

$N + mg = mv^2/r$: $N = 70(18 - 9.8) = 574$ N, pushing down on the upside-down rider.

6. Medium A car enters a circular loop of radius 5.0 m at the bottom at 18 m/s. What g-force does a rider feel at the bottom?

Show answer

$N/mg = 1 + v^2/(gr) = 1 + 324/49 = 7.6\,g$

7. Medium Why must each hill of a roller coaster (without extra motors) be lower than the first?

Show answer

Friction and air drag remove some energy along the way, so the train never has as much mechanical energy as it started with; it could not climb back to the first height.

8. Medium A coaster loses 15% of its energy between the top of a 50 m lift hill and the bottom. What is its speed at the bottom?

Show answer

$v = \sqrt{2(0.85)(9.8)(50)} = 28.9 \text{ m/s}$

9. Hard A Hot Wheels loop of radius 0.12 m needed a release height of 0.42 m. What fraction of the energy was lost before the top of the loop, assuming the car was just barely going fast enough there?

Show answer

Needed at top (per kg): $g(2r) + \frac{1}{2}gr = 2.5gr = 0.30g$; available: $0.42g$. Lost $0.12g$ out of $0.42g$, about 29%.

10. Hard Show that for a frictionless circular loop, a car released from $2.5r$ experiences $6\,g$ at the bottom and $0\,g$ at the top.

Show answer

Bottom: $v^2 = 2g(2.5r) = 5gr$, $N = mg + 5mg = 6mg$. Top: $v^2 = 2g(0.5r) = gr$, $N = mv^2/r - mg = 0$.

11. Challenge A clothoid loop is 30 m tall with a radius of curvature of only 6.0 m at the top. A train enters at the bottom at 28 m/s with no losses. Find its speed at the top and the riders' g-force there. Would a circular loop 30 m tall be gentler or harsher at the bottom for the same entry speed?

Show answer

Top: $v^2 = 28^2 - 2(9.8)(30) = 196$, $v = 14$ m/s. $N/mg = v^2/(gr) - 1 = 196/58.8 - 1 = 2.3\,g$ (pressing riders into their seats while upside down). A circular loop of radius 15 m would give $1 + 784/147 = 6.3\,g$ at the bottom, much harsher than the clothoid's gentle entry curve.

COMMON MISTAKES

- Measuring the release height from the floor instead of from the bottom of the loop (or measuring it to the top of the car instead of its center).
- Thinking the car needs zero speed at the top. It needs at least $\sqrt{gr}$, or it falls off the track.
- Forgetting that the normal force at the bottom adds to the rider's weight instead of replacing it.

> **Make it a game · Loop Height Prediction.** Using a toy track and a marble, predict the minimum starting height for a loop of a given radius, using the 2.5 times radius rule. Test and record whether it succeeds. Then try 2.0 and 3.0 times. Score each prediction, and estimate how much energy was lost to friction in the failing trials.

UNIT 5

Work, Energy and Power

Energy is the most useful idea in physics, because it is conserved: it changes form but never appears or disappears. This unit follows the discovery of that law, from 's Gravesande's balls dropped into clay, which showed that the effect of motion grows as v^2, and Galileo's pendulum, which always returns to its starting height, to Joule's paddle wheel, which turned falling weights into warm water at a fixed exchange rate. You will learn to calculate work, kinetic energy, gravitational and elastic potential energy, and power, and to use them to solve problems that would be hard with forces alone.

The engineering runs from James Watt's steam engine and his horsepower to pumped-storage power stations, brakes, elevators and roller coasters. The labs warm sand by flipping it, make craters in flour, launch a ball with a spring, measure your own horsepower on the stairs, and find the lowest start that sends a Hot Wheels car around a loop. The midterm exam follows this unit.

PREREQUISITES

- Units 2–4: kinematics, Newton's laws, free-body diagrams and centripetal force
- Hooke's law (Lesson 1.4)

MASTERY CHECKLIST

- ☐ I can calculate work with $W = Fd\cos\theta$ and kinetic energy with $\frac{1}{2}mv^2$, and use the work–energy theorem.
- ☐ I can calculate gravitational and elastic potential energy and solve problems with conservation of mechanical energy.
- ☐ I can explain Joule's experiment and convert mechanical energy to a temperature rise with $Q = mc\Delta T$.
- ☐ I can calculate power with $P = W/t$ and $P = Fv$, and convert between watts, horsepower and kilowatt-hours.
- ☐ I can find the minimum height for a loop-the-loop, the g-forces in a loop, and the efficiency of a real machine.

OPTIONAL KITS FOR THIS UNIT

- **K'NEX roller coaster building sets**: Ages 9 and up. Larger coaster sets have a motorized lift and adjustable track, useful for measuring drop heights, loops and efficiency in Lesson 5.5 and the portfolio.
- **Hot Wheels track sets with a loop**: Ages 4 and up. A basic loop set is enough for Lesson 5.5; extra straight track lengthens the ramp.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

5.1	Joule's Paddle Wheel: Energy Is Conserved	3 sessions of 50 minutes
5.2	Work and Kinetic Energy: Balls Dropped into Clay	3 sessions of 50 minutes
5.3	Potential Energy and Conservation: Galileo's Interrupted Pendulum 3 sessions of 50 minutes	
5.4	Power and Watt's Horsepower	3 sessions of 50 minutes
5.5	Roller Coasters, Loops and Efficiency	3 sessions of 50 minutes

Unit 5 Test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$, $c_{\text{water}} = 4186\ \mathrm{J/(kg \cdot K)}$ and $1\ \mathrm{hp} = 746\ \mathrm{W}$. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   State the law of conservation of energy and give an example of energy changing form three times.

2. **6 pts**
   A $5.0\ \mathrm{kg}$ mass falls $2.0\ \mathrm{m}$ ten times, turning a paddle wheel in $0.50\ \mathrm{kg}$ of water. If all the energy heats the water, find the temperature rise.

3. **5 pts**
   Explain why Joule's paddle-wheel result showed that heat is not a fluid.

4. **6 pts**
   A rope pulls a crate $15\ \mathrm{m}$ across a floor with a force of $120\ \mathrm{N}$ at 20° above the horizontal. How much work does the rope do?

5. **6 pts**
   A $0.060\ \mathrm{kg}$ tennis ball is served at $50\ \mathrm{m/s}$. Find its kinetic energy. How much work did the racket do on it (starting from rest)?

6. 7 pts

A 1500 kg car at 24 m/s brakes to a stop in 48 m. Find the average braking force. If the speed had been 12 m/s, what stopping distance would the same force give?

7. 5 pts

’s Gravesande dropped balls into clay. If a ball dropped from h makes a dent 1 cm deep, how deep is the dent from $4h$? Why did this support Leibniz rather than Descartes?

8. 6 pts

A pendulum bob is released 0.45 m above its lowest point. Find its speed at the bottom. If a nail catches the string, how high does it rise on the other side?

9. 7 pts

A spring ($k = 800$ N/m) is compressed 6.0 cm and launches a 40 g ball upward. Predict the maximum height. The ball actually rises 2.9 m; find the efficiency.

10. 6 pts

A roller-coaster car starts from rest at 45 m and passes over a 20 m hill. Find its speed on the hill (no friction).

11. 6 pts

A 70 kg student climbs 4.5 m of stairs in 5.0 s. Find her power in watts and horsepower.

12. 6 pts

A truck engine delivers 150 kW to its wheels at 25 m/s. What is the total resisting force?

13. 5 pts

A 2.0 kW heater runs for 3.0 h. How much energy does it use in kWh and in joules? At 15 cents per kWh, what does it cost?

14. 6 pts

Find the minimum release height and the minimum speed at the top for a frictionless loop of radius 0.20 m.

15. **6 pts**

A car enters a circular loop of radius 6.0 m at the bottom at 17 m/s. What is the rider's apparent weight in g there?

16. **4 pts**

Explain why modern roller-coaster loops are teardrop-shaped (clothoids).

17. **8 pts**

Essay. Using the stories of 's Gravesande, Galileo, Watt and Joule, explain how the idea of energy grew from an argument about moving balls into a law that engineers use to design machines.

ANSWER KEY

1. Energy is never created or destroyed; it only changes form or moves. Example: chemical energy in food → kinetic energy of a cyclist → thermal energy in the brakes when she stops (or gravitational potential energy on a hill).
2. $W = 10(5.0)(9.8)(2.0) = 980$ J; $\Delta T = 980/(0.50 \times 4186) = 0.47$ K
3. A fixed amount of mechanical work always produced the same amount of heating, and the heat was produced from nothing but motion; a conserved fluid could not be created that way. Heat must be a form of energy.
4. $120(15)\cos 20^\circ = 1690$ J
5. 75 J; 75 J
6. $KE = 432{,}000$ J; $F = 9000$ N; at half speed, a quarter of the distance: 12 m
7. 4 cm. From $4h$ the speed is only doubled, so the effect grew as v^2 (mv^2), not as v (mv).
8. $v = \sqrt{2(9.8)(0.45)} = 2.97$ m/s; it rises to 0.45 m (ignoring friction).
9. $PE_s = 1.44$ J; $h = 1.44/(0.040 \times 9.8) = 3.7$ m; efficiency $= 2.9/3.7 = 79\%$
10. $v = \sqrt{2(9.8)(25)} = 22$ m/s
11. $P = 70(9.8)(4.5)/5.0 = 617$ W $= 0.83$ hp
12. $F = P/v = 6000$ N
13. 6.0 kWh $= 2.16 \times 10^7$ J; 90 cents
14. $h = 0.50$ m; $v = \sqrt{9.8 \times 0.20} = 1.40$ m/s
15. $1 + 289/58.8 = 5.9\,g$
16. A clothoid has a large radius near the bottom and a small radius near the top. The train can enter fast on a gentle curve (low g-force) and pass over the top slowly in a tight curve (still enough centripetal force to stay on the track), so the g-forces stay moderate everywhere, unlike a circular loop, which needs about 6 g at the bottom.
17. Model answer: 's Gravesande's clay showed that the effect of a moving body grows with mv^2, the quantity now called kinetic energy. Galileo's interrupted pendulum showed that a falling body gains exactly enough speed to rise back to its starting height, the idea behind potential energy and its conservation. Watt, building engines to replace horses, defined power and showed that doing the same work with less fuel was possible, the beginning of efficiency. Joule proved that work and heat are two forms of one conserved quantity, completing the law. Engineers now use energy accounting to design brakes, elevators, power

stations and roller coasters, choosing materials and sizes so that every joule is accounted for.

PORTFOLIO PIECE: AN ENERGY-ACCOUNTED MARBLE COASTER

Design and build a marble roller coaster from foam pipe insulation (cut lengthwise), card stock or Hot Wheels track, with a first drop, at least one loop, a hill and a final stop in a catch cup. Before building, draw a scaled side view with every height labeled, predict the marble's speed at four points from energy conservation, and size the loop with $h_{\text{min}} = 2.5r$. After building, measure the marble's speed at the same four points with slow-motion video, calculate the efficiency from the start to each point, and adjust your predictions using the measured efficiency. Make the coaster run reliably 8 times out of 10. Write a two-page engineering report with your drawing, an energy table (predicted and measured), efficiency calculations and the changes you made.

LOOK FOR	WHAT IT MEANS
Design and predictions	Scaled drawing with heights; predicted speeds from energy conservation; loop sized by calculation.
Working coaster	Includes a drop, a loop, a hill and a safe stop; works at least 8 times out of 10.
Measurements	Speeds measured by video at four points, three trials each, with a table comparing predicted and measured values.
Energy accounting	Efficiency calculated at each point; losses explained (friction, rolling, air); corrected predictions.
Iteration and report	At least one documented design change with before-and-after data; clear, honest report with units.

UNIT 6

Momentum

Momentum, mass times velocity, is the quantity Newton called “quantity of motion,” and like energy it is conserved. Unlike energy, it is a vector and it is conserved in every collision, explosion and push-off, whether objects bounce, crumple or stick. This unit begins with Nils Bohlin’s three-point seat belt and the impulse–momentum theorem that explains why stretching a stop saves lives, then follows the Royal Society’s collision experiments and the Newton’s cradle, Norman Stingley’s Super Ball and Newton’s own measurements of bounciness, Robert Goddard’s first liquid-fueled rocket and the rocket equation, and Coriolis at the billiard table.

The labs drop eggs onto cushions, collide toy cars on a track, measure the coefficient of restitution of a dozen balls with a phone’s microphone, launch a water rocket in a field, and film coins colliding from above. The engineering runs from crumple zones and air bags to the Saturn V and accident reconstruction.

PREREQUISITES

- Unit 3: Newton’s three laws and free-body diagrams
- Unit 5: kinetic energy and its conservation
- Vectors and components (Lesson 1.5)

MASTERY CHECKLIST

- ☐ I can calculate momentum and impulse and use $F\Delta t = \Delta p$ to explain seat belts, air bags and helmets.
- ☐ I can use conservation of momentum to solve one-dimensional collision, explosion and recoil problems.
- ☐ I can distinguish elastic and inelastic collisions and measure a coefficient of restitution.
- ☐ I can calculate rocket thrust and use the rocket equation, and explain why rockets have stages.
- ☐ I can apply conservation of momentum to two-dimensional collisions using components.

OPTIONAL KITS FOR THIS UNIT

- **Estes model rockets**: Model rockets with solid motors are a step beyond the water rocket for Lesson 6.4. An adult must supervise every launch, following the National Association of Rocketry safety code, in an open field away from power lines.
- **Arbor Scientific collision carts and Newton's cradles**: Low-friction carts with magnetic or hook-and-loop bumpers make Lesson 6.2's momentum measurements much cleaner; a quality Newton's cradle fits Lessons 6.2–6.3.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

6.1	**Impulse and the Seat Belt**	3 sessions of 50 minutes
6.2	**Conservation of Momentum and Newton's Cradle**	3 sessions of 50 minutes
6.3	**Elastic and Inelastic Collisions: The Super Ball**	3 sessions of 50 minutes
6.4	**Rockets: Goddard and the Rocket Equation**	3 sessions of 50 minutes
6.5	**Momentum in Two Dimensions: Billiards and Accident Reconstruction**	3 sessions of 50 minutes

Unit 6 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$. Choose and state a positive direction for every one-dimensional problem; resolve into components for two-dimensional problems. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   Define momentum and impulse and state the impulse–momentum theorem.

2. **6 pts**
   A 65 kg passenger moving at 18 m/s is stopped by a seat belt in 0.12 s. Find the average force. What would it be if the stop took 0.015 s?

3. 5 pts

Explain in two or three sentences how a bicycle helmet protects the head.

4. 6 pts

A 0.40 kg soccer ball at 10 m/s is kicked back at 15 m/s in the opposite direction. The contact lasts 0.020 s. Find the impulse and average force.

5. 6 pts

A 3000 kg truck at 10 m/s collides with and sticks to a 1000 kg car at rest. Find their common speed and the kinetic energy lost.

6. 6 pts

A 55 kg skater throws a 4.0 kg bag forward at 6.0 m/s while standing at rest on ice. Find her recoil velocity.

7. 6 pts

Explain why only one ball swings out of a Newton's cradle when one ball is released, using both conservation laws.

8. 5 pts

A ball dropped from 1.50 m rebounds to 0.96 m. Find the coefficient of restitution and the fraction of energy lost.

9. 7 pts

A 0.30 kg cart at 4.0 m/s collides elastically head-on with a 0.10 kg cart at rest. Find both velocities afterward.

10. 5 pts

Classify each collision as elastic, inelastic or perfectly inelastic: (a) two lumps of clay that stick; (b) two steel balls of a Newton's cradle; (c) a car crash in which the cars bounce apart, both dented.

11. 6 pts

A rocket engine produces 2.0×10^6 N of thrust with an exhaust speed of 2500 m/s. What mass of exhaust does it expel per second?

12. 6 pts

Find the speed change of a rocket stage with $v_e = 3500\ \mathrm{m/s}$ and a mass ratio of 4.0.

13. 5 pts

Explain why rockets work in space, and why they are built in stages.

14. 7 pts

A 1200 kg car going east at $22\ \mathrm{m/s}$ collides with a 1800 kg van going north at $12\ \mathrm{m/s}$; they stick. Find the wreckage's speed and direction.

15. 6 pts

A cue ball at $2.5\ \mathrm{m/s}$ strikes an identical ball at rest elastically; the target ball leaves at 40° from the original direction. Find the cue ball's direction and both speeds.

16. 5 pts

In Goddard's 1926 flight the rocket rose only 41 feet. Explain why it is still regarded as a turning point in the history of engineering.

17. 8 pts

Essay. Choose two of these: the three-point seat belt, the crumple zone, the air bag, the bicycle helmet, the climbing rope. Explain how each uses impulse and momentum, and discuss why an engineer must consider how people will actually use a safety device.

ANSWER KEY

1. Momentum $\vec{p} = m\vec{v}$; impulse $\vec{J} = \vec{F}\Delta t$; the impulse on an object equals its change in momentum.
2. $\Delta p = 1170\ \mathrm{kg \cdot m/s}$; 9750 N; 78,000 N
3. Its foam crushes during an impact, lengthening the time over which the head's momentum is stopped. Since $F\Delta t = \Delta p$, a longer time means a smaller average force on the head, and the shell spreads the force over a larger area.
4. $\Delta p = 0.40(25) = 10\ \mathrm{kg \cdot m/s}$; $F = 500\ \mathrm{N}$
5. $7.5\ \mathrm{m/s}$; KE before 150,000 J, after 112,500 J; lost 37,500 J
6. $-0.44\ \mathrm{m/s}$ (backward)
7. One ball out at speed v conserves both momentum (mv) and kinetic energy ($\frac{1}{2}mv^2$). Two balls at $v/2$ would conserve momentum but have only half the kinetic energy, which would violate energy conservation in a nearly elastic collision.
8. $e = \sqrt{0.64} = 0.80$; 36% lost
9. $v_1' = (0.20/0.40)(4.0) = 2.0\ \mathrm{m/s}$; $v_2' = (0.60/0.40)(4.0) = 6.0\ \mathrm{m/s}$
10. (a) perfectly inelastic; (b) nearly elastic; (c) inelastic.
11. $800\ \mathrm{kg/s}$
12. $3500 \ln 4 = 4850\ \mathrm{m/s}$
13. A rocket pushes its exhaust backward and the exhaust pushes it forward (third law; momentum conservation); no air is needed. Stages let the rocket drop empty tanks and engines, so the remaining stages do not waste fuel accelerating dead weight, which the rocket equation makes very costly.
14. $p_x = 26{,}400$, $p_y = 21{,}600$; $p = 34{,}110$; $v = 11.4\ \mathrm{m/s}$ at 39.3° north of east
15. Cue ball at 50° on the other side. Target: $2.5 \cos 40^\circ = 1.92\ \mathrm{m/s}$; cue: $2.5 \cos 50^\circ = 1.61\ \mathrm{m/s}$
16. It was the first rocket powered by liquid fuel, which can produce far more energy per kilogram than the powder of earlier rockets and can be throttled, pumped and controlled. It proved that the approach worked; every large rocket since, including the Saturn V, descends from it.
17. Model answer: Each device lengthens the time over which a person's momentum is reduced, so that $F = \Delta p/\Delta t$ is smaller, and spreads the force over a strong part of the body. The seat belt stretches slightly and holds the pelvis and chest; the crumple zone crushes over a known distance; the air bag cushions

and vents; the helmet's foam crushes; the rope stretches. But a device helps only if it is used correctly: Bohlin designed his belt to buckle with one hand in seconds, helmets must be comfortable enough to wear, and ropes must be easy to inspect. Good safety engineering includes human habits as a design requirement.

PORTFOLIO PIECE: THE EGG LANDER

Design and build a lander that protects a raw egg dropped from 3.0 m (a second-floor window or a stair landing, with an adult) using no more than 20 sheets of paper, 1 m of tape and 10 drinking straws. Before testing, estimate the impact speed and the stopping distance your design provides, and calculate the average force on the egg. Test with a dummy weight first, film every drop in slow motion, measure the actual stopping time, and compare. Improve your design at least once. Write a report with a labeled sketch, your calculations of momentum, impulse and force, your video measurements and a discussion of the trade-off between mass, size and protection.

LOOK FOR	WHAT IT MEANS
Design and constraints	Labeled sketch; materials within the limits; a clear plan for lengthening the stopping time.
Calculations	Impact speed, change in momentum, predicted stopping time and average force, all with units.
Testing and measurement	Slow-motion video of each drop; measured stopping distance and time; survival recorded honestly.
Iteration	At least one documented improvement with before-and-after data.
Report	Explains the physics with $F\Delta t = \Delta p$ and connects the design to seat belts, air bags or helmets.

6.1 Impulse and the Seat Belt

OBJECTIVES

- Define momentum, $\vec{p} = m\vec{v}$, and impulse, $\vec{J} = \vec{F}\Delta t$, and use the impulse–momentum theorem $\vec{F}\Delta t = \Delta\vec{p}$.
- Explain why stretching out the time of a collision reduces the force, using seat belts, air bags, crumple zones and helmets.
- Find the impulse from the area under a force–time graph.
- Design and test a cushion that protects a dropped egg, and estimate the average force.

> **Warm-up.** Toss a raw egg (in a zip-top bag, over a sink) gently into a bedsheet held loosely by two people. Then imagine tossing it the same way at a wall. Why does the sheet save it? Write your answer using the words “force” and “time.”

WATCH

Impulse and Momentum - Formulas and Equations - College Physics

The Organic Chemistry Tutor · 15:24

https://www.youtube.com/watch?v=FYKoU0sb9ks

> **WHEN TO WATCH**
> After the physics paragraph, before the worked examples.

BACKUP VIDEO

1st place Egg Drop project ideas- using SCIENCE

Mark Rober · 9:48

https://www.youtube.com/watch?v=nsnyl8llfH4

Before you design the portfolio egg lander: an engineer’s ideas for stretching the stopping time.

KEY TERMS

momentum

Mass times velocity, $\vec{p} = m\vec{v}$, a vector measured in kg·m/s.

impulse

Force multiplied by the time it acts, $\vec{J} = \vec{F}\Delta t$, in N·s (the same as kg·m/s). For a changing force, the area under the force–time graph.

impulse–momentum theorem

The impulse on an object equals its change in momentum: $\vec{F}_{\text{avg}}\Delta t = \Delta\vec{p}$.

crumple zone

The front and rear parts of a car designed to crush in a controlled way, lengthening the time of a collision.

three-point seat belt

Nils Bohlin's 1959 design: a lap belt and a diagonal shoulder belt joined at a buckle beside the hip, holding the pelvis and chest.

CORE IDEAS

The story. Early automobile seat belts were simple lap belts, and in a hard crash they could injure the abdomen as the body folded over them. In 1958 the Swedish car maker Volvo hired Nils Bohlin, an engineer who had worked on aircraft safety systems, to design something better. Within a year he had a belt that crossed the chest diagonally and the hips horizontally, anchored at three points low beside the seat, that could be buckled with one hand in a few seconds. Volvo began fitting it as standard equipment in its cars in Sweden in 1959. Then the company did something unusual: it made the patent available for any manufacturer to use, so that the design would spread as quickly as possible. Today the three-point belt is in nearly every car in the world.

How they figured it out. Bohlin's rules were practical: the belt had to hold the body across the strongest bones, the pelvis and the rib cage; it had to stay in place in a crash; and it had to be so easy to use that people would actually wear it. Volvo then studied thousands of real accidents in Sweden to measure how the belts performed, the kind of honest data engineers need. Crash-test laboratories now use instrumented dummies, high-speed cameras and accelerometers, the descendants of Stapp's sled tests (Lesson 2.2), to measure the forces and times in every new design.

The physics. Momentum is $\vec{p} = m\vec{v}$. Newton actually wrote his second law in terms of momentum: the net force equals the rate of change of momentum. Multiply by time: $\vec{F}\Delta t = \Delta\vec{p}$. The left side is the **impulse**. In a crash, a passenger's change in momentum is fixed by her mass and speed; what the engineer can change is the time over which it happens. Double the stopping time and the average force is halved. A seat belt stretches slightly and a crumple zone crushes, extending the stop from a

few thousandths of a second (hitting the dashboard) to about a tenth of a second, and the belt spreads that force over the strongest parts of the body.

The engineering. The same physics designs air bags (which inflate in about 30 milliseconds and then vent as the passenger presses into them), bicycle helmets (whose foam crushes to lengthen the stop of the head), the plastic barrels of water and sand in front of highway bridge supports, the padding in gymnastics mats, and the stretchy "dynamic" ropes rock climbers use. Engineers sometimes want the opposite: a hammer, a golf club and a pile driver deliver a large force by making the contact time very short.

Try it. In the lab you will drop raw eggs onto different cushions, film the impacts, and calculate the average force on the egg from its change in momentum and the stopping time.

PICTURE STUDY

A Crash-Test Dummy After a Test, National Highway Traffic Safety Administration, 2013 · Photograph · Vehicle Research and Test Center, Ohio

Image: Wikimedia Commons, public domain

Every new car model is crash-tested with dummies full of sensors. This photograph shows the scene inside a test car moments after a collision.

Look closely

- Find the shoulder belt and the lap belt. Where do they cross the dummy's body?
- Find the air bag. What has happened to it?
- Look for wires and sensor marks on the dummy. What might they measure?

Talk about it

1. Why is the shoulder belt placed across the collarbone and chest instead of the neck or stomach?
2. Why does an air bag have vents that let it deflate as the dummy presses into it?
3. What two quantities must engineers measure to find the average force on the dummy's head?

Safety first. Do the egg drops outdoors or over a tray, with each egg in a sealed zip-top bag. Wash hands after handling raw eggs.

LAB

Protecting an Egg: Impulse and Stopping Distance

You will drop a raw egg in a bag from a fixed height onto different cushions, film each impact in slow motion, measure the stopping distance and time, and calculate the average force on the egg.

You will need

- Raw eggs (6–12), zip-top bags
- Cushions: a folded towel, a pillow, a block of foam, a stack of newspaper, a box of crumpled paper
- Tape measure, a ruler stood upright beside the cushion
- Phone with slow-motion video
- Kitchen scale

Steps

1. Weigh an egg in its bag. Choose a drop height of 1.00 m measured from the bottom of the egg.
2. **Hypothesis.** Rank the cushions from safest to least safe and give a reason in terms of stopping distance and time.
3. Drop the egg from rest onto the first cushion. Film the impact from the side, close up, with the upright ruler in view.
4. From the video, find how far the egg sank into the cushion before stopping (d), and count frames from first contact to the stop to find Δt.
5. Calculate the impact speed $v = \sqrt{2gh}$, the change in momentum mv, and the average force $F = mv/\Delta t + mg$. Check Δt against $2d/v$.
6. Record whether the egg survived. Repeat for each cushion.
7. **Fair test.** With the best cushion, raise the drop height to 1.5 m and 2.0 m. At what height does the egg break? Estimate the force it can survive.
8. **Improve.** Design a cushion of your own that uses the least material and still saves the egg from 2.0 m. Test it and calculate the force.

Tips

- A white egg against a dark background shows up best in video.
- Count frames carefully; at 240 frames per second, 10 frames is about 0.04 s.

Check your work

- ☐ I calculated the change in momentum and the average force for each cushion.
- ☐ I can explain with

$F\Delta t$why a longer stopping time lowers the force.

Δp

- ☐ I can explain how seat belts, air bags and helmets use the same idea.

LAB NOTEBOOK

1. Data table: cushion, d, Δt, impact speed, Δp, average force, survived?
2. Sketch force–time graphs for the egg landing on a pillow and on a hard floor, with the same area under each. Label which is taller and which is wider.
3. Draw a three-point seat belt on a simple outline of a person and label the strong bones it presses on.

KEY FORMULAS

$$\vec{p} = m\vec{v}$$

$$\vec{J} = \vec{F}_{\text{avg}}\Delta t = \Delta\vec{p}$$

$$\Delta t \approx \frac{2d}{v} \quad \text{(constant deceleration over stopping distance } d\text{)}$$

From history. Nils Bohlin was inducted into the U.S. National Inventors Hall of Fame in 2002. Seat-belt laws spread through the 1970s and 1980s, and the National Highway Traffic Safety Administration estimates that seat belts save thousands of lives in the United States every year.

Example 1. A 70 kg passenger moving at 15 m/s (about 34 mi/h) is brought to rest in a crash. Find the average force if (a) the seat belt and crumple zone stop her in 0.10 s, (b) she hits the dashboard and stops in 0.010 s.

Show solution

1. Change in momentum: $\Delta p = m\Delta v = (70\text{ kg})(0 - 15\text{ m/s}) = -1050\text{ kg}\cdot\text{m/s}$.
2. (a) $F = \dfrac{\Delta p}{\Delta t} = \dfrac{1050\text{ kg}\cdot\text{m/s}}{0.10\text{ s}} = 10{,}500\text{ N}$ (about 15 times her weight, spread over pelvis and chest).
3. (b) $F = \dfrac{1050\text{ kg}\cdot\text{m/s}}{0.010\text{ s}} = 105{,}000\text{ N}$, ten times larger, concentrated where she hits.

Answer: (a) 1.05×10^4 N; (b) 1.05×10^5 N. Same impulse, ten times the time, one tenth the force.

Example 2. A 0.145 kg baseball arrives at 40 m/s and leaves the bat at 45 m/s in the opposite direction. The contact lasts 0.70 ms. Find the impulse and the average force.

Show solution

1. Take the outgoing direction as positive: $v_i = -40\text{ m/s}$, $v_f = +45\text{ m/s}$.
2. $\Delta p = m(v_f - v_i) = (0.145\text{ kg})(45 - (-40))\text{ m/s} = (0.145)(85)\text{ kg}\cdot\text{m/s} = 12.3\text{ l}$
3. $F = \dfrac{12.3\text{ N}\cdot\text{s}}{7.0 \times 10^{-4}\text{ s}} = 1.76 \times 10^4\text{ N}$.

Answer: Impulse $12.3\text{ N}\cdot\text{s}$; average force about 17,600 N, more than 12,000 times the ball's weight.

Example 3. A 60 g egg falls 1.0 m onto a cushion that stops it in 2.0 cm. Estimate the stopping time and the average force from the cushion. What if a hard floor stops it in 2.0 mm?

Show solution

1. Impact speed: $v = \sqrt{2gh} = \sqrt{2(9.8)(1.0)}\ \mathrm{m/s} = 4.43\ \mathrm{m/s}$.
2. With steady deceleration the average speed while stopping is $v/2$, so $\Delta t = \dfrac{d}{v/2} = \dfrac{2(0.020\ \mathrm{m})}{4.43\ \mathrm{m/s}} = 9.0 \times 10^{-3}\ \mathrm{s}$.
3. Net force: $\dfrac{m\Delta v}{\Delta t} = \dfrac{(0.060)(4.43)}{0.0090}\ \mathrm{N} = 29.4\ \mathrm{N}$. The cushion must also hold up the egg's weight (0.59 N): $F \approx 30\ \mathrm{N}$.
4. Hard floor, $d = 2.0$ mm: ten times shorter time, so about 294 N.

Answer: About 9 ms and 30 N on the cushion; about 300 N on a hard floor, enough to break the shell.

PRACTICE

1. Easy Find the momentum of a 1200 kg car moving at 25 m/s.

   Show answer

   $3.0 \times 10^4\ \mathrm{kg \cdot m/s}$

2. Easy A 50 N force acts for 0.20 s. What is the impulse?

   Show answer

   $10\ \mathrm{N \cdot s}$

3. Easy Why do you bend your knees when you land from a jump?

   Show answer

   Bending your knees lengthens the time over which your momentum is stopped, which lowers the force on your legs.

4. Medium A 0.050 kg golf ball leaves the club at 70 m/s after a contact time of 0.50 ms. Find the average force.

   Show answer

   $\Delta p = 3.5\ \mathrm{kg \cdot m/s}$; $F = 7000\ \mathrm{N}$

5. Medium A 2.0 kg ball moving at 5.0 m/s hits a wall and bounces back at 4.0 m/s. Find the impulse from the wall.

Show answer

$2.0(4.0 - (-5.0)) = 18\ \mathrm{N \cdot s}$, directed away from the wall

6. Medium A force–time graph rises in a straight line from 0 to 600 N in 0.010 s and falls back to zero in another 0.010 s. Find the impulse. If it acts on a 0.40 kg ball at rest, what is the ball's speed afterward?

Show answer

$\text{Area} = \frac{1}{2}(0.020)(600) = 6.0\ \mathrm{N \cdot s}$; $v = 15\ \mathrm{m/s}$

7. Medium An air bag stops a 5.0 kg head moving at 12 m/s in 0.060 s. Find the average force. What would it be if the head hit the steering wheel and stopped in 0.0040 s?

Show answer

1000 N; 15,000 N

8. Medium Which has more momentum: a 1500 kg car at 2.0 m/s or a 0.010 kg object at 300 m/s? Which has more kinetic energy?

Show answer

The car: 3000 kg·m/s versus 3.0 kg·m/s. Kinetic energy: car 3000 J, object 450 J. The car has more of both.

9. Hard A 70 kg crash-test dummy moving at 13.4 m/s (30 mi/h) is stopped by its belt over 0.30 m of forward travel. Estimate the stopping time and average force, and express the force in multiples of the dummy's weight.

Show answer

$\Delta t = 2d/v = 0.045$ s; $F = 70(13.4)/0.045 = 2.1 \times 10^4$ N, about 30 times its weight (30 g).

10. Hard A rock climber of mass 80 kg falls 4.0 m before her rope begins to stretch, and the rope stops her in 0.50 s. Find the average force of the rope on her. Why are climbing ropes made to stretch?

Show answer

$v = 8.85$ m/s; $\Delta p = 708$ kg · m/s; net force 1416 N; rope force $= 1416 + 784 = 2200$ N. A stretchy rope lengthens the stopping time, which keeps the force on the climber and the anchors low.

11. Challenge Bohlin's design goals included "easy to use with one hand." Explain, from an engineering point of view, why a seat belt that people do not wear has zero safety value no matter how strong it is, and what this teaches about designing safety equipment.

Show answer

A safety device works only if it is used. If the belt is awkward, uncomfortable or slow, many people will skip it, and the impulse–momentum physics never comes into play. Good safety engineering includes human behavior: convenience and comfort are design requirements, not luxuries.

COMMON MISTAKES

- Forgetting that momentum is a vector. A ball that bounces back has a larger change in momentum than one that stops.
- Thinking a soft cushion reduces the change in momentum. It reduces the force by lengthening the time; the change in momentum is the same.
- Using kinetic energy where momentum belongs. Impulse changes momentum; work changes kinetic energy.

Make it a game · Seatbelt Time Stretch. Your parent names a stopping scenario: a 1000 kg car going from 10 m/s to rest in 0.1 seconds, then 1 second. Calculate each force with the impulse-momentum theorem in 60 seconds, then compare. Explain in one sentence how crumple zones help. Run three scenarios with different masses and speeds.

Scripture. "A prudent man foreseeth the evil, and hideth himself" (Proverbs 22:3, KJV)

Wearing a seat belt is simple prudence: foreseeing a danger and taking cover from it. Designing one is a way of loving your neighbor.

6.2 Conservation of Momentum and Newton's Cradle

OBJECTIVES

- State the law of conservation of momentum and explain why it follows from Newton's third law.
- Solve one-dimensional problems involving collisions, explosions and recoil.
- Explain why one ball, not two half-speed balls, swings out of a Newton's cradle.
- Measure momentum before and after a collision of two toy cars.

> **Warm-up.** Line up five marbles touching each other in the groove of a ruler (or along a book's spine). Roll a sixth marble into the end of the line. What happens? Now roll two marbles together into the line. Predict first, then try it, and write down what you saw.

WATCH

Introduction to Conservation of Momentum with Demonstrations

Flipping Physics · 3:58

https://www.youtube.com/watch?v=Kf0bBxmNeec

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Newton's Cradle Doesn't Work the Way You Think

The Action Lab · 8:44

https://www.youtube.com/watch?v=YXQUdYmTZIs

After the Newton's cradle worked example: why the cradle behaves as it does, with high-speed close-ups.

KEY TERMS

conservation of momentum
When no net external force acts on a system, its total momentum stays constant.

system
The set of objects you choose to study. Forces between them are internal; forces from outside are external.

isolated system

A system with no net external force, so its total momentum is conserved.

recoil

The backward motion of an object that pushes another object forward, as a skater does when throwing a ball.

Newton's cradle

A row of touching steel balls hung as pendulums; a ball swung into one end sends a ball out of the other.

CORE IDEAS

The story. In 1666 the Royal Society of London asked its members to find the laws of collision. Over the winter of 1668–1669, three answers arrived. John Wallis, a mathematician, treated soft bodies that stick together; Christopher Wren, the astronomer who would later build St Paul's Cathedral, and Christiaan Huygens in the Netherlands treated hard, bouncing bodies, and Wren showed his rules with balls hung as pendulums in front of the Society. In 1673 the French physicist Edme Mariotte published a whole book of collision experiments with rows of hanging balls. Newton used the same pendulum method in the *Principia*, colliding balls on 10-foot strings and measuring how far each swung, and found that in every collision the total "quantity of motion," mass times velocity, was the same before and after.

How they figured it out. The pendulum was the key instrument. A ball released from a measured height arrives at the bottom with a known speed, and after a collision the height to which each ball swings tells its new speed. Newton even corrected for air resistance by watching how much a single pendulum's swing shrank each time. With this he could compare momentum before and after collisions between balls of different sizes and materials. The toy now called Newton's cradle, a row of steel balls on strings, became a popular desk ornament in the 1960s, when the English actor Simon Prebble gave it its name in 1967; it is a living copy of the Royal Society's apparatus.

The physics. When two objects collide, Newton's third law says each pushes the other with an equal and opposite force for the same time, so their impulses are equal and opposite and their momentum changes cancel. The total momentum of the pair does not change: $m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$. This is true for any isolated system, in any kind of collision, explosion or push-off, whether or not kinetic energy is

conserved. In a Newton's cradle, both momentum and kinetic energy are (nearly) conserved; one ball swinging in at speed v could in principle send out two balls at $v/2$ (same momentum), but that would have only half the kinetic energy, so only one ball at speed v satisfies both laws.

The engineering. Momentum conservation lets accident investigators find the speeds of cars before a crash from where the wreckage went afterward. Spacecraft engineers use it to plan dockings, so that a capsule meeting the space station barely changes the station's motion. Railway yards couple freight cars by rolling one into another; the coupled cars move off at a speed set by momentum. And every rocket, jet and fire hose is a recoil problem, which is the subject of Lesson 6.4.

Try it. In the lab you will collide two toy cars on a straight track, one of which sticks to the other, film them, and check whether momentum before equals momentum after.

PICTURE STUDY

A Newton's Cradle, Unknown photographer, 2007 · Photograph · Wikimedia Commons (released into the public domain)

Image: Wikimedia Commons, public domain

This familiar toy is a small copy of the pendulum apparatus Wren, Mariotte and Newton used to discover the laws of collision.

Look closely

- Notice that each ball hangs from two strings in a V shape. Why two strings instead of one?
- Look at how the balls touch in a straight line. What would happen if they were not aligned?
- Find the frame. Why is it heavy and stiff?

Talk about it

1. When one ball strikes, why does exactly one ball fly out the other side?
2. What would happen if you swung two balls in? Three?
3. Why does the cradle eventually stop? Where does its energy go?

LAB

Momentum Before and After: Colliding Toy Cars

You will roll one toy car into another on a straight track, measure the speeds before and after with slow-motion video, and check whether the total momentum is conserved, both when the cars bounce and when they stick.

You will need

- Two toy cars that roll freely and straight (Hot Wheels on straight orange track, or two small toy trucks)
- A small piece of poster putty or hook-and-loop tape to make the cars stick
- Coins or washers taped on to change mass
- Kitchen scale, meter stick taped along the track, phone with slow-motion video
- Optional: a Newton's cradle

Steps

1. Weigh both cars. Lay the track flat and tape a meter stick beside it.
2. **Hypothesis.** If a moving car sticks to a stationary car of equal mass, predict their speed afterward as a fraction of the first car's speed.
3. Put putty on the front of car 1 and the back of car 2. Place car 2 at rest in the middle. Give car 1 a push so it rolls into car 2, filming from above or the side.
4. From the video, measure car 1's speed just before the collision and the pair's speed just after (distance over a short time, 10–20 cm).
5. Calculate the momentum before and after. Find the percent difference.
6. **Fair test.** Tape washers to car 2 to double its mass and repeat. Then remove the putty so the cars bounce, and measure both cars' speeds after the collision.
7. For the bouncing collisions, also compare the total kinetic energy before and after.
8. **Improve.** Friction slows the cars between your measurements. Measure the speeds as close to the collision as possible, or measure each car's slowing on its own and correct for it.

Tips

- Measure speeds over short distances right before and right after the collision.
- Choose the direction car 1 moves as positive and keep the sign of every velocity.

Check your work

- ☐ My momentum before and after agree within my measurement uncertainty.
- ☐ I can explain why momentum is conserved even when kinetic energy is not.
- ☐ I can explain the behavior of a Newton's cradle using both conservation laws.

LAB NOTEBOOK

1. Data table for each collision: masses, velocities before and after with signs, total momentum before and after, percent difference, kinetic energy before and after.
2. "Before" and "after" sketches of each collision with velocity arrows drawn to scale.
3. Explain, with the third law, why the momentum change of car 1 is exactly opposite to that of car 2.

KEY FORMULAS

$$\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}$$

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2'$$

$$v' = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} \quad \text{(objects stick together)}$$

From history. Wren's rules of collision, presented in December 1668, were among his last scientific papers before he turned fully to architecture. Mariotte is also remembered for Mariotte's law (Boyle's law, discovered independently) and for finding the blind spot of the eye.

Example 1. A 20,000 kg railroad car rolling at 3.0 m/s couples with a stationary 30,000 kg car. Find their speed afterward and the kinetic energy lost.

Show solution

1. Momentum before: $(20{,}000\ \text{kg})(3.0\ \text{m/s}) + 0 = 60{,}000\ \text{kg}\cdot\text{m/s}$.
2. After, moving together: $v' = \dfrac{60{,}000}{50{,}000}\ \text{m/s} = 1.2\ \text{m/s}$.
3. Kinetic energy: before $\frac{1}{2}(20{,}000)(3.0)^2 = 90{,}000\ \text{J}$; after $\frac{1}{2}(50{,}000)(1.2)^2 = 36{,}000\ \text{J}$; lost 54,000 J (to sound, heat and bending the couplers).

Answer: 1.2 m/s; 54 kJ of kinetic energy lost, though momentum is conserved.

Example 2. A 60 kg skater standing on ice throws a 2.0 kg ball forward at 8.0 m/s. Find her recoil velocity.

Show solution

1. Before: everything at rest, total momentum zero.
2. After: $(2.0\ \text{kg})(8.0\ \text{m/s}) + (60\ \text{kg})v = 0$.
3. $v = -\dfrac{16\ \text{kg}\cdot\text{m/s}}{60\ \text{kg}} = -0.27\ \text{m/s}$.

Answer: 0.27 m/s backward.

Example 3. In a Newton's cradle with balls of mass m, one ball strikes the row at speed v. Show that one ball leaving at v conserves both momentum and kinetic energy, but two balls leaving at $v/2$ would not.

Show solution

1. Before: momentum mv; kinetic energy $\frac{1}{2}mv^2$.
2. One ball out at v: momentum mv, energy $\frac{1}{2}mv^2$. Both match.
3. Two balls out at $v/2$: momentum $2m(v/2) = mv$ (matches), but energy $2 \cdot \frac{1}{2}m(v/2)^2 = \frac{1}{4}mv^2$, only half. Energy would vanish, which cannot happen in a nearly elastic collision of steel balls.

Answer: Only the one-ball result conserves both momentum and kinetic energy, so that is what the cradle does.

PRACTICE

1. Easy A 3.0 kg cart at 2.0 m/s sticks to a 1.0 kg cart at rest. Find their final speed.

   Show answer

   1.5 m/s

2. Easy Give two everyday examples of recoil.

   Show answer

   A rowboat moving backward when you step off it onto a dock; a fire hose pushing back on the firefighter; a skateboarder throwing a heavy ball.

3. Easy Is momentum conserved when a ball falls toward the Earth? Explain, choosing your system.

   Show answer

   For the ball alone, no: gravity is an external force. For the ball and the Earth together, yes: the Earth gains an equal and opposite (but undetectable) momentum.

4. Medium A 1500 kg car at 20 m/s rear-ends a 1000 kg car at 10 m/s, and they lock bumpers. Find their common speed.

   Show answer

   $(30{,}000 + 10{,}000)/2500 = 16 \text{ m/s}$

5. Medium A 50 kg girl and a 75 kg boy stand on skates facing each other and push apart. She moves off at 1.5 m/s. Find his velocity.

Show answer

1.0 m/s in the opposite direction

6. Medium A 0.20 kg ball moving at 4.0 m/s hits a 0.30 kg ball at rest. Afterward the first ball moves at 0.80 m/s backward. Find the second ball's velocity.

Show answer

$0.80 = -0.16 + 0.30v$, so $v = 3.2$ m/s forward

7. Medium A 2.0 kg model rocket body moving straight up at 10 m/s separates into a 1.2 kg nose section moving up at 25 m/s and a lower section. Find the lower section's velocity.

Show answer

$2.0(10) = 1.2(25) + 0.8v$, so $v = -12.5$ m/s (downward)

8. Medium In a perfectly inelastic collision, is kinetic energy conserved? Is momentum?

Show answer

Momentum is conserved; kinetic energy is not (some becomes heat, sound and deformation).

9. Hard A 70 kg astronaut floating at rest throws a 5.0 kg tool at 3.0 m/s to get back to the station 20 m away. How long does she take to reach it?

Show answer

$v = 15/70 = 0.214$ m/s; $t = 20/0.214 = 93$ s

10. Hard In Newton's pendulum experiments, a ball released from 0.10 m above its lowest point collides with an identical ball at rest, and they stick together. How high do they swing?

Show answer

Speed before: $\sqrt{2(9.8)(0.10)} = 1.40$ m/s; after: 0.70 m/s; height $= 0.70^2/19.6 = 0.025$ m, one quarter of the original.

11. **Challenge** A 2.0 kg cart at 3.0 m/s collides elastically with a 1.0 kg cart at rest. Use conservation of momentum and of kinetic energy to find both final velocities.

Show answer

$v_1' = \dfrac{m_1 - m_2}{m_1 + m_2} v = 1.0 \text{ m/s}$; $v_2' = \dfrac{2m_1}{m_1 + m_2} v = 4.0 \text{ m/s}$. Check: momentum $2 + 4 = 6$; energy $1 + 8 = 9$ J.

COMMON MISTAKES

- Dropping the signs: velocities in opposite directions must have opposite signs.
- Assuming kinetic energy is conserved in every collision. Only momentum is always conserved in an isolated system.
- Including an external force (gravity, friction) and still expecting momentum to be conserved.

> **Make it a game · Recoil Predictions.** Your parent describes a collision or explosion: a 2 kg cart at 3 m/s hitting a still 1 kg cart that sticks, or a skater pushing off a wall. Predict the final velocity by conservation of momentum on paper before you calculate. Closest guess wins. Try five scenarios, mixing sticky and bouncy collisions.

6.3 Elastic and Inelastic Collisions: The Super Ball

OBJECTIVES

- Distinguish elastic, inelastic and perfectly inelastic collisions by what happens to kinetic energy.
- Define the coefficient of restitution and measure it from bounce heights or bounce times.
- Solve one-dimensional elastic collisions with the standard formulas.
- Explain what makes a ball bouncy, using the Super Ball as a case study.

> **Warm-up.** Collect several balls: a tennis ball, a ping-pong ball, a golf ball, a rubber bouncy ball, and a ball of modeling clay. Predict the order from bounciest to least bouncy. Drop each from the same height onto a hard floor and watch how high it comes back. Were you right?

WATCH

Elastic and inelastic collisions | Impacts and linear momentum | Physics | Khan Academy

Khan Academy Physics · 14:24

https://www.youtube.com/watch?v=8OB8eIPgEkQ

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

The Physics of Bouncing a Ball | Science Project

Science Buddies · 3:16

https://www.youtube.com/watch?v=kzJ480muh4U

Before the lab: how to measure the coefficient of restitution of a bouncing ball.

KEY TERMS

elastic collision

A collision in which total kinetic energy is conserved (as well as momentum). Steel balls and billiard balls come close.

inelastic collision

A collision in which some kinetic energy becomes heat, sound or permanent deformation; momentum is still conserved.

perfectly inelastic collision

A collision in which the objects stick together, losing the most kinetic energy that momentum conservation allows.

coefficient of restitution

$e = \frac{\text{speed of separation}}{\text{speed of approach}}$. For a ball bouncing on a hard floor, $e = \sqrt{h_{\text{bounce}}/h_{\text{drop}}}$. $e = 1$ is perfectly elastic, $e = 0$ perfectly inelastic.

CORE IDEAS

The story. In 1964 Norman Stingley, a chemist at a small California rubber company, was experimenting with a synthetic rubber, polybutadiene. He found that when he squeezed it under enormous pressure, about 3,500 pounds per square inch, it made a ball that bounced almost as high as it was dropped. His own company was not interested, so he took it to the toy company Wham-O, which had sold the Hula-Hoop and the Frisbee. Wham-O's engineers worked with Stingley to solve the hard problem: the early balls tended to crack and break apart. In 1965 they sold the Super Ball, and millions were bought within months. Thrown hard at the ground, it bounced over houses.

How they figured it out. The first person to measure bounciness carefully was Newton. In the *Principia* he described colliding balls of different materials on long pendulums and measuring their speeds before and after. He found that the speed with which two balls separate is a fixed fraction of the speed with which they approach, a fraction that depends on the materials: about 5/9 for tightly wound balls of wool, about the same for steel, a little less for cork, and 15/16 for glass. That fraction is the **coefficient of restitution**. Stingley's compressed rubber had a coefficient of about 0.9: it gave back most of its energy instead of turning it into heat.

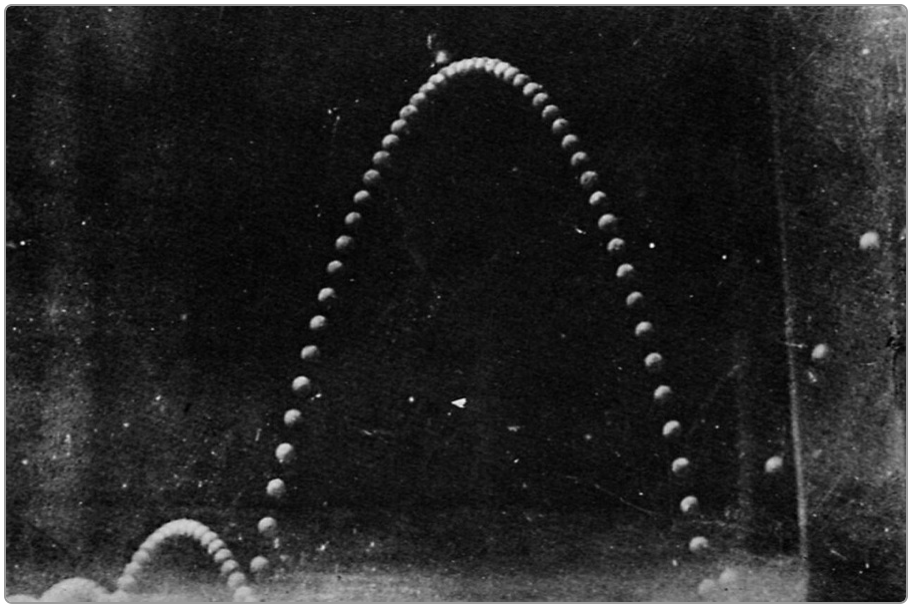
The physics. In every collision of an isolated system, momentum is conserved. Kinetic energy is conserved only in an **elastic** collision. When a ball hits the floor, it squashes; if the material springs back without internal friction, nearly all the stored

elastic energy returns as kinetic energy (high e); if the material is "lossy" like clay, the energy becomes heat (low e). For a ball dropped from h_1 that bounces to h_2, the speeds are $\sqrt{2gh}$, so $e = \sqrt{h_2/h_1}$. For a head-on elastic collision of m_1 moving at v_1 with m_2 at rest: $v_1' = \dfrac{m_1 - m_2}{m_1 + m_2} v_1$ and $v_2' = \dfrac{2m_1}{m_1 + m_2} v_1$. Equal masses simply exchange velocities, as in a Newton's cradle.

The engineering. Sports bodies regulate restitution. The United States Golf Association limits the coefficient of restitution of a driver's face to 0.830, so that clubs cannot be made too "springy." Baseballs, tennis balls and basketballs must bounce within set limits in standard tests. Car bumpers, by contrast, are designed to absorb energy in a low-speed collision rather than bounce. And engineers making rubber parts, from shoe soles to engine mounts, choose materials with the right amount of energy loss: too little, and they vibrate; too much, and they heat up.

Try it. In the lab you will measure the coefficient of restitution of several balls two ways: by bounce height, and by the time between bounces, measured with your phone's microphone.

PICTURE STUDY



The Rebound of a White Ball, Étienne-Jules Marey, 1886 · Chronophotograph · Collège de France, Paris

Image: Wikimedia Commons, public domain

Marey's camera recorded a bouncing ball at equal time intervals on one plate. The picture shows a projectile's parabola, and also the energy lost in each bounce.

Look closely

- Find the tall arch and the smaller arch after the bounce.
- Compare the spacing of the images near the top of an arch with the spacing near the floor.
- Estimate the ratio of the second arch's height to the first.

Talk about it

1. Use your height estimate to find the ball's coefficient of restitution.
2. Why are the images crowded at the tops of the arches?
3. Where did the energy lost in the bounce go?

LAB

How Bouncy Is It? Measuring the Coefficient of Restitution

You will measure the coefficient of restitution of several balls in two independent ways, from bounce heights and from the times between bounces, and compare.

You will need

- Several balls: a small high-bounce rubber ball, a tennis ball, a ping-pong ball, a golf ball, a basketball or soccer ball, a ball of modeling clay
- A hard floor (concrete or tile) and a meter stick taped to the wall
- Phone with slow-motion video and the free phyphox app (its "Inelastic collision" experiment times bounces by sound)
- A carpet or a foam mat for a surface test

Steps

1. **Hypothesis.** Rank the balls by expected e and give reasons in terms of the materials.
2. **Height method.** Drop each ball from 1.00 m (bottom of ball) beside the meter stick, filming in slow motion. Read the top of the first bounce. 3 trials each. Calculate $e = \sqrt{h_2/h_1}$.
3. **Timing method.** Start phyphox's "Inelastic collision" experiment, hold the phone near the floor, and drop each ball. Phyphox records the times between bounces. Calculate $e = t_2/t_1$ for the first few bounces.
4. Compare the two methods for each ball. Which do you trust more, and why?
5. Check whether e stays the same from bounce to bounce or changes as the ball slows.
6. **Fair test.** Keep the ball the same and change the surface: tile, wood, carpet, a foam mat. How does e change?
7. **Temperature test (optional).** Chill a tennis ball in the refrigerator for an hour and measure e again. Rubber becomes less elastic when cold.
8. **Improve.** Identify your largest error source (judging the bounce peak, background noise in phyphox) and reduce it.

Tips

- Release the ball, do not throw it; a spinning ball can bounce sideways.
- For the timing method, work in a quiet room.

Check your work

☐ I measured

e for several balls by two methods and compared them.

☐ I can explain why

e^2 is the fraction of energy kept in a bounce.

☐ I can solve a one-dimensional elastic collision.

LAB NOTEBOOK

1. Table of e for each ball by both methods, with a bar chart.
2. Sketch the height of a bouncing ball against time for $e = 0.9$ and $e = 0.5$, for the first four bounces.
3. Explain in a paragraph, with energy, why compressed polybutadiene made a better ball than ordinary rubber.

KEY FORMULAS

$$e = \frac{v_{\text{separation}}}{v_{\text{approach}}} = \sqrt{\frac{h_2}{h_1}} = \frac{t_2}{t_1}$$

$$v_1' = \frac{m_1 - m_2}{m_1 + m_2} v_1, \qquad v_2' = \frac{2m_1}{m_1 + m_2} v_1 \quad (\text{elastic, } m_2 \text{ at rest})$$

$$\frac{KE_{\text{lost}}}{KE_{\text{before}}} = \frac{m_2}{m_1 + m_2} \quad (\text{perfectly inelastic, } m_2 \text{ at rest})$$

From history. Wham-O eventually sold more than twenty million Super Balls. In the American Football League's early days, the name "Super Bowl" was suggested in 1966 by Lamar Hunt, who said he was thinking of his children's Super Ball. Polybutadiene is also used in the cores of most modern golf balls, for the same reason: high restitution.

Example 1. A bouncy ball dropped from 1.00 m rebounds to 0.81 m. Find the coefficient of restitution and the fraction of kinetic energy lost in the bounce.

Show solution

1. $e = \sqrt{\dfrac{h_2}{h_1}} = \sqrt{\dfrac{0.81}{1.00}} = 0.90.$
2. Energy after / energy before $= \dfrac{mgh_2}{mgh_1} = 0.81 = e^2$, so 19% of the kinetic energy is lost in each bounce.

Answer: $e = 0.90$; about 19% of the energy is lost per bounce.

Example 2. With phyphox listening to a bouncing ball, the time between the first and second bounces is 0.600 s and between the second and third is 0.540 s. Find e.

Show solution

1. Between bounces the ball rises and falls freely; the time in the air is $t = 2v/g$, proportional to the launch speed.
2. So $e = \dfrac{v_2}{v_1} = \dfrac{t_2}{t_1} = \dfrac{0.540}{0.600} = 0.90.$

Answer: $e = 0.90$, with no ruler needed.

Example 3. A 0.20 kg cart moving at 3.0 m/s collides elastically with a 0.30 kg cart at rest. Find both velocities afterward and check that kinetic energy is conserved.

Show solution

1. $v_1' = \dfrac{0.20 - 0.30}{0.50}(3.0\ \text{m/s}) = -0.60\ \text{m/s}$ (it bounces back).
2. $v_2' = \dfrac{2(0.20)}{0.50}(3.0\ \text{m/s}) = 2.4\ \text{m/s}$.
3. Momentum: before 0.60; after $(0.20)(-0.60) + (0.30)(2.4) = -0.12 + 0.72 = 0.60\ \text{kg}\cdot\text{m/s}$. ✓
4. Kinetic energy: before $\frac{1}{2}(0.20)(3.0)^2 = 0.90$ J; after $\frac{1}{2}(0.20)(0.36) + \frac{1}{2}(0.30)(5.76) = 0.036 + 0.864 = 0.90$ J. ✓

Answer: -0.60 m/s and $+2.4$ m/s; momentum and kinetic energy both conserved.

PRACTICE

1. Easy A ball dropped from 2.0 m bounces to 1.28 m. Find e.

   Show answer

   $\sqrt{0.64} = 0.80$

2. Easy Which loses more kinetic energy: two cars that bounce apart or two cars that lock together (same masses and speeds)?

   Show answer

   The cars that lock together (perfectly inelastic) lose the most.

3. Easy Is momentum conserved in an inelastic collision?

   Show answer

   Yes, if no external force acts; only kinetic energy is not conserved.

4. Medium A ball with $e = 0.75$ is dropped from 1.6 m. How high is its first bounce? Its second?

   Show answer

   $1.6(0.5625) = 0.90$ m; 0.51 m

5. Medium A 1.0 kg ball at 4.0 m/s collides elastically head-on with an identical ball at rest. Find both velocities afterward.

