

PLAYING THE ODDS

First edition

8 units

32 lessons

A video for every lesson

FIRST EDITION

COURSE GUIDE

Playing the Odds is a one-year elective for students in grades 4–7 about chance, counting and good decisions. Its big idea is simple: good players and coaches rarely win by luck. They win by choosing, again and again, the play most likely to pay off, while keeping the chance of a disaster small. A basketball player who takes the shot worth the most points per try, a goalkeeper who keeps the kicker guessing, a Yahtzee player who knows when to stop rolling, and a family that buckles its seat belts on a short trip are all playing the odds wisely.

Every lesson begins with a true story: the Wright brothers tossing a coin to see who would fly first, Blaise Pascal and Pierre de Fermat writing letters about an unfinished game in 1654, Francis Galton pouring shot through his board of pins, Henry Chadwick inventing the box score, Rick Barry shooting his free throws underhand, and Eric Liddell refusing to run on a Sunday. Then the student learns **how they figured it out, the mathematics** underneath, explained with fractions, tallies and simple multiplication, and how the idea is used **on the field** today.

Students do the mathematics with their own hands. They flip coins, roll dice hundreds of times, build a homemade Galton board, shoot a soft ball from two distances to find their own points per try, keep score of a real baseball game, chart shots, and play Yahtzee and Battleship with a scorebook beside them. Probability was born from questions about dice, and the course tells that history honestly, but it teaches chance for good decisions, never for staking money on the outcome: every game keeps score in points. The year ends with a family game day where every player keeps a scorebook.

Videos were chosen lesson by lesson from trusted teaching channels such as Math Antics, Khan Academy and Numberphile, from official sports and museum channels, and from a few smaller channels run by careful teachers and game designers. Every video was previewed frame by frame and its description read. None promotes games of chance for money, and channels built around such games, opinion

shows or public controversies were left out. Some channels end with an invitation to subscribe or a short sponsor message; you can stop the video when the teaching ends. The “when” note tells you when to watch, and a parent note flags anything to know in advance.

Start with Lesson 1.1 →

WHO IT IS FOR

Written for students in grades 4–7 (about ages 9–13). A 9-year-old can follow every lesson with a parent reading along; a 12-year-old can read it alone. Students should be able to add, subtract and multiply, and know what a simple fraction means; the course teaches decimals and percents as it goes, and a calculator is welcome.

Nothing needs to be bought beyond ordinary household items: dice, coins, a deck of cards, a soft ball, a bucket or basket, paper and pencil.

HOW TO USE THIS COURSE

1. **The weekly rhythm.** The year has 32 lessons, one a week, each in two sessions of about 45 minutes. Session 1 is the warm-up, the video, the teaching and the picture study. Session 2 is the project and the scorebook. In the last week of each unit, add a third session for the unit review and the portfolio project.
2. **Story, then mathematics, then the field.** The core ideas are written to be read aloud or read by the student. *The story* tells a real moment with names, dates and places. *How they figured it out* shows what people counted or noticed. *The mathematics* explains the idea with fractions, tallies and multiplication. *On the field* shows how players, coaches and families use it. After each paragraph, ask the student to tell it back in his or her own words.
3. **Picture study.** Each lesson has one public-domain painting, portrait, print or historic photograph. Look at it quietly for two minutes. Then put it away and have the student tell everything he or she remembers. Bring it back and work through the “look for” items and the questions, which point back to the lesson’s idea of chance, counting or choosing.
4. **The scorebook habit.** Keep one graph-paper notebook for the whole year: the scorebook. Every project follows the same four steps: *count* (tally real tries), *record* (a neat table or bar graph), *compare* (your results against the mathematics) and *decide* (what you would do next time, and why). Write a prediction before every

experiment and date every page. Record what really happened, not what should have happened.

5. **Chance and choices.** Every lesson names both the percentage (how often a choice pays off, and how much) and the risk (what can go wrong, and how bad it would be). The goal is not to guess the future but to choose well when the future is uncertain, and to judge a choice by how it was made, not only by how it turned out.
6. **How we handle games of chance.** Probability began with questions about dice, and the course tells that history plainly. But it teaches chance for good decisions, never for staking money on the outcome. No lesson, project or game uses money stakes of any kind; every game keeps score in points. Carnival booths appear in one lesson (3.3) as an example of a game built so that most players lose.
7. **Unit reviews and portfolios.** Each unit ends with a short review worth 50 points and a portfolio project that pulls the unit together, such as a fair game of the student's own design, a personal shot chart, a scored baseball game or a strategy guide for a family game. Reviews may be done aloud for a student who writes slowly.
8. **Optional books and games.** Some units list free websites, classic family games and books that fit the lessons. They are never required; every project works with household items.
9. **Safety for shooting and throwing.** Several projects use a soft ball and a basket or bucket. Warm up first, use only soft balls indoors, keep clear of windows, lamps and people, and wear proper shoes. A parent supervises any trip to a court, field or park.
10. **The family game day.** The last lesson of the year is a family game tournament. The student plans it, runs it and keeps score, and every player keeps a scorebook of his or her decisions. Set out the best project from each unit so guests can see the year's work.

SCOPE AND SEQUENCE

UNIT 1 • WEEKS 1–4

What Is Chance?

Likely and unlikely on the probability line; coin flips and fair choices, from the Wright brothers' coin toss to the start of a football game; Pascal and Fermat's letters about an unfinished game; and chance and providence (Proverbs 16:33).

- 1.1 Likely and Unlikely: The Probability Line
- 1.2 Heads or Tails: Fair Choices
- 1.3 Pascal and Fermat: The Unfinished Game
- 1.4 The Lot Is Cast: Chance and Providence

UNIT 3 • WEEKS 9–12

Fair and Unfair Games

What makes a game fair, from Abram and Lot to “I cut, you choose”; spinners and odd-or-even games with Milton Bradley's teetotum; why carnival games are hard to win; and designing and testing a fair game of your own, with Christiaan Huygens.

- 3.1 What Makes a Game Fair?
- 3.2 Spinners and Odd-or-Even
- 3.3 Why Carnival Games Are Hard to Win
- 3.4 Design a Fair Game

UNIT 2 • WEEKS 5–8

Dice and Counting

Rolling one die and the meaning of one in six; two dice and why 7 comes up most (and why Catan's 6 and 8 are red); tally charts, bar graphs and Galton's board; and why dice have no memory.

- 2.1 One Die, Six Faces
- 2.2 Two Dice: Why 7 Comes Most Often
- 2.3 Tally Charts and Galton's Board
- 2.4 Dice Have No Memory

UNIT 4 • WEEKS 13–16

Points per Try

Points per try: two-point and three-point shots at a home basket; free throws and Rick Barry's underhand shot; the extra-point kick versus going for two; and why the best shot is the one worth the most points per try.

- 4.1 Twos and Threes at a Home Basket
- 4.2 Free Throws and Rick Barry's Underhand Shot
- 4.3 The Kick or Going for Two
- 4.4 The Best Shot: Most Points per Try

UNIT 5 • WEEKS 17–20

Baseball by the Numbers

Henry Chadwick and the box score; batting average and on-base percentage, with Branch Rickey and Allan Roth; keeping score of a real game; and Ty Cobb, Babe Ruth and what their numbers mean.

- 5.1 Henry Chadwick's Box Score
- 5.2 Batting Average and On-Base Percentage
- 5.3 Keeping Score of a Real Game
- 5.4 Ty Cobb, Babe Ruth and the Numbers

UNIT 7 • WEEKS 25–28

Odds at the Game Table

Odds at the game table: keeping and rerolling in Yahtzee; hunting and targeting in Battleship; Monopoly's most-landed squares; and Can't Stop and knowing when to stop.

- 7.1 Yahtzee: Keep or Reroll?
- 7.2 Battleship: Hunt and Target
- 7.3 Monopoly's Most-Landed Squares
- 7.4 Can't Stop: Knowing When to Stop

UNIT 6 • WEEKS 21–24

The High-Percentage Shot

The high-percentage shot in other sports: shots close to the goal in soccer; the shot chart in hockey and basketball; golf's safe shot to the middle of the green; and tennis's first and second serve.

- 6.1 Close to the Goal: Shots in Soccer
- 6.2 The Shot Chart: Hockey's Slot
- 6.3 Aim for the Middle of the Green
- 6.4 First Serve, Second Serve

UNIT 8 • WEEKS 29–32

Wise Choices

Wise choices: safe and risky choices, from Jacques Plante's mask to seat belts; judging a good decision even when it loses; time and chance, and Eric Liddell; and a family game day where every player keeps a scorebook.

- 8.1 Safe and Risky Choices: Goalie Masks, Helmets and Seat Belts
- 8.2 A Good Decision Can Still Lose
- 8.3 Time and Chance: Eric Liddell
- 8.4 Family Game Day

WEEKLY PACING CALENDAR

WEEK	PLAN
1	- Session 1: Lesson 1.1 Likely and Unlikely: The Probability Line: warm-up, video, teaching and picture study - Session 2: Lesson 1.1 project and scorebook <i>Two sessions a week is the usual rhythm. Younger children can split a long session in two; older students can do both sessions in one sitting.</i>

WEEK	PLAN
2	- Session 1: Lesson 1.2 Heads or Tails: Fair Choices: warm-up, video, teaching and picture study - Session 2: Lesson 1.2 project and scorebook
3	- Session 1: Lesson 1.3 Pascal and Fermat: The Unfinished Game: warm-up, video, teaching and picture study - Session 2: Lesson 1.3 project and scorebook
4	- Session 1: Lesson 1.4 The Lot Is Cast: Chance and Providence: warm-up, video, teaching and picture study - Session 2: Lesson 1.4 project and scorebook - Session 3: Unit 1 review and portfolio project
5	- Session 1: Lesson 2.1 One Die, Six Faces: warm-up, video, teaching and picture study - Session 2: Lesson 2.1 project and scorebook
6	- Session 1: Lesson 2.2 Two Dice: Why 7 Comes Most Often: warm-up, video, teaching and picture study - Session 2: Lesson 2.2 project and scorebook
7	- Session 1: Lesson 2.3 Tally Charts and Galton's Board: warm-up, video, teaching and picture study - Session 2: Lesson 2.3 project and scorebook
8	- Session 1: Lesson 2.4 Dice Have No Memory: warm-up, video, teaching and picture study - Session 2: Lesson 2.4 project and scorebook - Session 3: Unit 2 review and portfolio project
9	- Session 1: Lesson 3.1 What Makes a Game Fair?: warm-up, video, teaching and picture study - Session 2: Lesson 3.1 project and scorebook
10	- Session 1: Lesson 3.2 Spinners and Odd-or-Even: warm-up, video, teaching and picture study - Session 2: Lesson 3.2 project and scorebook
11	Session 1: Lesson 3.3 Why Carnival Games Are Hard to Win: warm-up, video, teaching and picture study

WEEK	PLAN
	Session 2: Lesson 3.3 project and scorebook
12	- Session 1: Lesson 3.4 Design a Fair Game: warm-up, video, teaching and picture study - Session 2: Lesson 3.4 project and scorebook - Session 3: Unit 3 review and portfolio project
13	- Session 1: Lesson 4.1 Twos and Threes at a Home Basket: warm-up, video, teaching and picture study - Session 2: Lesson 4.1 project and scorebook
14	- Session 1: Lesson 4.2 Free Throws and Rick Barry's Underhand Shot: warm-up, video, teaching and picture study - Session 2: Lesson 4.2 project and scorebook
15	- Session 1: Lesson 4.3 The Kick or Going for Two: warm-up, video, teaching and picture study - Session 2: Lesson 4.3 project and scorebook
16	- Session 1: Lesson 4.4 The Best Shot: Most Points per Try: warm-up, video, teaching and picture study - Session 2: Lesson 4.4 project and scorebook - Session 3: Unit 4 review and portfolio project
17	- Session 1: Lesson 5.1 Henry Chadwick's Box Score: warm-up, video, teaching and picture study - Session 2: Lesson 5.1 project and scorebook
18	- Session 1: Lesson 5.2 Batting Average and On-Base Percentage: warm-up, video, teaching and picture study - Session 2: Lesson 5.2 project and scorebook
19	- Session 1: Lesson 5.3 Keeping Score of a Real Game: warm-up, video, teaching and picture study - Session 2: Lesson 5.3 project and scorebook

WEEK	PLAN
20	• Session 1: Lesson 5.4 Ty Cobb, Babe Ruth and the Numbers: warm-up, video, teaching and picture study • Session 2: Lesson 5.4 project and scorebook • Session 3: Unit 5 review and portfolio project
21	• Session 1: Lesson 6.1 Close to the Goal: Shots in Soccer: warm-up, video, teaching and picture study • Session 2: Lesson 6.1 project and scorebook
22	• Session 1: Lesson 6.2 The Shot Chart: Hockey's Slot: warm-up, video, teaching and picture study • Session 2: Lesson 6.2 project and scorebook
23	• Session 1: Lesson 6.3 Aim for the Middle of the Green: warm-up, video, teaching and picture study • Session 2: Lesson 6.3 project and scorebook
24	• Session 1: Lesson 6.4 First Serve, Second Serve: warm-up, video, teaching and picture study • Session 2: Lesson 6.4 project and scorebook • Session 3: Unit 6 review and portfolio project
25	• Session 1: Lesson 7.1 Yahtzee: Keep or Reroll?: warm-up, video, teaching and picture study • Session 2: Lesson 7.1 project and scorebook
26	• Session 1: Lesson 7.2 Battleship: Hunt and Target: warm-up, video, teaching and picture study • Session 2: Lesson 7.2 project and scorebook
27	• Session 1: Lesson 7.3 Monopoly's Most-Landed Squares: warm-up, video, teaching and picture study • Session 2: Lesson 7.3 project and scorebook
28	• Session 1: Lesson 7.4 Can't Stop: Knowing When to Stop: warm-up, video, teaching and picture study • Session 2: Lesson 7.4 project and scorebook • Session 3: Unit 7 review and portfolio project

WEEK	PLAN
29	• Session 1: Lesson 8.1 Safe and Risky Choices: Goalie Masks, Helmets and Seat Belts: warm-up, video, teaching and picture study • Session 2: Lesson 8.1 project and scorebook
30	• Session 1: Lesson 8.2 A Good Decision Can Still Lose: warm-up, video, teaching and picture study • Session 2: Lesson 8.2 project and scorebook
31	• Session 1: Lesson 8.3 Time and Chance: Eric Liddell: warm-up, video, teaching and picture study • Session 2: Lesson 8.3 project and scorebook
32	• Session 1: Lesson 8.4 Family Game Day: warm-up, video, teaching and picture study • Session 2: Lesson 8.4 project and scorebook • Session 3: Unit 8 review and portfolio project <i>Last week. Hold a family game day: the student runs the culminating event from Unit 8 and shows guests the scorebook and the best project from each unit.</i>

MATERIALS

The scorebook: a graph-paper composition book, pencils and a ruler

One for the whole year: predictions, tallies, tables, bar graphs, shot charts and “what I would decide and why.”

Five ordinary dice, two of them in different colors

For nearly every experiment. Two colors make it easy to see that a red 2 with a blue 5 is a different outcome from a red 5 with a blue 2.

Four coins

For coin-flip experiments and the homemade Galton board in Unit 2. Pennies work well.

A deck of ordinary playing cards

For drawing and counting experiments. Families who prefer can use 52 index cards numbered 1–13 in four colors.

Card stock, a paper clip, a pencil, scissors and tape

For making spinners, a homemade Galton board and game boards.

A soft ball (foam ball or rolled socks) and a basket, bucket or laundry hamper

For the shooting experiments in Units 4 and 6.

A tape measure or yardstick, and painter’s tape

For marking shooting lines and targets on the floor.

A calculator

For turning tallies into fractions, decimals and percents.

Family games you may already own

Yahtzee, Battleship and Monopoly fit Unit 7, but every lesson gives a paper-and-dice version, so nothing needs to be bought.

Optional: free spreadsheet software

Google Sheets or LibreOffice Calc can draw bar graphs from your tallies. Paper bar graphs are just as good.

GRADING

Grades come mostly from the projects and the scorebook, because chance is learned by counting real tries. A project earns full credit when the student predicts first, counts honestly, compares the results with the mathematics and writes a decision with a reason, even if the dice did something surprising.

PART	%
Projects (prediction, honest tallies, comparison, decision)	40
Scorebook (neat, dated, complete and honest)	20
Unit reviews	15
Portfolio projects	25

Scale: A 90–100, B 80–89, C 70–79, D 60–69. Below 60: redo the project or review with help.

- Grade the process, not the luck. A student who predicted carefully, counted honestly and explained a surprising result has done the work well.
- A scorebook page earns full marks when it has a date, a prediction, a tally or table with clear labels, and one sentence beginning “I would decide...”.

- Unit reviews may be done aloud for a student who writes slowly; the parent writes down the answers.
- Keep each unit's portfolio pages in one folder. They make a fine record of the year and a display for the family game day.

A NOTE TO PARENTS

You do not need to be a mathematician to teach this course. Read the story aloud together, watch the video, and then be the student's partner at the table: roll the dice, keep the tally, and ask two questions over and over. "What do you think will happen?" and "How do you know?" A child who can answer the second question from his or her own tallies is learning probability.

This course handles games of chance with care. It tells the history of probability honestly, including the dice questions that started it, but it never teaches or encourages playing for money. Every game keeps score in points, and the course never treats games of chance as a way to win money. One lesson looks at carnival booths, to show how a game can be built so that most players lose, and pairs it with Scripture on honest weights.

The course is also a course in wisdom. Scripture teaches that "the lot is cast into the lap; but the whole disposing thereof is of the LORD" (Proverbs 16:33), and also that the prudent man looks ahead and counts the cost. Good choices are careful and humble at the same time: we do the counting well, and we trust God with the outcome. Expect surprises, because chance is surprising, and praise careful counting and honest records more than lucky results.

UNIT 1

WHAT IS CHANCE?

Will it rain tomorrow? Will the next free throw go in? Will the coin land heads? Nobody knows for sure, but we are not completely in the dark either. Some things are certain, some are impossible, and most are somewhere in between. In this unit you will learn to put a number on chance, from 0 for impossible to 1 for certain, and to write it as a fraction, a decimal and a percent.

You will meet the people who first counted chances: Girolamo Cardano, an Italian doctor who wrote the first book about it, and Blaise Pascal and Pierre de Fermat, two Frenchmen who solved a puzzle about an unfinished game in a famous set of letters in 1654. You will see how the Wright brothers used a coin toss to decide who would fly first, and you will think about what Scripture says about the lot that is cast into the lap. Most of all, you will start your scorebook: predict, count, record and compare.

PREREQUISITES

- Can add, subtract and multiply whole numbers
- Knows what a simple fraction such as $\frac{1}{2}$ or $\frac{3}{4}$ means
- Can make tally marks and count them by fives

MASTERY CHECKLIST

- ☐ I can place events on a probability line from impossible (0) to certain (1).
- ☐ I can find a simple probability by counting equally likely outcomes, and write it as a fraction, a decimal and a percent.
- ☐ I can explain why a coin toss is a fair way to choose, and list the outcomes of flipping two coins.
- ☐ I can tell how Pascal and Fermat divided the prize of an unfinished game by looking at what could still happen.
- ☐ I can explain what Proverbs 16:33 teaches about chance and God's rule.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Khan Academy: Basic probability:** Free. Short videos and practice on simple probability, sample spaces and coin and dice experiments that match Lessons 1.1 and 1.2. Best for ages 11 and up; younger students can work through it with a parent.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

1.1 Likely and Unlikely: The Probability Line	2 sessions of 45 minutes
1.2 Heads or Tails: Fair Choices	2 sessions of 45 minutes
1.3 Pascal and Fermat: The Unfinished Game	2 sessions of 45 minutes
1.4 The Lot Is Cast: Chance and Providence	2 sessions of 45 minutes

UNIT 1 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. 4 pts

Place each event on the probability line as impossible, unlikely, even chance, likely or certain: (a) rolling a 6 on one die; (b) rolling a number from 1 to 6 on one die; (c) a coin lands heads; (d) rolling a 0 on one die.

2. 5 pts

A bag holds 2 green marbles and 3 yellow marbles. What is the probability of drawing a green marble without looking? Write it as a fraction, a decimal and a percent.

3. 4 pts

What did Girolamo Cardano write about in his *Book on Games of Chance*, and why was it new?

4. (5 pts)

List all the outcomes of flipping two coins. What is the probability of getting exactly one head?

5. (4 pts)

Why did the Wright brothers use a coin toss to decide who would fly first? Was it a fair way? Was it a wise way?

6. (5 pts)

A game to 3 points is stopped at 2 to 1. Use Fermat's method to find the leader's fair share of 12 marbles.

7. (4 pts)

Why is dividing the prize by the points already scored not fair?

8. (5 pts)

What would happen if you drew straws last instead of first, with 4 straws and one short? Show your reasoning.

9. (4 pts)

A coin is flipped and hidden under a cup. What is the probability it is heads for you? For someone who has peeked?

10. (4 pts)

In your 100 coin flips you got 46 heads. Does that prove your coin is unfair? Explain.

11. (6 pts)

Short answer: Write Proverbs 16:33 and explain in two or three sentences what it teaches about chance.

ANSWER KEY

1. (a) unlikely; (b) certain; (c) even chance; (d) impossible.
2. $\frac{2}{5} = 0.4 = 40\%$.
3. He wrote that chance could be measured with a number by counting equally likely outcomes, such as the six faces of a die. Before him, people described chance only with words like “often” or “hardly ever.”
4. HH, HT, TH, TT. Exactly one head: HT or TH, so $\frac{2}{4} = \frac{1}{2}$.
5. Both wanted to fly first and both had an equal claim. A coin gave each the same chance, so it was fair, and since the two choices were equally good, it was also a wise way to settle it.
6. Two more flips: HH, HT, TH, TT. The leader wins 3 of 4, so $\frac{3}{4} \times 12 = 9$ marbles; the other player gets 3.
7. Because the fair share depends on each player’s chance of winning from here. A leader of 1 to 0 in a game to 100 has not nearly won, yet dividing by points would give him everything.
8. Your chance is still $\frac{1}{4}$. For example, drawing last: the first three must all miss it, $\frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{4}$, and then the short straw is the one left.
9. For you, one half, because you have no information. For the person who peeked, it is 0 or 1, because he knows.
10. No. Chance results bounce around the counted probability; 46 out of 100 is well within the normal range for a fair coin.
11. “The lot is cast into the lap; but the whole disposing thereof is of the LORD.” Things that look like chance to us are still under God’s rule. Probability measures what we know, and we can count carefully while trusting God with the outcome.

PORTFOLIO PIECE: MY CHANCE POSTER

Make a poster titled “What Is Chance?” on a large sheet of paper. Draw a big probability line from 0 to 1 and place at least eight events on it, each with its probability written as a fraction, a decimal and a percent. Add a small table from one of your own experiments in this unit (coin flips, unfinished games or straws) that shows what you predicted, what happened and how close they were. Include a short paragraph about one person from the unit (Cardano, the Wright brothers, or Pascal and Fermat) and what that person teaches about chance. Finish with Proverbs 16:33 written at the bottom.

LOOK FOR	WHAT IT MEANS
Probability line	At least eight events placed sensibly, each with a correct fraction, decimal and percent.
Experiment data	A neat table with a prediction, honest tallies, and a comparison with the counted probability.
Story	A clear paragraph telling a true story from the unit with correct names and dates.
Neatness and Scripture	The poster is easy to read, labeled, and ends with Proverbs 16:33 copied correctly.

1.1 LIKELY AND UNLIKELY: THE PROBABILITY LINE

OBJECTIVES

- Place everyday events on a probability line from impossible to certain.
- Find a probability by counting equally likely outcomes.
- Write the same probability as a fraction, a decimal and a percent.

Warm-up. Draw a long line across a page. Write “impossible” at the left end, “certain” at the right end and “even chance” in the middle. Now place these on the line: the sun will rise tomorrow; a coin lands heads; you roll a 7 on one ordinary die; it snows where you live next July; you roll a number bigger than 1 on one die. Talk about any you disagree on.

WATCH

Math Antics - Basic Probability

[mathantics](#) · 11:28

<https://www.youtube.com/watch?v=KzfWUEJjG18>

WHEN TO WATCH

After the warm-up, before the core ideas. Pause at each example and place it on your own probability line.

BACKUP VIDEO

Probability! | Mini Math Movies | Scratch Garden

[Scratch Garden](#) · 6:13

<https://www.youtube.com/watch?v=7XuNVVID98g>

A short, playful review for younger students.

KEY TERMS

chance

How likely it is that something will happen.

outcome

One possible result of an experiment, such as “heads” when you flip a coin or “4” when you roll a die.

probability

A number from 0 to 1 that tells how likely something is: the number of ways it can happen divided by the number of equally likely outcomes.

impossible

Cannot happen. Its probability is 0.

certain

Must happen. Its probability is 1.

equally likely

Having the same chance. The six faces of a fair die are equally likely.

CORE IDEAS

The story. Around the year 1564, in Italy, a doctor named Girolamo Cardano wrote a short book with a long Latin name, *Liber de ludo aleae*, which means *The Book on Games of Chance*. Cardano was a famous physician and mathematician who loved puzzles about dice. In his book he did something no one had written down before: he said that chance could be *measured with a number*. The book sat unpublished for about a hundred years. It was finally printed in 1663, long after Cardano had died in 1576.

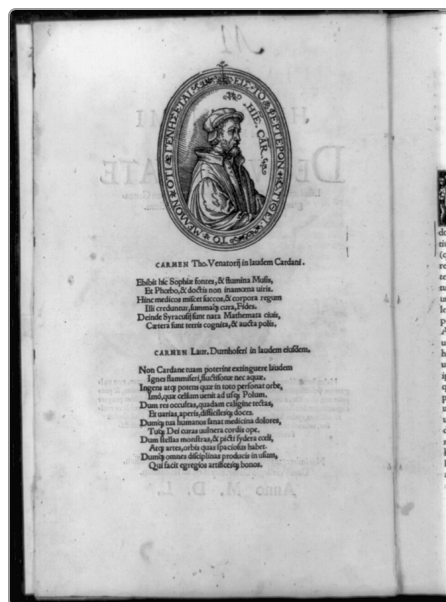
How they figured it out. Before Cardano, people talked about chance with words like “maybe,” “often” and “hardly ever,” or they said it was just fortune. Cardano counted instead. A fair die has six faces, and each one is just as likely to land on top as any other. So the chance of rolling a 4 is one way out of six ways. He also saw that you can count the ways something *can* happen and compare them with all the ways that *might* happen. That simple idea, counting equally likely outcomes, is still the first step in every probability problem today.

The mathematics. Probability is a number on a line from 0 to 1. Something impossible has a probability of 0, something certain has a probability of 1, and an even chance, like heads on a coin, sits right in the middle at $\frac{1}{2}$. When all the outcomes are equally likely, $\text{probability} = \frac{\text{ways it can happen}}{\text{all possible outcomes}}$. Rolling a 4 has probability $\frac{1}{6}$. Rolling an even number (2, 4 or 6) has probability $\frac{3}{6} = \frac{1}{2}$. The same probability can be written three ways: $\frac{1}{2} = 0.5 = 50\%$, and $\frac{3}{4} = 0.75 = 75\%$.

On the field. Coaches and players think on the probability line all the time. A player who makes 3 of every 4 free throws has a probability of about $\frac{3}{4}$, or 75%, so a free throw for her is *likely*, but not certain. A weather forecast that says “70% chance of

rain” means rain is likely, so a wise coach brings the tarp and plans an indoor practice just in case. Notice both sides of every chance: **the percentage** (how likely the good result is) and **the risk** (what happens if it does not come, and how bad that would be). Likely things sometimes fail, and unlikely things sometimes happen. That is why we plan for both.

PICTURE STUDY



Girolamo Cardano: A Portrait Page, Unknown engraver, 16th century · Engraved portrait printed on a book page with Latin verses · Library of Congress, Washington, D.C.

Image: [Wikimedia Commons](#), public domain

Girolamo Cardano (1501–1576) was a doctor, a mathematician and an inventor from Milan in Italy. He wrote more than a hundred books on medicine, mathematics and science. This page, kept today at the Library of Congress, shows his portrait in a small oval with Latin poems in his honor printed beneath it. His little book on games of chance was the first to measure chance by counting equally likely outcomes.

Look closely

- Find Cardano's portrait in the oval. Look at his cap, his beard and his robe.
- Find the words that run around the edge of the oval. Old portraits often told who the person was there.
- Look at the Latin poems printed below. Find the word “CARMEN,” which means “poem” or “song.” Why would someone write poems praising a scientist?
- Notice how small the portrait is compared with the page. Printers fitted pictures and words together carefully on every page.

Talk about it

1. Cardano said chance could be measured with a number. What does that change about how you talk about a coin toss or a free throw?
2. Cardano's book waited about a hundred years to be printed. Why might a good idea take a long time to spread?

PROJECT

My Probability Line

You will build a probability line for your scorebook, predict where ten events belong, and then test three of them with real tries to see whether your predictions were right.

You will need

- Your scorebook and a pencil
- A long strip of paper (tape two sheets end to end) and a ruler
- One coin, one ordinary die and a deck of playing cards
- Small sticky notes or slips of paper

Steps

1. Draw a line 20 centimeters long on your paper strip. Mark 0 at the left (impossible), 1 at the right (certain) and $\frac{1}{2}$ in the middle (even chance). Mark $\frac{1}{4}$ and $\frac{3}{4}$ too.
2. Write ten events on sticky notes. Use these five and make up five of your own: a coin lands tails; a die shows 6; a die shows a number from 1 to 6; you draw a red card from a full deck; a die shows 9.
3. Place each note on the line where you think it belongs. Copy your line into your scorebook as your **prediction**.
4. Now count. For each of the five events above, write the probability as a fraction by counting outcomes (for the cards, 26 of the 52 cards are red).
5. **Test 1.** Flip a coin 20 times. Make a tally of heads and tails.
6. **Test 2.** Roll a die 30 times. Tally how many times it shows 6.
7. **Test 3.** Shuffle the deck and draw one card 20 times, putting it back and shuffling each time. Tally red and black.
8. For each test, write the result as a fraction (for example, 9 heads out of 20 is $\frac{9}{20}$), then as a decimal and a percent with a calculator.
9. Compare each result with the probability you counted. Were they close? Move any sticky note that was in the wrong place, and explain why in one sentence.

Tips

- Make your tally marks in groups of five so they are easy to count.
- Do not expect a perfect match. Twenty flips might give 8 heads or 12 heads. That is normal, and Unit 2 will show why.
- Say the probability out loud three ways: “one half, zero point five, fifty percent.”

Check your work

- ☐ My probability line has 0, one half and 1 marked, with ten events placed.
- ☐ I wrote each test result as a fraction, a decimal and a percent.
- ☐ I compared my results with the counted probabilities and moved any note that was wrong.

SCOREBOOK

1. Copy your finished probability line neatly onto a full page of your scorebook. Color the impossible end gray, the certain end green and the middle yellow.
2. Make a table with three columns: Event, Probability I counted, What happened in my tries. Fill it in for your three tests.
3. Write one sentence that starts “I would decide...” about a real choice, such as whether to bring a raincoat when the forecast says 30% chance of rain.

From history. Cardano also helped find a way to solve hard equations called cubic equations, and he invented a kind of joint that lets a turning rod bend around a corner. That joint is still called a cardan joint, and it is used in cars and trucks today.

WORKED EXAMPLES

Example 1. A bag holds 3 red marbles and 1 blue marble. You reach in without looking. What is the probability of pulling out a red marble? Write it three ways.

Show solution

1. Count all the outcomes: 3 red + 1 blue = 4 marbles, each equally likely.
2. Count the ways to get red: 3.
3. Probability of red = $\frac{3}{4}$. Divide 3 by 4 to get 0.75, and multiply by 100 to get 75%.

Answer: $\frac{3}{4} = 0.75 = 75\%$. Red is likely, but not certain.

Example 2. You roll one ordinary die. Where on the probability line does “rolling a number less than 3” belong?

Show solution

1. All outcomes: 1, 2, 3, 4, 5, 6. That is 6 equally likely outcomes.
2. Numbers less than 3: 1 and 2. That is 2 ways.
3. Probability = $\frac{2}{6} = \frac{1}{3}$, about 0.33 or 33%.

Answer: About one third of the way from impossible to certain: unlikely, but it will happen about once in every three rolls.

COMMON MISTAKES

- Thinking “unlikely” means “impossible.” An event with probability $\frac{1}{6}$ happens about once in every six tries.
- Forgetting to count all the outcomes. A die has six faces, so the bottom of the fraction is 6.
- Expecting a short experiment to match the counted probability exactly. Small numbers of tries bounce around.

Make it a game • Line Up the Chances. Write ten everyday events on slips, such as the sun rising tomorrow or rolling a 7 on one die. Stretch a string across the table as a probability line, labeled impossible at one end and certain at the other. Race to place each slip and defend your spot out loud. Your parent may challenge two placements. Fewest changes wins.

1.2 HEADS OR TAILS: FAIR CHOICES

OBJECTIVES

- Explain why a coin toss gives each side an equal chance.
- List all the outcomes of flipping two coins and find their probabilities.
- Tell when a fair choice by chance is wise, and when a choice needs judgment instead.

Warm-up. Ask someone to flip a coin 10 times while you call each flip before it lands. How many did you call correctly? Now ask: was there any way to do better than about half?

WATCH

How random is a coin toss? - Numberphile

Numberphile · 7:40

<https://www.youtube.com/watch?v=AYnJv68T3MM>

WHEN TO WATCH

After “How they figured it out.” Professor Diaconis is the Stanford mathematician mentioned in the lesson.

BACKUP VIDEO

Biography of the Wright Brothers for Children: Orville and Wilbur Wright for Kids - FreeSchool

Free School · 7:28

<https://www.youtube.com/watch?v=Qt3pry6cWJM>

The Wright brothers’ story for children; watch it before the core ideas if the student does not know it.

KEY TERMS

fair

Giving each player the same chance, or treating each player by the same rule.

coin toss

Flipping a coin to make a choice. Heads and tails each have probability one half.

even chance

A probability of one half: as likely to happen as not.

lot

An object drawn or cast to make a choice by chance, like drawing straws. In the Bible, lots were cast to settle questions.

CORE IDEAS

The story. On December 14, 1903, Wilbur and Orville Wright stood on the windy sand hills near Kitty Hawk, North Carolina, beside the airplane they had built. Both brothers wanted to be the first pilot, and both had worked equally hard. So they settled it the fair way: they tossed a coin. Wilbur won. His first try lasted only about three seconds before the airplane stalled and bumped into the sand, and it needed repairs. Three days later, on December 17, it was Orville's turn. At 10:35 in the morning he flew for 12 seconds and about 120 feet, the first powered, controlled flight of an airplane. Later that same day Wilbur flew 852 feet in 59 seconds.

How they figured it out. People have used coins to choose for at least two thousand years, because a coin has two sides that are just about equally likely to land up. Scientists have tested this carefully. A mathematician at Stanford University, Persi Diaconis, who was once a stage magician, showed that a flipped coin lands on the same side it started from very slightly more often, about 51 times in 100. That is so close to even that a coin toss is still one of the fairest ways people know to make a choice.

The mathematics. A coin toss has two equally likely outcomes, so heads has probability $\frac{1}{2} = 0.5 = 50\%$. With two coins (or one coin flipped twice) there are four outcomes: heads-heads, heads-tails, tails-heads and tails-tails. Each one has probability $\frac{1}{4}$. Getting one head and one tail can happen two ways (HT or TH), so its probability is $\frac{2}{4} = \frac{1}{2}$, while two heads has only $\frac{1}{4}$. To list outcomes without missing any, imagine the two coins are different, such as a penny and a dime.

On the field. Many games begin with a coin toss. In American football the winner of the toss chooses whether to kick, receive or pick a goal to defend. In soccer the winner chooses which goal to attack, and in cricket the toss decides who bats first. A coin toss is fair because it gives each side the same chance and no one can complain that the referee chose a favorite. **The risk side:** a coin is fair, but it is not wise. It is a good way to choose between two equally good options, like who flies first. It is a poor

way to choose between a good option and a bad one. Use a coin when the choices are equal, and use your head when they are not.

PICTURE STUDY



The First Flight, John T. Daniels, December 17, 1903 · Glass-plate photograph · Near Kitty Hawk, North Carolina

Image: [Wikimedia Commons](#), public domain

Orville Wright set up his camera on a tripod, aimed it at the end of the launching rail, and asked John T. Daniels, a man from the nearby lifesaving station, to squeeze the shutter bulb if the airplane left the ground. Daniels took this photograph at the moment of the first flight. It is one of the most famous photographs ever made.

Look closely

- Find Orville lying at the controls of the airplane, and Wilbur standing on the sand to the right. Which brother had won the coin toss three days earlier?
- Look at the wooden rail on the ground. The airplane ran along it to take off. What could go wrong at this moment?
- Look at the empty, flat sand and the sky. Why did the brothers choose a windy, open, sandy place for their tests?

Talk about it

1. Why was a coin toss a fair way for the brothers to choose who would fly first?
2. Wilbur won the toss but his flight failed, and Orville's succeeded. Did that make the coin toss a bad way to choose? Why or why not?

3. Name one decision where you would never use a coin toss. What would you use instead?

PROJECT

One Hundred Flips

Is a coin really fair? You will flip a coin 100 times, then flip two coins together 40 times, and compare what happened with the probabilities you counted.

You will need

- Two different coins, such as a penny and a dime
- Your scorebook, a pencil and a calculator
- A towel or tray to flip onto, so the coins do not roll away

Steps

1. **Predict.** Out of 100 flips, how many heads do you expect? Write your prediction, and write the smallest and largest numbers you think would still be normal.
2. Flip the penny 100 times, in ten rounds of 10. After each round write the number of heads in a table.
3. Add up the heads. Write your total as a fraction of 100, a decimal and a percent.
4. Look at your ten rounds. Which round had the most heads? The fewest? Were any rounds far from 5?
5. **Two coins.** Before flipping, write the four outcomes (HH, HT, TH, TT) and the probability of each.
6. Flip the penny and the dime together 40 times. Tally which of the four outcomes you get each time.
7. Combine HT and TH into “one of each.” Compare your tallies for two heads, one of each and two tails with what you expect from 40 tries: about 10, 20 and 10.
8. **Decide.** Write whether your coin seems fair, and how you know.

Tips

- Flip the coin high enough to turn over many times. A coin that barely turns is not a fair flip.
- Write each result as soon as it lands. Do not try to remember several at once.

- If you get 45 or 55 heads, that is normal. Getting exactly 50 is actually not very common.

Check your work

- ☐ I flipped 100 times and recorded every round.
- ☐ I listed the four outcomes for two coins and compared my 40 tries with 10, 20 and 10.
- ☐ I wrote whether my coin seems fair, with a reason from my data.

SCOREBOOK

1. Draw a bar graph of your ten rounds: round number along the bottom, number of heads up the side. Draw a dotted line at 5.
2. Draw a tree diagram for two coins: two branches for the penny, then two branches from each for the dime. Write the four outcomes at the ends.
3. Write two decisions: one that is fine to settle with a coin toss, and one that is not. Explain each in a sentence.

From history. The Wright brothers flew four times on December 17, 1903, taking turns. The airplane they flew, the 1903 Wright Flyer, now hangs in the Smithsonian's National Air and Space Museum in Washington, D.C.

WORKED EXAMPLES

Example 1. Two players flip a penny and a dime together. Player A scores a point if both coins match (two heads or two tails). Player B scores a point if they do not match. Is this fair?

Show solution

1. List the four equally likely outcomes: HH, HT, TH, TT.
2. Matches: HH and TT, 2 outcomes. Probability $\frac{2}{4} = \frac{1}{2}$.
3. No match: HT and TH, 2 outcomes. Probability $\frac{2}{4} = \frac{1}{2}$.

Answer: Yes. Each player has probability one half, so the game is fair.

COMMON MISTAKES

- Thinking there are only three outcomes for two coins (two heads, two tails, one of each). There are four, because heads-tails and tails-heads are different ways.

- Calling a result “wrong” because it is not exactly half. Chance results always bounce around the counted probability.
- Thinking a fair method always gives the result you wanted. Fair means equal chances, not equal results.

Make it a game • Two-Coin Prediction. Flip two coins and call the result before each toss: two heads, two tails or one of each. Keep a bean tally for twenty flips. Before starting, write the four outcomes from memory and say each one's probability. Then compare your counts with the prediction. Score a bean for each correct call. Most beans wins.

Scripture. “The lot causeth contentions to cease, and parteth between the mighty.” (Proverbs 18:18, KJV)

Casting a lot, like tossing a coin, can end an argument between two people who both have a fair claim, as it did for Wilbur and Orville.

1.3 PASCAL AND FERMAT: THE UNFINISHED GAME

OBJECTIVES

- Tell the story of Pascal and Fermat's letters of 1654.
- List every way an unfinished game could still end, and count how many each player would win.
- Divide a prize fairly by looking at what could still happen, not only at the score so far.

Warm-up. Two players are flipping a coin. Heads scores for Anna, tails for Ben, and the first to 3 points wins. The score is Anna 2, Ben 1, when supper is called and the game must stop. They had agreed to play for a bag of 24 marbles. How would you divide the marbles? Write your answer and your reason before you read on.

WATCH

Problem of Points (Part 1/2) | Math for fun and glory | Khan Academy

Khan Academy Labs • 1:29

<https://www.youtube.com/watch?v=o3y7CzT-LRk>

WHEN TO WATCH

After “The story,” before “How they figured it out.” Pause before the end and predict how to divide the prize.

KEY TERMS

problem of points

The old puzzle of how to divide a prize fairly when a game is stopped before anyone has won.

sample space

The list of all possible outcomes.

fair share

The part of the prize equal to a player's chance of winning, if the game had been finished.

CORE IDEAS

The story. In 1654 a French nobleman called the Chevalier de Méré brought a puzzle to a brilliant young mathematician in Paris, Blaise Pascal. Two players agree that

whoever first wins a certain number of rounds will take the whole prize. But the game is stopped before anyone has won. How should the prize be divided? Pascal wrote about it to Pierre de Fermat, a judge in the city of Toulouse who did mathematics in his spare time. The two men never met, but that summer and autumn they sent letters back and forth and solved the puzzle in two different ways. Their letters are often called the birth of probability.

How they figured it out. The puzzle was already more than 150 years old. In 1494 an Italian friar, Luca Pacioli, had said to divide the prize by the points each player already had. But that does not work: if the game is to 100 points and it stops at 1 to 0, the leader would get the whole prize, even though the other player still had a good chance! Pascal and Fermat saw what everyone had missed. **What matters is not what has already happened, but what could still happen.** Fermat listed every way the rest of the game could go. Pascal worked backward from the end. Both got the same answer, and Pascal wrote to Fermat that he could see “the truth is the same at Toulouse and at Paris.”

The mathematics. Take the warm-up game: first to 3 points, and the score is Anna 2, Ben 1. At most two more flips would finish the game. Fermat’s trick is to list both flips, even if the first one ends the game: HH, HT, TH, TT. Anna wins in every list that has at least one H, which is 3 of the 4. Ben wins only with TT, 1 of the 4. So Anna’s fair share is $\frac{3}{4}$ of the prize and Ben’s is $\frac{1}{4}$. With 24 marbles, Anna gets 18 and Ben gets 6. Pascal checked it a different way: if the next flip is heads, Anna wins everything; if it is tails, the score is 2 to 2 and they should split evenly. Half of all plus half of a half is $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$.

On the field. Coaches use Pascal’s idea every time they ask, “From here, how often do we win?” instead of “Who is ahead right now?” A two-goal lead with five minutes left is very safe; a two-goal lead with eighty minutes left is not. **The risk side:** a team that thinks the prize is already won has forgotten the rounds still to be played. A team that is behind should remember that it still has a real chance, and keep playing well. Pascal’s lesson is a habit of mind: judge a game by all the ways it can still turn out.

PICTURE STUDY



Portrait of Blaise Pascal, Unknown artist, after François Quesnel, About 1690 · Oil on canvas · Palace of Versailles, France

Image: [Wikimedia Commons](#), public domain

Blaise Pascal (1623–1662) was a French mathematician, scientist and writer. At nineteen he built one of the first mechanical calculating machines to help his father, a tax official, with long sums. He was also a devoted Christian, and his notes on the Christian faith, published after his death as the *Pensées* (“Thoughts”), are still read today.

Look closely

- Look at Pascal’s eyes and face. Does he look like someone who would rush to an answer, or someone who would think a problem all the way through?
- Notice his plain dark clothes and white collar. Pascal cared more about ideas than about show.
- Imagine Pascal at a desk writing to Fermat, far away in Toulouse. What would he need to explain clearly so that Fermat could follow him without ever meeting him?

Talk about it

1. Pascal and Fermat solved the puzzle in two different ways and got the same answer. Why is it helpful to check an answer a second way?

2. Pascal looked at what could still happen, not just at the score. How could that habit help a team that is losing at halftime?

PROJECT

Finish the Unfinished Game

Pascal and Fermat counted. You will count too, and then test their answer by actually playing out the unfinished game many times with a coin.

You will need

- A coin
- Your scorebook and a pencil
- 24 small counters, such as dried beans or buttons, to stand for the prize

Steps

1. **Count first.** For a game to 3 points stopped at 2 to 1, list the four outcomes of two more flips and find each player's fair share. Divide your 24 counters into the two shares.
2. **Predict.** If you start at 2 to 1 and play the game out 40 times, about how many times will the leader win? (Hint: $\frac{3}{4}$ of 40.)
3. **Play it out.** Start at 2 to 1, flip until someone reaches 3, and tally who won. Repeat 40 times.
4. Write the leader's wins as a fraction of 40, then as a percent. Compare with 75%.
5. **Second test.** Count the fair shares for a game stopped at 2 to 0 (see the worked example). Then play it out 40 times from 2 to 0 and compare with $\frac{7}{8}$, which is 87.5%.
6. **Your own puzzle.** Make up a stopped game of your own, such as a game to 4 stopped at 3 to 2. List the outcomes, find the fair shares and test it 20 times.
7. Write a short letter, as Pascal might have written to Fermat, explaining how to divide the prize in your own puzzle and why.

Tips

- When listing outcomes, list every flip that *might* be needed, even if the game would end early. That is Fermat's trick, and it keeps every outcome equally likely.

- Use the counters to show the shares. Seeing 18 counters next to 6 makes the fraction $\frac{3}{4}$ easy to understand.

Check your work

- ☐ I listed all the outcomes and found the fair shares before playing.
- ☐ I played out each stopped game the required number of times and compared my results with the counted probability.
- ☐ My letter explains the method in my own words.

SCOREBOOK

1. Copy the table of outcomes for the 2-to-1 game and circle the ones Anna wins in one color and the one Ben wins in another.
2. Make a results table: Starting score, Leader's wins, Total games, Fraction, Percent, Counted probability.
3. Write: "I would divide the prize of a stopped game by..." and finish the sentence.

From history. Pierre de Fermat (1601–1665) was a lawyer and judge who did mathematics for joy. He is also famous for a puzzle he wrote in the margin of a book, saying he had a proof that would not fit there. Mathematicians worked on "Fermat's Last Theorem" for more than 350 years until it was finally proved in 1994.

WORKED EXAMPLES

Example 1. First to 3 points wins. The score is Anna 2, Ben 0 when the game stops. What is each player's fair share of 24 marbles?

Show solution

1. Ben needs 3 more points and Anna needs 1. At most 3 more flips would finish the game.
2. List all 3 flips: HHH, HHT, HTH, HTT, THH, THT, TTH, TTT. That is 8 equally likely outcomes.
3. Ben wins only with TTT: 1 of 8. Anna wins the other 7.
4. Anna's share: $\frac{7}{8} \times 24 = 21$. Ben's share: $\frac{1}{8} \times 24 = 3$.

Answer: Anna gets 21 marbles and Ben gets 3.

Example 2. The score is 2 to 2 in a game to 3. How should 24 marbles be divided?

Show solution

1. The next flip decides the game: heads for Anna, tails for Ben.
2. Each has probability $\frac{1}{2}$.

Answer: 12 marbles each. When the chances are equal, the shares are equal.

COMMON MISTAKES

- Dividing the prize by the points already scored. The fair share depends on the chances of winning from here.
- Stopping the list as soon as someone would win. To keep the outcomes equally likely, list every flip that might be needed.
- Forgetting that the shares must add up to the whole prize. Check that the fractions add to 1.

Make it a game • Finish the Game. Pascal and Fermat's players stopped a game early. Set up your own: first to three coin-flip wins, stopped when one player has two wins and the other has one. List every way the game could still end on paper, then split ten beans fairly. Tell your parent how you counted. Swap and make a new unfinished game.

1.4 THE LOT IS CAST: CHANCE AND PROVIDENCE

OBJECTIVES

- Tell how lots were used in the Bible, including the choice of Matthias.
- Explain that probability measures what we know, not what God can do.
- Show by experiment that the order of drawing straws does not change anyone's chance.

Warm-up. Flip a coin and quickly cover it with a cup before anyone sees it. Ask: “What is the probability that it is heads?” The coin has already landed, so is it still a half? Talk about it, then lift the cup.

WATCH

Should we cast lots today to help us make decisions?

Ligonier Ministries · 2:28

<https://www.youtube.com/watch?v=ZXoQVNkoGWA>

WHEN TO WATCH

After the core ideas. Talk together about why Christians today do not usually cast lots to make decisions.

BACKUP VIDEO

What do you think of casting lots to determine God's will?

Desiring God · 3:28

https://www.youtube.com/watch?v=Yd_JpmBEczk

Another short answer from a pastor to the same question; good for family discussion.

KEY TERMS

casting lots

Making a choice by drawing or throwing marked objects, as the people of the Bible often did.

providence

God's care and rule over all things, including the things that look like chance to us.

random

Not able to be predicted by us ahead of time, though every outcome has its chance.

information

What you know. When you learn something new, the chances you should use can change.