Show answer

They exchange velocities: the first stops and the second moves at 4.0 m/s.

6. Medium A 2.0 kg cart at 6.0 m/s sticks to a 4.0 kg cart at rest. Find the final speed and the fraction of kinetic energy lost.

Show answer

$2.0\ \mathrm{m/s}$; lost fraction $= 4/6 = 2/3$

7. Medium Newton found $e \approx 15/16$ for glass balls. If a glass ball were dropped from 1.0 m onto glass, how high would it bounce?

Show answer

$(15/16)^2 = 0.88\ \mathrm{m}$

8. Medium Phyphox measures successive bounce intervals of 0.80, 0.64 and 0.51 s. Find e for each bounce. Is it constant?

Show answer

$0.64/0.80 = 0.80$; $0.51/0.64 = 0.80$. Yes, about constant.

9. Hard A 0.10 kg ball at 5.0 m/s hits a 1.0 kg ball at rest elastically. Find both final velocities.

Show answer

$v_1' = (-0.9/1.1)(5.0) = -4.1\ \mathrm{m/s}$; $v_2' = (0.2/1.1)(5.0) = 0.91\ \mathrm{m/s}$

10. Hard A ball with $e = 0.80$ is dropped from 1.0 m. Find the total distance it travels before coming to rest (use the geometric series $1 + 2r + 2r^2 + \ldots$ with $r = e^2$).

Show answer

$d = h(1 + 2r/(1 - r)) = 1.0(1 + 1.28/0.36) = 4.6\ \mathrm{m}$

11. **Challenge** Stack a tennis ball on top of a basketball and drop them together. In the ideal case (the basketball much heavier than the tennis ball, both bounces perfectly elastic), show that the tennis ball rebounds at three times the impact speed and rises to nine times the drop height.

Show answer

Both hit the ground at speed v. The basketball rebounds at v upward and meets the tennis ball still moving down at v: relative speed $2v$. In an elastic collision with a very heavy partner, the light ball leaves with relative speed $2v$ upward relative to the basketball, so $v + 2v = 3v$. Height $\propto v^2$: nine times. (Real balls lose energy, so they go less high, but still far higher than the drop height.)

COMMON MISTAKES

- Thinking a bouncing ball's momentum is conserved. The floor (and Earth) exerts an external force on the ball.
- Using $e = h_2/h_1$ instead of $\sqrt{h_2/h_1}$.
- Applying the elastic-collision formulas to collisions in which the objects stick or deform.

> **Make it a game · Bounce Height Ranking.** Gather four balls: a tennis ball, a rubber ball, a marble and a wad of paper. Predict their bounce ranking from a one-meter drop, then measure each with a ruler. Calculate the coefficient of restitution as the square root of the height ratio. Score points for correct ranking and closest estimates.

6.4 Rockets: Goddard and the Rocket Equation

OBJECTIVES

- Explain rocket propulsion as conservation of momentum, and why a rocket works in empty space.
- Calculate thrust from the exhaust's mass flow and speed, $F = \dot{m} v_e$.
- Use the rocket equation, $\Delta v = v_e \ln(m_0/m_f)$, and explain why rockets are built in stages.
- Launch a water rocket safely and find the best amount of water by a fair test.

> **Warm-up.** Blow up a balloon, pinch the neck, and let it go across the room. Then think about this: in 1920 a newspaper said a rocket could never work in outer space because there is no air there for it to push against. Was the balloon pushing on the air in the room? Write your answer before reading on.

WATCH

Rocket Propulsion Physics & Mass Flow Rate - Newton's 3rd Law of Motion

The Organic Chemistry Tutor · 10:20

https://www.youtube.com/watch?v=kvAf-Xouj8s

> **WHEN TO WATCH**
> After the physics paragraph. Pause before each calculation.

BACKUP VIDEO

The Story of Robert Goddard, Father of Modern Rocketry

NASA Goddard · 2:49

https://www.youtube.com/watch?v=sGZZ2QHprHw

With the story paragraph: Goddard's life and his first liquid-fueled rocket.

KEY TERMS

thrust

The forward force on a rocket produced by throwing exhaust backward: $F = \dot{m}v_e$, where $\dot{m}$ is the mass of exhaust per second and v_e its speed relative to the rocket.

exhaust velocity

The speed of the exhaust gas relative to the rocket, typically 2,500–4,500 m/s for chemical rockets.

rocket equation

Tsiolkovsky's result (1903): the change in a rocket's velocity is $\Delta v = v_e \ln(m_0/m_f)$, where m_0 and m_f are its starting and final masses.

mass ratio

The ratio of a rocket's full mass to its empty mass, m_0/m_f.

staging

Building a rocket in sections that are dropped when their fuel is used, so the rest no longer carries their empty weight.

CORE IDEAS

The story. Robert H. Goddard, a physics professor at Clark University in Worcester, Massachusetts, had dreamed since boyhood of traveling into space. In 1919 the Smithsonian Institution published his paper *A Method of Reaching Extreme Altitudes*, which suggested that a rocket could someday reach the Moon. The *New York Times* mocked him in an editorial, saying a rocket could not work in a vacuum because it would have nothing to push against. Goddard kept working. On March 16, 1926, on his aunt's farm in Auburn, Massachusetts, he launched the world's first liquid-fueled rocket, burning gasoline and liquid oxygen. It flew for 2.5 seconds, rose 41 feet and landed 184 feet away. On July 17, 1969, as Apollo 11 flew toward the Moon, the *Times* printed a correction.

How they figured it out. Goddard tested his ideas in the laboratory before he flew them. He fired small rockets inside a sealed, evacuated chamber and measured their recoil, proving that a rocket produces thrust in a vacuum, even better than in air. He measured exhaust speeds, improved nozzle shapes (using the converging–diverging

de Laval nozzle from steam turbines, which roughly doubled his exhaust speed), and later developed gyroscopic steering and fuel pumps in New Mexico. He held more than 200 patents. The Russian schoolteacher Konstantin Tsiolkovsky had independently worked out the mathematics of rocket flight in 1903.

The physics. A rocket is a momentum machine. Before the engine fires, the rocket and its fuel have zero total momentum (in their own frame). As the engine throws exhaust backward, the rocket gains equal forward momentum. It does not push on the air; it pushes on its own exhaust, and the exhaust pushes back (Newton's third law). The thrust is the momentum given to the exhaust each second: $F = \dot{m} v_e$. Because the rocket gets lighter as it burns fuel, its speed gain is given by the rocket equation, $\Delta v = v_e \ln(m_0/m_f)$. The logarithm is the hard part: to double Δv you must square the mass ratio, which is why most of a rocket on the launch pad is fuel.

The engineering. Rocket engineers fight the rocket equation in two ways: raise the exhaust velocity (better fuels and nozzles) and drop dead weight (staging). The Saturn V that carried Apollo crews to the Moon had three stages; at liftoff it weighed about 2.9 million kg, and its first stage's five F-1 engines produced about 33 million newtons of thrust for about two and a half minutes before the stage was dropped. Model and water rockets follow the same physics on a small scale, which is why the amount of water in a water rocket matters so much: too little gives too little "exhaust," too much makes the rocket too heavy.

Try it. In the lab you will launch a water rocket outdoors, measure its height, and find the best fill of water by a fair test.

PICTURE STUDY

Goddard and His First Liquid-Fueled Rocket, Esther C. Goddard (attributed), March 1926 · Photograph · Auburn, Massachusetts (NASA)

Image: Wikimedia Commons, public domain

Goddard's 1926 rocket looked nothing like a modern rocket: the small engine is at the top and the fuel tanks are at the bottom, connected by thin pipes. The photograph is thought to have been taken by his wife, Esther.

Look closely

- Find the engine at the top of the frame and the tanks below.
- Find the launch frame. Why does the rocket need one?
- Look at the setting: snow, a field, no buildings nearby. Why would Goddard choose such a place?

Talk about it

1. Why might Goddard have put the engine above the tanks?
2. His rocket rose only 41 feet. Why was this flight still historic?
3. What would you need to measure to find this rocket's thrust?

> **Safety first.** Water rockets are launched only outdoors in an open field with an adult. Never exceed 60 psi (check with the pump's gauge), wear safety glasses, stand at least 10 m away and to the side, launch straight up or away from everyone, never lean over a pressurized bottle, and never use a bottle that is scratched, dented or not made for carbonated drinks.

LAB

The Best Water Fill for a Water Rocket

A pressurized soda bottle partly filled with water is a real rocket: compressed air throws the water out of the nozzle, and the bottle recoils. You will measure the height reached for different amounts of water and find the best fill.

You will need

- A 2-liter carbonated-drink bottle (with fins and a nose of cardboard or foam taped on, if you like)
- A rubber stopper fitted with a ball-inflation needle (or a commercial water-rocket launcher)
- A bicycle pump with a pressure gauge
- Measuring cup, water, tape measure
- A protractor with a straw taped along its straight edge and a string with a weight (a homemade inclinometer)
- Stopwatch, safety glasses, an open field, an adult

Steps

1. Read the safety box. Choose a launch site at least 30 m from buildings, trees, power lines and people.
2. **Hypothesis.** Predict which fill (0.25, 0.50, 0.75 or 1.00 L) will fly highest, and why.
3. Fill the bottle with 0.50 L of water, insert the stopper, set it nose-up on the launcher, and pump to 40 psi (do not exceed 60 psi). Everyone stands back.
4. Release. A partner standing a measured distance d (for example, 20 m) from the launch point sights the highest point through the inclinometer straw and reads the angle θ. Height $h = d \tan \theta$ (plus the partner's eye height). Another partner times the whole flight.
5. Do 3 launches at each fill: 0.25, 0.50, 0.75 and 1.00 L, always at the same pressure.
6. Graph height against water volume. Which fill is best?

7. Check your heights another way: for a flight time t (up and down, ignoring drag and the short powered phase), $h \approx \frac{1}{8}gt^2$. Compare.
8. **Explain.** Using momentum, explain why too little water and too much water both give low flights.
9. **Improve.** Keep the best fill and change one design feature: fins, a nose weight of clay, or a smaller nozzle. Test it 3 times.

Tips

- Keep pressure the same for every launch; it is a controlled variable.
- Wear clothes you do not mind getting wet.

Check your work

- ☐ I launched safely and measured height by sighting angle and by flight time.
- ☐ My graph shows an optimum water fill, and I can explain it with momentum and mass.
- ☐ I can calculate thrust and use the rocket equation.

LAB NOTEBOOK

1. Data table and graph of height against water fill, with both height methods compared.
2. Free-body diagram of the water rocket during the push phase and during coasting.
3. Explain in one paragraph why the *New York Times* was wrong in 1920, using Newton's third law and conservation of momentum.

KEY FORMULAS

$$F_{\text{thrust}} = \dot{m} v_e$$

$$\Delta v = v_e \ln \frac{m_0}{m_f}$$

$$a = \frac{F_{\text{thrust}} - mg}{m} \quad \text{(vertical launch)}$$

$$h = d \tan \theta \quad \text{(height by sighting angle)}$$

From history. Goddard moved his work to Roswell, New Mexico, in 1930 with support from the Guggenheim family, and by 1937 his rockets reached about 2.7 km. NASA's Goddard Space Flight Center in Maryland, founded in 1959, is named for him. After his death in 1945, his widow, Esther, spent years organizing his papers and seeing that his patents were recognized.

WORKED EXAMPLES

Example 1. A rocket engine expels 250 kg of exhaust each second at 2500 m/s. Find its thrust.

Show solution

1. $F = \dot{m}v_e = (250\ \text{kg/s})(2500\ \text{m/s}) = 6.25 \times 10^5\ \text{N}$.

Answer: 6.3×10^5 N (about 140,000 pounds of thrust).

Example 2. At liftoff the Saturn V had a mass of about 2.95×10^6 kg and a thrust of about 3.34×10^7 N. Find its initial acceleration.

Show solution

1. Net force:
   $F - mg = 3.34 \times 10^7 - (2.95 \times 10^6)(9.8) = 3.34 \times 10^7 - 2.89 \times 10^7 = 4.5 \times 10^6\ \text{N}$.
2. $a = \dfrac{4.5 \times 10^6\ \text{N}}{2.95 \times 10^6\ \text{kg}} = 1.5\ \text{m/s}^2$.
3. The acceleration grew steadily as fuel burned and the mass fell, reaching about $4\,g$ near the end of the first stage.

Answer: About $1.5\ \text{m/s}^2$ at liftoff, a slow, majestic start.

Example 3. A rocket stage with exhaust velocity 3000 m/s has a mass ratio $m_0/m_f = 5$. What speed change can it give? What mass ratio would double that speed change?

Show solution

1. $\Delta v = v_e \ln \dfrac{m_0}{m_f} = (3000\ \mathrm{m/s}) \ln 5 = (3000)(1.609)\ \mathrm{m/s} = 4830\ \mathrm{m/s}$.
2. Doubling Δv needs $\ln(m_0/m_f) = 2 \ln 5 = \ln 25$, a mass ratio of 25: 96% of the rocket would have to be fuel.

Answer: 4.8 km/s; doubling it needs a mass ratio of 25, which is why rockets use stages.

PRACTICE

1. Easy An engine expels 10 kg/s of exhaust at 2000 m/s. Find the thrust.

   Show answer

   2.0×10^4 N

2. Easy Does a rocket need air to push against? Explain.

   Show answer

   No. It pushes its exhaust backward, and by the third law the exhaust pushes it forward; air actually gets in the way.

3. Easy A model rocket of mass 0.10 kg has a thrust of 6.0 N. Find its initial acceleration straight up.

   Show answer

   $a = (6.0 - 0.98)/0.10 = 50\ \mathrm{m/s^2}$

4. Medium What mass flow rate at 3000 m/s is needed for 1.0×10^6 N of thrust?

   Show answer

   333 kg/s

5. Medium Find Δv for a stage with $v_e = 2500$ m/s and a mass ratio of 3.0.

   Show answer

   $2500 \ln 3 = 2750$ m/s

6. Medium Goddard's 1926 rocket flew 56 m (184 ft) in 2.5 s. What was its average horizontal speed?

Show answer

$22\ \mathrm{m/s}$

7. Medium A water rocket is sighted from 20 m away at an angle of 55° at its highest point. The observer's eye is 1.5 m above the ground. How high did it go?

Show answer

$20 \tan 55^\circ + 1.5 = 28.6 + 1.5 = 30\ \mathrm{m}$

8. Medium A water rocket's total flight time is 4.8 s. Estimate its maximum height, ignoring drag.

Show answer

$h = gt^2/8 = 28\ \mathrm{m}$

9. Hard To reach low Earth orbit a rocket needs a Δv of about 9.4 km/s (including losses). With $v_e = 3.0\ \mathrm{km/s}$ in a single stage, what mass ratio would be required? Why is this impractical?

Show answer

$m_0/m_f = e^{9.4/3.0} = e^{3.13} = 23$: about 96% of the rocket would have to be fuel, leaving almost nothing for tanks, engines and payload. Stages solve this.

10. Hard A 0.50 kg water rocket expels 0.30 kg of water in 0.20 s at an average speed of 15 m/s. Estimate the average thrust and the average acceleration during the push, using the average mass.

Show answer

$\dot{m} = 1.5\ \mathrm{kg/s}$; $F = 22.5\ \mathrm{N}$; average mass 0.35 kg; $a = (22.5 - 3.4)/0.35 = 55\ \mathrm{m/s^2}$

11. **Challenge** Show that the rocket equation gives a speed change proportional to v_e and explain why engineers value a high exhaust speed so much. Compare a kerosene–oxygen engine ($v_e \approx 3.0$ km/s) with a hydrogen–oxygen engine ($v_e \approx 4.4$ km/s) for the same mass ratio of 4.

Show answer

$\Delta v = v_e \ln 4$: $3.0(1.386) = 4.2$ km/s versus $4.4(1.386) = 6.1$ km/s. The higher exhaust speed gives 47% more Δv from the same tank, which is why upper stages often burn hydrogen.

COMMON MISTAKES

- Thinking a rocket pushes on the air or the ground. It pushes on its exhaust.
- Forgetting to subtract the rocket's weight when finding its vertical acceleration.
- Treating the rocket's mass as constant. It drops steadily as fuel burns, which is why the rocket equation has a logarithm.

Make it a game · Rocket Thrust Explain-Off. Teach your parent why a rocket works in empty space, using a balloon on a string line indoors as a demo. Your parent plays doubter and asks 'what does it push against?' Answer using momentum. Then calculate thrust for 2 kg/s of exhaust at 3000 m/s. Swap roles and see who explains clearer.

Scripture. "they shall mount up with wings as eagles" (Isaiah 40:31, KJV)

Goddard spent his life on the dream of rising above the Earth. Isaiah's promise is of a greater strength, given to those who wait upon the Lord.

6.5 Momentum in Two Dimensions: Billiards and Accident Reconstruction

OBJECTIVES

- Apply conservation of momentum separately to the x and y components in two-dimensional collisions.
- Solve two-dimensional perfectly inelastic collisions, as accident reconstructionists do.
- Show that in a glancing elastic collision of equal masses (one at rest) the two objects leave at right angles.
- Measure a two-dimensional collision of coins or marbles from video.

> **Warm-up.** On a smooth table, flick a coin so it strikes an identical coin slightly off-center. Watch the paths. Try it several times with different aims. What do you notice about the angle between the two coins' paths after the collision?

WATCH

Momentum Collisions in 2D

Physics with Professor Matt Anderson · 11:12

https://www.youtube.com/watch?v=p19i0gk3uFE

> **WHEN TO WATCH**
> After the physics paragraph, before the worked examples.

BACKUP VIDEO

Collisions Demo: Billiard Balls

Physics Demos · 1:31

https://www.youtube.com/watch?v=yFXUzf0ZC7c

With the warm-up: real billiard-ball collisions filmed from above.

KEY TERMS

glancing collision

A collision that is not head-on, so the objects move off at angles to the original direction.

component conservation

In two dimensions, the total x-momentum and the total y-momentum are each conserved separately.

accident reconstruction

The engineering work of finding vehicles' speeds and directions before a crash from the evidence afterward, using momentum and energy.

impact parameter

How far off-center one ball strikes another; it sets the angles at which they leave.

CORE IDEAS

The story. Billiards was enormously popular in France in the early 1800s, and one of its keenest students was the engineer Gaspard-Gustave Coriolis, the same man who gave us the words "work" and "kinetic energy" (Lesson 5.2). In 1835 he published *Théorie mathématique des effets du jeu de billard* (*The Mathematical Theory of the Game of Billiards*), explaining with Newton's laws how a ball's spin and the point where the cue strikes it control the paths of the balls after they collide. Expert players had learned the angles by experience; Coriolis showed that they followed from momentum, energy and friction.

How they figured it out. Coriolis treated each collision in two parts: the brief impact, during which the balls push on each other only along the line joining their centers, and the rolling afterward, during which friction with the cloth acts on spinning balls. For the impact, he applied conservation of momentum in each direction. He found the rule every pool player knows: when a ball rolling without spin strikes an identical ball at rest off-center, the two balls separate at a right angle. Spin changes the later paths, which is how skilled players make the cue ball follow, draw back or curve.

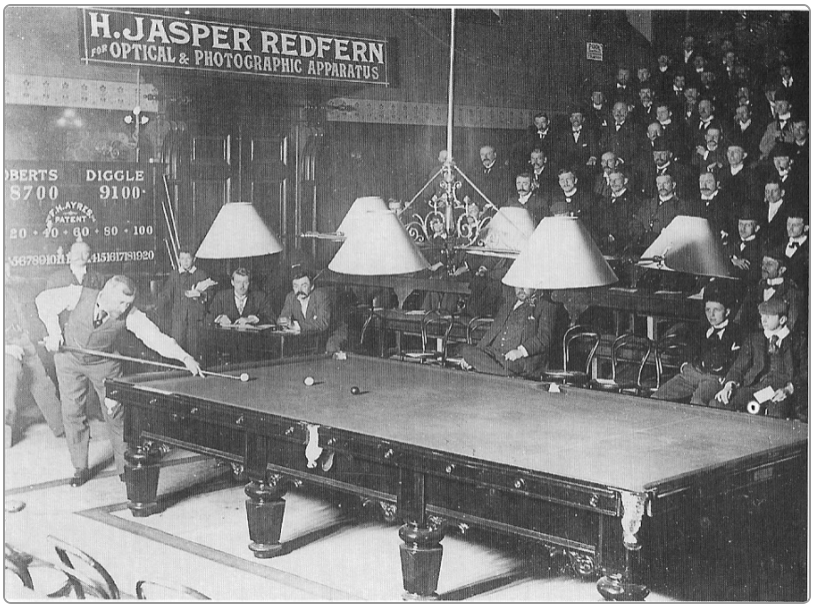
The physics. Momentum is a vector, so in two dimensions conservation means two equations: $\sum p_x$ before $= \sum p_x$ after and $\sum p_y$ before $= \sum p_y$ after. Resolve every velocity into components, write the two equations, and solve. When two objects stick together, the combined momentum vector after the crash equals the vector sum of the momenta before; its size and direction give the speed and direction of the wreckage. For an elastic glancing collision of equal masses with one at rest, momentum and energy conservation together require the two velocities afterward to be perpendicular.

The engineering. Police and insurance engineers reconstruct crashes at road junctions this way. Skid marks after impact, measured friction coefficients and the work–energy theorem give the speed of the wreckage; momentum conservation in

two directions then gives each car's speed before impact. Such careful, honest reconstructions are used in courts and to redesign dangerous junctions. Particle physicists and engineers designing air-hockey tables, bowling lanes and pinball machines use the same component equations.

Try it. In the lab you will film collisions of coins or marbles from above, measure velocities and angles, and check momentum conservation in both directions.

PICTURE STUDY



A Billiards Match, Unknown photographer, Early 1900s · Photograph · England (Wikimedia Commons)

Image: Wikimedia Commons, public domain

Professional English billiards drew large crowds in the early twentieth century. Every shot the player makes is a problem in two-dimensional momentum.

Look closely

- Find the player, his cue and the balls on the table.
- Look at the size of the table and the cushions around its edge.
- Find the scoreboard on the wall.

Talk about it

1. If the player strikes a ball at rest slightly off-center with no spin, at what angle will the two balls separate?
2. What does a cushion do to the component of a ball's momentum perpendicular to it? Parallel to it?
3. How could spin change the path of the cue ball after the collision?

LAB

Collisions on a Tabletop, Filmed from Above

You will collide identical coins (or marbles on a smooth tray), film from directly above, and measure the velocity components before and after to test conservation of momentum in two dimensions.

You will need

- Several identical large coins (or marbles, or air-hockey pucks) and one heavier coin
- A smooth table covered with a sheet of paper on which you draw a 5 cm grid
- A phone held directly above the table (on a stack of books across two chairs, or a selfie stick)
- Protractor, ruler, kitchen scale

Steps

1. Weigh the coins. Set the phone looking straight down at the grid and record in slow motion.
2. **Hypothesis.** Predict the angle between the two coins' paths after a glancing collision of identical coins.
3. Place one coin at rest in the center. Flick the other so it strikes off-center. Film 5 different collisions.
4. For each, step through the video and mark positions on the grid at equal frame intervals just before and just after the collision. Measure each coin's velocity components (cm per frame, converted to cm/s).
5. Add the x-momenta and the y-momenta before and after. Find the percent differences.
6. Measure the angle between the two paths after collision. Compare with 90°.
7. **Fair test.** Replace the target coin with a heavier coin (or stack two coins taped together). How do the angles change? Is momentum still conserved?
8. **Improve.** Sliding coins slow down quickly because of friction. Measure velocities in the two or three frames closest to the collision, or switch to

marbles on a smooth tray, which roll with much less friction.

Tips

- A dark coin on white paper, or white paper dots stuck on the coins, makes tracking easier.
- Keep the phone perfectly parallel to the table so the grid is not distorted.

Check your work

- ☐ I measured momentum components before and after and found them conserved within my uncertainty.
- ☐ I measured the separation angle for equal masses and compared it with 90°.
- ☐ I can solve a two-dimensional perfectly inelastic collision.

LAB NOTEBOOK

1. For one collision, a scale drawing of the momentum vectors before and after, arranged tip to tail to show that they add to the same total.
2. A data table of components for all trials.
3. Sketch a crash at a crossroads, label the momentum vectors, and write the two equations an investigator would use.

KEY FORMULAS

$$\sum p_x : \; m_1 v_{1x} + m_2 v_{2x} = m_1 v'_{1x} + m_2 v'_{2x}$$

$$\sum p_y : \; m_1 v_{1y} + m_2 v_{2y} = m_1 v'_{1y} + m_2 v'_{2y}$$

$$\theta'_1 + \theta'_2 = 90^\circ \quad \text{(elastic, equal masses, one at rest)}$$

From history. Coriolis is better known for the Coriolis effect, the apparent sideways deflection of moving objects on a rotating Earth, which he described in 1835, the same year as his billiards book. The two works share a theme: careful analysis of motion where rotation matters.

Example 1. At a crossroads, a 1500 kg car traveling east at 20 m/s collides with a 1000 kg car traveling north at 25 m/s. They lock together. Find the speed and direction of the wreckage just after impact.

Show solution

1. $p_x = (1500\text{ kg})(20\text{ m/s}) = 30{,}000\text{ kg}\cdot\text{m/s}$ east; $p_y = (1000\text{ kg})(25\text{ m/s}) = 25{,}000\text{ kg}\cdot\text{m/s}$ north.
2. Total momentum: $p = \sqrt{30{,}000^2 + 25{,}000^2} = 39{,}050\text{ kg}\cdot\text{m/s}$.
3. $v = \dfrac{p}{m_{\text{total}}} = \dfrac{39{,}050}{2500}\text{ m/s} = 15.6\text{ m/s}$.
4. Direction: $\theta = \tan^{-1}\left(\dfrac{25{,}000}{30{,}000}\right) = 39.8^\circ$ north of east.

Answer: 15.6 m/s at 39.8° north of east.

Example 2. A cue ball moving at 2.0 m/s strikes an identical ball at rest. The target ball leaves at 30° to the cue ball's original path with speed 1.73 m/s. Find the cue ball's velocity afterward, and check whether kinetic energy is conserved.

Show solution

1. Let the original path be $+x$. Before: $p_x = 2.0m$, $p_y = 0$.
2. Target after: $v_x = 1.73\cos 30^\circ = 1.50\text{ m/s}$, $v_y = 1.73\sin 30^\circ = 0.865\text{ m/s}$.
3. Cue ball after: $v_x = 2.0 - 1.50 = 0.50\text{ m/s}$, $v_y = 0 - 0.865 = -0.865\text{ m/s}$; speed $\sqrt{0.25 + 0.75} = 1.0\text{ m/s}$ at $\tan^{-1}(0.865/0.50) = 60^\circ$ on the other side.
4. Energy (per unit mass, times 2): before $2.0^2 = 4.0$; after $1.73^2 + 1.0^2 = 3.0 + 1.0 = 4.0$. Conserved; and $30^\circ + 60^\circ = 90^\circ$.

Answer: 1.0 m/s at 60° on the opposite side; the balls separate at 90° and kinetic energy is conserved.

Example 3. Show that in an elastic collision of equal masses with one at rest, the final velocities must be perpendicular.

Show solution

1. Momentum (divide by m): $\vec{v} = \vec{v}_1' + \vec{v}_2'$.
2. Energy (multiply by $2/m$): $v^2 = v_1'^2 + v_2'^2$.
3. Square the momentum equation: $v^2 = v_1'^2 + v_2'^2 + 2\vec{v}_1' \cdot \vec{v}_2'$.
4. Compare: $\vec{v}_1' \cdot \vec{v}_2' = 0$, so the velocities are perpendicular (unless one is zero, a head-on hit).

Answer: The final velocities are at right angles, which is why billiard players can predict the cue ball's path.

PRACTICE

1. Easy A 2.0 kg object moves at 3.0 m/s east and a 1.0 kg object at 4.0 m/s north. Find the total momentum (size and direction).

   Show answer

   $\sqrt{36 + 16} = 7.2$ kg · m/s at 33.7° north of east

2. Easy Why must you use components to apply conservation of momentum in two dimensions?

   Show answer

   Momentum is a vector; conservation applies to each component separately. Adding sizes without directions gives wrong answers.

3. Easy Two equal billiard balls separate at right angles after a collision. What does that suggest about the collision?

   Show answer

   It was (nearly) elastic, with one ball initially at rest.

4. Medium A 0.17 kg cue ball at 3.0 m/s hits an identical ball at rest; the cue ball moves off at 1.5 m/s at 60° to its original path. Find the other ball's speed and direction.

   Show answer

   $v_x = 3.0 - 0.75 = 2.25$, $v_y = -1.30$: 2.6 m/s at 30° on the other side

5. Medium A 1200 kg car going north at 15 m/s and a 1600 kg truck going west at 10 m/s collide and stick. Find the speed and direction of the wreckage.

Show answer

$p_y = 18{,}000$, $p_x = -16{,}000$; $p = 24{,}080$; $v = 8.6$ m/s at 48.4° north of west

6. Medium A puck slides into a smooth wall at an angle and bounces off elastically. Which component of its momentum changes?

Show answer

Only the component perpendicular to the wall reverses; the parallel component is unchanged, so the angle of reflection equals the angle of incidence.

7. Medium In the crossroads example, investigators find the wreckage slid at 39.8° north of east. If they had measured 45° instead, what would that tell them about the cars' momenta?

Show answer

At 45°, the two momenta would be equal; the northbound car's momentum would have to be as large as the eastbound car's, meaning one of the speed estimates is wrong.

8. Medium An object at rest explodes into three equal pieces. Two fly off at 10 m/s, one east and one north. Find the velocity of the third.

Show answer

14.1 m/s toward the southwest (45° south of west)

9. Hard After a crash at a crossroads, two locked cars (1400 kg and 1100 kg) skid 18 m at 35° north of east on a road with $\mu_k = 0.70$. The lighter car had been traveling north and the heavier east. Find the wreckage's speed just after impact and each car's speed before.

Show answer

$v = \sqrt{2(0.70)(9.8)(18)} = 15.7$ m/s; $p = 2500(15.7) = 39{,}300$; east car: $39{,}300 \cos 35^\circ / 1400 = 23$ m/s; north car: $39{,}300 \sin 35^\circ / 1100 = 20.5$ m/s

10. Hard A 4.0 kg ball moving at 6.0 m/s along $+x$ collides with a 2.0 kg ball at rest. Afterward the 4.0 kg ball moves at 4.0 m/s at 20° above the x-axis. Find the 2.0 kg ball's velocity, and decide whether the collision was elastic.

Show answer

p_x: $24 = 15.04 + 2v_x$, $v_x = 4.48$; p_y: $0 = 5.47 + 2v_y$, $v_y = -2.74$; $v = 5.25$ m/s at 31.4° below the axis. KE before 72 J, after $32 + 27.6 = 59.6$ J: inelastic.

11. Challenge Explain why a skilled billiards player can make the cue ball stop dead after a straight-on hit, but cannot make it stop dead after a glancing hit (without spin).

Show answer

Head-on, an elastic collision of equal masses transfers all the cue ball's momentum to the target ball, so the cue ball stops. In a glancing hit, the target ball leaves along the line of centers and receives only the momentum component along that line; the perpendicular component stays with the cue ball, so it must keep moving (at right angles to the target ball).

COMMON MISTAKES

- Adding speeds instead of vector components.
- Using the angle with the wrong axis when resolving components. Draw a sketch and label each angle.
- Expecting the 90° rule for unequal masses or inelastic collisions; it holds only for equal masses, elastic collisions, one at rest.

> **Make it a game · Two-D Momentum Drawing.** Draw a two-dimensional collision on paper as arrows: a 2 kg mass at 4 m/s east hits a 2 kg mass at rest, sticking together. Split momentum into x and y parts, calculate the final velocity and angle, and check your parent's version of a similar problem. Score a point per correct component.

UNIT 6

Momentum

Momentum, mass times velocity, is the quantity Newton called “quantity of motion,” and like energy it is conserved. Unlike energy, it is a vector and it is conserved in every collision, explosion and push-off, whether objects bounce, crumple or stick. This unit begins with Nils Bohlin’s three-point seat belt and the impulse–momentum theorem that explains why stretching a stop saves lives, then follows the Royal Society’s collision experiments and the Newton’s cradle, Norman Stingley’s Super Ball and Newton’s own measurements of bounciness, Robert Goddard’s first liquid-fueled rocket and the rocket equation, and Coriolis at the billiard table.

The labs drop eggs onto cushions, collide toy cars on a track, measure the coefficient of restitution of a dozen balls with a phone’s microphone, launch a water rocket in a field, and film coins colliding from above. The engineering runs from crumple zones and air bags to the Saturn V and accident reconstruction.

PREREQUISITES

- Unit 3: Newton’s three laws and free-body diagrams
- Unit 5: kinetic energy and its conservation
- Vectors and components (Lesson 1.5)

MASTERY CHECKLIST

☐ I can calculate momentum and impulse and use

$F\Delta t$ ~~to~~ explain seat belts, air bags and

Δp helmets.

☐ I can use conservation of momentum to solve one-dimensional collision, explosion and recoil problems.

☐ I can distinguish elastic and inelastic collisions and measure a coefficient of restitution.

☐ I can calculate rocket thrust and use the rocket equation, and explain why rockets have stages.

☐ I can apply conservation of momentum to two-dimensional collisions using components.

OPTIONAL KITS FOR THIS UNIT

- **Estes model rockets**: Model rockets with solid motors are a step beyond the water rocket for Lesson 6.4. An adult must supervise every launch, following the National Association of Rocketry safety code, in an open field away from power lines.
- **Arbor Scientific collision carts and Newton's cradles**: Low-friction carts with magnetic or hook-and-loop bumpers make Lesson 6.2's momentum measurements much cleaner; a quality Newton's cradle fits Lessons 6.2–6.3.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

6.1	**Impulse and the Seat Belt**	3 sessions of 50 minutes
6.2	**Conservation of Momentum and Newton's Cradle**	3 sessions of 50 minutes
6.3	**Elastic and Inelastic Collisions: The Super Ball**	3 sessions of 50 minutes
6.4	**Rockets: Goddard and the Rocket Equation**	3 sessions of 50 minutes
6.5	**Momentum in Two Dimensions: Billiards and Accident Reconstruction**	3 sessions of 50 minutes

Unit 6 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$. Choose and state a positive direction for every one-dimensional problem; resolve into components for two-dimensional problems. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   Define momentum and impulse and state the impulse–momentum theorem.

2. **6 pts**
   A 65 kg passenger moving at 18 m/s is stopped by a seat belt in 0.12 s. Find the average force. What would it be if the stop took 0.015 s?

3. 5 pts

Explain in two or three sentences how a bicycle helmet protects the head.

4. 6 pts

A 0.40 kg soccer ball at 10 m/s is kicked back at 15 m/s in the opposite direction. The contact lasts 0.020 s. Find the impulse and average force.

5. 6 pts

A 3000 kg truck at 10 m/s collides with and sticks to a 1000 kg car at rest. Find their common speed and the kinetic energy lost.

6. 6 pts

A 55 kg skater throws a 4.0 kg bag forward at 6.0 m/s while standing at rest on ice. Find her recoil velocity.

7. 6 pts

Explain why only one ball swings out of a Newton's cradle when one ball is released, using both conservation laws.

8. 5 pts

A ball dropped from 1.50 m rebounds to 0.96 m. Find the coefficient of restitution and the fraction of energy lost.

9. 7 pts

A 0.30 kg cart at 4.0 m/s collides elastically head-on with a 0.10 kg cart at rest. Find both velocities afterward.

10. 5 pts

Classify each collision as elastic, inelastic or perfectly inelastic: (a) two lumps of clay that stick; (b) two steel balls of a Newton's cradle; (c) a car crash in which the cars bounce apart, both dented.

11. 6 pts

A rocket engine produces 2.0×10^6 N of thrust with an exhaust speed of 2500 m/s. What mass of exhaust does it expel per second?

12. 6 pts

Find the speed change of a rocket stage with $v_e = 3500\ \mathrm{m/s}$ and a mass ratio of 4.0.

13. 5 pts

Explain why rockets work in space, and why they are built in stages.

14. 7 pts

A 1200 kg car going east at $22\ \mathrm{m/s}$ collides with a 1800 kg van going north at $12\ \mathrm{m/s}$; they stick. Find the wreckage's speed and direction.

15. 6 pts

A cue ball at $2.5\ \mathrm{m/s}$ strikes an identical ball at rest elastically; the target ball leaves at 40° from the original direction. Find the cue ball's direction and both speeds.

16. 5 pts

In Goddard's 1926 flight the rocket rose only 41 feet. Explain why it is still regarded as a turning point in the history of engineering.

17. 8 pts

Essay. Choose two of these: the three-point seat belt, the crumple zone, the air bag, the bicycle helmet, the climbing rope. Explain how each uses impulse and momentum, and discuss why an engineer must consider how people will actually use a safety device.

ANSWER KEY

1. Momentum $\vec{p} = m\vec{v}$; impulse $\vec{J} = \vec{F}\Delta t$; the impulse on an object equals its change in momentum.
2. $\Delta p = 1170\ \mathrm{kg \cdot m/s}$; 9750 N; 78,000 N
3. Its foam crushes during an impact, lengthening the time over which the head's momentum is stopped. Since $F\Delta t = \Delta p$, a longer time means a smaller average force on the head, and the shell spreads the force over a larger area.
4. $\Delta p = 0.40(25) = 10\ \mathrm{kg \cdot m/s}$; $F = 500\ \mathrm{N}$
5. $7.5\ \mathrm{m/s}$; KE before 150,000 J, after 112,500 J; lost 37,500 J
6. $-0.44\ \mathrm{m/s}$ (backward)
7. One ball out at speed v conserves both momentum (mv) and kinetic energy ($\frac{1}{2}mv^2$). Two balls at $v/2$ would conserve momentum but have only half the kinetic energy, which would violate energy conservation in a nearly elastic collision.
8. $e = \sqrt{0.64} = 0.80$; 36% lost
9. $v_1' = (0.20/0.40)(4.0) = 2.0\ \mathrm{m/s}$; $v_2' = (0.60/0.40)(4.0) = 6.0\ \mathrm{m/s}$
10. (a) perfectly inelastic; (b) nearly elastic; (c) inelastic.
11. $800\ \mathrm{kg/s}$
12. $3500 \ln 4 = 4850\ \mathrm{m/s}$
13. A rocket pushes its exhaust backward and the exhaust pushes it forward (third law; momentum conservation); no air is needed. Stages let the rocket drop empty tanks and engines, so the remaining stages do not waste fuel accelerating dead weight, which the rocket equation makes very costly.
14. $p_x = 26{,}400$, $p_y = 21{,}600$; $p = 34{,}110$; $v = 11.4\ \mathrm{m/s}$ at 39.3° north of east
15. Cue ball at 50° on the other side. Target: $2.5 \cos 40^\circ = 1.92\ \mathrm{m/s}$; cue: $2.5 \cos 50^\circ = 1.61\ \mathrm{m/s}$
16. It was the first rocket powered by liquid fuel, which can produce far more energy per kilogram than the powder of earlier rockets and can be throttled, pumped and controlled. It proved that the approach worked; every large rocket since, including the Saturn V, descends from it.
17. Model answer: Each device lengthens the time over which a person's momentum is reduced, so that $F = \Delta p/\Delta t$ is smaller, and spreads the force over a strong part of the body. The seat belt stretches slightly and holds the pelvis and chest; the crumple zone crushes over a known distance; the air bag cushions

and vents; the helmet's foam crushes; the rope stretches. But a device helps only if it is used correctly: Bohlin designed his belt to buckle with one hand in seconds, helmets must be comfortable enough to wear, and ropes must be easy to inspect. Good safety engineering includes human habits as a design requirement.

PORTFOLIO PIECE: THE EGG LANDER

Design and build a lander that protects a raw egg dropped from 3.0 m (a second-floor window or a stair landing, with an adult) using no more than 20 sheets of paper, 1 m of tape and 10 drinking straws. Before testing, estimate the impact speed and the stopping distance your design provides, and calculate the average force on the egg. Test with a dummy weight first, film every drop in slow motion, measure the actual stopping time, and compare. Improve your design at least once. Write a report with a labeled sketch, your calculations of momentum, impulse and force, your video measurements and a discussion of the trade-off between mass, size and protection.

LOOK FOR	WHAT IT MEANS
Design and constraints	Labeled sketch; materials within the limits; a clear plan for lengthening the stopping time.
Calculations	Impact speed, change in momentum, predicted stopping time and average force, all with units.
Testing and measurement	Slow-motion video of each drop; measured stopping distance and time; survival recorded honestly.
Iteration	At least one documented improvement with before-and-after data.
Report	Explains the physics with $F\Delta t = \Delta p$ and connects the design to seat belts, air bags or helmets.

UNIT 7

Rotation and Statics

Everything so far has moved as a point. Real objects turn, tip, balance and spin, and this unit gives you the tools for them: torque, center of mass, the conditions for equilibrium, rotational inertia and angular momentum. They are the physics of every lever and crane, every bridge and building that must stand still, every flywheel and every gyroscope.

The stories run from Archimedes' lever and the Leaning Tower of Pisa, which engineers stopped from falling in 2001, to Benjamin Baker's human cantilever model of the Forth Bridge, Pedro Flores's yo-yo and Foucault's gyroscope. The labs use toys and household tools as apparatus: a meter-stick balance, a balancing-bird toy, a beam on two kitchen scales, a rolling contest and a yo-yo, and a spinning office chair.

PREREQUISITES

- Units 3 and 5: Newton's laws, free-body diagrams and energy
- Unit 4: circular motion (angular speed and $v = \omega r$)
- Right-triangle trigonometry

MASTERY CHECKLIST

- ☐ I can calculate torque with $\tau = rF\sin\theta$ and use the law of the lever.
- ☐ I can find a center of mass by calculation and by experiment, and predict when an object will tip.
- ☐ I can solve static-equilibrium problems for beams, cantilevers, cranes and hanging signs using $\sum F = 0$ and $\sum \tau = 0$.
- ☐ I can use rotational inertia, $\tau = I\alpha$ and rotational kinetic energy, and predict which rolling object wins.
- ☐ I can apply conservation of angular momentum and explain gyroscopic precession.

OPTIONAL KITS FOR THIS UNIT

- **Arbor Scientific rotational motion and gyroscope apparatus**: A classroom gyroscope, a rotating platform or a set of rolling objects (hoop, disk, sphere of matched size) make Lessons 7.4–7.5 more precise. Optional; household objects work.
- **KEVA Planks**: Identical maple planks for building towers, cantilevers and balanced structures; excellent for exploring center of mass and equilibrium in Lessons 7.2–7.3 and the portfolio.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

7.1	**Torque: Archimedes' Lever**	3 sessions of 50 minutes
7.2	**Center of Mass and the Leaning Tower**	3 sessions of 50 minutes
7.3	**Equilibrium: Cranes and the Forth Bridge**	3 sessions of 50 minutes
7.4	**Rotational Inertia: The Yo-Yo and the Flywheel**	3 sessions of 50 minutes
7.5	**Angular Momentum: Tops, Gyroscopes and Foucault**	3 sessions of 50 minutes

Unit 7 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$. For torque problems, state your pivot and your sign convention. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   A 60 N force is applied at the end of a 0.40 m wrench at 90° and then at 30° to the wrench. Find both torques.

2. **6 pts**
   On a meter stick balanced at 50 cm, 100 g hangs at 20 cm. Where must 75 g hang to balance it?

3. 5 pts

State the law of the lever and explain why a crowbar can lift a heavy rock.

4. 6 pts

Find the center of mass of $3.0\ \mathrm{kg}$ at $x = 0$, $1.0\ \mathrm{kg}$ at $x = 2.0\ \mathrm{m}$ and $4.0\ \mathrm{kg}$ at $x = 5.0\ \mathrm{m}$.

5. 6 pts

A uniform crate $0.60\ \mathrm{m}$ wide and $1.5\ \mathrm{m}$ tall stands on a tilting truck bed. At what angle does it tip (if it does not slide)?

6. 5 pts

Explain why the Leaning Tower of Pisa has not fallen and how engineers made it safer.

7. 8 pts

A uniform $6.0\ \mathrm{m}$ beam weighing $400\ \mathrm{N}$ rests on supports at its ends. An $800\ \mathrm{N}$ load sits $1.5\ \mathrm{m}$ from the left end. Find both support forces.

8. 7 pts

A $25\ \mathrm{kg}$ sign hangs from the end of a light $2.0\ \mathrm{m}$ horizontal rod hinged at a wall, held by a cable at 40° above the rod. Find the cable tension.

9. 5 pts

Describe Baker's human cantilever model of the Forth Bridge and what it showed.

10. 6 pts

A tower crane carries $1500\ \mathrm{kg}$ at $30\ \mathrm{m}$. Its counterweight is $12\ \mathrm{m}$ behind the tower. What counterweight balances the load (ignore the jib's weight)?

11. 6 pts

A solid cylinder and a hoop roll from rest down a 15° ramp. Find each one's acceleration. Which wins?

12. 7 pts

A $40\ \mathrm{g}$ yo-yo falls with an acceleration of $0.60\ \mathrm{m/s^2}$ while unwinding from an axle of radius $0.35\ \mathrm{cm}$. Find its rotational inertia.

13. **6 pts**

A flywheel (solid disk, 40 kg, radius 0.30 m) spins at 1800 rev/min. Find its kinetic energy.

14. **6 pts**

A skater spinning at 1.5 rev/s with $I = 3.6\ \mathrm{kg \cdot m^2}$ pulls in to $I = 1.2\ \mathrm{kg \cdot m^2}$. Find her new spin rate. Is her kinetic energy conserved?

15. **6 pts**

A gyroscope with $L = 0.40\ \mathrm{kg \cdot m^2/s}$ has a gravitational torque of $0.12\ \mathrm{N \cdot m}$ acting on it. Find its precession rate.

16. **10 pts**

Essay. Explain how angular momentum is used in two of the following: a gyrocompass, a space telescope's reaction wheels, a helicopter's tail rotor, a spinning football. For each, say what would happen without it.

ANSWER KEY

1. $24\ \mathrm{N \cdot m}$; $12\ \mathrm{N \cdot m}$
2. $100(30) = 75d$, $d = 40$ cm from the pivot: the 90 cm mark
3. At balance $F_1 d_1 = F_2 d_2$. With the fulcrum close to the rock, the short arm under the rock and the long arm under the hand let a small force on the long arm produce a large force on the rock.
4. $(0 + 2 + 20)/8 = 2.75\ \mathrm{m}$
5. $\tan\theta = 0.30/0.75 = 0.40$; $\theta = 21.8^\circ$
6. Its center of mass is still above its base of support, so gravity's torque does not overturn it. Engineers removed small amounts of soil under the high side so it settled, rotating the tower back by about half a degree and moving the center of mass farther inside the base.
7. Right: $(400 \times 3 + 800 \times 1.5)/6 = 400$ N; left: 800 N
8. $T \sin 40^\circ = 245$ N; $T = 381$ N
9. Two men sat on chairs with arms outstretched holding sticks braced against the chairs (arms in tension, sticks in compression), supporting a plank with a person on it between them, with brick counterweights tied to their other hands. It showed that each tower of the bridge is a balanced cantilever, with the suspended span carried between the arms and the torques balanced by anchorage.
10. 3750 kg
11. $g \sin 15^\circ = 2.54\ \mathrm{m/s^2}$; cylinder 1.69, hoop $1.27\ \mathrm{m/s^2}$; the cylinder wins.
12. $I = mr^2(g/a - 1) = 0.040(0.0035)^2(15.33) = 7.5 \times 10^{-6}\ \mathrm{kg \cdot m^2}$
13. $I = 1.8\ \mathrm{kg \cdot m^2}$; $\omega = 188.5$ rad/s; $KE = 3.2 \times 10^4$ J
14. 4.5 rev/s. No: it triples; her muscles do the work.
15. 0.30 rad/s
16. Model answer: A gyrocompass uses a rapidly spinning rotor whose large angular momentum keeps its axis steady; with friction and gravity arranged to make it settle north, it points to true north regardless of a steel ship's magnetism. Without the spin it would simply be a pendulum. Reaction wheels change a spacecraft's direction by spinning a wheel one way so the spacecraft turns the other way, keeping total angular momentum zero; without them the telescope would need thrusters, which use fuel and blur images. A helicopter's tail rotor cancels the reaction torque of the main rotor; without it the body would spin

opposite the blades. A spinning football's angular momentum keeps its axis pointed along its path; without spin it tumbles and loses distance and accuracy.

PORTFOLIO PIECE: A COUNTERWEIGHTED CANTILEVER

Design and build a model cantilever, a balcony or a bridge arm, from craft sticks, KEVA planks, cardboard or balsa, that reaches at least 40 cm out from a support and carries a load of coins at its tip. Use a counterweight or an anchor at the back, and calculate in advance, with torques, the counterweight needed for loads of 50, 100 and 150 g. Measure the actual counterweight at which the model is balanced for each load, using a kitchen scale under the support to measure the support force as well, and compare. Write a report with an extended free-body diagram, calculations, a table of predicted and measured values, photos and an explanation connecting your model to the Forth Bridge or a tower crane.

LOOK FOR	WHAT IT MEANS
Design	Model reaches at least 40 cm; drawing shows dimensions, pivot, load point and counterweight point.
Calculations	Torque balance and support-force calculations for three loads, with units.
Measurements	Measured counterweights and support forces, compared with predictions in a table with percent differences.
Explanation	Clear account of equilibrium ($\sum F = 0$, $\sum \tau = 0$) and the role of the counterweight, with connection to a real structure.
Iteration	At least one design change (stiffer arm, different counterweight position) with before-and-after results.

7.1 Torque: Archimedes' Lever

OBJECTIVES

- Define torque, $\tau = rF\sin\theta$, and identify the lever arm of a force.
- State the law of the lever and use it to balance masses on a beam.
- Use a sign convention (counterclockwise positive) to add torques.
- Measure an unknown mass with a meter-stick balance.

> **Warm-up.** Open a door by pushing on it near the handle, then by pushing at the middle, then by pushing a hand's width from the hinges. Use the same gentle push each time. What changes? Then try pushing near the handle but straight toward the hinges. Write a sentence that explains all four results.

WATCH

Torque Introduction

Flipping Physics · 9:58

https://www.youtube.com/watch?v=Tu6e9_SrzBA

> **WHEN TO WATCH**
> After the physics paragraph, before the worked examples.

BACKUP VIDEO

The mighty mathematics of the lever - Andy Peterson and Zack Patterson

TED-Ed · 4:45

https://www.youtube.com/watch?v=YlYEi0PgG1g

With the story paragraph: Archimedes' law of the lever, animated.

KEY TERMS

torque

The turning effect of a force about an axis: $\tau = rF\sin\theta$, measured in newton-meters (N·m).

lever arm

The perpendicular distance from the axis to the line along which a force acts; torque = force × lever arm.

fulcrum

The pivot point of a lever.

law of the lever

A lever balances when the torques on the two sides are equal: $F_1 d_1 = F_2 d_2$.

mechanical advantage

The ratio of the output force to the input force of a machine; for a lever, the ratio of the lever arms.

CORE IDEAS

The story. Archimedes of Syracuse (about 287–212 BC), the greatest mathematician and engineer of the ancient world, was the first to prove the law of the lever. In his book *On the Equilibrium of Planes*, he showed that unequal weights balance on a beam when they are at distances inversely proportional to their weights. The later writer Pappus of Alexandria reports him saying, "Give me a place to stand, and I will move the Earth." Plutarch tells how King Hiero, unbelieving, asked for proof, and Archimedes, sitting at a distance and pulling gently on a system of pulleys, drew a fully loaded ship onto the shore by himself.

How they figured it out. Archimedes' proof began from a few self-evident statements: equal weights at equal distances balance; if weight is added to one side, that side goes down. Then he argued that a weight can be imagined as split into equal parts spread along the beam without changing the balance, and from this he proved that a weight of 2 at distance 1 balances a weight of 1 at distance 2. Merchants had long used steelyards, balances with unequal arms and a sliding weight, but Archimedes turned the practice into a theorem, the first law of physics stated and proved mathematically.

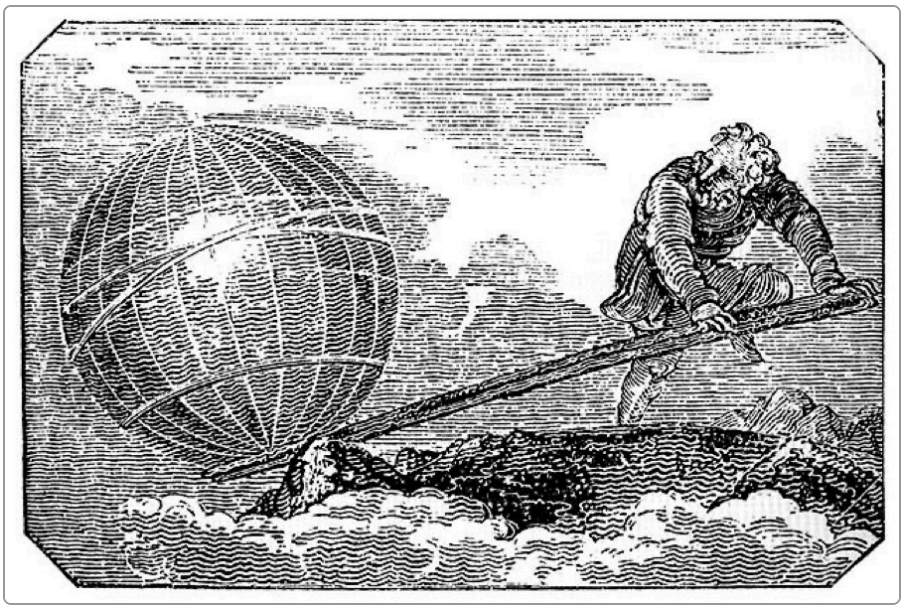
The physics. A force's turning effect, its **torque**, depends on three things: how big the force is, how far from the axis it acts, and its direction. $\tau = rF\sin\theta$, where r is the distance from the axis to where the force acts and θ is the angle between $\vec{r}$ and $\vec{F}$. Equivalently, torque is force times **lever arm**, the perpendicular distance from the axis to the force's line of action. A push straight toward the hinge has zero lever arm and no torque. Choose counterclockwise torques as positive and clockwise as negative. A lever balances when the total torque about the fulcrum is zero: $F_1 d_1 = F_2 d_2$, Archimedes' law.

The engineering. Torque is everywhere in engineering. Bolts on cars and aircraft are tightened to a specified torque with a torque wrench, because too little lets them loosen and too much stretches or breaks them. A longer wrench gives more torque from the same pull. Door handles are placed far from the hinges, bicycle pedals turn

on long cranks, and the engines of cars are rated by the torque they deliver to the crankshaft. Cranes, seesaws, wheelbarrows, scissors and the bones and muscles of your arm are all levers.

Try it. In the lab you will balance coins on a meter stick, test the law of the lever, and use it to weigh an unknown object, as merchants did with a steelyard.

PICTURE STUDY



Archimedes Moving the Earth, Unknown engraver, 1824 · Engraving · *Mechanics Magazine*, London
Image: Wikimedia Commons, public domain

This engraving appeared on the title page of a London magazine for working mechanics, illustrating Archimedes' famous boast. It is a joke with a serious point: a long enough lever multiplies force without limit.

Look closely

- Find the fulcrum. Is it closer to Archimedes or to the Earth?
- Compare the lengths of the two arms of the lever.
- Find the "place to stand."

Talk about it

1. Why must the fulcrum be very close to the Earth for Archimedes to lift it?
2. If the long arm were a million times longer than the short arm, how far would Archimedes have to push down to raise the Earth 1 cm?
3. What does this tell you about the work done by a lever?

LAB

The Meter-Stick Balance

You will build a lever from a meter stick, test the law of the lever with coins of known mass, and then use it to weigh unknown objects.

You will need

- A meter stick (or a straight 1 m dowel marked in cm)
- A pivot: a pencil taped to the table, a triangular block, or a string loop hung from a hook
- Coins (U.S. nickels have a mass of 5.00 g; quarters 5.67 g; pennies made after 1982, 2.50 g)
- Small paper clips or tape to hold coin stacks
- Unknown objects (an eraser, a key, a spoon)
- Kitchen scale for checking

Steps

1. Balance the empty meter stick on the pivot and record the balance point (it may not be exactly 50 cm).
2. **Hypothesis.** Predict where 2 nickels must sit to balance 4 nickels placed 10 cm from the pivot.
3. Place stacks of coins on both sides and slide one stack until the stick balances. Record masses and distances from the pivot. Do at least 6 different arrangements.
4. For each, compare the torques $m_1 d_1$ and $m_2 d_2$. Find the percent difference.
5. **Three weights.** Put two stacks on one side and one on the other. Find the balance position and check that the sum of torques is zero.
6. **Weigh unknowns.** Balance each unknown object against a known stack, calculate its mass, and check it on the kitchen scale.
7. **The stick's own weight.** Move the pivot to the 30 cm mark. Now the stick's own weight (acting at its balance point) has a torque. Balance a known mass on the short side, and use the law of the lever to find the mass of the meter stick itself. Check on the scale.

8. **Improve.** What limits your accuracy (friction at the pivot, coin positions, reading the scale)? Change one thing and repeat the worst measurement.

Tips

- Measure distances to the center of each coin stack.
- A sharp pivot (a triangular prism) is more sensitive than a round pencil.

Check your work

☐ I verified that

$m_1 d_1 = m_2 d_2$ at balance within my uncertainty.

☐ I weighed unknown objects with the law of the lever.

☐ I can calculate torque for a force at an angle.

LAB NOTEBOOK

1. Diagram of each arrangement with masses and distances, and a table of torques on each side.
2. Draw a door from above with three forces of equal size: at the handle perpendicular, at the handle at 30°, and pointing at the hinge. Mark each lever arm.
3. Sketch your forearm as a lever: elbow as fulcrum, biceps attached 4 cm from the elbow, a weight in your hand 35 cm away. Calculate the biceps force needed to hold a 5 kg weight.

KEY FORMULAS

$$\tau = rF\sin\theta = F \times (\text{lever arm})$$

$$\sum \tau = 0 \quad (\text{balance})$$

$$F_1 d_1 = F_2 d_2$$

From history. Archimedes also discovered the principle of buoyancy (Lesson 8.2), found the volume of a sphere, and estimated π between $3\frac{10}{71}$ and $3\frac{1}{7}$. He died during the Roman capture of Syracuse in 212 BC. Over his tomb, as he had asked, was carved a sphere inside a cylinder, the figure of his favorite theorem.

WORKED EXAMPLES

Example 1. A mechanic pulls with 80 N on the end of a 0.25 m wrench. Find the torque if she pulls (a) perpendicular to the wrench, (b) at 60° to the wrench.

Show solution

1. (a) $\tau = rF\sin 90^\circ = (0.25\text{ m})(80\text{ N}) = 20\text{ N}\cdot\text{m}$.
2. (b) $\tau = rF\sin 60^\circ = (0.25\text{ m})(80\text{ N})(0.866) = 17.3\text{ N}\cdot\text{m}$.

Answer: (a) 20 N · m; (b) 17 N · m. Pulling at an angle wastes part of the force.

Example 2. A 30 kg child sits 2.0 m from the pivot of a seesaw. Where must a 45 kg child sit to balance?

Show solution

1. Balance: $m_1 g d_1 = m_2 g d_2$ (g cancels).
2. $d_2 = \dfrac{m_1 d_1}{m_2} = \dfrac{(30\text{ kg})(2.0\text{ m})}{45\text{ kg}} = 1.33\text{ m}$ on the other side.