CORE IDEAS

The story. After Jesus rose and ascended into heaven, His eleven apostles needed someone to take the place of Judas. The Book of Acts tells that they put forward two men who had followed Jesus from the beginning, Joseph called Barsabas and Matthias. They prayed and asked the Lord to show which one He had chosen. Then “they gave forth their lots; and the lot fell upon Matthias; and he was numbered with the eleven apostles” (Acts 1:26). Long before, Joshua had divided the Promised Land among the tribes of Israel by casting lots “before the LORD” at Shiloh (Joshua 18:10).

How they figured it out. Many ancient people believed that chance was ruled by a goddess of fortune, or by fate. The Bible teaches something very different: nothing is outside God’s rule, even a lot thrown into someone’s lap. Centuries later, Christian mathematicians such as Blaise Pascal saw no conflict between careful counting and trust in God. To them, probability did not measure what God could do. It measured *what we know*. When we say a coin has a one-half chance of heads, we mean that we do not know which way it will land, and that the two sides are equally likely as far as we can tell.

The mathematics. Think about the warm-up. Once the coin is under the cup, it is already heads or tails. But for you, who cannot see it, the probability is still $\frac{1}{2}$, because you have no information that makes one side more likely. As soon as you lift the cup, the probability for you becomes 0 or 1. Probability changes when information changes. Drawing straws shows another surprise: if 5 people draw from 5 straws and one is short, each person has a $\frac{1}{5}$ chance of the short straw, **no matter whether he draws first or last**. The first drawer has $\frac{1}{5}$ right away. The second drawer gets the short straw only if the first did not ($\frac{4}{5}$) and then he picks it from the 4 left ($\frac{1}{4}$): $\frac{4}{5} \times \frac{1}{4} = \frac{1}{5}$.

On the field. A wise athlete prepares as hard as he can, plays as well as he can, and then accepts the bounce of the ball without blaming “bad luck” or trusting a special pair of socks. The outcome is in God’s hands, but the preparation is in ours. **The risk side:** people who think chance is a mysterious force often make poor choices, like

believing they are “due” for a win or that a ritual will help. People who understand chance count carefully, prepare well and stay humble, because they know that even a likely result is not certain.

PICTURE STUDY



Matthias Is Chosen as an Apostle by Lot, Unknown engraver, after Johann Jakob von Sandrart, 1697 ·

Engraving · Rijksmuseum, Amsterdam

Image: [Wikimedia Commons](#), public domain

This engraving was made in Germany in 1697 for a picture Bible. It shows the apostles choosing Matthias by lot, as told in Acts 1:23-26. The German words at the bottom say, “Through the lot ruled by God, Matthias becomes an apostle,” which is just what Proverbs 16:33 teaches.

Look closely

- Find the boy holding the urn. The lots were drawn from a vessel like this so that no one could choose for himself.
- Find the man bowing with his hand on his chest. Who might he be, and how might he feel?
- Look at the hands of the apostles. Where are they pointing? Where does the artist want your eye to go?
- Read the date and the Bible reference at the bottom (“Apost. Gesch. I. 26” means Acts 1:26).

Talk about it

1. The apostles prayed before they cast lots. Why does that matter for how we think about the result?

2. From where the apostles stood, each man had a one-in-two chance. From God's side, was it chance at all? Use Proverbs 16:33 in your answer.

PROJECT

Drawing Straws: Does Order Matter?

Many people think it is better to draw first, or last. You will test it. Then you will run a short experiment about how information changes probability.

You will need

- Five drinking straws or paper strips, one cut shorter than the others
- A cup to hold them so only the tops show
- A coin and a second cup
- Your scorebook

Steps

1. **Predict.** Five people draw one straw each, in order. Which position, first through fifth, do you think gets the short straw most often? Write your prediction.
2. Set up a table with columns 1st, 2nd, 3rd, 4th and 5th.
3. Have a helper mix the straws in the cup so the short one cannot be seen. Draw the straws one at a time, keeping them in order, and put a tally under the position that got the short straw.
4. Repeat 50 times. (You can draw all five yourself, laying them out in order.)
5. Add up each column. Each position should get about $\frac{1}{5}$ of 50, which is 10. Write each count as a fraction of 50.
6. **Information test.** Flip a coin under a cup 20 times. Each time, before lifting, a helper peeks and says only “It is not tails” or says nothing. Record your probability for heads before the hint and after the hint.
7. Write two sentences: one about whether drawing order matters, and one about how a hint changed the probability.

Tips

- Mix the straws well each time, and do not let the short one stick up lower than the others.
- Your counts will not all be exactly 10. A spread from about 5 to 15 is normal for 50 rounds.

Check your work

- ☐ I drew 50 rounds and tallied the position of the short straw each time.
- ☐ I compared each position with one fifth of 50.
- ☐ I explained in my own words why probability measures what we know.

SCOREBOOK

1. Draw a bar graph of your straw results, one bar for each position. Draw a dotted line at 10.
2. Write Proverbs 16:33 neatly at the top of a page. Under it, draw the probability line from Lesson 1.1 and write: “This line measures what I know. God knows the end from the beginning.”
3. Write about a time you did your best and the result went against you. What would you do the same next time?

From history. In the Old Testament, the word for “lot,” *goral*, came to mean a person’s portion or share in life. The psalmist writes, “Thou maintainest my lot” (Psalm 16:5), meaning that God holds his future.

WORKED EXAMPLES

Example 1. Three straws, one short. You draw second. What is your chance of getting the short straw?

Show solution

1. The first person misses the short straw with probability $\frac{2}{3}$.
2. Then 2 straws are left, and you pick the short one with probability $\frac{1}{2}$.
3. Multiply: $\frac{2}{3} \times \frac{1}{2} = \frac{2}{6} = \frac{1}{3}$.

Answer: $\frac{1}{3}$, exactly the same as drawing first.

COMMON MISTAKES

- Thinking the first person to draw has a better or worse chance. Every position has the same chance.
- Thinking that because a result is already decided, its probability must be 0 or 1 for everyone. Probability depends on what you know.
- Thinking chance means God is not in control. Scripture teaches that He rules even the lot.

Make it a game • Fair Lot Draw. Tell the story of Matthias in your own words, then run three lot draws from a cup using four numbered paper chips. Before each draw, say what the chance is for each chip. After the third, explain to your parent why probability measures what we know, not what God can do. A correct explanation earns a bean.

Scripture. “The lot is cast into the lap; but the whole disposing thereof is of the LORD.” (Proverbs 16:33, KJV)

To us a lot or a coin toss is uncertain. To God nothing is uncertain. We count chances carefully, and we trust Him with the outcome.

UNIT 1

WHAT IS CHANCE?

Will it rain tomorrow? Will the next free throw go in? Will the coin land heads? Nobody knows for sure, but we are not completely in the dark either. Some things are certain, some are impossible, and most are somewhere in between. In this unit you will learn to put a number on chance, from 0 for impossible to 1 for certain, and to write it as a fraction, a decimal and a percent.

You will meet the people who first counted chances: Girolamo Cardano, an Italian doctor who wrote the first book about it, and Blaise Pascal and Pierre de Fermat, two Frenchmen who solved a puzzle about an unfinished game in a famous set of letters in 1654. You will see how the Wright brothers used a coin toss to decide who would fly first, and you will think about what Scripture says about the lot that is cast into the lap. Most of all, you will start your scorebook: predict, count, record and compare.

PREREQUISITES

- Can add, subtract and multiply whole numbers
- Knows what a simple fraction such as $\frac{1}{2}$ or $\frac{3}{4}$ means
- Can make tally marks and count them by fives

MASTERY CHECKLIST

- ☐ I can place events on a probability line from impossible (0) to certain (1).
- ☐ I can find a simple probability by counting equally likely outcomes, and write it as a fraction, a decimal and a percent.
- ☐ I can explain why a coin toss is a fair way to choose, and list the outcomes of flipping two coins.
- ☐ I can tell how Pascal and Fermat divided the prize of an unfinished game by looking at what could still happen.
- ☐ I can explain what Proverbs 16:33 teaches about chance and God's rule.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Khan Academy: Basic probability**: Free. Short videos and practice on simple probability, sample spaces and coin and dice experiments that match Lessons 1.1 and 1.2. Best for ages 11 and up; younger students can work through it with a parent.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

1.1 Likely and Unlikely: The Probability Line	2 sessions of 45 minutes
1.2 Heads or Tails: Fair Choices	2 sessions of 45 minutes
1.3 Pascal and Fermat: The Unfinished Game	2 sessions of 45 minutes
1.4 The Lot Is Cast: Chance and Providence	2 sessions of 45 minutes

UNIT 1 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. **4 pts**

Place each event on the probability line as impossible, unlikely, even chance, likely or certain: (a) rolling a 6 on one die; (b) rolling a number from 1 to 6 on one die; (c) a coin lands heads; (d) rolling a 0 on one die.

2. **5 pts**

A bag holds 2 green marbles and 3 yellow marbles. What is the probability of drawing a green marble without looking? Write it as a fraction, a decimal and a percent.

3. **4 pts**

What did Girolamo Cardano write about in his *Book on Games of Chance*, and why was it new?

4. (5 pts)

List all the outcomes of flipping two coins. What is the probability of getting exactly one head?

5. (4 pts)

Why did the Wright brothers use a coin toss to decide who would fly first? Was it a fair way? Was it a wise way?

6. (5 pts)

A game to 3 points is stopped at 2 to 1. Use Fermat's method to find the leader's fair share of 12 marbles.

7. (4 pts)

Why is dividing the prize by the points already scored not fair?

8. (5 pts)

What would happen if you drew straws last instead of first, with 4 straws and one short? Show your reasoning.

9. (4 pts)

A coin is flipped and hidden under a cup. What is the probability it is heads for you? For someone who has peeked?

10. (4 pts)

In your 100 coin flips you got 46 heads. Does that prove your coin is unfair? Explain.

11. (6 pts)

Short answer: Write Proverbs 16:33 and explain in two or three sentences what it teaches about chance.

ANSWER KEY

1. (a) unlikely; (b) certain; (c) even chance; (d) impossible.
2. $\frac{2}{5} = 0.4 = 40\%$.
3. He wrote that chance could be measured with a number by counting equally likely outcomes, such as the six faces of a die. Before him, people described chance only with words like “often” or “hardly ever.”
4. HH, HT, TH, TT. Exactly one head: HT or TH, so $\frac{2}{4} = \frac{1}{2}$.
5. Both wanted to fly first and both had an equal claim. A coin gave each the same chance, so it was fair, and since the two choices were equally good, it was also a wise way to settle it.
6. Two more flips: HH, HT, TH, TT. The leader wins 3 of 4, so $\frac{3}{4} \times 12 = 9$ marbles; the other player gets 3.
7. Because the fair share depends on each player’s chance of winning from here. A leader of 1 to 0 in a game to 100 has not nearly won, yet dividing by points would give him everything.
8. Your chance is still $\frac{1}{4}$. For example, drawing last: the first three must all miss it, $\frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} = \frac{1}{4}$, and then the short straw is the one left.
9. For you, one half, because you have no information. For the person who peeked, it is 0 or 1, because he knows.
10. No. Chance results bounce around the counted probability; 46 out of 100 is well within the normal range for a fair coin.
11. “The lot is cast into the lap; but the whole disposing thereof is of the LORD.” Things that look like chance to us are still under God’s rule. Probability measures what we know, and we can count carefully while trusting God with the outcome.

PORTFOLIO PIECE: MY CHANCE POSTER

Make a poster titled “What Is Chance?” on a large sheet of paper. Draw a big probability line from 0 to 1 and place at least eight events on it, each with its probability written as a fraction, a decimal and a percent. Add a small table from one of your own experiments in this unit (coin flips, unfinished games or straws) that shows what you predicted, what happened and how close they were. Include a short paragraph about one person from the unit (Cardano, the Wright brothers, or Pascal and Fermat) and what that person teaches about chance. Finish with Proverbs 16:33 written at the bottom.

LOOK FOR	WHAT IT MEANS
Probability line	At least eight events placed sensibly, each with a correct fraction, decimal and percent.
Experiment data	A neat table with a prediction, honest tallies, and a comparison with the counted probability.
Story	A clear paragraph telling a true story from the unit with correct names and dates.
Neatness and Scripture	The poster is easy to read, labeled, and ends with Proverbs 16:33 copied correctly.

UNIT 2

DICE AND COUNTING

Dice are some of the oldest game pieces in the world, and they are also a perfect tool for learning about chance. In this unit you will roll one die hundreds of times and see that each face comes up about one time in six. Then you will roll two dice and discover something that surprises many people: the sums are not equally likely at all. A 7 comes up six times as often as a 2 or a 12, and that is exactly why the 6 and 8 tokens in the game *Catan* are printed in red.

You will learn to keep a tally chart and draw a bar graph, the two most useful tools in your scorebook, and you will build a homemade version of Francis Galton's famous board of pins to watch chance make a beautiful, predictable shape. Finally you will learn one of the most important facts in this whole course: dice and coins have no memory. A streak of sixes does not make the next roll more or less likely to be a six.

PREREQUISITES

- Can write a probability as a fraction, a decimal and a percent (Unit 1)
- Can list the outcomes of a simple experiment
- Can multiply small whole numbers

MASTERY CHECKLIST

- ☐ I can explain what makes a die fair and test a die by rolling it many times.
- ☐ I can draw the 6-by-6 grid of two-dice outcomes and find the probability of any sum.
- ☐ I can explain why 7 is the most common sum of two dice and why *Catan*'s 6 and 8 are red.
- ☐ I can keep a tally chart and turn it into a bar graph.
- ☐ I can explain why dice and coins have no memory, and why streaks still happen.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **CATAN**: The publisher's site for the family game Catan, whose red 6 and 8 tokens are explained in Lesson 2.2. Ages 10 and up. A good game for practicing two-dice counting, but never required.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

2.1 One Die, Six Faces	2 sessions of 45 minutes
2.2 Two Dice: Why 7 Comes Most Often	2 sessions of 45 minutes
2.3 Tally Charts and Galton's Board	2 sessions of 45 minutes
2.4 Dice Have No Memory	2 sessions of 45 minutes

UNIT 2 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. 4 pts

What makes a die fair? Give two things a die maker must get right.

2. 4 pts

You roll a fair die 90 times. About how many times do you expect a 3? About how many times a number greater than 4?

3. 5 pts

How many outcomes are there when you roll two dice? List all the ways to roll a total of 8.

4. 4 pts

Why are the 6 and 8 tokens printed in red in Catan?

5. (5 pts)

Which is more likely with two dice: a total of 3, or a total of 11? Show your counting.

6. (4 pts)

What did the dice players who asked Galileo get wrong when they counted the ways to make 9 and 10 with three dice?

7. (5 pts)

Four coins are flipped. How many outcomes are there? How many give exactly 2 heads?

8. (4 pts)

Why do more balls land in the middle of a Galton board than at the edges?

9. (5 pts)

What would happen if you planned your game around a 6 being “due” because none has come up in 12 rolls? Explain what “independent” means.

10. (4 pts)

A shooter has made 4 free throws in a row. Is the next one sure to go in? What did Miller and Sanjurjo find about the hot hand?

11. (6 pts)

Short answer: You flip a coin 50 times and get a streak of 6 heads. A friend says the coin must be broken. Write three or four sentences explaining what you would tell him.

ANSWER KEY

1. Every face must be equally likely to land on top. The die must be a true cube with equal faces, and evenly heavy so no side is weighted.
2. A 3: $\frac{1}{6} \times 90 = 15$. Greater than 4 (5 or 6): $\frac{2}{6} \times 90 = 30$.
3. 36. A total of 8: 2-6, 3-5, 4-4, 5-3, 6-2 (5 ways), so $P(8) = \frac{5}{36}$.
4. They are the most likely numbers that produce cards: each comes up in 5 of 36 outcomes. Only 7 is more likely, and a 7 moves the robber instead of producing.
5. Equally likely. A total of 3: 1-2, 2-1 (2 ways). A total of 11: 5-6, 6-5 (2 ways). Each is $\frac{2}{36} = \frac{1}{18}$.
6. They counted ways to write the total (groups like $3 + 3 + 3$) instead of ways the dice can fall. When each die is treated as different, some groups can fall in several orders and others in only one.
7. $2 \times 2 \times 2 \times 2 = 16$ outcomes; exactly 2 heads happens in 6 of them (HHTT, HTHT, HTTH, THHT, THTH, TTHH).
8. Many different paths of left and right bounces lead to the middle, but only one path leads to each far edge (all left or all right).
9. The 6 would still come up only one time in six on each roll; it is never due. Independent means each roll is not affected by earlier rolls, so waiting for a “due” number is a poor plan.
10. No; it is still uncertain. They found that when the counting was corrected, there was a small real hot hand, but much smaller than fans believe.
11. Streaks are a normal part of randomness; long runs happen far more often than people expect. Each flip is independent, so a run of heads does not change the next flip. To test the coin, look at the total number of heads over many flips, not one streak. A run of 6 in 50 flips is not unusual for a fair coin.

PORTFOLIO PIECE: THE DICE REPORT

Write and illustrate a three-page “Dice Report.” Page 1: your 120-roll test of one die, with its bar graph and your decision about whether it is fair. Page 2: your colored 6-by-6 outcome grid, your two-dice bar graph with the expected staircase drawn on it, and a short explanation of why 7 comes up most. Page 3: either your Galton board results or your streak-hunting results, with one paragraph explaining what they show about chance. Finish with three “rules for dice” in your own words that would help a younger brother or sister play family games more wisely.

LOOK FOR	WHAT IT MEANS
Honest data	Every experiment has a prediction, complete tallies and correct totals.
Graphs	Bar graphs are neat, titled and labeled, with expected counts shown.
Explanations	The report correctly explains fairness, the 36 outcomes and why 7 is most common, using fractions.
Rules for dice	Three sensible rules, including that dice have no memory, written clearly for a younger reader.

2.1 ONE DIE, SIX FACES

OBJECTIVES

- Explain what makes a die fair.
- Find the probability of rolling any number, or group of numbers, on one die.
- Test a die by rolling it 120 times and comparing the results with the counted probability.

Warm-up. Look closely at an ordinary die. Add the dots on the top face and the bottom face. Now try another pair of opposite faces, and another. What do you notice? Why might die makers do it this way?

WATCH

[Probability](#) | [Rolling a Die](#) | [Basic Probability](#) | [Math with Mr. J](#)

[Math with Mr. J](#) • 10:47

<https://www.youtube.com/watch?v=7PexM-QD3Mc>

WHEN TO WATCH

After “The mathematics.” Try each example before Mr. J solves it.

KEY TERMS

die

A small cube (or other shape) with a different number on each face, used to make a random choice. The plural is “dice.”

fair die

A die whose faces are all equally likely to land on top.

pips

The dots on a die that show its numbers.

tally

A quick count made with marks, usually bundled in fives.

expected count

How many times you expect something to happen in a number of tries: the probability times the number of tries.

CORE IDEAS

The story. About eleven hundred years ago, in the busy city of Nishapur in Persia (now Iran), someone carved small dice from bone or ivory and marked each face with little circles cut into the surface. The dice were lost and buried for centuries. In the 1930s, archaeologists from the Metropolitan Museum of Art in New York dug at Nishapur and found them. They look very much like the dice in your game cupboard: six faces, one to six spots. People in ancient Egypt, Rome, India and China all used dice too, and the oldest ever found are more than four thousand years old.

How they figured it out. If you look closely at old dice, many are not perfect cubes. Some are a little long or lopsided, and a lopsided die lands on its big flat sides more often than on its small ones. Over the centuries, die makers learned to make the faces the same size and the whole die evenly heavy, so that every face has the same chance. On a modern die the opposite faces add up to 7 (1 and 6, 2 and 5, 3 and 4), which spreads the dots evenly. Thinkers like Cardano (Lesson 1.1) and Galileo saw that if a die is fair, you can count its six faces to find any probability.

The mathematics. A fair die has six equally likely outcomes, so each number has probability $\frac{1}{6}$, about 0.17 or 17%. To find the chance of a group of numbers, count the faces in the group. Rolling a 5 or 6 has probability $\frac{2}{6} = \frac{1}{3}$. To find how many times to expect something, multiply the probability by the number of tries. In 120 rolls you expect each face about $\frac{1}{6} \times 120 = 20$ times. You will not get exactly 20 of each, but a fair die should give numbers near 20, while a lopsided die will show one face again and again.

On the field. Many family games are built on one die: how far you move, which turn you get, whether you can start. A good player always asks, “What are my chances on this roll?” If you need a 6 to escape, you have only a 1-in-6 chance, so you should not plan as if it will happen. If you need anything but a 1, you have a 5-in-6 chance, which is very likely. **The risk side:** a plan that needs one exact number to come up is a fragile plan. A plan that works on most numbers is a strong one.

PICTURE STUDY



Dice, Unknown maker, 9th–10th century · Ivory or bone, incised and inlaid with paint · Found at Nishapur, Iran; Metropolitan Museum of Art, New York

Image: [Wikimedia Commons](#), public domain

These three dice were dug up at Nishapur, an important trading city in Persia, by archaeologists from the Metropolitan Museum of Art. They are about eleven hundred years old. Their makers marked each number with small circles, each with a dot in the middle.

Look closely

- Look at the shapes of the three dice. Are they all perfect cubes? Which one looks the most even?
- Count the circles on the top face of each die. How are the three spots arranged on the middle and right dice?
- Find the crack in the die on the right. How might a crack or a chip change whether a die is fair?

Talk about it

1. If a die were longer in one direction than the others, which faces do you think would land on top most often? Why?
2. How could you test one of these old dice to find out whether it is fair, without damaging it?

PROJECT

Is My Die Fair?

You will test an ordinary die by rolling it 120 times. Then you will make a paper die, make a “lopsided” paper die on purpose, and see whether your tally can catch the difference.

You will need

- An ordinary die and a cup or tray to roll into
- Card stock, scissors, tape and a ruler (for paper dice)
- A coin or a few paper clips to tape inside one paper die
- Your scorebook and a calculator

Steps

1. **Predict.** In 120 rolls of a fair die, how many of each number do you expect? Write the expected count (20) and the range you think would still be normal.
2. Make a tally chart with rows for 1, 2, 3, 4, 5 and 6.
3. Roll the ordinary die 120 times, shaking it in the cup each time, and tally every roll.
4. Count each row. Write each count as a fraction of 120 and as a percent. Which face came up most? Least?
5. **Paper dice.** Draw a cross-shaped net of six squares, each 3 centimeters on a side, on card stock. Number the faces so that opposite faces add to 7. Cut it out and tape it into a cube.
6. Make a second paper die the same way, but tape a coin or a few paper clips inside, against the face marked 1, before you close it.
7. Roll each paper die 60 times and tally. Expect about 10 of each number from a fair die.
8. **Decide.** Which die seems fair and which does not? Which face did the weighted die land on most? Explain why the weight changed the chances.

Tips

- Shake the die in a cup and roll it onto a towel so it tumbles the same way each time.

- A fair die will not give exactly 20 of each in 120 rolls. Counts from about 12 to 28 are common. A face that comes up 40 times is a sign of trouble.
- A weighted paper die usually lands with the heavy face *down*, so the opposite face (the 6) comes up on top.

Check your work

- ☐ I rolled 120 times and recorded every roll in a tally chart.
- ☐ I compared my counts with the expected count of 20 and wrote them as fractions and percents.
- ☐ I tested a fair paper die and a weighted one and explained the difference.

SCOREBOOK

1. Turn your 120-roll tally into a bar graph: faces 1 to 6 along the bottom, counts up the side. Draw a dotted line at 20.
2. Draw the net of a die with the faces numbered so that opposite faces add to 7.
3. Write: “I would decide a die is fair if...” and finish the sentence using your data.

From history. The Metropolitan Museum of Art dug at Nishapur from 1935 to 1940 and found pottery, glass, tools and game pieces from everyday life in the city more than a thousand years ago. Many of the finds are now in the museum’s galleries in New York.

WORKED EXAMPLES

Example 1. You roll a fair die 60 times. About how many times should you expect an even number?

Show solution

1. Even numbers: 2, 4, 6. That is 3 of the 6 faces.
2. Probability: $\frac{3}{6} = \frac{1}{2}$.
3. Expected count: $\frac{1}{2} \times 60 = 30$.

Answer: About 30 times, though 25 or 35 would not be surprising.

Example 2. In a game, you can only start moving when you roll a 6. What is the chance you can start on your first turn? What is the chance you cannot?

Show solution

1. Rolling a 6: $\frac{1}{6}$.
 2. Not rolling a 6: all the other faces, $\frac{5}{6}$.
-

Answer: You start with probability $\frac{1}{6}$ (about 17%) and wait with probability $\frac{5}{6}$ (about 83%). The two always add to 1.

COMMON MISTAKES

- Expecting exactly 20 of each number in 120 rolls. Fair dice still bounce around the expected count.
- Thinking a die is unfair after only a few rolls. It takes many rolls to tell a slightly unfair die from a fair one.
- Forgetting that the probabilities of all six faces must add up to 1.

Make it a game • Is It Fair Tally. Roll one die thirty times, keeping a tally on paper with beans as markers. Before you roll, write down how many of each number you expect. Afterward, compare. Your parent guesses which number came up most, and you explain why a fair die could still show surprises. Closest prediction wins a bean.