Answer: 1.33 m from the pivot.

Example 3. On a meter stick balanced at its 50 cm mark, a 50 g stack of coins sits at the 30 cm mark. An unknown mass at the 85 cm mark balances it. Find the unknown mass.

Show solution

1. Distances from the pivot: $d_1 = 50 - 30 = 20\text{ cm}$; $d_2 = 85 - 50 = 35\text{ cm}$.
2. $m_1 d_1 = m_2 d_2$: $m_2 = \dfrac{(50\text{ g})(20\text{ cm})}{35\text{ cm}} = 28.6\text{ g}$.

Answer: 28.6 g.

PRACTICE

1. Easy Find the torque of a 40 N force applied perpendicular to a door 0.90 m from the hinges.

   Show answer

   36 N · m

2. Easy Why is it nearly impossible to open a door by pushing near the hinges?

Show answer

The lever arm is very small, so even a large force produces little torque.

3. Easy A 2.0 kg mass sits 0.30 m left of a fulcrum. What mass 0.20 m to the right balances it?

Show answer

3.0 kg

4. Medium A 15 N force acts on a wrench 0.30 m long at 40° to the handle. Find the torque.

Show answer

$0.30(15)\sin 40^\circ = 2.9\ \mathrm{N \cdot m}$

5. Medium A torque specification calls for $110\ \mathrm{N \cdot m}$ on a car's wheel nuts. With a 0.45 m wrench, what perpendicular force is needed?

Show answer

244 N

6. Medium Three children sit on a seesaw: 25 kg at 2.0 m left, 20 kg at 1.0 m left, and one 30 kg child on the right. Where must the right child sit?

Show answer

$(50 + 20)/30 = 2.33$ m right

7. Medium In the forearm problem from the notebook, find the biceps force (biceps 4.0 cm from the elbow, 5.0 kg in the hand 35 cm from the elbow; ignore the forearm's own weight).

Show answer

$F = 5.0(9.8)(35)/4.0 = 429$ N, nearly nine times the weight held.

8. Medium A steelyard has a short arm holding the goods and a long arm with a sliding weight. Why can a small sliding weight balance a heavy load?

Show answer

The sliding weight can be placed far from the pivot, so its large lever arm gives a torque equal to that of the heavy load on its short arm.

9. Hard A uniform 2.0 kg meter stick is supported at its 25 cm mark. What mass hung at the 0 cm mark will balance it?

Show answer

The stick's weight acts at 50 cm, 25 cm from the pivot: $m(25) = 2.0(25)$, so $m = 2.0$ kg.

10. Hard A 70 kg person uses a 2.0 m crowbar to lift one end of a rock, with the fulcrum 0.20 m from the rock end. If she pushes down with her full weight, what force can she apply to the rock?

Show answer

$F = 686(1.80)/0.20 = 6170$ N

11. Challenge Archimedes claimed he could move the Earth with a long enough lever. If he could push with 500 N and wanted to lift a load of 6.0×10^{24} kg $\times$ 9.8 m/s^2 (pretending the Earth sat in a uniform gravity), what ratio of lever arms would he need? How far would he have to push his end to raise the Earth by 1 mm?

Show answer

Ratio $= 5.9 \times 10^{25}/500 = 1.2 \times 10^{23}$. To lift 1 mm he would push 1.2×10^{20} m, about 13,000 light-years. A lever multiplies force, but not work.

COMMON MISTAKES

- Using the distance to the force along a slant instead of the perpendicular lever arm.
- Measuring distances from the end of the stick instead of from the pivot.
- Forgetting the weight of the lever itself when the pivot is not at its center of mass.

> **Make it a game · Balance the Beam.** Lay a ruler across a pencil fulcrum. Your parent places a stack of coins at 5 cm from the pivot; predict how many coins at 15 cm balance it using torque. Test it. Try three setups. Score one point per prediction within one coin, and a bonus for naming the lever arm each time.

7.2 Center of Mass and the Leaning Tower

OBJECTIVES

- Calculate the center of mass of a set of objects, $x_{cm} = \sum m_i x_i / \sum m_i$.
- Find the center of mass of a flat shape experimentally with a plumb line.
- Explain stability: an object stays upright while its center of mass is above its base of support, and find the tipping angle.
- Explain how a balancing-bird toy works.

> **Warm-up.** Stand with your heels and back against a wall and try to pick up a coin from the floor in front of your toes without bending your knees or moving your feet. Then step away from the wall and try again. What does your body do differently away from the wall? Write your explanation.

WATCH

Center of Mass

Physics with Professor Matt Anderson · 11:53

https://www.youtube.com/watch?v=a4KhrJJvD3w

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

How Engineers Straightened the Leaning Tower of Pisa

Practical Engineering · 13:16

https://www.youtube.com/watch?v=0ZhHoyqQEhA

With the engineering paragraph: how engineers stopped the Leaning Tower's lean from growing.

KEY TERMS

center of mass

The average position of an object's mass, weighted by mass: $x_{cm} = \dfrac{\sum m_i x_i}{\sum m_i}$. For a uniform object, its geometric center.

center of gravity

The point where the whole weight of an object can be considered to act; in uniform gravity, the same as the center of mass.

base of support

The area enclosed by the points where an object touches the ground.

stable equilibrium

A balance to which an object returns after a small push, because its center of mass rises when it is tipped.

tipping angle

The angle at which an object's center of mass passes over the edge of its base; for a uniform block, $\tan\theta = \dfrac{\text{half-width}}{\text{height of center of mass}}$.

CORE IDEAS

The story. In 1173 the city of Pisa began a bell tower for its cathedral. By 1178, when the builders had reached the third story, the tower had begun to sink and lean: it stands on soft clay and sand, and the ground under the south side gave way more than the north. Work stopped for nearly a century, which let the soil compress and strengthen; the builders who resumed tried to correct the lean by making the upper stories slightly taller on the low side, so the tower actually curves. The bell chamber was finished around 1372. The lean kept growing, slowly, for six hundred years. By 1990 the top overhung the base by more than 4.5 meters and the tower was closed to visitors for fear it would fall.

How they figured it out. An international committee of engineers, with the British soil engineer John Burland as a leading member, studied the tower's soil and built computer models of how it moved. First they placed 900 tonnes of lead weights on the north side of the base to stop the lean temporarily. Then, from 1999 to 2001, they removed small amounts of soil from beneath the north side with long drills, a few liters at a time, letting the north side settle gently. The tower straightened by about 0.5°, enough to move its top back roughly 45 cm, and it has been stable since. The tower stands because its center of mass is still well inside its base. The engineers' job was to keep it there.

The physics. The **center of mass** is the balance point of an object: $x_{cm} = \dfrac{\sum m_i x_i}{\sum m_i}$, and similarly for y. Gravity acts on the whole object as if all its weight were at that point. An object resting on a base is stable as long as the vertical line through its center of mass falls inside the base; tip it past the point where that line reaches the

edge, and the weight's torque turns it over. For a uniform block of width b and height h, the tipping angle is $\tan\theta = \dfrac{b/2}{h/2} = \dfrac{b}{h}$. A low, wide object is hard to tip. A balancing-bird toy, with heavy weights in its wing tips hanging below its beak, has its center of mass *below* the pivot, so any tilt raises the center of mass and gravity pulls it back.

The engineering. Engineers design vehicles, cranes and buildings around the center of mass. Car makers measure each model's "static stability factor," half the track width divided by the height of the center of mass, which predicts how easily it rolls over. Mobile cranes extend outriggers to widen their base before lifting, and ships carry ballast low in the hull to keep the center of mass down. Tall buildings are built on deep foundations so that even in strong wind the resultant of weight and wind load stays well within the base.

Try it. In the lab you will find the center of mass of an irregular piece of cardboard with a plumb line, build a balancing-bird toy, and measure the tipping angle of blocks.

PICTURE STUDY

The Leaning Tower, Pisa, Detroit Publishing Company, c. 1890–1900 · Photochrom print · Library of Congress

Image: Wikimedia Commons, public domain

This color photochrom shows the tower about a century before engineers stabilized it. Its lean is easy to see, and so is the slight curve of its upper stories.

Look closely

- Hold a ruler vertically against the picture. Which way does the tower lean?
- Look at the top stories. Are they in line with the lower ones, or bent?
- Find the base. Where would a plumb line from the top land?

Talk about it

1. Why has the tower not fallen over, even though it leans?
2. Why did the builders make the upper stories slightly taller on the low side?
3. Why was removing soil from under the high side a better solution than pushing on the tower?

LAB

Centers of Mass, a Balancing Bird and Tipping Blocks

You will find centers of mass by experiment and by calculation, build a balancing-bird toy that stays balanced on its beak, and measure tipping angles.

You will need

- Stiff cardboard, scissors (adult help), a pushpin, string with a small weight (a plumb line), a ruler
- Card stock for the bird, two coins or washers, tape or glue
- A rectangular block or box, a cereal box, a soup can; a protractor; a board to tilt
- Optional: a balancing-bird toy from a toy store

Steps

1. Cut an irregular shape from cardboard. Make three small holes near its edge.
2. Hang the shape from a pushpin through one hole, so it swings freely, and hang the plumb line from the same pin. Trace the string's line on the cardboard. Repeat from the other two holes. The lines cross at the center of mass. Check by balancing the shape on a pencil tip there.
3. **Hypothesis.** Predict where the center of mass of an L-shaped piece is, then calculate it by splitting it into two rectangles, and test with the plumb line.
4. **Balancing bird.** Cut a bird shape with long wings swept forward so the wing tips are in front of the beak. Tape a coin to each wing tip. Balance the beak on your fingertip. Adjust until it balances level; then tip it and watch it rock back.
5. Explain with center of mass why the bird is stable, and find (by plumb line, hanging the bird from its beak) where its center of mass is.
6. **Tipping.** Stand a block on a board with a rough surface (so it tips rather than slides). Slowly tilt the board and measure the angle at which the block tips. Compare with $\tan\theta = b/h$. Repeat with the block on its side.
7. Tape a heavy washer to the top of the block (raising its center of mass) and repeat. Then tape it to the bottom.

8. **Improve.** If the block slides before tipping, add sandpaper. If your measured angle differs from theory, explain why.

Tips

- The plumb line and the pin must hang freely; any friction gives a wrong line.
- For the bird, the coins must be below and in front of the beak, not level with it.

Check your work

☐ I found a center of mass with a plumb line and by calculation, and they agree.

☐ I can explain why a balancing bird is stable.

☐ I measured tipping angles and compared them with

$\tan\theta = b/h$.

LAB NOTEBOOK

1. Your cardboard shape with the three plumb lines; the L-shape calculation.
2. Sketch the balancing bird from the side, marking the pivot and center of mass, and show what happens when it tilts.
3. Table of tipping angles: predicted and measured, with and without the extra washer.

KEY FORMULAS

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + \cdots}{m_1 + m_2 + \cdots}$$

$$\tan\theta_{\text{tip}} = \frac{\text{half-width of base}}{\text{height of center of mass}}$$

$$\text{SSF} = \frac{t}{2h}$$

From history. Galileo, born in Pisa in 1564, is said to have used the tower for falling-body experiments (Lesson 2.3). The stabilization project won engineering awards, and monitoring shows the tower has continued to straighten very slightly on its own since 2001. Engineers expect it to remain stable for at least two hundred years.

Example 1. A 2.0 kg mass sits at $x = 0$ and a 3.0 kg mass at $x = 0.60$ m on a light rod. Find the center of mass.

Show solution

1. $x_{cm} = \dfrac{(2.0\text{ kg})(0) + (3.0\text{ kg})(0.60\text{ m})}{5.0\text{ kg}} = \dfrac{1.8\text{ kg}\cdot\text{m}}{5.0\text{ kg}} = 0.36\text{ m}.$
2. It lies closer to the heavier mass, as expected.

Answer: 0.36 m from the 2.0 kg mass.

Example 2. Model the Leaning Tower as a uniform cylinder 56 m tall and 15.5 m across, on rigid ground. At what angle of lean would its center of mass pass over the edge of its base?

Show solution

1. The center of mass is at half the height, 28 m; the half-width of the base is 7.75 m.
2. $\tan\theta = \dfrac{7.75\text{ m}}{28\text{ m}} = 0.277$, so $\theta = 15.5^\circ$.

Answer: About 15°. The real tower, leaning about 5.5° in 1990, was in danger long before that because its soft soil, not rigid ground, was giving way under the low side.

Example 3. An SUV has a track width (distance between left and right wheels) of 1.6 m and its center of mass is 0.70 m above the road. Find its static stability factor and the sideways acceleration at which, in theory, it would begin to roll on a flat curve.

Show solution

1. $\text{SSF} = \dfrac{t}{2h} = \dfrac{1.6\ \text{m}}{2(0.70\ \text{m})} = 1.14.$
2. On a curve, rollover starts when the torque of the centripetal force about the outer wheels equals that of the weight: $ma\,h = mg\,(t/2)$, so $a = g\,\text{SSF} = (9.8\ \text{m/s}^2)(1.14) = 11.2\ \text{m/s}^2$.
3. A sports car with $\text{SSF} = 1.5$ would need $14.7\ \text{m/s}^2$; the tires would skid before it rolled.

Answer: $\text{SSF} = 1.14$; about $1.1\,g$ sideways. Lower and wider means more stable.

PRACTICE

1. Easy Masses of 1.0 kg at $x = 0$ and 1.0 kg at $x = 2.0$ m: where is the center of mass?

   Show answer

   $x = 1.0\ \text{m}$

2. Easy Why is a pyramid more stable than a tall, thin column of the same mass?

   Show answer

   Its center of mass is lower and its base wider, so it must be tipped through a much larger angle before the center of mass passes over the edge.

3. Easy Why do you lean forward when you carry a heavy backpack?

   Show answer

   To keep the combined center of mass of you and the backpack above your feet.

4. Medium Find the center of mass of 4.0 kg at $x = 1.0$ m, 2.0 kg at $x = 3.0$ m and 2.0 kg at $x = 6.0$ m.

   Show answer

   $(4 + 6 + 12)/8 = 2.75\ \text{m}$

5. Medium A box 0.40 m wide and 1.2 m tall with its center of mass at its center. At what tilt does it tip?

Show answer

$\tan\theta = 0.20/0.60$, $\theta = 18.4^\circ$

6. Medium Earth (5.97×10^{24} kg) and Moon (7.35×10^{22} kg) are 3.84×10^8 m apart. Where is their center of mass? Is it inside the Earth?

Show answer

$x_{cm} = (7.35 \times 10^{22})(3.84 \times 10^8)/6.04 \times 10^{24} = 4.67 \times 10^6$ m from the Earth's center, inside the Earth (radius 6.37×10^6 m).

7. Medium Explain how the Leaning Tower's engineers straightened it without pushing on it.

Show answer

They removed small amounts of soil from beneath the high (north) side, so that side settled slightly, rotating the tower back toward vertical and moving its center of mass farther inside its base.

8. Medium A car has SSF $= 1.40$. In theory, what sideways acceleration would make it begin to roll?

Show answer

$1.40 \times 9.8 = 13.7\ \mathrm{m/s^2}$

9. Hard An L-shaped plate is made of two uniform rectangles of the same material: one 4 cm $\times$ 1 cm with center at $(2, 0.5)$, and one 1 cm $\times$ 3 cm standing on it with center at $(0.5, 2.5)$ (all in cm). Find the center of mass.

Show answer

Areas 4 and 3: $x = (8 + 1.5)/7 = 1.36$ cm, $y = (2 + 7.5)/7 = 1.36$ cm

10. Hard A stack of identical books 20 cm long can overhang a table edge: the top book's center of mass must be over the book below, the top two over the third, and so on. How far can the top book of a three-book stack overhang the table?

Show answer

Overhangs of $L/2$, $L/4$, $L/6$: $10 + 5 + 3.3 = 18.3$ cm, almost a full book length.

11. **Challenge** Explain why a balancing bird with its center of mass below the beak is stable, while a pencil balanced on its point is not, using what happens to the height of the center of mass and the torque of gravity when each is tipped slightly.

Show answer

Tipping the bird raises its center of mass; gravity's torque about the beak then turns it back, so it is stable. Tipping the pencil lowers its center of mass, and gravity's torque turns it farther from upright: unstable.

COMMON MISTAKES

- Thinking the center of mass must be inside the object. For a ring, a boomerang or a balancing bird it can be in empty space.
- Forgetting to weight positions by mass.
- Confusing the base of support with the object's width at the top.

> **Make it a game · Find the Center.** Hand your parent a cardboard cutout of an irregular shape. Predict where its center of mass is, mark it, then check by balancing on a fingertip or finding the plumb-line intersection. Score by distance in centimeters. Then predict whether a tall box tips at a certain angle. Swap shapes, aim to shrink your error.

7.3 Equilibrium: Cranes and the Forth Bridge

OBJECTIVES

- State the two conditions for static equilibrium: $\sum \vec{F} = 0$ and $\sum \tau = 0$.
- Solve for unknown support forces on beams by choosing a convenient pivot.
- Explain how a cantilever and a counterweight keep a structure in balance.
- Measure the support forces on a loaded beam with two kitchen scales and compare with calculation.

> **Warm-up.** Lay a ruler across two stacks of books, one at each end. Press down with a finger at the middle, then near one end. Which book stack do you think is pushing up harder in each case? Now try to imagine being one of the stacks. Write down your prediction; you will measure it in the lab.

WATCH

Physics, Torque (11 of 13) Static Equilibrium, Hanging Sign No. 5

Step by Step Science · 11:56

https://www.youtube.com/watch?v=z4pd_K44QUo

> **WHEN TO WATCH**
> After the physics paragraph. Pause at each example and choose your own pivot first.

BACKUP VIDEO

Forth Bridges: Spanning Three Centuries of Engineering Innovation

The B1M · 7:37

https://www.youtube.com/watch?v=tgeFKVX2C-o

With the story paragraph: the Forth Bridge, its cantilevers and its history.

KEY TERMS

static equilibrium

The state of an object at rest with no tendency to start moving or rotating: the net force and the net torque on it are both zero.

support (reaction) force

The force a support exerts on a structure, often found by applying the equilibrium conditions.

cantilever

A beam or structure supported at only one end, held in balance by a torque at the support, often provided by an anchor or counterweight.

counterweight

A weight placed to balance the torque of a load, as on a tower crane's rear jib.

free-body diagram (extended object)

A diagram of an object showing each force acting at its actual point of application, needed for torque calculations.

CORE IDEAS

The story. In December 1879 the first railway bridge over the Firth of Tay in Scotland collapsed in a violent storm while a train was crossing, with the loss of everyone aboard. Its designer had not properly allowed for the force of the wind. The design for a second great Scottish bridge, over the Firth of Forth, was abandoned, and the engineers John Fowler and Benjamin Baker were asked for something unquestionably strong. Their answer was a cantilever bridge: three huge double-cantilever towers of steel tubes and lattice girders, each balanced on its own pier, with short suspended spans linking their arms. Built from 1882 to 1890 with about 54,000 tonnes of steel, its two main spans of 521 meters were the longest in the world. It still carries trains today and is a World Heritage Site.

How they figured it out. Baker had to explain the strange-looking design to the public, so in 1887 he staged a living model. Two men sat on chairs with their arms outstretched, gripping sticks that pressed back against the chair seats: their arms were in tension, the sticks in compression. Their outer hands held up a short plank, the suspended span, on which sat the engineer Kaichi Watanabe, one of the first Japanese engineers trained in Britain. Their inner hands were tied to piles of bricks, the counterweights at the bridge's ends. Each person-and-chair was a cantilever in equilibrium. The photograph was printed in newspapers everywhere, and people understood at once how the bridge would stand.

The physics. An object is in **static equilibrium** when it has no acceleration and no angular acceleration: $\sum \vec{F} = 0$ and $\sum \tau = 0$ about *any* point. To find unknown support forces on a beam, draw the beam with every force at its point of application

(its weight at its center of mass), choose a pivot where an unknown force acts (so that force has no torque), and set the sum of torques to zero. Then use $\sum F = 0$ for the remaining unknown. For a cantilever, the torque of the load must be balanced by a torque at the support, provided by an anchor, a counterweight or a fixed joint.

The engineering. Every building, bridge and crane is designed with these two equations. A tower crane's rear jib carries concrete counterweights so that the torques of load and counterweight about the tower nearly cancel; its load chart limits the load more strictly the farther out it is carried. Bridge engineers calculate the reactions at every pier and bearing; architects calculate the forces in every beam and column. A structure that satisfies the equilibrium equations under every expected load, with a safety margin, stands.

Try it. In the lab you will rest a meter stick on two kitchen scales, slide a load along it, measure both support forces and compare them with your calculations.

PICTURE STUDY

The Forth Bridge's Three Cantilevers, Unknown photographer (contractors' construction record), 1889 · Photograph · Construction record of the Forth Bridge

Image: Wikimedia Commons, public domain

This photograph from the contractors' record shows the three towers of the bridge at full height, about 1888. Next, each tower's arms would be built outward in both directions at once, so that each stayed balanced on its own pier like Baker's human model.

Look closely

- Find the three towers and the wide gaps of water between them.
- Look at the tower in the foreground. Is it symmetrical about its pier?
- Find the cranes and platforms at the tops of the towers. What were they used for?

Talk about it

1. Why did the engineers build each tower's arms outward in both directions at once?
2. When the arms are complete and a train crosses one of them, what must happen at the other end of that cantilever to keep the tower in equilibrium?
3. Why would a wide base on each pier make the towers safer in a storm?

LAB

A Beam on Two Kitchen Scales

You will support a meter stick (or a board) on two kitchen scales, move a load along it, and measure how the support forces change, comparing every reading with the equilibrium equations.

You will need

- Two kitchen scales of the same height (or one scale and a block of the same height)
- A meter stick or a light board about 1 m long
- Two small wooden blocks or pencils to make knife-edge supports on the scales
- A load: a can of food or a bag of coins (weighed)
- Tape measure

Steps

1. Place the supports on the scales 80 cm apart (or the full length of your board) and lay the stick across them. Zero the scales with the supports on them, then record the readings with the bare stick. Weigh the stick.
2. **Hypothesis.** Predict the two readings when the load is at the middle, and when it is one quarter of the way from the left support.
3. Place the load at 0, 10, 20, ... 80 cm from the left support. At each position record both readings (in grams, then convert to newtons).
4. For each position, calculate the predicted reactions with $\sum \tau = 0$ about the left support and $\sum F = 0$.
5. Graph both measured readings against the load position on one graph, and draw the predicted lines.
6. Check: at every position, does the sum of the two readings equal the total weight?
7. **Cantilever.** Move the right support inward to 50 cm so the stick overhangs. Place the load on the overhang. What happens to the left reading? Predict, then measure. How far out can the load go before the stick tips?

8. **Improve.** Identify sources of disagreement (support positions, the stick's own weight, scale accuracy) and fix the largest.

Tips

- Use knife-edge supports so you know exactly where each force acts.
- Read the scales with the load at rest; tap the stick gently to settle friction.

Check your work

- ☐ My measured support forces match the equilibrium calculation within my uncertainty.
- ☐ I can choose a pivot that eliminates an unknown force.
- ☐ I can explain how a cantilever or a counterweight keeps a structure in balance.

LAB NOTEBOOK

1. Extended free-body diagram of the beam with every force at its point of application; data table; graph of both reactions against load position.
2. Sketch Baker's human cantilever model and label tension, compression, the counterweight and the suspended span.
3. Draw a tower crane, label the load, the counterweight and the tower, and calculate the counterweight that balances a 1000 kg load at 30 m if the counterweight is 12 m behind the tower.

KEY FORMULAS

$$\sum \vec{F} = 0, \qquad \sum \tau = 0$$

$$R_B = \frac{\sum(\text{load} \times \text{distance from } A)}{L} \quad (\text{beam on supports } A, B)$$

From history. Benjamin Baker went on to help design the first Aswan Dam on the Nile and the first London Underground deep-level tube lines. Kaichi Watanabe returned to Japan and built bridges and railways there. Painting the Forth Bridge became a British expression for a never-ending job, until a long-lasting coating applied from 2002 to 2011 made repainting rare.

Example 1. A uniform 4.0 m beam weighing 200 N rests on supports at its two ends, A (left) and B (right). A 600 N load sits 1.0 m from A. Find both support forces.

Show solution

1. Choose A as the pivot (the unknown R_A has no torque there). Counterclockwise positive.
2. Torques about A: $R_B(4.0\text{ m}) - (200\text{ N})(2.0\text{ m}) - (600\text{ N})(1.0\text{ m}) = 0$, so $R_B = \dfrac{1000\text{ N}\cdot\text{m}}{4.0\text{ m}} = 250\text{ N}$.
3. Forces: $R_A + R_B = 200 + 600 = 800\text{ N}$, so $R_A = 550\text{ N}$.
4. Check with torques about B: $-R_A(4.0) + 200(2.0) + 600(3.0) = -2200 + 400 + 1800 = 0$. ✓

Answer: $R_A = 550\text{ N}$, $R_B = 250\text{ N}$; the support nearer the load carries more.

Example 2. A tower crane lifts a 2000 kg load at 25 m from the tower. Its counterweight hangs 10 m from the tower on the other side. What counterweight mass balances the load's torque (ignoring the jibs' own weight)?

Show solution

1. Balance about the tower: $m_{\text{load}}g(25\text{ m}) = m_{\text{cw}}g(10\text{ m})$.
2. $m_{\text{cw}} = \dfrac{(2000\text{ kg})(25\text{ m})}{10\text{ m}} = 5000\text{ kg}$.

Answer: 5000 kg. Real cranes choose a counterweight that balances about half the maximum load torque, so the tower is never bent too far in either direction, loaded or empty.

Example 3. A 20 kg sign hangs from the end of a light horizontal rod 1.5 m long hinged to a wall. A cable from the rod's end to the wall makes 30° with the rod. Find the cable tension and the hinge force.

Show solution

1. Torques about the hinge: the tension's vertical component $T \sin 30^\circ$ acts at 1.5 m; the sign's weight 196 N acts at 1.5 m. So $T \sin 30^\circ = 196$ N and $T = 392$ N.
2. Horizontal: the hinge pushes outward with $H = T \cos 30^\circ = 339$ N.
3. Vertical: $V + T \sin 30^\circ - 196 = 0$, so $V = 0$ (the rod is weightless and the load is at the end).

Answer: $T = 392$ N; the hinge pushes horizontally with 339 N.

PRACTICE

1. Easy State the two conditions for static equilibrium.

   Show answer

   $\sum \vec{F} = 0$ and $\sum \tau = 0$ (about any point).

2. Easy A 600 N person stands in the middle of a light plank resting on two supports. What does each support push?

   Show answer

   300 N each

3. Easy Why is it useful to choose the pivot at the point where an unknown force acts?

   Show answer

   That force has zero lever arm about that point, so it drops out of the torque equation, leaving fewer unknowns.

4. Medium A light 3.0 m plank rests on supports at its ends. A 500 N painter stands 1.0 m from the left end. Find both support forces.

   Show answer

   Right: $500(1.0)/3.0 = 167$ N; left: 333 N

5. Medium A uniform 10 m, 2000 N bridge deck rests on piers at its ends. A 15,000 N truck is 3.0 m from the left pier. Find the pier forces.

Show answer

Right: $(2000 \times 5 + 15{,}000 \times 3)/10 = 5500$ N; left: 11,500 N

6. Medium A diving board 4.0 m long is held by a bolt at its left end and a support 1.0 m from that end. A 600 N diver stands at the right end. Ignoring the board's weight, find the forces at the bolt and at the support.

Show answer

About the bolt: $S(1.0) = 600(4.0)$, $S = 2400$ N up. Bolt: $2400 - 600 = 1800$ N down.

7. Medium In Baker's human cantilever, which parts were in tension and which in compression?

Show answer

The men's arms (pulling between the chairs and the loads) were in tension; the sticks pushing against the chair seats were in compression; the brick piles were counterweights balancing the torque of the suspended span.

8. Medium A 0.50 kg meter stick is supported at its 40 cm mark. Where must a 0.20 kg mass be placed to balance it?

Show answer

Stick's weight acts 10 cm right of the pivot: $0.50(10) = 0.20d$, so $d = 25$ cm left, at the 15 cm mark.

9. Hard A 5.0 m ladder weighing 200 N leans against a smooth wall with its foot 1.5 m from the wall. Find the force of the wall and the friction needed at the floor.

Show answer

Height $= 4.77$ m. Torques about the foot: $W_{\text{wall}}(4.77) = 200(0.75)$, so $W_{\text{wall}} = 31.4$ N; floor friction $= 31.4$ N; floor normal force $= 200$ N.

10. Hard A crane's horizontal jib is uniform, 40 m long and 8000 kg, centered 10 m in front of the tower. Its counterweight is 15 m behind. If it carries 3000 kg at the jib's tip (30 m out), what counterweight makes the net torque on the tower zero?

Show answer

$8000(10) + 3000(30) = m(15)$: $m = 170{,}000/15 = 11{,}300$ kg

11. **Challenge** The Tay Bridge's designer underestimated wind loads. Explain, with torques, why a strong side wind on a tall, narrow bridge pier and the train on it can overturn the structure, and what designers do to prevent it.

Show answer

Wind pushes sideways on the girders and train at a height above the pier's base, creating an overturning torque about the downwind edge. The weight of the structure acts at its center, giving a restoring torque equal to the weight times half the base width. If the wind torque exceeds the restoring torque, the structure tips. Designers use measured wind pressures with safety factors, widen and anchor the bases, and lower the height where wind acts, so the restoring torque always wins.

COMMON MISTAKES

- Putting the beam's weight at the wrong point. A uniform beam's weight acts at its center.
- Forgetting that $\sum \tau = 0$ must use the same pivot for every torque in one equation.
- Mixing up signs: decide which rotation is positive and keep it.

Make it a game · Support Force Duel. Your parent places a 2 m plank on two supports with a heavy book at some point. Without a calculator, estimate which support holds more and by how much, by choosing a pivot at one support. Then compute exactly. Closest estimate wins a point. Play with the book moved to three new places.

Scripture. "Through wisdom is an house builded; and by understanding it is established" (Proverbs 24:3, KJV)

A structure stands when the forces and torques on it are understood and balanced. The engineers of the Forth Bridge built with that kind of understanding after a costly lesson.

7.4 Rotational Inertia: The Yo-Yo and the Flywheel

OBJECTIVES

- Define rotational inertia, $I = \sum mr^2$, and compare hoops, disks and spheres.
- Use the rotational form of Newton's second law, $\tau = I\alpha$, and rotational kinetic energy, $\frac{1}{2}I\omega^2$.
- Predict which of several rolling objects reaches the bottom of a ramp first.
- Measure the rotational inertia of a yo-yo from its acceleration.

> **Warm-up.** Take two identical cans of soup, or a can of soup and a can of broth. Predict which will roll down a gently sloping board faster. Then release them side by side, three times. Now try a full can against an empty can of the same size. What do you notice?

WATCH

Rotational Inertia

Bozeman Science · 5:38

https://www.youtube.com/watch?v=W9fPGXyvrVM

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Rotational Inertia: The Race Between a Ring and a Disc

North Carolina School of Science and Mathematics · 3:11

https://www.youtube.com/watch?v=CHQOctEvtTY

With the warm-up: a ring and a disc rolled side by side.

KEY TERMS

angular velocity

How fast an object turns, ω, in radians per second (2π rad = one turn).

angular acceleration

The rate of change of angular velocity, α, in rad/s^2.

rotational inertia (moment of inertia)

An object's resistance to changes in its rotation, $I = \sum mr^2$, which depends on how far its mass is from the axis. Hoop: MR^2; solid disk or cylinder: $\frac{1}{2}MR^2$; solid sphere: $\frac{2}{5}MR^2$.

flywheel

A heavy wheel with large rotational inertia, used to smooth out rotation or store energy.

rotational kinetic energy

The energy of rotation, $\frac{1}{2}I\omega^2$.

CORE IDEAS

The story. The yo-yo is one of the oldest toys: Greek vase paintings from about 440 BC show boys playing with them. But the modern yo-yo began in 1928 in Santa Barbara, California, when Pedro Flores, a young man from the Philippines, started a company to make wooden yo-yos. His key change was the string: instead of tying it to the axle, he looped it loosely around the axle, so the yo-yo could keep spinning at the bottom of the string, "sleeping," until a tug made it climb back. Within a year his small factory was turning out thousands a day. The businessman Donald Duncan bought the company soon after and made the yo-yo a national craze.

How they figured it out. Why does a yo-yo fall so slowly? A dropped yo-yo, unlike a dropped ball, must also spin up as it falls, and most of the energy from falling goes into spinning, not into falling speed. How much depends on how the mass is arranged. The mathematician Leonhard Euler worked out the theory of rotating rigid bodies in the 1750s and gave the name "moment of inertia" to the quantity that measures it. Flores and Duncan learned by experiment what Euler's theory predicts: a yo-yo with its mass concentrated in a heavy rim (large rotational inertia) and a thin axle spins longer and falls more slowly.

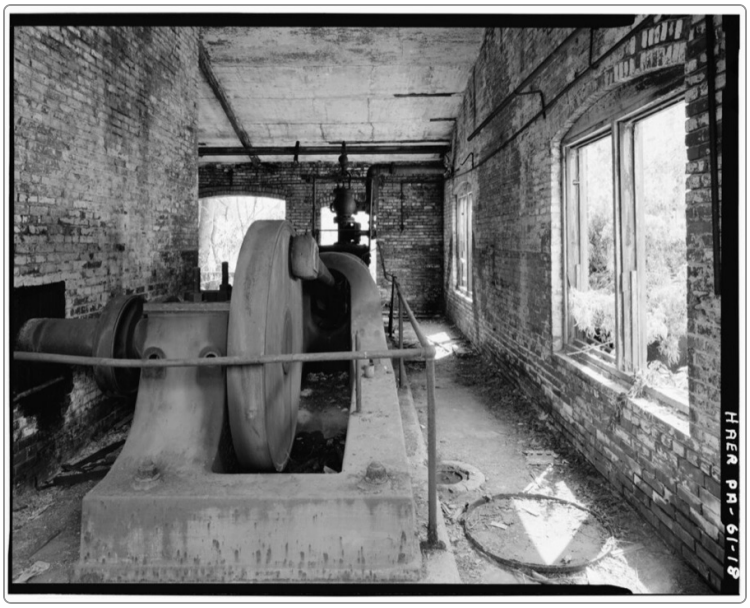
The physics. Rotational motion has its own version of Newton's second law: $\tau = I\alpha$. The rotational inertia $I = \sum mr^2$ depends on mass *and* on how far that mass is from the axis; mass far from the axis counts much more. A hoop of mass M and radius R has $I = MR^2$; a solid disk, $\frac{1}{2}MR^2$; a solid sphere, $\frac{2}{5}MR^2$. A rolling object has kinetic energy of both kinds, $\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ with $v = \omega r$, so on a ramp $a = \dfrac{g\sin\theta}{1 + I/(mr^2)}$. Size

and mass cancel: any solid sphere beats any solid cylinder, which beats any hoop. A yo-yo unwinding from a thin axle of radius r falls with $a = \dfrac{g}{1 + I/(mr^2)}$, much less than g because r is small.

The engineering. Flywheels store energy in rotation. Every piston engine has one to carry the crankshaft smoothly between power strokes, and the great steam engines of the nineteenth century had flywheels many meters across. Modern flywheel energy-storage systems spin carbon-fiber rotors at tens of thousands of turns per minute in vacuum chambers to smooth out supply on electric grids. Engineers designing anything that must start and stop quickly, such as robot arms, wind-turbine blades or bicycle wheels, try to keep rotational inertia low by keeping mass close to the axis.

Try it. In the lab you will roll objects side by side down a ramp to test the predictions, then measure a yo-yo's acceleration and calculate its rotational inertia.

PICTURE STUDY



A Steam Engine's Flywheel, Historic American Engineering Record, Engine of 1908; photographed late 20th century · Photograph · Baltimore Fan House, Wilkes-Barre, Pennsylvania (Library of Congress)
Image: Wikimedia Commons, public domain

The Historic American Engineering Record photographs and measures important machines before they disappear. This flywheel smoothed the motion of a Corliss steam engine that drove a mine ventilation fan.

Look closely

- Find the flywheel's rim and spokes. Where is most of its mass?
- Compare its size with the rest of the engine.
- Find the crank or shaft that connects the engine to the wheel.

Talk about it

1. Why is a flywheel built with a heavy rim and light spokes, rather than as a solid disk?
2. What job does the flywheel do between the piston's strokes?
3. If this wheel spun twice as fast, how much more energy would it store?

Safety first. Keep the ramp on the floor or a low table and catch the rolling objects before they hit anything breakable.

LAB

Rolling Contests and the Yo-Yo

You will test the prediction that sphere beats cylinder beats hoop on a ramp, regardless of size and mass, and then measure the rotational inertia of a yo-yo from how fast it falls.

You will need

- A smooth ramp at least 1 m long and books to prop it
- Rolling objects: a solid ball (golf or rubber), a full can of soup (solid-like cylinder), a ring or hoop (a roll of tape, a jar lid ring or a bangle), a second ball of different size, an empty can
- A yo-yo, a meter stick taped to a wall, kitchen scale, ruler or calipers
- Phone with slow-motion video

Steps

1. **Hypothesis.** Predict the finishing order of each pair, with reasons.
2. Raise the ramp to a modest angle (about 10°). Hold two objects behind a ruler at the top and release them together. Roll every pair 3 times and record the winner.
3. Time each object alone over the full ramp length with video. Find $a = 2d/t^2$ and compare with $g \sin\theta/(1 + I/mr^2)$ for its shape.
4. Roll two balls of different size and mass, and a full can against an empty can. Explain each result.
5. **Yo-yo.** Weigh the yo-yo and measure its axle radius (the radius the string unwinds from). Hold the end of the string still against the meter stick and release the yo-yo from rest. Film the fall over at least 0.60 m. Find a.
6. Calculate $I = mr^2(g/a - 1)$. Then estimate I another way, by treating the yo-yo as two solid disks ($\frac{1}{2}MR^2$ with its outer radius), and compare.
7. **Fair test.** Tape coins to the rim of the yo-yo (increasing I) and measure again. Does it fall more slowly, as predicted?
8. **Improve.** Identify what limits your accuracy (string slipping, hand moving, release) and repeat one measurement.

Tips

- Start each trial with both objects touching the ruler so they leave together.
- Hold the yo-yo string perfectly still; any movement of your hand changes the acceleration.

Check your work

- ☐ My rolling results agree with the predicted order, and I can explain why size and mass do not matter.
- ☐ I measured a yo-yo's acceleration and calculated its rotational inertia.
- ☐ I can calculate the rotational kinetic energy of a flywheel.

LAB NOTEBOOK

1. Table of rolling results and measured accelerations, compared with predictions for each shape.
2. Energy bar charts (translational KE, rotational KE, PE) for a hoop and a sphere at the bottom of the same ramp.
3. Sketch a yo-yo, mark the axle radius and the outer radius, and explain why a thin axle makes it fall slowly.

KEY FORMULAS

$$I = \sum mr^2; \quad I_{\text{hoop}} = MR^2,\ I_{\text{disk}} = \tfrac{1}{2}MR^2,\ I_{\text{sphere}} = \tfrac{2}{5}MR^2$$

$$\tau = I\alpha$$

$$KE = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2, \qquad v = \omega r$$

$$a_{\text{roll}} = \frac{g\sin\theta}{1 + I/(mr^2)}, \qquad a_{\text{yo-yo}} = \frac{g}{1 + I/(mr^2)}$$

From history. Pedro Flores is said to have learned the yo-yo as a boy in the Philippines. Duncan's company held the "Yo-Yo" trademark until a court ruled in 1965 that the word had become a common name. In 1985 a yo-yo flew on the Space Shuttle *Discovery*, where, with no gravity to unwind it, it would not sleep at all.

Example 1. A hoop, a solid cylinder and a solid sphere roll without slipping down a 10° ramp. Find each one's acceleration.

Show solution

1. $g \sin 10^\circ = (9.8\ \mathrm{m/s^2})(0.174) = 1.70\ \mathrm{m/s^2}$.
2. Hoop: $I/(mr^2) = 1$, so $a = 1.70/2 = 0.85\ \mathrm{m/s^2}$.
3. Cylinder: $I/(mr^2) = \frac{1}{2}$, so $a = 1.70/1.5 = 1.13\ \mathrm{m/s^2}$.
4. Sphere: $I/(mr^2) = \frac{2}{5}$, so $a = 1.70/1.4 = 1.22\ \mathrm{m/s^2}$.

Answer: Sphere 1.22, cylinder 1.13, hoop $0.85\ \mathrm{m/s^2}$. The sphere wins, whatever the sizes or masses.

Example 2. A 50 g yo-yo with an axle radius of 0.40 cm falls 0.80 m from rest in 1.8 s as it unwinds. Find its acceleration and its rotational inertia.

Show solution

1. $a = \dfrac{2d}{t^2} = \dfrac{2(0.80\ \mathrm{m})}{(1.8\ \mathrm{s})^2} = 0.494\ \mathrm{m/s^2}$.
2. From $a = \dfrac{g}{1 + I/(mr^2)}$: $I = mr^2\left(\dfrac{g}{a} - 1\right)$.
3. $I = (0.050\ \mathrm{kg})(0.0040\ \mathrm{m})^2\left(\dfrac{9.8}{0.494} - 1\right) = (8.0 \times 10^{-7})(18.8)\ \mathrm{kg \cdot m^2} = 1.5 \times 10$

Answer: $a = 0.49\ \mathrm{m/s^2}$ (one-twentieth of g); $I = 1.5 \times 10^{-5}\ \mathrm{kg \cdot m^2}$.

Example 3. A flywheel is a solid steel disk of mass 100 kg and radius 0.50 m spinning at 3000 rev/min. How much energy does it store?

Show solution

1. $I = \frac{1}{2}MR^2 = \frac{1}{2}(100\ \mathrm{kg})(0.50\ \mathrm{m})^2 = 12.5\ \mathrm{kg \cdot m^2}$.
2. $\omega = 3000\ \mathrm{rev/min} \times \dfrac{2\pi\ \mathrm{rad}}{1\ \mathrm{rev}} \times \dfrac{1\ \mathrm{min}}{60\ \mathrm{s}} = 314\ \mathrm{rad/s}$.
3. $KE = \frac{1}{2}I\omega^2 = \frac{1}{2}(12.5)(314)^2\ \mathrm{J} = 6.2 \times 10^5\ \mathrm{J}$, about 0.17 kWh.

Answer: About 620 kJ, the kinetic energy of a car at about 30 m/s.

PRACTICE

1. Easy Find the rotational inertia of a 2.0 kg hoop of radius 0.30 m.

Show answer

$0.18\ \mathrm{kg \cdot m^2}$

2. Easy Convert 120 rev/min to rad/s.

Show answer

$12.6\ \mathrm{rad/s}$

3. Easy Which is harder to spin up: a wheel with its mass at the rim or one with its mass near the hub (same total mass and size)?

Show answer

The one with mass at the rim; it has larger rotational inertia.

4. Medium A torque of $6.0\ \mathrm{N \cdot m}$ acts on a wheel with $I = 0.40\ \mathrm{kg \cdot m^2}$. Find its angular acceleration and its angular velocity after 2.0 s from rest.

Show answer

$\alpha = 15\ \mathrm{rad/s^2}$; $\omega = 30\ \mathrm{rad/s}$

5. Medium A solid ball rolls from rest down a ramp 0.50 m high. Find its speed at the bottom.

Show answer

$mgh = \frac{1}{2}mv^2(1 + 2/5)$: $v = \sqrt{10gh/7} = 2.65\ \mathrm{m/s}$

6. Medium What fraction of a rolling solid cylinder's kinetic energy is rotational?

Show answer

$\frac{1}{4}mv^2$ out of $\frac{3}{4}mv^2$: one third

7. Medium Why does an empty can reach the bottom after a full can of soup of the same size?

Show answer

The empty can's mass is in its walls, far from the axis, like a hoop; the full can behaves more like a solid cylinder, with a smaller ratio I/mr^2, so it accelerates faster. (The liquid inside also does not have to rotate fully.)

8. Medium Find the kinetic energy of a 0.20 kg solid ball rolling at 3.0 m/s.

Show answer

$\frac{1}{2}(0.20)(9)(1.4) = 1.26$ J

9. Hard A yo-yo has mass 60 g, axle radius 0.50 cm and rotational inertia 2.4×10^{-5} kg · m^2. Find its falling acceleration and the string tension.

Show answer

$I/(mr^2) = 2.4 \times 10^{-5}/1.5 \times 10^{-6} = 16$; $a = 9.8/17 = 0.58$ m/s^2; $T = m(g - a) = 0.553$ N

10. Hard A flywheel storage unit is a 500 kg hoop-like rotor of radius 0.40 m spinning at 20,000 rev/min. How much energy does it store, in kWh?

Show answer

$I = 80$ kg · m^2; $\omega = 2094$ rad/s; $KE = 1.75 \times 10^8$ J $= 49$ kWh

11. Challenge Show that the acceleration of an object rolling down a ramp is $a = g\sin\theta/(1 + I/mr^2)$, using energy conservation and $v^2 = 2ad$.

Show answer

After rolling a distance d along the ramp it has dropped $h = d\sin\theta$: $mgd\sin\theta = \frac{1}{2}mv^2(1 + I/mr^2)$, so $v^2 = \dfrac{2gd\sin\theta}{1 + I/mr^2}$. Comparing with $v^2 = 2ad$ gives the result.

COMMON MISTAKES

- Using turns per minute in $\frac{1}{2}I\omega^2$. Convert to radians per second first.
- Forgetting the rotational kinetic energy of rolling objects.
- Using the outer radius of a yo-yo where the axle radius belongs: the string unwinds from the axle.

Make it a game · Rolling Contest Forecast. Roll a full soup can and a can of different contents down a gentle ramp, or compare a hoop and a disk. Predict the winner first and explain with rotational inertia. Run three races with your parent also predicting. Pretend points go to the correct predictor, with a bonus for the right explanation.

7.5 Angular Momentum: Tops, Gyroscopes and Foucault

OBJECTIVES

- Define angular momentum, $L = I\omega$, and state its conservation law.
- Explain why a spinning skater or a person on a spinning chair speeds up when pulling in the arms.
- Describe gyroscopic stability and precession, and calculate a precession rate.
- Explain how gyroscopes are used in navigation and to point spacecraft.

> **Warm-up.** Spin a top (or a coin on its edge) on a table and watch what happens as it slows down. Then, if you have a bicycle, lift the front wheel, spin it fast and try to turn the handlebars. What do you feel? Write down both observations; you will explain them by the end of the lesson.

WATCH

Wheel Conservation of Angular Momentum Demonstration and Solution

Flipping Physics · 5:19

https://www.youtube.com/watch?v=vM6G-NgN0PY

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Gyroscopic Precession

Veritasium · 3:49

https://www.youtube.com/watch?v=ty9QSiVC2g0

With the gyroscope paragraph: precession explained with a spinning wheel.

KEY TERMS

angular momentum

The rotational counterpart of momentum, $L = I\omega$, a vector along the axis of rotation, measured in kg·m²/s.

conservation of angular momentum

When no net external torque acts, a system's angular momentum stays constant.

gyroscope

A rapidly spinning wheel mounted so its axis can point in any direction; its large angular momentum resists changes in direction.

precession

The slow turning of a spinning object's axis when a torque acts on it, as when a tilted top's axis circles around the vertical.

reaction wheel

A wheel inside a spacecraft that is sped up or slowed down to turn the spacecraft the other way, conserving angular momentum.

CORE IDEAS

The story. In 1851 the French physicist Léon Foucault showed the Earth's rotation with a long pendulum in the Panthéon in Paris, whose swing slowly turned through the day. A year later he looked for a second proof. He built a heavy bronze wheel mounted in rings (gimbals) so that its axis could point in any direction, spun it to high speed with a gear train, and watched its axis through a microscope. As the Earth turned beneath it, the wheel's axis stayed fixed in space, so relative to the room it appeared to drift slowly. Foucault named his invention the *gyroscope*, from Greek words meaning "to see the rotation."

How they figured it out. Foucault's wheel ran for only about ten minutes before friction slowed it, so he had to observe quickly and carefully. The principle behind it, that a spinning body keeps its axis pointing the same way unless a torque acts on it, had been worked out by Euler and others in the 1700s. Engineers soon put it to work. In 1908 Hermann Anschütz-Kaempfe in Germany built the first practical gyrocompass, a gyroscope kept spinning by an electric motor that settles to point to true north, unaffected by the iron of a steel ship; in 1911 Elmer Sperry built one for the United States Navy. Gyroscopes then went into aircraft instruments, autopilots and spacecraft.

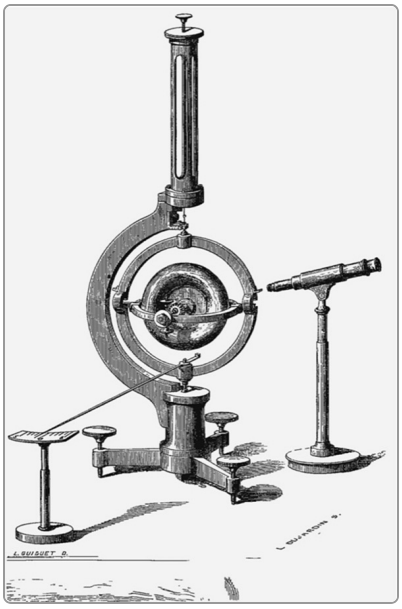
The physics. Angular momentum is $L = I\omega$, and Newton's second law for rotation can be written $\tau = \Delta L/\Delta t$. With no external torque, L is conserved. A skater who pulls her arms in reduces I, so ω must increase: $I_1\omega_1 = I_2\omega_2$. Her kinetic energy increases, supplied by the work her arms do pulling in. Angular momentum is a vector along the spin axis. A torque perpendicular to L changes its direction, not its

size, so the axis swings sideways: this is **precession**. A gyroscope supported at one end, with gravity pulling down on its center, precesses at $\Omega = \frac{\tau}{L} = \frac{mgr}{I\omega}$; the faster it spins, the slower it precesses.

The engineering. The Hubble Space Telescope points at its targets with reaction wheels: to turn the telescope one way, a motor spins a wheel the other way, and total angular momentum stays zero. The International Space Station uses large control-moment gyroscopes the same way. A helicopter needs a tail rotor because its main rotor's torque would otherwise spin the body the opposite way. Phones, game controllers and drones use tiny vibrating-structure gyroscopes on chips to sense rotation. And spinning bullets, footballs and Frisbees fly straight because their angular momentum keeps their axes steady.

Try it. In the lab you will sit on a spinning chair holding weights and measure how your rotation rate changes as you pull them in, and watch a top or a bicycle wheel precess.

PICTURE STUDY



Foucault's Gyroscope, From Camille Flammarion, *Astronomie populaire*, 1894 (instrument of 1852) · Engraving · Paris

Image: Wikimedia Commons, public domain

Flammarion's popular astronomy book showed readers the instrument with which Foucault made the Earth's rotation visible in a single room.

Look closely

- Find the heavy wheel (the rotor) and the rings that hold it.
- Find the microscope or pointer used to watch the axis.
- Look at how the rings are pivoted. Which ways can the rotor's axis turn?

Talk about it

1. Why must the rings be so free to turn?
2. Why did Foucault need the wheel to spin very fast?
3. If the gyroscope's axis stays fixed in space, what does an observer in the room see as the Earth turns?

Safety first. On the spinning chair, keep the speed modest, keep weights close to your body when stopping, and have a partner spot you. Stop if you feel dizzy. Keep fingers clear of a spinning bicycle wheel's spokes.

LAB

The Spinning Chair and the Precessing Top

You will measure how your rotation rate changes as you pull weights in on a spinning office chair, compare with conservation of angular momentum, and observe gyroscopic precession.

You will need

- A swivel office chair that spins freely
- Two equal weights (full water bottles or books, 1–2 kg each)
- Phone with slow-motion video or a stopwatch; a partner
- A top, a gyroscope toy or a bicycle wheel with handles on its axle
- Tape measure, string

Steps

1. Sit on the chair holding the weights at arm's length. Your partner gives you a gentle spin and lifts their hands away.
2. **Hypothesis.** Predict how many times faster you will spin when you pull the weights to your chest.
3. Film from above (or count). Measure the time for one rotation with arms out, then pull the weights in and measure again. Repeat 3 times.
4. Estimate the rotational inertias: model each weight as a point mass, mr^2, at arm's length and at chest distance, plus an estimate for your body and the chair (about 1.5–3 kg·m^2). Predict the ratio of rotation rates and compare.
5. Repeat without weights. Is the effect smaller? Why?
6. **Precession.** Spin a top and watch its axis circle. Time one precession circle as the top slows. Does precession speed up or slow down as the spin slows?
7. If you have a bicycle wheel: spin it, hold the axle horizontal by one end with a string loop, and release the other end. Time the precession, measure r, m and estimate ω, and compare with $\Omega = mgr/(I\omega)$.
8. **Improve.** Friction in the chair slows you during the measurement. Measure the rotation rate just before and just after pulling in, within one or two turns.

Tips

- Pull the weights in quickly and smoothly.
- A spinning top precesses faster as it slows down, until it falls.

Check your work

☐ I measured the change in rotation rate and compared it with

$I_1\omega_1 = I_2\omega_2$.

☐ I can explain why a skater's kinetic energy increases while her angular momentum stays the same.

☐ I can explain precession and why a spinning gyroscope does not fall.

LAB NOTEBOOK

1. Data and estimates for the chair experiment; predicted and measured ratios.
2. Draw a tilted spinning top with arrows for $\vec{L}$, the weight, and the torque, and show which way the axis precesses.
3. Explain in a paragraph how a space telescope can turn to a new target with no thrusters, using reaction wheels.

KEY FORMULAS

$$L = I\omega$$

$$\tau = \frac{\Delta L}{\Delta t}$$

$$I_1\omega_1 = I_2\omega_2 \quad \text{(no external torque)}$$

$$\Omega_{\text{precession}} = \frac{mgr}{I\omega}$$

From history. Foucault became physicist to the Paris Observatory and also measured the speed of light in air and in water (1850), showing it is slower in water, as the wave theory predicted (Lesson 9.5). Modern ring-laser and fiber-optic gyroscopes in airliners have no moving parts at all; they detect rotation from light sent both ways around a loop.

Example 1. A skater spins at 2.0 rev/s with her arms out ($I = 4.0\ \text{kg}\cdot\text{m}^2$). She pulls them in to $I = 1.6\ \text{kg}\cdot\text{m}^2$. Find her new spin rate and compare her kinetic energies.

Show solution

1. $I_1\omega_1 = I_2\omega_2$: $\omega_2 = \dfrac{(4.0)(2.0\ \text{rev/s})}{1.6} = 5.0\ \text{rev/s}$.
2. In rad/s: $\omega_1 = 4\pi = 12.6\ \text{rad/s}$, $\omega_2 = 10\pi = 31.4\ \text{rad/s}$.
3. $KE_1 = \frac{1}{2}(4.0)(12.6)^2 = 316\ \text{J}$; $KE_2 = \frac{1}{2}(1.6)(31.4)^2 = 790\ \text{J}$.

Answer: 5.0 rev/s. Angular momentum is conserved but kinetic energy rises from 316 to 790 J: her muscles did 474 J of work pulling her arms in.

Example 2. A bicycle wheel ($m = 2.0$ kg, nearly all at radius 0.30 m) spins at 20 rad/s. It is held by one end of its axle, 0.20 m from the wheel's center, with the axle horizontal. Find its angular momentum, the torque from gravity and the precession rate.

Show solution

1. $I \approx MR^2 = (2.0)(0.30)^2 = 0.18\ \text{kg}\cdot\text{m}^2$; $L = I\omega = (0.18)(20) = 3.6\ \text{kg}\cdot\text{m}^2/\text{s}$.
2. Torque of gravity about the support: $\tau = mgr = (2.0)(9.8)(0.20) = 3.92\ \text{N}\cdot\text{m}$.
3. $\Omega = \dfrac{\tau}{L} = \dfrac{3.92}{3.6}\ \text{rad/s} = 1.09\ \text{rad/s}$, about one turn every 5.8 s.

Answer: $L = 3.6\ \text{kg}\cdot\text{m}^2/\text{s}$; $\tau = 3.9\ \text{N}\cdot\text{m}$; it precesses at about 1.1 rad/s instead of falling.

Example 3. Kepler's second law is conservation of angular momentum. A comet is 0.60 AU from the Sun at perihelion moving at 54 km/s. How fast is it moving at aphelion, 35 AU away?

Show solution

1. At perihelion and aphelion the velocity is perpendicular to the radius, so $L = mvr$ is conserved: $v_p r_p = v_a r_a$.
2. $v_a = \dfrac{v_p r_p}{r_a} = \dfrac{(54\ \text{km/s})(0.60)}{35} = 0.93\ \text{km/s}$.

Answer: About 0.93 km/s, nearly sixty times slower.

PRACTICE

1. Easy Find the angular momentum of a disk with $I = 0.50\ \text{kg} \cdot \text{m}^2$ spinning at 8.0 rad/s.

   Show answer

   $4.0\ \text{kg} \cdot \text{m}^2/\text{s}$

2. Easy Why does a diver tuck into a ball to spin faster?

   Show answer

   Tucking reduces her rotational inertia; with angular momentum conserved, her rotation rate increases.

3. Easy Why does a helicopter need a tail rotor?

   Show answer

   The engine's torque spinning the main rotor one way would spin the body the other way (conservation of angular momentum); the tail rotor pushes sideways to cancel that torque.

4. Medium A turntable with $I = 0.020\ \text{kg} \cdot \text{m}^2$ spins at 3.5 rad/s. A 0.10 kg lump of clay is dropped onto it 0.15 m from the center. Find the new angular velocity.

   Show answer

   $I_2 = 0.020 + 0.10(0.15)^2 = 0.02225;\ \omega_2 = 0.070/0.02225 = 3.1\ \text{rad/s}$

5. Medium A spinning-chair student has $I = 3.0\ \mathrm{kg \cdot m^2}$ with weights out and $1.2\ \mathrm{kg \cdot m^2}$ with weights in. If he spins once every 2.0 s with weights out, how long does one turn take with weights in?

Show answer

$0.80\ \mathrm{s}$

6. Medium A planet moves at $30\ \mathrm{km/s}$ at $1.0\ \mathrm{AU}$ at perihelion. Its aphelion is $1.2\ \mathrm{AU}$. What is its speed at aphelion?

Show answer

$25\ \mathrm{km/s}$

7. Medium Why does a spinning top not fall over while it spins fast, but falls when it slows?

Show answer

Its large angular momentum means gravity's torque only makes the axis precess slowly. As the spin slows, L shrinks, the precession becomes faster and wobblier, and finally the top cannot keep upright.

8. Medium A torque of $0.50\ \mathrm{N \cdot m}$ acts for 4.0 s on a wheel at rest. Find its final angular momentum, and its angular velocity if $I = 0.25\ \mathrm{kg \cdot m^2}$.

Show answer

$L = 2.0\ \mathrm{kg \cdot m^2/s}$; $\omega = 8.0\ \mathrm{rad/s}$

9. Hard A toy gyroscope's rotor has $I = 5.0 \times 10^{-5}\ \mathrm{kg \cdot m^2}$ and spins at $200\ \mathrm{rad/s}$. Its total mass is $0.10\ \mathrm{kg}$, with its center $3.0\ \mathrm{cm}$ from the pivot. Find the precession rate and the time per precession circle.