2.2 TWO DICE: WHY 7 COMES MOST OFTEN

OBJECTIVES

- Draw the 6-by-6 grid of all 36 outcomes for two dice.
- Find the probability of every sum from 2 to 12.
- Explain why Catan prints its 6 and 8 tokens in red, and use counting to choose a good spot.

Warm-up. Before you count anything, guess: when you roll two dice and add them, which total comes up most often? Which comes up least? Roll two dice 20 times and tally the totals to see whether your guess looks right.

WATCH

Probability Distribution - Sum of Two Dice

Brian Veitch · 4:17

<https://www.youtube.com/watch?v=8ZsEYKN0ht8>

WHEN TO WATCH

After “The mathematics.” Build your own 6-by-6 grid as the video builds its table.

BACKUP VIDEO

Probability of Dice

MooMooMath and Science · 2:43

<https://www.youtube.com/watch?v=aydipyWwcd8>

A shorter explanation of the 36 outcomes with a table of sums.

KEY TERMS

sum

The total you get by adding numbers, such as the two numbers on a pair of dice.

outcome grid

A table that shows every way two dice can land, with one die across the top and the other down the side.

multiplication principle

If one thing can happen in 6 ways and another in 6 ways, together they can happen in $6 \times 6 = 36$ ways.

distribution

The pattern of how often each result happens.

CORE IDEAS

The story. Around the year 1620 a nobleman in Italy brought a puzzle to the great scientist Galileo Galilei. Players who rolled three dice had noticed, after many years of play, that a total of 10 came up a little more often than a total of 9. That seemed strange, because both totals can be made from six different groups of numbers. Galileo wrote a short paper to explain it. Almost four hundred years later, in 1995, a German game designer named Klaus Teuber published a board game called *The Settlers of Catan*. Every hex on its board has a number token, and two of the numbers, 6 and 8, are printed in red. Galileo would have understood exactly why.

How they figured it out. The players had counted the wrong thing. They counted ways to *write* a total, like $3 + 3 + 3$, instead of ways the dice can *fall*. Galileo imagined each die was a different color. Then a red 1 with a blue 2 is a different fall from a red 2 with a blue 1. Counted this way, 10 can happen in 27 ways and 9 in only 25 out of 216, so 10 really is a little more likely. The lesson works for two dice too: imagine one die is red and the other is blue, and count every way they can land.

The mathematics. Two dice can land in $6 \times 6 = 36$ equally likely ways. A total of 2 happens only one way (1 and 1), so its probability is $\frac{1}{36}$. A total of 7 happens six ways: 1–6, 2–5, 3–4, 4–3, 5–2 and 6–1. So $P(7) = \frac{6}{36} = \frac{1}{6}$, the most likely total. The number of ways for the totals 2 through 12 goes 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1: up like a staircase and down again. Totals of 6 and 8 each happen 5 ways out of 36.

On the field. In *Catan*, every time the dice show a hex's number, the settlements next to it collect a card. A 7 produces nothing (it moves the robber instead), so the best producing numbers are 6 and 8, at 5 chances in 36 each. That is why they are red: they are the hot numbers. Under each number token are small dots, one for each way to roll it, so you can add the dots to compare spots. A settlement touching a 6, an 8 and a 5 collects on $5 + 5 + 4 = 14$ of the 36 outcomes. **The risk side:** everyone wants the red numbers, the robber is often moved there, and a player whose spots all share one number may wait a long time if that number goes cold. Wise players weigh the best chances against having some variety.

PICTURE STUDY



Galileo Galilei, Ottavio Leoni, 1624 · Engraving · National Gallery of Art, Washington, D.C.

Image: [Wikimedia Commons](#), public domain

Ottavio Leoni was a Roman artist famous for portraits of the great people of his day. He drew Galileo from life and then made this engraving in 1624, at about the time Galileo was explaining the puzzle of the three dice.

Look closely

- Look at Galileo's eyes. Leoni was known for capturing a person's real expression. What kind of man does this seem to be?
- Read the words around the oval. Find "Galileus Galileus" and "Florentinus," which tell who he was and where his family came from.
- Find the date 1624 in the corners of the picture.

Talk about it

1. Galileo is famous for looking at the moons of Jupiter through a telescope. How is counting dice outcomes like looking closely through a telescope?
2. The dice players had noticed the difference between 9 and 10 only after years of play. Why does a small difference in chances take many, many rolls to see?

PROJECT

The Two-Dice Staircase

You will draw the outcome grid, predict how often each total should come up, and then roll two dice 72 times (or 144 times if you are keen) to see the staircase shape appear.

You will need

- Two dice of different colors
- Your scorebook, graph paper, colored pencils and a calculator

Steps

1. Draw a 6-by-6 grid. Write 1 to 6 across the top for the red die and 1 to 6 down the side for the blue die. Fill each box with the total.
2. Color all the boxes that show 7 in one color. Then color each other total in its own color. Count the boxes for each total from 2 to 12.
3. **Predict.** Write the expected count of each total for 72 rolls: 2, 4, 6, 8, 10, 12, 10, 8, 6, 4, 2.
4. Make a tally chart with rows for the totals 2 through 12.
5. Roll both dice 72 times and tally each total. (Roll 144 times to double every expected count.)
6. Draw a bar graph of your results. Draw the expected staircase on top of it as a dotted line.
7. Which total came up most? Did any total come up much more or less often than expected?
8. **Catan challenge.** Imagine three spots on a game board: Spot A touches 6, 8 and 3; Spot B touches 5, 9 and 10; Spot C touches 2, 12 and 11. Count the ways for each spot and rank them.

Tips

- Use different colored dice so you can see that red 2 with blue 5 and red 5 with blue 2 are two different outcomes.
- If your 7 does not come up most in 72 rolls, do not worry. Roll another 72 and add them together; the staircase gets clearer with more rolls.

Check your work

- ☐ My outcome grid shows all 36 outcomes, and I counted the ways for every total.
- ☐ My bar graph shows my 72 (or 144) rolls with the expected staircase drawn on it.
- ☐ I ranked the three Catan spots by counting the ways.

SCOREBOOK

1. Copy your colored 6-by-6 outcome grid neatly. Under it write the probability of each total as a fraction of 36.
2. Make a table: Total, Ways, Expected in 72 rolls, What I rolled.
3. Answer the Catan challenge (Spot A: $5 + 5 + 2 = 12$ ways; Spot B: $4 + 4 + 3 = 11$ ways; Spot C: $1 + 1 + 2 = 4$ ways) and write which spot you would choose and why.

From history. Galileo's little paper on dice was written sometime between 1613 and 1623 but not printed until 1718, long after his death. Klaus Teuber, the designer of Catan, was a dental technician who designed games in his spare time; Catan became one of the best-loved family games in the world.

WORKED EXAMPLES

Example 1. Roll two dice. What is the probability that the total is 10 or more?

Show solution

1. Total 10: 4–6, 5–5, 6–4 (3 ways). Total 11: 5–6, 6–5 (2 ways). Total 12: 6–6 (1 way).
2. Add the ways: $3 + 2 + 1 = 6$.
3. Probability: $\frac{6}{36} = \frac{1}{6}$, about 17%.

Answer: $\frac{1}{6}$, the same chance as rolling exactly 7.

Example 2. In 72 rolls of two dice, how many 7s do you expect? How many 2s?

Show solution

1. 7s: $\frac{6}{36} \times 72 = 12$.

2. 2s: $\frac{1}{36} \times 72 = 2$.

Answer: About 12 sevens and about 2 twos. In 72 rolls the expected counts for 2 through 12 are 2, 4, 6, 8, 10, 12, 10, 8, 6, 4, 2.

COMMON MISTAKES

- Thinking there are only 11 outcomes (the totals 2 to 12) and that each is equally likely. There are 36 equally likely outcomes, and the totals are not equally likely.
- Counting 2–5 and 5–2 as the same outcome. With two dice of different colors, you can see they are different.
- Expecting the staircase to be perfect in only 72 rolls.

Make it a game • Grid in a Minute. Draw the 6-by-6 two-dice grid from memory in sixty seconds, timed by a parent. Circle every box that makes a sum of seven. Then name the probability of seven, and of two or twelve. Parents call out a random sum and you give its count of ways. Score a bean for each correct answer. Beat your own best time.

2.3 TALLY CHARTS AND GALTON'S BOARD

OBJECTIVES

- Keep an accurate tally chart and turn it into a bar graph.
- Explain how Galton's board of pins makes a hump-shaped pattern.
- Count the outcomes of flipping four coins and predict the shape of the results.

Warm-up. Flip four coins at once and count the heads. Do it ten times and write the number of heads each time. Which number of heads (0, 1, 2, 3 or 4) showed up most? Why do you think that is?

WATCH

Galton Board

IMAmaths · 2:25

<https://www.youtube.com/watch?v=6YDHBFIvIvIs>

WHEN TO WATCH

After “The story.” Watch the shot pile up, and predict the shape before it finishes.

BACKUP VIDEO

The Galton Board

D!NG · 10:50

<https://www.youtube.com/watch?v=UCmPmkHqHXk>

A longer look at the Galton board and Pascal's triangle for older students.

KEY TERMS

tally chart

A table where each result gets a mark, bundled in groups of five, so it can be counted quickly.

bar graph

A drawing that shows counts as bars, so you can compare them at a glance.

Galton board

A board of pins where balls bounce left or right at each pin and pile up in slots at the bottom. Also called a bean machine or quincunx.

path

One complete set of left-and-right bounces a ball can take from the top of the board to a slot.

CORE IDEAS

The story. Francis Galton was an English scientist of the 1800s who loved to measure everything: fingerprints, weather, people's heights and much more. In the 1870s he built a wooden box with a glass front and rows of pins inside, set in a staggered pattern like the dots on the five face of a die. When he poured small lead shot in at the top, each little ball bounced off pin after pin and landed in one of the slots at the bottom. Every time, the balls piled up in the same shape: a tall hump in the middle and low piles at the edges. In 1889 he printed a drawing of his board in a book called *Natural Inheritance*.

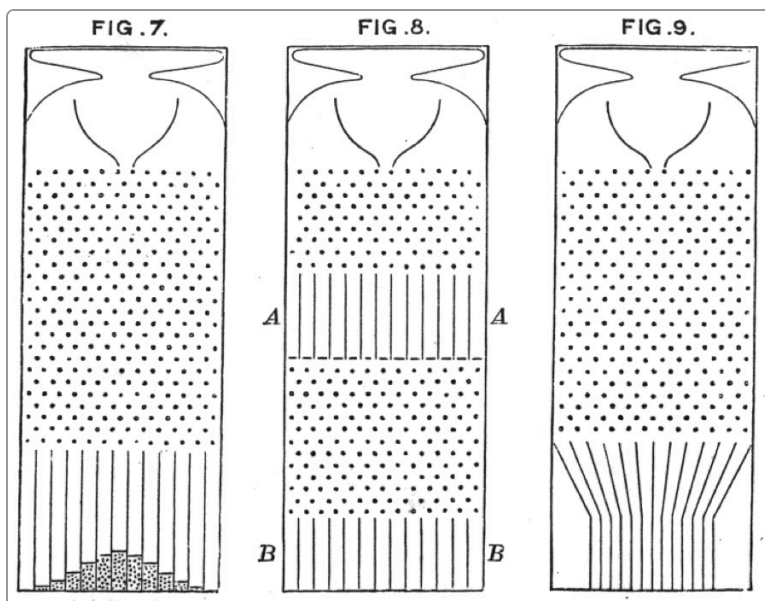
How they figured it out. No one can say where any single ball will land. Yet the pile as a whole always takes the same shape. Galton saw why. At every pin, a ball goes left or right, like a coin flip. To land at the far left, a ball must bounce left at every single pin, and there is only one path that does that. To land in the middle, it can take many different paths, some left bounces and some right in any order. More paths means more balls. Galton was amazed that so much chance could make so much order.

The mathematics. Flipping four coins is like dropping one ball through four rows of pins. Each coin is heads or tails, so there are $2 \times 2 \times 2 \times 2 = 16$ equally likely outcomes. Zero heads happens 1 way, one head 4 ways, two heads 6 ways, three heads 4 ways and four heads 1 way. The numbers 1, 4, 6, 4, 1 make a hump, just like Galton's board. So two heads has probability $\frac{6}{16} = \frac{3}{8}$, while four heads has only $\frac{1}{16}$. A tally chart is how you see the hump appear: one mark per try, then count, then draw a bar graph.

On the field. The hump shape appears whenever many small chances add up. A basketball player who takes 10 free throws in each practice will not make the same number every day; most days she will be near her usual number, and only rarely far above or far below. A coach who keeps a tally of her makes every day can see the hump and knows that one bad day is not a reason to panic. **The risk side:** results at

the edges of the hump do happen, just not often. A wise player expects ordinary days most of the time, and is not shaken when an unusual one comes.

PICTURE STUDY



The Quincunx, Francis Galton, 1889 · Printed diagrams from the book *Natural Inheritance* · London
Image: [Wikimedia Commons](#), public domain

Galton drew these diagrams to explain his board of pins. He called it a quincunx, after the Latin word for the pattern of five dots, like the five on a die, that the pins make.

Look closely

- Find the funnel at the top of each drawing, where the shot is poured in.
- Look at the rows of dots. Each dot is a pin. Notice how each row is shifted half a space from the row above it.
- Look at the bottom of Fig. 7. What shape does the pile of shot make in the slots?
- Compare Fig. 8 and Fig. 9 with Fig. 7. How are they different?

Talk about it

1. Why do more balls land in the middle slots than at the edges?
2. If you added more rows of pins, would the hump get wider or narrower?
Make a guess, then test it with more coins in the project.

PROJECT

A Homemade Galton Board

You will run Galton's experiment two ways: with four coins, and with a paper board of your own. Then you will make a careful tally chart and bar graph.

You will need

- Four coins
- A piece of cardboard about the size of a sheet of paper, a pencil, a ruler and a marker
- A small button, bead or dried bean to be the "ball," and one coin to decide each bounce
- Your scorebook

Steps

1. **Predict.** Write the 16 outcomes of four coins (HHHH, HHHT, and so on), and count how many give 0, 1, 2, 3 and 4 heads.
2. Flip four coins together 80 times. Tally the number of heads each time in a chart with rows for 0, 1, 2, 3 and 4.
3. Count your tallies and draw a bar graph. Draw the expected counts (5, 20, 30, 20, 5) as dots above each bar.
4. **Paper board.** On the cardboard, draw 4 rows of dots (pins) in a triangle: 1 dot in the top row, 2 in the next, then 3, then 4. Under the last row draw 5 slots and number them 0 to 4.
5. Put your button at the top dot. Flip a coin: heads, move it down-right to the next dot; tails, move it down-left. Repeat until it falls into a slot.
6. Do this 40 times and tally which slot the button lands in each time.
7. Compare your board results with your four-coin results. Write why they make the same kind of shape.

Tips

- Bundle your tally marks in fives with a slanted fifth line so you can count quickly and accurately.

- Keep the coin flips honest: flip high, let the coin land, and record before you move the button.
- If your hump is lopsided after 80 flips, that is normal. Your data are real, and honest data are better than perfect-looking data.

Check your work

- ☐ My tally chart has 80 four-coin flips, counted correctly.
- ☐ My bar graph is labeled and shows the expected counts too.
- ☐ I explained in my own words why the middle results happen most often.

SCOREBOOK

1. Draw your bar graph neatly on graph paper with a title, labeled axes and the expected counts marked.
2. Draw a copy of Galton's board with 4 rows of pins. Trace three different paths that end in the middle slot.
3. Write: "One bad day of practice does not worry me because..." and finish the sentence using what you learned about the hump.

From history. Galton also studied fingerprints and showed that they stay the same for a whole lifetime and differ from person to person, which helped police begin using fingerprints in the early 1900s. He was a half-cousin of the naturalist Charles Darwin.

WORKED EXAMPLES

Example 1. You flip four coins 80 times and count the heads each time. How many times do you expect exactly 2 heads? Exactly 4 heads?

Show solution

1. Two heads: 6 of 16 outcomes, so $\frac{6}{16} \times 80 = 30$.
2. Four heads: 1 of 16 outcomes, so $\frac{1}{16} \times 80 = 5$.

Answer: About 30 times for two heads and about 5 times for four heads. The full expected counts for 0 to 4 heads are 5, 20, 30, 20, 5.

COMMON MISTAKES

- Thinking each number of heads (0 to 4) is equally likely. Two heads is six times as likely as four heads.

- Making tally marks in long rows without bundling them in fives, which makes counting errors easy.
- Drawing bar graphs without labels. Every graph needs a title and labels on both edges.

Make it a game • Bean Hump Race. Drop twenty beans, one at a time, down a tilted tray with a few buttons stuck as pins, or simply drop them between your fingers onto a drawn grid. Tally where they land and draw a bar graph. Predict the hump shape before you start. Explain to your parent why the middle gets more. Beat your own graph accuracy.

2.4 DICE HAVE NO MEMORY

OBJECTIVES

- Explain what it means for coin flips and dice rolls to be independent.
- Show by experiment that streaks happen often in random results.
- Tell how scientists studied the “hot hand” in basketball and corrected each other.

Warm-up. A coin has just landed heads five times in a row. Ask three people in your family: “Is the next flip more likely to be heads, more likely to be tails, or even?” Write their answers. Then flip it and see.

WATCH

[Coin flipping probability | Probability and Statistics | Khan Academy](https://www.youtube.com/watch?v=mkyZ45KQYi4)

[Khan Academy](https://www.youtube.com/watch?v=mkyZ45KQYi4) • 9:20

<https://www.youtube.com/watch?v=mkyZ45KQYi4>

WHEN TO WATCH

After “The mathematics.” It shows how to find the chance of several heads in a row by multiplying; younger students may stop halfway.

KEY TERMS

independent

Not affected by what happened before. Each flip of a fair coin is independent of the others.

streak

A run of the same result several times in a row, such as four heads or five made shots.

“due” mistake

The false idea that a result is more likely because it has not happened for a while.

hot hand

The idea that a player who has made several shots in a row is more likely to make the next one.

CORE IDEAS

The story. Basketball fans have long believed in the “hot hand”: a player who makes three or four shots in a row is “on fire” and should get the ball again. In 1985 three scientists, Thomas Gilovich, Robert Vallone and Amos Tversky, decided to test it. They studied the shots of the Philadelphia 76ers, the free throws of the Boston Celtics, and an experiment with players from Cornell University. They reported that

players were no more likely to make a shot after a make than after a miss, and that the streaks looked just like the streaks a coin makes.

How they figured it out. For about thirty years many people treated the hot hand as a mistake of the mind. Then in 2018 two other scientists, Joshua Miller and Adam Sanjurjo, found a tiny trap in the way the 1985 study had counted streaks in short series of shots. When they fixed the counting, the Cornell players did show a small hot hand after all. It was real, but much smaller than fans believed. This is a fine example of good science: careful people checked the counting, corrected it, and everyone learned something.

The mathematics. A coin or a die has no memory. After five heads in a row, the next flip is still heads with probability $\frac{1}{2}$, because the coin does not know what came before. We call such flips **independent**. For independent events, the chance of several in a row is found by multiplying: three heads in a row has probability $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$. That sounds rare, but in a long series of flips streaks are everywhere. In 20 flips of a fair coin, there is about a 3-in-4 chance of a run of four or more of the same side somewhere. Streaks are what randomness looks like.

On the field. Two opposite mistakes come from forgetting this. The first is thinking a result is “due”: “I have missed five in a row, so the next one has to go in.” The second is thinking a streak must go on: “He has made four, so he can’t miss.” A wise player keeps taking good shots and does not change his plan because of a short streak, good or bad. A wise coach does not bench a good shooter for a cold stretch that chance can explain. **The risk side:** believing a result is “due” leads people to take foolish chances, sure that their turn must come. The dice do not keep score; only you do.

PICTURE STUDY



Snap-the-Whip, Winslow Homer, 1872 · Oil on canvas · Butler Institute of American Art, Youngstown, Ohio

Image: [Wikimedia Commons](#), public domain

Winslow Homer painted this scene of country schoolboys at recess in 1872. In the game of snap-the-whip, a line of children run holding hands; the leader stops and swings the line, and the children at the end must hold on as they are whipped around.

Look closely

- Count the boys in the line. Which ones are still standing and which have fallen?
- Find the boys at the end of the line. Why is it hardest to hold on at the end?
- Look at the schoolhouse and the hills behind. What time of year do you think it is?

Talk about it

1. Some boys have held on round after round, and two have just fallen. Does holding on four times in a row mean a boy is sure to hold on the fifth time? What else could matter?
2. In this game, is falling off just chance, or does skill matter too? How is that like the hot hand in basketball: real, but smaller than people think?

PROJECT

Streak Hunters

People often think a fair coin should switch back and forth a lot. You will find out how long streaks really are, and whether heads is more likely after heads.

You will need

- A coin
- Your scorebook and a pencil
- A partner (optional)

Steps

1. **Fake flips.** Before touching a coin, write down a list of 50 made-up flips (H and T) that you think looks random. Do not flip anything yet.
2. **Real flips.** Now flip a real coin 50 times and write down every result in order.
3. In each list, find the longest streak of the same side. Circle it. Which list has the longer streak?
4. Count how many streaks of 4 or more each list has. Real coins usually make more long streaks than people expect, and made-up lists usually switch too often.
5. **After heads.** In your real list, look at every flip that comes right after a head. Tally whether that next flip was H or T.
6. Write the fraction of “after heads” flips that were heads. Is it close to one half?
7. Flip 50 more times and add them to your counts. Is it closer to one half now?
8. If you have a partner, have each of you make a fake list and a real list, swap them, and see whether you can tell which is real by looking for long streaks.

Tips

- Write each flip as soon as it lands, in a long row of letters, so you can see the streaks.
- Do not be surprised by a run of 5 or 6. In 100 flips, a run of 6 or more happens about four times in five.

Check your work

- ☐ I made a fake list and a real list and compared their longest streaks.
- ☐ I tallied what came after each head and found that it was close to one half.
- ☐ I can explain why a result is never “due.”

SCOREBOOK

1. Copy your real list of flips with the streaks of 4 or more circled in color.
2. Make a table: Flip after a head, H or T, Count, Fraction.
3. Write: “If I miss three shots in a row, I would decide...” and finish the sentence.

From history. Amos Tversky worked for many years with Daniel Kahneman, studying the ways people’s minds misjudge chance. Kahneman received the Nobel Prize in economics in 2002 for their work together.

WORKED EXAMPLES

Example 1. A die has not shown a 6 in its last 10 rolls. What is the probability that the next roll is a 6?

Show solution

1. The die has no memory; each roll is independent.
2. A fair die shows 6 with probability $\frac{1}{6}$ on every roll.

Answer: $\frac{1}{6}$, exactly the same as always. A 6 is never “due.”

Example 2. What is the probability of rolling a 6 on a die twice in a row?

Show solution

1. Each roll is independent, so multiply.
2. $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

Answer: $\frac{1}{36}$, the same as the chance of a 12 with two dice (Lesson 2.2).

COMMON MISTAKES

- Believing a result is “due” after it has not happened for a while.
- Thinking a random list should switch back and forth often. Real random lists have more long streaks than most people expect.

- Thinking the hot hand is either completely fake or very strong. The best evidence says it is real but small.

Make it a game • The Due Trap. Flip a coin ten times, then have your parent ask, are tails due? Write what you answer and why. Repeat with a die. Your parent tries to fool you with one story about a lucky streak, and you must name the due mistake. Play three rounds. Score a bean each time you spot the trap.

Scripture. “The simple believeth every word: but the prudent man looketh well to his going.” (Proverbs 14:15, KJV)

A prudent person does not believe a feeling like “I’m due” or “I can’t miss” just because it is strong. He checks it against the facts.

UNIT 2

DICE AND COUNTING

Dice are some of the oldest game pieces in the world, and they are also a perfect tool for learning about chance. In this unit you will roll one die hundreds of times and see that each face comes up about one time in six. Then you will roll two dice and discover something that surprises many people: the sums are not equally likely at all. A 7 comes up six times as often as a 2 or a 12, and that is exactly why the 6 and 8 tokens in the game *Catan* are printed in red.

You will learn to keep a tally chart and draw a bar graph, the two most useful tools in your scorebook, and you will build a homemade version of Francis Galton's famous board of pins to watch chance make a beautiful, predictable shape. Finally you will learn one of the most important facts in this whole course: dice and coins have no memory. A streak of sixes does not make the next roll more or less likely to be a six.

PREREQUISITES

- Can write a probability as a fraction, a decimal and a percent (Unit 1)
- Can list the outcomes of a simple experiment
- Can multiply small whole numbers

MASTERY CHECKLIST

- ☐ I can explain what makes a die fair and test a die by rolling it many times.
- ☐ I can draw the 6-by-6 grid of two-dice outcomes and find the probability of any sum.
- ☐ I can explain why 7 is the most common sum of two dice and why *Catan*'s 6 and 8 are red.
- ☐ I can keep a tally chart and turn it into a bar graph.
- ☐ I can explain why dice and coins have no memory, and why streaks still happen.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **CATAN**: The publisher's site for the family game Catan, whose red 6 and 8 tokens are explained in Lesson 2.2. Ages 10 and up. A good game for practicing two-dice counting, but never required.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

2.1 One Die, Six Faces	2 sessions of 45 minutes
2.2 Two Dice: Why 7 Comes Most Often	2 sessions of 45 minutes
2.3 Tally Charts and Galton's Board	2 sessions of 45 minutes
2.4 Dice Have No Memory	2 sessions of 45 minutes

UNIT 2 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. 4 pts

What makes a die fair? Give two things a die maker must get right.

2. 4 pts

You roll a fair die 90 times. About how many times do you expect a 3? About how many times a number greater than 4?

3. 5 pts

How many outcomes are there when you roll two dice? List all the ways to roll a total of 8.

4. 4 pts

Why are the 6 and 8 tokens printed in red in Catan?

5. (5 pts)

Which is more likely with two dice: a total of 3, or a total of 11? Show your counting.

6. (4 pts)

What did the dice players who asked Galileo get wrong when they counted the ways to make 9 and 10 with three dice?

7. (5 pts)

Four coins are flipped. How many outcomes are there? How many give exactly 2 heads?

8. (4 pts)

Why do more balls land in the middle of a Galton board than at the edges?

9. (5 pts)

What would happen if you planned your game around a 6 being “due” because none has come up in 12 rolls? Explain what “independent” means.

10. (4 pts)

A shooter has made 4 free throws in a row. Is the next one sure to go in? What did Miller and Sanjurjo find about the hot hand?

11. (6 pts)

Short answer: You flip a coin 50 times and get a streak of 6 heads. A friend says the coin must be broken. Write three or four sentences explaining what you would tell him.

ANSWER KEY

1. Every face must be equally likely to land on top. The die must be a true cube with equal faces, and evenly heavy so no side is weighted.
2. A 3: $\frac{1}{6} \times 90 = 15$. Greater than 4 (5 or 6): $\frac{2}{6} \times 90 = 30$.
3. 36. A total of 8: 2-6, 3-5, 4-4, 5-3, 6-2 (5 ways), so $P(8) = \frac{5}{36}$.
4. They are the most likely numbers that produce cards: each comes up in 5 of 36 outcomes. Only 7 is more likely, and a 7 moves the robber instead of producing.
5. Equally likely. A total of 3: 1-2, 2-1 (2 ways). A total of 11: 5-6, 6-5 (2 ways). Each is $\frac{2}{36} = \frac{1}{18}$.
6. They counted ways to write the total (groups like $3 + 3 + 3$) instead of ways the dice can fall. When each die is treated as different, some groups can fall in several orders and others in only one.
7. $2 \times 2 \times 2 \times 2 = 16$ outcomes; exactly 2 heads happens in 6 of them (HHTT, HTHT, HTTH, THHT, THTH, TTHH).
8. Many different paths of left and right bounces lead to the middle, but only one path leads to each far edge (all left or all right).
9. The 6 would still come up only one time in six on each roll; it is never due. Independent means each roll is not affected by earlier rolls, so waiting for a “due” number is a poor plan.
10. No; it is still uncertain. They found that when the counting was corrected, there was a small real hot hand, but much smaller than fans believe.
11. Streaks are a normal part of randomness; long runs happen far more often than people expect. Each flip is independent, so a run of heads does not change the next flip. To test the coin, look at the total number of heads over many flips, not one streak. A run of 6 in 50 flips is not unusual for a fair coin.

PORTFOLIO PIECE: THE DICE REPORT

Write and illustrate a three-page “Dice Report.” Page 1: your 120-roll test of one die, with its bar graph and your decision about whether it is fair. Page 2: your colored 6-by-6 outcome grid, your two-dice bar graph with the expected staircase drawn on it, and a short explanation of why 7 comes up most. Page 3: either your Galton board results or your streak-hunting results, with one paragraph explaining what they show about chance. Finish with three “rules for dice” in your own words that would help a younger brother or sister play family games more wisely.

LOOK FOR	WHAT IT MEANS
Honest data	Every experiment has a prediction, complete tallies and correct totals.
Graphs	Bar graphs are neat, titled and labeled, with expected counts shown.
Explanations	The report correctly explains fairness, the 36 outcomes and why 7 is most common, using fractions.
Rules for dice	Three sensible rules, including that dice have no memory, written clearly for a younger reader.

UNIT 3

FAIR AND UNFAIR GAMES

Every good game needs fair rules. But what does “fair” really mean? Sometimes it means that every player has the same chance. Sometimes it means a clever rule, like “I cut, you choose,” that leaves everyone satisfied. And sometimes a game that looks fair is not fair at all. In this unit you will test simple dice and spinner games, find out which ones secretly favor one player, and learn how to fix them.

You will meet Abram and Lot dividing the land, Milton Bradley and the spinning top he used in his first board game, the great Chicago fair of 1893 where the word “midway” was born, and Christiaan Huygens, who wrote the first printed book about the mathematics of chance. You will also learn why carnival booth games are so hard to win. At the end you will design, test and polish a fair game of your own.

PREREQUISITES

- Can find the probability of a simple event by counting outcomes (Units 1–2)
- Can draw the 6-by-6 grid of two-dice outcomes (Unit 2)
- Can find an expected count by multiplying a probability by a number of tries

MASTERY CHECKLIST

- ☐ I can explain what makes a game fair and give an example of “I cut, you choose.”
- ☐ I can test a dice or spinner game for fairness by counting outcomes and by playing it many times.
- ☐ I can make a spinner with sections of the size I want and find the probability of each section.
- ☐ I can explain why carnival booth games are hard to win.
- ☐ I can fix an unfair game by changing the points so that each player’s points per round are equal.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Khan Academy: Probability (7th grade)**: Free. Short videos and practice on probability, experiments and simulations that match Lessons 3.1 and 3.2. Best for ages 11 and up; younger students can work through it with a parent.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

3.1 What Makes a Game Fair?	2 sessions of 45 minutes
3.2 Spinners and Odd-or-Even	2 sessions of 45 minutes
3.3 Why Carnival Games Are Hard to Win	2 sessions of 45 minutes
3.4 Design a Fair Game	2 sessions of 45 minutes

UNIT 3 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. **4 pts**
Describe two different ways a game or a choice can be fair. Give an example of each.
2. **5 pts**
Two dice are rolled. A scores if the product is even; B scores if the product is odd. Is it fair? Show your counting.
3. **5 pts**
Fix the game in question 2 by changing B's points. How many points should B get per win if A gets 1?
4. **4 pts**
A spinner has a section of 90 degrees. What is its probability? In 40 spins, how many times do you expect it?

5. (4 pts)

Why did Milton Bradley's *Checkered Game of Life* use a teetotum?

6. (4 pts)

What would happen in odds and evens if one player always held out one finger? Explain.

7. (5 pts)

You make 3 of 30 tosses at a carnival-style ring toss. What is your win rate as a fraction, a decimal and a percent? How many tries per win is that?

8. (4 pts)

Name two ways a booth game can be made harder than it looks.

9. (4 pts)

What question did Christiaan Huygens ask about a chance, and what did he call the answer?

10. (5 pts)

A scores 1 point on a die roll of 1, 2, 3, 4 or 5. B scores on a 6. How many points should B get per win to make the game fair?

11. (6 pts)

Short answer: Explain in three or four sentences what Proverbs 11:1 and 2 Timothy 2:5 teach about games and rules.

ANSWER KEY

1. Every player has the same chance (a coin toss, or odd-or-even totals with two dice); or a rule like “I cut, you choose” that leaves both satisfied, as when Abram let Lot choose first. Other good examples accepted.
2. Not fair. The product is odd only when both dice are odd: $3 \times 3 = 9$ outcomes. A wins 27 of 36 (three quarters) and B only 9 of 36 (one quarter).
3. 3 points. A: $\frac{3}{4} \times 1 = \frac{3}{4}$; B: $\frac{1}{4} \times 3 = \frac{3}{4}$.
4. $\frac{90}{360} = \frac{1}{4}$; about 10 times in 40 spins.
5. Many families of his day would not keep dice in the house, so he used a six-sided spinning top numbered 1 to 6, which gives each number the same one-in-six chance as a die.
6. An opponent who noticed could choose so that he always won (for example, if he wins on odd totals, always show two). The game is only fair when both players mix their choices.
7. $\frac{3}{30} = \frac{1}{10} = 0.1 = 10\%$; about 10 tries per win.
8. Any two: a ring barely wider than the bottle neck; a smaller or oval hoop; a very hard, bouncy ball; bottles weighted at the bottom; a tilted target. Other sensible answers accepted.
9. He asked what a chance is worth. He called it the expectation: the prize times the probability, which is the average per round over many rounds.
10. A expects $\frac{5}{6} \times 1 = \frac{5}{6}$. B needs $\frac{1}{6} \times 5 = \frac{5}{6}$, so 5 points per win.
11. God delights in honest measures and hates false ones, so games should have honest chances and honest scorekeeping. Paul says a victory only counts if the player strives lawfully, by the rules. A fair game tells its real chances, and good players keep the rules even when no one is watching.

PORTFOLIO PIECE: MY FAIR GAME

Finish the game you designed in Lesson 3.4 and present it as a complete, playable game. Include: a neat board or playing pieces if the game needs them; a typed or neatly written rule book; a “fairness page” that shows the outcomes each player can score on, each player’s probability, and the expected points per round for each player; and your playtest tables from at least 45 rounds, with a short note on the rule you changed and why. Teach the game to someone outside your family, or to a grandparent, and write two sentences about how it went.

LOOK FOR	WHAT IT MEANS
Fairness	The fairness page counts the outcomes correctly and shows equal expected points per round for every player.
Rule book	Rules are complete and clear enough for a new player to learn the game alone.
Playtesting	At least 45 rounds are recorded, with one rule improved for a stated reason.
Choices and fun	The game gives players at least one real decision and was enjoyed by the players who tested it.

3.1 WHAT MAKES A GAME FAIR?

OBJECTIVES

- Describe three ways a game or a choice can be fair.
- Test simple two-dice games for fairness by counting the 36 outcomes.
- Fix an unfair game by giving the less likely player more points.

Warm-up. Two children must share one cookie. One suggests: “I’ll cut it, and you choose which half you want.” Why does this rule make the cutter try very hard to cut evenly? Try it with a piece of bread or a sheet of paper.

WATCH

The last banana: A thought experiment in probability - Leonardo Barichello

TED-Ed • 4:10

<https://www.youtube.com/watch?v=Kgudt4PXs28>

WHEN TO WATCH

After the warm-up. This is the “Big Die” game from the project; pause and guess who is favored before the answer.

BACKUP VIDEO

Using probability to make fair decisions

Khan Academy • 3:33

https://www.youtube.com/watch?v=Z_5f3kys52o

A short Khan Academy example of making a fair choice with probability.

KEY TERMS

fair game

A game in which every player has the same chance to win, or the same points per round on average.

unfair game

A game whose rules give one player a better chance or more points on average.

“I cut, you choose”

An old way to divide something fairly between two people: one divides it, and the other picks first.

points per round

The points a player wins on average each round: the points for a win times the probability of winning.

CORE IDEAS

The story. In the Book of Genesis, Abram and his nephew Lot traveled together with so many sheep, goats and cattle that the land could not feed them all, and their herdsmen began to quarrel. Abram, the older man, could have taken the best land for himself. Instead he said to Lot, “Is not the whole land before thee? separate thyself, I pray thee, from me: if thou wilt take the left hand, then I will go to the right” (Genesis 13:9). Abram let Lot choose first. Lot looked and chose the green, well-watered plain of the Jordan River, and Abram went the other way.

How they figured it out. People have divided things fairly by “one divides, the other chooses” for thousands of years. The divider knows he will get whichever part is left, so he divides as evenly as he can. In the 1940s a Polish mathematician, Hugo Steinhaus, asked how to divide a cake fairly among three or more people, and mathematicians have studied fair division ever since. For games, the most common kind of fairness is simpler: give every player the same chance. Mathematicians test that by counting outcomes, just as you did with dice.

The mathematics. To test a dice game, count the outcomes each player wins. Suppose Player A scores when the total of two dice is even, and Player B scores when it is odd. Counting the 36-box grid gives 18 even totals and 18 odd totals, so the game is fair. Now suppose A scores when the *product* of the two dice is even and B when it is odd. A product is odd only when both dice are odd: $3 \times 3 = 9$ outcomes. So A wins $\frac{27}{36} = \frac{3}{4}$ of the time and B only $\frac{9}{36} = \frac{1}{4}$. That is unfair. One way to fix it is with points: give B 3 points for each win and A 1 point. Then A averages $\frac{3}{4} \times 1 = \frac{3}{4}$ point per round and B averages $\frac{1}{4} \times 3 = \frac{3}{4}$. The points per round are equal, so the game is fair again.

On the field. Sports work hard to stay fair. Soccer and football teams switch ends at halftime so neither side keeps the wind or the sun at its back all game. Tennis players take turns serving, and chess players take turns playing White, which moves first. In family games, the youngest might go first, or players take turns starting. **The risk side:** a game that looks fair may not be. Before you agree to a game’s rules, count

the outcomes. And when you are the stronger player, remember Abram: being generous with a younger or weaker player is part of playing well.

PICTURE STUDY



Abraham and Lot Divide the Land, Claes Cornelisz. Moeyaert, 1646 · Oil on panel · Private collection

Image: [Wikimedia Commons](#), public domain

Claes Moeyaert was a Dutch painter in Amsterdam who loved to paint Bible stories set in wide landscapes full of animals and people. Here he shows the moment when Abram and Lot agree to part, with their great herds all around them.

Look closely

- Find Abram and Lot talking on the left. Which one do you think is Abram, the older man? Which way is each one pointing?
- Count the kinds of animals you can see. Why would so many animals cause quarrels between the herdsmen?
- Look at the land in the background. If you were Lot, which way would you choose?

Talk about it

1. Abram let Lot choose first. Was that fair to Abram? Was it generous? What is the difference?
2. How is Abram's offer like "I cut, you choose"?

PROJECT

The Fairness Lab

You will test three two-dice games for fairness, first by counting and then by playing each 36 times. Then you will fix the unfair ones with points.

You will need

- Two dice of different colors
- Your 6-by-6 outcome grid from Lesson 2.2
- Your scorebook, a pencil and a partner

Steps

1. **Game 1, Odd or Even Total.** A scores on an even total, B on an odd total.
2. **Game 2, Odd or Even Product.** Multiply the two dice. A scores on an even product, B on an odd product.
3. **Game 3, Big Die.** A scores if the larger die is 1–4; B scores if it is 5 or 6.
4. **Predict.** Before counting, guess which games are fair. Write your guesses.
5. **Count.** On copies of your 6-by-6 grid, color the boxes A wins in one color and B wins in another. Count each color for each game.
6. **Play.** Play each game 36 rounds with a partner and tally the wins.
7. Compare your tallies with your counts. Which games were unfair? Did the play show it clearly, or did chance hide it?
8. **Fix.** For each unfair game, choose points for a win so that each player's points per round are equal. Play 36 more rounds with the new points and add up the scores.
9. Write a one-sentence verdict on each game: fair, unfair, or fixed.

Tips

- When you count products, remember that any even number times anything is even.
- After 36 rounds a fair game can still end 21 to 15. That is why counting the outcomes is the stronger test.
- Keep score in points, and let the younger player choose which side to be.

Check your work

- ☐ I counted the outcomes for all three games before playing.
- ☐ I played each game 36 rounds and compared the tallies with my counts.
- ☐ I fixed each unfair game with points and checked that the points per round are equal.

SCOREBOOK

1. Paste or draw your three colored grids with a title and the counts for A and B under each.
2. Make a table: Game, A's chance, B's chance, Fair?, My fix.
3. Write about a time when someone was generous like Abram in a game or a choice.

From history. Hugo Steinhaus worked out a fair way to divide a cake among three people while hiding from danger during the Second World War. Fair-division methods are now used to divide inheritances, schedules and even shared rooms in a house.

WORKED EXAMPLES

Example 1. Roll two dice. Player A scores if the larger number is 1, 2, 3 or 4. Player B scores if the larger number is 5 or 6. Is the game fair?

Show solution

1. A scores only when both dice are 4 or less: $4 \times 4 = 16$ outcomes.
2. B scores in all the others: $36 - 16 = 20$ outcomes.
3. A wins $\frac{16}{36}$, about 44%; B wins $\frac{20}{36}$, about 56%.

Answer: Not fair: B is favored, even though A has four numbers and B only two.

Example 2. A wins a round with probability $\frac{1}{3}$ and B with probability $\frac{2}{3}$. B gets 1 point for a win. How many points should A get for a win to make the game fair?

Show solution

1. B's points per round: $\frac{2}{3} \times 1 = \frac{2}{3}$.
2. A needs $\frac{1}{3} \times \text{points} = \frac{2}{3}$, so points = 2.

Answer: A should get 2 points for each win.

COMMON MISTAKES

- Thinking a game is fair because each player has the same number of numbers. What matters is how many outcomes each player wins.
- Deciding a game is fair or unfair from a few rounds. Count the outcomes first.
- Fixing an unfair game by giving the favored player more points. The less likely player needs more points.

Make it a game • Fair or Foul Court. Your parent invents three two-dice games, such as I win on sums under 7 and you win on sums over 7. You act as judge and decide if each is fair by counting outcomes in the grid. Say what you counted. Then invent your own fair game, and your parent tries to break it.

Scripture. “Is not the whole land before thee? separate thyself, I pray thee, from me: if thou wilt take the left hand, then I will go to the right; or if thou depart to the right hand, then I will go to the left.” (Genesis 13:9, KJV)

Abram used a fair and generous rule to end a quarrel: he let Lot choose first. Fair rules keep peace among players and families.

3.2 SPINNERS AND ODD-OR-EVEN

OBJECTIVES

- Find the probability of each section of a spinner from its size.
- Build a fair spinner and an unfair one and test them.
- Explain why the schoolyard game of odds and evens is fair.

Warm-up. Draw a circle and divide it into two halves. Color one half red. Divide the other half into two quarters, one blue and one yellow. If you spin a pointer on this circle, which color is most likely? How much more likely is it than blue?

WATCH

Theoretical vs. Experimental Probability | What's the Difference? | Math with Mr. J.

Math with Mr. J. • 7:43

<https://www.youtube.com/watch?v=Ay5pnC-0bVI>

WHEN TO WATCH

Before the project. It shows how to compare counted and experimental probability, just as you will with your spinners.

BACKUP VIDEO

Collections Video: The Checkered Game of Life

Vermont Historical Society. • 1:43

https://www.youtube.com/watch?v=_FH40THKzbc

A short museum video showing an original Checkered Game of Life with its teetotum.

KEY TERMS

spinner

A pointer that spins on a circle divided into sections, used to make a random choice.

teetotum

A small spinning top with numbered sides, used like a die in old board games.

section

One part of a spinner's circle. A bigger section has a bigger chance.

experimental probability

The fraction of times something actually happened in your tries, such as 14 reds in 40 spins.

counted probability

The probability found by reasoning, such as one half for a section that is half the circle. It is also called theoretical probability.

CORE IDEAS

The story. In 1860 a young printer in Springfield, Massachusetts, named Milton Bradley made his first board game, *The Checkered Game of Life*. Players moved across a checkered board from Infancy toward Happy Old Age, with squares for Honesty, Bravery and Industry along the way, and squares like Idleness and Ruin to avoid. Many families of his day would not keep dice in the house, so instead of dice Bradley gave his game a **teetotum**, a small six-sided spinning top numbered 1 to 6. The game sold very well, and the Milton Bradley Company went on to make games for more than a hundred years.

How they figured it out. A teetotum or spinner works because every part of the circle has the same chance to stop under the pointer. A six-sided teetotum with equal sides works just like a die: each number has a chance of one in six. But a spinner does not have to have equal parts. If one section takes up half the circle, the pointer stops there half the time. Game makers learned that by changing the sizes of the sections, they could make some results common and others rare, on purpose.

The mathematics. On a spinner, the probability of a section is the fraction of the circle it covers. A whole circle is 360 degrees, so a section of 90 degrees covers $\frac{90}{360} = \frac{1}{4}$ of the circle and has probability $\frac{1}{4}$. The warm-up spinner has red $\frac{1}{2}$, blue $\frac{1}{4}$ and yellow $\frac{1}{4}$. The fractions always add to 1. Now think about odds and evens: two players each hold out one or two fingers at the same moment, and one player wins if the total is odd, the other if it is even. The outcomes are 1+1, 1+2, 2+1 and 2+2. Two totals are odd and two are even, so if both players choose without a pattern, the game is fair.

On the field. Many games use spinners or wheels because they are easy to see and hard to cheat. Coaches use the same idea when they pick drills at random to keep practice fresh. In odds and evens, notice something important: the game is only fair if both players mix their choices. A player who always shows one finger can be beaten by an opponent who notices. That is your first taste of an idea that matters in sports, such as penalty kicks and pitching: **be unpredictable.** **The risk side:** any pattern you fall into is a pattern your opponent can learn.

PICTURE STUDY



A Child with a Teetotum, Jean-Siméon Chardin, 1738 · Oil on canvas · Musée du Louvre, Paris

Image: [Wikimedia Commons](#), public domain

Jean-Siméon Chardin was a French painter who loved quiet scenes of everyday life. This boy, the son of a jeweler, has stopped his lessons to watch a teetotum spin on the table. Chardin painted the moment just before the top falls and shows its number.

Look closely

- Find the teetotum on the table. Is it still spinning? How can you tell?
- Look at the boy's eyes and hands. He is very still. Why is he watching so closely?
- Find the pen, the inkwell and the books. What was the boy supposed to be doing?

Talk about it

1. Can the boy do anything to make the teetotum stop on the number he wants? Why does that make it a good tool for a fair game?
2. Milton Bradley used a teetotum instead of dice. If one side of a teetotum were wider than the others, what would happen?

PROJECT

Make and Test Your Own Spinners

You will make a fair four-color spinner and an unfair spinner, predict how often each color should come up, test them, and then play odds and evens to see whether patterns can be beaten.

You will need

- Card stock, a compass or round lid, a protractor (or folded paper) and a ruler
- A paper clip and a pencil (the paper clip spins around the pencil point)
- Colored pencils
- Your scorebook and a partner

Steps

1. Trace two circles about 12 centimeters across on card stock.
2. **Spinner 1 (fair):** divide a circle into four equal sections of 90 degrees (fold it in half twice to find the lines). Color them red, blue, yellow and green.
3. **Spinner 2 (unfair):** divide the other circle into a red section of 180 degrees, a blue section of 120 degrees and a green section of 60 degrees.
4. **Predict.** Write the probability of each section and the expected count in 60 spins (fair spinner: 15 each; unfair spinner: 30, 20, 10).
5. To spin, hold a pencil point at the center through one end of a paper clip and flick the clip. Spin each spinner 60 times and tally where the clip points.
6. Write each result as a fraction of 60 and compare with your predictions.
7. **Odds and evens.** Play 20 rounds with a partner. First let your partner choose freely. Then ask your partner to show one finger every time while you try to win. Record both sets of results.
8. Write what you learned: When is a spinner fair? Why must odds-and-evens players mix their choices?

Tips

- Flick the paper clip hard enough to spin at least twice around. A weak spin is not a fair spin.

- If the clip lands on a line, spin again.
- Use a protractor if you have one; otherwise fold paper circles to find halves, quarters and sixths.

Check your work

- ☐ I made one fair and one unfair spinner with the sections measured.
- ☐ I spun each 60 times and compared my counts with the expected counts.
- ☐ I explained why a pattern in odds and evens can be beaten.

SCOREBOOK

1. Draw both spinners with each section labeled with its angle and its probability as a fraction.
2. Make a table for each spinner: Color, Expected in 60 spins, What I spun.
3. Write: “In a game against a smart opponent, I would decide to...” and explain why being unpredictable matters.

From history. Milton Bradley later became a leader of the kindergarten movement in America and printed books and materials for teaching young children. His company went on to make famous family games such as *Battleship* (Lesson 7.2). Teetotums had been used in children’s games in Europe for hundreds of years before Bradley’s game.

WORKED EXAMPLES

Example 1. A spinner has a red section of 180 degrees, a blue section of 120 degrees and a green section of 60 degrees. Find each probability. In 60 spins, how many blues do you expect?

Show solution

1. Red: $\frac{180}{360} = \frac{1}{2}$. Blue: $\frac{120}{360} = \frac{1}{3}$. Green: $\frac{60}{360} = \frac{1}{6}$.
2. Check: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$.
3. Expected blues in 60 spins: $\frac{1}{3} \times 60 = 20$.

Answer: Red $\frac{1}{2}$, blue $\frac{1}{3}$, green $\frac{1}{6}$; about 20 blues in 60 spins.

COMMON MISTAKES

- Thinking every color on a spinner is equally likely just because there is one section of each. Size matters.

- Forgetting to check that the fractions add to 1.
- Thinking odds and evens is fair even when one player always shows the same number of fingers. It is only fair when both players mix.

Make it a game · Spin and Judge. Draw a spinner on a paper plate with unequal sections, secretly picking the sizes. Your parent guesses each section's probability by looking at it. Spin twenty times with a paperclip and pencil, tally the beans, and compare. Whose estimate came closest? Explain the difference between counted and experimental probability, then swap roles.