Show answer

$\Omega = (0.10)(9.8)(0.030)/(0.010) = 2.94\ \mathrm{rad/s}$; period $= 2.1\ \mathrm{s}$

10. Hard If the Sun (radius $7.0 \times 10^8\ \mathrm{m}$, rotation period about 25 days) collapsed to a radius of $10\ \mathrm{km}$ while keeping its mass and angular momentum (and the same shape factor), what would its rotation period be?

Show answer

$I \propto R^2$, so $T_2 = T_1(R_2/R_1)^2 = (25 \times 86{,}400)(10^4/7.0 \times 10^8)^2 = 4.4 \times 10^{-4}\ \mathrm{s}$, more than 2000 turns per second. (Collapsed stars called pulsars spin this fast.)

11. **Challenge** Using $\tau = \Delta L/\Delta t$, explain why a bicycle wheel's precession is slower when it spins faster, and show that the precession rate is $\Omega = mgr/(I\omega)$.

Show answer

In time Δt gravity's torque adds $\Delta L = \tau \Delta t$ perpendicular to $\vec{L}$, turning it by an angle $\Delta\phi = \Delta L/L = \tau\Delta t/L$. So $\Omega = \Delta\phi/\Delta t = \tau/L = mgr/(I\omega)$; a larger ω means a larger L and a smaller turning angle per second.

COMMON MISTAKES

- Thinking kinetic energy is conserved when a spinner pulls in. Only angular momentum is; the spinner does work.
- Expecting a supported gyroscope to fall in the direction of gravity. A torque perpendicular to $\vec{L}$ turns the axis sideways.
- Using rev/s in $L = I\omega$ when the answer should be in kg·m²/s. Use rad/s.

Make it a game · Spin Faster Explain. Sit on a swivel chair, holding two heavy books. Your parent turns you gently while you hold your arms out, then you pull the books in. Predict, then notice what happens to the spin, and explain using L = I omega. Record your guess, then swap roles. Score one point for each accurate explanation.

UNIT 7

Rotation and Statics

Everything so far has moved as a point. Real objects turn, tip, balance and spin, and this unit gives you the tools for them: torque, center of mass, the conditions for equilibrium, rotational inertia and angular momentum. They are the physics of every lever and crane, every bridge and building that must stand still, every flywheel and every gyroscope.

The stories run from Archimedes' lever and the Leaning Tower of Pisa, which engineers stopped from falling in 2001, to Benjamin Baker's human cantilever model of the Forth Bridge, Pedro Flores's yo-yo and Foucault's gyroscope. The labs use toys and household tools as apparatus: a meter-stick balance, a balancing-bird toy, a beam on two kitchen scales, a rolling contest and a yo-yo, and a spinning office chair.

PREREQUISITES

- Units 3 and 5: Newton's laws, free-body diagrams and energy
- Unit 4: circular motion (angular speed and $v = \omega r$)
- Right-triangle trigonometry

MASTERY CHECKLIST

- ☐ I can calculate torque with $\tau = rF\sin\theta$ and use the law of the lever.
- ☐ I can find a center of mass by calculation and by experiment, and predict when an object will tip.
- ☐ I can solve static-equilibrium problems for beams, cantilevers, cranes and hanging signs using $\sum F = 0$ and $\sum \tau = 0$.
- ☐ I can use rotational inertia, $\tau = I\alpha$ and rotational kinetic energy, and predict which rolling object wins.
- ☐ I can apply conservation of angular momentum and explain gyroscopic precession.

OPTIONAL KITS FOR THIS UNIT

- **Arbor Scientific rotational motion and gyroscope apparatus**: A classroom gyroscope, a rotating platform or a set of rolling objects (hoop, disk, sphere of matched size) make Lessons 7.4–7.5 more precise. Optional; household objects work.
- **KEVA Planks**: Identical maple planks for building towers, cantilevers and balanced structures; excellent for exploring center of mass and equilibrium in Lessons 7.2–7.3 and the portfolio.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

7.1	**Torque: Archimedes' Lever**	3 sessions of 50 minutes
7.2	**Center of Mass and the Leaning Tower**	3 sessions of 50 minutes
7.3	**Equilibrium: Cranes and the Forth Bridge**	3 sessions of 50 minutes
7.4	**Rotational Inertia: The Yo-Yo and the Flywheel**	3 sessions of 50 minutes
7.5	**Angular Momentum: Tops, Gyroscopes and Foucault**	3 sessions of 50 minutes

Unit 7 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$. For torque problems, state your pivot and your sign convention. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   A 60 N force is applied at the end of a 0.40 m wrench at 90° and then at 30° to the wrench. Find both torques.

2. **6 pts**
   On a meter stick balanced at 50 cm, 100 g hangs at 20 cm. Where must 75 g hang to balance it?

3. 5 pts

State the law of the lever and explain why a crowbar can lift a heavy rock.

4. 6 pts

Find the center of mass of $3.0\ \mathrm{kg}$ at $x = 0$, $1.0\ \mathrm{kg}$ at $x = 2.0\ \mathrm{m}$ and $4.0\ \mathrm{kg}$ at $x = 5.0\ \mathrm{m}$.

5. 6 pts

A uniform crate $0.60\ \mathrm{m}$ wide and $1.5\ \mathrm{m}$ tall stands on a tilting truck bed. At what angle does it tip (if it does not slide)?

6. 5 pts

Explain why the Leaning Tower of Pisa has not fallen and how engineers made it safer.

7. 8 pts

A uniform $6.0\ \mathrm{m}$ beam weighing $400\ \mathrm{N}$ rests on supports at its ends. An $800\ \mathrm{N}$ load sits $1.5\ \mathrm{m}$ from the left end. Find both support forces.

8. 7 pts

A $25\ \mathrm{kg}$ sign hangs from the end of a light $2.0\ \mathrm{m}$ horizontal rod hinged at a wall, held by a cable at 40° above the rod. Find the cable tension.

9. 5 pts

Describe Baker's human cantilever model of the Forth Bridge and what it showed.

10. 6 pts

A tower crane carries $1500\ \mathrm{kg}$ at $30\ \mathrm{m}$. Its counterweight is $12\ \mathrm{m}$ behind the tower. What counterweight balances the load (ignore the jib's weight)?

11. 6 pts

A solid cylinder and a hoop roll from rest down a 15° ramp. Find each one's acceleration. Which wins?

12. 7 pts

A $40\ \mathrm{g}$ yo-yo falls with an acceleration of $0.60\ \mathrm{m/s^2}$ while unwinding from an axle of radius $0.35\ \mathrm{cm}$. Find its rotational inertia.

13. 6 pts

A flywheel (solid disk, 40 kg, radius 0.30 m) spins at 1800 rev/min. Find its kinetic energy.

14. 6 pts

A skater spinning at 1.5 rev/s with $I = 3.6\ \mathrm{kg \cdot m^2}$ pulls in to $I = 1.2\ \mathrm{kg \cdot m^2}$. Find her new spin rate. Is her kinetic energy conserved?

15. 6 pts

A gyroscope with $L = 0.40\ \mathrm{kg \cdot m^2/s}$ has a gravitational torque of $0.12\ \mathrm{N \cdot m}$ acting on it. Find its precession rate.

16. 10 pts

Essay. Explain how angular momentum is used in two of the following: a gyrocompass, a space telescope's reaction wheels, a helicopter's tail rotor, a spinning football. For each, say what would happen without it.

ANSWER KEY

1. $24\ \mathrm{N \cdot m}$; $12\ \mathrm{N \cdot m}$
2. $100(30) = 75d$, $d = 40$ cm from the pivot: the 90 cm mark
3. At balance $F_1 d_1 = F_2 d_2$. With the fulcrum close to the rock, the short arm under the rock and the long arm under the hand let a small force on the long arm produce a large force on the rock.
4. $(0 + 2 + 20)/8 = 2.75\ \mathrm{m}$
5. $\tan\theta = 0.30/0.75 = 0.40$; $\theta = 21.8^\circ$
6. Its center of mass is still above its base of support, so gravity's torque does not overturn it. Engineers removed small amounts of soil under the high side so it settled, rotating the tower back by about half a degree and moving the center of mass farther inside the base.
7. Right: $(400 \times 3 + 800 \times 1.5)/6 = 400$ N; left: 800 N
8. $T \sin 40^\circ = 245$ N; $T = 381$ N
9. Two men sat on chairs with arms outstretched holding sticks braced against the chairs (arms in tension, sticks in compression), supporting a plank with a person on it between them, with brick counterweights tied to their other hands. It showed that each tower of the bridge is a balanced cantilever, with the suspended span carried between the arms and the torques balanced by anchorage.
10. 3750 kg
11. $g \sin 15^\circ = 2.54\ \mathrm{m/s^2}$; cylinder 1.69, hoop $1.27\ \mathrm{m/s^2}$; the cylinder wins.
12. $I = mr^2(g/a - 1) = 0.040(0.0035)^2(15.33) = 7.5 \times 10^{-6}\ \mathrm{kg \cdot m^2}$
13. $I = 1.8\ \mathrm{kg \cdot m^2}$; $\omega = 188.5$ rad/s; $KE = 3.2 \times 10^4$ J
14. 4.5 rev/s. No: it triples; her muscles do the work.
15. 0.30 rad/s
16. Model answer: A gyrocompass uses a rapidly spinning rotor whose large angular momentum keeps its axis steady; with friction and gravity arranged to make it settle north, it points to true north regardless of a steel ship's magnetism. Without the spin it would simply be a pendulum. Reaction wheels change a spacecraft's direction by spinning a wheel one way so the spacecraft turns the other way, keeping total angular momentum zero; without them the telescope would need thrusters, which use fuel and blur images. A helicopter's tail rotor cancels the reaction torque of the main rotor; without it the body would spin

opposite the blades. A spinning football's angular momentum keeps its axis pointed along its path; without spin it tumbles and loses distance and accuracy.

PORTFOLIO PIECE: A COUNTERWEIGHTED CANTILEVER

Design and build a model cantilever, a balcony or a bridge arm, from craft sticks, KEVA planks, cardboard or balsa, that reaches at least 40 cm out from a support and carries a load of coins at its tip. Use a counterweight or an anchor at the back, and calculate in advance, with torques, the counterweight needed for loads of 50, 100 and 150 g. Measure the actual counterweight at which the model is balanced for each load, using a kitchen scale under the support to measure the support force as well, and compare. Write a report with an extended free-body diagram, calculations, a table of predicted and measured values, photos and an explanation connecting your model to the Forth Bridge or a tower crane.

LOOK FOR	WHAT IT MEANS
Design	Model reaches at least 40 cm; drawing shows dimensions, pivot, load point and counterweight point.
Calculations	Torque balance and support-force calculations for three loads, with units.
Measurements	Measured counterweights and support forces, compared with predictions in a table with percent differences.
Explanation	Clear account of equilibrium ($\sum F = 0$, $\sum \tau = 0$) and the role of the counterweight, with connection to a real structure.
Iteration	At least one design change (stiffer arm, different counterweight position) with before-and-after results.

UNIT 8

Fluids and Heat

Fluids, liquids and gases, push, lift and flow, and heat moves between them. This unit begins with the pressure of the "ocean of air" we live beneath, measured by Torricelli and Pascal's brother-in-law on the Puy de Dôme, and with the buoyancy Archimedes discovered. It continues with fluids in motion, from Bernoulli's pipes and Torricelli's jets to the wing of the Wright Flyer, explained correctly: wings turn air downward, and the pressure above them is lower. Then it turns to heat: Joseph Black's latent heat, the calorimeter, and finally Carnot's and Kelvin's laws of thermodynamics, which govern every engine and power plant.

The labs build a two-syringe hydraulic press, weigh objects in water to test a "crown," measure the speed of water jets from a bottle and a toy water blaster, measure the specific heat of steel, and find absolute zero with a syringe of air in water baths.

PREREQUISITES

- Unit 5: work, energy and power; Joule's mechanical equivalent of heat (Lesson 5.1)
- Unit 2: projectile motion (for water jets)
- Unit 3: forces and free-body diagrams

MASTERY CHECKLIST

- ☐ I can calculate pressure and pressure at depth, and apply Pascal's principle to hydraulic machines.
- ☐ I can use Archimedes' principle to find buoyant forces, densities and the fraction of a floating object submerged.
- ☐ I can use the continuity equation, Bernoulli's equation and Torricelli's law, and explain lift correctly.
- ☐ I can solve heating, phase-change and calorimetry problems with

$Q =$and
$mc\Delta T$

$Q = mL.$

- ☐ I can apply the first and second laws of thermodynamics and calculate Carnot efficiencies in kelvins.

OPTIONAL KITS FOR THIS UNIT

- **Vernier Go Direct temperature and pressure sensors**: Wireless sensors that graph temperature and gas pressure live on a phone or computer; they make the calorimetry and absolute-zero labs (Lessons 8.4–8.5) faster and more precise. Optional.
- **Thames & Kosmos hydraulics kits**: Ages 10 and up. Syringe-powered hydraulic machines (a crane, an excavator) that extend Lesson 8.1's two-syringe press into working models.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

8.1	**Pressure: Pascal and the Puy de Dôme**	3 sessions of 50 minutes
8.2	**Buoyancy: Archimedes' Principle**	3 sessions of 50 minutes
8.3	**Moving Fluids: Bernoulli, Torricelli and the Wing**	3 sessions of 50 minutes
8.4	**Temperature and Heat: Joseph Black and the Calorimeter**	3 sessions of 50 minutes
8.5	**Thermodynamics and Engines: Carnot and Kelvin**	3 sessions of 50 minutes

Unit 8 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$, $\rho_{\text{water}} = 1000\ \mathrm{kg/m^3}$, $c_{\text{water}} = 4186\ \mathrm{J/(kg \cdot K)}$, $L_f = 334\ \mathrm{kJ/kg}$ and $L_v = 2260\ \mathrm{kJ/kg}$. Use kelvins in thermodynamic formulas. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   A 60 kg person stands on a board of area $0.50\ \mathrm{m^2}$ on soft ground. What pressure does the board exert?

2. **6 pts**
   Find the absolute pressure $25\ \mathrm{m}$ below the surface of a freshwater lake.

3. **6 pts**
   A hydraulic lift has a small piston $3.0\ \mathrm{cm}$ across and a large one $30\ \mathrm{cm}$ across. What force on the small piston lifts a $2000\ \mathrm{kg}$ truck? How far must the small piston move to raise the truck $0.10\ \mathrm{m}$?

4. **5 pts**
   Describe the Puy de Dôme experiment and explain what it proved.

5. **6 pts**

   A metal object weighs 4.90 N in air and 4.40 N in water. Find its volume and density. What might it be?

6. **5 pts**

   A block of wood floats with 70% of its volume submerged in fresh water. What is its density? Would it float higher or lower in seawater?

7. **5 pts**

   Explain how a steel ship floats.

8. **6 pts**

   Water flows at $3.0\ \mathrm{m/s}$ through a pipe $4.0\ \mathrm{cm}$ across, which narrows to $2.0\ \mathrm{cm}$. Find the speed in the narrow section and the drop in pressure (horizontal pipe).

9. **5 pts**

   A hole is $0.45\ \mathrm{m}$ below the water surface in a tank. Find the speed of the jet.

10. **7 pts**

    Explain how a wing produces lift. Then explain what is wrong with the claim that air over the top must travel faster to meet the air from below at the trailing edge.

11. **6 pts**

    How much heat is needed to warm $3.0\ \mathrm{kg}$ of water from 15 to $65^\circ\mathrm{C}$?

12. **7 pts**

    $300\ \mathrm{g}$ of aluminum ($c = 900$) at $95^\circ\mathrm{C}$ is dropped into $400\ \mathrm{g}$ of water at $18^\circ\mathrm{C}$. Find the final temperature.

13. **6 pts**

    How much heat melts $0.20\ \mathrm{kg}$ of ice at $0^\circ\mathrm{C}$ and then boils the water away (from 0 to $100^\circ\mathrm{C}$ and vaporized)?

14. **5 pts**

    What is latent heat, and how did Joseph Black discover it?

15. 6 pts

Find the Carnot efficiency of an engine between $400^\circ\mathrm{C}$ and $40^\circ\mathrm{C}$.

16. 6 pts

A heat engine takes in 5000 J per cycle and rejects 3200 J. Find the work done and the efficiency.

17. 8 pts

Essay. State the first and second laws of thermodynamics and explain, using a power plant or a car engine as an example, what each law means for engineers trying to make engines more efficient.

ANSWER KEY

1. $1176\ \mathrm{Pa}$
2. $101.3 + 245 = 346\ \mathrm{kPa}$
3. $196\ \mathrm{N}$; $10\ \mathrm{m}$ (in many short strokes)
4. Two identical mercury barometers were filled in Clermont; one stayed there with an observer while the other was carried up the mountain. At the summit the mercury was about 8.5 cm lower, and it returned to the original height on the way down. This showed that air pressure is caused by the weight of the air above and decreases with height.
5. $F_b = 0.50\ \mathrm{N}$; $V = 5.1 \times 10^{-5}\ \mathrm{m}^3$; $\rho = 9800\ \mathrm{kg/m^3}$; close to lead or a dense bronze, not gold
6. $700\ \mathrm{kg/m^3}$; higher (seawater is denser)
7. Its hull encloses a large volume of air, so its average density is less than water's. It sinks until the water it displaces weighs as much as the whole ship.
8. $12\ \mathrm{m/s}$; $\Delta P = \frac{1}{2}(1000)(144 - 9) = 67.5\ \mathrm{kPa}$
9. $2.97\ \mathrm{m/s}$
10. A wing's shape and angle of attack turn the oncoming air downward; by Newton's third law the air pushes the wing up, and this appears as lower pressure above the wing and higher pressure below. The air above does move faster, but there is no rule that parcels separated at the front must rejoin at the back; in reality the upper air arrives well before the lower air, so the "equal transit time" argument is false and gives the wrong amount of lift.
11. $628\ \mathrm{kJ}$
12. $270(95 - T) = 1674(T - 18)$; $T = 28.7^\circ\mathrm{C}$
13. $66.8 + 83.7 + 452 = 602\ \mathrm{kJ}$
14. Latent heat is energy absorbed or released during a change of phase without a change in temperature. Black noticed that ice in a warm room stays at 0 °C while it melts slowly, so it must be absorbing heat that does not raise its temperature; he measured that melting ice took as much heat as warming the same mass of water by about 140 °F.
15. $1 - 313/673 = 53\%$
16. $1800\ \mathrm{J}$; 36%
17. Model answer: The first law says energy is conserved: the work an engine does plus the heat it rejects equals the heat it takes in. Engineers cannot get more

work out than energy in, so efficiency is always less than 100%, and every joule of fuel must be accounted for. The second law says heat flows naturally only from hot to cold and no engine can convert heat entirely into work; even an ideal engine must reject heat, with efficiency limited to $1 - T_C/T_H$. So engineers raise the hot temperature (better alloys, higher steam pressures), lower the cold temperature (cooling towers, large condensers), reduce friction and heat leaks to approach the Carnot limit, and recover rejected heat, for example in combined-cycle plants that pass gas-turbine exhaust to a steam turbine.

PORTFOLIO PIECE: A CARTESIAN DIVER AND A FLOATING CARGO BARGE

Build two buoyancy devices and explain each with numbers. First, a Cartesian diver: a medicine dropper or a folded condiment packet in a sealed 2-liter bottle of water that sinks when you squeeze the bottle and rises when you let go. Measure how the air bubble inside the diver changes volume when it sinks, and explain the result with Pascal's principle, Boyle's law and Archimedes' principle. Second, a cargo barge folded from aluminum foil with a measured flat bottom: predict from Archimedes' principle how many pennies (2.5 g each) it will carry before sinking, then test it in fresh water and in salt water. Write a report with sketches, calculations, measurements and a discussion of the differences.

LOOK FOR	WHAT IT MEANS
Cartesian diver	Works reliably; bubble volume measured or estimated at the surface and when sinking; explanation uses pressure, gas compression and buoyancy correctly.
Barge prediction	Capacity predicted from the barge's dimensions and Archimedes' principle before testing, with units.
Testing	Pennies counted in fresh and salt water, three trials each; results compared with predictions.
Explanation	Report correctly relates density, displacement and buoyant force, and explains why salt water carries more.
Honest analysis	Sources of error discussed (leaks, uneven loading, foil weight) and one improvement made.

8.1 Pressure: Pascal and the Puy de Dôme

OBJECTIVES

- Define pressure, $P = F/A$, and use pascals, kilopascals and atmospheres.
- Calculate pressure at a depth in a fluid, $P = P_0 + \rho g h$, and distinguish gauge from absolute pressure.
- Apply Pascal's principle to hydraulic machines.
- Measure the force multiplication of a two-syringe hydraulic system.

> **Warm-up.** Fill a plastic bottle with water and poke three small holes in its side with a pushpin, one near the bottom, one in the middle and one near the top (do this over a sink). Before you uncap it, predict which stream will shoot out farthest. Then uncap it and watch. What does the result tell you about water pressure and depth?

WATCH

Pascal's Principle, Hydraulic Lift System, Pascal's Law of Pressure, Fluid Mechanics Problems

The Organic Chemistry Tutor · 21:04

https://www.youtube.com/watch?v=MZ6GCH2nLy0

> **WHEN TO WATCH**
> After the physics paragraph. Pause and work each example first.

BACKUP VIDEO

The history of the barometer (and how it works) - Asaf Bar-Yosef

TED-Ed · 4:46

https://www.youtube.com/watch?v=EkDhlzA-lwI

With the story paragraph: Torricelli's and Pascal's barometer experiments, animated.

KEY TERMS

pressure

Force per unit area, $P = F/A$, measured in pascals: $1\ \mathrm{Pa} = 1\ \mathrm{N/m^2}$.

atmospheric pressure

The pressure of the air at sea level, about 101.3 kPa (1 atmosphere, 760 mm of mercury, 14.7 psi).

gauge pressure

Pressure above atmospheric pressure, as read by a tire gauge; absolute pressure = gauge pressure + atmospheric pressure.

Pascal's principle

A pressure applied to an enclosed fluid is transmitted undiminished to every part of the fluid and the walls of its container.

barometer

An instrument that measures atmospheric pressure, originally by the height of a column of mercury.

CORE IDEAS

The story. In 1643 Evangelista Torricelli, Galileo's last student, filled a glass tube with mercury, closed its end, and turned it upside down in a bowl of mercury. The mercury fell to a height of about 76 cm and stopped, leaving an empty space above it. Torricelli argued that the column was held up by the weight of the air pressing on the mercury in the bowl: "We live submerged at the bottom of an ocean of air." In France, the young mathematician Blaise Pascal saw a way to test this. If air has weight, there should be less of it pressing down on a mountaintop. Pascal could not make the climb himself, so he asked his brother-in-law, Florin Périer, who lived in Clermont at the foot of the Puy de Dôme, a mountain about 1,000 meters higher.

How they figured it out. On September 19, 1648, Périer and several friends filled two identical barometer tubes in Clermont and found them equal. One was left with a watcher, who checked it all day. The others carried the second tube up the mountain. At the summit the mercury stood about 8.5 cm lower than in the town; at stops on the way down it rose step by step, and back in Clermont it matched the tube that had stayed behind. The experiment had a control, repeated measurements and a clear prediction, and it showed that the air's pressure falls with height. Pascal went on to state the principle that pressure in a fluid is transmitted equally in all directions. A devout Christian, he is also remembered for his *Pensées*, his reflections on faith.

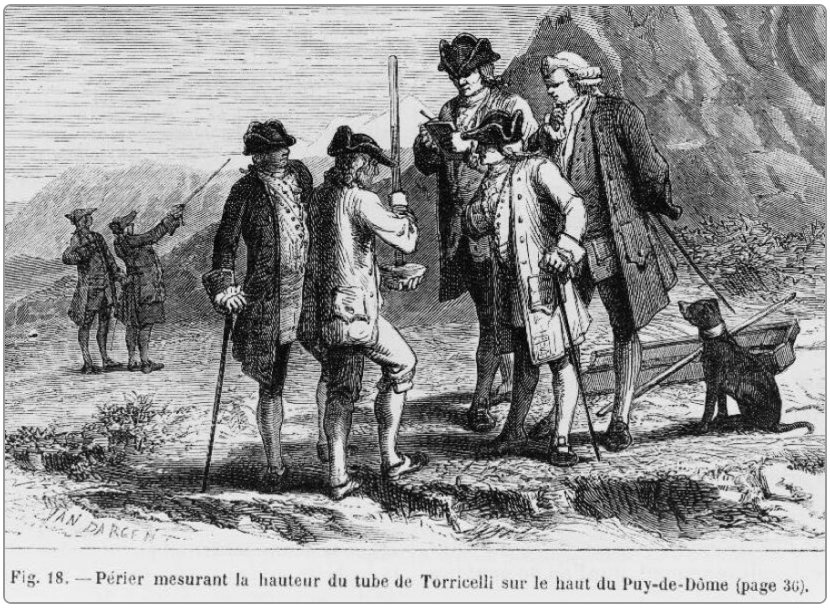
The physics. Pressure is force per area: $P = F/A$. A fluid at rest pushes equally in all directions at a given depth, and its pressure grows with depth because of the weight of fluid above: $P = P_0 + \rho g h$. In water, pressure rises by about 1 atmosphere (101 kPa) for every 10 m of depth. A mercury column 76 cm high has the same pressure at its base as the whole atmosphere: $\rho g h = (13{,}600)(9.8)(0.76) = 101\ \text{kPa}$.

Pascal's principle says that a pressure added anywhere in an enclosed fluid is felt everywhere in it. So in a hydraulic press, a small force on a small piston produces the same pressure on a large piston, giving a large force: $\frac{F_1}{A_1} = \frac{F_2}{A_2}$.

The engineering. Hydraulics moves the heavy world. Car brakes use a small master cylinder pushed by the pedal to drive larger pistons at each wheel. Excavators, garbage trucks, aircraft landing gear and car lifts all use hydraulic cylinders; Joseph Bramah patented the hydraulic press in 1795. Engineers designing dams make them thicker at the bottom, where the water pressure is greatest, and water towers use the height of water to provide pressure to a town's taps. Divers and submarine designers must reckon with the pressure of the deep sea.

Try it. In the lab you will build a hydraulic system from two syringes of different sizes and measure how it multiplies force.

PICTURE STUDY



Périer on the Puy de Dôme, Yan' Dargent, 1867 · Wood engraving · French popular-science book (Wikimedia Commons)

Image: Wikimedia Commons, public domain

This nineteenth-century engraving imagines the moment on September 19, 1648, when Périer and his friends measured the barometer on the summit. (The artist dressed them in the fashions of a century later.)

Look closely

- Find the tall tube and the dish it stands in.
- Find the person measuring. What tool might he be using?
- Look at the landscape. How does the artist show that they are high up?

Talk about it

1. Why did Périer leave a second barometer in the town with a watcher?
2. What would the experiment have shown if the mercury had stood at the same height on the summit?
3. Why is mercury, rather than water, used in a barometer? (A water barometer would need a tube over 10 m tall.)

LAB

A Two-Syringe Hydraulic Press

You will connect a small syringe and a large syringe with water-filled tubing and measure how much force the large piston can exert for a given push on the small one, testing Pascal's principle.

You will need

- Two plastic syringes of different sizes (for example, 10 mL and 60 mL), no needles
- About 30 cm of clear tubing that fits both tips
- Water with a drop of food coloring
- Kitchen scale; a stack of books; a ruler
- A plastic bottle and a pushpin for the depth test

Steps

1. Fill both syringes and the tubing with water, pushing out all air bubbles. Connect them.
2. Measure the inside diameter of each syringe barrel (or calculate the area from the volume markings: area = volume ÷ length of that volume) and find the area ratio.
3. **Hypothesis.** Predict the force multiplication and the ratio of distances the two pistons move.
4. Stand the large syringe upright on the kitchen scale, its plunger against a fixed board above (a stack of books). Push the small syringe's plunger down with a measured force (press it on a second scale, or hang a known weight from it).
5. Record the force on the large piston (the scale reading) for at least four different input forces. Graph output force against input force; the slope is the force multiplication.
6. Measure how far each plunger moves for one push. Compare the distance ratio with the area ratio, and compare work in with work out.

7. **Reverse it.** Push the large syringe and measure the force from the small one. What do you gain and what do you lose?
8. **Depth test.** Fill the bottle with three holes at different heights, and measure how far each stream lands. Relate the results to $P = \rho g h$.
9. **Improve.** Friction in the syringes makes the output smaller than predicted. Measure it by pushing the large syringe alone, and correct your results.

Tips

- Air bubbles make the system spongy; tap the tubing to free them.
- Hold the syringes level with each other, or the water's weight adds to the pressure.

Check your work

- ☐ My measured force multiplication is close to the area ratio.
- ☐ I showed that the small piston moves farther, so work out does not exceed work in.
- ☐ I can calculate pressure at a depth and explain a barometer.

LAB NOTEBOOK

1. Sketch and data table for the hydraulic press, graph of output against input force, and the work comparison.
2. Draw a mercury barometer and label the heights and pressures, showing why the column is 76 cm tall.
3. Sketch a dam in cross-section and draw arrows for the water's pressure at several depths. Explain its shape.

KEY FORMULAS

$$P = \frac{F}{A}$$

$$P = P_0 + \rho g h$$

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{(Pascal's principle)}$$

$$1 \text{ atm} = 101.3 \text{ kPa} = 760 \text{ mm Hg}$$

From history. Pascal also built one of the first mechanical calculating machines, the Pascaline (1642), to help his father with tax accounts. The SI unit of pressure was named the pascal in 1971. Torricelli's barometer became the first weather instrument: falling pressure warns of an approaching storm.

WORKED EXAMPLES

Example 1. A 70 kg person stands on one narrow heel of area 1.0 cm^2, then on one flat shoe of area 150 cm^2. Find the pressure on the floor in each case.

Show solution

1. Weight: $F = mg = (70\ \text{kg})(9.8\ \text{m/s}^2) = 686\ \text{N}$.
2. Heel: $P = \dfrac{686\ \text{N}}{1.0 \times 10^{-4}\ \text{m}^2} = 6.9 \times 10^6\ \text{Pa}$, about 68 atmospheres.
3. Flat shoe: $P = \dfrac{686\ \text{N}}{1.5 \times 10^{-2}\ \text{m}^2} = 4.6 \times 10^4\ \text{Pa}$.

Answer: 6.9 MPa versus 46 kPa: the same force, 150 times the pressure, which is why narrow heels dent wooden floors.

Example 2. Find the gauge pressure and absolute pressure 10 m below the surface of a freshwater lake.

Show solution

1. Gauge: $\rho gh = (1000\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(10\ \text{m}) = 9.8 \times 10^4\ \text{Pa} = 98\ \text{kPa}$.
2. Absolute: $101.3\ \text{kPa} + 98\ \text{kPa} = 199\ \text{kPa}$, about 2 atmospheres.

Answer: 98 kPa gauge; 199 kPa absolute.

Example 3. A car lift has a small piston 2.0 cm across and a large piston 20 cm across. What force on the small piston lifts a 1500 kg car?

Show solution

1. Area ratio: $\dfrac{A_2}{A_1} = \left(\dfrac{20}{2.0}\right)^2 = 100.$
2. $F_1 = F_2 \dfrac{A_1}{A_2} = \dfrac{(1500\ \text{kg})(9.8\ \text{m/s}^2)}{100} = 147\ \text{N}.$
3. The small piston must move 100 times as far as the large one: the work in equals the work out.

Answer: 147 N, about the weight of a 15 kg bag.

Example 4. On the Puy de Dôme the mercury fell about 8.5 cm. What drop in air pressure is that? If the air's density is about 1.1 kg/m^3, roughly how high was the climb?

Show solution

1. $\Delta P = \rho_{\text{Hg}} g \Delta h = (13{,}600\ \text{kg/m}^3)(9.8\ \text{m/s}^2)(0.085\ \text{m}) = 1.13 \times 10^4\ \text{Pa}.$
2. For the air column: $\Delta P = \rho_{\text{air}} g H$, so $H = \dfrac{1.13 \times 10^4\ \text{Pa}}{(1.1\ \text{kg/m}^3)(9.8\ \text{m/s}^2)} = 1050\ \text{m}.$

Answer: About 11 kPa (11% of an atmosphere), consistent with a climb of about 1000 m.

PRACTICE

1. Easy A 500 N force acts on 0.25 m^2. Find the pressure.

   Show answer

   2000 Pa

2. Easy What is the gauge pressure 5.0 m deep in water?

   Show answer

   49 kPa

3. Easy Why do snowshoes keep you from sinking into snow?

   Show answer

   They spread your weight over a large area, lowering the pressure on the snow.

4. Medium A tire gauge reads 220 kPa. What is the absolute pressure in the tire?

Show answer

321 kPa

5. Medium A hydraulic jack has pistons of area $5.0\ \mathrm{cm}^2$ and $200\ \mathrm{cm}^2$. What force on the small piston lifts 8000 N?

Show answer

200 N

6. Medium How tall would a water barometer be at sea level?

Show answer

$h = 101{,}300/(1000 \times 9.8) = 10.3\ \mathrm{m}$

7. Medium The pressure at the bottom of a swimming pool is 30 kPa gauge. How deep is the pool?

Show answer

3.1 m

8. Medium A water tower stands 40 m above a town. What gauge pressure does it give at a faucet at ground level? Why do engineers put water towers on hills?

Show answer

$\rho g h = 392$ kPa (about 57 psi). Height is pressure: a tank on a hill supplies more pressure without pumps.

9. Hard Atmospheric pressure acts on a $2.0\ \mathrm{m} \times 1.0\ \mathrm{m}$ table top. What force is that? Why does the table not collapse?

Show answer

2.0×10^5 N (the weight of about 20 tonnes). Air pushes up on the underside almost exactly as hard, so the net force is nearly zero.

10. Hard A diver descends to 30 m in seawater ($\rho = 1025\ \mathrm{kg/m^3}$). Find the absolute pressure. By what factor is a balloon of air she carries down squeezed (Boyle's law: PV constant)?

Show answer

$P = 101 + 301 = 403$ kPa, about 4 atm; the balloon's volume shrinks to about one quarter.

11. **Challenge** Show that a liquid in a U-tube with one side open and the other connected to a gas tank measures the tank's gauge pressure as $\rho g \Delta h$, where Δh is the difference in liquid levels. If water stands 12 cm higher on the open side, find the gauge pressure.

Show answer

At the same level in the liquid the pressures are equal: $P_{\text{gas}} = P_{\text{atm}} + \rho g \Delta h$. Gauge pressure $= 1000(9.8)(0.12) = 1180\ \text{Pa}$.

COMMON MISTAKES

- Using diameters instead of areas in hydraulic problems. Area grows as diameter squared.
- Forgetting atmospheric pressure when absolute pressure is asked for (or adding it when gauge pressure is asked for).
- Thinking pressure depends on the shape or width of a container. At a given depth it depends only on depth and density.

Make it a game · Pressure Pop Quiz. Your parent calls a scenario: a person standing versus on snowshoes, a thumbtack, a scuba diver at 10 m. Give the pressure in pascals or compare with atmospheres in 30 seconds. Also distinguish gauge from absolute. Parent slips in one wrong fact; catch it for a bonus. Seven correct calls beats the high mark.

8.2 Buoyancy: Archimedes' Principle

OBJECTIVES

- State Archimedes' principle: the buoyant force equals the weight of the fluid displaced.
- Explain why an object sinks, floats or hovers by comparing densities.
- Calculate buoyant forces, apparent weights and the fraction of a floating object below the surface.
- Measure the density of objects, including a "crown," by weighing them in water.

> **Warm-up.** Put a lump of modeling clay in a bowl of water: it sinks. Now shape the same clay into a little boat and set it on the water. Why does the same clay float now? Add coins one at a time until it sinks, and count them. Write an explanation using the word "displace."

WATCH

Archimedes Principle, Buoyant Force, Basic Introduction - Buoyancy & Density - Fluid Statics

The Organic Chemistry Tutor · 15:19

https://www.youtube.com/watch?v=Qgl_l0H7Qgc

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Buoyant force | AP Physics | Khan Academy

Khan Academy · 12:41

https://www.youtube.com/watch?v=NYWG5eXRBFQ

Optional, for a second explanation of buoyant force from the pressure difference.

KEY TERMS

buoyant force

The upward force a fluid exerts on an object in it, caused by the greater pressure at the object's bottom than at its top.

Archimedes' principle

The buoyant force on an object equals the weight of the fluid it displaces: $F_b = \rho_{\text{fluid}} V_{\text{displaced}} g$.

displacement

The volume of fluid pushed aside by an object, equal to the volume of the part of the object below the surface.

apparent weight

An object's weight minus the buoyant force on it; what a scale reads when the object hangs in a fluid.

specific gravity

The ratio of a substance's density to that of water; gold is 19.3, iron 7.9, ice 0.92.

CORE IDEAS

The story. The Roman architect Vitruvius, writing two centuries after Archimedes, tells the most famous story in science. King Hiero of Syracuse had given a goldsmith a weighed amount of gold to make a crown. The crown weighed the right amount, but the king suspected that some gold had been replaced with silver. He asked Archimedes to find out without damaging the crown. Archimedes, stepping into a full bath, noticed the water overflowing, and realized that a body pushes aside its own volume of water. He is said to have run home crying "Eureka!" ("I have found it!"). A crown partly of silver would be bulkier than the same weight of pure gold, and would displace more water.

How they figured it out. In 1586 the young Galileo pointed out that the overflow method would hardly work: the difference in volume would be a fraction of a milliliter, too small to see in spilled water. He proposed what he thought Archimedes really did: hang the crown and an equal weight of pure gold from the two ends of a balance, and then lower both into water. The bulkier crown is buoyed up more, so the balance tips toward the gold. Whether or not that is what Archimedes did, his book *On Floating Bodies* states the principle exactly: a body immersed in a fluid is buoyed up by a force equal to the weight of the fluid it displaces.

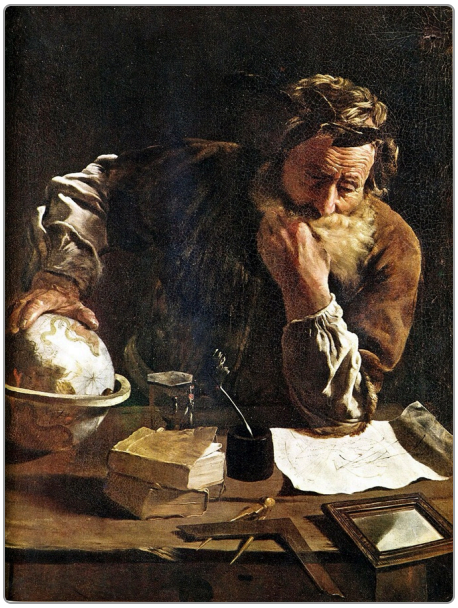
The physics. The pressure in a fluid is greater at the bottom of an object than at its top, so the fluid pushes up harder than it pushes down: the net upward push is the **buoyant force**, $F_b = \rho_{\text{fluid}} V_{\text{sub}} g$, exactly the weight of the displaced fluid. An object denser than the fluid sinks (its weight exceeds the buoyant force even when fully submerged); an object less dense floats, sinking only until it has displaced its own weight of fluid. The fraction submerged is $\rho_{\text{object}}/\rho_{\text{fluid}}$: ice ($917\ \text{kg/m}^3$) in seawater (

1025 $\mathrm{kg/m^3}$) floats about 89% under. A hanging object's apparent weight is its weight minus F_b, which lets you find its volume and density with a simple scale.

The engineering. A steel ship floats because its hull encloses so much air that its average density is less than water's; it sinks until it displaces water equal to its weight, which is why ship sizes are given as displacement in tonnes. Load lines painted on hulls (the Plimsoll line) show how deep a ship may safely ride in different waters, since warm fresh water is less dense than cold seawater. Submarines flood or empty ballast tanks to sink or rise, and hot-air and helium balloons float in the "ocean of air" by the same principle.

Try it. In the lab you will measure buoyant forces with a kitchen scale, find the densities of metal objects, and test a "crown" the way Galileo suggested.

PICTURE STUDY



Archimedes Thoughtful, Domenico Fetti, c. 1620 · Oil on canvas · Gemäldegalerie Alte Meister, Dresden

Image: Wikimedia Commons, public domain

The Italian painter Domenico Fetti imagined Archimedes not in his bath but at his desk, thinking. Mathematicians' tools surround him.

Look closely

- Find the globe. What might Archimedes have been studying?
- Find the dividers and the papers with drawings.
- Look at his face and hands. How has the painter shown concentration?

Talk about it

1. Archimedes proved his principle with geometry, not just by experiment. Why would a mathematical proof be convincing in a way a single experiment is not?
2. Which measurement would you make to test whether a crown is pure gold?
3. Why is a "Eureka" moment usually the result of long thinking, as this painting suggests?

LAB

Weighing in Water: Buoyancy and the Crown Test

You will measure buoyant forces with a kitchen scale, use them to find the volumes and densities of solid objects, and test a "crown" against pure metal, as Galileo suggested.

You will need

- Kitchen scale (to 1 g)
- A container of water that fits on the scale
- Thread, and a stand or ruler to hang objects from
- Objects: a steel bolt, a stack of identical coins, an aluminum object, a rock, a lump of clay, a piece of wood
- A "crown": a ring or pendant, or a disguised object made by your parent from two metals
- Salt

Steps

1. Weigh each object dry (mass m).
2. Put the container of water on the scale and press tare (zero). Hang an object by its thread so it is fully under water but not touching the sides or bottom. The scale now reads the mass of water displaced, m_w (because the water pushes up on the object with F_b and the object pushes down on the water just as hard).
3. **Hypothesis.** Predict which of your objects has the greatest density.
4. For each object: volume $V = m_w/(1.00\ \mathrm{g/cm^3})$, density $\rho = m/V$. Compare with accepted values.
5. **Crown test.** Measure the "crown's" density the same way. Compare it with the density of pure gold, silver, copper or steel. What could it be made of?
6. **Galileo's balance.** Hang the crown and an equal mass of a known metal from the ends of your meter-stick balance (Lesson 7.1), balance them in air, then lower both into water. Which way does it tip, and why?

7. **Floating.** Float the wood in water and measure what fraction is submerged. Compare with $\rho_{\text{wood}}/\rho_{\text{water}}$. Then dissolve salt in the water and float it again.
8. **Improve.** Small objects give small, uncertain readings; weigh several identical coins together, or use a narrower container.

Tips

- Shake off air bubbles; they add buoyancy.
- Make sure the thread holds all the object's weight and the object does not touch the container.

Check your work

- ☐ I measured volumes and densities by weighing in water.
- ☐ I can explain Archimedes' principle from the pressure difference between top and bottom.
- ☐ I can predict whether an object floats and how much of it is submerged.

LAB NOTEBOOK

1. Data table: object, dry mass, mass of water displaced, volume, density, accepted density, percent error.
2. Free-body diagrams of an object hanging in water (tension, weight, buoyant force) and of a floating object.
3. Explain in a paragraph how a steel ship floats while a steel nail sinks.

KEY FORMULAS

$$F_b = \rho_{\text{fluid}} V_{\text{sub}} g$$

$$W_{\text{apparent}} = W - F_b$$

$$\frac{V_{\text{sub}}}{V} = \frac{\rho_{\text{object}}}{\rho_{\text{fluid}}} \quad \text{(floating)}$$

$$\rho_{\text{object}} = \rho_{\text{water}} \frac{W}{W - W_{\text{apparent}}}$$

From history. Archimedes' *On Floating Bodies* also worked out when a floating shape is stable, the beginning of naval architecture. In 1906 a long-lost copy of several of his works, including *On Floating Bodies*, was found beneath the text of a medieval prayer book; modern imaging in the 2000s recovered most of it.

Example 1. A "gold" crown has a mass of 1.000 kg. Hung from a scale in water it reads 0.940 kg. Find its volume and density. Is it pure gold ($19.3\ \mathrm{g/cm^3}$)?

Show solution

1. The buoyant force corresponds to $1.000 - 0.940 = 0.060$ kg of water, so the crown displaces $60\ \mathrm{cm^3}$ (60 g of water).
2. Density: $\rho = \dfrac{1000\ \mathrm{g}}{60\ \mathrm{cm^3}} = 16.7\ \mathrm{g/cm^3}$.
3. Pure gold of that mass would have volume $1000/19.3 = 51.8\ \mathrm{cm^3}$ and would read 0.948 kg in water.

Answer: $60\ \mathrm{cm^3}$; $16.7\ \mathrm{g/cm^3}$. Less dense than gold, so it is not pure: some lighter metal has been mixed in.

Example 2. What fraction of an iceberg ($917\ \mathrm{kg/m^3}$) floats below the surface of seawater ($1025\ \mathrm{kg/m^3}$)?

Show solution

1. Floating: weight = buoyant force, so $\rho_{\mathrm{ice}} V g = \rho_{\mathrm{sea}} V_{\mathrm{sub}} g$.
2. $\dfrac{V_{\mathrm{sub}}}{V} = \dfrac{917}{1025} = 0.895$.

Answer: About 89.5% below; only about a tenth shows.

Example 3. A ship has a mass of 1.00×10^7 kg (10,000 tonnes). What volume of seawater does it displace? How much more does it displace when loaded with 2.0×10^6 kg of cargo?

Show solution

1. $V = \dfrac{m}{\rho} = \dfrac{1.00 \times 10^7\ \mathrm{kg}}{1025\ \mathrm{kg/m^3}} = 9760\ \mathrm{m^3}$.
2. Extra: $\dfrac{2.0 \times 10^6}{1025}\ \mathrm{m^3} = 1950\ \mathrm{m^3}$ more.

Answer: $9760\ \mathrm{m^3}$ empty; $1950\ \mathrm{m^3}$ more when loaded, so it sinks deeper in the water.

Example 4. A helium balloon has a volume of $0.010\ \mathrm{m}^3$ (10 L). Air has density $1.20\ \mathrm{kg/m}^3$ and helium $0.18\ \mathrm{kg/m}^3$. Ignoring the rubber, what net upward force does it have?

Show solution

1. Buoyant force: $\rho_{\text{air}} V g = (1.20)(0.010)(9.8)\ \mathrm{N} = 0.118\ \mathrm{N}$.
2. Helium's weight: $(0.18)(0.010)(9.8)\ \mathrm{N} = 0.018\ \mathrm{N}$.
3. Net: $0.100\ \mathrm{N}$, enough to lift about 10 g.

Answer: About $0.10\ \mathrm{N}$ upward, so the balloon can carry about 10 g of rubber and string.

PRACTICE

1. Easy An object displaces $200\ \mathrm{cm}^3$ of water. What is the buoyant force on it?

   Show answer

   $0.200\ \mathrm{kg} \times 9.8 = 1.96\ \mathrm{N}$

2. Easy Will an object with density $0.6\ \mathrm{g/cm}^3$ float in water? What fraction will be submerged?

   Show answer

   Yes; 60% submerged.

3. Easy Why is it easier to float in the Dead Sea than in a swimming pool?

   Show answer

   The very salty water is denser, so a smaller submerged volume displaces your weight.

4. Medium A rock weighs 30 N in air and 18 N in water. Find its volume and density.

   Show answer

   $F_b = 12\ \mathrm{N}$, $V = 1.22 \times 10^{-3}\ \mathrm{m}^3$; $\rho = 30/(9.8 \times 1.22 \times 10^{-3}) = 2500\ \mathrm{kg/m}^3$

5. Medium A block of wood $0.20\ \mathrm{m} \times 0.10\ \mathrm{m} \times 0.10\ \mathrm{m}$ floats with $0.06\ \mathrm{m}$ of its height submerged. Find its density.

   Show answer

   $600\ \mathrm{kg/m}^3$

6. Medium An aluminum block ($2700\ \mathrm{kg/m^3}$) of volume $100\ \mathrm{cm^3}$ hangs from a spring scale in water. What does the scale read?

Show answer

$W = 2.65\ \mathrm{N}$; $F_b = 0.98\ \mathrm{N}$; reading $1.67\ \mathrm{N}$

7. Medium A barge with a flat bottom $20\ \mathrm{m} \times 8.0\ \mathrm{m}$ floats in fresh water. How much deeper does it sink when 40,000 kg of cargo is loaded?

Show answer

$V = 40\ \mathrm{m^3}$; depth $= 40/160 = 0.25\ \mathrm{m}$

8. Medium Galileo said the overflow method could not detect silver in a crown. Explain why, with an estimate for a 1 kg crown with 10% silver ($10.5\ \mathrm{g/cm^3}$).

Show answer

Pure gold: 51.8 $\mathrm{cm^3}$. With 100 g silver and 900 g gold: 9.5 + 46.6 = 56.1 $\mathrm{cm^3}$, only about 4 $\mathrm{cm^3}$ more, a thin film of spilled water from a bowl, too small to measure reliably by overflow. Weighing in water detects the difference easily.

9. Hard A submarine has a volume of $3000\ \mathrm{m^3}$ and a mass of 2.9×10^6 kg with empty ballast tanks. What mass of seawater must it take on to hover fully submerged?

Show answer

Displaced water: $3000 \times 1025 = 3.075 \times 10^6$ kg; take on 1.75×10^5 kg.

10. Hard A 1.0 kg hollow steel sphere just barely floats, fully submerged, in fresh water. What is its outer volume? If steel has density $7900\ \mathrm{kg/m^3}$, what volume of steel does it contain and how much is empty space?

Show answer

Outer volume $1.0 \times 10^{-3}\ \mathrm{m^3}$ (1.0 L); steel $1.27 \times 10^{-4}\ \mathrm{m^3}$; empty $8.7 \times 10^{-4}\ \mathrm{m^3}$.

11. Challenge Show, using pressures, that the buoyant force on a submerged box of height h and top area A is $\rho_{\text{fluid}} g A h$, whatever the depth of the box.

Show answer

Top pressure $P_0 + \rho g d$, bottom pressure $P_0 + \rho g(d + h)$. Net upward force $= A[\rho g(d + h) - \rho g d] = \rho g A h = \rho g V$, the weight of the displaced fluid, independent of d.

COMMON MISTAKES

- Using the object's density instead of the fluid's in $F_b = \rho V g$.
- Thinking heavy objects always sink. Floating depends on average density, not weight.
- Using the whole volume of a floating object for V. Only the submerged part displaces fluid.

Make it a game · Sink Float Hunt. Gather ten small household objects and a bowl of water. Predict sink or float for each, with a one-sentence density reason, before placing it in. Then compute the specific gravity of one object by weighing in water. Score one point per correct prediction, and aim for ten out of ten.

Scripture. "They that go down to the sea in ships, that do business in great waters; These see the works of the LORD, and his wonders in the deep." (Psalm 107:23-24, KJV)

Every ship that floats depends on the law Archimedes found. Sailors have long seen in the sea the works and wonders of the Lord.

8.3 Moving Fluids: Bernoulli, Torricelli and the Wing

OBJECTIVES

- Use the continuity equation, $A_1 v_1 = A_2 v_2$, for flowing liquids.
- State Bernoulli's equation and explain why pressure is lower where a fluid moves faster.
- Use Torricelli's law, $v = \sqrt{2gh}$, for liquid flowing out of a tank.
- Explain correctly how a wing produces lift, and measure the speed of a water jet.

> **Warm-up.** Put your thumb partly over the end of a running garden hose (outdoors) or a faucet. What happens to the speed of the water? Then hold two sheets of paper hanging a few centimeters apart in front of your face and blow between them. Predict whether they will move apart or together before you try.

WATCH

Understanding Bernoulli's Equation

The Efficient Engineer · 13:43

https://www.youtube.com/watch?v=DW4rItB20h4

> **WHEN TO WATCH**
> After the physics paragraph, before the worked examples.

BACKUP VIDEO

Understanding Aerodynamic Lift

The Efficient Engineer · 14:19

https://www.youtube.com/watch?v=E3i_XHlVCeU

With the paragraph on lift. The video explains both the pressure difference and the downwash, and why the "equal transit time" story is wrong.

KEY TERMS

continuity equation

For an incompressible fluid in a pipe, the volume flow rate is the same everywhere: $A_1 v_1 = A_2 v_2$.

Bernoulli's equation

Along a streamline in steady flow, $P + \frac{1}{2}\rho v^2 + \rho g h$ is constant: where a fluid speeds up, its pressure drops.

Torricelli's law

Liquid flows out of a hole a depth h below the surface at speed $v = \sqrt{2gh}$, the same as if it had fallen freely through h.

lift

The upward force on a wing. The wing turns the air flowing past it downward, and by Newton's third law the air pushes the wing up; this shows up as lower pressure above the wing than below.

Venturi tube

A pipe with a narrow throat, where the fluid moves faster and its pressure is lower; used to measure flow rates.

CORE IDEAS

The story. Daniel Bernoulli came from a family of Swiss mathematicians and was trained as a physician. In 1738 he published *Hydrodynamica*, the first great book on moving fluids. He had studied the flow of water through pipes and found that where a pipe narrows and the water speeds up, the pressure on the walls is lower. To measure it, he poked small open glass tubes into the side of a flowing pipe and watched how high the water rose in each; the faster the flow, the lower the column. Doctors adopted his method to measure blood pressure, sticking a glass tube directly into an artery, for more than a century. A hundred years earlier, Torricelli had found that water spurts from a hole in a tank as fast as if it had fallen from the water's surface.

How they figured it out. Bernoulli reasoned with energy, as Huygens and Leibniz had for falling bodies: a parcel of water speeds up only if it is pushed from behind harder than from in front, so it must be moving from higher pressure to lower. His friend Leonhard Euler then wrote the equations of fluid motion in 1752–1757, from which the modern form of "Bernoulli's equation" follows: along a streamline, $P + \frac{1}{2}\rho v^2 + \rho g h$ stays the same. In 1732 the French engineer Henri Pitot had already invented the tube, now called a pitot tube, that measures the speed of a river (and today of an airplane) from the pressure of flow pushing into it.

The physics. For a liquid in a pipe, what flows in must flow out: $A_1 v_1 = A_2 v_2$, so the fluid speeds up where the pipe narrows. **Bernoulli's equation** is conservation of energy per unit volume: pressure energy P, kinetic energy $\frac{1}{2}\rho v^2$ and potential

energy $\rho g h$. Where speed rises, pressure falls. **Torricelli's law** follows directly: the pressure at the surface and at the hole are both atmospheric, so $\rho g h = \frac{1}{2}\rho v^2$ and $v = \sqrt{2gh}$. A wing works because its shape and its angle to the oncoming air turn the airflow downward; by Newton's third law the air pushes the wing up. Turning the flow requires the pressure above the wing to be lower than the pressure below, and the air above does move faster. But it is *not* true that air over the top must travel farther and "meet up" with the air below at the trailing edge; in fact the upper air arrives well before the lower air. The "equal transit time" explanation is a myth.

The engineering. Every airliner has pitot tubes to measure airspeed. Venturi meters measure water flow in city mains; carburetors used the Venturi effect to draw fuel into an engine's air stream; chimneys draw better when wind blows across their tops. Hydraulic engineers use Bernoulli's equation, with corrections for friction, to size pipes and pumps for buildings and towns. And aircraft designers calculate lift from $L = \frac{1}{2}\rho v^2 S C_L$, testing wing shapes in wind tunnels as the Wright brothers did in 1901.

Try it. In the lab you will measure the speed of water jets from a bottle and from a squeeze bottle or toy water blaster, and test Torricelli's law.

PICTURE STUDY

First Flight, John T. Daniels, December 17, 1903 · Glass-plate photograph · Library of Congress
Image: Wikimedia Commons, public domain

The Wrights designed their wings from data they gathered in their own wind tunnel in 1901, measuring the lift of about two hundred shapes. This photograph shows the result at the moment of takeoff.

Look closely

- Find the curved (cambered) wings. Look at their front edges and how they slope toward the back.
- Find the elevator at the front. What does it do?
- Find the rail and Wilbur running at the wing tip.

Talk about it

1. How do the wings turn the oncoming air downward, and why does that produce lift?
2. Why did the Wrights take off into a strong headwind?
3. Why did they need their own wind-tunnel data instead of the published tables?

Safety first. Do the jet experiments outdoors or in a tub. Aim water blasters only at targets, never at people's faces.

LAB

Torricelli's Jets and the Water Blaster

You will test Torricelli's law by measuring how far water spurts from a hole in a bottle at different water depths, and then use projectile motion to measure the jet speed of a squeeze bottle or a toy water blaster.

You will need

- A tall plastic bottle (2 L), a pushpin or small nail, tape
- A tray or tub, a meter stick, a marker
- A squeeze bottle or a toy water blaster (Super Soaker or similar)
- Phone with slow-motion video; a partner

Steps

1. Make one small, round hole near the bottom of the bottle. Cover it with tape and fill the bottle. Mark the water level every 2 cm above the hole.
2. Set the bottle on a block so the hole is a measured height H above the tray.
3. **Hypothesis.** If the water depth above the hole is cut to one quarter, predict what happens to the jet's speed and to its landing distance.
4. Remove the tape. As the water level passes each mark, have your partner mark where the jet lands. (Filming makes this easier.)
5. For each depth h, the predicted speed is $v = \sqrt{2gh}$ and, using projectile motion, the landing distance is $x = v\sqrt{2H/g} = 2\sqrt{hH}$. Compare measured and predicted distances.
6. Graph landing distance against $\sqrt{h}$. Is it a straight line? Compare its slope with $2\sqrt{H}$.
7. **Water blaster.** Fire the blaster horizontally from a measured height (pump it the same number of times each trial). Measure where the jet lands and calculate its speed from projectile motion. Then calculate the pressure it needed, $\frac{1}{2}\rho v^2$.
8. **Fair test.** Change the number of pumps (5, 10, 15) and find the jet speed and pressure for each.

9. **Improve.** Real jets land a bit short of the prediction. Estimate the fraction of the ideal speed you actually get (the "discharge coefficient"), and explain where the lost energy goes.

Tips

- A small, clean, round hole gives a smooth jet.
- Measure h from the hole to the water surface at the moment the jet lands at each mark.

Check your work

☐ My measured landing distances follow

$x =$closely.
$2\sqrt{hH}$

☐ I calculated a water blaster's jet speed and pressure from projectile motion and Bernoulli's equation.

☐ I can explain lift correctly and say what is wrong with the "equal transit time" explanation.

LAB NOTEBOOK

1. Data table and the graph of landing distance against $\sqrt{h}$ with the predicted line.
2. Sketch air flowing past a tilted wing, with streamlines turning downward behind it, low pressure above and high pressure below, and the lift and downwash forces.
3. Draw a Venturi tube with three small vertical tubes, showing how the water levels in them reveal the pressure at each point.

KEY FORMULAS

$$A_1 v_1 = A_2 v_2$$

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g h_2$$

$$v = \sqrt{2gh} \quad \text{(Torricelli)}$$

$$\Delta P = \tfrac{1}{2}\rho v^2 \quad \text{(pressure needed for jet speed } v\text{)}$$

From history. Daniel Bernoulli also founded the kinetic theory of gases in *Hydrodynamica*, explaining gas pressure as the impacts of countless moving particles. The Wright brothers' 1901 wind tunnel, a wooden box 6 feet long with a fan driven by their shop's engine, produced the first reliable tables of wing lift and drag.

Example 1. Water moves at $1.5\ \mathrm{m/s}$ in a garden hose $2.0\ \mathrm{cm}$ in diameter. How fast does it leave a nozzle $0.50\ \mathrm{cm}$ across?