3.3 WHY CARNIVAL GAMES ARE HARD TO WIN

OBJECTIVES

- Explain how a booth game can look easy but have a small chance of winning.
- Measure your own chance of winning a homemade booth game in a fair version and a hard version.
- Explain why honest rules and honest measures matter.

Warm-up. Set a plastic bottle on the floor and stand about 2 meters away. Toss a jar ring, a bracelet or a ring made from a paper plate at the bottle 10 times. How many times did it land over the neck? Does the game look easier than it is?

WATCH

Carnival Scam Science (and how to WIN).

Mark Rober · 10:52

https://www.youtube.com/watch?v=tk_ZIWJ3qJI

WHEN TO WATCH

After “How they figured it out.”

Note: Parents: the engineer measures real carnival booths, says how much prizes cost the booth, and calls some games “scams.” Mention of money is brief. The video ends with his own backyard booth versions, similar to this lesson’s project.

KEY TERMS

midway

The part of a fair or carnival with rides, food and booth games. The name comes from the Midway Plaisance at the Chicago fair of 1893.

booth game

A throwing or tossing game at a fair where a player tries to win a prize.

win rate

The fraction of tries that win: wins divided by tries.

just weight

An honest measure. In the Bible, merchants were to use honest weights and balances, not ones that cheated the buyer.

CORE IDEAS

The story. In 1893 the city of Chicago held a giant world's fair, the World's Columbian Exposition, to mark four hundred years since Columbus sailed to America. Millions of visitors came. Alongside the grand white buildings ran a long strip of park called the Midway Plaisance, full of shows, food and amusements. Its star was a huge wheel designed by the engineer George Washington Gale Ferris Jr., about 264 feet tall, with 36 cars that could each carry dozens of riders. Ever since, the part of a fair with rides and games has been called the **midway**, and the big wheel is called a Ferris wheel.

How they figured it out. Midway booth games look easy: toss a ring over a bottle, sink a ball in a basket, knock down three bottles. But the people who run the booths need many more players to lose than to win. Over the years they learned many ways to make a game look easier than it is. A ring may be barely wider than the bottle's neck, so it only lands flat over it at a perfect angle. A basketball hoop may be a little smaller than usual, or the ball pumped very hard so it bounces out. Bottles may be heavier at the bottom than they look. The engineer Mark Rober, who once worked at NASA, measured and tested many of these games for his video and showed how they are built.

The mathematics. Your win rate is $\frac{\text{wins}}{\text{tries}}$. If you sink 1 shot in 20, your win rate is $\frac{1}{20} = 0.05 = 5\%$, so on average you need about 20 tries for each win. A game designer can change the win rate by changing the target: a slightly smaller hoop or narrower bottle neck can turn a 1-in-2 game into a 1-in-20 game while looking almost the same. Because the booth runs thousands of tries, its results are very predictable even though each player's result is chance (remember Galton's board). That is why the booth always comes out ahead over a whole summer.

On the field. The same skills you use on a real court do not work the same way on a midway, because the equipment is different. A wise person enjoys the rides and the food, and if he plays a booth game at all, he plays it knowing the chances, for fun, without thinking "next time I'm sure to win." Dice have no memory, and neither does a ring toss. **The risk side:** the danger is not the game itself but believing the game is easier than it is. Scripture praises honest measures. A fair game tells you its real chances; a game that hides them is like a merchant's false balance.

PICTURE STUDY



The Ferris Wheel, C. D. Arnold, 1893 · Photograph · World's Columbian Exposition, Chicago

Image: [Wikimedia Commons](#), public domain

Charles Dudley Arnold was the official photographer of the 1893 Chicago fair. This is one of his views of the first Ferris wheel on the Midway Plaisance. It was the fair's answer to the Eiffel Tower, which had been built for the Paris fair four years before.

Look closely

- Count as many of the wheel's cars as you can see. There were 36 in all.
- Find the buildings below. How does the wheel compare with them in height?
- Look at the thin spokes. Like a bicycle wheel, the Ferris wheel hung from its center on rods in tension.

Talk about it

1. The Ferris wheel was built to amaze people and draw them to the Midway. How do booth games also try to draw people in?
2. The wheel gave every rider the same ride. Booth games do not give every player a prize. What makes one a fair deal and the other a game of chance?

Safety first. Use only soft balls, paper rings and plastic bottles or paper cups. Keep the throwing area clear of windows, lamps and people, and let only one person throw at a time.

PROJECT

Our Family Midway

You will build two homemade booth games, each in a “fair” version and a “carnival” version, and measure how much harder the small changes make them. Keep score in points only.

You will need

- Plastic bottles and paper cups
- Rings: jar rings, bracelets or rings cut from paper plates, in two sizes
- A soft ball and a bucket or laundry basket, plus a smaller bucket or a pot
- Painter’s tape and a tape measure
- Your scorebook

Steps

1. Mark a throwing line with painter’s tape about 2 meters from the targets.
2. **Ring toss.** Fair version: a big ring over a bottle. Carnival version: a ring just a little wider than the bottle neck.
3. **Basket toss.** Fair version: a soft ball into a wide basket. Carnival version: a smaller pot, or the same basket tilted back so balls bounce out.
4. **Predict.** For each game, guess your win rate for both versions before you throw.
5. Throw 20 times at each version of each game. Tally wins and misses.
6. Write each win rate as a fraction, a decimal and a percent. Find tries per win.
7. Ask two family members to play the carnival versions without telling them about the changes. Then tell them, and ask what they think of the games.
8. **Decide.** Write a sign for each booth that tells its real win rate honestly, as an honest booth would.

Tips

- Throw from the same line and in the same way each time, so the only change is the target.
- Warm up your arm with a few easy throws first.

- Keep score in points: one point for each win. No prizes are needed to learn the lesson.

Check your work

- ☐ I built a fair and a carnival version of two booth games.
- ☐ I threw 20 times at each version and found the win rates.
- ☐ I made an honest sign for each booth and explained why honest rules matter.

SCOREBOOK

1. Draw your two booth games, fair and carnival versions side by side, with the measurements that changed.
2. Make a table: Game, Version, Wins, Tries, Win rate, Tries per win.
3. Copy Proverbs 11:1 and write one sentence about why a just weight matters in games.

From history. The first Ferris wheel was taken down after the fair, put up again for the 1904 World's Fair in St. Louis, and finally taken apart in 1906. Today's giant observation wheels, like the London Eye, follow Ferris's idea.

WORKED EXAMPLES

Example 1. On your fair ring toss you win 8 times in 20 tries. On the hard version (a smaller ring) you win 1 time in 20. Find each win rate. About how many tries does each version take per win?

Show solution

1. Fair version: $\frac{8}{20} = 0.40 = 40\%$. Tries per win: $20 \div 8 = 2.5$.

2. Hard version: $\frac{1}{20} = 0.05 = 5\%$. Tries per win: $20 \div 1 = 20$.

Answer: 40% (about $2\frac{1}{2}$ tries per win) against 5% (about 20 tries per win). A small change in the ring made the game eight times harder.

COMMON MISTAKES

- Thinking a booth game is easy because it looks easy. Measure the win rate.
- Believing a win is “due” after several misses. Each try is independent.
- Thinking the booth is lucky. With thousands of tries, its results follow the win rate very closely.

Make it a game • Booth Win Rate. Build a toss-the-button-into-a-cup booth with two difficulty settings: an easy line and a hard line. Take ten tosses at each, and record your win rate as a fraction. Your parent predicts it first. Explain why the hard line looks easy but is hard to win. Awards use paper chips and nothing else.

Scripture. “A false balance is abomination to the LORD: but a just weight is his delight.” (Proverbs 11:1, KJV)

God cares about honest measures. Honest rules, honest scorekeeping and honest chances are part of playing any game well.

3.4 DESIGN A FAIR GAME

OBJECTIVES

- Tell how Christiaan Huygens worked out the value of a chance.
- Use points per round to check whether a game is fair.
- Design, test and improve a fair dice or spinner game of your own.

Warm-up. Invent a quick game with one die for two players, such as “I score on 1 or 2, you score on 3, 4, 5 or 6.” Play 10 rounds. Does one player seem to have an advantage? How could you change the points to even it out?

WATCH

10 Steps to ACTUALLY Design a Board Game

Dave Jeltema · 9:52

https://www.youtube.com/watch?v=ZUuudT9Vn_w

WHEN TO WATCH

Before the project, as you begin designing your own game. Younger students can stop after the first few steps.

Note: Parents: a game designer’s advice aimed at teens and adults; it mentions online groups and crowdfunding near the end, which you may skip.

KEY TERMS

value of a chance

What a chance is worth on average: the points for a win times the probability of winning. Huygens called it the expectation.

expected points

The average number of points a player earns per round over many rounds.

rule book

The written rules of a game, clear enough that someone who has never played can learn it.

playtest

Playing a new game many times to find and fix problems in its rules.

CORE IDEAS

The story. In 1655 a young Dutch scientist named Christiaan Huygens visited Paris and heard that Pascal and Fermat had solved puzzles about games of chance. He did not meet them and could not learn their methods, so he went home to The Hague and worked the problems out for himself. In 1657 he published a short book, *On Reasoning in Games of Chance*. It was the first printed book about the mathematics of chance, and for fifty years it was the book every student of probability learned from. Huygens later became famous for inventing the pendulum clock and for discovering Saturn's largest moon, Titan.

How they figured it out. Huygens asked a new question: what is a chance *worth*? If you have one chance in three to win 6 points, your chance is worth 2 points, because over many rounds you would average 2 points a round. He called this the “expectation,” and today we call it expected value. With it, Huygens could check any game for fairness at once: a game is fair when every player's expectation is the same. He did not have to play it a thousand times; he could count.

The mathematics. Expected points per round

= points for a win $\times$ probability of winning. Suppose in your game A scores 1 point on a die roll of 3, 4, 5 or 6 (probability $\frac{4}{6}$), and B scores on 1 or 2 (probability $\frac{2}{6}$). A's expected points are $\frac{4}{6} \times 1 = \frac{4}{6}$. If B gets 2 points per win, B's are $\frac{2}{6} \times 2 = \frac{4}{6}$. Equal expectations: the game is fair. So the rule for fixing a game is simple: **the rarer win is worth more points**, in just the right amount.

On the field. Sports use this idea all the time. In basketball a long shot is worth 3 points because it goes in less often than a short one. In football a touchdown is worth more than a field goal because it is harder to get. Game designers playtest their games again and again, changing points and rules until good choices are rewarded and no player has an unfair edge. **The risk side:** a game with fair chances can still be spoiled by unclear rules. Write your rules so clearly that no one can argue, and keep them the same for everyone.

PICTURE STUDY



Portrait of Christiaan Huygens, Attributed to Jean Charles François, 1760s · Crayon-manner engraving (book illustration) · Rijksmuseum, Amsterdam

Image: [Wikimedia Commons](#), public domain

This engraving was made in Paris in the 1760s for a book of portraits of famous thinkers, about a century after Huygens lived. Huygens (1629–1695) was one of the greatest scientists of his age, and his little book on chance was the first ever printed on the subject.

Look closely

- Find Huygens's name printed at the bottom of the portrait.
- Look at his long curled wig. Fashionable men of the 1600s wore wigs like this.
- Look at his face. Does he seem in a hurry, or patient and thoughtful?

Talk about it

1. Huygens worked out the problems himself instead of waiting to learn the answers from Pascal and Fermat. Why is it valuable to work something out yourself?
2. Huygens could tell whether a game was fair without playing it. How?

PROJECT

My Fair Game

You will design a new game for two to four players that uses dice or a spinner, check its fairness with expected points, playtest it with your family, and write a clear rule book. This is the start of your unit portfolio.

You will need

- Dice, coins or your spinners from Lesson 3.2
- Paper, card stock, markers and counters for a board if your game needs one
- Your scorebook and calculator

Steps

1. **Idea.** Decide what players do on a turn and how they score points. Keep it simple: one or two choices per turn.
2. **Count.** List the outcomes that score for each player and find each player's probability of scoring.
3. **Balance.** Find each player's expected points per round. If they are not equal, change the points until they are.
4. **Add one choice.** Give players at least one real decision, such as "roll one die or two" or "stop or roll again." A game with choices is more fun than one that is all chance.
5. **Playtest.** Play at least 30 rounds with family members. Tally each player's points.
6. Ask each player: Was it fun? Was it fair? Was anything confusing?
7. Change one rule to fix any problem, and playtest 15 more rounds.
8. **Rule book.** Write the final rules: name, players, materials, how to play a turn, how to score, how to win.

Tips

- Test your rule book by giving it to someone who has never played and watching whether they can learn the game without your help.
- Name your game. Good names make people want to play.

- If one player is younger, let the game reward choices that a younger player can understand.

Check your work

- ☐ I calculated each player's expected points and made them equal.
- ☐ My game includes at least one real choice.
- ☐ I playtested at least 30 rounds, changed one rule, and wrote a clear rule book.

SCOREBOOK

1. Write your game's outcome list and expected-points calculation for each player.
2. Make a playtest table: Round, Player, Points. Add up the totals.
3. Write: "I changed the rule... because..." and explain how the change made the game better.

From history. Huygens first wrote his book in Dutch; his teacher, Frans van Schooten, translated it into Latin and printed it in 1657 at the end of his own mathematics textbook. Jakob Bernoulli later reprinted the whole book with notes of his own.

WORKED EXAMPLES

Example 1. Two dice are rolled. A scores when the total is 6, 7 or 8. B scores when the total is 2, 3, 4, 10, 11 or 12. Nobody scores on 5 or 9. Choose points that make the game fair.

Show solution

1. A's outcomes: $5 + 6 + 5 = 16$ of 36. B's outcomes: $1 + 2 + 3 + 3 + 2 + 1 = 12$ of 36.
2. Try $A = 3$ points and $B = 4$ points per win.
3. A: $\frac{16}{36} \times 3 = \frac{48}{36}$. B: $\frac{12}{36} \times 4 = \frac{48}{36}$.

Answer: Give A 3 points and B 4 points per win. Both expect $\frac{48}{36}$, about 1.33 points per round.

COMMON MISTAKES

- Giving the player who wins more often more points as well. The rarer win should be worth more.
- Writing rules that only make sense to the designer. Test them on someone new.
- Making a game that is fair but has no choices at all. Fair and fun go together.

Make it a game • Value of a Chance. Play a coin game: heads wins three chips, tails loses one. Before ten flips, work out whether it is fair, using points per round. Check afterward with your chip count. Tell your parent how Huygens thought about the value of a chance. Then adjust one rule so the game becomes fair, and test it again.

Scripture. “And if a man also strive for masteries, yet is he not crowned, except he strive lawfully.” (2 Timothy 2:5, KJV)

Paul used the games of his day to teach that victory only counts when it is won by the rules. Good rules, kept by everyone, make a game worth winning.

UNIT 3

FAIR AND UNFAIR GAMES

Every good game needs fair rules. But what does “fair” really mean? Sometimes it means that every player has the same chance. Sometimes it means a clever rule, like “I cut, you choose,” that leaves everyone satisfied. And sometimes a game that looks fair is not fair at all. In this unit you will test simple dice and spinner games, find out which ones secretly favor one player, and learn how to fix them.

You will meet Abram and Lot dividing the land, Milton Bradley and the spinning top he used in his first board game, the great Chicago fair of 1893 where the word “midway” was born, and Christiaan Huygens, who wrote the first printed book about the mathematics of chance. You will also learn why carnival booth games are so hard to win. At the end you will design, test and polish a fair game of your own.

PREREQUISITES

- Can find the probability of a simple event by counting outcomes (Units 1–2)
- Can draw the 6-by-6 grid of two-dice outcomes (Unit 2)
- Can find an expected count by multiplying a probability by a number of tries

MASTERY CHECKLIST

- ☐ I can explain what makes a game fair and give an example of “I cut, you choose.”
- ☐ I can test a dice or spinner game for fairness by counting outcomes and by playing it many times.
- ☐ I can make a spinner with sections of the size I want and find the probability of each section.
- ☐ I can explain why carnival booth games are hard to win.
- ☐ I can fix an unfair game by changing the points so that each player’s points per round are equal.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Khan Academy: Probability (7th grade)**: Free. Short videos and practice on probability, experiments and simulations that match Lessons 3.1 and 3.2. Best for ages 11 and up; younger students can work through it with a parent.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

3.1 What Makes a Game Fair?	2 sessions of 45 minutes
3.2 Spinners and Odd-or-Even	2 sessions of 45 minutes
3.3 Why Carnival Games Are Hard to Win	2 sessions of 45 minutes
3.4 Design a Fair Game	2 sessions of 45 minutes

UNIT 3 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. **4 pts**
Describe two different ways a game or a choice can be fair. Give an example of each.
2. **5 pts**
Two dice are rolled. A scores if the product is even; B scores if the product is odd. Is it fair? Show your counting.
3. **5 pts**
Fix the game in question 2 by changing B's points. How many points should B get per win if A gets 1?
4. **4 pts**
A spinner has a section of 90 degrees. What is its probability? In 40 spins, how many times do you expect it?

5. (4 pts)

Why did Milton Bradley's *Checkered Game of Life* use a teetotum?

6. (4 pts)

What would happen in odds and evens if one player always held out one finger? Explain.

7. (5 pts)

You make 3 of 30 tosses at a carnival-style ring toss. What is your win rate as a fraction, a decimal and a percent? How many tries per win is that?

8. (4 pts)

Name two ways a booth game can be made harder than it looks.

9. (4 pts)

What question did Christiaan Huygens ask about a chance, and what did he call the answer?

10. (5 pts)

A scores 1 point on a die roll of 1, 2, 3, 4 or 5. B scores on a 6. How many points should B get per win to make the game fair?

11. (6 pts)

Short answer: Explain in three or four sentences what Proverbs 11:1 and 2 Timothy 2:5 teach about games and rules.

ANSWER KEY

1. Every player has the same chance (a coin toss, or odd-or-even totals with two dice); or a rule like “I cut, you choose” that leaves both satisfied, as when Abram let Lot choose first. Other good examples accepted.
2. Not fair. The product is odd only when both dice are odd: $3 \times 3 = 9$ outcomes. A wins 27 of 36 (three quarters) and B only 9 of 36 (one quarter).
3. 3 points. A: $\frac{3}{4} \times 1 = \frac{3}{4}$; B: $\frac{1}{4} \times 3 = \frac{3}{4}$.
4. $\frac{90}{360} = \frac{1}{4}$; about 10 times in 40 spins.
5. Many families of his day would not keep dice in the house, so he used a six-sided spinning top numbered 1 to 6, which gives each number the same one-in-six chance as a die.
6. An opponent who noticed could choose so that he always won (for example, if he wins on odd totals, always show two). The game is only fair when both players mix their choices.
7. $\frac{3}{30} = \frac{1}{10} = 0.1 = 10\%$; about 10 tries per win.
8. Any two: a ring barely wider than the bottle neck; a smaller or oval hoop; a very hard, bouncy ball; bottles weighted at the bottom; a tilted target. Other sensible answers accepted.
9. He asked what a chance is worth. He called it the expectation: the prize times the probability, which is the average per round over many rounds.
10. A expects $\frac{5}{6} \times 1 = \frac{5}{6}$. B needs $\frac{1}{6} \times 5 = \frac{5}{6}$, so 5 points per win.
11. God delights in honest measures and hates false ones, so games should have honest chances and honest scorekeeping. Paul says a victory only counts if the player strives lawfully, by the rules. A fair game tells its real chances, and good players keep the rules even when no one is watching.

PORTFOLIO PIECE: MY FAIR GAME

Finish the game you designed in Lesson 3.4 and present it as a complete, playable game. Include: a neat board or playing pieces if the game needs them; a typed or neatly written rule book; a “fairness page” that shows the outcomes each player can score on, each player’s probability, and the expected points per round for each player; and your playtest tables from at least 45 rounds, with a short note on the rule you changed and why. Teach the game to someone outside your family, or to a grandparent, and write two sentences about how it went.

LOOK FOR	WHAT IT MEANS
Fairness	The fairness page counts the outcomes correctly and shows equal expected points per round for every player.
Rule book	Rules are complete and clear enough for a new player to learn the game alone.
Playtesting	At least 45 rounds are recorded, with one rule improved for a stated reason.
Choices and fun	The game gives players at least one real decision and was enjoyed by the players who tested it.

POINTS PER TRY

Which is the better shot: an easy one worth 2 points, or a harder one worth 3? Many people say the easy one, because it goes in more often. But a good coach asks a different question: *how many points does each shot earn per try, on average?* A shot that misses more often can still be the better shot if it is worth enough more. That simple idea, points per try, has changed basketball, football and many other games.

In this unit you will measure your own points per try at a home basket from two distances. You will learn the story of the three-point line, of Rick Barry and his underhand free throws, and of the football coach's choice between kicking the extra point and going for two. Along the way you will always ask two questions: what is the percentage, and what is the risk?

PREREQUISITES

- Can find a win rate as a fraction, a decimal and a percent (Unit 3)
- Can multiply a whole number by a decimal, with a calculator if needed
- Knows the idea of expected points per round (Lesson 3.4)

MASTERY CHECKLIST

- ☐ I can find points per try by multiplying the points for a success by the make rate.
- ☐ I can measure my own points per try from two distances and decide which shot is better for me.
- ☐ I can explain why Rick Barry's underhand free throw worked, and why few players copy it.
- ☐ I can compare the extra-point kick with the two-point try and explain why the score can change the choice.
- ☐ I can explain why the best shot is the one worth the most points per try, not always the one that goes in most often.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Basketball-Reference: NBA league averages:** Free data. Season-by-season averages of three-point tries and make rates since the line began, for Lessons 4.1 and 4.4. Contains advertising; a parent should look at it with the student.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

4.1	Twos and Threes at a Home Basket	2 sessions of 45 minutes
4.2	Free Throws and Rick Barry's Underhand Shot	2 sessions of 45 minutes
4.3	The Kick or Going for Two	2 sessions of 45 minutes
4.4	The Best Shot: Most Points per Try	2 sessions of 45 minutes

UNIT 4 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. **4 pts**
Write the formula for points per try and explain it in your own words.
2. **5 pts**
A shooter makes 45% of twos and 35% of threes. Find the points per try for each. Which is the better shot?
3. **4 pts**
What three-point make rate breaks even with a 50% two-point shot?
4. **5 pts**
Rick Barry made 90% of free throws; Wilt Chamberlain made about 51% in his career. Find the points per trip of two free throws for each. How many more points would Barry score in 50 trips?

5. (4 pts)

Why did Wilt Chamberlain stop shooting underhand, even though it worked better for him? What lesson does that teach?

6. (4 pts)

Why is alternating rounds of two shooting styles a fairer test than shooting all of one style first?

7. (5 pts)

A kicker makes 94% of extra points. A team succeeds on 45% of two-point tries. Which choice earns more on average? What is the break-even rate?

8. (5 pts)

A team trails by 8 points. On the last play of the game it scores a touchdown (6 points). What would happen if it kicked the extra point? Which choice should it make, and why?

9. (4 pts)

Stephen Curry made 45.4% of his threes in 2015–16. Find his points per try. What two-point make rate would match it?

10. (4 pts)

Rank by points per try: at the basket 60% (2 points), middle 40% (2 points), three 34% (3 points).

11. (6 pts)

Short answer: In three or four sentences, explain what “the high-percentage shot” means in this course, and name one risk that goes with it.

ANSWER KEY

1. Points per try = points for a success $\times$ make rate. It is the average number of points each try earns over many tries.
2. Two: $2 \times 0.45 = 0.90$. Three: $3 \times 0.35 = 1.05$. The three is better.
3. One third, about 33%, because $3 \times \frac{1}{3} = 1 = 2 \times 0.5$.
4. Barry $2 \times 0.90 = 1.80$; Chamberlain $2 \times 0.51 = 1.02$. Difference 0.78 per trip, so about 39 points in 50 trips.
5. He said it made him feel foolish. A wise player chooses what works, even if it looks unusual.
6. Tiredness and warming up then affect both styles equally, so the only difference is the style.
7. Kick 0.94; two-point try $2 \times 0.45 = 0.90$. Kicking earns slightly more. The break-even is 47%.
8. After the touchdown it trails by 2. A kick would leave it 1 point behind, and it would lose. It must go for two: a two-point try ties the game about half the time, while the kick can never tie it. The goal (tying the game) decides the choice, not average points.
9. $3 \times 0.454 \approx 1.36$ points per try. A two would need $1.36 \div 2 = 0.68$, or 68%.
10. Basket 1.20, three 1.02, middle 0.80.
11. The high-percentage shot is the one that earns the most points per try on average, not simply the one that goes in most often. It is found by multiplying the points by the make rate. One risk: even the best shot misses often, so streaks of misses are normal; another is that a make rate from a small sample may be wrong, or that a defense will take the shot away. Good answers name the percentage and the risk.

PORTFOLIO PIECE: MY SHOT PLAN

Prepare a two-page “Shot Plan” for yourself as a player. Page 1 is your shot chart from Lesson 4.4, redrawn neatly with at least four spots, their distances, make rates and points per try, colored by points per try. Page 2 has three short sections: your free-throw test from Lesson 4.2 with your chosen style and why; your kick-or-two decision chart from Lesson 4.3; and a paragraph explaining the high-percentage shot to a younger player, naming both the percentage and the risk. Then shoot 20 more shots from your best spot and 20 from your worst spot and report whether the ranking held up.

LOOK FOR	WHAT IT MEANS
Shot chart	Neat, labeled chart with distances, make rates and correct points per try for at least four spots.
Fair tests	Free-throw styles and kick-or-two tested with enough tries and recorded honestly.
Decision making	The plan names the best shots, the decision chart has sensible reasons, and the risk side is stated.
Follow-up	The extra 40 shots are recorded and compared with the original ranking.

4.1 TWOS AND THREES AT A HOME BASKET

OBJECTIVES

- Find points per try for a two-point shot and a three-point shot.
- Measure your own make rate from two distances.
- Decide which shot is better for you by comparing points per try.

Warm-up. Stand close to a laundry basket or bucket and take 10 shots with a soft ball. Then step back to a far line and take 10 more. Before you shoot, decide that close makes are worth 2 points and far makes are worth 3. Which distance earned more points? Was it the one where you made more shots?

WATCH

The Basketball Scoring System | Basketball

SIKANA English • 1:42

<https://www.youtube.com/watch?v=dV-jZKwQHjA>

WHEN TO WATCH

Before the core ideas, as a quick reminder of how basketball scoring works.

KEY TERMS

make rate

The fraction of tries that go in, also called the shooting percentage.

points per try

The points a shot is worth times its make rate: the average points each try earns.

three-point line

An arc on a basketball court; a shot made from behind it counts 3 points instead of 2.

break-even

The make rate at which two choices earn the same points per try.

CORE IDEAS

The story. When James Naismith invented basketball in 1891, every basket counted the same, so the best shot was simply the one most likely to go in. Many years later, professional leagues added a line far from the basket and made shots from behind it worth 3 points. On October 12, 1979, in the first game of the season, Chris Ford of the Boston Celtics made the first three-point basket in the history of the National

Basketball Association. At first, coaches treated the three as a trick shot. That season NBA teams tried fewer than three of them a game and made only 28 percent.

How they figured it out. For years many coaches judged a shot by how often it went in. A 45% two-point shot looked better than a 35% three. Then people who studied the numbers asked a new question: how many points does each try earn on average? Over 100 tries, the 45% two-pointer scores $45 \times 2 = 90$ points, while the 35% three-pointer scores $35 \times 3 = 105$ points. The three is the better shot, even though it misses more often! As teams learned this, they took more and more threes. By the 2024–25 season, NBA teams were taking about 37 threes a game and making about 36 percent.

The mathematics. $\text{points per try} = \text{points} \times \text{make rate}$. A 2-point shot made half the time earns $2 \times 0.5 = 1$ point per try. A 3-point shot made one third of the time also earns $3 \times \frac{1}{3} = 1$. So a three that goes in one time in three **breaks even** with a two that goes in half the time. If you make threes more often than that, the three is the better shot; if less often, the two is better.

On the field. Coaches now design plays to end in the shots that earn the most points per try: shots right at the basket, free throws, and open threes. A young player can do the same thing with her own numbers. **The percentage:** the shot with the most points per try. **The risk side:** a make rate measured from only 10 shots can fool you, so measure from many tries. And when your team needs just one basket to tie, a sure 2 may matter more than a risky 3. Know your own numbers, not just the league's.

PICTURE STUDY



James Naismith with a Ball and a Peach Basket, Unknown photographer, Before 1939 · Photograph · United States

Image: [Wikimedia Commons](#), public domain

James Naismith was a Canadian-born teacher and a Presbyterian minister. In December 1891, at the YMCA training school in Springfield, Massachusetts, he invented basketball so his students would have a lively game to play indoors in winter. The first goals were two peach baskets nailed to the balcony of the gym. Later he taught for many years at the University of Kansas.

Look closely

- Find the ball and the basket. How is this basket different from a modern hoop and net?
- Look at the bottom of the basket. What would players have needed to do after every goal?
- Look at Naismith's face and clothes. He was a teacher for his whole life. Does he look proud of his game?

Talk about it

1. In Naismith's first game every goal counted the same. In that game, what would the best shot be?
2. Adding the three-point line changed which shots were best. If you could invent a new scoring rule for basketball, what would it be, and how would

players change?

Safety first. Warm up your arms before shooting. Use only a soft ball indoors, keep clear of windows, lamps and people, and wear shoes with a good grip.

PROJECT

My Points per Try

League numbers describe other players. Your best shot depends on your own make rates. You will measure them from two distances and find your break-even.

You will need

- A soft ball and a basket, bucket or laundry hamper
- Painter's tape and a tape measure
- Your scorebook and a calculator

Steps

1. Mark a close line about 1.5 meters from the basket and a far line about 3.5 meters away. (Adjust so you make some, but not all, from each line.)
2. **Predict.** Guess your make rate from each line, and which line will earn more points per try.
3. Take 30 shots from the close line, in three rounds of 10. Tally makes and misses.
4. Take 30 shots from the far line, in three rounds of 10. Tally makes and misses.
5. Find your make rate from each line as a fraction, a decimal and a percent.
6. Find your points per try: close shots are worth 2 points, far shots 3 points.
7. Find your break-even: what far make rate would equal your close points per try? (Divide your close points per try by 3.)
8. **Decide.** Which is your better shot? What would have to change for the other shot to become better?
9. If a family member wants to try, measure his or her points per try too, and compare.

Tips

- Shoot the same way each time: same ball, same stance, same kind of throw.
- Rest between rounds so you are not tired for the far shots only.

- Thirty shots is a small sample. If your two numbers are very close, shoot another 30 from each line before deciding.

Check your work

- ☐ I recorded 30 shots from each line and found both make rates.
- ☐ I found my points per try for each shot and my break-even.
- ☐ I wrote which shot is better for me and why.

SCOREBOOK

1. Make a table: Line, Makes, Tries, Make rate, Points per make, Points per try.
2. Draw a bird's-eye view of your shooting area with the two lines and the basket, marked with your make rates.
3. Write: "In a game, I would decide to shoot from... because..." and finish the sentence.

From history. The American Basketball League tried a three-point line in 1961, and the American Basketball Association used one from 1967. The NBA added it in 1979, and American college basketball followed in 1986.

WORKED EXAMPLES

Example 1. Close to the basket you make 12 of 20 shots worth 2 points. From the far line you make 7 of 20 shots worth 3 points. Which shot earns more points per try?

Show solution

1. Close make rate: $\frac{12}{20} = 0.60$. Points per try: $2 \times 0.60 = 1.20$.
2. Far make rate: $\frac{7}{20} = 0.35$. Points per try: $3 \times 0.35 = 1.05$.

Answer: The close shot earns more: 1.20 points per try against 1.05. For this shooter, the two is the better shot right now.

Example 2. A player makes 50% of her twos. What make rate from three would earn the same points per try?

Show solution

1. Two-point points per try: $2 \times 0.50 = 1.00$.

2. We need $3 \times \text{rate} = 1.00$, so $\text{rate} = \frac{1}{3} \approx 33\%$.

Answer: About 33%. If she makes more than one three in three, the three is her better shot.

COMMON MISTAKES

- Choosing the shot that goes in most often instead of the one that earns the most points per try.
- Forgetting to multiply by the points. A make rate alone does not tell the whole story.
- Trusting a make rate from just a few shots.

Make it a game • Two or Three Duel. Toss a soft ball into a bucket from a near line worth two points and a far line worth three. Take ten shots from each. Before shooting, predict which line pays more points per try. Compute it out loud. Your parent keeps score with beans. The line with more points per try wins, even if you missed more.

4.2 FREE THROWS AND RICK BARRY'S UNDERHAND SHOT

OBJECTIVES

- Find the points earned by a trip to the free-throw line.
- Tell the story of Rick Barry's underhand free throw and compare his numbers with Wilt Chamberlain's.
- Test two shooting styles fairly and decide which works better for you.

Warm-up. Stand at a line about 2 meters from a basket. Shoot 10 soft-ball shots the usual way, with one hand. Then shoot 10 underhand, holding the ball with both hands between your knees and lifting it softly. Which way did you make more? Which way felt silly?

WATCH

Rick Barry Underhanded Free Throw Technique

Hoops Movement • 3:36

https://www.youtube.com/watch?v=UmAB_iAO_iM

WHEN TO WATCH

After “How they figured it out.” Try the motion slowly with a soft ball before the project.

BACKUP VIDEO

Rick Barry Talks Underhand Free-Throw Shooting

ThePostGame • 2:07

<https://www.youtube.com/watch?v=m2MJLfEeh-A>

Rick Barry explains in his own words why the underhand shot works.

KEY TERMS

free throw

An unguarded shot from a set line, given after a foul. Each make is worth 1 point.

underhand free throw

A free throw shot with both hands from below the waist, lifting the ball softly toward the basket.

trip to the line

A set of free throws, usually two, given after a foul on a shot.

fair test

An experiment in which only one thing is changed, so you can tell what made the difference.

CORE IDEAS

The story. Rick Barry was one of the great scorers in basketball history, and he shot his free throws underhand, the way his father had taught him: ball held low with both hands, a smooth, soft lift. Fans sometimes laughed, and people called it a “granny shot.” But Barry made 90 percent of his free throws in the NBA, 3,818 of 4,243, one of the best records of all time, and in the 1978–79 season he made 94.7 percent. Wilt Chamberlain, the strongest player of his time, was a poor free-throw shooter. In the 1961–62 season he tried shooting underhand and made 61.3 percent, his best ever. On March 2, 1962, he scored 100 points in one game and made 28 of 32 free throws. Then he gave up the underhand shot, because he said it made him feel foolish. His free-throw percentage fell back to about 51 percent for his career.

How they figured it out. Barry explained why his shot worked. Holding the ball with two hands keeps it straight. The arms swing like a pendulum, which is easy to repeat the same way every time. And the soft, high arc gives the ball a better chance to drop in if it is a little off. Coaches and scientists who studied it agreed that it is a sound way to shoot. Yet very few players copy it, even today. Barry’s son Canyon also shot underhand in college and did very well. The numbers were clear; what stopped other players was how the shot looked.

The mathematics. A free throw is worth 1 point, so its points per try equal its make rate. A trip to the line for two shots earns $2 \times$ make rate points on average. For Barry: $2 \times 0.90 = 1.80$ points per trip. For Chamberlain’s career: $2 \times 0.51 = 1.02$. Over 100 trips that is a difference of about 78 points. If Chamberlain had kept his underhand rate of 61%, he would have earned $2 \times 0.61 = 1.22$ per trip, about 20 more points every 100 trips than with his usual shot.

On the field. Free throws are the highest-percentage shot in basketball, so a few extra points on them can decide close games. The wise player chooses the method that works best *for him* and practices it, even if it looks unusual. **The risk side:** any new method needs a fair test with many tries before you trust it, and changing your

shot in the middle of a season can hurt at first. Courage to do what works, and patience to test it fairly, belong together.

PICTURE STUDY



An 1896 University of Washington Basketball, Unknown maker, 1896 · Leather ball with laces, inscribed with game scores · Museum of History & Industry (MOHAI), Seattle

Image: [Wikimedia Commons](#), public domain

This leather ball was used by a University of Washington team in 1896, only five years after Naismith invented the game. Someone wrote the team's game scores on it, including "U of W vs. Y.M.C.A." Balls like this had a rubber bladder inside and were laced shut, so they were not perfectly round.

Look closely

- Find the laces at the top. How might the laces change the way the ball bounces or spins?
- Look for the scores written on the ball. Do they look high or low compared with games today?
- Look at the shape. Is it a perfect sphere?

Talk about it

1. With a lumpy, laced ball like this, would a soft, high shot or a hard, flat shot be more likely to go in? Why?
2. Early players often shot free throws underhand with two hands. Why do you think that style was common then?

Safety first. Warm up first. Use a soft ball indoors, clear the area around the basket, and do not shoot toward windows or lamps.

PROJECT

A Fair Test of Two Free-Throw Styles

Was Rick Barry right? You will run a fair test of the one-hand shot and the two-hand underhand shot and decide which earns you more points per trip.

You will need

- A soft ball and a basket or bucket
- Painter's tape to mark a free-throw line about 2 to 3 meters from the basket
- Your scorebook and a calculator

Steps

1. Mark your free-throw line and do not change it during the test.
2. **Predict.** Which style will you make more often? Write your guess.
3. Shoot in rounds of 10, switching styles each round: overhand, underhand, overhand, underhand, until you have 40 shots of each style.
4. Tally makes for each style in each round.
5. Find your make rate for each style and your points per trip of two free throws.
6. Find the difference in points per 100 trips between the two styles.
7. **Decide.** Which style is better for you? Did the result surprise you? Would you be willing to use the better style in front of others?
8. Write a short paragraph comparing your results with Barry's and Chamberlain's.

Tips

- Switching styles every round makes the test fair: tiredness and warming up affect both styles equally.
- For the underhand shot, bend your knees, keep your elbows straight and lift with a smooth swing.
- If the difference is small, shoot 40 more of each before you decide.

Check your work

- ☐ I shot 40 of each style in alternating rounds.
- ☐ I found make rates and points per trip for each style.
- ☐ I made a decision and explained it, and compared my results with Barry's.

SCOREBOOK

1. Make a table: Round, Style, Makes out of 10. Add totals and make rates at the bottom.
2. Draw a bar graph comparing your two styles.
3. Write: "I would decide to use the ... shot, even if..." and finish the sentence honestly.

From history. Rick Barry was elected to the Naismith Memorial Basketball Hall of Fame in 1987. Wilt Chamberlain's 100-point game, played in Hershey, Pennsylvania, against the New York Knicks, is still the NBA record for points in one game.

WORKED EXAMPLES

Example 1. A player makes 70% of free throws. How many points does she earn per trip of two free throws? Over 20 trips?

Show solution

1. Per trip: $2 \times 0.70 = 1.40$ points.
2. Over 20 trips: $20 \times 1.40 = 28$ points.

Answer: 1.40 points per trip, about 28 points in 20 trips.

COMMON MISTAKES

- Testing one style when fresh and the other when tired. Alternate rounds.
- Deciding after only 10 shots of each style.
- Choosing a method because it looks good instead of because it works.

Make it a game • Overhand or Underhand. Take twenty free-throw style tosses at a bucket, half overhand and half underhand. Predict which style will make more before you start. Tally the makes with beans. Then tell your parent the Rick Barry story and compare his numbers with Wilt Chamberlain's. Explain what makes a fair test, and try it again with another thirty shots.

4.3 THE KICK OR GOING FOR TWO

OBJECTIVES

- Compare the points per try of the extra-point kick and the two-point try.
- Find the break-even success rate for going for two.
- Explain why the score and time left can change the right choice.

Warm-up. Set a paper cup on its side as a small goal and roll or kick a soft ball at it 10 times from 1 meter (worth 1 point each) and 10 times from 3 meters (worth 2 points each). Which earned more points?

WATCH

How the two-point conversion made its way to the NFL | 1st

Secret Base • 3:09

<https://www.youtube.com/watch?v=lcXrNUiG1WE>

WHEN TO WATCH

After “The story.”

Note: Parents: a sports-history explainer from SB Nation; previewed for language.

KEY TERMS

touchdown

In American football, carrying or catching the ball in the opponent’s end zone. It is worth 6 points.

extra point

A kick through the goalposts after a touchdown, worth 1 point.

two-point try

Instead of kicking, a team runs or passes the ball into the end zone from close range after a touchdown, for 2 points.

conversion

The try for extra points after a touchdown, by kick or by play.

CORE IDEAS

The story. After a touchdown in American football, a team gets a bonus try. For many years in the National Football League, it simply kicked the ball through the goalposts for 1 point, and kickers almost never missed. College football began

allowing a team to run or pass for 2 points instead in 1958, and the NFL added the two-point try in 1994. Then in 2015 the NFL moved the extra-point kick back, so it became a kick of about 33 yards instead of about 20. Kickers started missing more often, and coaches had a real choice to make.

How they figured it out. Coaches and people who studied the game counted. After the change, NFL kickers made about 94 of every 100 extra points. Two-point tries succeeded about half the time. So which earns more? A kick made 94% of the time is worth $1 \times 0.94 = 0.94$ points per try. A two-point try made 50% of the time is worth $2 \times 0.50 = 1.00$ point per try. The two choices are almost equal on average! That means the right choice depends on the situation: the score, the time left, the weather and how good your team is near the goal line.

The mathematics. To find the break-even, ask what two-point success rate would equal the kick: $2 \times \text{rate} = 0.94$, so $\text{rate} = 0.47$, or 47%. If your team succeeds on two-point tries more than 47% of the time, going for two earns more points on average. If less, kick. Notice how close this is. A small change, like a strong wind that makes kicking harder, or a great short-yardage play, can tip the balance.

On the field. Coaches now carry a chart that tells them when to go for two. Some situations are clear: if you are behind by 8 points late in the game after scoring a touchdown to make it 2, you go for two to tie. **The percentage:** nearly equal on average. **The risk side:** the kick is a steady, low-risk choice that almost always gives 1 point; going for two is a bigger risk, giving 2 points half the time and 0 the other half. When a team needs exactly 2 points to tie or win, the bigger risk is the right one. When a team just wants to add a little to a lead, the steady kick is often wiser.

PICTURE STUDY



Otis Guernsey Kicking at Yale, Bain News Service, 1915 · Glass-plate photograph · Yale Bowl, New Haven, Connecticut; Library of Congress

Image: [Wikimedia Commons](#), public domain

Otis Guernsey was a kicking fullback for Yale University. The Bain News Service, a New York photo agency, took this picture of him practicing in 1915 in the Yale Bowl, a huge new stadium that had opened the year before. In those days, kicking was an even bigger part of football than it is today.

Look closely

- Find the ball. Where is it in the picture? Did the kick just happen, or is it about to?
- Look at Guernsey's leg and arms. How does he keep his balance?
- Find the goalposts behind him. How are they different from goalposts today?

Talk about it

1. A kicker practices the same kick hundreds of times. How does practice change a team's choice between kicking and going for two?
2. If kicking is very reliable, why would a coach ever go for two?

Safety first. Kick only a soft ball, outdoors or in a clear room, and never toward people, windows or pets.

PROJECT

Kick or Go for Two: A Decision Chart

You will measure your own “kick” and “two-point play” with a soft ball, find the break-even, and make a decision chart for different game situations.

You will need

- A soft ball
- Two targets: a wide goal (two chairs 1 meter apart) and a narrow target (a laundry basket on its side)
- Painter’s tape and a tape measure
- Your scorebook

Steps

1. **Kick (1 point).** From about 3 meters, kick or roll the soft ball between the two chairs. Take 20 tries and tally the makes.
2. **Two-point play (2 points).** From about 3 meters, kick or roll the ball into the basket on its side. Take 20 tries and tally the makes.
3. Find your make rate and points per try for each.
4. Find the break-even rate for your two-point play: divide your kick points per try by 2.
5. **Decide.** Which earns more on average for you?
6. Make a decision chart with three situations: (a) ahead by 7 late in the game, (b) behind by 2 after a touchdown with no time left, (c) tied early in the game. For each, write “kick” or “go for two” and a reason.
7. Play a short game with a family member: each “touchdown” is a coin flip landing heads, then you choose kick or two. Use your chart and keep score in points.

Tips

- Adjust the distances so the kick goes in most of the time and the two-point play about half the time, like the real game.
- For situation (b), remember the goal is to tie the game, not to earn the most points on average.

Check your work

- ☐ I measured both choices with 20 tries each and found points per try.
- ☐ I found my break-even rate.
- ☐ My decision chart gives a choice and a reason for three different situations.

SCOREBOOK

1. Make a table: Choice, Points, Makes, Tries, Make rate, Points per try.
2. Draw your decision chart neatly as a table with three rows.
3. Write: “When my team needs exactly two points, I would decide to... because...”

From history. Before 1912, a touchdown in college football was worth only 5 points, and a field goal was once worth more than a touchdown. The point values of football have changed many times, and each change made coaches redo their arithmetic.

WORKED EXAMPLES

Example 1. A team’s kicker makes 90% of extra points in the wind. The team makes 50% of two-point tries. Which earns more per try?

Show solution

1. Kick: $1 \times 0.90 = 0.90$.
2. Two-point try: $2 \times 0.50 = 1.00$.

Answer: Going for two earns more on average, 1.00 against 0.90 points per try.

Example 2. Over 10 touchdowns, how many points would a team expect from always kicking (94%)? From always going for two (48%)?

Show solution

1. Kicking: $10 \times 0.94 = 9.4$ points.
2. Going for two: $10 \times 2 \times 0.48 = 9.6$ points.

Answer: About 9.4 against 9.6 points: almost the same. The situation should decide.

COMMON MISTAKES

- Thinking the safer choice is always better. When you need exactly 2 points, the kick cannot help.

- Forgetting that the kick is not certain. Since 2015, NFL kickers have missed several extra points in every hundred.
- Comparing make rates instead of points per try.

Make it a game · Kick or Two Chart. Kick a paper ball at a goal made from two shoes, ten tries. Then toss a ball from a closer line, ten tries. Work out points per try for the one-point kick and the two-point try. Find the break-even success rate. Make a chart your parent can use. Explain your recommendation in two sentences.

4.4 THE BEST SHOT: MOST POINTS PER TRY

OBJECTIVES

- Compare several shots by points per try and rank them.
- Explain why the best shot is the one worth the most points per try, not always the one made most often.
- Name the risk that goes with each shot.

Warm-up. Look at a dartboard picture or an archery target online with a parent, or draw a target with three rings: the center worth 5, the middle ring worth 3 and the outer ring worth 1. If you hit the outer ring almost every time but the center only sometimes, where should you aim?

WATCH

Stephen Curry from 30 feet | Sport Science | ESPN Archives

ESPN · 1:58

<https://www.youtube.com/watch?v=W0044xEaZaA>

WHEN TO WATCH

After “The story.” Notice how much practice lies behind a high-percentage long shot.

KEY TERMS

high-percentage shot

In this course, the shot that earns the most points per try, not simply the one that goes in most often.

shot choice

A player’s choices about which shots to take and which to pass up.

efficiency

How many points a player or team earns for each try.

sample

The set of tries you measured. A bigger sample gives a more trustworthy make rate.

CORE IDEAS

The story. In the 2015–16 NBA season, Stephen Curry of the Golden State Warriors made 402 three-point shots, the first time any player had made even 300 in a season, let alone 400. He took 886 threes and made 45.4 percent of them, many from far

beyond the line. His team won 73 games, the most ever in one NBA season. Many fans thought he was taking wild shots. But the numbers said something else: Curry's long threes were among the best shots in the whole league.

How they figured it out. Analysts looked at points per try. Curry's threes earned $3 \times 0.454 \approx 1.36$ points per try. Very few shots in basketball earn that much, except shots right at the rim and free throws. A two-pointer would have to go in 68% of the time to match it. So for Curry, a long three was a high-percentage shot, even though it missed more than half the time. That was true for Curry because of years of careful practice; for most players, the same shot would earn far less.

The mathematics. To find the best shot, list your shots, find each one's points per try, and rank them. Suppose a player makes 60% at the basket (2 points), 40% from the middle distance (2 points) and 35% from three (3 points). Points per try:
 $2 \times 0.60 = 1.20$, $2 \times 0.40 = 0.80$, and $3 \times 0.35 = 1.05$. The best shot is at the basket, then the three, and the middle shot is last, even though it goes in more often than the three.

On the field. Good teams pass up a fair shot to get a better one, because every try is precious. **The percentage:** take the shots that earn the most points per try. **The risk side:** a shot with high points per try may still miss often, so a long run of misses is normal and not a reason to panic. Also, if a defense knows you will only take one kind of shot, it will try to take that shot away. And a player should measure his *own* numbers; copying a great player's shot without his skill is not playing the percentages. As in archery, the best aim depends on how steady your hand is.

PICTURE STUDY



Archery Meeting in Bradgate Park, Leicestershire, John Ferneley, About 1850 · Oil on canvas ·
Leicestershire, England

Image: [Wikimedia Commons](#), public domain

John Ferneley was an English painter best known for horses and country scenes. Here he shows an archery meeting in a park in Leicestershire about 1850. Archery was a popular sport for both ladies and gentlemen in England at that time, and clubs held meetings where archers shot at targets for points.

Look closely

- Find the round targets. Each archery target has rings, and the center ring is worth the most points.
- Find the archers and the people watching. Who is shooting, and who is keeping score?
- Look at how far the targets are from the archers. Why would distance matter for where an archer aims?

Talk about it

1. An archer who hits the outer ring every time but rarely hits the center might score more by aiming at the center anyway. How is that like choosing between a two and a three in basketball?
2. How would a steady archer and a shaky archer choose their aim differently?

Safety first. Use soft balls only, clear the area, and warm up first.

PROJECT

My Shot Chart

You will make a shot chart of your home court with three or four shooting spots, find your points per try from each spot, and write a short “shot plan” for yourself. This is the start of your unit portfolio.