Show solution

1. $A_1 v_1 = A_2 v_2$, and area is proportional to diameter squared.
2. $v_2 = v_1 \left(\dfrac{d_1}{d_2}\right)^2 = (1.5\ \mathrm{m/s}) \left(\dfrac{2.0}{0.50}\right)^2 = (1.5)(16)\ \mathrm{m/s} = 24\ \mathrm{m/s}.$

Answer: $24\ \mathrm{m/s}$, sixteen times faster.

Example 2. A hole is punched $0.20\ \mathrm{m}$ below the water surface in a tall bottle. How fast does water leave the hole?

Show solution

1. Torricelli: $v = \sqrt{2gh} = \sqrt{2(9.8\ \mathrm{m/s^2})(0.20\ \mathrm{m})} = 1.98\ \mathrm{m/s}.$

Answer: About $2.0\ \mathrm{m/s}$.

Example 3. A toy water blaster shoots a horizontal jet at $12\ \mathrm{m/s}$. What gauge pressure must the air chamber have, ignoring friction in the nozzle?

Show solution

1. Bernoulli from the chamber (speed ≈ 0, pressure $P_0 + \Delta P$) to the jet (speed v, pressure P_0): $\Delta P = \frac{1}{2}\rho v^2$.
2. $\Delta P = \frac{1}{2}(1000\ \mathrm{kg/m^3})(12\ \mathrm{m/s})^2 = 72{,}000\ \mathrm{Pa} = 72\ \mathrm{kPa}$, about 10 psi.

Answer: About $72\ \mathrm{kPa}$ (10 psi) above atmospheric; real blasters need somewhat more to overcome friction.

Example 4. A small airplane of mass 1100 kg has a wing area of $16.2\ \mathrm{m}^2$. In level flight, what is the average pressure difference between the lower and upper wing surfaces?

Show solution

1. In level flight, lift equals weight: $L = (1100\ \mathrm{kg})(9.8\ \mathrm{m/s}^2) = 10{,}800\ \mathrm{N}$.
2. $\Delta P = \dfrac{L}{A} = \dfrac{10{,}800\ \mathrm{N}}{16.2\ \mathrm{m}^2} = 665\ \mathrm{Pa}$.

Answer: About 670 Pa, less than 1% of atmospheric pressure, acting over a large area.

PRACTICE

1. Easy Water flows at $2.0\ \mathrm{m/s}$ through a pipe of area $4.0\ \mathrm{cm}^2$ into a pipe of area $1.0\ \mathrm{cm}^2$. Find its new speed.

   Show answer

   $8.0\ \mathrm{m/s}$

2. Easy How fast does water leave a hole $1.25\ \mathrm{m}$ below the surface of a tank?

   Show answer

   $4.95\ \mathrm{m/s}$

3. Easy When you blow between two hanging sheets of paper, they move together. Why?

   Show answer

   The moving air between them has lower pressure than the still air outside, so the outside air pushes them together.

4. Medium Water flows through a horizontal pipe at $2.0\ \mathrm{m/s}$ with pressure 180 kPa. It enters a narrow section where it moves at $6.0\ \mathrm{m/s}$. Find the pressure there.

   Show answer

   $P_2 = 180{,}000 + \frac{1}{2}(1000)(4 - 36) = 164\ \mathrm{kPa}$

5. **Medium** A fire hose nozzle shoots water straight up to 20 m. What was the speed at the nozzle, and what gauge pressure is needed behind it (ignoring friction)?

Show answer

$v = \sqrt{2(9.8)(20)} = 19.8\ \text{m/s}$; $\Delta P = 196\ \text{kPa}$

6. **Medium** A water tank has a hole 0.80 m below the surface and 1.2 m above the ground. How far from the tank does the jet land?

Show answer

$x = 2\sqrt{(0.80)(1.2)} = 1.96\ \text{m}$

7. **Medium** Explain, without the equal-transit-time idea, why an airplane wing produces lift.

Show answer

The wing's shape and angle of attack turn the oncoming air downward. To turn the flow, the air must push on the wing: by Newton's third law, deflecting air down pushes the wing up. The curved flow requires lower pressure above the wing than below, and that pressure difference over the wing area is the lift.

8. **Medium** A shower curtain blows inward when the shower runs. Give a Bernoulli explanation.

Show answer

The moving water and air inside the shower have lower pressure than the still air outside, so the outside air pushes the curtain in. (Warm rising air also contributes.)

9. **Hard** A Venturi meter in a water main has a pipe area of $0.020\ \text{m}^2$ and a throat area of $0.010\ \text{m}^2$. The pressure in the throat is 15 kPa lower than in the pipe. Find the flow speed in the pipe and the flow rate.

Show answer

$v_2 = 2v_1$; $15{,}000 = \frac{1}{2}(1000)(4v_1^2 - v_1^2)$, so $v_1 = 3.16\ \text{m/s}$; flow rate $= 0.063\ \text{m}^3/\text{s}$ (63 L/s).

10. Hard An airliner's pitot tube measures a pressure difference of 4500 Pa between the air brought to rest in the tube and the still-air (static) pressure. At high altitude the air density is $0.41\ \mathrm{kg/m^3}$. Find the airspeed.

Show answer

$$v = \sqrt{2(4500)/0.41} = 148\ \mathrm{m/s}$$

11. Challenge Show that Torricelli's law follows from Bernoulli's equation for a large open tank with a small hole. Why can the speed of the water surface be taken as zero?

Show answer

Between the surface (pressure P_0, height h, speed ≈ 0) and the hole (pressure P_0, height 0, speed v): $P_0 + \rho g h = P_0 + \frac{1}{2}\rho v^2$, so $v = \sqrt{2gh}$. By continuity the surface moves at $vA_{\text{hole}}/A_{\text{tank}}$, which is tiny when the tank is much wider than the hole.

COMMON MISTAKES

- Teaching that air over a wing must meet air from under the wing at the trailing edge. It does not; this "equal transit time" idea is false.
- Applying Bernoulli's equation between points not on the same streamline, or across a pump or fan that adds energy.
- Using diameters directly in the continuity equation; areas go as diameter squared.

Make it a game · Flow Speed Estimator. Your parent describes a hose narrowing from 2 cm diameter to 1 cm. In 20 seconds, estimate the speed ratio using continuity, then say where pressure is lower and why. Add a Torricelli question: how fast does water leave a hole 0.5 m below the surface? Score a point per correct prediction.

Scripture. "The way of an eagle in the air" (Proverbs 30:19, KJV)

Agur counted the way of an eagle in the air among the things too wonderful for him. Bernoulli, Euler and the Wrights spent their lives learning a little of how it works.

8.4 Temperature and Heat: Joseph Black and the Calorimeter

OBJECTIVES

- Distinguish temperature from heat and from thermal energy, and convert between Celsius, Fahrenheit and kelvin.
- Use $Q = mc\Delta T$ and latent heat, $Q = mL$, in heating and phase-change problems.
- Solve calorimetry problems by setting heat lost equal to heat gained.
- Measure the specific heat of a metal by calorimetry.

> **Warm-up.** Put a thermometer (or your finger) in a glass of ice water and stir. Keep checking the temperature every minute while the ice melts. Does the temperature rise while ice remains? Write down what you see and a guess at where the heat from the room is going.

WATCH

Calorimetry Problems, Thermochemistry Practice, Specific Heat Capacity, Enthalpy Fusion, Chemistry

The Organic Chemistry Tutor · 27:36

https://www.youtube.com/watch?v=HlvllF6Ml9c

> **WHEN TO WATCH**
> After the physics paragraph. Use it for the calorimetry worked examples.

BACKUP VIDEO

Specific Heat Capacity | Matter | Physics | FuseSchool

FuseSchool - Global Education · 3:13

https://www.youtube.com/watch?v=TqJFIBODrjM

Before the lab, for a short review of specific heat capacity.

KEY TERMS

temperature

A measure of how hot something is, related to the average kinetic energy of its molecules; measured in °C, °F or kelvin (K).

heat

Energy transferred because of a temperature difference, from hotter to colder.

specific heat capacity

The energy to raise 1 kg of a substance by 1 K: $Q = mc\Delta T$. Water: 4186 J/(kg·K); aluminum 900; steel 450; copper 385.

latent heat

The energy absorbed or released per kilogram during a change of phase at constant temperature: $Q = mL$. For water, $L_f = 334$ kJ/kg (melting) and $L_v = 2260$ kJ/kg (boiling).

calorimeter

An insulated container in which heat exchanges are measured; in it, heat lost by hot objects equals heat gained by cold ones.

CORE IDEAS

The story. In the 1700s "heat" and "temperature" meant the same thing to most people. Joseph Black, a physician and chemistry professor at Glasgow and later Edinburgh, separated them. Around 1761 he noticed that ice in a warm room melts slowly, and that while it melts, its temperature stays at the freezing point; yet it must be taking in heat all the time. He measured it: melting a pound of ice took as much heat as would warm a pound of water by about 140 °F. That heat, which went into the ice without raising its temperature, he called *latent*, or hidden, heat. He found the same for boiling water. He also showed that equal masses of different substances need different amounts of heat to warm by the same amount, the idea of specific heat.

How they figured it out. Black's method was careful comparison: the same heater, the same masses, different substances, and a thermometer read over time. His friend and lab assistant James Watt used Black's latent heat of steam to understand why the Newcomen engine wasted so much fuel, which led to the separate condenser (Lesson 5.4). In 1782–1783, the French chemist Antoine Lavoisier and the mathematician Pierre-Simon Laplace built an ice calorimeter: a hot object placed in an inner chamber surrounded by ice melts some of the ice, and weighing the meltwater measures the heat it gave off. Marie-Anne Paulze Lavoisier drew the engraved plates that show it.

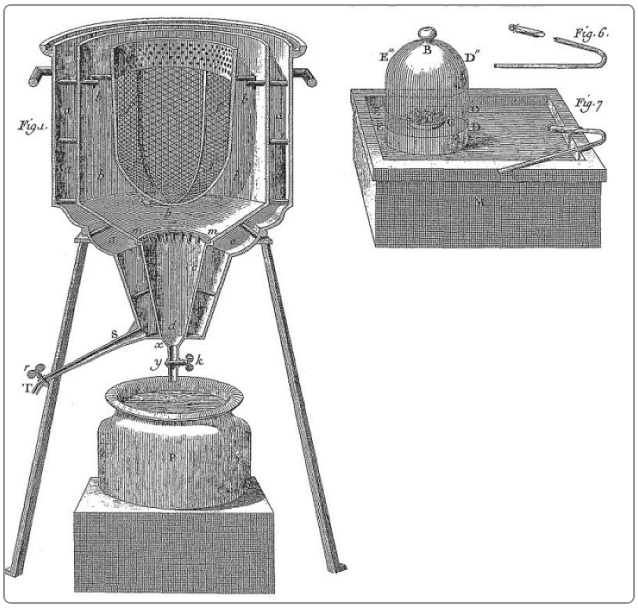
The physics. Temperature tells which way heat will flow; **heat** is the energy that flows. Celsius and kelvin degrees are the same size; $T(\mathrm{K}) = T(^\circ\mathrm{C}) + 273.15$ and

$T(^\circ\mathrm{F}) = \frac{9}{5}T(^\circ\mathrm{C}) + 32$. Warming a mass m by ΔT takes $Q = mc\Delta T$; melting or boiling it takes $Q = mL$ with no change in temperature. In an insulated calorimeter, energy is conserved: the heat lost by the hot objects equals the heat gained by the cold ones, $\sum Q = 0$. Heat moves by conduction (through materials), convection (carried by moving fluid) and radiation (as infrared light).

The engineering. Water's very large specific heat makes it the best common coolant: car engines, power plants and computer servers are cooled by water or water-based fluids. Its large latent heat of vaporization is why sweating cools you and why steam heating carries so much energy through pipes. Ice and refrigeration engineers use latent heat; builders use the "thermal mass" of brick and concrete to even out the temperature of a house between day and night; and electronics engineers fit heat sinks to chips to spread heat by conduction and shed it by convection.

Try it. In the lab you will heat steel washers in boiling water, drop them into cool water in a foam-cup calorimeter, and calculate the specific heat of steel.

PICTURE STUDY



The Ice Calorimeter of Lavoisier and Laplace, Marie-Anne Paulze Lavoisier (engraved plate), 1789 · Engraving · Antoine Lavoisier, *Traité élémentaire de chimie*, Paris

Image: Wikimedia Commons, public domain

The ice calorimeter measured heat with a balance instead of a thermometer: the hotter the object, the more ice it melted. Madame Lavoisier, who trained as an artist to record her husband's experiments, drew the plates.

Look closely

- Find the inner chamber where the hot object is placed.
- Find the two layers of ice. Which one does the measuring, and which one protects it?
- Find the tap and the jar that collect the meltwater.

Talk about it

1. Why is the outer layer of ice needed?
2. If an object melts 100 g of ice, how much heat did it give off? (Use $L_f = 334\ \mathrm{J/g}$.)
3. Why does this method work only when the ice is exactly at its melting point?

Safety first. An adult handles the boiling water and transfers the hot washers with tongs. Use oven mitts. Keep the foam cup on a stable surface.

LAB

The Specific Heat of Steel

You will measure the specific heat of steel washers (or bolts) by the method of mixtures in a foam-cup calorimeter, and then measure the latent heat of melting ice.

You will need

- About 200 g of steel washers or bolts tied on a string
- A pot of boiling water (adult)
- Two nested foam cups with a lid, a digital thermometer reading to 0.1 °C
- Kitchen scale, tongs, oven mitts
- Ice cubes and paper towels

Steps

1. Weigh the washers. Put about 200 g of cool tap water in the nested foam cups and weigh it; record its temperature.
2. **Hypothesis.** Predict the final temperature, using the accepted $c_{\text{steel}} = 450\ \text{J}/(\text{kg}\cdot\text{K})$.
3. Lower the washers into boiling water for 5 minutes so they reach 100 °C (adjust for altitude if needed).
4. Quickly lift them out, shake off the drops, and drop them into the cup. Put on the lid, stir gently with the thermometer, and record the highest steady temperature.
5. Calculate $c_{\text{steel}} = \dfrac{m_w c_w (T_f - T_w)}{m_s (100 - T_f)}$. Find the percent error.
6. Repeat twice. Then try another metal (aluminum or copper objects) if you have them.
7. **Latent heat.** Put 200 g of warm water (about 40 °C) in the cup. Add two or three ice cubes, patted dry and weighed. Stir until they melt and record the lowest temperature. Calculate L_f from heat lost by the warm water = heat to melt the ice + heat to warm the meltwater.

8. **Improve.** Identify where heat leaks (water carried on the washers, the cup itself absorbing heat, heat lost while transferring). Change one thing and repeat.

Tips

- Work fast in the transfer; every second the washers cool.
- The cup absorbs a little heat too; a lid and two nested cups make it negligible.

Check your work

- ☐ I measured the specific heat of steel within about 15% of the accepted value.
- ☐ I can solve a calorimetry problem by setting heat lost equal to heat gained.
- ☐ I can explain latent heat and why ice water stays at 0 °C while the ice melts.

LAB NOTEBOOK

1. Data table and calculation for each trial; percent errors.
2. A temperature-against-time sketch for ice heated from −10 °C to steam at 110 °C, with the flat parts labeled “melting” and “boiling.”
3. Paragraph: why is water used to cool car engines and power plants? Use numbers.

KEY FORMULAS

$$Q = mc\Delta T$$

$$Q = mL$$

$$\sum Q = 0 \quad \text{(calorimeter)}$$

$$T_K = T_C + 273.15, \qquad T_F = \tfrac{9}{5}T_C + 32$$

From history. Black also discovered carbon dioxide, which he called “fixed air,” in the 1750s. The Fahrenheit scale (1724) set 32° for freezing water and about 96° for body temperature; Anders Celsius’s scale (1742) originally put 100 at freezing and 0 at boiling, and was turned around soon after his death. Lavoisier died in 1794; his wife saved and published his last papers.

Example 1. How much energy does it take to heat 1.5 kg of water from 20°C to 100°C? How much more to boil it all away?

Show solution

1. Heating: $Q_1 = mc\Delta T = (1.5\text{ kg})(4186\text{ J/(kg}\cdot\text{K)})(80\text{ K}) = 5.02\times10^5\text{ J}$.
2. Boiling: $Q_2 = mL_v = (1.5\text{ kg})(2.26\times10^6\text{ J/kg}) = 3.39\times10^6\text{ J}$, almost seven times more.

Answer: 502 kJ to heat, then 3.39 MJ to boil away.

Example 2. 200 g of steel washers at 100°C are dropped into 300 g of water at 20.0°C in an insulated cup. Find the final temperature ($c_{\text{steel}} = 450\text{ J/(kg}\cdot\text{K)}$).

Show solution

1. Heat lost by steel = heat gained by water: $(0.200)(450)(100-T) = (0.300)(4186)(T-20.0)$.
2. $90(100-T) = 1256(T-20.0)$, so $9000 + 25{,}120 = 1346T$.
3. $T = \dfrac{34{,}120}{1346} = 25.3°\text{C}$.

Answer: 25.3°C: the water warms only about 5 degrees, because of its large specific heat.

Example 3. How much heat is needed to melt 0.50 kg of ice at 0°C and warm the water to 20°C? Compare with Black's observation.

Show solution

1. Melting: $Q_1 = mL_f = (0.50)(334{,}000)\text{ J} = 1.67\times10^5\text{ J}$.
2. Warming: $Q_2 = (0.50)(4186)(20)\text{ J} = 4.19\times10^4\text{ J}$.
3. Total: 2.09×10^5 J. Melting took four times as much heat as warming the water 20 degrees, and the heat to melt the ice could have warmed the same water by $334{,}000/4186 = 80°\text{C}$ (144°F), close to Black's 140 °F.

Answer: 209 kJ, most of it latent heat.

PRACTICE

1. Easy Convert 25°C to kelvin and to Fahrenheit.

   Show answer

   $298\ \text{K};\ 77^\circ\text{F}$

2. Easy How much heat raises $2.0\ \text{kg}$ of aluminum from 20 to 70°C?

   Show answer

   $2.0(900)(50) = 90\ \text{kJ}$

3. Easy Why does a swimming pool stay cool on a hot day while the concrete around it gets hot?

   Show answer

   Water has a much larger specific heat (and mixes heat downward), so the same sunlight raises its temperature much less.

4. Medium How much energy melts $250\ \text{g}$ of ice at 0°C?

   Show answer

   $83.5\ \text{kJ}$

5. Medium A $0.50\ \text{kg}$ copper block at 90°C is dropped into $1.0\ \text{kg}$ of water at 20°C. Find the final temperature ($c_{\text{Cu}} = 385$).

   Show answer

   $192.5(90 - T) = 4186(T - 20);\ T = 23.1^\circ\text{C}$

6. Medium In the lab, $200\ \text{g}$ of washers at 100°C warm $200\ \text{g}$ of water from 20.0 to 27.8°C. Find the specific heat of the washers.

   Show answer

   $c = (0.200)(4186)(7.8)/(0.200 \times 72.2) = 452\ \text{J/(kg}\cdot\text{K)}$

7. Medium Why does sweating cool you, and why does it work less well on a humid day?

   Show answer

   Evaporating sweat absorbs latent heat of vaporization from your skin. In humid air, sweat evaporates slowly, so less heat is carried away.

8. Medium How many kilograms of ice at $0°\text{C}$ must melt to cool $2.0\ \text{kg}$ of lemonade (treat as water) from $30°\text{C}$ to $0°\text{C}$?

Show answer

$$m = 2.0(4186)(30)/334{,}000 = 0.75\ \text{kg}$$

9. Hard A $1200\ \text{kg}$ car brakes from $25\ \text{m/s}$ to rest. Its four iron brake discs (total $36\ \text{kg}$, $c = 450$) absorb 90% of the energy. How much do they warm?

Show answer

$$KE = 375\ \text{kJ};\ Q = 337.5\ \text{kJ};\ \Delta T = 20.8\ \text{K}$$

10. Hard How much energy is needed to turn $1.0\ \text{kg}$ of ice at $-10°\text{C}$ into steam at $100°\text{C}$ ($c_{\text{ice}} = 2100$)?

Show answer

$$21 + 334 + 418.6 + 2260 = 3034\ \text{kJ} \approx 3.0\ \text{MJ}$$

11. Challenge A $50\ \text{g}$ ice cube at $0°\text{C}$ is dropped into $200\ \text{g}$ of water at $25°\text{C}$. Does all the ice melt? Find the final temperature.

Show answer

Heat available to cool the water to 0 °C: $0.200(4186)(25) = 20.9\ \text{kJ}$; to melt all the ice: $0.050(334{,}000) = 16.7\ \text{kJ}$. All melts, leaving $4.2\ \text{kJ}$ to warm $0.250\ \text{kg}$: $T = 4200/(0.25 \times 4186) = 4.0°\text{C}$.

COMMON MISTAKES

- Using temperature and heat interchangeably. A bathtub of warm water holds more thermal energy than a cup of boiling water.
- Forgetting latent heat when a substance changes phase, or using $mc\Delta T$ across a melting point.
- Getting the signs mixed up in calorimetry. Write heat lost = heat gained with each ΔT positive.

> **Make it a game · Heat Estimate Race.** Your parent names a heating job, such as 1 kg of water from 20 to 40 degrees Celsius. Estimate the heat needed with Q = mc delta T before reaching for a calculator. Closest within ten percent earns a point. Add a phase change round with melting ice. Convert one temperature among Celsius, Fahrenheit and kelvin.

8.5 Thermodynamics and Engines: Carnot and Kelvin

OBJECTIVES

- State the first law of thermodynamics, $\Delta U = Q - W$, and apply it to engines.
- Calculate the efficiency of a heat engine and the Carnot limit, $e = 1 - T_C/T_H$, using kelvin temperatures.
- State the second law of thermodynamics and explain why no engine can be 100% efficient.
- Find absolute zero by measuring how the volume of air changes with temperature.

> **Warm-up.** Rub your hands and they warm: work turns into heat easily. Now think of the reverse. Could you cool your hands and make them do work by themselves? Could a hot cup of coffee cool down and lift itself off the table? Write down why you think one direction is easy and the other never happens.

WATCH

Efficiency of a Carnot engine | Thermodynamics | Physics | Khan Academy

Khan Academy · 14:03

https://www.youtube.com/watch?v=M_5KYncYNyc

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

How Absolute Zero Was Discovered

Rational Thinker · 8:41

https://www.youtube.com/watch?v=gwQGXDD1aOU

With the story paragraph: how the idea of absolute zero was discovered.

KEY TERMS

first law of thermodynamics

Energy is conserved: the change in a system's internal energy equals the heat added minus the work done by the system, $\Delta U = Q - W$.

heat engine

A device that takes in heat Q_H from a hot source, does work W, and rejects heat Q_C to a cold sink: $W = Q_H - Q_C$.

efficiency

For a heat engine, $e = W/Q_H$, the fraction of the heat input turned into work.

Carnot efficiency

The greatest possible efficiency of any engine working between T_H and T_C (in kelvins): $e_{\max} = 1 - T_C/T_H$.

second law of thermodynamics

Heat flows by itself only from hotter to colder objects; equivalently, no engine can turn heat entirely into work.

absolute zero

The lowest possible temperature, 0 K or −273.15 °C, where a gas's pressure (at constant volume) would fall to zero.

CORE IDEAS

The story. By the 1820s steam engines powered Britain's mines and mills, but no one knew how good an engine could possibly be. In 1824 a young French army engineer, Sadi Carnot, published a short book, *Reflections on the Motive Power of Fire*, that asked the question in its most general form. He imagined an ideal engine, perfectly reversible, running between a hot and a cold body, and proved that no engine could do better, whatever its working substance. Its efficiency depended only on the two temperatures. The book sold few copies, and Carnot died young in 1832. In 1848 William Thomson, a 24-year-old professor at Glasgow later made Lord Kelvin, read about Carnot's work and saw that it gave a way to define temperature itself, independent of any thermometer liquid.

How they figured it out. Thomson's absolute temperature scale put its zero at the point where an ideal engine would turn all heat into work, which matched the temperature at which the pressure of a gas, cooled at constant volume, would fall to zero: about −273 °C. Experimenters had already found that gases shrink by about 1/273 of their volume at 0 °C for each degree of cooling. Then, with Joule's measurements of the mechanical equivalent of heat (Lesson 5.1), Rudolf Clausius (1850) and Thomson (1851) put Carnot's idea together with energy conservation into

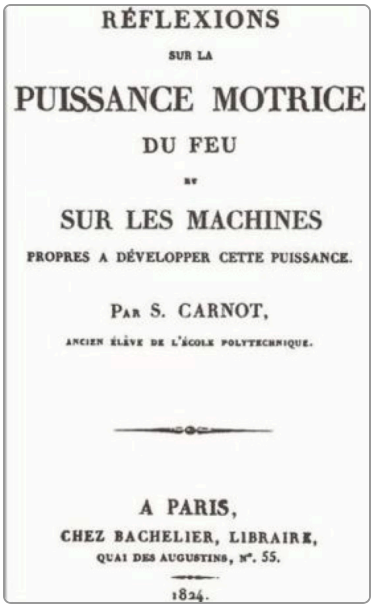
the two laws of thermodynamics. Kelvin, a lifelong Christian, connected this running-down of usable energy with the Bible's picture of the heavens growing old like a garment.

The physics. The **first law**: energy is conserved, $\Delta U = Q - W$. An engine running in a cycle returns to its starting state ($\Delta U = 0$), so the work it does is $W = Q_H - Q_C$, and its efficiency is $e = W/Q_H$. The **second law**: heat flows by itself only from hot to cold, and no engine can turn all its heat input into work; some heat must always be dumped to a cold sink. The best possible efficiency is Carnot's, $e_{\max} = 1 - \dfrac{T_C}{T_H}$, with both temperatures in **kelvins**. A power-plant turbine with steam at 550 °C and a condenser at 30 °C can be at most 63% efficient. Refrigerators and heat pumps run engines backward, using work to move heat from cold to hot.

The engineering. Carnot's formula tells engineers how to improve engines: raise the hot temperature or lower the cold one. Modern steam power plants run at about 33–45% efficiency; combined-cycle gas plants, which use hot exhaust from a gas turbine to make steam for a second turbine, reach about 60%; car engines about 25–35%. The rejected heat goes into cooling towers, rivers or radiators. Engineers improve efficiency with high-temperature alloys and ceramics for turbine blades, and recover "waste" heat for heating buildings. Every kilowatt of electricity from heat comes with roughly two kilowatts of heat that must go somewhere.

Try it. In the lab you will measure the volume of a trapped sample of air at several temperatures and extrapolate your graph back to find absolute zero, as Kelvin's predecessors did.

PICTURE STUDY



RÉFLEXIONS

SUR LA

PUISSANCE MOTRICE

DU FEU

ET

SUR LES MACHINES

PROPRES A DÉVELOPPER CETTE PUISSANCE.

PAR S. CARNOT,

ANCIEN ÉLÈVE DE L'ÉCOLE POLYTECHNIQUE.

A PARIS,

CHEZ BACHELIER, LIBRAIRE,

QUAI DES AUGUSTINS, N°. 55.

1824.

Reflections on the Motive Power of Fire, Sadi Carnot, 1824 · Printed title page · Bachelier, Paris

Image: Wikimedia Commons, public domain

This modest little book, published at the author's own expense, founded the science of thermodynamics. It contains almost no equations; Carnot reasoned in words about an ideal engine.

Look closely

- Find the title: "Reflections on the motive power of fire and on machines fitted to develop that power."
- Find the author's name and how he describes himself.
- Find the place and date.

Talk about it

1. Why might a book about engines be called "motive power of fire"?
2. Carnot proved that an engine's best efficiency depends only on two temperatures. Why is that surprising?
3. Why do you think this book was nearly forgotten for twenty years?

Safety first. An adult handles hot water. Use hot tap water (not boiling) for the hot bath, and keep the syringe tip sealed with a cap or clay so no water enters.

LAB

Finding Absolute Zero

You will trap air in a syringe, measure its volume in water baths at several temperatures, and extrapolate the straight-line graph of volume against temperature to the temperature at which the volume would be zero.

You will need

- A 60 mL plastic syringe (no needle) with a cap, or the tip sealed with modeling clay
- A digital thermometer
- Four water baths in tall containers: ice water, cool tap water, room-temperature water, and hot tap water (about 55–60 °C)
- Graph paper or a spreadsheet

Steps

1. Draw about 30 mL of air into the syringe and seal the tip. Lubricate the plunger with a little vegetable oil so it slides freely.
2. **Hypothesis.** Predict how the volume will change between the ice bath and the hot bath, and where the graph line will cross zero volume.
3. Hold the syringe fully under water in the ice bath (plunger up) for 3 minutes. Gently tap the plunger to free it, then read the volume and the bath temperature.
4. Repeat in each of the other baths, from cold to hot. Then repeat the whole set to check.
5. Graph volume against temperature in °C. Draw the best straight line and extend it to the left until the volume is zero. Read the temperature there.
6. Calculate the percent difference from −273 °C.
7. **Carnot calculation.** Using your ice and hot baths as T_C and T_H, find the best possible efficiency of an engine running between them. Why are real engines run much hotter?
8. **Improve.** Friction makes the plunger stick, giving pressures slightly different from atmospheric. Tap the plunger before each reading, and use a larger

syringe for larger volume changes. Repeat and compare your extrapolations.

Tips

- Wait for the air to reach the bath temperature; air warms and cools slowly through plastic.
- A long extrapolation magnifies small errors; a result within 30–40 degrees of −273 °C is good work.

Check your work

- ☐ My volume–temperature graph is a straight line that extrapolates close to −273 °C.
- ☐ I can calculate Carnot efficiencies with kelvin temperatures.
- ☐ I can explain the first and second laws of thermodynamics in my own words.

LAB NOTEBOOK

1. Data table and the extrapolated graph with absolute zero marked.
2. Energy-flow diagram of a power plant: heat in from the boiler, work out to the generator, heat out to the cooling tower, with numbers for a 40%-efficient plant producing 1000 MW.
3. Explain in a paragraph why a refrigerator needs electricity to move heat from inside (cold) to the kitchen (warm).

KEY FORMULAS

$$\Delta U = Q - W$$

$$e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

$$e_{\text{max}} = 1 - \frac{T_C}{T_H} \quad (T \text{ in K})$$

$$\text{COP}_{\text{fridge, max}} = \frac{T_C}{T_H - T_C}$$

$$\frac{V}{T} = \text{constant} \quad (\text{gas at constant pressure})$$

From history. Kelvin later supervised the engineering of the first successful transatlantic telegraph cable (1866) and was knighted for it. The SI unit of temperature, the kelvin, is named for him. Carnot's brother Hippolyte preserved his notes, which showed that before he died Sadi had begun to reject the caloric theory and had even estimated the mechanical equivalent of heat.

WORKED EXAMPLES

Example 1. A steam turbine receives steam at $550^\circ\mathrm{C}$ and rejects heat to a condenser at $30^\circ\mathrm{C}$. Find its Carnot efficiency. If a real plant achieves 40%, what fraction of the Carnot limit is that?

Show solution

1. Convert to kelvins: $T_H = 550 + 273 = 823\ \mathrm{K}$; $T_C = 30 + 273 = 303\ \mathrm{K}$.
2. $e_{\max} = 1 - \dfrac{303}{823} = 1 - 0.368 = 0.632$.
3. Real: $\dfrac{0.40}{0.632} = 0.63$, about 63% of the ideal.

Answer: 63% maximum; a real 40% plant reaches about two-thirds of Carnot's limit.

Example 2. An engine takes in 1000 J of heat each cycle and does 250 J of work. Find its efficiency and the heat it rejects.

Show solution

1. $e = \dfrac{W}{Q_H} = \dfrac{250}{1000} = 0.25 = 25\%$.
2. $Q_C = Q_H - W = 1000 - 250 = 750\ \mathrm{J}$, rejected to the surroundings.

Answer: 25%; 750 J rejected per cycle.

Example 3. A trapped sample of air has a volume of $50.0\ \mathrm{mL}$ at $0°\mathrm{C}$ and $63.7\ \mathrm{mL}$ at $75°\mathrm{C}$ at constant pressure. Extrapolate to find the temperature at which its volume would be zero.

Show solution

1. Slope of the volume–temperature line: $\dfrac{63.7 - 50.0}{75 - 0}\ \mathrm{mL}/°\mathrm{C} = 0.183\ \mathrm{mL}/°\mathrm{C}$.
2. $V = 50.0 + 0.183T$; set $V = 0$: $T = -\dfrac{50.0}{0.183} = -274°\mathrm{C}$.

Answer: About $-274°\mathrm{C}$, close to absolute zero, $-273.15°\mathrm{C}$.

Example 4. A freezer keeps food at $-18°\mathrm{C}$ in a $25°\mathrm{C}$ kitchen. What is the best possible coefficient of performance (heat removed per joule of work)?

Show solution

1. $T_C = 255\ \mathrm{K}$, $T_H = 298\ \mathrm{K}$.
2. $\mathrm{COP}_{\max} = \dfrac{T_C}{T_H - T_C} = \dfrac{255}{43} = 5.9$.

Answer: At most about 5.9 J of heat removed per joule of electrical work; real freezers manage about 2 to 3.

PRACTICE

1. Easy Convert $-40°\mathrm{C}$ and $100°\mathrm{C}$ to kelvins.

   Show answer

   233 K; 373 K

2. Easy An engine does 300 J of work from 1200 J of heat. Find its efficiency.

   Show answer

   25%

3. Easy Why can no engine be 100% efficient?

   Show answer

   By the second law, some heat must always be rejected to a cold sink; an efficiency of 100% would require a cold sink at absolute zero.

4. **Medium** Find the Carnot efficiency of an engine between 500 K and 300 K.

Show answer

40%

5. **Medium** A gas absorbs 500 J of heat and does 200 J of work. What is the change in its internal energy?

Show answer

$+300$ J

6. **Medium** A power plant produces 1000 MW of electricity at 38% efficiency. How much heat does it reject per second?

Show answer

Heat in $= 2630$ MW; rejected $= 1630$ MW

7. **Medium** A car engine's combustion gases reach about 2300 K and its exhaust leaves at about 1000 K. What is the Carnot limit? Why is a real car engine only about 30% efficient?

Show answer

$1 - 1000/2300 = 57\%$. Friction, heat lost through the cylinder walls, incomplete expansion and the fact that real cycles are far from reversible all lower it.

8. **Medium** Why must temperatures be in kelvins in the Carnot formula?

Show answer

The formula depends on ratios of temperatures, which make sense only on an absolute scale with zero at absolute zero. On the Celsius scale, for example, 0 °C would give a meaningless ratio.

9. **Hard** A heat pump moves heat from outdoor air at $0°$C into a house at $20°$C. What is its best possible COP for heating ($T_H/(T_H - T_C)$)? If it uses 2.0 kW of electricity at one-third of that COP, how much heat does it deliver?

Show answer

$293/20 = 14.7$; one third is 4.9; heat delivered $= 9.8$ kW

10. Hard Air in a syringe has volume 40.0 mL at 20°C. Predict its volume at 80°C (constant pressure).

Show answer

$$V = 40.0(353/293) = 48.2 \text{ mL}$$

11. Challenge Two engines run in series: the first between 800 K and 500 K, the second using the first's rejected heat between 500 K and 300 K. Both are Carnot engines. Show that the combined efficiency equals that of one Carnot engine between 800 K and 300 K.

Show answer

Engine 1: $e_1 = 3/8$, so $W_1 = \frac{3}{8}Q$, rejecting $\frac{5}{8}Q$. Engine 2: $e_2 = 2/5$, $W_2 = \frac{2}{5} \cdot \frac{5}{8}Q = \frac{1}{4}Q$. Total $= \frac{3}{8} + \frac{2}{8} = \frac{5}{8}Q$; and $1 - 300/800 = 5/8$. Equal, which is the idea behind combined-cycle power plants.

COMMON MISTAKES

- Using Celsius temperatures in $1 - T_C/T_H$.
- Thinking an engine's rejected heat is a design failure that better engineering can eliminate. Some rejected heat is required by the second law.
- Confusing the sign of W: in $\Delta U = Q - W$, W is the work done *by* the system.

Make it a game · Carnot Limit Duel. Your parent gives a hot and cold reservoir, such as 500 K and 300 K. Both of you estimate the maximum possible efficiency before calculating. Then explain why a real engine always does worse. Switch temperatures to Celsius first, and see if you convert to kelvin correctly. Five rounds; track mistakes.

Scripture. "They shall perish, but thou shalt endure: yea, all of them shall wax old like a garment" (Psalm 102:26, KJV)

The second law says that the usable energy of the world runs down, like a garment growing old. The psalmist contrasts that with the One who endures.

UNIT 8

Fluids and Heat

Fluids, liquids and gases, push, lift and flow, and heat moves between them. This unit begins with the pressure of the "ocean of air" we live beneath, measured by Torricelli and Pascal's brother-in-law on the Puy de Dôme, and with the buoyancy Archimedes discovered. It continues with fluids in motion, from Bernoulli's pipes and Torricelli's jets to the wing of the Wright Flyer, explained correctly: wings turn air downward, and the pressure above them is lower. Then it turns to heat: Joseph Black's latent heat, the calorimeter, and finally Carnot's and Kelvin's laws of thermodynamics, which govern every engine and power plant.

The labs build a two-syringe hydraulic press, weigh objects in water to test a "crown," measure the speed of water jets from a bottle and a toy water blaster, measure the specific heat of steel, and find absolute zero with a syringe of air in water baths.

PREREQUISITES

- Unit 5: work, energy and power; Joule's mechanical equivalent of heat (Lesson 5.1)
- Unit 2: projectile motion (for water jets)
- Unit 3: forces and free-body diagrams

MASTERY CHECKLIST

- ☐ I can calculate pressure and pressure at depth, and apply Pascal's principle to hydraulic machines.
- ☐ I can use Archimedes' principle to find buoyant forces, densities and the fraction of a floating object submerged.
- ☐ I can use the continuity equation, Bernoulli's equation and Torricelli's law, and explain lift correctly.
- ☐ I can solve heating, phase-change and calorimetry problems with

$Q =$and
$mc\Delta T$

$Q = mL.$

- ☐ I can apply the first and second laws of thermodynamics and calculate Carnot efficiencies in kelvins.

OPTIONAL KITS FOR THIS UNIT

- **Vernier Go Direct temperature and pressure sensors**: Wireless sensors that graph temperature and gas pressure live on a phone or computer; they make the calorimetry and absolute-zero labs (Lessons 8.4–8.5) faster and more precise. Optional.
- **Thames & Kosmos hydraulics kits**: Ages 10 and up. Syringe-powered hydraulic machines (a crane, an excavator) that extend Lesson 8.1's two-syringe press into working models.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

8.1	**Pressure: Pascal and the Puy de Dôme**	3 sessions of 50 minutes
8.2	**Buoyancy: Archimedes' Principle**	3 sessions of 50 minutes
8.3	**Moving Fluids: Bernoulli, Torricelli and the Wing**	3 sessions of 50 minutes
8.4	**Temperature and Heat: Joseph Black and the Calorimeter**	3 sessions of 50 minutes
8.5	**Thermodynamics and Engines: Carnot and Kelvin**	3 sessions of 50 minutes

Unit 8 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$, $\rho_{\text{water}} = 1000\ \mathrm{kg/m^3}$, $c_{\text{water}} = 4186\ \mathrm{J/(kg \cdot K)}$, $L_f = 334\ \mathrm{kJ/kg}$ and $L_v = 2260\ \mathrm{kJ/kg}$. Use kelvins in thermodynamic formulas. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   A 60 kg person stands on a board of area $0.50\ \mathrm{m^2}$ on soft ground. What pressure does the board exert?

2. **6 pts**
   Find the absolute pressure 25 m below the surface of a freshwater lake.

3. **6 pts**
   A hydraulic lift has a small piston 3.0 cm across and a large one 30 cm across. What force on the small piston lifts a 2000 kg truck? How far must the small piston move to raise the truck 0.10 m?

4. **5 pts**
   Describe the Puy de Dôme experiment and explain what it proved.

5. **6 pts**

   A metal object weighs 4.90 N in air and 4.40 N in water. Find its volume and density. What might it be?

6. **5 pts**

   A block of wood floats with 70% of its volume submerged in fresh water. What is its density? Would it float higher or lower in seawater?

7. **5 pts**

   Explain how a steel ship floats.

8. **6 pts**

   Water flows at $3.0\ \mathrm{m/s}$ through a pipe $4.0\ \mathrm{cm}$ across, which narrows to $2.0\ \mathrm{cm}$. Find the speed in the narrow section and the drop in pressure (horizontal pipe).

9. **5 pts**

   A hole is $0.45\ \mathrm{m}$ below the water surface in a tank. Find the speed of the jet.

10. **7 pts**

    Explain how a wing produces lift. Then explain what is wrong with the claim that air over the top must travel faster to meet the air from below at the trailing edge.

11. **6 pts**

    How much heat is needed to warm $3.0\ \mathrm{kg}$ of water from 15 to $65^\circ\mathrm{C}$?

12. **7 pts**

    $300\ \mathrm{g}$ of aluminum ($c = 900$) at $95^\circ\mathrm{C}$ is dropped into $400\ \mathrm{g}$ of water at $18^\circ\mathrm{C}$. Find the final temperature.

13. **6 pts**

    How much heat melts $0.20\ \mathrm{kg}$ of ice at $0^\circ\mathrm{C}$ and then boils the water away (from 0 to $100^\circ\mathrm{C}$ and vaporized)?

14. **5 pts**

    What is latent heat, and how did Joseph Black discover it?

15. 6 pts

Find the Carnot efficiency of an engine between 400°C and 40°C.

16. 6 pts

A heat engine takes in 5000 J per cycle and rejects 3200 J. Find the work done and the efficiency.

17. 8 pts

Essay. State the first and second laws of thermodynamics and explain, using a power plant or a car engine as an example, what each law means for engineers trying to make engines more efficient.

ANSWER KEY

1. $1176\ \mathrm{Pa}$
2. $101.3 + 245 = 346\ \mathrm{kPa}$
3. $196\ \mathrm{N}$; $10\ \mathrm{m}$ (in many short strokes)
4. Two identical mercury barometers were filled in Clermont; one stayed there with an observer while the other was carried up the mountain. At the summit the mercury was about 8.5 cm lower, and it returned to the original height on the way down. This showed that air pressure is caused by the weight of the air above and decreases with height.
5. $F_b = 0.50\ \mathrm{N}$; $V = 5.1 \times 10^{-5}\ \mathrm{m}^3$; $\rho = 9800\ \mathrm{kg/m^3}$; close to lead or a dense bronze, not gold
6. $700\ \mathrm{kg/m^3}$; higher (seawater is denser)
7. Its hull encloses a large volume of air, so its average density is less than water's. It sinks until the water it displaces weighs as much as the whole ship.
8. $12\ \mathrm{m/s}$; $\Delta P = \frac{1}{2}(1000)(144 - 9) = 67.5\ \mathrm{kPa}$
9. $2.97\ \mathrm{m/s}$
10. A wing's shape and angle of attack turn the oncoming air downward; by Newton's third law the air pushes the wing up, and this appears as lower pressure above the wing and higher pressure below. The air above does move faster, but there is no rule that parcels separated at the front must rejoin at the back; in reality the upper air arrives well before the lower air, so the "equal transit time" argument is false and gives the wrong amount of lift.
11. $628\ \mathrm{kJ}$
12. $270(95 - T) = 1674(T - 18)$; $T = 28.7^\circ\mathrm{C}$
13. $66.8 + 83.7 + 452 = 602\ \mathrm{kJ}$
14. Latent heat is energy absorbed or released during a change of phase without a change in temperature. Black noticed that ice in a warm room stays at 0 °C while it melts slowly, so it must be absorbing heat that does not raise its temperature; he measured that melting ice took as much heat as warming the same mass of water by about 140 °F.
15. $1 - 313/673 = 53\%$
16. $1800\ \mathrm{J}$; 36%
17. Model answer: The first law says energy is conserved: the work an engine does plus the heat it rejects equals the heat it takes in. Engineers cannot get more

work out than energy in, so efficiency is always less than 100%, and every joule of fuel must be accounted for. The second law says heat flows naturally only from hot to cold and no engine can convert heat entirely into work; even an ideal engine must reject heat, with efficiency limited to $1 - T_C/T_H$. So engineers raise the hot temperature (better alloys, higher steam pressures), lower the cold temperature (cooling towers, large condensers), reduce friction and heat leaks to approach the Carnot limit, and recover rejected heat, for example in combined-cycle plants that pass gas-turbine exhaust to a steam turbine.

PORTFOLIO PIECE: A CARTESIAN DIVER AND A FLOATING CARGO BARGE

Build two buoyancy devices and explain each with numbers. First, a Cartesian diver: a medicine dropper or a folded condiment packet in a sealed 2-liter bottle of water that sinks when you squeeze the bottle and rises when you let go. Measure how the air bubble inside the diver changes volume when it sinks, and explain the result with Pascal's principle, Boyle's law and Archimedes' principle. Second, a cargo barge folded from aluminum foil with a measured flat bottom: predict from Archimedes' principle how many pennies (2.5 g each) it will carry before sinking, then test it in fresh water and in salt water. Write a report with sketches, calculations, measurements and a discussion of the differences.

LOOK FOR	WHAT IT MEANS
Cartesian diver	Works reliably; bubble volume measured or estimated at the surface and when sinking; explanation uses pressure, gas compression and buoyancy correctly.
Barge prediction	Capacity predicted from the barge's dimensions and Archimedes' principle before testing, with units.
Testing	Pennies counted in fresh and salt water, three trials each; results compared with predictions.
Explanation	Report correctly relates density, displacement and buoyant force, and explains why salt water carries more.
Honest analysis	Sources of error discussed (leaks, uneven loading, foil weight) and one improvement made.

UNIT 9

Waves, Sound and Light

A wave carries energy without carrying matter, and much of what we know about the world comes to us by waves: sound, light, radio, the shaking of the ground. This unit starts with the simplest oscillation, the pendulum, which Huygens turned into the first accurate clock, and moves on to waves on a Slinky and on Chladni's singing plates, the Doppler effect that Buys Ballot tested with horn players on a train, the reflection and refraction of light that Newton mastered in his mirror telescope, and finally Thomas Young's double-slit experiment, which showed that light itself is a wave.

The labs measure g with a pendulum, time a toy spring, send pulses down a Slinky, measure the speed of sound with a phone tone and a tube of water, find a speed from a Doppler shift, measure the index of refraction of water, build a periscope or kaleidoscope, and measure the wavelength of a laser with an old CD.

PREREQUISITES

- Unit 1: graphing and linearizing; Unit 2: kinematics
- Lesson 1.4 (Hooke's law) and Unit 5 (energy)
- Right-triangle trigonometry, including sine and inverse sine

MASTERY CHECKLIST

- ☐ I can use $T = 2\pi\sqrt{L/g}$ and $T = 2\pi\sqrt{m/k}$ and explain what makes motion simple harmonic.
- ☐ I can use $v = f\lambda$, find standing-wave frequencies for strings and tubes, and explain resonance.
- ☐ I can apply the Doppler equations for sound and explain Doppler radar and sonic booms.
- ☐ I can use the law of reflection, Snell's law, critical angles and the thin-lens equation.
- ☐ I can explain Young's experiment and use $d\sin\theta = m\lambda$ to find wavelengths.

OPTIONAL KITS FOR THIS UNIT

- **Arbor Scientific optics and waves sets**: Inexpensive diffraction gratings, slit slides, lens sets and spectroscope glasses for Lessons 9.4–9.5, and a long spring for waves in Lesson 9.2.
- **Thames & Kosmos optics kits**: Ages 10 and up. Lens, mirror and prism kits for building telescopes, periscopes and kaleidoscopes in Lesson 9.4.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

9.1	**Oscillations: Huygens and the Pendulum Clock**	3 sessions of 50 minutes
9.2	**Waves and Sound: The Slinky and Chladni's Plates**	3 sessions of 50 minutes

9.3	**The Doppler Effect: Horns on a Train**	3 sessions of 50 minutes
9.4	**Reflection, Refraction and Lenses: Newton's Telescope**	3 sessions of 50 minutes
9.5	**Light as a Wave: Young's Double Slit**	3 sessions of 50 minutes

Unit 9 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$, the speed of sound $343\ \mathrm{m/s}$ (unless told otherwise) and $c = 3.00 \times 10^8\ \mathrm{m/s}$. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   A pendulum is $1.20\ \mathrm{m}$ long. Find its period.

2. **5 pts**
   A $0.30\ \mathrm{kg}$ mass on a spring oscillates with a period of $0.60\ \mathrm{s}$. Find the spring constant.

3. **5 pts**
   Why did Huygens' pendulum clock keep much better time than earlier clocks? What did it not do well?

4. **5 pts**
   A wave on a rope has a frequency of $4.0\ \mathrm{Hz}$ and a wavelength of $1.5\ \mathrm{m}$. Find its speed.

5. **6 pts**
   A string $0.70\ \mathrm{m}$ long fixed at both ends has a wave speed of $280\ \mathrm{m/s}$. Find its fundamental frequency and the wavelength of its third harmonic.

6. **6 pts**
   A tube closed at one end resonates first at a length of $0.25\ \mathrm{m}$ for a $340\ \mathrm{Hz}$ tone. Find the speed of sound (ignore the end correction).

7. 5 pts

Explain why sand collects in lines on a vibrating Chladni plate.

8. 7 pts

A siren of 800 Hz moves at 30 m/s. What frequency does a person at rest hear as it approaches and as it moves away?

9. 5 pts

Describe Buys Ballot's 1845 test of the Doppler effect and what it showed.

10. 6 pts

A radar gun at 10.5 GHz detects a 1750 Hz shift from a car. How fast is the car moving?

11. 6 pts

Light in air enters glass ($n = 1.52$) at 45°. Find the angle of refraction.

12. 5 pts

Find the critical angle for light leaving glass ($n = 1.52$) into air, and explain how optical fibers use it.

13. 6 pts

An object is 25 cm from a converging lens of focal length 10 cm. Find the image distance and magnification, and say whether the image is real or virtual.

14. 5 pts

Why did Newton build a reflecting telescope?

15. 6 pts

Light passes through two slits 0.25 mm apart onto a screen 3.0 m away. The bright fringes are 7.2 mm apart. Find the wavelength.

16. 6 pts

A grating has 500 lines per millimeter. Find the first-order angle for 600 nm light.

17. **11 pts**

Essay. Explain how Thomas Young's double-slit experiment showed that light is a wave. Include what he saw, why particles could not explain it, and how he used the pattern to measure the wavelength of light.

ANSWER KEY

1. $2.20\ \mathrm{s}$
2. $32.9\ \mathrm{N/m}$
3. A pendulum's period depends only on its length and g, not on the size of the swing (for small swings) or the driving force, so it regulated the clock steadily. It worked poorly on ships, whose motion disturbed the pendulum.
4. $6.0\ \mathrm{m/s}$
5. $200\ \mathrm{Hz}$; $0.467\ \mathrm{m}$
6. $v = 4Lf = 340\ \mathrm{m/s}$
7. The plate vibrates in a standing-wave pattern; sand is bounced away from the moving antinodes and settles on the nodal lines, where the plate stays still.
8. $877\ \mathrm{Hz}$; $736\ \mathrm{Hz}$
9. Horn players on an open railway car played a steady note while the train passed listeners with perfect pitch at measured speeds. The listeners heard the pitch rise as the car approached and fall as it receded, by about the amount Doppler's formula predicted.
10. $v = c\Delta f/(2f) = 25\ \mathrm{m/s}$
11. 27.7°
12. 41.1°. Light inside a fiber strikes its wall at angles larger than critical, so it is totally reflected and stays inside the fiber.
13. $d_i = 16.7\ \mathrm{cm}$; $m = -0.67$; real and inverted
14. Glass lenses split white light into colors (chromatic aberration), blurring images; a mirror reflects all colors alike, so it focuses them together.
15. $600\ \mathrm{nm}$
16. $d = 2.0\ \mu\mathrm{m}$; $\sin\theta = 0.30$; 17.5°
17. Model answer: Young split a narrow beam of sunlight with a thin card so the two halves overlapped on a screen. Instead of a single bright patch he saw alternating bright and dark bands, with a bright band in the center. Particles traveling in straight lines would simply add, giving brightness wherever either beam reached; they cannot cancel to make darkness. Waves can: where crests from one side meet troughs from the other they cancel (dark), and where they meet in step they reinforce (bright). Blocking one beam made the bands vanish, confirming that both were needed. The spacing of the bands depends on the wavelength, the separation of the sources and the screen distance ($\Delta y = \lambda L/d$),

so by measuring them he calculated the wavelength of red light as about 1/36,000 inch (0.7 μm).

PORTFOLIO PIECE: A MUSICAL INSTRUMENT, DESIGNED BY PHYSICS

Design and build a simple instrument that plays at least five notes of a scale: a set of tuned bottles or tubes (air columns), a box guitar with strings of adjustable tension, or a xylophone of wooden or metal bars. Use physics to choose the dimensions before building: for tubes, $\lambda = 4L$ and $v = f\lambda$; for strings, $f = \frac{1}{2L}\sqrt{T/\mu}$. Measure each note's frequency with phyphox's audio spectrum, compare with your design, and adjust to tune it. Play a short hymn or folk tune on it. Write a report with your design calculations, a table of target and measured frequencies, a sketch of the standing waves for two of your notes, and what you changed to tune it.

LOOK FOR	WHAT IT MEANS
Design calculations	Dimensions for at least five notes calculated from wave physics before building, with units.
Measurement	Each note's frequency measured, compared with the target, and percent differences given.
Tuning and iteration	Documented adjustments that brought notes closer to target.
Physics explanation	Standing-wave diagrams and a clear explanation of resonance, nodes and antinodes for the instrument.
Performance	Plays a recognizable short tune.

9.1 Oscillations: Huygens and the Pendulum Clock

OBJECTIVES

- Describe simple harmonic motion and define period, frequency and amplitude.
- Use $T = 2\pi\sqrt{L/g}$ for a pendulum and $T = 2\pi\sqrt{m/k}$ for a mass on a spring.
- Explain why the pendulum made clocks accurate, and what limits that accuracy.
- Measure g from a graph of pendulum period squared against length, and measure a spring's period.

> **Warm-up.** Tie a washer to a string about 1 m long and let it swing. Count 10 swings (back and forth = 1) and time them. Now pull it farther to the side and time 10 swings again. Then shorten the string to about 25 cm and time 10 swings. Before each new trial, predict whether the time will be longer, shorter or the same.

WATCH

When is a Pendulum in Simple Harmonic Motion?(SHM)

Flipping Physics · 6:32

https://www.youtube.com/watch?v=vQypNfbga5U

> **WHEN TO WATCH**
> After the physics paragraph: why a pendulum is simple harmonic only for small swings.

BACKUP VIDEO

SHM of simple pendulums | AP Physics | Khan Academy

Khan Academy · 12:13

https://www.youtube.com/watch?v=Dv5R543h3XM

Optional, after the worked examples: a second derivation of the pendulum's period.

KEY TERMS

simple harmonic motion

Back-and-forth motion in which the restoring force is proportional to the displacement, $F = -kx$; its period does not depend on the amplitude.

period

The time for one complete oscillation, T, in seconds.

frequency

The number of oscillations per second, $f = 1/T$, in hertz (Hz).

amplitude

The largest displacement from the equilibrium position.

isochronism

The property of taking the same time for every swing, whatever its size; a pendulum has it only for small swings.

escapement

The clock mechanism that releases the gear train one tooth per swing and gives the pendulum a small push to keep it going.

CORE IDEAS

The story. According to his first biographer, Galileo as a young student in Pisa watched a lamp swinging from the cathedral ceiling and, timing it against his pulse, saw that wide swings and narrow ones took the same time. For seventy years no one made a clock that used the fact. Then, on Christmas Day 1656, Christiaan Huygens in The Hague finished the first pendulum clock, and the next year a local clockmaker, Salomon Coster, began building them under Huygens' patent. The best clocks before had drifted by perhaps a quarter of an hour a day; Huygens' clocks kept time to within about 15 seconds a day.

How they figured it out. Huygens did not trust the pendulum blindly. He found by experiment and then proved mathematically that a pendulum's swings are only *nearly* equal in time: large swings take slightly longer. He showed that a bob following a cycloid, a curve traced by a point on a rolling wheel, would be perfectly isochronous, and he fitted his clocks with curved metal "cheeks" to bend the string into that path. In *Horologium Oscillatorium* (1673) he derived the period of a pendulum, $T = 2\pi\sqrt{L/g}$, and used it to measure g precisely. He also hoped pendulum clocks would solve the problem of finding longitude at sea, but a ship's motion upset them; that problem was solved in the 1760s by John Harrison's spring-driven chronometers.

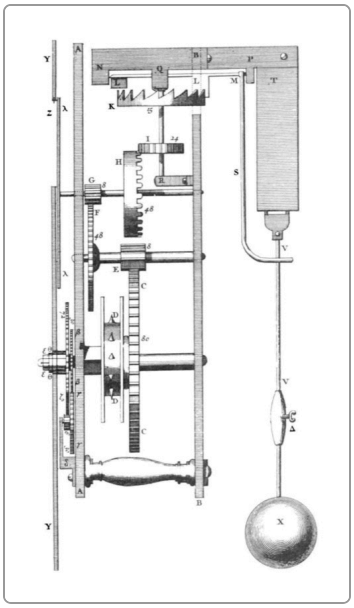
The physics. When a restoring force is proportional to displacement, $F = -kx$, the motion is **simple harmonic**: the object swings back and forth with a period that does not depend on the amplitude. A mass on a spring has $T = 2\pi\sqrt{m/k}$. A pendulum is nearly simple harmonic for small swings (less than about 15°), with $T = 2\pi\sqrt{L/g}$, independent of the bob's mass. A pendulum 0.994 m long beats once a second (period 2 s), the "seconds pendulum" of old clocks. Energy sloshes between kinetic

and potential twice per cycle, and friction slowly reduces the amplitude unless the clock's escapement gives a tiny push each swing.

The engineering. Oscillators keep time everywhere. A quartz watch counts the vibrations of a tiny quartz tuning fork at 32,768 Hz; atomic clocks count oscillations of cesium atoms, about 9.19 billion per second, and GPS depends on them. Engineers also fight unwanted oscillations: the 660-tonne pendulum hanging near the top of the Taipei 101 skyscraper is a tuned mass damper that swings against the building's sway in wind, and car suspensions use springs and shock absorbers to keep bounces from building up.

Try it. In the lab you will measure a pendulum's period at several lengths and find g from the slope of T^2 against L, then time a mass bouncing on a toy spring.

PICTURE STUDY



Huygens' Pendulum Clock, Christiaan Huygens, 1673 · Engraving · *Horologium Oscillatorium*, Paris
Image: Wikimedia Commons, public domain

Huygens drew his clock in detail so that clockmakers could build it. Look for the curved cheeks that made the pendulum follow a cycloid.

Look closely

- Find the pendulum and its bob.
- Find where the pendulum hangs from the clock. (In some of his clocks Huygens added curved "cheeks" here to bend the suspension into a cycloid.)
- Find the escapement, where the gear train meets the pendulum.
- Find the weight that drives the clock.

Talk about it

1. Why does a clock need a pendulum at all, if the falling weight already turns the gears?
2. What would happen to the clock if the pendulum were made longer? Shorter?
3. Why did Huygens want the bob to follow a cycloid instead of a circle?

LAB

The Pendulum and the Spring

You will measure a pendulum's period at several lengths, find g from the slope of a graph of T^2 against L, test whether the period depends on mass and amplitude, and then time a mass bouncing on a toy spring.

You will need

- String, several identical washers or a heavy nut, tape, a hook or doorframe
- Meter stick, protractor, stopwatch or phone
- The toy spring from Lesson 1.4 (or any spring), coins or washers as masses, kitchen scale

Steps

1. Hang a pendulum with a length (pivot to the center of the bob) of 1.00 m.
2. **Hypotheses.** Predict how the period depends on length, on mass and on amplitude.
3. Pull the bob aside about 10° and release. Time 20 complete swings; divide by 20. Repeat twice.
4. Repeat at lengths of 0.80, 0.60, 0.40 and 0.20 m.
5. Graph T^2 against L. Draw the best-fit line through the origin. Since $T^2 = (4\pi^2/g)L$, $g = 4\pi^2/\text{slope}$. Find your percent error.
6. **Fair tests.** At 1.00 m, change only the mass (one washer, then four). Then change only the amplitude: 5°, 15°, 30°, 45°. Which matters?
7. **Spring.** Hang masses of 50, 100, 150 and 200 g from the toy spring, set each bouncing gently, and time 20 bounces. Graph T^2 against m; the slope is $4\pi^2/k$. Compare this k with the one you measured by stretching in Lesson 1.4.
8. **Improve.** Reduce timing error by counting more swings or starting the stopwatch as the bob passes the lowest point (where it moves fastest and the moment is easiest to judge).

Tips

- Measure length to the center of the bob.
- Keep swings small for the length tests; large swings are slower, as Huygens found.

Check your work

☐ My graph of

T^2 against

L gives

g within a few percent.

☐ I showed that the period does not depend on mass, and depends only slightly on amplitude for small swings.

☐ I found a spring constant from oscillations and compared it with Hooke's law.

LAB NOTEBOOK

1. Data tables and the T^2–L graph with slope and g; the amplitude test results.
2. Graph of T^2 against m for the spring, and the spring constant compared with Lesson 1.4.
3. Sketch position, velocity and acceleration against time for one pendulum cycle, and mark where kinetic and potential energy are greatest.

KEY FORMULAS

$$T = 2\pi\sqrt{\frac{L}{g}} \quad \text{(pendulum, small swings)}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \text{(mass on a spring)}$$

$$f = \frac{1}{T}$$

$$T^2 = \frac{4\pi^2}{g}L$$

From history. Richer's 1672 discovery that pendulums swing slower near the equator was one of the first clues that gravity varies over the Earth. Huygens' pendulum clocks remained the world's most accurate timekeepers until quartz clocks in the 1930s.

Example 1. How long must a pendulum be to have a period of exactly 2.00 s (the "seconds pendulum")?

Show solution

1. From $T = 2\pi\sqrt{L/g}$: $L = \dfrac{gT^2}{4\pi^2}$.
2. $L = \dfrac{(9.80\ \mathrm{m/s^2})(2.00\ \mathrm{s})^2}{4\pi^2} = \dfrac{39.2}{39.48}\ \mathrm{m} = 0.993\ \mathrm{m}$.

Answer: 0.993 m, just under a meter.

Example 2. On a hot day a clock's brass pendulum rod expands, making it 0.10% longer. By what fraction does the period change, and how many seconds does the clock lose per day?

Show solution

1. $T \propto \sqrt{L}$, so a small fractional change in L gives half that fractional change in T: $\dfrac{\Delta T}{T} = \frac{1}{2}(0.0010) = 0.00050$.
2. Each swing takes 0.050% longer, so the clock runs slow by $0.00050 \times 86{,}400\ \mathrm{s} = 43\ \mathrm{s}$ per day.

Answer: About 43 s lost per day, which is why fine clocks used temperature-compensated pendulums.

Example 3. A 0.20 kg mass hangs from the toy spring of Lesson 1.4 ($k = 24.5\ \mathrm{N/m}$). Find its period and frequency of bouncing.

Show solution

1. $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{0.20\ \mathrm{kg}}{24.5\ \mathrm{N/m}}} = 2\pi(0.0904\ \mathrm{s}) = 0.57\ \mathrm{s}$.
2. $f = \dfrac{1}{T} = 1.76\ \mathrm{Hz}$.

Answer: 0.57 s; 1.8 Hz.

Example 4. A student's pendulum is 0.750 m long and has a period of 1.74 s. What value of g does this give?

Show solution

1. $g = \dfrac{4\pi^2 L}{T^2} = \dfrac{4\pi^2(0.750\ \text{m})}{(1.74\ \text{s})^2} = \dfrac{29.6}{3.03}\ \text{m/s}^2 = 9.78\ \text{m/s}^2$.

Answer: $9.78\ \text{m/s}^2$, within 0.2% of 9.80.

PRACTICE

1. Easy A pendulum makes 30 swings in 45 s. Find its period and frequency.

   Show answer

   1.5 s; 0.67 Hz

2. Easy Find the period of a 2.5 m pendulum.

   Show answer

   3.17 s

3. Easy Does a heavier bob change a pendulum's period? Why not?

   Show answer

   No. A heavier bob feels a larger force of gravity but also has more inertia; the two effects cancel, just as all objects fall with the same acceleration.

4. Medium What length of pendulum has a period of 1.0 s?

   Show answer

   0.248 m

5. Medium A 0.50 kg mass on a spring oscillates with a period of 0.80 s. Find the spring constant.

   Show answer

   $k = 4\pi^2(0.50)/0.64 = 30.8\ \text{N/m}$

6. Medium If the mass on a spring is quadrupled, what happens to the period? If the pendulum's length is quadrupled?

   Show answer

   Both periods double.

7. Medium A pendulum clock that keeps perfect time in New York ($g = 9.803\ \mathrm{m/s^2}$) is taken to the Moon ($g = 1.62$). What is its new period if it beat seconds ($T = 2.00$ s) on Earth?

Show answer

$$T = 2.00\sqrt{9.803/1.62} = 4.92\ \mathrm{s}$$

8. Medium Why did pendulum clocks fail at sea?

Show answer

The ship's rolling and pitching accelerated the clock, adding to or subtracting from gravity and pushing the pendulum, so its period kept changing.

9. Hard In 1672 the astronomer Jean Richer found that a seconds pendulum had to be shortened by about 2.8 mm near the equator in South America. What fractional change in g does that show?

Show answer

$g \propto L$ for a fixed period: $\Delta g/g = 2.8/994 = 0.28\%$ smaller, evidence (as Newton explained) of the Earth's spin and equatorial bulge.

10. Hard A building's tuned mass damper is a 660-tonne pendulum that must swing with the building's natural period of about 7 s. How long must its cables be?

Show answer

$$L = g(7/2\pi)^2 = 12\ \mathrm{m}$$

11. Challenge Show that a pendulum is simple harmonic for small angles: the restoring force along the arc is $mg\sin\theta \approx mg\theta = (mg/L)s$, where $s = L\theta$ is the arc length. Find the effective spring constant and derive $T = 2\pi\sqrt{L/g}$.

Show answer

Effective $k = mg/L$. Then $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{m/(mg/L)} = 2\pi\sqrt{L/g}$. The approximation $\sin\theta \approx \theta$ holds for small angles, which is why large swings are slower.

COMMON MISTAKES

- Measuring one swing instead of many. Time 10 or 20 swings and divide.
- Measuring the string length to the top of the bob instead of its center.
- Confusing period (seconds per swing) with frequency (swings per second).

Make it a game · Pendulum Period Prediction. Build a pendulum from string and a washer. Predict its period for a 1 m length using T = 2 pi sqrt(L/g), then time ten swings and divide. Change the length to 25 centimeters and predict again. Score by percent difference. Then ask: does a heavier washer matter? Predict, then test.

Scripture. “To every thing there is a season, and a time to every purpose under the heaven” (Ecclesiastes 3:1, KJV)

A clock divides time into equal parts because God made a world in which a pendulum keeps its beat. Measuring time well is a way of living by the seasons He set.

9.2 Waves and Sound: The Slinky and Chladni's Plates

OBJECTIVES

- Distinguish transverse from longitudinal waves and use $v = f\lambda$.
- Describe standing waves, nodes and antinodes, and find the resonant wavelengths of strings and air columns.
- Explain what determines the speed of sound and calculate distances from echo and delay times.
- Measure wave speed on a Slinky and the speed of sound with a resonance tube.

> **Warm-up.** Stretch a Slinky along a smooth floor with a partner holding the other end. Give your end a quick sideways flick and watch the pulse travel. Then push your end sharply forward once. How are the two pulses different? Do they reflect from your partner's end? Write down what you see.

WATCH

Introduction to Waves

Flipping Physics · 8:23

https://www.youtube.com/watch?v=drOnyIIe5Eg

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Chladni Figures - random couscous snaps into beautiful patterns

Steve Mould · 3:50

https://www.youtube.com/watch?v=CR_XL192wXw

With the story paragraph: Chladni figures made with a vibrating plate.

KEY TERMS

wave

A disturbance that carries energy from place to place without carrying matter along with it.

transverse and longitudinal waves

In a transverse wave the medium moves perpendicular to the wave's direction (a flicked Slinky, light); in a longitudinal wave it moves along it (a pushed Slinky, sound).

wavelength

The distance λ between successive crests (or compressions); $v = f\lambda$.

standing wave

A pattern formed by waves reflecting back and forth, with fixed points of no motion (nodes) and points of greatest motion (antinodes).

resonance

The large response of a system driven at one of its natural frequencies.

CORE IDEAS

The story. Ernst Chladni, a German lawyer turned physicist and musician, wanted to *see* sound. In 1787 he sprinkled fine sand on a thin brass plate clamped at its center and drew a violin bow along its edge. The plate sang, and the sand jumped away from the vibrating parts and gathered along lines where the plate stood still, forming beautiful patterns that changed with every note. He published them in *Discoveries in the Theory of Sound* and toured Europe showing them. In 1809 he showed them in Paris, and a prize was offered for a mathematical theory of the patterns; it was won in 1816 by the self-taught mathematician Sophie Germain.

How they figured it out. Chladni's sand lines are **nodes**, the places where waves traveling across the plate and reflecting from its edges cancel each other, leaving the plate still. Between them are **antinodes**, where the waves add and the plate shakes hardest. Each note the plate can sing is one of its natural frequencies, with its own pattern. The same idea, waves reflecting and combining into standing waves, explains why a guitar string or an organ pipe sounds only certain notes. Instrument makers have used Chladni's method ever since to test violin and guitar tops, and

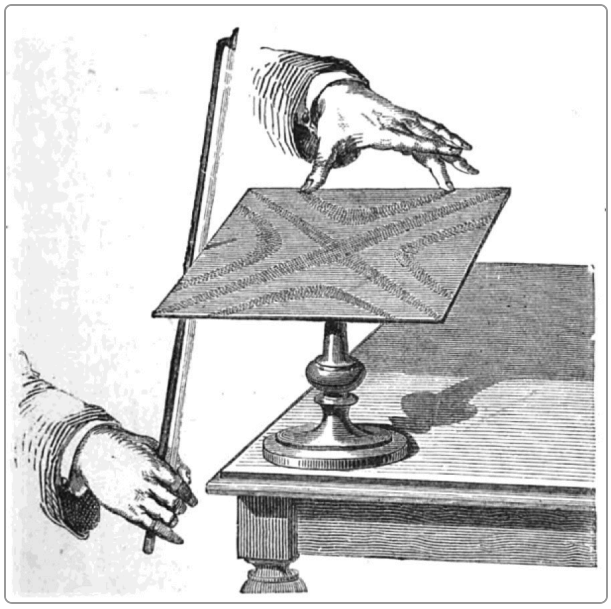
engineers use its modern form, modal analysis, to find how car bodies, aircraft panels and bridges vibrate.

The physics. A wave carries energy, not matter. Its speed, frequency and wavelength are related by $v = f\lambda$. The speed is set by the medium: on a string, $v = \sqrt{T/\mu}$ (tension over mass per length); sound in air travels about $343\ \mathrm{m/s}$ at 20 °C, faster in warm air (about $331 + 0.6T_C$ m/s), faster still in water (about 1500 m/s) and steel (about 5000 m/s). A string fixed at both ends resonates when a whole number of half-wavelengths fits: $\lambda_n = 2L/n$. A tube closed at one end resonates when a quarter-wavelength fits: $\lambda_1 = 4L$, then $4L/3$, $4L/5$. Sound is a longitudinal pressure wave; we hear frequencies from about 20 Hz to 20,000 Hz.

The engineering. Acoustic engineers design concert halls so that reflections arrive at the right times and no seat sits in a node. Ultrasound scanners send 2–15 MHz sound into the body and time the echoes; sonar finds the depth of the sea the same way. Structural engineers keep machines and buildings from being driven at their natural frequencies, which can build up dangerous resonant vibrations, and they measure those frequencies with Chladni's idea, now using accelerometers and lasers.

Try it. In the lab you will time pulses on a Slinky, make standing waves on it, and measure the speed of sound with a tone from your phone and a tube of water.

PICTURE STUDY



Bowing a Chladni Plate, Unknown engraver, 1879 · Engraving · Nineteenth-century physics text (Wikimedia Commons)

Image: Wikimedia Commons, public domain

Physics books of the 1800s showed students how to make Chladni's figures themselves. Notice where the bow touches and where a finger holds the plate still.

Look closely

- Find the clamp at the plate's center.
- Find where the bow strokes the edge and where a finger touches it.
- Look for lines or patterns in the sand.

Talk about it

1. Why does the sand collect on some lines and leave other places bare?
2. Why does touching the plate at a particular point change the pattern?
3. If the plate sang a higher note, would you expect more lines or fewer?

Safety first. Keep the Slinky from snapping back toward anyone's face. Fill the resonance container in a sink or tub.

LAB

Waves on a Slinky and the Speed of Sound

You will measure the speed of pulses on a Slinky, make standing waves and check $v = f\lambda$, and then find the speed of sound from the resonance of an air column.

You will need

- A metal Slinky (full size) and a partner, a smooth floor, tape measure, stopwatch, phone with slow-motion video
- A tall jar or a tall clear plastic bottle with the top cut off, and a pitcher of water
- A tube open at both ends (a paper-towel tube or a length of PVC pipe) that fits inside the jar
- A phone with a tone-generator app (phyphox has one) or a tuning fork
- Thermometer

Steps

1. Stretch the Slinky to 4–5 m along the floor. Send a single transverse pulse and time several trips back and forth with video. Find $v =$ distance ÷ time.
2. **Hypothesis.** Predict what happens to the pulse speed if you stretch the Slinky farther (more tension).
3. Stretch it farther and measure again. Then send a longitudinal pulse and compare its speed.
4. **Standing waves.** Shake one end steadily side to side so that the Slinky forms one loop, then two, then three. Time 10 shakes for each to find f, measure λ from the loop length ($\lambda = 2\times$ loop length), and check that $f\lambda$ is the same each time and equals your pulse speed.
5. **Speed of sound.** Put the tube in the jar of water. Hold the phone playing a steady tone (start with 500 Hz) just above the tube. Slowly raise the tube until the sound gets suddenly loud. Measure the length L of the air column.
6. Calculate $v = 4Lf$ (adding about 0.3 × the tube diameter to L for the open-end correction). Repeat at two other frequencies.

7. Compare with $331 + 0.6T_C$ for the air temperature you measure.
8. **Improve.** Find the second resonance (about 3 times the first length) for one frequency; the difference between the two lengths is exactly $\lambda/2$, which avoids the end correction.

Tips

- Keep the Slinky on the floor to reduce sagging.
- Move the tube slowly near resonance; the loudness peak is sharp.

Check your work

- ☐ I measured wave speed on a Slinky two ways and they agree.
- ☐ I can explain nodes and antinodes and find resonant wavelengths for strings and tubes.
- ☐ I measured the speed of sound within a few percent of the expected value.

LAB NOTEBOOK

1. Data tables for pulse speeds, standing waves (f, λ, $f\lambda$) and the resonance tube.
2. Sketch the first three standing-wave patterns on a string fixed at both ends and in a tube closed at one end, marking nodes and antinodes.
3. Draw a longitudinal wave on a Slinky with compressions and rarefactions labeled, and a transverse wave with crests and troughs.

KEY FORMULAS

$$v = f\lambda$$

$$v_{\text{string}} = \sqrt{\frac{T}{\mu}}$$

$$\lambda_n = \frac{2L}{n} \quad \text{(string fixed at both ends)}$$

$$\lambda_1 = 4L \quad \text{(tube closed at one end)}, \qquad v_{\text{sound}} \approx (331 + 0.6T_C)\ \text{m/s}$$

From history. Chladni also collected meteorites and was among the first to argue, in 1794, that they fall from space, an idea scientists then laughed at. Sophie Germain, who won the prize for the theory of Chladni's figures, had taught herself mathematics from her father's books and corresponded with the great mathematician Carl Friedrich Gauss.

Example 1. Waves on a stretched Slinky have a wavelength of 2.0 m when you shake one end at 1.5 Hz. Find the wave speed.

Show solution

1. $v = f\lambda = (1.5\ \text{Hz})(2.0\ \text{m}) = 3.0\ \text{m/s}$.

Answer: 3.0 m/s.

Example 2. A guitar string 0.65 m long sounds its fundamental at 110 Hz. Find the wavelength and wave speed on the string, and the frequency of the next harmonic.

Show solution

1. Fundamental: $\lambda_1 = 2L = 2(0.65\ \text{m}) = 1.30\ \text{m}$.
2. $v = f\lambda = (110\ \text{Hz})(1.30\ \text{m}) = 143\ \text{m/s}$.
3. Second harmonic: $\lambda_2 = L = 0.65\ \text{m}$, so $f_2 = 220\ \text{Hz}$, an octave higher.

Answer: $\lambda = 1.30\ \text{m}$; $v = 143\ \text{m/s}$; next harmonic 220 Hz.

Example 3. A phone plays a 440 Hz tone over a tube whose bottom end is closed by water. The first loud resonance occurs when the air column is 19.5 cm long. Find the speed of sound.

Show solution

1. For a tube closed at one end, the first resonance is a quarter-wavelength: $\lambda = 4L = 4(0.195\ \text{m}) = 0.780\ \text{m}$.
2. $v = f\lambda = (440\ \text{Hz})(0.780\ \text{m}) = 343\ \text{m/s}$.

Answer: 343 m/s. (A small correction for the open end, about 0.3 times the tube's diameter, makes real tubes slightly shorter than $\lambda/4$.)

Example 4. You see lightning and hear the thunder 3.0 s later. How far away was the strike?

Show solution

1. Light arrives almost instantly; sound takes 3.0 s.
2. $d = vt = (343\ \mathrm{m/s})(3.0\ \mathrm{s}) = 1030\ \mathrm{m}$, about 1 km.

Answer: About 1.0 km (roughly 3 seconds per kilometer).

PRACTICE

1. Easy A wave has frequency 50 Hz and wavelength 0.40 m. Find its speed.

   Show answer

   $20\ \mathrm{m/s}$

2. Easy Find the wavelength of a 256 Hz sound in air at $20^\circ\mathrm{C}$.

   Show answer

   $1.34\ \mathrm{m}$

3. Easy Is sound transverse or longitudinal? Light?

   Show answer

   Sound in air is longitudinal; light is transverse.

4. Medium Find the speed of sound at $35^\circ\mathrm{C}$ and at $-10^\circ\mathrm{C}$.

   Show answer

   $352\ \mathrm{m/s}$; $325\ \mathrm{m/s}$

5. Medium A string 0.80 m long has a wave speed of 240 m/s. Find its first three resonant frequencies.

   Show answer

   150, 300, 450 Hz

6. Medium A ship's sonar pulse returns from the seabed 1.6 s after it is sent. How deep is the water ($v = 1500\ \mathrm{m/s}$)?

   Show answer

   $1200\ \mathrm{m}$

7. Medium A tube closed at one end is 0.30 m long. Find its fundamental frequency at 20°C (ignore the end correction).

Show answer

$343/1.2 = 286$ Hz

8. Medium Explain why the sand on a Chladni plate collects along lines.

Show answer

The plate vibrates in a standing-wave pattern. The sand is bounced off the vibrating regions (antinodes) and comes to rest on the nodal lines, where the plate does not move.

9. Hard A guitar string has mass per length 5.0×10^{-3} kg/m and length 0.65 m. What tension gives a fundamental of 196 Hz?

Show answer

$v = 2Lf = 255$ m/s; $T = \mu v^2 = 325$ N

10. Hard In the resonance-tube lab, resonances at 440 Hz occur at air-column lengths of 17.8 cm and 56.8 cm. Find the wavelength, the speed of sound and the end correction.

Show answer

$\lambda/2 = 39.0$ cm, $\lambda = 0.780$ m, $v = 343$ m/s; $\lambda/4 = 19.5$ cm, so the end correction is 1.7 cm.

11. Challenge Two tuning forks of 440 Hz and 444 Hz sound together. Explain why you hear the loudness rise and fall, and find how often. How do piano tuners use this?

Show answer

The two waves drift in and out of step, alternately adding and canceling: beats at the difference frequency, 4 per second. A tuner adjusts a string until the beats with a reference tone slow and disappear.

COMMON MISTAKES

- Thinking the medium travels with the wave. Each bit of the Slinky moves back and forth; only the disturbance travels.
- Using $\lambda = 2L$ for a tube closed at one end. That tube's fundamental is $\lambda = 4L$.

- Thinking frequency depends on the medium. The source sets the frequency; the medium sets the speed, and so the wavelength.

> **Make it a game · Wavelength Quick Calc.** Your parent names a frequency, such as 440 Hz, and a speed of sound near 343 m/s. Find the wavelength in 30 seconds. Then a pipe closed at one end 0.5 m long: find its fundamental wavelength and frequency. Hum or whistle to test a note against the answer. Five correct answers is a perfect round.

9.3 The Doppler Effect: Horns on a Train

OBJECTIVES

- Explain why the pitch of a moving source rises as it approaches and falls as it recedes.
- Use the Doppler equations for a moving source and a moving observer.
- Describe how Doppler radar and ultrasound measure speed, and what a sonic boom is.
- Measure a speed from the Doppler shift of a tone recorded with a phone.

> **Warm-up.** Stand beside a road (safely back from it) and listen to cars passing. Describe how the sound changes as each car approaches, passes and drives away. If an ambulance or fire truck passes with its siren on, notice the sudden drop in pitch. Write a guess at the cause before reading on.

WATCH

Doppler Effect Demonstrations and Animations

Flipping Physics · 7:24

https://www.youtube.com/watch?v=SqGtp6RQy_8

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

Brass band on train demonstrates Doppler effect

Health & Science Mania · 3:09

https://www.youtube.com/watch?v=WgMxtT_jYf0

With the story paragraph: a modern re-creation of Buys Ballot's experiment with musicians on a train.

KEY TERMS

Doppler effect

The change in observed frequency of a wave because the source or the observer is moving: higher when they approach each other, lower when they separate.

source and observer

The source emits the wave; the observer detects it. Their speeds are measured relative to the medium (the air, for sound).

redshift

The lowering of the frequency (lengthening of the wavelength) of light from a source moving away; blueshift for approach.

sonic boom

The shock wave produced when a source travels faster than sound, piling up its sound waves into a cone.

Mach number

Speed divided by the speed of sound.

CORE IDEAS

The story. In 1842 the Austrian physicist Christian Doppler, teaching in Prague, published a paper with a bold claim: the color of light from a star should change if the star moves toward or away from us, and the pitch of a sound should change if its source moves. Many scientists doubted him. In 1845 the Dutch meteorologist Christophorus Buys Ballot decided to test the idea with sound on the new railway near Utrecht. He put horn players on an open railway car, pulled by a locomotive at measured speeds, and stationed musicians with perfect pitch beside the track to name the note they heard as the car approached and as it receded.

How they figured it out. The listeners heard the horns' steady note rise as the train came toward them and fall as it went away, by about a half step at the highest speeds, close to what Doppler's formula predicted. Buys Ballot also did the reverse, with musicians on the train listening to horns on the platform. The effect was real. Doppler's prediction about starlight was much harder to test, because stars move slowly compared with light; it was first measured in 1868 by William Huggins in the spectrum of the star Sirius, and in the 1920s Doppler shifts of distant galaxies became one of the great measurements of astronomy.

The physics. A source emitting waves at frequency f while moving toward you crowds its wave crests together ahead of it: each crest is emitted a little closer to you than the last. The wavelength in front is shorter and you hear a higher frequency: $f' = f\dfrac{v}{v - v_s}$, where v is the wave speed and v_s the source's speed. Moving away, $f' = f\dfrac{v}{v + v_s}$. A moving observer meets crests more often: $f' = f\dfrac{v + v_o}{v}$ approaching, $f\dfrac{v - v_o}{v}$ receding. For light and radar, where the speeds are tiny compared with c, the fractional shift is about v/c, and for a wave reflected from a moving object it is doubled: $\Delta f \approx 2vf/c$. A source faster than sound outruns its own waves, which pile up into a cone-shaped shock wave, heard as a sonic boom.

The engineering. Police radar guns measure a car's speed from the Doppler shift of microwaves reflected from it. Weather radar measures the winds inside storms, including the rotation that warns of tornadoes. Doppler ultrasound measures the speed of blood in arteries and a baby's heartbeat. In 1957, engineers at Johns Hopkins University tracked the first artificial satellite by the Doppler shift of its radio beeps, and then reversed the method to let ships find their own positions from satellites, a forerunner of GPS.

Try it. In the lab you will record a steady tone from a phone swung in a circle on a cord and use the measured Doppler shift to find its speed.

PICTURE STUDY

Rain, Steam and Speed – The Great Western Railway, J. M. W. Turner, 1844 · Oil on canvas · National Gallery, London

Image: Wikimedia Commons, public domain

Turner painted this in 1844, the year before Buys Ballot's horn players rode the rails in the Netherlands. The railway had made speed something people could feel and hear for the first time.

Look closely

- Find the locomotive and its dark chimney. Which way is it moving?
- Look for the bridge and the river below.
- Find the small hare running on the track ahead of the train.

Talk about it

1. If the train whistled as it approached you, how would its pitch change as it passed?
2. How does Turner show speed in a still picture?
3. Why might the railways have made the Doppler effect easier to test than anything before?

Safety first. Tie the phone into a sock or pouch securely on a strong cord, swing it outdoors in a clear area, and keep everyone well outside the circle. Never stand near railway tracks or traffic to make recordings.

LAB

Measuring Speed with the Doppler Effect

You will swing a phone playing a steady tone in a circle on a cord, record the sound with a second phone running phyphox, measure the highest and lowest frequencies, and calculate the speed. Then you will check it against the speed measured from the radius and the time per turn.

You will need

- Two phones: one with a tone-generator app (phyphox's "Tone generator"), one with phyphox's "Audio spectrum" or "Doppler effect" experiment
- A thick sock or padded pouch and about 1.0 m of strong cord
- Tape measure, stopwatch
- A quiet, open outdoor space

Steps

1. Set the tone generator to a steady 2000 Hz (higher frequencies give larger, easier-to-measure shifts). Put the phone securely in the sock and tie the cord tightly.
2. **Hypothesis.** Predict the largest frequency the microphone will record if the phone moves at 6 m/s.
3. Place the recording phone on the ground 5 m from the swinger, in the plane of the circle. Start recording the spectrum or frequency against time.
4. Swing the phone in a steady horizontal circle of measured radius. Have a partner time 10 turns.
5. From the recording, find the highest and lowest frequencies. Calculate the speed from each with the moving-source formulas, and average.
6. Calculate the speed from the circle: $v = 2\pi r/T$. Compare with the Doppler speed.
7. **Fair test.** Swing faster and slower. Plot the Doppler speed against the circle speed. Is it a straight line with slope 1?

8. **Improve.** Identify errors (the circle not being horizontal, the phone not passing straight toward the microphone, the radius changing) and repeat your best trial.

Tips

- The phone is moving straight toward the microphone only for an instant each turn; the peak frequency is the one to use.
- Phyphox's audio spectrum view shows the frequency peak shifting back and forth as the phone circles.

Check your work

☐ I measured a Doppler shift and calculated a speed that agrees with $2\pi r/T$.

☐ I can use the Doppler formulas for moving sources and observers.

☐ I can explain how radar guns and weather radar measure speed.

LAB NOTEBOOK

1. Data table: radius, period, circle speed, highest and lowest frequencies, Doppler speeds.
2. Draw wave crests (circles) from a source moving to the right, showing them crowded in front and spread out behind.
3. Sketch the cone of a sonic boom and explain why you hear the boom after the airplane has passed overhead.

KEY FORMULAS

$$f' = f\frac{v \pm v_o}{v \mp v_s} \quad \text{(upper signs: approaching)}$$

$$f' = f\frac{v}{v - v_s} \quad \text{(source approaching a still observer)}$$

$$\Delta f \approx \frac{2v}{c} f \quad \text{(radar reflected from a moving target)}$$

$$\sin\theta = \frac{v}{v_s} \quad \text{(shock cone)}$$

From history. Buys Ballot is better known for a rule of weather: in the Northern Hemisphere, if you stand with your back to the wind, low pressure is on your left. Doppler's work on starlight inspired astronomers to measure the speeds of stars, and every modern weather radar station is, in effect, a Doppler instrument.

WORKED EXAMPLES

Example 1. A train sounds a 440 Hz horn while moving at 25 m/s. What frequency does a person beside the track hear as it approaches and as it recedes? (Use $v = 343$ m/s.)

Show solution

1. Approaching:
$$f' = f\frac{v}{v - v_s} = (440\text{ Hz})\frac{343}{343 - 25} = (440)(1.0786)\text{ Hz} = 475\text{ Hz}.$$
2. Receding: $f' = (440\text{ Hz})\dfrac{343}{343 + 25} = (440)(0.9321)\text{ Hz} = 410\text{ Hz}.$

Answer: 475 Hz approaching and 410 Hz receding; the pitch drops by about 65 Hz as it passes.

Example 2. A musical half step is a frequency ratio of 1.0595. How fast must Buys Ballot's horn players approach for listeners to hear the note raised by a half step?

Show solution

1. $\dfrac{f'}{f} = \dfrac{v}{v - v_s} = 1.0595$, so $v - v_s = \dfrac{v}{1.0595}$.
2. $v_s = v\left(1 - \dfrac{1}{1.0595}\right) = (343\text{ m/s})(0.0562) = 19.3\text{ m/s}$, about 43 mi/h.

Answer: About 19 m/s (43 mi/h), a speed the trains of 1845 could reach.

Example 3. A police radar gun emits 10.5 GHz microwaves. A car approaches at 30 m/s. What frequency shift does the gun measure in the reflected waves?

Show solution

1. For a reflection from a moving target,
   $\Delta f \approx \frac{2v}{c} f = \frac{2(30\ \mathrm{m/s})}{3.00 \times 10^8\ \mathrm{m/s}}(10.5 \times 10^9\ \mathrm{Hz})$.
2. $\Delta f = 2100$ Hz, a shift of only 2 parts in 10 million, easily measured by mixing the outgoing and returning waves.

Answer: 2.1 kHz.

Example 4. A phone playing 1000 Hz is swung in a circle. A microphone at a distance records a highest frequency of 1018 Hz. How fast is the phone moving?

Show solution

1. At the highest frequency the phone is moving straight toward the microphone: $f' = f\frac{v}{v - v_s}$.
2. $v_s = v\left(1 - \frac{f}{f'}\right) = (343\ \mathrm{m/s})\left(1 - \frac{1000}{1018}\right) = (343)(0.0177)\ \mathrm{m/s} = 6.1\ \mathrm{m/s}$.

Answer: About 6.1 m/s.

PRACTICE

1. Easy An ambulance siren sounds higher as it approaches and lower as it moves away. Is the siren actually changing its pitch?

   Show answer

   No. The siren's frequency is constant; the motion changes the frequency you receive.

2. Easy A source of 500 Hz approaches you at 20 m/s. What frequency do you hear?

   Show answer

   $500(343/323) = 531$ Hz

3. Easy If you move toward a stationary siren, does its pitch rise or fall?

Show answer

It rises.

4. Medium A car horn of 400 Hz moves away from you at 30 m/s. What frequency do you hear?

Show answer

$400(343/373) = 368\ \text{Hz}$

5. Medium You ride a bicycle at 8.0 m/s toward a stationary 600 Hz alarm. What frequency do you hear?

Show answer

$600(351/343) = 614\ \text{Hz}$

6. Medium A train's whistle is heard at 520 Hz as it approaches and 480 Hz as it recedes. Find the train's speed and the whistle's true frequency.

Show answer

$520(343 - v) = 480(343 + v)$: $v = 13.7\ \text{m/s}$; $f = 520(329.3/343) = 499\ \text{Hz}$

7. Medium A radar gun at 24 GHz measures a shift of 4800 Hz from a pitched baseball. How fast is the ball moving?

Show answer

$v = c\Delta f/(2f) = 30\ \text{m/s}$ (67 mi/h)

8. Medium Why does a sonic boom happen only when an aircraft flies faster than sound?

Show answer

Only then does the aircraft outrun its own sound waves, so the waves pile up along a cone-shaped shock front; slower aircraft send their waves out ahead of them.

9. Hard A jet flies at Mach 2 (686 m/s). Find the half-angle of its shock cone.

Show answer

$\sin\theta = 343/686 = 0.5$; $\theta = 30^\circ$

10. Hard A star's spectral line normally at 656.3 nm is observed at 656.6 nm. Is the star approaching or receding, and how fast? (Use $\Delta\lambda/\lambda \approx v/c$.)

Show answer

Receding (redshift); $v = (0.3/656.3)(3.00 \times 10^8) = 1.4 \times 10^5$ m/s (137 km/s)

11. Challenge Doppler ultrasound sends 5.0 MHz sound into the body ($v =$ 1540 m/s in tissue). Blood moving at 0.30 m/s directly toward the probe reflects it. Estimate the frequency shift, and explain why the shift falls in the range of human hearing, which is useful to doctors.

Show answer

$\Delta f \approx 2vf/v_{\text{sound}} = 2(0.30)(5.0 \times 10^6)/1540 = 1950$ Hz. The shift is in the audible range, so the machine can play it as a sound: doctors literally listen to the blood flow.

COMMON MISTAKES

- Getting the signs backward. Approach always raises the frequency; check that your answer does.
- Thinking the loudness change is the Doppler effect. The Doppler effect is the change in pitch (frequency).
- Using the moving-source formula for a moving observer. They are different (though nearly equal at low speeds).

> **Make it a game · Pitch Shift Predictions.** Your parent describes a siren moving toward or away from you at a given speed. Predict, before calculating, whether the pitch rises or falls, and by roughly what fraction. Then compute with the Doppler formula. Close estimates earn points. Act out the effect with a hummed tone and your parent walking past.

9.4 Reflection, Refraction and Lenses: Newton's Telescope

OBJECTIVES

- State the law of reflection and Snell's law of refraction, and calculate angles with them.
- Find critical angles and explain total internal reflection and fiber optics.
- Use the thin-lens equation and magnification to locate images.
- Measure the index of refraction of water and the focal length of a lens; build a periscope or kaleidoscope.

> **Warm-up.** Put a pencil in a glass of water and look at it from the side: it seems bent at the surface. Then put a coin in the bottom of an empty mug, move your head back until the coin just disappears behind the rim, and have someone slowly pour in water without moving the coin. What happens? Write down both observations.

WATCH

Refraction and Snell's law | Geometric optics | Physics | Khan Academy

Khan Academy · 14:24

https://www.youtube.com/watch?v=y55tzg_jW9I

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

How Isaac Newton Changed Telescopes Forever

Adam Savage's Tested · 10:54

https://www.youtube.com/watch?v=de8NxIOvdmY

With the story paragraph: Newton's reflecting telescope and why it mattered.

KEY TERMS

law of reflection

The angle of reflection equals the angle of incidence, both measured from the normal (the line perpendicular to the surface).

Snell's law

$n_1 \sin\theta_1 = n_2 \sin\theta_2$: light bends toward the normal entering a slower medium.

focal length

The distance from a lens or curved mirror to the point where parallel rays meet; for a mirror, $f = R/2$.

index of refraction

$n = c/v$, how many times slower light travels in a material than in a vacuum: water 1.33, glass about 1.5, diamond 2.42.

total internal reflection

Complete reflection of light trying to leave a slower medium at more than the critical angle, $\sin\theta_c = n_2/n_1$.

chromatic aberration

Colored fringes in a lens image, caused by different colors refracting by slightly different amounts.

CORE IDEAS

The story. In the 1660s the best telescopes were long tubes with glass lenses, and every bright object they showed was fringed with colored blur. In 1666 Isaac Newton, experimenting with prisms in a darkened room, found the reason: white light is a mixture of colors, and glass bends each color by a slightly different amount, so a lens cannot bring them all to one focus. He concluded (too pessimistically, it turned out) that lens telescopes could never be perfected. So in 1668 he built a telescope with no lens for its main element: a curved mirror of polished metal at the bottom of a tube, which reflected all colors alike to a focus, and a small flat mirror at 45° near the top to turn the light out the side to an eyepiece. It was about 15 cm long and magnified about 40 times, as well as a lens telescope ten times longer.

How they figured it out. Newton ground and polished the mirror himself from an alloy of copper and tin, a difficult job, and even made his own tools. In late 1671 the Royal Society asked to see it; it caused a sensation and he was elected a Fellow in 1672. The laws he relied on were older. The law of reflection was known to the ancient Greeks; the law of refraction was found by the Persian scholar Ibn Sahl around 984, and again by Willebrord Snell in the Netherlands in 1621. Today nearly

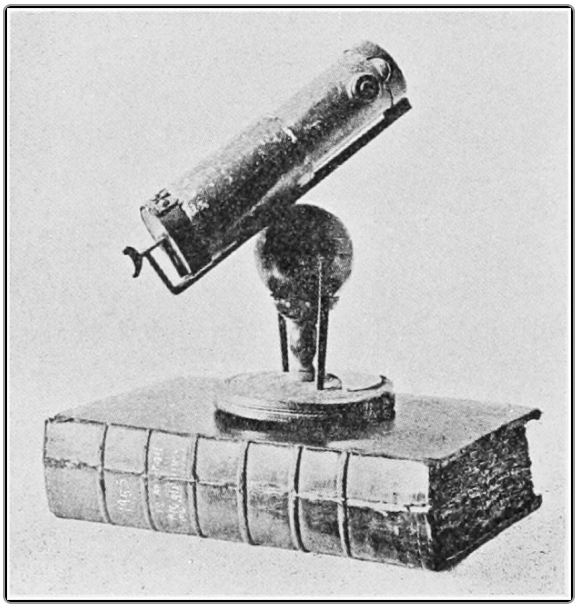
every large research telescope, from the 2.4 m Hubble to the 6.5 m James Webb Space Telescope, is a reflector, as Newton's was.

The physics. Light reflects at an equal angle: $\theta_r = \theta_i$, measured from the normal. When light passes into a material where it is slower, it bends toward the normal: $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Going the other way, from glass or water into air, it bends away from the normal, and beyond the **critical angle** ($\sin\theta_c = n_2/n_1$; 48.8° for water, about 41.8° for glass) none escapes: it is totally reflected. A converging lens or a concave mirror forms images obeying $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$, with magnification $m = -d_i/d_o$. A negative image distance means a virtual image, on the same side as the object, as in a magnifying glass.

The engineering. Optical fibers carry the internet as pulses of infrared light trapped by total internal reflection in glass threads thinner than a hair. Periscopes use mirrors or prisms (prisms reflect totally with no silver coating to tarnish). Camera and eyeglass designers combine lenses of different glasses to cancel chromatic aberration, which disproved Newton's pessimism in the 1750s. Kaleidoscopes, patented by a Scottish physicist in 1817, use two or three mirrors at angles to multiply an image into symmetrical patterns.

Try it. In the lab you will measure the index of refraction of water with a low-power laser pointer, find the focal length of a magnifying glass, and build a mirror periscope or kaleidoscope.

PICTURE STUDY



Newton's Reflecting Telescope, Unknown illustrator, 1906 · Halftone photograph · *Popular Science Monthly*, vol. 69

Image: Wikimedia Commons, public domain

This picture shows the small reflecting telescope kept by the Royal Society, believed to be made by Newton himself. It is short and stubby compared with the long lens telescopes of its day.

Look closely

- Find the eyepiece. Why is it on the side of the tube instead of at the end?
- Find the ball mount. How would the user point the telescope?
- Estimate the tube's length from its proportions.

Talk about it

1. Where inside the tube is the main mirror, and what does it do?
2. Why does a mirror not suffer from the colored fringes that a lens does?
3. Why are almost all large telescopes today reflectors?

Safety first. Use only a low-power laser pointer (5 mW or less, Class 2 or 3R). Never look into the beam or point it at anyone, at a mirror toward a face, or at aircraft. Handle mirrors and glass carefully; tape their edges.

LAB

Refraction, a Magnifier and a Periscope

You will measure the index of refraction of water with a laser beam, find a lens's focal length, and build a periscope (or a kaleidoscope) from mirrors.

You will need

- A clear rectangular plastic box or a straight-sided glass dish of water with a drop of milk (to make the beam visible), a laser pointer of 5 mW or less
- Protractor, paper, pencil
- A magnifying glass, a ruler, a sunny window or a bright lamp across the room, white card
- Two small flat mirrors and a long cardboard box or milk carton, tape, scissors (for the periscope); or three mirror strips taped into a triangle (for the kaleidoscope)

Steps

1. Set the water box on a sheet of paper. Draw the outline of the box and a normal line where the beam will enter.
2. **Hypothesis.** Predict the angle inside the water for a beam entering at 40° from the normal.
3. Shine the laser into the side of the box at several angles (10° to 60°). Mark the entry and exit points of the beam inside the water and measure the angle of refraction for each.
4. Graph $\sin\theta_1$ against $\sin\theta_2$. The slope is n_{water}. Compare with 1.33.
5. **Total internal reflection.** Aim the beam up through the water toward the surface at increasing angles. Find the angle at which the beam no longer leaves the water, and compare with the critical angle you calculate from your n.
6. **Lens.** Use the magnifying glass to focus the image of a distant window on a white card; the lens-to-card distance is f. Then focus a nearby lamp at a measured distance and check $1/f = 1/d_o + 1/d_i$.

7. **Build.** Cut openings at opposite ends and sides of a long box, tape a mirror at 45° behind each opening, and look over a wall with your periscope. (Or tape three mirror strips into a triangular tube for a kaleidoscope and count the reflected images.)
8. **Improve.** Identify your largest angle-measurement error and reduce it (a longer beam path, a finer pencil line). Repeat two angles.

Tips

- A little milk or a puff of chalk dust makes the laser beam visible.
- Measure all angles from the normal, not from the surface.

Check your work

- ☐ I measured the index of refraction of water from the slope of a graph.
- ☐ I observed total internal reflection and compared its angle with the critical angle.
- ☐ I measured a focal length and built a working periscope or kaleidoscope.

LAB NOTEBOOK

1. Ray diagram of each beam measurement and the $\sin\theta_1$–$\sin\theta_2$ graph.
2. Ray diagrams for a converging lens with the object (a) beyond $2f$, (b) between f and $2f$, (c) inside f.
3. Diagram of your periscope showing the 45° mirrors and the path of the light.

KEY FORMULAS

$$\theta_r = \theta_i$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}, \qquad m = -\frac{d_i}{d_o}$$

From history. In 1758 John Dollond showed that a lens made of two glasses, crown and flint, could cancel most chromatic aberration, proving Newton wrong about lens telescopes. Newton's own optical book, *Opticks* (1704), was written in English and full of experiments that readers could repeat; its closing "Queries" asked questions about light that took two centuries to answer.

Example 1. A light ray in air strikes a water surface ($n = 1.33$) at 30° from the normal. Find the angle of refraction.

Show solution

1. $n_1 \sin\theta_1 = n_2 \sin\theta_2$: $(1.00)\sin 30^\circ = 1.33 \sin\theta_2$.
2. $\sin\theta_2 = \dfrac{0.500}{1.33} = 0.376$, so $\theta_2 = 22.1^\circ$.

Answer: 22.1° from the normal, bent toward it.

Example 2. Find the critical angle for light going from water ($n = 1.33$) into air, and from glass ($n = 1.50$) into air.

Show solution

1. Water: $\sin\theta_c = \dfrac{1.00}{1.33} = 0.752$, $\theta_c = 48.8^\circ$.
2. Glass: $\sin\theta_c = \dfrac{1.00}{1.50} = 0.667$, $\theta_c = 41.8^\circ$.
3. A 45° prism therefore reflects light totally, which is why periscopes and binoculars use prisms.

Answer: 48.8° for water; 41.8° for glass.

Example 3. An object 30 cm from a converging lens of focal length 10 cm. Find the image distance and magnification.

Show solution

1. $\dfrac{1}{d_i} = \dfrac{1}{f} - \dfrac{1}{d_o} = \dfrac{1}{10} - \dfrac{1}{30} = \dfrac{2}{30}\ \text{cm}^{-1}$, so $d_i = 15$ cm.
2. $m = -\dfrac{d_i}{d_o} = -\dfrac{15}{30} = -0.50$: the image is real, inverted and half size.

Answer: $d_i = 15$ cm; $m = -0.50$.

Example 4. The same lens is used as a magnifier with an object 6.0 cm away. Find the image distance and magnification.

Show solution

1. $\frac{1}{d_i} = \frac{1}{10} - \frac{1}{6.0} = -\frac{1}{15}\ \text{cm}^{-1}$, so $d_i = -15$ cm.
2. $m = -\frac{-15}{6.0} = +2.5$: the image is virtual, upright and 2.5 times larger, on the same side as the object.

Answer: $d_i = -15$ cm (virtual); $m = +2.5$.

PRACTICE

1. Easy Light hits a mirror at 35° from the normal. At what angle does it reflect?

   Show answer

   35°

2. Easy How fast does light travel in glass with $n = 1.50$?

   Show answer

   $2.0 \times 10^8\ \text{m/s}$

3. Easy Why does a straight stick look bent where it enters water?

   Show answer

   Light from the underwater part bends away from the normal as it leaves the water, so it seems to come from a shallower place than it does.

4. Medium Light passes from water ($n = 1.33$) into glass ($n = 1.50$) at 40° from the normal. Find the angle in the glass.

   Show answer

   $\sin\theta_2 = 1.33 \sin 40^\circ / 1.50 = 0.570$; 34.8°

5. Medium Find the critical angle for diamond ($n = 2.42$). Why does this make diamonds sparkle?

   Show answer

   24.4°. Light inside a cut diamond is totally reflected at most of its facets and bounces around before escaping, often split into colors.

6. Medium An object is 20 cm from a lens of focal length 15 cm. Find the image distance and magnification.

Show answer

$d_i = 60$ cm; $m = -3.0$

7. Medium A concave mirror has a radius of curvature of 1.2 m. What is its focal length? Where does it focus starlight?

Show answer

$f = 0.60$ m; at its focal point, 0.60 m in front of it.

8. Medium Why did Newton build a telescope with a mirror?

Show answer

Lenses split white light into colors (chromatic aberration), blurring images; a mirror reflects all colors at the same angle, so it focuses them together.

9. Hard An optical fiber has a core of $n = 1.48$ and cladding of $n = 1.46$. Find the critical angle at the core–cladding boundary. Why is light trapped even when the fiber bends gently?

Show answer

$\sin\theta_c = 1.46/1.48$, $\theta_c = 80.6°$. Light traveling nearly along the fiber strikes the wall at glancing angles greater than this, so it is totally reflected over and over.

10. Hard A camera lens with $f = 50$ mm photographs a person 3.0 m away. How far from the lens is the sensor, and how tall is the image of a 1.8 m person?

Show answer

$d_i = 50.8$ mm; $m = -0.0169$; image height 3.0 cm (inverted)

11. Challenge Two mirrors meet at 60°, as in a kaleidoscope. How many images of an object between them do you see? Explain the rule $N = 360°/\theta - 1$.

Show answer

Five images. Each reflection rotates the scene by twice the mirror angle; the full circle of 360° contains 360/60 = 6 copies, one of which is the object itself, leaving 5 images.

COMMON MISTAKES

- Measuring angles from the surface instead of from the normal.

- Bending light the wrong way: into a slower (higher-n) material it bends toward the normal.
- Ignoring the sign of d_i: a negative image distance means a virtual image.

> **Make it a game · Ray Diagram Showdown.** Draw a ray bending into water on paper with an incident angle of 30 degrees and n of 1.33. Predict the refracted angle is smaller or larger, then compute it with Snell's law. Your parent sets a new angle. Add a critical-angle question for a bonus. Fastest accurate diagram wins the round.

9.5 Light as a Wave: Young's Double Slit

OBJECTIVES

- Explain interference and how Young's double-slit experiment showed that light is a wave.
- Use $d \sin\theta = m\lambda$ and $\Delta y = \lambda L/d$ to find wavelengths and fringe spacings.
- Explain how a diffraction grating separates colors.
- Measure the wavelength of a laser with a CD or DVD used as a diffraction grating.

> **Warm-up.** Look at the shiny side of an old CD or DVD under a bright light and tilt it slowly. Where do the rainbow colors come from? There is no dye or paint on the disc. Then look at a distant lamp through the narrow gap between two fingers held almost touching. Describe the fine dark lines you see.

WATCH

The Original Double Slit Experiment

Veritasium · 7:39

https://www.youtube.com/watch?v=Iuv6hY6zsd0

> **WHEN TO WATCH**
> After the story paragraph. The video re-creates Young's original experiment with sunlight and a card.

BACKUP VIDEO

Lasers measured with CD measured with lasers!?!

Brainiac75 · 12:21

https://www.youtube.com/watch?v=ZUszmEDm3FU

Before the lab: measuring a laser's wavelength with a CD and a DVD.

KEY TERMS

interference

The adding of two or more waves: crest on crest gives a bigger wave (constructive), crest on trough cancels (destructive).

path difference

The difference in the distances two waves travel to reach a point; constructive interference when it is a whole number of wavelengths.

diffraction

The spreading of waves around obstacles and through openings.

diffraction grating

A surface with many evenly spaced lines or grooves; light of wavelength λ is sent strongly at angles where $d \sin\theta = m\lambda$.

fringe

One of the bright or dark bands in an interference pattern.

CORE IDEAS

The story. For a century after Newton, most scientists thought light was a stream of tiny particles, as Newton had suggested. Thomas Young, an English physician who also helped decipher the Rosetta Stone, disagreed. In a lecture to the Royal Society in November 1803 he described an experiment anyone could repeat. He let a narrow beam of sunlight into a darkened room through a small hole in a window shutter and held a thin card, about a thirtieth of an inch thick, edge-on in the beam, splitting it into two parts that passed on either side of the card. On a screen beyond, where the two beams overlapped, he saw not a single bright patch but a row of bright and dark bands, with a bright band in the very center, behind the card.

How they figured it out. Particles cannot cancel each other out; waves can. Young explained the dark bands as places where the waves from the two sides of the card arrived out of step, crest meeting trough, and the bright bands as places where they arrived in step. When he blocked the light on one side of the card, the bands disappeared. From the spacing of the bands he calculated the wavelength of light, about 1/36,000 of an inch (0.7 micrometers) for red and about half that for violet. The particle view did not give way at once. In 1818 Augustin Fresnel's wave theory predicted a bright spot at the very center of the shadow of a round disk; François Arago looked, and found it.

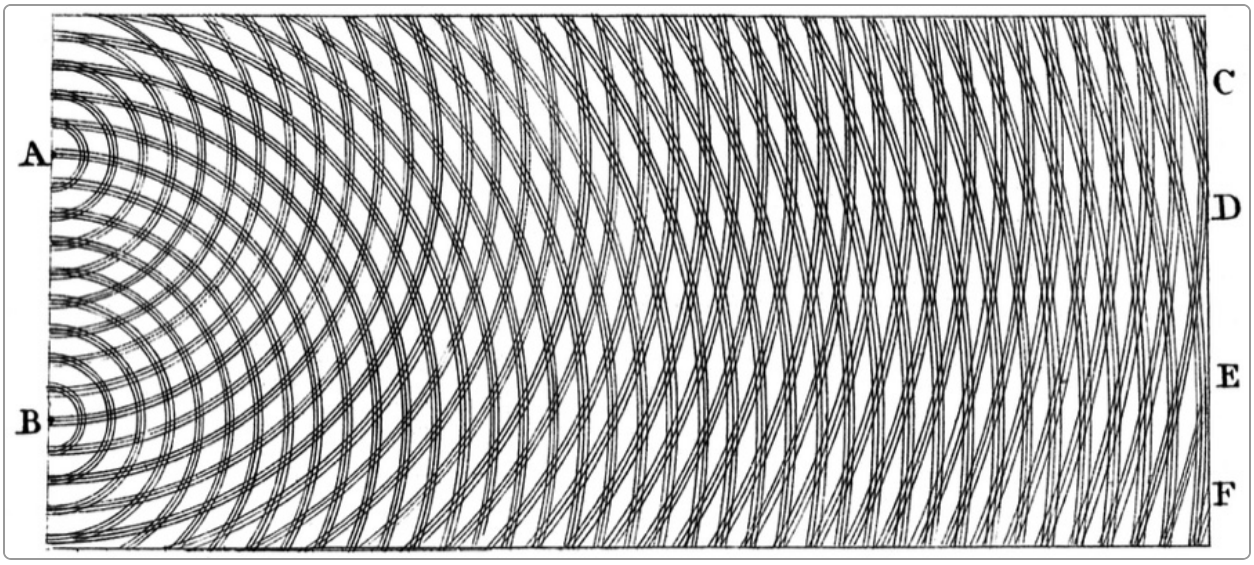
The physics. Two waves of the same wavelength from sources a distance d apart reach a distant point with a path difference of $d \sin\theta$. Where that is a whole number of wavelengths, they add (bright fringes): $d \sin\theta = m\lambda$. Halfway between, they cancel (dark fringes). On a screen a distance L away, for small angles, the bright fringes are spaced $\Delta y = \dfrac{\lambda L}{d}$. A **diffraction grating**, with thousands of slits per

millimeter, makes much sharper, brighter maxima at the same angles. The grooves of a CD are 1.6 μm apart; those of a DVD 0.74 μm. Each color goes to a different angle, which is why discs shine with rainbows.

The engineering. Spectrometers use gratings to split light into its wavelengths and identify what a star, a flame or a medicine is made of. Anti-reflection coatings on eyeglasses and camera lenses are thin films that make reflected waves interfere destructively. CD, DVD and Blu-ray players read their discs with lasers whose spot size is set by diffraction, which is why Blu-ray uses a shorter, blue wavelength to pack data more tightly. The LIGO detectors, the most sensitive rulers ever built, use the interference of laser light in 4-km arms to sense stretches smaller than a proton.

Try it. In the lab you will shine a laser pointer on a CD and a DVD, measure the angles of the bright spots, and calculate the laser's wavelength.

PICTURE STUDY



Young's Drawing of Interference, Thomas Young, 1807 · Engraving · *A Course of Lectures on Natural Philosophy and the Mechanical Arts*, London

Image: Wikimedia Commons, public domain

Young drew this figure to explain interference, using water waves as his model. You can see it best by holding the page level with your eye and sighting along it from the left edge.

Look closely

- Find the two sources on the left edge.
- Find the circular wave crests from each.
- Hold the picture at eye level and look along it. Find the lines where the waves seem to vanish and the lines where they are strong.

Talk about it

1. Where do crests from one source fall on troughs from the other?
2. What would change in the pattern if the sources were moved farther apart?
3. Why does this pattern show that light behaves like a wave?

Safety first. Use only a laser pointer of 5 mW or less. Never look into the beam or its reflections, and never point it at a person, an animal, a vehicle or an aircraft. Keep the setup low, below eye level, and have an adult present.

LAB

Measuring a Laser's Wavelength with a CD

The tracks of a CD and a DVD act as diffraction gratings with known spacings. You will measure the angles of the bright spots a laser makes and calculate its wavelength.

You will need

- A red or green laser pointer of 5 mW or less
- An old CD and DVD (a DVD may need its two layers pried apart by an adult, or use it whole)
- A large sheet of white paper taped to a wall, a meter stick, tape
- Books to hold the laser and disc steady

Steps

1. Prop the CD upright so the laser beam hits it at right angles and reflects straight back near the laser. Tape white paper on a wall facing the disc.
2. **Hypothesis.** Predict, from $d = 1.6\ \mu\text{m}$ and the laser's labeled wavelength, how far the first-order spots will be from the center if the wall is 0.50 m away.
3. Turn on the laser. You will see a central reflected spot and first-order spots on each side (on the wall or on a card held near the laser). Measure the distance L from disc to screen and the distance y from the central spot to each first-order spot.
4. Calculate $\theta = \tan^{-1}(y/L)$ and $\lambda = d \sin\theta$. Average the two sides.
5. Repeat at two other distances L. Your wavelength should not change.
6. Repeat with the DVD ($d = 0.74\ \mu\text{m}$). Why are its spots so much farther out?
7. Compare your result with the wavelength printed on the laser's label. Find the percent error.
8. **Without a disc.** Hold a single hair across the beam and look at the pattern on a wall 2 m away. The dark gaps are spaced about $\lambda L/w$, where w is the hair's width. Use your measured λ to estimate the width of the hair.
9. **Improve.** The main error is usually measuring y and L. Use a larger L (where the spots are farther apart) and mark spot centers carefully.

Tips

- Do this in a dim room so the spots are easy to see.
- Keep your eyes above the beam's height at all times.

Check your work

☐ I measured a laser's wavelength within about 5% of its labeled value.

☐ I can explain interference and Young's experiment in my own words.

☐ I can calculate fringe spacing and grating angles.

LAB NOTEBOOK

1. A labeled top-view diagram of the setup with L, y and θ; the data table and wavelength calculations.
2. Draw two sets of circular waves from two sources and mark the lines of constructive and destructive interference, as Young did.
3. Write a paragraph explaining why Young's dark fringes could not be explained if light were made of particles that simply travel in straight lines.

KEY FORMULAS

$$d \sin\theta = m\lambda \quad \text{(bright fringes)}$$

$$\Delta y = \frac{\lambda L}{d} \quad \text{(small angles)}$$

$$\tan\theta = \frac{y}{L}$$

From history. Young was a child prodigy who read at two and learned more than a dozen languages; he also proposed that the eye sees color with three kinds of receptors, red, green and blue, which is how every screen works today. His wave theory of light was later completed by Maxwell, who showed that light is an electromagnetic wave (Lesson 10.5).

WORKED EXAMPLES

Example 1. A red laser shines through two slits 0.20 mm apart onto a wall 2.0 m away. The bright fringes are 6.5 mm apart. Find the wavelength.

Show solution

1. $\lambda = \dfrac{\Delta y\, d}{L} = \dfrac{(6.5 \times 10^{-3}\ \text{m})(0.20 \times 10^{-3}\ \text{m})}{2.0\ \text{m}}$.
2. $\lambda = 6.5 \times 10^{-7}\ \text{m} = 650\ \text{nm}$.

Answer: 650 nm, red light.

Example 2. A CD's tracks are 1.6 μm apart. A 650 nm laser reflects from it. At what angle is the first-order bright spot, and how far from the central spot does it land on a wall 1.0 m away?

Show solution

1. $\sin\theta = \dfrac{m\lambda}{d} = \dfrac{(1)(650 \times 10^{-9}\ \text{m})}{1.6 \times 10^{-6}\ \text{m}} = 0.406$, so $\theta = 24.0^\circ$.
2. $y = L\tan\theta = (1.0\ \text{m})\tan 24.0^\circ = 0.445\ \text{m}$.

Answer: 24.0°; about 44 cm from the central spot.

Example 3. Young estimated red light's wavelength as about 1/36,000 of an inch. Express that in nanometers (1 in = 2.54 cm).

Show solution

1. $\lambda = \dfrac{0.0254\ \text{m}}{36{,}000} = 7.06 \times 10^{-7}\ \text{m} = 706\ \text{nm}$.

Answer: About 706 nm, a good value for deep red light (visible light spans about 400–700 nm).

PRACTICE

1. Easy What kind of interference makes a bright fringe? A dark fringe?

   Show answer

   Bright: constructive (waves in step). Dark: destructive (crest meets trough).

2. Easy Find the fringe spacing for 500 nm light, slits 0.10 mm apart, screen 1.5 m away.

Show answer

7.5 mm

3. Easy If the slits are moved closer together, do the fringes get closer together or farther apart?

Show answer

Farther apart ($\Delta y = \lambda L/d$).

4. Medium Green light (532 nm) falls on a grating with 600 lines per millimeter. Find the angle of the first-order maximum.

Show answer

$d = 1.67\ \mu\text{m}$; $\sin\theta = 0.319$; $\theta = 18.6^\circ$

5. Medium How many orders can be seen for 650 nm light on a CD ($d = 1.6\ \mu\text{m}$)?

Show answer

$m \leq d/\lambda = 2.46$, so orders 1 and 2 (plus the central spot).

6. Medium A DVD ($0.74\ \mu\text{m}$) sends a red laser's first-order spot to 61°. What is the wavelength?

Show answer

$\lambda = 0.74 \sin 61^\circ = 0.647\ \mu\text{m} = 647\ \text{nm}$

7. Medium Why does a CD show rainbow colors in white light?

Show answer

Each wavelength is sent to a different angle by the grating of tracks ($d \sin\theta = m\lambda$), so the colors separate.

8. Medium In Young's experiment with red light (700 nm), the two edges of his card were about 0.85 mm apart and the screen about 2.5 m away. Estimate the fringe spacing.

Show answer

$\Delta y = (700 \times 10^{-9})(2.5)/(0.85 \times 10^{-3}) = 2.1\ \text{mm}$

9. Hard A thin hair placed in a laser beam (650 nm) produces dark gaps 9.0 mm apart on a wall 1.0 m away. Estimate the hair's width.

Show answer

$$w \approx \lambda L/\Delta y = 72\ \mu\text{m}$$

10. Hard Blu-ray players use a 405 nm laser and DVDs a 650 nm laser. If the smallest readable spot is roughly proportional to wavelength, by about what factor can a Blu-ray disc of the same area hold more data?

Show answer

Spot area scales as λ^2: $(650/405)^2 = 2.6$ times (better lenses add more, so real Blu-ray discs hold about five times as much).

11. Challenge Show that for small angles the bright fringes in a double-slit pattern are equally spaced by $\Delta y = \lambda L/d$, starting from $d \sin\theta = m\lambda$ and $y = L\tan\theta$.

Show answer

For small angles $\sin\theta \approx \tan\theta \approx \theta$, so $y_m = L\theta_m = mL\lambda/d$. Successive fringes differ by $y_{m+1} - y_m = \lambda L/d$.

COMMON MISTAKES

- Using the small-angle formula $\Delta y = \lambda L/d$ for gratings, where angles are large. Use $d\sin\theta = m\lambda$ and $\tan\theta = y/L$.
- Forgetting to convert lines per millimeter into spacing d in meters.
- Thinking interference makes light disappear. Energy that misses the dark fringes goes into the bright ones.

Make it a game · Fringe Spacing Estimate. Your parent names a wavelength, a slit spacing and a screen distance, for example 650 nm, 0.1 mm and 2 m. Estimate the fringe spacing mentally using delta y = lambda L / d, then calculate precisely. Beat your own estimate error across five rounds. Explain why red light spreads wider than blue.

Scripture. "Who coverest thyself with light as with a garment" (Psalm 104:2, KJV)

Light is one of the most beautiful and mysterious things God made. The more carefully scientists like Young and Maxwell studied it, the more wonderful it proved to be.

UNIT 9

Waves, Sound and Light

A wave carries energy without carrying matter, and much of what we know about the world comes to us by waves: sound, light, radio, the shaking of the ground. This unit starts with the simplest oscillation, the pendulum, which Huygens turned into the first accurate clock, and moves on to waves on a Slinky and on Chladni's singing plates, the Doppler effect that Buys Ballot tested with horn players on a train, the reflection and refraction of light that Newton mastered in his mirror telescope, and finally Thomas Young's double-slit experiment, which showed that light itself is a wave.

The labs measure g with a pendulum, time a toy spring, send pulses down a Slinky, measure the speed of sound with a phone tone and a tube of water, find a speed from a Doppler shift, measure the index of refraction of water, build a periscope or kaleidoscope, and measure the wavelength of a laser with an old CD.

PREREQUISITES

- Unit 1: graphing and linearizing; Unit 2: kinematics
- Lesson 1.4 (Hooke's law) and Unit 5 (energy)
- Right-triangle trigonometry, including sine and inverse sine

MASTERY CHECKLIST

☐ I can use $T = 2\pi\sqrt{L/g}$ and $T = 2\pi\sqrt{m/k}$ and explain what makes motion simple harmonic.

☐ I can use $v = f\lambda$, find standing-wave frequencies for strings and tubes, and explain resonance.

☐ I can apply the Doppler equations for sound and explain Doppler radar and sonic booms.

☐ I can use the law of reflection, Snell's law, critical angles and the thin-lens equation.

☐ I can explain Young's experiment and use $d\sin\theta = m\lambda$ to find wavelengths.

OPTIONAL KITS FOR THIS UNIT

- **Arbor Scientific optics and waves sets**: Inexpensive diffraction gratings, slit slides, lens sets and spectroscope glasses for Lessons 9.4–9.5, and a long spring for waves in Lesson 9.2.
- **Thames & Kosmos optics kits**: Ages 10 and up. Lens, mirror and prism kits for building telescopes, periscopes and kaleidoscopes in Lesson 9.4.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

9.1	**Oscillations: Huygens and the Pendulum Clock**	3 sessions of 50 minutes
9.2	**Waves and Sound: The Slinky and Chladni's Plates**	3 sessions of 50 minutes

9.3	**The Doppler Effect: Horns on a Train**	3 sessions of 50 minutes
9.4	**Reflection, Refraction and Lenses: Newton's Telescope**	3 sessions of 50 minutes
9.5	**Light as a Wave: Young's Double Slit**	3 sessions of 50 minutes

Unit 9 test

Calculators are allowed. Use $g = 9.8\ \mathrm{m/s^2}$, the speed of sound $343\ \mathrm{m/s}$ (unless told otherwise) and $c = 3.00 \times 10^8\ \mathrm{m/s}$. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **5 pts**
   A pendulum is $1.20\ \mathrm{m}$ long. Find its period.

2. **5 pts**
   A $0.30\ \mathrm{kg}$ mass on a spring oscillates with a period of $0.60\ \mathrm{s}$. Find the spring constant.

3. **5 pts**
   Why did Huygens' pendulum clock keep much better time than earlier clocks? What did it not do well?

4. **5 pts**
   A wave on a rope has a frequency of $4.0\ \mathrm{Hz}$ and a wavelength of $1.5\ \mathrm{m}$. Find its speed.

5. **6 pts**
   A string $0.70\ \mathrm{m}$ long fixed at both ends has a wave speed of $280\ \mathrm{m/s}$. Find its fundamental frequency and the wavelength of its third harmonic.

6. **6 pts**
   A tube closed at one end resonates first at a length of $0.25\ \mathrm{m}$ for a $340\ \mathrm{Hz}$ tone. Find the speed of sound (ignore the end correction).

7. 5 pts

Explain why sand collects in lines on a vibrating Chladni plate.

8. 7 pts

A siren of 800 Hz moves at 30 m/s. What frequency does a person at rest hear as it approaches and as it moves away?

9. 5 pts

Describe Buys Ballot's 1845 test of the Doppler effect and what it showed.

10. 6 pts

A radar gun at 10.5 GHz detects a 1750 Hz shift from a car. How fast is the car moving?

11. 6 pts

Light in air enters glass ($n = 1.52$) at 45°. Find the angle of refraction.

12. 5 pts

Find the critical angle for light leaving glass ($n = 1.52$) into air, and explain how optical fibers use it.

13. 6 pts

An object is 25 cm from a converging lens of focal length 10 cm. Find the image distance and magnification, and say whether the image is real or virtual.

14. 5 pts

Why did Newton build a reflecting telescope?

15. 6 pts

Light passes through two slits 0.25 mm apart onto a screen 3.0 m away. The bright fringes are 7.2 mm apart. Find the wavelength.

16. 6 pts

A grating has 500 lines per millimeter. Find the first-order angle for 600 nm light.

17. **11 pts**

Essay. Explain how Thomas Young's double-slit experiment showed that light is a wave. Include what he saw, why particles could not explain it, and how he used the pattern to measure the wavelength of light.

ANSWER KEY

1. $2.20\ \mathrm{s}$
2. $32.9\ \mathrm{N/m}$
3. A pendulum's period depends only on its length and g, not on the size of the swing (for small swings) or the driving force, so it regulated the clock steadily. It worked poorly on ships, whose motion disturbed the pendulum.
4. $6.0\ \mathrm{m/s}$
5. $200\ \mathrm{Hz}$; $0.467\ \mathrm{m}$
6. $v = 4Lf = 340\ \mathrm{m/s}$
7. The plate vibrates in a standing-wave pattern; sand is bounced away from the moving antinodes and settles on the nodal lines, where the plate stays still.
8. $877\ \mathrm{Hz}$; $736\ \mathrm{Hz}$
9. Horn players on an open railway car played a steady note while the train passed listeners with perfect pitch at measured speeds. The listeners heard the pitch rise as the car approached and fall as it receded, by about the amount Doppler's formula predicted.
10. $v = c\Delta f/(2f) = 25\ \mathrm{m/s}$
11. 27.7°
12. 41.1°. Light inside a fiber strikes its wall at angles larger than critical, so it is totally reflected and stays inside the fiber.
13. $d_i = 16.7\ \mathrm{cm}$; $m = -0.67$; real and inverted
14. Glass lenses split white light into colors (chromatic aberration), blurring images; a mirror reflects all colors alike, so it focuses them together.
15. $600\ \mathrm{nm}$
16. $d = 2.0\ \mu\mathrm{m}$; $\sin\theta = 0.30$; 17.5°
17. Model answer: Young split a narrow beam of sunlight with a thin card so the two halves overlapped on a screen. Instead of a single bright patch he saw alternating bright and dark bands, with a bright band in the center. Particles traveling in straight lines would simply add, giving brightness wherever either beam reached; they cannot cancel to make darkness. Waves can: where crests from one side meet troughs from the other they cancel (dark), and where they meet in step they reinforce (bright). Blocking one beam made the bands vanish, confirming that both were needed. The spacing of the bands depends on the wavelength, the separation of the sources and the screen distance ($\Delta y = \lambda L/d$),

so by measuring them he calculated the wavelength of red light as about 1/36,000 inch (0.7 μm).

PORTFOLIO PIECE: A MUSICAL INSTRUMENT, DESIGNED BY PHYSICS

Design and build a simple instrument that plays at least five notes of a scale: a set of tuned bottles or tubes (air columns), a box guitar with strings of adjustable tension, or a xylophone of wooden or metal bars. Use physics to choose the dimensions before building: for tubes, $\lambda = 4L$ and $v = f\lambda$; for strings, $f = \frac{1}{2L}\sqrt{T/\mu}$. Measure each note's frequency with phyphox's audio spectrum, compare with your design, and adjust to tune it. Play a short hymn or folk tune on it. Write a report with your design calculations, a table of target and measured frequencies, a sketch of the standing waves for two of your notes, and what you changed to tune it.

LOOK FOR	WHAT IT MEANS
Design calculations	Dimensions for at least five notes calculated from wave physics before building, with units.
Measurement	Each note's frequency measured, compared with the target, and percent differences given.
Tuning and iteration	Documented adjustments that brought notes closer to target.
Physics explanation	Standing-wave diagrams and a clear explanation of resonance, nodes and antinodes for the instrument.
Performance	Plays a recognizable short tune.

UNIT 10

Electricity and Magnetism

The last unit traces one of the greatest stories in science: the discovery that electricity, magnetism and light are one thing. It begins with Coulomb measuring the force between charges with a torsion balance, and Volta's pile and Ohm's law, which turned the electric current into a tool. Then Oersted's compass needle swung beside a wire, Faraday found that a moving magnet makes a current, and Maxwell wrote the equations that showed light to be an electromagnetic wave. The course ends at the doorway of modern physics, with Planck's and Einstein's photons and Bohr's atom.

The labs build an electroscope, measure the charge on balloons, graph current against voltage with a multimeter, repeat Oersted's experiment, build an electromagnet and a simple motor, induce a current with a magnet, and make a spectroscope from a CD. Every circuit runs on batteries; none touches the outlets in the wall.

PREREQUISITES

- Unit 4: inverse-square laws and circular motion
- Unit 5: energy and power
- Unit 9: waves, $c = f\lambda$ and diffraction gratings
- Scientific notation

MASTERY CHECKLIST

- ☐ I can use Coulomb's law and explain charging by friction, contact and induction.
- ☐ I can use Ohm's law and the series and parallel rules, and calculate electric power.
- ☐ I can find the magnetic fields of wires and solenoids and the magnetic force on currents and moving charges.
- ☐ I can use Faraday's and Lenz's laws and explain generators, motors, transformers and high-voltage transmission.
- ☐ I can explain how Maxwell's equations predicted light, and use $E = hf$ and the Bohr energy levels.

OPTIONAL KITS FOR THIS UNIT

- **Snap Circuits (Elenco)**: Ages 8 and up. Snap-together circuit parts that make Lesson 10.2's series and parallel circuits, and motors and electromagnets for Lessons 10.3–10.4, quick and reliable. Battery-powered.
- **Arbor Scientific electricity and magnetism sets**: Inexpensive coils, strong magnets, galvanometers, electroscopes and diffraction gratings for Lessons 10.1–10.5.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

10.1 Charge and Coulomb's Law	3 sessions of 50 minutes
10.2 Circuits and Ohm's Law	3 sessions of 50 minutes
10.3 Magnetism: Gilbert, Oersted and the Electric Current	3 sessions of 50 minutes
10.4 Faraday's Induction: Generators, Motors and Transformers 3 sessions of 50 minutes	
10.5 Maxwell, Light and the Photon	3 sessions of 50 minutes

Unit 10 test

Calculators are allowed. Use $k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$, $e = 1.602 \times 10^{-19}\ \mathrm{C}$, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$, $h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$, $c = 3.00 \times 10^8\ \mathrm{m/s}$ and $E(\mathrm{eV}) = 1240/\lambda(\mathrm{nm})$. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **6 pts**
   Find the force between charges of $+4.0\ \mu\mathrm{C}$ and $+2.0\ \mu\mathrm{C}$ placed $0.20\ \mathrm{m}$ apart. Is it attractive or repulsive?

2. **5 pts**
   Explain how a rubbed balloon can stick to a neutral wall.

3. 5 pts

How did Coulomb measure the force between charges, and what law did he find?

4. 5 pts

A 12 V battery drives 0.40 A through a lamp. Find its resistance and power.

5. 7 pts

Resistors of 30 Ω and 60 Ω are connected (a) in series and (b) in parallel to a 9.0 V battery. Find the total current in each case.

6. 5 pts

What resistor should be placed in series with an LED that drops 2.2 V at 15 mA, run from a 6.0 V battery?

7. 5 pts

Describe Oersted's 1820 discovery and why it was important.

8. 6 pts

Find the magnetic field 3.0 cm from a long wire carrying 12 A.

9. 6 pts

A solenoid of 800 turns and length 0.20 m carries 1.5 A. Find the field inside.

10. 5 pts

A 0.30 m wire with 4.0 A lies perpendicular to a 0.25 T field. Find the force on it.

11. 6 pts

A 300-turn coil of area 2.0×10^{-3} m^2 is perpendicular to a field that drops from 0.50 T to zero in 0.020 s. Find the average induced emf.

12. 5 pts

State Lenz's law and use it to explain why a magnet falls slowly through a copper pipe.

13. 6 pts

A transformer steps 240 V down to 12 V for a 36 W device. Find the turns ratio and the primary current.

14. **6 pts**

Explain, with a calculation, why power is transmitted at high voltage: compare the line loss for 2.0 MW sent through a 4.0 Ω line at 20 kV and at 200 kV.

15. **5 pts**

Find the wavelength of a 1.9 GHz cell-phone signal.

16. **6 pts**

Find the energy in eV of a 480 nm photon. Light of this wavelength falls on a metal with work function 2.3 eV; find the maximum kinetic energy of ejected electrons.

17. **11 pts**

Essay. Trace the chain of discoveries from Oersted (1820) through Faraday (1831) to Maxwell (1865) and Hertz (1887). Explain how each step built on the one before, and why Maxwell's conclusion that light is an electromagnetic wave was such a turning point.

ANSWER KEY

1. 1.8 N, repulsive
2. The balloon's charge polarizes the wall by induction, pulling opposite charges slightly closer to the surface; the attraction to the nearer opposite charges outweighs the repulsion from the farther like charges.
3. With a torsion balance: a charged ball on a needle hung from a fine wire was repelled by another charged ball, and the twist of the wire needed to hold them at a given distance measured the force. He found the force proportional to each charge and inversely proportional to the square of the distance.
4. 30 Ω; 4.8 W
5. (a) 90 Ω, 0.10 A; (b) 20 Ω, 0.45 A
6. 253 Ω (use 270 Ω)
7. A compass needle near a wire swung when current flowed and reversed when the current reversed: an electric current makes a magnetic field. It was the first link between electricity and magnetism, leading directly to electromagnets, motors and Faraday's induction.
8. 8.0×10^{-5} T
9. 7.5×10^{-3} T
10. 0.30 N
11. 15 V
12. Induced currents flow so as to oppose the change in flux that causes them. The falling magnet induces eddy currents in the pipe whose fields oppose its motion, slowing it.
13. 20 to 1; 0.15 A
14. At 20 kV: $I = 100$ A, loss $= 40$ kW. At 200 kV: $I = 10$ A, loss $= 400$ W. Ten times the voltage, one hundredth the loss.
15. 0.158 m
16. 2.58 eV; 0.28 eV
17. Model answer: Oersted showed that an electric current produces a magnetic field. Faraday, reasoning that the reverse should hold, found that a changing magnetic field produces a current (induction), and he described fields as lines of force filling space. Maxwell put Faraday's fields into equations and added that a changing electric field produces a magnetic field; the equations then predicted self-sustaining waves of electric and magnetic fields, and their calculated speed

matched the measured speed of light, so light must be such a wave. Hertz generated and detected these waves electrically, confirming the theory and opening the way to radio. The result united electricity, magnetism and optics in one theory, gave engineers the tools for every wireless technology, and raised the questions about light and matter that led to quantum physics.

PORTFOLIO PIECE: A WORKING ELECTRIC MACHINE

Design, build and test one battery-powered electric machine and explain it with this unit's physics: a simple motor that turns a small fan, a hand-cranked generator that lights an LED (a hobby motor turned by hand works as a generator), a buzzer or doorbell made with an electromagnet and a spring contact, or a speaker made from a coil, a magnet and a paper cup. Make measurements: the motor's speed (turns per second by slow-motion video) and current for two battery voltages; the generator's voltage at two cranking speeds; the electromagnet's pull against the number of turns. Draw a labeled diagram and a circuit diagram, and write a report explaining how Ohm's law, magnetic forces and induction make your machine work, with your data and one improvement you made.

LOOK FOR	WHAT IT MEANS
Working machine	The machine works reliably on batteries only, with safe wiring.
Diagrams	Labeled drawing and a correct circuit diagram with standard symbols.
Measurements	At least two variables measured with units and repeated trials, presented in a table or graph.
Physics explanation	Correct use of Ohm's law, magnetic force on currents and/or Faraday's law to explain the results.
Improvement	One design change tested, with before-and-after data.

10.1 Charge and Coulomb's Law

OBJECTIVES

- Describe electric charge, its conservation and its quantization, and explain charging by friction, conduction and induction.
- Use Coulomb's law, $F = kq_1q_2/r^2$, and compare electric and gravitational forces.
- Explain how Coulomb's torsion balance measured the inverse-square law.
- Build an electroscope and estimate the charge on two balloons from their repulsion.

> **Warm-up.** Rub a balloon on your hair or a wool sock and hold it near small bits of paper, then near a thin stream of water from a faucet, then against a wall. Describe what happens in each case. Then rub two balloons and hang them side by side on strings. What do they do?

WATCH

Introduction to Coulomb's Law or the Electric Force

Flipping Physics · 12:10

https://www.youtube.com/watch?v=4ubqby1Id4g

> **WHEN TO WATCH**
> After the physics paragraph, before the worked examples.

BACKUP VIDEO

Measure Static Electricity With An Electroscope! | STEM Activity

Science Buddies · 3:35

https://www.youtube.com/watch?v=wso0FqcnG7g

Before the lab: building a simple electroscope.

KEY TERMS

electric charge

A basic property of matter, positive (protons) or negative (electrons), measured in coulombs (C). Like charges repel; unlike charges attract.

elementary charge

The charge of one proton, $e = 1.602 \times 10^{-19}$ C; an electron has $-e$. All free charges are whole multiples of e.

conservation of charge

Charge is never created or destroyed; rubbing only moves electrons from one object to another.

conductor and insulator

In a conductor (metals, salt water) charges move freely; in an insulator (rubber, plastic, glass, dry air) they stay put.

induction

Charging or polarizing an object by bringing a charge near it, without touching, so its own charges shift.

Coulomb's law

$F = k\frac{q_1 q_2}{r^2}$ with $k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$.

CORE IDEAS

The story. In the 1700s electricity was a parlor wonder: glass rods rubbed with silk, sparks from spinning glass globes, Benjamin Franklin's dangerous 1752 kite experiment in a thunderstorm (which no one should ever repeat). People knew that charges attract and repel, but not by what law. In 1785 the French engineer Charles-Augustin de Coulomb, who had already studied friction (Lesson 3.4) and the twisting of wires, built a torsion balance for electricity: a light needle hung by a fine silver wire inside a glass cylinder, with a small gilded pith ball on one end. A second charged ball was brought near; the repulsion twisted the wire, and the twist, read on a scale, measured the force.

How they figured it out. Coulomb charged both balls alike and measured the twist needed to hold them at different distances. Halving the distance made the force four times larger; the force followed an inverse square, like Newton's gravity. He also showed that the force was proportional to each charge, by touching a charged ball to an identical uncharged one to split its charge exactly in half. Joseph Priestley had guessed the inverse-square law in 1767 from the fact that there is no electric force inside a hollow charged sphere, and Henry Cavendish had measured it even more precisely in 1773 but never published. Coulomb's careful measurements convinced the world, and the unit of charge bears his name.

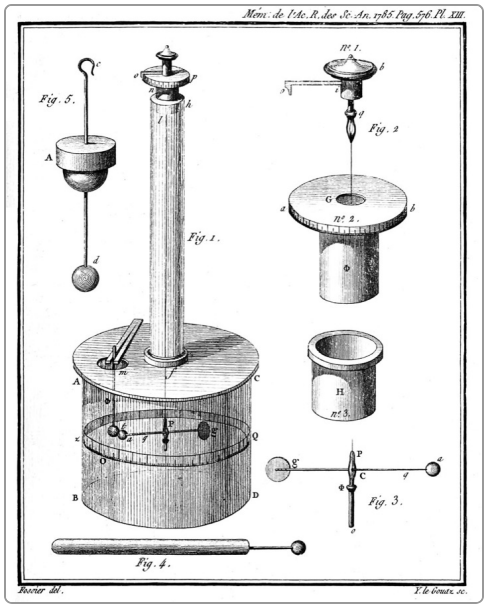
The physics. Matter is made of positive protons and negative electrons, normally in balance. Rubbing transfers electrons, leaving one object negative and the other equally positive: charge is **conserved**. Charges come in whole numbers of $e = 1.602 \times 10^{-19}$ C, so a coulomb is about 6.24×10^{18} electrons. **Coulomb's law**, $F = kq_1q_2/r^2$, has the same form as Newton's gravity, but it is enormously stronger:

between a proton and an electron the electric attraction is about 10^{39} times the gravitational one. A charged object attracts a neutral one by **induction**, pulling its opposite charges slightly nearer, which is why a charged balloon picks up paper.

The engineering. Electrostatics is everywhere in technology. Laser printers and photocopiers, descendants of Chester Carlson's 1938 invention of xerography, use charged drums to pick up toner exactly where light has not discharged them. Electrostatic precipitators remove ash from smokestack gases, and electrostatic spray guns wrap paint around car bodies. Electronics engineers protect chips from static discharge with grounding straps and special packaging, because a spark you cannot even feel can destroy a transistor.

Try it. In the lab you will build a foil electroscope, test charging by contact and by induction, and estimate the charge on two rubbed balloons from how far apart they hang.

PICTURE STUDY



Coulomb's Torsion Balance, Charles-Augustin de Coulomb (engraved plate), 1785 · Engraving · *Mémoires de l'Académie Royale des Sciences*, Paris

Image: Wikimedia Commons, public domain

Coulomb published this plate with his first memoir on electricity. The balance is a cousin of Cavendish's instrument for gravity (Lesson 4.4), built on the same idea: measure a tiny force by the twist of a thin wire.

Look closely

- Find the twist head at the top, where the wire is turned and the angle read.
- Find the needle and the small ball at one end of it.
- Find the scale around the outside of the large cylinder.
- Find the separate detail drawings. What parts do they show?

Talk about it

1. Why is the apparatus enclosed in glass?
2. How could Coulomb halve the charge on a ball without measuring it?
3. What would the twist angle do if the distance between the balls were halved?

Safety first. Never use electricity from the outlets in the wall. Static sparks from balloons and wool are harmless, but keep everything away from electronic devices and computer parts.

LAB

An Electroscope and Two Charged Balloons

You will build an electroscope from a jar and foil, use it to explore charging by contact and induction, and then estimate the charge on two balloons from their repulsion using Coulomb's law.

You will need

- A clear glass jar with a cardboard lid, a large paper clip, aluminum foil
- Two balloons, thread, tape, a wool sock or cloth
- Meter stick, protractor, kitchen scale
- A plastic comb or pen, a glass or acrylic rod (optional)

Steps

1. Straighten the paper clip into a hook, push it through the lid, and hang two light strips of foil (about 1 × 3 cm) from the hook inside the jar. Crumple a ball of foil on top of the clip as the electroscope's "knob."
2. **Hypothesis.** Predict what the foil leaves will do when a charged balloon is brought near the knob without touching, and when it touches.
3. Charge a balloon with wool and bring it slowly near the knob. Record what the leaves do. Remove it. Then touch the knob with the balloon and remove it. Record again.
4. **Charging by induction.** Bring the balloon near (not touching), touch the knob briefly with your finger, remove your finger, then remove the balloon. Do the leaves stay apart? Explain where the charge came from and what sign it has.
5. Weigh two balloons. Hang them from one point on equal threads 0.50 m long. Charge both equally by rubbing each the same number of times, and let them hang.
6. Measure the distance between their centers. Calculate the angle, the electric force $mg \tan\theta$ and the charge on each with Coulomb's law.
7. **Fair test.** Rub 5, 10 and 20 times. Does the separation grow? Calculate the charge each time.

8. **Improve.** Humidity lets charge leak away. Work quickly, on a dry day, and note how the separation changes over a few minutes.

Tips

- Dry air works best; static experiments often fail on humid days.
- Handle charged balloons by their knots so your hand does not discharge them.

Check your work

- ☐ I built an electroscope and explained charging by contact and by induction.
- ☐ I estimated the charge on a balloon with Coulomb's law.
- ☐ I can compare electric and gravitational forces.

LAB NOTEBOOK

1. Diagrams of the electroscope with plus and minus signs showing where the charges are: (a) balloon near, (b) after touching, (c) after induction.
2. Free-body diagram of one hanging balloon with weight, tension and electric force, and your calculation.
3. A short explanation of why a charged balloon sticks to a neutral wall.

KEY FORMULAS

$$F = k\frac{q_1 q_2}{r^2}, \qquad k = 8.99 \times 10^9 \ \mathrm{N \cdot m^2/C^2}$$

$$q = ne, \qquad e = 1.602 \times 10^{-19} \ \mathrm{C}$$

$$F_e = mg \tan\theta \quad \text{(hanging charged balloon)}$$

From history. Coulomb also published seven memoirs on electricity and magnetism and showed that magnetic poles obey an inverse-square law too. Franklin's choice of which kind of charge to call "positive" was made before anyone knew of electrons, which is why electrons, the charges that actually move in wires, ended up negative.

Example 1. Two small spheres each carry $+1.0\ \mu\text{C}$ and are 10 cm apart. Find the force between them.

Show solution

1. $F = k\dfrac{q_1 q_2}{r^2} = (8.99 \times 10^9)\dfrac{(1.0 \times 10^{-6})(1.0 \times 10^{-6})}{(0.10)^2}\ \text{N}.$
2. $F = (8.99 \times 10^9)(1.0 \times 10^{-12})/0.010\ \text{N} = 0.90\ \text{N}$, repulsive.

Answer: 0.90 N repulsion, about the weight of a 90 g apple, from a millionth of a coulomb each.

Example 2. In a hydrogen atom the electron and proton are about 5.3×10^{-11} m apart. Compare the electric and gravitational forces between them ($m_e = 9.11 \times 10^{-31}$ kg, $m_p = 1.67 \times 10^{-27}$ kg).

Show solution

1. Electric: $F_e = \dfrac{(8.99 \times 10^9)(1.602 \times 10^{-19})^2}{(5.3 \times 10^{-11})^2}\ \text{N} = 8.2 \times 10^{-8}\ \text{N}.$
2. Gravitational:
   $F_g = \dfrac{(6.674 \times 10^{-11})(9.11 \times 10^{-31})(1.67 \times 10^{-27})}{(5.3 \times 10^{-11})^2}\ \text{N} = 3.6 \times 10^{-47}\ \text{N}.$
3. Ratio: $\dfrac{8.2 \times 10^{-8}}{3.6 \times 10^{-47}} = 2.3 \times 10^{39}.$

Answer: The electric force is about 2×10^{39} times stronger; gravity is utterly negligible inside atoms.

Example 3. Two identical rubbed balloons, each 3.0 g, hang from the same point on threads 0.50 m long. They repel and come to rest 0.20 m apart. Estimate the charge on each, assuming equal charges.

Show solution

1. Each thread makes an angle θ with the vertical, where $\sin\theta = \dfrac{0.10}{0.50} = 0.20$, so $\theta = 11.5^\circ$.
2. Balance on one balloon: $T\cos\theta = mg$ and $T\sin\theta = F_e$, so $F_e = mg\tan\theta = (0.0030)(9.8)(0.204)\ \text{N} = 6.0\times10^{-3}\ \text{N}$.
3. Coulomb: $F_e = kq^2/r^2$, so $q = r\sqrt{F_e/k} = (0.20)\sqrt{\dfrac{6.0\times10^{-3}}{8.99\times10^{9}}}\ \text{C} = 1.6\times10^{-7}\ \text{C}$.

Answer: About 0.16 μC each, around 10^{12} extra electrons. (Treating balloons as points is rough, so this is an estimate.)

PRACTICE

1. Easy How many electrons make up a charge of -1.0 C?

   Show answer

   6.24×10^{18}

2. Easy If the distance between two charges doubles, what happens to the force between them?

   Show answer

   It becomes one quarter as large.

3. Easy A balloon rubbed on wool becomes negative. What charge does the wool have, and how much?

   Show answer

   Positive, equal in size: the electrons moved from the wool to the balloon.

4. Medium Find the force between charges of $+2.0\ \mu$C and $-3.0\ \mu$C placed 0.30 m apart.

   Show answer

   0.60 N, attractive

5. **Medium** Two equal charges repel with 0.10 N when 5.0 cm apart. Find each charge.

Show answer

$$q = r\sqrt{F/k} = 0.050\sqrt{0.10/8.99 \times 10^9} = 1.7 \times 10^{-7}\ \text{C}$$

6. **Medium** Coulomb found the torsion needed at 36 units of separation was 36 degrees of twist. By the inverse-square law, what twist would he expect at 18 units? At 9 units?

Show answer

144 degrees; 576 degrees (four times as much each time the distance halves).

7. **Medium** Why does a charged comb attract small, uncharged pieces of paper?

Show answer

The comb's charge pulls opposite charges in the paper slightly closer (induction or polarization). The nearer opposite charges are attracted more strongly than the farther like charges are repelled, so the net force is an attraction.

8. **Medium** A metal sphere with $+6.0$ nC touches an identical neutral sphere. What charge does each have afterward?

Show answer

$+3.0$ nC each

9. **Hard** Three charges lie on a line: $+2.0\ \mu\text{C}$ at $x = 0$, $-1.0\ \mu\text{C}$ at $x = 0.10$ m, and $+3.0\ \mu\text{C}$ at $x = 0.30$ m. Find the net force on the $-1.0\ \mu\text{C}$ charge.

Show answer

From $+2$: 1.80 N toward $x = 0$ (left). From $+3$: 0.674 N toward $x = 0.30$ (right). Net 1.12 N to the left.

10. **Hard** How far apart must two electrons be for the repulsion between them to equal the weight of one electron?

Show answer

$$r = e\sqrt{k/(m_e g)} = 5.1\ \text{m}$$

11. **Challenge** Why can an electric force of about 10^{39} times gravity hold atoms together, while gravity rules the motions of planets and stars?

Show answer

Large objects contain almost exactly equal numbers of positive and negative charges, so their electric forces cancel. Gravity has only one sign, attraction, so it adds up over enormous masses and dominates on large scales.

COMMON MISTAKES

- Forgetting to convert microcoulombs (10^{-6}) and nanocoulombs (10^{-9}) to coulombs.
- Thinking rubbing creates charge. It only separates charges that were already there.
- Squaring the charges instead of the distance.

> **Make it a game · Charge or Not.** Rub a balloon on your hair or a sweater and pick up scraps of tissue paper. Predict what happens with the balloon near a thin stream of tap water, then test. Explain each outcome with charge induction in 20 seconds. Your parent challenges one explanation per round; each defended claim earns a point.

10.2 Circuits and Ohm's Law

OBJECTIVES

- Define current, voltage and resistance, and use Ohm's law, $V = IR$.
- Find the equivalent resistance, currents and voltages in series and parallel circuits.
- Calculate electric power, $P = IV = I^2R = V^2/R$, and energy use.
- Measure current–voltage graphs for a resistor and a bulb with a multimeter.

> **Warm-up.** Connect a small flashlight bulb to one AA battery with two wires, then to two AA batteries in a row (series). Compare the brightness. Then connect two bulbs one after the other to the same two batteries, and then two bulbs side by side. Predict the brightness each time before connecting. Batteries only; never the outlets in the wall.

WATCH

Ohm's Law

The Organic Chemistry Tutor · 14:00

https://www.youtube.com/watch?v=_rSHqvjDksg

> **WHEN TO WATCH**
> After the physics paragraph. Work each circuit with the presenter.

KEY TERMS

current

The rate of flow of charge, $I = \Delta q/\Delta t$, measured in amperes (1 A = 1 C/s).

voltage (potential difference)

The energy given to (or used by) each coulomb of charge, measured in volts (1 V = 1 J/C).

resistance

Opposition to current, $R = V/I$, measured in ohms (Ω); for a wire, $R = \rho L/A$.

Ohm's law

For many conductors at constant temperature, current is proportional to voltage: $V = IR$.

series and parallel

In series, components share one path and the same current; in parallel, they share the same voltage and the current divides.

electric power

The rate of energy transfer, $P = IV$, in watts.

CORE IDEAS

The story. In 1800 Alessandro Volta, a professor at Pavia in Italy, announced in a letter to the Royal Society of London that a stack of zinc and copper (or silver) disks separated by cardboard soaked in salt water produced a steady flow of electricity, not just a spark. Volta's "pile," the first battery, made the electric current a laboratory tool. In the 1820s Georg Simon Ohm, a schoolteacher in Cologne, set out to find how the current in a wire depends on the wire. Volta's piles weakened as they ran, spoiling his measurements, so on a friend's advice he used a thermocouple, two joined metals with one junction in boiling water and the other in ice, which gave a very steady voltage.

How they figured it out. Ohm measured the current by the deflection of a magnetic needle hung by a torsion wire above the circuit, and he tested wires of different lengths and thicknesses. He found that the current was proportional to the driving voltage and inversely proportional to the wire's resistance, which grew with length and fell with thickness. He published the law in 1827. At first German scientists dismissed it, and Ohm lost hope of a university post; but in 1841 the Royal Society awarded him its Copley Medal, and today the unit of resistance is the ohm.

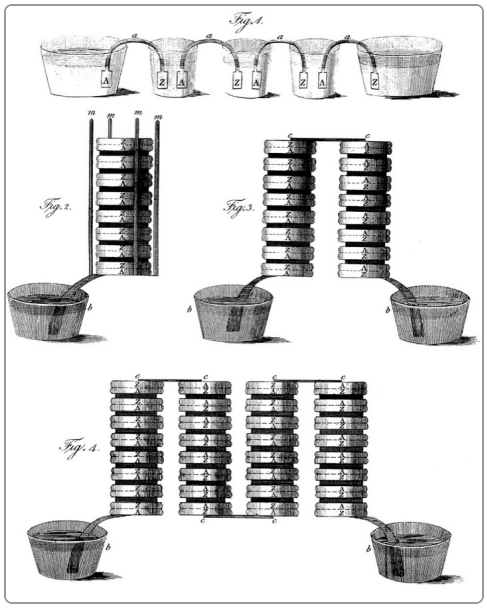
The physics. A battery gives each coulomb of charge some energy (its **voltage**); the charges flow around a closed loop (the **current**) and give that energy up in the circuit's components. **Ohm's law**: $V = IR$. In **series**, resistances add, $R_s = R_1 + R_2 + \cdots$, the current is the same everywhere and the voltage divides. In **parallel**, $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$, each branch has the full voltage and the currents add. **Power** is $P = IV$; using Ohm's law, $P = I^2R = V^2/R$. A light bulb's filament is not "ohmic": its resistance rises as it heats, so its current–voltage graph curves.

The engineering. House wiring connects outlets and lights in parallel, so each gets the full voltage and one can be switched off without affecting the others; fuses and circuit breakers open the circuit if the total current gets dangerously large. (Household wiring carries lethal voltages and is the job of licensed electricians; your

experiments use batteries only.) Electronics designers use series resistors to limit the current through LEDs, voltage dividers to set signal levels, and wires thick enough that I^2R heating stays small. Electric companies bill energy in kilowatt-hours: power times time.

Try it. In the lab you will measure current–voltage graphs for a resistor and a small bulb with a digital multimeter and battery packs, and test the series and parallel rules.

PICTURE STUDY



Volta's Pile and Crown of Cups, Alessandro Volta, 1800 · Engraving · *Philosophical Transactions of the Royal Society*

Image: Wikimedia Commons, public domain

Volta sent this plate with the letter announcing his invention. It shows two forms of the first battery: a "crown of cups" of salt water with metal strips, and the stacked "pile" of disks.

Look closely

- Find the cups and the bent strips joining them. What do the letters on the strips stand for?
- Find the tall stacks of disks. How are the disks arranged?
- Find the dishes at the bottom and the connections between stacks.

Talk about it

1. Why do more disks (or cups) in a row give a stronger effect?
2. What would Ohm have found if he had used Volta's pile, whose voltage slowly dropped, for his experiments?
3. Which arrangement in the picture is like batteries in series in a flashlight?

> **Safety first.** Use only batteries (AA packs and 9 V). Never connect a battery's terminals directly with a bare wire for more than a moment: it gets hot quickly. Never connect anything to the outlets in the wall, and do not open appliances. Keep coin cells away from small children.

LAB

Current and Voltage: Resistors, Bulbs, Series and Parallel

You will measure the current through a resistor and a flashlight bulb at different voltages, graph the results, and test the rules for series and parallel circuits.

You will need

- Four AA batteries in holders (to make 1.5, 3.0, 4.5 and 6.0 V)
- A digital multimeter
- Two resistors (for example 100 Ω and 220 Ω), two small bulbs with holders (rated near 3–6 V)
- Alligator clip leads
- A soft pencil and paper (for a graphite resistor, optional)

Steps

1. Set the multimeter to DC volts and measure each battery and the packs of 2, 3 and 4. Record the actual voltages.
2. **Hypothesis.** Sketch what you expect the current–voltage graph to look like for a resistor and for a bulb.
3. Connect the 100 Ω resistor to one battery with the meter (set to DC milliamps) in series, in the loop. Record the current. Repeat with 2, 3 and 4 batteries.
4. Graph I against V. Find the slope; $R = 1/\text{slope}$. Compare with the color code.
5. Repeat with a bulb. Calculate $R = V/I$ at each point. How does the bulb's resistance change as it gets brighter, and why?
6. **Series.** Connect the two resistors in series to 6 V. Measure the current and each voltage. Check $R_s = R_1 + R_2$ and $V_1 + V_2 = V$.
7. **Parallel.** Connect them in parallel. Measure each branch current and the total. Check $I = I_1 + I_2$ and $1/R_p = 1/R_1 + 1/R_2$.
8. Calculate the power in each resistor and bulb in your circuits.
9. **Optional.** Draw a thick graphite line with a soft pencil. Measure its resistance at different lengths with the meter on ohms. Is $R \propto L$?

Tips

- To measure current, the meter must be in the loop (series); to measure voltage, connect it across the part (parallel).
- Start with the meter on its highest current range to avoid blowing its fuse.

Check your work

- ☐ My resistor's current–voltage graph is a straight line through the origin, and its slope gives the resistance.
- ☐ I verified the series and parallel rules with measurements.
- ☐ I can calculate current, voltage and power anywhere in a simple circuit.

LAB NOTEBOOK

1. Circuit diagrams (with standard symbols) for every circuit you built, labeled with measured currents and voltages.
2. The I–V graphs for the resistor and the bulb on one set of axes.
3. Explain why the lights in a house are wired in parallel and why a short circuit is dangerous.

KEY FORMULAS

$$V = IR$$

$$R_s = R_1 + R_2 + \cdots, \qquad \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots$$

$$P = IV = I^2 R = \frac{V^2}{R}$$

$$R = \rho \frac{L}{A}$$

From history. Volta was made a count by Napoleon, and the volt is named for him. Ohm finally became a professor at Munich in 1852, two years before his death. Gustav Kirchhoff, in 1845 while still a student, generalized Ohm's results into the two circuit rules engineers use for every network: currents into a junction equal currents out, and voltages around any loop add to zero.

Example 1. A 450 Ω resistor is connected across a 9.0 V battery. Find the current and the power.

Show solution

1. $I = \dfrac{V}{R} = \dfrac{9.0\ \text{V}}{450\ \Omega} = 0.020\ \text{A} = 20\ \text{mA}.$
2. $P = IV = (0.020\ \text{A})(9.0\ \text{V}) = 0.18\ \text{W}.$

Answer: 20 mA; 0.18 W.

Example 2. Resistors of 100 Ω and 220 Ω are connected in series to a 6.0 V battery. Find the current and the voltage across each.

Show solution

1. $R_s = 100 + 220 = 320\ \Omega$; $I = \dfrac{6.0\ \text{V}}{320\ \Omega} = 0.01875\ \text{A} = 18.8\ \text{mA}.$
2. $V_1 = IR_1 = (0.01875)(100)\ \text{V} = 1.88\ \text{V}$; $V_2 = (0.01875)(220)\ \text{V} = 4.13\ \text{V}.$
3. Check: $1.88 + 4.13 = 6.0$ V (to rounding).

Answer: 18.8 mA; 1.88 V and 4.13 V, the larger resistor taking the larger share.

Example 3. The same two resistors are connected in parallel to the 6.0 V battery. Find the equivalent resistance and all three currents.

Show solution

1. $\dfrac{1}{R_p} = \dfrac{1}{100} + \dfrac{1}{220} = 0.01455\ \Omega^{-1}$, so $R_p = 68.8\ \Omega$.
2. Branch currents: $I_1 = \dfrac{6.0}{100}\ \text{A} = 60\ \text{mA}$; $I_2 = \dfrac{6.0}{220}\ \text{A} = 27.3\ \text{mA}.$
3. Total: 87.3 mA; check $6.0/68.8 = 87.2$ mA.

Answer: $R_p = 68.8\ \Omega$, less than either resistor; branch currents 60 and 27 mA, total 87 mA.

Example 4. A red LED needs 20 mA and drops 2.0 V. What series resistor lets it run from a 9.0 V battery? What power does the resistor waste?

Show solution

1. The resistor must drop the rest of the voltage: $9.0 - 2.0 = 7.0\ \text{V}$.
2. $R = \dfrac{7.0\ \text{V}}{0.020\ \text{A}} = 350\ \Omega$ (use the next standard value, 360 or 390 Ω).
3. $P = IV = (0.020)(7.0)\ \text{W} = 0.14\ \text{W}$, so a common $\frac{1}{4}$-watt resistor is fine.

Answer: About 350 Ω; it dissipates 0.14 W.

PRACTICE

1. Easy A current of 0.50 A flows through a 12 Ω resistor. Find the voltage across it.

   Show answer

   6.0 V

2. Easy How much charge passes a point in a wire carrying 2.0 A for 1.0 min?

   Show answer

   120 C

3. Easy Find the equivalent resistance of 10 Ω, 20 Ω and 30 Ω in series, and in parallel.

   Show answer

   60 Ω; 5.45 Ω

4. Medium A 3.0 V flashlight bulb draws 0.30 A. Find its resistance and power.

   Show answer

   10 Ω; 0.90 W

5. Medium Two identical bulbs are connected in series to a battery, then in parallel to the same battery. In which arrangement is each bulb brighter, and by what factor is the power per bulb larger (ignoring changes in bulb resistance)?

   Show answer

   Parallel: each gets the full voltage, so $P = V^2/R$; in series each gets $V/2$, so $P = V^2/(4R)$. Four times brighter in parallel.

6. Medium A 12 V car battery runs a 55 W headlight. Find the current and the bulb's resistance.

Show answer

4.6 A; 2.6 Ω

7. Medium A 100 Ω and a 300 Ω resistor are in series across 8.0 V. Find the voltage across each.

Show answer

2.0 V and 6.0 V

8. Medium Why did Ohm switch from Volta's pile to a thermocouple?

Show answer

The pile's voltage dropped as it was used, so the current changed during each measurement. A thermocouple with junctions in boiling water and ice gave a steady voltage, allowing a fair comparison of different wires.

9. Hard A 6.0 V battery feeds a 40 Ω resistor in series with a parallel pair of 60 Ω and 30 Ω. Find the total current and the current in the 30 Ω resistor.

Show answer

Parallel pair $= 20\ \Omega$; total $60\ \Omega$; $I = 0.10$ A; voltage across pair 2.0 V; $I_{30} = 0.067$ A

10. Hard A copper wire ($\rho = 1.7 \times 10^{-8}\ \Omega \cdot \mathrm{m}$) is 10 m long and 1.0 mm in diameter. Find its resistance and the power wasted when it carries 5.0 A.

Show answer

$A = 7.85 \times 10^{-7}\ \mathrm{m}^2$; $R = 0.22\ \Omega$; $P = I^2 R = 5.4$ W

11. Challenge Show that two resistors in parallel always have an equivalent resistance less than the smaller of them, and find the resistor that must be put in parallel with 100 Ω to make 75 Ω.

Show answer

$1/R_p = 1/R_1 + 1/R_2 > 1/R_1$, so $R_p < R_1$ (and likewise $< R_2$).
$1/R_2 = 1/75 - 1/100 = 1/300$: $R_2 = 300\ \Omega$.

COMMON MISTAKES

- Putting an ammeter across a component (in parallel). It must be in series, in the loop.

- Adding parallel resistances directly. Add their reciprocals.
- Thinking current is “used up” in a circuit. The same current flows into and out of every series component; energy, not charge, is used.

> **Make it a game · Circuit Calc Relay.** Your parent writes a circuit on paper: 6 V across 2 ohm and 4 ohm in series, then in parallel. Calculate current through each resistor and equivalent resistance in 90 seconds per version. Pass the pencil back and forth to check each other's arithmetic. Fastest correct pair wins a point.

10.3 Magnetism: Gilbert, Oersted and the Electric Current

OBJECTIVES

- Describe magnetic poles, magnetic fields and the Earth's magnetism.
- Explain Oersted's discovery and find the magnetic field of a long wire, $B = \mu_0 I/(2\pi r)$, and of a solenoid, $B = \mu_0 n I$.
- Calculate the magnetic force on a current-carrying wire and on a moving charge, and use the right-hand rules.
- Map the field around a wire with a compass and build an electromagnet.

> **Warm-up.** Lay a compass on the table and let it settle. Bring a magnet slowly toward it from different directions and watch the needle. Then slide a steel paper clip near it. Finally, hold a wire connected to a battery for just a moment (one second) directly over the compass, parallel to the needle. What happens? Disconnect the wire quickly so it does not heat.

WATCH

Magnetic Field of a Wire

Bozeman Science · 7:33

https://www.youtube.com/watch?v=nfSJ62mzKyY

> **WHEN TO WATCH**
> After the physics paragraph.

BACKUP VIDEO

What Oersted Discovered with his Compass

National MagLab · 3:00

https://www.youtube.com/watch?v=RwilgsQ9xaM

With the story paragraph: Oersted's 1820 discovery re-created.

KEY TERMS

magnetic field

The region around a magnet or current where magnetic forces act, measured in teslas (T); the Earth's field is about 5×10^{-5} T.

magnetic poles

The two ends of a magnet, north and south; like poles repel, unlike poles attract, and poles always come in pairs.

solenoid

A long coil of wire; with current it makes a nearly uniform field inside, $B = \mu_0 n I$, like a bar magnet.

permeability of free space

$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$, the constant in the formulas for magnetic fields of currents.

right-hand rule

For a straight wire: point the right thumb along the current, and the fingers curl in the direction of the field.

magnetic force on a current

$F = BIL \sin\theta$ on a wire of length L; on a moving charge, $F = qvB \sin\theta$, perpendicular to both the velocity and the field.

CORE IDEAS

The story. In 1600 William Gilbert, physician to Queen Elizabeth I, published *De Magnete*, the first great book of experimental physics. He carved a sphere of lodestone, a *terrella* or "little Earth," and showed that a small compass needle moved over its surface points and dips just as compasses do on the real Earth: the Earth itself is a giant magnet. For two centuries afterward electricity and magnetism were thought to be separate. Then, in the spring of 1820, Hans Christian Oersted, a professor at Copenhagen, set up a battery, a wire and a compass during a lecture, and when he closed the circuit, the compass needle swung away from north. An electric current makes a magnetic field.

How they figured it out. Oersted spent the next months investigating: the needle turned to point across the wire, not along it, and reversed when the current reversed; the effect passed through glass, wood and water. He published a four-page paper in Latin in July 1820, and within weeks André-Marie Ampère in Paris had shown that two parallel wires carrying currents attract or repel each other, and that a coil of wire acts like a bar magnet. In 1825 the English inventor William Sturgeon

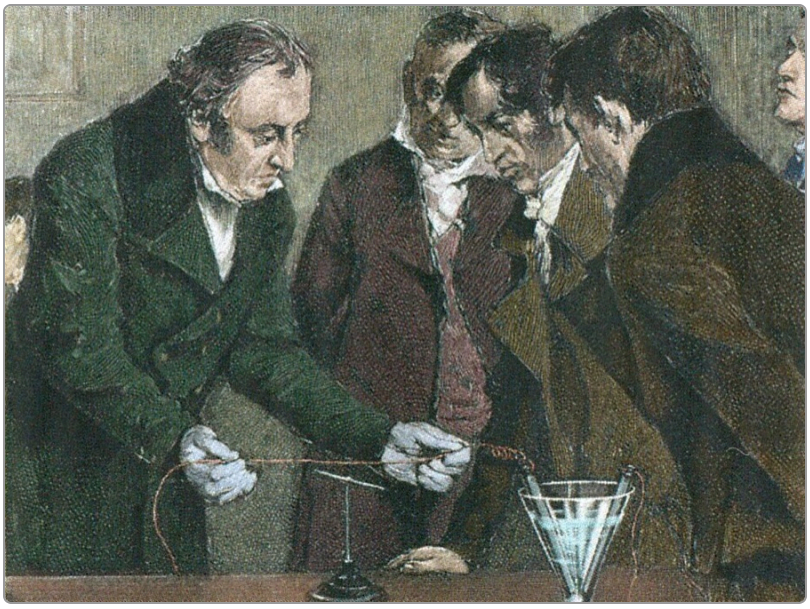
wrapped wire around an iron bar to make the first electromagnet, and in America Joseph Henry soon built electromagnets that could lift a ton.

The physics. A magnet is surrounded by a **magnetic field**, drawn as lines from north to south poles. A current in a long straight wire makes circular field lines around it, $B = \frac{\mu_0 I}{2\pi r}$, with direction given by the right-hand rule. A coil (solenoid) concentrates the field inside: $B = \mu_0 n I$, where n is turns per meter; an iron core multiplies it hundreds of times. Fields push on currents: a wire carrying current I across a field B feels $F = BIL\sin\theta$, and a moving charge feels $F = qvB\sin\theta$, always perpendicular to its motion, so charges in a uniform field move in circles. This force on currents is what turns every electric motor.

The engineering. Electromagnets lift scrap steel, ring doorbells, switch relays and lock doors. Loudspeakers push a coil in a magnet's field back and forth with the audio current. MRI scanners use superconducting solenoids with fields of 1.5 to 3 teslas, about 50,000 times the Earth's field. Maglev trains float and ride on magnetic fields. And because a current-carrying wire makes a field, electrical engineers route power and signal wires carefully so their fields do not disturb sensitive instruments or compasses.

Try it. In the lab you will repeat Oersted's experiment, map the field around a wire with a compass (or phyphox's magnetometer), and build an electromagnet.

PICTURE STUDY



Oersted Discovers Electromagnetism, Unknown artist, 19th century · Colored engraving · Wikimedia Commons

Image: Wikimedia Commons, public domain

This later picture imagines the moment in 1820 when Oersted and his audience watched a compass needle swing as current flowed. Notice how close everyone leans to see a tiny motion.

Look closely

- Find the compass and the wire above or beside it.
- Find the battery or the cups of the battery.
- Look at the faces. Who seems surprised?

Talk about it

1. Why does the compass needle turn to point across the wire, not along it?
2. What would happen if the current were reversed?
3. Why do you think no one noticed this effect earlier, even though batteries had existed for twenty years?

Safety first. Use only batteries. Connect the bare wire to the battery for only a few seconds at a time; it heats quickly. Strong magnets can pinch fingers and should be kept away from phones, cards and pacemakers.

LAB

Oersted's Experiment and an Electromagnet

You will repeat Oersted's discovery, map the magnetic field around a current-carrying wire, and build an electromagnet whose strength you measure by counting paper clips.

You will need

- A compass (or phone with phyphox's magnetometer)
- A D-cell battery in a holder, alligator clip leads, a length of insulated wire
- A large iron nail (about 10 cm) and 2 m of thin insulated or enameled magnet wire (sand the ends)
- Small steel paper clips
- Cardboard, pencil

Steps

1. **Oersted.** Lay the compass flat. Stretch a wire straight over it, along the needle's north–south line. Connect the battery for 2 seconds. Record the needle's swing. Reverse the battery and repeat. Place the wire under the compass instead and repeat.
2. **Hypothesis.** Predict the shape of the field lines around a straight wire.
3. Push the wire vertically through a hole in a sheet of cardboard. With current flowing (a few seconds at a time), place the compass at several points around the wire and draw the needle's direction at each. Check the right-hand rule.
4. If you have phyphox: hold the phone's magnetometer at 1, 2, 3 and 4 cm from the wire (measuring the field component around it) and see whether B falls as $1/r$.
5. **Electromagnet.** Wind 20 turns of magnet wire around the nail, connect the battery and count how many paper clips the nail picks up. Disconnect after each test.
6. **Fair test.** Increase to 40 and 60 turns (same battery). Graph paper clips against number of turns.

7. Use two D cells in series and repeat at 40 turns. Remove the nail and test the coil alone.
8. **Improve.** The battery weakens and the wire warms as you work. Test in random order and repeat your first measurement at the end.

Tips

- Keep the compass away from steel tables, which bend the field.
- Number of turns per length matters; wind the turns tightly side by side.

Check your work

- ☐ I repeated Oersted's experiment and found that reversing the current reverses the needle.
- ☐ My field map shows circles around the wire, matching the right-hand rule.
- ☐ My graph shows how an electromagnet's strength depends on the number of turns.

LAB NOTEBOOK

1. Top-view drawing of the compass directions around the wire, with the current direction marked.
2. Graph of paper clips against turns; note the effect of the iron core and of a second battery.
3. Draw the field lines of a bar magnet and of a solenoid side by side and explain why they look alike.

KEY FORMULAS

$$B = \frac{\mu_0 I}{2\pi r} \quad \text{(long straight wire)}$$

$$B = \mu_0 n I \quad \text{(solenoid)}$$

$$F = BIL\sin\theta, \qquad F = qvB\sin\theta$$

$$r = \frac{mv}{qB} \quad \text{(charge circling in a field)}$$

From history. The unit of magnetic field, the tesla, honors Nikola Tesla; an older unit, the gauss (1 T = 10,000 G), honors Carl Friedrich Gauss, who in 1832 made the first absolute measurement of the Earth's magnetic field. The ampere, until 2019, was defined by the force between two parallel wires, Ampère's own discovery.

Example 1. A long wire carries 5.0 A. Find the magnetic field 2.0 cm from it and compare it with the Earth's field (5×10^{-5} T).

Show solution

1. $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{(4\pi \times 10^{-7})(5.0)}{2\pi(0.020)}\ \mathrm{T} = 5.0 \times 10^{-5}\ \mathrm{T}$.
2. That equals the Earth's field, so a compass 2 cm away is pushed noticeably.

Answer: 5.0×10^{-5} T, about the same as the Earth's field, which is why Oersted's compass swung.

Example 2. A solenoid 10 cm long has 500 turns and carries 2.0 A. Find the field inside (no iron core).

Show solution

1. $n = \dfrac{500}{0.10\ \mathrm{m}} = 5000\ \mathrm{m}^{-1}$.
2. $B = \mu_0 n I = (4\pi \times 10^{-7})(5000)(2.0)\ \mathrm{T} = 1.26 \times 10^{-2}\ \mathrm{T}$.

Answer: 0.0126 T, about 250 times the Earth's field; an iron core would multiply it many times more.

Example 3. A 0.20 m length of wire carrying 3.0 A lies perpendicular to a 0.50 T magnetic field. Find the force on it.

Show solution

1. $F = BIL \sin 90^\circ = (0.50\ \mathrm{T})(3.0\ \mathrm{A})(0.20\ \mathrm{m}) = 0.30\ \mathrm{N}$.

Answer: 0.30 N, perpendicular to both the wire and the field.

Example 4. An electron moves at 1.0×10^6 m/s perpendicular to a 0.010 T field. Find the radius of its circular path.

Show solution

1. $r = \dfrac{mv}{qB} = \dfrac{(9.11 \times 10^{-31})(1.0 \times 10^6)}{(1.602 \times 10^{-19})(0.010)}\ \text{m} = 5.7 \times 10^{-4}\ \text{m}.$

Answer: About 0.57 mm; magnetic fields steer electron beams this way in instruments.

PRACTICE

1. Easy Find the magnetic field 5.0 cm from a long wire carrying 10 A.

   Show answer

   4.0×10^{-5} T

2. Easy What did Oersted discover in 1820?

   Show answer

   That an electric current produces a magnetic field, deflecting a nearby compass needle.

3. Easy A compass needle's north pole points toward the Earth's geographic north. Is the magnetic pole near there a north or a south magnetic pole?

   Show answer

   A magnetic south pole, since it attracts the needle's north pole.

4. Medium A solenoid with 2000 turns per meter carries 0.50 A. Find the field inside.

   Show answer

   1.26×10^{-3} T

5. Medium What current in a long wire gives a field of 1.0×10^{-4} T at 1.0 cm?

   Show answer

   5.0 A

6. Medium A 0.50 m wire carrying 8.0 A crosses a 0.20 T field at 30°. Find the force.

Show answer

0.40 N

7. Medium A proton (1.67×10^{-27} kg) moves at 2.0×10^5 m/s perpendicular to a 0.10 T field. Find the force and the radius of its path.

Show answer

$F = 3.2 \times 10^{-15}$ N; $r = 2.1$ cm

8. Medium Why does a magnetic force on a moving charge never change its speed?

Show answer

The force is always perpendicular to the velocity, so it does no work; it changes only the direction of motion.

9. Hard Two parallel wires 5.0 cm apart each carry 20 A in the same direction. Find the field of one at the other and the force per meter between them. Attract or repel?

Show answer

$B = 8.0 \times 10^{-5}$ T; $F/L = BI = 1.6 \times 10^{-3}$ N/m; they attract.

10. Hard A loudspeaker coil has 50 turns of radius 1.5 cm in a radial field of 1.2 T and carries 0.80 A. Find the force on it (the total wire length is $50 \times 2\pi r$).

Show answer

$L = 4.71$ m; $F = BIL = 4.5$ N

11. Challenge Explain how Gilbert's terrella experiment supported his claim that the Earth is a magnet, and what the "dip" of a compass needle tells a navigator.

Show answer

A small compass needle carried over the surface of the magnetized sphere pointed toward its poles and tilted (dipped) more steeply nearer the poles, exactly as compass needles behave on the Earth. So the Earth behaves like a large magnet. The dip angle grows with latitude, so in principle measuring it tells roughly how far north or south you are.

COMMON MISTAKES

- Using the left hand for the right-hand rule, or applying the rule for electron flow without reversing it.
- Forgetting $\sin\theta$: a wire parallel to a field feels no force.
- Using the total number of turns instead of turns per meter in $B = \mu_0 n I$.

> **Make it a game · Right-Hand Rule Charades.** Using your right hand, show the field direction around a current-carrying wire, with a thumb for current and curled fingers for the field. Your parent names a current direction and you show the field. Reverse: your parent shows, you name current. Five correct calls wins. Then compare field of a wire versus solenoid.

10.4 Faraday's Induction: Generators, Motors and Transformers

OBJECTIVES

- Define magnetic flux and use Faraday's law, $\mathcal{E} = -N\Delta\Phi/\Delta t$, and Lenz's law.
- Explain how a generator, a motor and a transformer work, and calculate a generator's peak voltage.
- Use the transformer ratio and explain why power is transmitted at high voltage.
- Induce a current with a magnet and a coil, and build a simple motor.

> **Warm-up.** Wrap about 50 turns of thin insulated wire around a toilet-paper tube and connect the ends to a multimeter set to millivolts (or to an LED). Push a strong magnet quickly into the tube, hold it still, and pull it out. Watch the meter each time. What happens while the magnet is still?

WATCH

Faraday's Law of Electromagnetic Induction, Magnetic Flux & Induced EMF - Physics & Electromagnetism

The Organic Chemistry Tutor · 11:53

https://www.youtube.com/watch?v=LDOa7UdfcMQ

> **WHEN TO WATCH**
> After the physics paragraph, before the worked examples.

BACKUP VIDEO

AC Electrical Generator Basics - How electricity is generated

The Engineering Mindset · 5:56

https://www.youtube.com/watch?v=WhATjUHgzxQ

With the engineering paragraph: how a generator turns motion into alternating current.

KEY TERMS

magnetic flux

The amount of magnetic field passing through an area, $\Phi = BA\cos\theta$, measured in webers (Wb = T·m^2).

electromagnetic induction

The production of a voltage (emf) in a circuit by a changing magnetic flux through it.

Faraday's law

The induced emf equals the rate of change of flux linkage: $\mathcal{E} = -N\dfrac{\Delta\Phi}{\Delta t}$.

Lenz's law

An induced current flows in the direction that opposes the change that caused it (the minus sign in Faraday's law).

generator

A machine that turns a coil in a magnetic field (or a magnet near a coil) to produce an alternating emf, $\mathcal{E} = NBA\omega \sin \omega t$.

transformer

Two coils on an iron core; an alternating current in one induces a voltage in the other in proportion to their turns, $V_s/V_p = N_s/N_p$.

CORE IDEAS

The story. Michael Faraday, the son of a London blacksmith, left school at thirteen and became an apprentice bookbinder. He read the science books he bound, attended lectures by Humphry Davy at the Royal Institution, and was hired as Davy's laboratory assistant in 1813. In 1821, a year after Oersted's discovery, he made a wire carrying current spin continuously around a magnet: the first electric motor. If a current could produce magnetism, he reasoned, magnetism should be able to produce a current. For ten years he tried and failed. On August 29, 1831, he wound two coils of wire on opposite sides of an iron ring and connected one to a battery; at the instant he closed or opened the switch, a galvanometer connected to the other coil kicked, and then fell back to zero.

How they figured it out. The key was *change*. A steady current in the first coil did nothing to the second; only switching it on or off, changing the magnetism in the ring, produced a current. That autumn Faraday showed that pushing a magnet into a coil, or pulling it out, produced a current, and that a copper disk spinning between the poles of a magnet produced a steady current: the first generator. He did all this without mathematics; he thought in pictures of "lines of force," an idea Maxwell later turned into equations. Faraday was a devout member of a small Christian church, the Sandemanians, and he saw his work as reading the laws God had written into nature. He also began the Royal Institution's Christmas lectures for young people, which still continue.

The physics. Magnetic flux $\Phi = BA\cos\theta$ measures how much field threads a loop. **Faraday's law**: when the flux through a coil of N turns changes, an emf is induced,

$\mathcal{E} = -N\Delta\Phi/\Delta t$. Faster change, more turns, stronger magnets: more voltage. **Lenz's law** (the minus sign) says the induced current opposes the change; a magnet falling through a copper pipe is slowed by the currents it induces. A **generator** turns a coil steadily in a field, giving $\mathcal{E} = NBA\omega \sin \omega t$, alternating current. A **motor** is a generator run backward: current in a coil in a field produces forces that turn it. A **transformer** changes alternating voltages in the ratio of its turns, $V_s/V_p = N_s/N_p$, with $V_p I_p \approx V_s I_s$.

The engineering. Almost all the world's electricity is made by Faraday's law: turbines spun by steam, water or wind turn huge coils past magnets (or magnets past coils). Transformers step the voltage up to hundreds of thousands of volts for long transmission lines, because for the same power a higher voltage means a smaller current and much smaller I^2R losses; then other transformers step it down for homes. Electric motors, from phone vibrators to locomotives, run on the force Oersted and Ampère found. Induction also charges phones wirelessly, heats pans on induction cooktops, detects metal in airport scanners, and lets electric cars recover energy when braking.

Try it. In the lab you will measure the voltage a moving magnet induces in a coil, test how it depends on speed and turns, and build a simple motor from a battery, a magnet and a coil of wire.

PICTURE STUDY

Michael Faraday in His Laboratory, Harriet Jane Moore, c. 1850–1855 · Watercolor · The Royal Institution, London

Image: Wikimedia Commons, public domain

Harriet Moore, a friend of the Faraday family, painted the basement laboratory of the Royal Institution where Faraday made his great discoveries. It is a picture of a working scientist in his everyday surroundings.

Look closely

- Find Faraday at his bench.
- Look along the shelves. What do the rows of bottles suggest about his work?
- Find any coils, magnets or electrical apparatus on the tables.

Talk about it

1. Faraday had little formal schooling and no university degree. What does this picture suggest about how he learned?
2. Why might a basement be a good place for delicate experiments?
3. What equipment would Faraday have needed for his 1831 induction experiments?

> **Safety first.** Use only batteries. The simple motor's coil and the battery can get hot within a minute; disconnect between trials. Strong neodymium magnets can snap together and pinch skin, and they must be kept away from small children, phones and pacemakers.

LAB

Making Electricity and Motion: A Coil, a Magnet and a Motor

You will induce a voltage by moving a magnet through a coil and find how it depends on speed and number of turns; then you will build a simple electric motor and explain it with the same physics.

You will need

- A cardboard tube, about 10 m of thin magnet wire (enameled), sandpaper
- A strong neodymium magnet (or a stack of them) that fits inside the tube
- A digital multimeter (DC millivolts) or a red LED
- A D-cell battery, two large paper clips, a rubber band, a small ceramic or neodymium magnet (for the motor)
- Phone with slow-motion video (optional)

Steps

1. Wind 50 turns of magnet wire on the tube, leaving long ends. Sand the enamel off the ends and connect them to the meter on its millivolt range.
2. **Hypothesis.** Predict how the induced voltage will change if you move the magnet twice as fast, and if you double the number of turns.
3. Drop or push the magnet through the coil at three different speeds. Record the largest reading each time. Note the sign as the magnet enters and as it leaves.
4. Rewind with 100 and then 150 turns (or add turns to the same coil) and repeat at a similar speed. Graph peak voltage against number of turns.
5. Hold the magnet still inside the coil. What does the meter read? Explain with Faraday's law.
6. **Lenz's law (optional).** Drop the magnet through a copper or aluminum pipe and compare the fall time with an unmagnetized steel object of similar size.
7. **Motor.** Wind 10 turns of magnet wire around a D cell to make a loop about 2.5 cm across, leaving straight ends as an axle. Sand the enamel completely off one end and only off the top half of the other. Bend two paper clips into

supports, hold them to the battery terminals with a rubber band, and set the magnet on the battery under the coil. Give the coil a gentle push.

8. Explain why sanding only half of one end keeps the coil spinning the same way. Then reverse the magnet or the battery and observe.
9. **Improve.** Balance the coil, straighten the axle and try a stronger magnet; time how many turns per second it makes with slow-motion video.

Tips

- The induced voltage is small (millivolts); a fast pass and many turns help.
- The motor needs a balanced coil and clean contacts; small adjustments make a big difference.

Check your work

- ☐ I induced a voltage with a magnet and showed that it depends on speed and number of turns.
- ☐ I can explain with Faraday's and Lenz's laws why a still magnet induces nothing.
- ☐ I built a working motor and can explain the force that turns it.

LAB NOTEBOOK

1. Data table and graph of induced voltage against turns; notes on speed and sign.
2. Labeled diagram of your motor showing the current direction, the magnet's field and the forces on the two sides of the coil.
3. Sketch the path of electricity from a power station generator to a lamp, labeling the transformers and the voltage at each stage.

KEY FORMULAS

$$\Phi = BA\cos\theta$$

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t}$$

$$\mathcal{E}_{\max} = NBA\omega$$

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}, \qquad P_{\text{loss}} = I^2R$$

From history. Faraday was offered a knighthood and the presidency of the Royal Society and declined both, preferring to remain, as he said, plain Michael Faraday. When asked by a government minister what use his discoveries might be, he is said to have replied that one day the minister might tax them. Albert Einstein kept pictures of Faraday, Newton and Maxwell on his study wall.

WORKED EXAMPLES

Example 1. A coil of 200 turns and area $4.0\ \text{cm}^2$ is perpendicular to a field that rises from 0 to $0.30\ \text{T}$ in $0.10\ \text{s}$. Find the average induced emf.

Show solution

1. $\Delta\Phi = A\Delta B = (4.0 \times 10^{-4}\ \text{m}^2)(0.30\ \text{T}) = 1.2 \times 10^{-4}\ \text{Wb}$.
2. $|\mathcal{E}| = N\dfrac{\Delta\Phi}{\Delta t} = (200)\dfrac{1.2 \times 10^{-4}\ \text{Wb}}{0.10\ \text{s}} = 0.24\ \text{V}$.

Answer: $0.24\ \text{V}$ while the field is changing, and zero once it is steady.

Example 2. A generator coil has 100 turns of area $0.020\ \text{m}^2$ and turns 60 times a second in a $0.20\ \text{T}$ field. Find its peak emf.

Show solution

1. $\omega = 2\pi f = 2\pi(60\ \text{Hz}) = 377\ \text{rad/s}$.
2. $\mathcal{E}_{\text{max}} = NBA\omega = (100)(0.20\ \text{T})(0.020\ \text{m}^2)(377\ \text{rad/s}) = 151\ \text{V}$.

Answer: About $151\ \text{V}$ peak, alternating 60 times a second.

Example 3. A transformer on a utility pole steps $7200\ \text{V}$ down to $240\ \text{V}$. What is its turns ratio? If the house draws $50\ \text{A}$, what current flows in the primary (ideal transformer)?

Show solution

1. $\dfrac{N_p}{N_s} = \dfrac{V_p}{V_s} = \dfrac{7200}{240} = 30$.
2. Power in = power out: $I_p = I_s\dfrac{V_s}{V_p} = (50\ \text{A})\dfrac{240}{7200} = 1.7\ \text{A}$.

Answer: 30 to 1; about $1.7\ \text{A}$ in the primary.

Example 4. A power line with resistance 5.0 Ω carries 1.0 MW. Compare the power lost as heat if the line runs at 10 kV and at 100 kV.

Show solution

1. At 10 kV: $I = \dfrac{P}{V} = \dfrac{1.0 \times 10^6}{1.0 \times 10^4}\ \text{A} = 100\ \text{A}$; loss $= I^2R = (100)^2(5.0)\ \text{W} =$ 50 kW (5%).
2. At 100 kV: $I = 10\ \text{A}$; loss $= (10)^2(5.0)\ \text{W} = 500\ \text{W}$ (0.05%).

Answer: Ten times the voltage cuts the loss a hundredfold, from 50 kW to 0.5 kW: the reason for high-voltage transmission.

PRACTICE

1. Easy A 0.50 T field passes perpendicularly through a loop of area 0.010 m^2. Find the flux.

   Show answer

   5.0×10^{-3} Wb

2. Easy Why does a steady current in Faraday's first coil induce no current in the second?

   Show answer

   Induction requires a changing flux. A steady current makes a constant flux, so nothing is induced.

3. Easy What is the difference between a motor and a generator?

   Show answer

   A motor turns electrical energy into motion (current in a field makes a force); a generator turns motion into electrical energy (moving a coil in a field induces a voltage). The same machine can do either.

4. Medium The flux through a 50-turn coil falls from 4.0×10^{-3} to 1.0×10^{-3} Wb in 0.050 s. Find the induced emf.

   Show answer

   3.0 V

5. Medium A transformer has 400 primary turns and 20 secondary turns. With 120 V in, what is the output voltage?

Show answer

6.0 V

6. Medium A generator produces a peak of 170 V at 60 Hz. If its coil has 200 turns of area $0.015\ \mathrm{m}^2$, what field does it turn in?

Show answer

$B = \mathcal{E}/(NA\omega) = 170/(200 \times 0.015 \times 377) = 0.15\ \mathrm{T}$

7. Medium A magnet falls through a copper pipe much more slowly than through a cardboard tube. Explain using Lenz's law.

Show answer

The falling magnet changes the flux through each ring of the pipe, inducing currents (eddy currents) that make their own magnetic fields opposing the magnet's motion, so it is slowed. Cardboard does not conduct, so no currents flow.

8. Medium A 60 W device on a 12 V supply is fed by a transformer from 120 V. What current flows in the primary (ideal)?

Show answer

0.50 A

9. Hard A 500 MW power plant sends its output over lines with a total resistance of 10 Ω. What fraction is lost if the voltage is 345 kV?

Show answer

$I = 1450\ \mathrm{A}$; loss $= I^2R = 21\ \mathrm{MW}$, about 4.2%

10. Hard A square coil of 100 turns, 5.0 cm on a side, is pulled out of a 0.40 T field at 2.0 m/s. While one side is outside the field, find the emf.

Show answer

$\mathcal{E} = NBLv = 100(0.40)(0.050)(2.0) = 4.0\ \mathrm{V}$

11. **Challenge** Explain why transformers work with alternating current but not with steady direct current, and what this had to do with the choice of AC for power grids in the 1890s.

Show answer

A transformer depends on a changing flux in its core; steady DC gives a constant flux and induces nothing in the secondary. AC changes constantly, so transformers can step its voltage up for efficient long-distance transmission and down for safe use. That ability gave AC systems a decisive advantage for large grids.

COMMON MISTAKES

- Thinking a strong magnet near a coil makes a current by itself. Only a changing flux induces an emf.
- Forgetting the number of turns N in Faraday's law.
- Expecting a transformer to increase power. It trades voltage for current; the power out is at most the power in.

> **Make it a game · Induction Why Race.** Your parent names a change, such as moving a magnet into a coil faster, doubling the turns or reversing the pole. Predict whether the induced voltage rises, falls or flips, and justify with Faraday's law and Lenz's law in 15 seconds. Swap roles. Three of four correct wins the round.

10.5 Maxwell, Light and the Photon

OBJECTIVES

- Describe what Maxwell's equations say in words and how they predicted electromagnetic waves.
- Calculate the speed of light from μ_0 and ε_0, and use $c = f\lambda$ across the electromagnetic spectrum.
- Use the photon energy $E = hf$ and the photoelectric equation, and explain why light also behaves as particles.
- Explain atomic line spectra with the Bohr model, and measure spectral lines with a homemade spectroscope.

> **Warm-up.** Look at an ordinary incandescent or halogen bulb, then a fluorescent tube or compact fluorescent lamp, then a white LED, through the rainbow reflection of a CD tilted near your eye (never look at the Sun). Do the colors look the same? Sketch what you see for each light.

WATCH

Let There Be Light: Maxwell's Equation EXPLAINED for BEGINNERS

Parth G · 10:38

https://www.youtube.com/watch?v=0jW74lrpeM0

> **WHEN TO WATCH**
> After the physics paragraph's first half, on Maxwell's equations.

BACKUP VIDEO

Photoelectric Effect Explained | Perimeter Institute for Theoretical Physics

Perimeter Institute for Theoretical Physics · 5:31

https://www.youtube.com/watch?v=jWbwDTPju-M

With the paragraph on photons: why the photoelectric effect needs light to come in packets.

KEY TERMS

electromagnetic wave

A traveling wave of linked electric and magnetic fields, each regenerating the other, moving through empty space at $c = 3.00 \times 10^8$ m/s.

Maxwell's equations

Four laws (from Gauss, Gauss for magnetism, Faraday, and Ampère with Maxwell's addition) that together describe all of classical electricity, magnetism and light.

electromagnetic spectrum

The full range of electromagnetic waves: radio, microwaves, infrared, visible light (about 400–700 nm), ultraviolet, X-rays and gamma rays.

photon

A packet (quantum) of light energy, $E = hf$, with $h = 6.626 \times 10^{-34}$ J · s.

photoelectric effect

The ejection of electrons from a metal by light; their maximum kinetic energy is $hf - \phi$, where ϕ is the metal's work function.

line spectrum

The set of sharp wavelengths emitted by a gas of one element, its "fingerprint"; for hydrogen, explained by electrons jumping between energy levels.

CORE IDEAS

The story. James Clerk Maxwell, a Scottish physicist and devout Christian, took Faraday's pictures of lines of force and turned them into mathematics. Between 1861 and 1865 he wrote down a set of equations for electric and magnetic fields and noticed that they predicted waves: a changing electric field makes a magnetic field, a changing magnetic field makes an electric field (Faraday's law), and the two could sustain each other and travel through empty space. When Maxwell calculated the speed of these waves from two constants measured in electrical laboratories, he got about 310,000 km/s, very close to the measured speed of light. In 1865 he concluded that light itself is "an electromagnetic disturbance." In 1887 Heinrich Hertz made such waves with an electric spark and detected them across his laboratory: radio.

How they figured it out. Maxwell's theory was the crowning success of classical physics, but by 1900 two puzzles remained about light and matter. The glow of hot objects could not be explained until Max Planck proposed in 1900 that light energy is

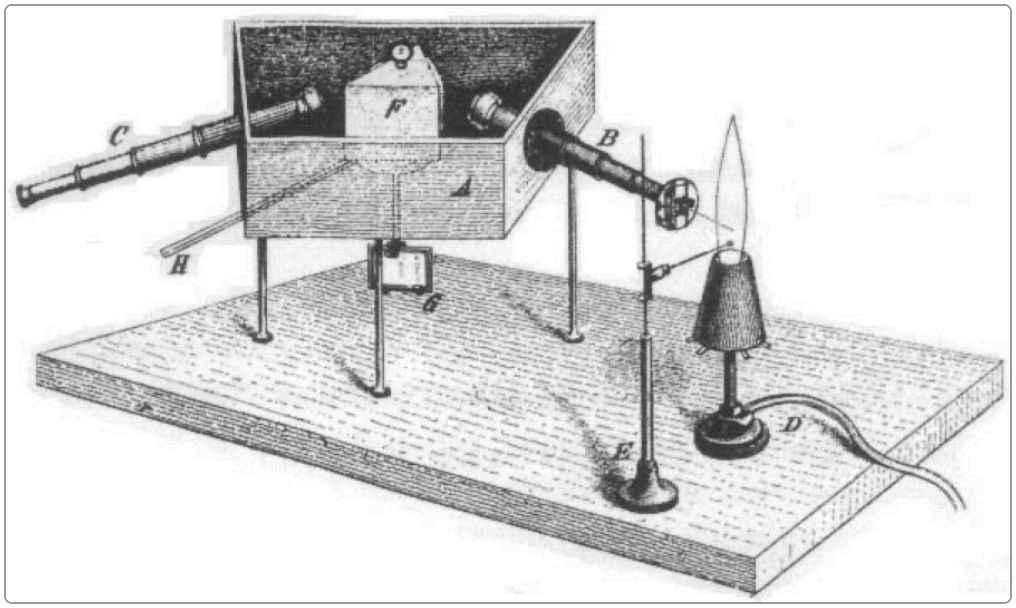
emitted in small packets, $E = hf$. In 1905 Albert Einstein used the same idea to explain the photoelectric effect: light knocks electrons out of metal only if each packet, each **photon**, has enough energy, however bright the light. Robert Millikan, who expected to disprove Einstein, confirmed his equation precisely in 1916. And in 1913 Niels Bohr explained why hydrogen glows at only certain wavelengths: its electron can have only certain energies, and it emits a photon when it jumps between them. Light is a wave and, in its exchanges of energy, a stream of particles; that double nature opened the door to quantum physics.

The physics. Maxwell's theory gives the speed of electromagnetic waves as $c = 1/\sqrt{\mu_0 \varepsilon_0} = 3.00 \times 10^8$ m/s. All electromagnetic waves travel at c in a vacuum, and $c = f\lambda$ sorts them into the spectrum: an FM radio wave at 100 MHz is 3 m long; green light at 5.5×10^{14} Hz is 550 nm. A photon's energy is $E = hf = hc/\lambda$; a handy form is $E(\text{eV}) = 1240/\lambda(\text{nm})$. In the photoelectric effect, $KE_{\text{max}} = hf - \phi$. In Bohr's hydrogen atom the energy levels are $E_n = -13.6\ \text{eV}/n^2$, and a jump from level n_i to n_f emits a photon of energy $E_i - E_f$: from 3 to 2 gives the red line at 656 nm.

The engineering. Maxwell's equations are the working equations of radio, television, radar, Wi-Fi, cell phones, GPS, microwave ovens and fiber optics; every antenna is designed with them. The photon picture is the working idea of solar cells, digital camera sensors, LEDs and lasers. Spectroscopy, reading the line "fingerprints" of elements, tells astronomers what stars are made of, lets chemists identify substances and lets engineers monitor the purity of steel and semiconductors.

Try it. In the lab you will build a spectroscope from a cardboard box and a CD and measure the wavelengths of the bright lines of a fluorescent lamp, comparing them with the known lines of mercury.

PICTURE STUDY



Kirchhoff and Bunsen's Spectroscope, Gustav Kirchhoff and Robert Bunsen, 1860 · Engraving · *Annalen der Physik und Chemie*, vol. 110

Image: Wikimedia Commons, public domain

With this instrument, Kirchhoff and Bunsen showed in 1859–1860 that each element, heated in a flame, gives its own pattern of bright lines, and they used it to discover two new elements, cesium and rubidium. Their work became the starting point for Bohr's atom.

Look closely

- Find the flame where a sample is heated.
- Find the prism and the tube that brings light to it.
- Find the telescope used to look at the spectrum.

Talk about it

1. Why does a prism (or grating) spread light into a spectrum?
2. Why would each element give a different set of lines?
3. How could this instrument tell what a distant star is made of?

Safety first. Never look at the Sun, directly or through any optical device. Use ordinary room lamps as light sources, and do not open or break fluorescent tubes or bulbs (they contain mercury).

LAB

A CD Spectroscope and the Lines of Mercury

You will build a spectroscope from a box and a piece of a CD, calibrate it, and measure the wavelengths of the bright lines in a fluorescent lamp's light, which come from mercury atoms, comparing them with the known values.

You will need

- A small cardboard box (a cereal or tea box), black tape, a razor blade or two straight-edged cards for a slit (adult helps with cutting)
- An old CD (a quarter of it, cut by an adult)
- A fluorescent lamp or compact fluorescent bulb, an incandescent or halogen bulb, a white LED
- A protractor and a ruler
- Phone camera (optional, to photograph the spectra)

Steps

1. Cut a narrow slit (about 1 mm) at one end of the box, made with two straight card edges taped close together. At the other end, cut a viewing hole and tape the CD piece at about 60° so you can look at its reflection of the slit.
2. **Hypothesis.** Predict how the spectra of an incandescent bulb, a fluorescent lamp and an LED will differ.
3. Point the slit at an incandescent bulb and look at the CD. Sketch the spectrum. Repeat for the fluorescent lamp and the LED.
4. For the fluorescent lamp, identify the bright violet, green and yellow-orange lines. Mercury's brightest visible lines are at 436 nm (violet-blue), 546 nm (green) and 577–579 nm (yellow).
5. **Measure.** Mount the CD flat on a table under the lamp with a protractor. Shine the lamp's light through the slit onto the CD and measure the angle at which each line appears in first order. Calculate $\lambda = d \sin\theta$ with $d = 1.6\ \mu\text{m}$ (use the geometry of your setup; a parent can help draw it).
6. Calibrate: if your green line does not come out at 546 nm, find the correction factor and apply it to the other lines.

7. Calculate the photon energy of each mercury line in eV.
8. **Compare.** Explain the continuous spectrum of the incandescent bulb, the separate lines of the fluorescent lamp, and the broad bands of the LED.
9. **Improve.** A narrower slit gives sharper lines; a darker box gives more contrast. Improve one and remeasure.

Tips

- Look at a light that is several feet away so the slit is evenly lit.
- Photographing the spectrum through the viewing hole with a phone makes it easier to measure.

Check your work

- ☐ I built a spectroscope and saw continuous and line spectra.
- ☐ I measured the wavelengths of mercury lines and compared them with the known values.
- ☐ I can explain line spectra with photons and energy levels.

LAB NOTEBOOK

1. Colored sketches of the three spectra you observed, labeled.
2. A table of mercury lines: measured wavelength, accepted wavelength, percent difference, photon energy in eV.
3. Draw the hydrogen energy levels for $n = 1$ to 4 and arrows for the jumps that give the visible lines; label each with its wavelength.

KEY FORMULAS

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$c = f\lambda$$

$$E = hf = \frac{hc}{\lambda}, \qquad E(\text{eV}) \approx \frac{1240}{\lambda(\text{nm})}$$

$$KE_{\max} = hf - \phi$$

$$E_n = -\frac{13.6\ \text{eV}}{n^2} \quad (\text{hydrogen})$$

From history. Maxwell also founded the statistical theory of gases, made the first color photograph (1861) and became the first director of the Cavendish Laboratory at Cambridge. A prayer found among his papers asks that he might use his knowledge to serve others. Einstein said that the special theory of relativity owed its origins to Maxwell's equations.

WORKED EXAMPLES

Example 1. Calculate the speed of electromagnetic waves from $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$ and $\varepsilon_0 = 8.854 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}$.

Show solution

1. $\mu_0\varepsilon_0 = (1.2566 \times 10^{-6})(8.854 \times 10^{-12}) = 1.1127 \times 10^{-17}\ \mathrm{s^2/m^2}$.
2. $c = \dfrac{1}{\sqrt{1.1127 \times 10^{-17}}}\ \mathrm{m/s} = 3.00 \times 10^8\ \mathrm{m/s}$.

Answer: $3.00 \times 10^8\ \mathrm{m/s}$, the speed of light, from two constants measured with coils and capacitors: Maxwell's great discovery.

Example 2. An FM station broadcasts at 101.1 MHz. Find the wavelength.

Show solution

1. $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8\ \mathrm{m/s}}{101.1 \times 10^6\ \mathrm{Hz}} = 2.97\ \mathrm{m}$.

Answer: About 3.0 m, which is why FM antennas are about 0.75 m (a quarter-wavelength) long.

Example 3. Find the energy of a photon of green light (550 nm) in joules and electron-volts ($1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$).

Show solution

1. $E = \dfrac{hc}{\lambda} = \dfrac{(6.626 \times 10^{-34})(3.00 \times 10^8)}{550 \times 10^{-9}}\ \mathrm{J} = 3.61 \times 10^{-19}\ \mathrm{J}$.
2. In eV: $\dfrac{3.61 \times 10^{-19}}{1.602 \times 10^{-19}} = 2.26\ \mathrm{eV}$ (or $1240/550 = 2.25\ \mathrm{eV}$).

Answer: $3.6 \times 10^{-19}\ \mathrm{J}$, about 2.3 eV.

Example 4. Use the Bohr model to find the wavelength of the light emitted when hydrogen's electron drops from $n = 3$ to $n = 2$.

Show solution

1. $E_3 = -\dfrac{13.6}{9}\ \text{eV} = -1.51\ \text{eV}$; $E_2 = -\dfrac{13.6}{4}\ \text{eV} = -3.40\ \text{eV}$.
2. Photon energy: $E_3 - E_2 = 1.89\ \text{eV}$.
3. $\lambda = \dfrac{1240}{1.89}\ \text{nm} = 656\ \text{nm}$, red.

Answer: 656 nm, the bright red line of hydrogen seen in the Sun and in glowing nebulae.

PRACTICE

1. Easy What is the frequency of red light with $\lambda = 700$ nm?

   Show answer

   4.29×10^{14} Hz

2. Easy Name the parts of the electromagnetic spectrum from longest to shortest wavelength.

   Show answer

   Radio, microwaves, infrared, visible, ultraviolet, X-rays, gamma rays.

3. Easy What did Hertz show in 1887?

   Show answer

   That electromagnetic waves, predicted by Maxwell, really exist: he produced radio waves with a spark and detected them across his laboratory.

4. Medium A microwave oven uses 2.45 GHz. Find the wavelength.

   Show answer

   12.2 cm

5. Medium Find the energy in eV of a 400 nm (violet) photon and a 700 nm (red) photon.

   Show answer

   3.1 eV; 1.77 eV

6. Medium Light of 250 nm shines on a metal with work function 4.3 eV. Find the maximum kinetic energy of the ejected electrons. Would red light eject any?

Show answer

$4.96 - 4.3 = 0.66$ eV. No: a red photon (about 1.8 eV) has less energy than the work function, however bright the light.

7. Medium Why did the photoelectric effect show that light behaves as particles?

Show answer

The energy of ejected electrons depended on the light's frequency, not its brightness, and below a threshold frequency no electrons came out at all. That makes sense only if light delivers energy in packets of hf, each absorbed by one electron.

8. Medium Find the wavelength of the hydrogen line from $n = 4$ to $n = 2$.

Show answer

$E = 13.6(1/4 - 1/16) = 2.55$ eV; $\lambda = 486$ nm (blue-green)

9. Hard A 1.0 mW red laser (650 nm) emits how many photons per second?

Show answer

$E = 3.06 \times 10^{-19}$ J per photon; $N = 10^{-3}/3.06 \times 10^{-19} = 3.3 \times 10^{15}$ per second

10. Hard What is the longest wavelength that can ionize a hydrogen atom from its ground state (13.6 eV)? In what part of the spectrum is it?

Show answer

$\lambda = 1240/13.6 = 91$ nm, ultraviolet

11. **Challenge** Maxwell calculated the speed of his waves from electrical measurements and found it equal to the measured speed of light. Explain why that agreement was such strong evidence that light is electromagnetic, and how Hertz's experiment completed the argument.

Show answer

The two constants came from experiments with charges, currents, coils and capacitors that had nothing to do with light, yet they gave a wave speed matching the separately measured speed of light within experimental error, an unlikely coincidence if light were something else. Hertz then generated electromagnetic waves electrically and showed they reflect, refract and interfere like light, travel at the same speed, and differ only in wavelength.

COMMON MISTAKES

- Thinking brighter light gives more energetic photoelectrons. Brightness gives more electrons; frequency sets their energy.
- Mixing up eV and joules. $1\ \mathrm{eV} = 1.602 \times 10^{-19}\ \mathrm{J}$.
- Thinking radio waves and light are different kinds of things. They are the same kind of wave with different wavelengths.

Make it a game · Spectrum Sort Race. Write radio, microwave, infrared, visible, ultraviolet, X-ray and gamma on seven cards. Shuffle and sort them by frequency in 20 seconds, then add one use for each. Then calculate the frequency of a 500 nm light with c = f lambda. Beat your own sorting time on the next round.

Scripture. "And God said, Let there be light: and there was light." (Genesis 1:3, KJV)

The story of physics that began with Galileo's rolling balls ends, in this course, with the nature of light. The more we learn about it, the more wonderful the first words spoken over creation become.

UNIT 10

Electricity and Magnetism

The last unit traces one of the greatest stories in science: the discovery that electricity, magnetism and light are one thing. It begins with Coulomb measuring the force between charges with a torsion balance, and Volta's pile and Ohm's law, which turned the electric current into a tool. Then Oersted's compass needle swung beside a wire, Faraday found that a moving magnet makes a current, and Maxwell wrote the equations that showed light to be an electromagnetic wave. The course ends at the doorway of modern physics, with Planck's and Einstein's photons and Bohr's atom.

The labs build an electroscope, measure the charge on balloons, graph current against voltage with a multimeter, repeat Oersted's experiment, build an electromagnet and a simple motor, induce a current with a magnet, and make a spectroscope from a CD. Every circuit runs on batteries; none touches the outlets in the wall.

PREREQUISITES

- Unit 4: inverse-square laws and circular motion
- Unit 5: energy and power
- Unit 9: waves, $c = f\lambda$ and diffraction gratings
- Scientific notation

MASTERY CHECKLIST

- ☐ I can use Coulomb's law and explain charging by friction, contact and induction.
- ☐ I can use Ohm's law and the series and parallel rules, and calculate electric power.
- ☐ I can find the magnetic fields of wires and solenoids and the magnetic force on currents and moving charges.
- ☐ I can use Faraday's and Lenz's laws and explain generators, motors, transformers and high-voltage transmission.
- ☐ I can explain how Maxwell's equations predicted light, and use $E = hf$ and the Bohr energy levels.

OPTIONAL KITS FOR THIS UNIT

- **Snap Circuits (Elenco)**: Ages 8 and up. Snap-together circuit parts that make Lesson 10.2's series and parallel circuits, and motors and electromagnets for Lessons 10.3–10.4, quick and reliable. Battery-powered.
- **Arbor Scientific electricity and magnetism sets**: Inexpensive coils, strong magnets, galvanometers, electroscopes and diffraction gratings for Lessons 10.1–10.5.

Not required. Every project in this unit can be built with household materials. School Launch has no ties to these companies.

LESSONS IN THIS UNIT

10.1	Charge and Coulomb's Law	3 sessions of 50 minutes
10.2	Circuits and Ohm's Law	3 sessions of 50 minutes
10.3	Magnetism: Gilbert, Oersted and the Electric Current	3 sessions of 50 minutes
10.4	Faraday's Induction: Generators, Motors and Transformers	3 sessions of 50 minutes
10.5	Maxwell, Light and the Photon	3 sessions of 50 minutes

Unit 10 test

Calculators are allowed. Use $k = 8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2}$, $e = 1.602 \times 10^{-19}\ \mathrm{C}$, $\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$, $h = 6.626 \times 10^{-34}\ \mathrm{J \cdot s}$, $c = 3.00 \times 10^8\ \mathrm{m/s}$ and $E(\mathrm{eV}) = 1240/\lambda(\mathrm{nm})$. Show the equation, substitution with units and the answer with its unit. The test is worth 100 points.

1. **6 pts**
   Find the force between charges of $+4.0\ \mu\mathrm{C}$ and $+2.0\ \mu\mathrm{C}$ placed $0.20\ \mathrm{m}$ apart. Is it attractive or repulsive?

2. **5 pts**
   Explain how a rubbed balloon can stick to a neutral wall.

3. 5 pts

How did Coulomb measure the force between charges, and what law did he find?

4. 5 pts

A 12 V battery drives 0.40 A through a lamp. Find its resistance and power.

5. 7 pts

Resistors of 30 Ω and 60 Ω are connected (a) in series and (b) in parallel to a 9.0 V battery. Find the total current in each case.

6. 5 pts

What resistor should be placed in series with an LED that drops 2.2 V at 15 mA, run from a 6.0 V battery?

7. 5 pts

Describe Oersted's 1820 discovery and why it was important.

8. 6 pts

Find the magnetic field 3.0 cm from a long wire carrying 12 A.

9. 6 pts

A solenoid of 800 turns and length 0.20 m carries 1.5 A. Find the field inside.

10. 5 pts

A 0.30 m wire with 4.0 A lies perpendicular to a 0.25 T field. Find the force on it.

11. 6 pts

A 300-turn coil of area $2.0 \times 10^{-3}\ \mathrm{m}^2$ is perpendicular to a field that drops from 0.50 T to zero in 0.020 s. Find the average induced emf.

12. 5 pts

State Lenz's law and use it to explain why a magnet falls slowly through a copper pipe.

13. 6 pts

A transformer steps 240 V down to 12 V for a 36 W device. Find the turns ratio and the primary current.

14. **6 pts**

Explain, with a calculation, why power is transmitted at high voltage: compare the line loss for 2.0 MW sent through a 4.0 Ω line at 20 kV and at 200 kV.

15. **5 pts**

Find the wavelength of a 1.9 GHz cell-phone signal.

16. **6 pts**

Find the energy in eV of a 480 nm photon. Light of this wavelength falls on a metal with work function 2.3 eV; find the maximum kinetic energy of ejected electrons.

17. **11 pts**

Essay. Trace the chain of discoveries from Oersted (1820) through Faraday (1831) to Maxwell (1865) and Hertz (1887). Explain how each step built on the one before, and why Maxwell's conclusion that light is an electromagnetic wave was such a turning point.

ANSWER KEY

1. 1.8 N, repulsive
2. The balloon's charge polarizes the wall by induction, pulling opposite charges slightly closer to the surface; the attraction to the nearer opposite charges outweighs the repulsion from the farther like charges.
3. With a torsion balance: a charged ball on a needle hung from a fine wire was repelled by another charged ball, and the twist of the wire needed to hold them at a given distance measured the force. He found the force proportional to each charge and inversely proportional to the square of the distance.
4. 30 Ω; 4.8 W
5. (a) 90 Ω, 0.10 A; (b) 20 Ω, 0.45 A
6. 253 Ω (use 270 Ω)
7. A compass needle near a wire swung when current flowed and reversed when the current reversed: an electric current makes a magnetic field. It was the first link between electricity and magnetism, leading directly to electromagnets, motors and Faraday's induction.
8. 8.0×10^{-5} T
9. 7.5×10^{-3} T
10. 0.30 N
11. 15 V
12. Induced currents flow so as to oppose the change in flux that causes them. The falling magnet induces eddy currents in the pipe whose fields oppose its motion, slowing it.
13. 20 to 1; 0.15 A
14. At 20 kV: $I = 100$ A, loss $= 40$ kW. At 200 kV: $I = 10$ A, loss $= 400$ W. Ten times the voltage, one hundredth the loss.
15. 0.158 m
16. 2.58 eV; 0.28 eV
17. Model answer: Oersted showed that an electric current produces a magnetic field. Faraday, reasoning that the reverse should hold, found that a changing magnetic field produces a current (induction), and he described fields as lines of force filling space. Maxwell put Faraday's fields into equations and added that a changing electric field produces a magnetic field; the equations then predicted self-sustaining waves of electric and magnetic fields, and their calculated speed

matched the measured speed of light, so light must be such a wave. Hertz generated and detected these waves electrically, confirming the theory and opening the way to radio. The result united electricity, magnetism and optics in one theory, gave engineers the tools for every wireless technology, and raised the questions about light and matter that led to quantum physics.

PORTFOLIO PIECE: A WORKING ELECTRIC MACHINE

Design, build and test one battery-powered electric machine and explain it with this unit's physics: a simple motor that turns a small fan, a hand-cranked generator that lights an LED (a hobby motor turned by hand works as a generator), a buzzer or doorbell made with an electromagnet and a spring contact, or a speaker made from a coil, a magnet and a paper cup. Make measurements: the motor's speed (turns per second by slow-motion video) and current for two battery voltages; the generator's voltage at two cranking speeds; the electromagnet's pull against the number of turns. Draw a labeled diagram and a circuit diagram, and write a report explaining how Ohm's law, magnetic forces and induction make your machine work, with your data and one improvement you made.

LOOK FOR	WHAT IT MEANS
Working machine	The machine works reliably on batteries only, with safe wiring.
Diagrams	Labeled drawing and a correct circuit diagram with standard symbols.
Measurements	At least two variables measured with units and repeated trials, presented in a table or graph.
Physics explanation	Correct use of Ohm's law, magnetic force on currents and/or Faraday's law to explain the results.
Improvement	One design change tested, with before-and-after data.

COURSE EXAM

Midterm Exam

Covers Units 1–5. Time: 100 minutes.

You have 100 minutes. A scientific calculator is allowed; no notes or formula sheets. Use $g = 9.8\ \mathrm{m/s^2}$ unless told otherwise. For every calculation write the equation, substitute with units, and give the answer with its unit and sensible significant figures; correct work earns partial credit even if the final number slips. Draw a free-body diagram for every force problem. The exam has 26 problems worth a total of 100 points.

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1. **3 pts** **Lesson 1.2**

   Convert 90 km/h to m/s, and a density of $2.70\ \mathrm{g/cm^3}$ to $\mathrm{kg/m^3}$.

2. **3 pts** **Lesson 1.3**

   Report the result of $4.52\ \mathrm{m} \times 2.1\ \mathrm{m}$ and of $13.27\ \mathrm{s} + 0.4\ \mathrm{s}$ with correct significant figures.

3. **3 pts** **Lesson 1.3**

   Five measurements of a period are 1.42, 1.45, 1.40, 1.44 and 1.44 s. Find the mean, and the percent error if the accepted value is 1.40 s.

4. **3 pts** **Lesson 1.4**

   A spring stretches 4.0, 8.1 and 11.9 cm under 2.0, 4.0 and 6.0 N. Find the spring constant from the best-fit line through the origin.

5. **4 pts** **Lesson 1.5**

   Find the resultant of 12 N east and 16 N north, and the components of a 50 N force at 37° above the horizontal.

6. **4 pts** **Lesson 2.1**

   A ball rolls from rest down a ramp and covers 0.45 m in 1.5 s. Find its acceleration and its distance after 3.0 s.

7. **4 pts** **Lesson 2.2**

   A car at 20 m/s needs 40 m to stop after the brakes are applied. Find its deceleration and the stopping time.

8. 4 pts **Lesson 2.3**

A ball is thrown straight up at 14.7 m/s. Find its maximum height and its time in the air.

9. 4 pts **Lesson 2.4**

A velocity–time graph rises from 0 to 10 m/s in 5.0 s, stays at 10 m/s for 5.0 s and falls to 0 in 2.0 s. Find the total displacement and the final deceleration.

10. 4 pts **Lesson 2.5**

A ball rolls off a 1.10 m table at 2.5 m/s. How far from the table does it land?

11. 4 pts **Lesson 2.5**

A ball is kicked at 18 m/s at 40° above level ground. Find its range.

12. 3 pts **Lesson 3.1**

State Newton's third law, and identify the third-law partner of the Earth's gravitational pull on a falling apple.

13. 4 pts **Lesson 3.3**

A 55 kg person rides an elevator accelerating downward at 1.8 $\mathrm{m/s^2}$. What does a bathroom scale read in newtons?

14. 4 pts **Lesson 3.4**

A 12 kg box is pulled at constant speed across a floor by a horizontal 47 N force. Find μ_k. What force is needed to accelerate it at 1.5 $\mathrm{m/s^2}$?

15. 4 pts **Lesson 3.5**

An Atwood machine holds 1.5 kg and 1.0 kg. Find the acceleration and the string tension.

16. 4 pts **Lesson 3.5**

A 4.0 kg block slides down a 25° ramp with $\mu_k = 0.20$. Find its acceleration.

17. 4 pts **Lesson 4.1**

A 900 kg car rounds a curve of radius 60 m at 18 m/s. Find the friction force required and the minimum μ_s.

18. 3 pts **Lesson 4.2**

At what altitude above the Earth's surface is g one quarter of its surface value? ($R_E = 6370\ \mathrm{km}$)

19. 4 pts **Lesson 4.3**

An asteroid orbits the Sun with a period of 5.2 years. Find its semi-major axis in AU.

20. 4 pts **Lesson 4.5**

Find the orbital speed and period of a satellite $1000\ \mathrm{km}$ above the Earth ($GM_E = 3.99 \times 10^{14}\ \mathrm{m^3/s^2}$, $R_E = 6.37 \times 10^6\ \mathrm{m}$).

21. 4 pts **Lesson 5.1**

A $10\ \mathrm{kg}$ weight falls $2.0\ \mathrm{m}$ fifteen times, stirring $1.5\ \mathrm{kg}$ of water. If all the energy heats the water, find the temperature rise.

22. 4 pts **Lesson 5.2**

A $1300\ \mathrm{kg}$ car slows from $25\ \mathrm{m/s}$ to $10\ \mathrm{m/s}$. How much work do the brakes do?

23. 4 pts **Lesson 5.3**

A $0.050\ \mathrm{kg}$ ball is launched straight up by a spring ($k = 300\ \mathrm{N/m}$) compressed $8.0\ \mathrm{cm}$. Ignoring losses, how high does it rise?

24. 4 pts **Lesson 5.4**

A motor lifts a $250\ \mathrm{kg}$ load $12\ \mathrm{m}$ in $20\ \mathrm{s}$. Find its power output in watts and horsepower.

25. 4 pts **Lesson 5.5**

Find the minimum release height for a frictionless loop of radius $0.25\ \mathrm{m}$, and the rider's g-force at the bottom when released from that height.

26. 6 pts **Lesson 1.1**

Short answer. Using one example from Units 1–5 (Kepler and Mars, Galileo's ramp, Michelson, Joule), explain what it means to let measurements decide between theories, and why honest records matter.

ANSWER KEY

1. $25\ \text{m/s}$; $2700\ \text{kg/m}^3$
2. $9.5\ \text{m}^2$; $13.7\ \text{s}$
3. Mean $1.43\ \text{s}$; percent error 2.1%
4. Slope of F against x is about $6.0\ \text{N}/0.119\ \text{m} \approx 50\ \text{N/m}$
5. $20\ \text{N}$ at 53° north of east; $F_x = 39.9\ \text{N}$, $F_y = 30.1\ \text{N}$
6. $a = 0.40\ \text{m/s}^2$; $1.8\ \text{m}$ (four times as far)
7. $5.0\ \text{m/s}^2$; $4.0\ \text{s}$
8. $11.0\ \text{m}$; $3.0\ \text{s}$
9. $25 + 50 + 10 = 85\ \text{m}$; $5.0\ \text{m/s}^2$
10. $t = 0.474\ \text{s}$; $x = 1.18\ \text{m}$
11. $R = 18^2 \sin 80^\circ / 9.8 = 32.6\ \text{m}$
12. Forces come in equal and opposite pairs acting on different objects. The partner is the apple's equal upward gravitational pull on the Earth.
13. $55(9.8 - 1.8) = 440\ \text{N}$
14. $\mu_k = 47/117.6 = 0.40$; $47 + 18 = 65\ \text{N}$
15. $a = 1.96\ \text{m/s}^2$; $T = 11.8\ \text{N}$
16. $9.8(\sin 25^\circ - 0.20 \cos 25^\circ) = 2.36\ \text{m/s}^2$
17. $F = 4860\ \text{N}$; $\mu_s = 0.55$
18. At $2R_E$ from the center: altitude $6370\ \text{km}$
19. $a = 5.2^{2/3} = 3.0\ \text{AU}$
20. $r = 7.37 \times 10^6\ \text{m}$; $v = 7.36\ \text{km/s}$; $T = 6290\ \text{s} = 105\ \text{min}$
21. $W = 2940\ \text{J}$; $\Delta T = 0.47\ \text{K}$
22. $\Delta KE = \frac{1}{2}(1300)(100 - 625) = -3.4 \times 10^5\ \text{J}$
23. $PE_s = 0.96\ \text{J}$; $h = 1.96\ \text{m}$
24. $1470\ \text{W}$; $1.97\ \text{hp}$
25. $0.625\ \text{m}$; $6\,g$
26. Model answer: Kepler's circle model missed Tycho's Mars data by 8 arcminutes, several times the data's uncertainty, so he discarded the model and found the ellipse. A model must fit careful measurements within their uncertainty; if the

records were altered to fit, the error would never be discovered. Honest records, including failed trials, let others check and build on the work.

COURSE EXAM

Final Exam

Covers Units 1–10. Time: 120 minutes.

You have 120 minutes. A scientific calculator is allowed; no notes. Use $g = 9.8\ \mathrm{m/s^2}$, $G = 6.674 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$, sound speed $343\ \mathrm{m/s}$, $c = 3.00 \times 10^8\ \mathrm{m/s}$, $c_{\text{water}} = 4186\ \mathrm{J/(kg \cdot K)}$ and $E(\mathrm{eV}) = 1240/\lambda(\mathrm{nm})$. Show the equation, substitution with units and the answer with its unit for every calculation. The exam has 30 problems worth a total of 100 points.

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1. 3 pts **Lesson 1.5**

   A boat heads due north at $4.0\ \mathrm{m/s}$ across a river flowing east at $3.0\ \mathrm{m/s}$. Find its velocity relative to the bank.

2. 3 pts **Lesson 2.2**

   A train accelerates from 5.0 to $25\ \mathrm{m/s}$ over $600\ \mathrm{m}$. Find its acceleration and the time taken.

3. 4 pts **Lesson 2.5**

   A stone is thrown horizontally at $12\ \mathrm{m/s}$ from a $45\ \mathrm{m}$ cliff. Find its time of flight, landing distance and speed at impact.

4. 3 pts **Lesson 3.3**

   A $2.0\ \mathrm{kg}$ box on a frictionless table is pulled by a $10\ \mathrm{N}$ force at 30° above the horizontal. Find its acceleration and the normal force.

5. 3 pts **Lesson 3.4**

   A crate starts to slide when a ramp is tilted to 32°. Find μ_s.

6. 3 pts **Lesson 4.1**

   Find the bank angle for a curve of radius $150\ \mathrm{m}$ designed for $22\ \mathrm{m/s}$ with no friction needed.

7. 4 pts **Lesson 4.4**

   Use $g = 3.71\ \mathrm{m/s^2}$ and $R = 3.39 \times 10^6\ \mathrm{m}$ to find the mass of Mars.

8. 3 pts **Lesson 5.3**

A roller-coaster car starts from rest 35 m high. With 12% of its energy lost to friction by the bottom, how fast is it moving there?

9. 3 pts **Lesson 6.1**

A 0.16 kg hockey puck moving at 30 m/s is stopped by a goalie's glove in 0.025 s. Find the average force. Why does the goalie let the glove move back?

10. 3 pts **Lesson 6.2**

A 70 kg skater moving at 3.0 m/s catches a 5.0 kg ball coming toward him at 10 m/s. Find his speed afterward.

11. 3 pts **Lesson 6.3**

A 0.50 kg cart at 4.0 m/s collides elastically head-on with a stationary 1.5 kg cart. Find both velocities afterward.

12. 3 pts **Lesson 6.4**

A rocket stage with exhaust speed 3200 m/s and mass ratio 3.5 gives what change in speed?

13. 4 pts **Lesson 6.5**

A 1000 kg car going east at 18 m/s and a 1500 kg car going north at 14 m/s collide and stick. Find the speed and direction of the wreckage.

14. 3 pts **Lesson 7.1**

On a meter stick balanced at 50 cm, 120 g hangs at 15 cm. Where must 200 g hang to balance it?

15. 4 pts **Lesson 7.3**

A uniform 5.0 m, 300 N plank rests on supports at its ends. A 700 N person stands 1.0 m from the left end. Find both support forces.

16. 3 pts **Lesson 7.4**

A solid sphere rolls from rest down a ramp 1.2 m high. Find its speed at the bottom.

17. 3 pts **Lesson 7.5**

A spinning chair rider with $I = 2.8\ \mathrm{kg \cdot m^2}$ turns once every $1.5\ \mathrm{s}$. Pulling in weights reduces I to $1.4\ \mathrm{kg \cdot m^2}$. Find the new period.

18. 3 pts **Lesson 8.1**

Find the gauge pressure at the bottom of a $3.5\ \mathrm{m}$ deep pool, and the force on a $0.20\ \mathrm{m}$-square drain cover there.

19. 3 pts **Lesson 8.2**

A $250\ \mathrm{g}$ object weighs $1.47\ \mathrm{N}$ when hung in water. Find its volume and density.

20. 4 pts **Lesson 8.3**

Water leaves a hole $0.80\ \mathrm{m}$ below the surface of a tank. Find its speed. If the hole is $1.0\ \mathrm{m}$ above the ground, how far away does the jet land?

21. 3 pts **Lesson 8.4**

$400\ \mathrm{g}$ of water at $80^\circ\mathrm{C}$ is mixed with $600\ \mathrm{g}$ of water at $20^\circ\mathrm{C}$ in an insulated container. Find the final temperature.

22. 3 pts **Lesson 8.5**

Find the Carnot efficiency of an engine running between $600\ \mathrm{K}$ and $300\ \mathrm{K}$. A real engine between these temperatures takes in $2000\ \mathrm{J}$ and rejects $1300\ \mathrm{J}$; find its efficiency.

23. 3 pts **Lesson 9.1**

Find the length of a pendulum with a period of $1.5\ \mathrm{s}$, and the period of a $0.40\ \mathrm{kg}$ mass on a spring with $k = 40\ \mathrm{N/m}$.

24. 3 pts **Lesson 9.3**

A siren of $600\ \mathrm{Hz}$ approaches you at $25\ \mathrm{m/s}$. What frequency do you hear? What after it passes?

25. 3 pts **Lesson 9.4**

Light enters water ($n = 1.33$) from air at 50°. Find the refraction angle, and the critical angle for light leaving the water.

26. 3 pts **Lesson 9.5**

A grating with 600 lines/mm sends a laser's first-order beam to 22.9°. Find the wavelength.

27. 4 pts **Lesson 10.2**

A 6.0 V battery is connected to 20 Ω in series with a parallel pair of 30 Ω and 60 Ω. Find the total current and the power delivered by the battery.

28. 3 pts **Lesson 10.4**

A coil of 150 turns and area $5.0 \times 10^{-3}\ \mathrm{m}^2$ lies perpendicular to a field that changes by 0.40 T in 0.050 s. Find the induced emf. Explain why a transformer will not work on a steady battery current.

29. 3 pts **Lesson 10.5**

Find the energy in eV of a 620 nm photon, and the wavelength of the hydrogen line from $n = 3$ to $n = 2$.

30. 7 pts **Lesson all**

Essay. Choose one law of physics from this course (for example, conservation of energy, conservation of momentum, universal gravitation or Faraday's law). Tell the story of its discovery, show with one calculation how it is used, and explain one engineering application that serves people. Close with a sentence on what the order of the physical world suggests to you.

ANSWER KEY

1. $5.0\ \mathrm{m/s}$ at 36.9° east of north
2. $a = 0.50\ \mathrm{m/s^2}$; $t = 40\ \mathrm{s}$
3. $t = 3.03\ \mathrm{s}$; $x = 36.4\ \mathrm{m}$; $v = \sqrt{144 + 29.7^2} = 32.0\ \mathrm{m/s}$
4. $a = 4.33\ \mathrm{m/s^2}$; $N = 19.6 - 5.0 = 14.6\ \mathrm{N}$
5. $\tan 32^\circ = 0.62$
6. $\tan\theta = 484/1470 = 0.329$; $\theta = 18.2^\circ$
7. $M = gR^2/G = 6.39 \times 10^{23}\ \mathrm{kg}$
8. $v = \sqrt{2(0.88)(9.8)(35)} = 24.6\ \mathrm{m/s}$
9. $F = 4.8/0.025 = 192\ \mathrm{N}$. Letting the glove move back lengthens the stopping time and lowers the force.
10. $(210 - 50)/75 = 2.13\ \mathrm{m/s}$ in his original direction
11. $v_1' = -2.0\ \mathrm{m/s}$; $v_2' = 2.0\ \mathrm{m/s}$
12. $3200 \ln 3.5 = 4010\ \mathrm{m/s}$
13. $p_x = 18{,}000$, $p_y = 21{,}000$; $v = 11.1\ \mathrm{m/s}$ at 49.4° north of east
14. $120(35) = 200d$, $d = 21\ \mathrm{cm}$: the 71 cm mark
15. Right: $(300 \times 2.5 + 700 \times 1.0)/5.0 = 290\ \mathrm{N}$; left: $710\ \mathrm{N}$
16. $v = \sqrt{10gh/7} = 4.10\ \mathrm{m/s}$
17. $0.75\ \mathrm{s}$
18. $34.3\ \mathrm{kPa}$; $1370\ \mathrm{N}$
19. $F_b = 2.45 - 1.47 = 0.98\ \mathrm{N}$; $V = 100\ \mathrm{cm^3}$; $\rho = 2.5\ \mathrm{g/cm^3}$
20. $v = 3.96\ \mathrm{m/s}$; $x = 2\sqrt{0.80 \times 1.0} = 1.79\ \mathrm{m}$
21. $44^\circ\mathrm{C}$
22. 50%; 35%
23. $0.559\ \mathrm{m}$; $0.63\ \mathrm{s}$
24. $647\ \mathrm{Hz}$; $559\ \mathrm{Hz}$
25. 35.2°; 48.8°
26. $\lambda = (1.667 \times 10^{-6}) \sin 22.9^\circ = 648\ \mathrm{nm}$
27. $R = 40\ \Omega$; $I = 0.15\ \mathrm{A}$; $P = 0.90\ \mathrm{W}$
28. $6.0\ \mathrm{V}$. A steady current makes a constant flux, and only a changing flux induces a voltage.

29. 2.0 eV; 656 nm
30. Model answer will vary. Full credit for an accurate historical account (who, when, what was measured), a correct calculation with units, a clear engineering application, and a thoughtful closing reflection.