You will need

- A soft ball and a basket or bucket
- Painter’s tape and a tape measure
- Graph paper and colored pencils
- Your scorebook and calculator

Steps

1. Choose four spots: right next to the basket (2 points), a middle spot (2 points), a far spot (3 points) and a free-throw line (1 point).
2. Mark each spot with tape and measure its distance from the basket.
3. **Predict.** Guess which spot will give the most points per try.
4. Take 20 shots from each spot. Tally makes.
5. Find make rate and points per try for each spot.
6. Draw a shot chart: a bird’s-eye drawing of your court with each spot marked and colored by points per try (for example, green for the best, yellow for the middle, red for the worst).
7. Rank the spots. Write your shot plan: which shots you will take when you have a choice, and which you will pass up.
8. Write the risk side of your plan: what could go wrong, and when you would change it.

Tips

- Shoot from the spots in a mixed order so tiredness does not affect only one spot.
- Use the same colors on every chart you make this year so they are easy to compare.

Check your work

- ☐ My chart shows at least four spots with distances, make rates and points per try.
- ☐ My spots are colored by points per try.
- ☐ My shot plan names the best shot and the risk side.

SCOREBOOK

1. Draw your shot chart neatly with a key that explains the colors.
2. Make a table ranking your spots from best to worst by points per try.
3. Write: “The high-percentage shot for me is... because...”

From history. Before the 2015–16 season, the record for threes in one NBA season was 286, set by Curry himself the year before. Shot charts drawn from cameras that track every shot became common in the NBA in the 2010s and changed how teams plan their offenses.

WORKED EXAMPLES

Example 1. Rank these three shots by points per try: a 2-point shot made 55% of the time, a 3-point shot made 30% of the time, and a 1-point free throw made 85% of the time.

Show solution

1. 2-pointer: $2 \times 0.55 = 1.10$.
2. 3-pointer: $3 \times 0.30 = 0.90$.
3. Free throw: $1 \times 0.85 = 0.85$.

Answer: Best is the 2-pointer (1.10), then the 3-pointer (0.90), then a single free throw (0.85). A trip for two free throws, $2 \times 0.85 = 1.70$, beats them all.

COMMON MISTAKES

- Thinking a high make rate always means a good shot.
- Copying a great player’s shot choices without his skill.
- Forgetting that even the best shot misses often. Judge a shot by many tries.

Make it a game • Rank the Shots. Stand at three distances from a bucket and take ten tosses from each. Before shooting, write a ranking of the shots by expected points per try. Afterward, compute the real numbers. Score a bean for every shot ranked in the right place. Then explain why the best shot is not always the one made most often.

Scripture. “And every man that striveth for the mastery is temperate in all things. Now they do it to obtain a corruptible crown; but we an incorruptible.” (1 Corinthians 9:25, KJV)

Choosing the good shot over the tempting one takes self-control. Paul says athletes practice that self-control for a prize that fades; Christians practice it for one that lasts.

POINTS PER TRY

Which is the better shot: an easy one worth 2 points, or a harder one worth 3? Many people say the easy one, because it goes in more often. But a good coach asks a different question: *how many points does each shot earn per try, on average?* A shot that misses more often can still be the better shot if it is worth enough more. That simple idea, points per try, has changed basketball, football and many other games.

In this unit you will measure your own points per try at a home basket from two distances. You will learn the story of the three-point line, of Rick Barry and his underhand free throws, and of the football coach's choice between kicking the extra point and going for two. Along the way you will always ask two questions: what is the percentage, and what is the risk?

PREREQUISITES

- Can find a win rate as a fraction, a decimal and a percent (Unit 3)
- Can multiply a whole number by a decimal, with a calculator if needed
- Knows the idea of expected points per round (Lesson 3.4)

MASTERY CHECKLIST

- ☐ I can find points per try by multiplying the points for a success by the make rate.
- ☐ I can measure my own points per try from two distances and decide which shot is better for me.
- ☐ I can explain why Rick Barry's underhand free throw worked, and why few players copy it.
- ☐ I can compare the extra-point kick with the two-point try and explain why the score can change the choice.
- ☐ I can explain why the best shot is the one worth the most points per try, not always the one that goes in most often.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Basketball-Reference: NBA league averages:** Free data. Season-by-season averages of three-point tries and make rates since the line began, for Lessons 4.1 and 4.4. Contains advertising; a parent should look at it with the student.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

4.1	Twos and Threes at a Home Basket	2 sessions of 45 minutes
4.2	Free Throws and Rick Barry's Underhand Shot	2 sessions of 45 minutes
4.3	The Kick or Going for Two	2 sessions of 45 minutes
4.4	The Best Shot: Most Points per Try	2 sessions of 45 minutes

UNIT 4 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. **4 pts**
Write the formula for points per try and explain it in your own words.
2. **5 pts**
A shooter makes 45% of twos and 35% of threes. Find the points per try for each. Which is the better shot?
3. **4 pts**
What three-point make rate breaks even with a 50% two-point shot?
4. **5 pts**
Rick Barry made 90% of free throws; Wilt Chamberlain made about 51% in his career. Find the points per trip of two free throws for each. How many more points would Barry score in 50 trips?

5. **4 pts**

Why did Wilt Chamberlain stop shooting underhand, even though it worked better for him? What lesson does that teach?

6. **4 pts**

Why is alternating rounds of two shooting styles a fairer test than shooting all of one style first?

7. **5 pts**

A kicker makes 94% of extra points. A team succeeds on 45% of two-point tries. Which choice earns more on average? What is the break-even rate?

8. **5 pts**

A team trails by 8 points. On the last play of the game it scores a touchdown (6 points). What would happen if it kicked the extra point? Which choice should it make, and why?

9. **4 pts**

Stephen Curry made 45.4% of his threes in 2015–16. Find his points per try. What two-point make rate would match it?

10. **4 pts**

Rank by points per try: at the basket 60% (2 points), middle 40% (2 points), three 34% (3 points).

11. **6 pts**

Short answer: In three or four sentences, explain what “the high-percentage shot” means in this course, and name one risk that goes with it.

ANSWER KEY

1. Points per try = points for a success $\times$ make rate. It is the average number of points each try earns over many tries.
2. Two: $2 \times 0.45 = 0.90$. Three: $3 \times 0.35 = 1.05$. The three is better.
3. One third, about 33%, because $3 \times \frac{1}{3} = 1 = 2 \times 0.5$.
4. Barry $2 \times 0.90 = 1.80$; Chamberlain $2 \times 0.51 = 1.02$. Difference 0.78 per trip, so about 39 points in 50 trips.
5. He said it made him feel foolish. A wise player chooses what works, even if it looks unusual.
6. Tiredness and warming up then affect both styles equally, so the only difference is the style.
7. Kick 0.94; two-point try $2 \times 0.45 = 0.90$. Kicking earns slightly more. The break-even is 47%.
8. After the touchdown it trails by 2. A kick would leave it 1 point behind, and it would lose. It must go for two: a two-point try ties the game about half the time, while the kick can never tie it. The goal (tying the game) decides the choice, not average points.
9. $3 \times 0.454 \approx 1.36$ points per try. A two would need $1.36 \div 2 = 0.68$, or 68%.
10. Basket 1.20, three 1.02, middle 0.80.
11. The high-percentage shot is the one that earns the most points per try on average, not simply the one that goes in most often. It is found by multiplying the points by the make rate. One risk: even the best shot misses often, so streaks of misses are normal; another is that a make rate from a small sample may be wrong, or that a defense will take the shot away. Good answers name the percentage and the risk.

PORTFOLIO PIECE: MY SHOT PLAN

Prepare a two-page “Shot Plan” for yourself as a player. Page 1 is your shot chart from Lesson 4.4, redrawn neatly with at least four spots, their distances, make rates and points per try, colored by points per try. Page 2 has three short sections: your free-throw test from Lesson 4.2 with your chosen style and why; your kick-or-two decision chart from Lesson 4.3; and a paragraph explaining the high-percentage shot to a younger player, naming both the percentage and the risk. Then shoot 20 more shots from your best spot and 20 from your worst spot and report whether the ranking held up.

LOOK FOR	WHAT IT MEANS
Shot chart	Neat, labeled chart with distances, make rates and correct points per try for at least four spots.
Fair tests	Free-throw styles and kick-or-two tested with enough tries and recorded honestly.
Decision making	The plan names the best shots, the decision chart has sensible reasons, and the risk side is stated.
Follow-up	The extra 40 shots are recorded and compared with the original ranking.

UNIT 5

BASEBALL BY THE NUMBERS

No sport keeps track of itself like baseball. Every pitch, every hit and every out can be written down, and fans have been writing them down for more than 160 years. In this unit you will meet Henry Chadwick, who invented the box score, and learn to read one. You will learn the two most famous numbers about a hitter, the batting average and the on-base percentage, and why Branch Rickey argued that the second one matters more.

Then you will keep score of a real game yourself with a simple scorecard, the way fans and volunteers have done for generations, and use your own records to compute real statistics. Finally you will compare two of the greatest players who ever lived, Ty Cobb and Babe Ruth, and see how their numbers tell the story of two very different ways to play the odds at the plate.

PREREQUISITES

- Can find a rate as a fraction, a decimal and a percent (Units 1–4)
- Can divide with a calculator and round to three decimal places
- Knows the basic rules of baseball or softball: outs, bases, hits and walks

MASTERY CHECKLIST

- ☐ I can read a simple box score and tell what each column means.
- ☐ I can compute a batting average and an on-base percentage and write them the baseball way (such as .300).
- ☐ I can explain why getting on base matters so much when each team has only 27 outs.
- ☐ I can keep score of a real game using position numbers and simple symbols.
- ☐ I can compare Ty Cobb and Babe Ruth using their real numbers.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Baseball-Reference:** Free statistics for every major-league player and season, used in Lessons 5.2 and 5.4. Contains advertising; a parent should look at it with the student.
- **Retrosheet:** Free. Volunteer-built play-by-play records and box scores of past major-league games, mentioned in Lesson 5.3. Best for ages 11 and up with a parent.
- **National Baseball Hall of Fame and Museum:** Free online biographies of Henry Chadwick, Ty Cobb, Babe Ruth and Branch Rickey, which fit Lessons 5.1, 5.2 and 5.4.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

5.1	Henry Chadwick's Box Score	2 sessions of 45 minutes
5.2	Batting Average and On-Base Percentage	2 sessions of 45 minutes
5.3	Keeping Score of a Real Game	2 sessions of 45 minutes
5.4	Ty Cobb, Babe Ruth and the Numbers	2 sessions of 45 minutes

UNIT 5 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. 4 pts

Who was Henry Chadwick, and what did he give baseball?

2. 5 pts

A batter has 4 hits in 12 at bats and 3 walks. Find his batting average and simple on-base percentage.

3. 4 pts

Why does a walk help a team even though it does not count as a hit?

4. (4 pts)

What did Branch Rickey argue in his 1954 *Life* article, and whose numbers did he use?

5. (4 pts)

Write the position numbers for the pitcher, catcher, shortstop and center fielder. What does “4-3” mean?

6. (5 pts)

A batter’s scorecard boxes read: 1B, K, BB, HR, 6-3. Find AB, H, BB, BA and simple OBP.

7. (5 pts)

In 20 at bats a hitter has 3 singles, 2 doubles and 1 home run. Find his slugging percentage.

8. (4 pts)

Cobb’s career BA was .366 and Ruth’s .342. Does that prove Cobb was the better hitter? Explain using OBP and SLG.

9. (5 pts)

What would happen to a young player’s statistics if he swung hard at every pitch, like Ruth, without Ruth’s strength?

10. (4 pts)

Why is a statistic from one game less trustworthy than one from a whole season?

11. (6 pts)

Short answer: In three or four sentences, explain how keeping an honest scorebook connects to Proverbs 20:23.

ANSWER KEY

1. An English-born New York sportswriter (1824–1908) who created the baseball box score around 1859, gave us K for a strikeout, wrote guides and rules, and was elected to the Hall of Fame in 1938.
2. $BA\ 4 \div 12 \approx .333$. $OBP\ (4 + 3) \div (12 + 3) = 7 \div 15 \approx .467$.
3. It puts a runner on base and avoids an out. A team has only 27 outs, so every out saved keeps the inning alive.
4. That getting on base matters more than batting average; he used the numbers of Allan Roth, the Dodgers' statistician.
5. Pitcher 1, catcher 2, shortstop 6, center fielder 8. A 4-3 is a ground ball fielded by the second baseman and thrown to first for an out.
6. AB 4 (1B, K, HR, 6-3), H 2, BB 1. $BA\ 2 \div 4 = .500$; $OBP\ 3 \div 5 = .600$.
7. Total bases $3 + 4 + 4 = 11$; $SLG\ 11 \div 20 = .550$.
8. No. Ruth's OBP (.474 to .433) and SLG (.690 to .512) were both higher: he reached base more often and his hits were worth far more bases.
9. He would make many strikeouts without hitting many home runs or drawing many walks, so his BA, OBP and SLG would likely all fall. A style must fit the player's gifts.
10. One game is a very small sample; chance can make a good hitter go 0 for 4. Many games give a larger sample and a more trustworthy rate.
11. Proverbs 20:23 says false weights and balances are not good in God's sight. A scorebook is a kind of balance that measures what players did. Writing down exactly what happened, even when it is not what we hoped, is honest measuring, and every statistic built from it can be trusted.

PORTFOLIO PIECE: MY SEASON SCOREBOOK

Keep score of three games (real games watched with a parent, or radio or television broadcasts, or your own backyard games played with full rules). For each game include your complete scorecard for at least three innings. Then make a “season” table for one team or for your backyard players that adds up all three games: AB, H, BB, HR, BA, OBP and SLG for each player. Finish with a one-page report that names the team’s most valuable hitter and explains why, using at least two statistics, and that says how a one-game number differed from the three-game number for at least one player.

LOOK FOR	WHAT IT MEANS
Scorecards	Three games scored neatly with correct symbols and runs that match the final scores.
Season table	Totals added correctly across games, with BA, OBP and SLG computed and rounded to three places.
Reasoning	The most valuable hitter is chosen with at least two statistics, and the report explains why a larger sample is more trustworthy.
Honesty and neatness	Missed plays are marked honestly, not guessed, and the work is easy to read.

5.1 HENRY CHADWICK'S BOX SCORE

OBJECTIVES

- Tell the story of Henry Chadwick and the first box scores.
- Read the columns of a simple box score.
- Make a box score of a backyard game from your own records.

Warm-up. Play three short “innings” of a game with a family member, such as tossing a soft ball into a bucket from three distances. Afterward, try to remember exactly how many points each player scored in each inning. Was it easy? What would have helped you remember?

WATCH

Inside the art of baseball scorekeeping with just pen and paper

TODAY • 4:11

<https://www.youtube.com/watch?v=548bprlVMHI>

WHEN TO WATCH

After “The story.” It shows fans who still keep score with pencil and paper, as Chadwick did.

KEY TERMS

box score

A small table that tells, player by player, what happened in a game.

at bat (AB)

A batter’s turn at the plate that ends in a hit or an out. Walks do not count as at bats.

hit (H)

Reaching base safely by hitting the ball, without an error or a fielder’s choice.

run (R)

A point in baseball, scored when a runner touches home plate.

statistic

A number made by counting and computing, used to describe what happened.

CORE IDEAS

The story. Henry Chadwick was born in Exeter, England, in 1824 and came to America as a boy. He grew up loving cricket, a game whose newspaper reports always printed a scorecard. In the 1850s he started writing about a new game, baseball, for the New York newspapers. Around 1859 he took the cricket scorecard and turned it into the baseball **box score**, a small grid that showed what each player did in a game. He also gave us the letter K for a strikeout. For fifty years he wrote guides, helped shape the rules and urged players to play honestly. He was elected to the Baseball Hall of Fame in 1938, and he is often called “the Father of Baseball.”

How they figured it out. Before box scores, a newspaper could only say who won and tell a few exciting plays. Chadwick saw that if every game was written down in the same way, readers could compare players and teams across a whole season. A box score is a record kept by *counting*: runs, hits, at bats, outs. Once the counts were kept, people could divide them to make rates, like the fraction of at bats that became hits. That is how baseball statistics began.

The mathematics. A box score is a table of counts. Each row is a player and each column is something counted, such as AB (at bats), R (runs), H (hits) and RBI (runs batted in). Totals at the bottom must agree with the game: the team’s runs in the box score must equal the runs on the scoreboard. From the counts you can compute rates. A player who has 2 hits in 4 at bats has hit safely in $\frac{2}{4} = 0.500$ of his at bats. Baseball writes this as **.500**, read “five hundred.”

On the field. Coaches still study box scores and scorebooks to decide who should bat where and who needs extra practice. A careful record lets a coach see patterns that memory forgets. **The risk side:** one game’s box score is a very small sample. A player who goes 0 for 4 today may be a fine hitter having an ordinary bad day. Good coaches look at many games before deciding anything, just as you learned with coins and dice.

PICTURE STUDY



The American National Game of Base Ball, Currier & Ives, 1866 · Hand-colored lithograph · Metropolitan Museum of Art, New York

Image: [Wikimedia Commons](#), public domain

Currier & Ives was a New York printing company that sold colorful pictures of American life. This print shows a championship game at the Elysian Fields in Hoboken, New Jersey, a park where some of the earliest organized baseball games were played. It was made in 1866, when Henry Chadwick was already writing about the game and printing box scores.

Look closely

- Find the batter, the catcher and the pitcher. How is the pitcher throwing?
- Count the fielders. Are they in the same places as fielders today?
- Look at the crowd. Do you see anyone who might be writing down the game?
- Find the umpire. Where is he standing?

Talk about it

1. If you were Henry Chadwick at this game, what would you count and write down for each player?
2. Why would a newspaper reader in another city want a box score of this game instead of just the final score?

PROJECT

A Backyard Box Score

You will invent a simple batting game for the backyard or living room, keep a careful record of every turn, and turn your record into a box score.

You will need

- A soft ball, and a foam bat or your hand to hit with (or a soft ball tossed into buckets as “hits”)
- Four markers for bases, or four buckets at different distances
- Your scorebook and a pencil
- Two to four players

Steps

1. Agree on simple rules. For example: a pitcher tosses gently; a ball hit past a line is a hit; a ball caught or a third miss is an out; three outs end an inning. (Or play the bucket version: a toss in the near bucket is a single, the far bucket a double, a miss is an out.)
2. In your scorebook, draw a box score with columns for Player, AB, R, H and Outs.
3. Play three innings. After every turn, write it down immediately.
4. At the end, total each column. Check that the runs in your box score add up to the final score.
5. For each player, compute hits divided by at bats and write it the baseball way.
6. Write a short newspaper report of your game, three or four sentences long, using the numbers in your box score.
7. **Real box score.** With a parent, find a box score of a real game in a newspaper or on a baseball website. Copy one team’s box score and check that the hits add up.

Tips

- Keep score as you go. Even great memories forget by the third inning.
- If a player is hit by a pitch or walks, write it down but do not count it as an at bat.

- Have one person be the official scorer each inning, and take turns.

Check your work

- ☐ My box score has a row for every player and columns for AB, R and H.
- ☐ My totals match the final score.
- ☐ I computed each player's hits divided by at bats and wrote a short game report.

SCOREBOOK

1. Copy your backyard box score neatly with totals at the bottom.
2. Copy a real box score and circle the column you think tells the most about a player. Explain why.
3. Write: "One game's box score would not make me decide... because..."

From history. Chadwick also wrote the first rule books and yearly guides for baseball and campaigned against cheating and rough play. He died in 1908 at the age of 83, after catching a chill at a ball game on opening day.

WORKED EXAMPLES

Example 1. In a box score, a player has AB 5, R 2, H 3. What fraction of his at bats were hits? Write it the baseball way.

Show solution

1. Hits divided by at bats: $\frac{3}{5}$.
2. As a decimal: $3 \div 5 = 0.600$.

Answer: $\frac{3}{5}$, written .600 ("six hundred") in baseball.

COMMON MISTAKES

- Counting a walk as an at bat. A walk is a plate appearance, but not an at bat.
- Forgetting to check that the runs in the box score add up to the final score.
- Judging a player from one game's line.

Make it a game • Box Score Detective. Draw a blank box score with columns AB, H and R. Your parent reads a made-up inning of a game aloud, and you record it as fast as you can. Then your parent asks three questions from the sheet, such as who got a hit. Add one fact about Henry Chadwick. Beat your accuracy on the second inning.

5.2 BATTING AVERAGE AND ON-BASE PERCENTAGE

OBJECTIVES

- Compute a batting average and a simple on-base percentage.
- Explain why every out matters when a team has only 27 outs.
- Tell the story of Branch Rickey, Allan Roth and the idea that getting on base is what counts.

Warm-up. Two hitters each come to bat 10 times. Hitter A gets 3 hits and makes 7 outs. Hitter B gets 2 hits, walks 3 times and makes 5 outs. Which hitter helped his team more? Talk about it before reading on.

WATCH

Batting Average | Baseball Explained

Outta Here Baseball · 5:02

<https://www.youtube.com/watch?v=vk327e2S9pE>

WHEN TO WATCH

Before “The mathematics.” Then compute on-base percentage yourself, which the video does not cover.

BACKUP VIDEO

Comparing Batting Average and On Base Percentage

TheOnDeckCircle · 4:28

<https://www.youtube.com/watch?v=NLZEC3gO1dE>

Compares batting average and on-base percentage directly; good after the worked examples.

KEY TERMS

batting average (BA)

Hits divided by at bats. It tells how often a batter gets a hit.

walk (BB)

Reaching first base after four balls. It is written BB, for “base on balls.”

plate appearance (PA)

Every completed turn at the plate, including walks.

on-base percentage (OBP)

How often a batter reaches base by a hit, a walk or being hit by a pitch, out of his plate appearances. A simple version is $(\text{hits} + \text{walks}) \div (\text{at bats} + \text{walks})$.

out

A batter or runner put out by the fielders. Each team gets 3 outs an inning and 27 in a nine-inning game.

CORE IDEAS

The story. Branch Rickey was one of the most important leaders in baseball history. He was a devout Christian who would not go to the ballpark on Sundays, and as head of the Brooklyn Dodgers he signed Jackie Robinson in 1945, a choice that took great courage. Rickey also wanted to know what really wins games. In the late 1940s he hired Allan Roth as the first full-time statistician for a major-league team. Roth recorded every pitch and every plate appearance. On August 2, 1954, *Life* magazine printed an article by Rickey called “Goodbye to Some Old Baseball Ideas,” which used Roth’s numbers to argue that getting on base mattered more than batting average.

How they figured it out. For a hundred years the favorite number in baseball had been the batting average: hits divided by at bats. But batting average leaves out walks completely, as though a batter who reached first base on four balls had done nothing. Rickey and Roth saw that runs come from runners, and runners come from batters who get on base by any means. Every team has only 27 outs in a game. A batter who avoids making an out keeps the inning alive and gives the next batter a chance. Baseball made on-base percentage an official statistic in 1984.

The mathematics. $BA = \frac{H}{AB}$. A simple $OBP = \frac{H + BB}{AB + BB}$. (The official formula also counts batters hit by a pitch and sacrifice flies.) In the warm-up, Hitter A has $BA \frac{3}{10} = .300$ and $OBP \frac{3}{10} = .300$. Hitter B has 2 hits in 7 at bats, $BA \frac{2}{7} \approx .286$, but $OBP \frac{2+3}{7+3} = \frac{5}{10} = .500$. B reached base half the time and made only 5 outs. In 1941 Ted Williams of the Boston Red Sox batted .406, but his on-base percentage was .553, because he also drew 147 walks: he avoided an out more than half the time.

On the field. A wise hitter does not swing at bad pitches just to try for a hit. He waits for a good pitch, and if none comes, he takes his walk. That is playing the percentages: a swing at a bad pitch usually makes an out, while patience often puts a runner on base. Managers now put their best on-base hitters near the top of the lineup, so they bat most often. **The risk side:** a batter who never swings can be struck out by a pitcher who throws strikes. Patience is wise only when it is joined with being ready to hit the good pitch.

PICTURE STUDY



Branch Rickey and Burt Shotton, Underwood & Underwood, 1924 · Newspaper photograph and caption
· St. Louis, Missouri

Image: [Wikimedia Commons](#), public domain

In 1924 Branch Rickey was the manager of the St. Louis Cardinals. This newspaper clipping explains that he would not go to the ballpark on Sundays, “it being a matter of principle with him,” and that his coach Burt Shotton ran the team on those days. (The caption misspells Shotton’s name as “Shotten.”)

Look closely

- Read the headline. What made Rickey different from every other manager?
- Find Rickey and Shotton in the photograph. How are they dressed differently?
- Look at their faces. Do they seem relaxed or serious about the game?

Talk about it

1. Rickey was willing to be different on Sundays and different in how he judged hitters. How might those two kinds of courage be connected?
2. Rickey did not just trust his feelings about players; he hired someone to count. Why is counting a good partner to experience?

PROJECT

Who Is the Better Hitter?

You will compute batting averages and on-base percentages for real players with a parent's help, and for a backyard game of your own, and decide whether the two numbers tell the same story.

You will need

- Your scorebook and a calculator
- A soft ball and a foam bat, or the bucket game from Lesson 5.1
- With a parent: a baseball statistics website or the back of baseball cards

Steps

1. Copy this table: Player, AB, H, BB, BA, OBP.
2. Fill in the warm-up hitters A and B, and Ted Williams in 1941 (456 AB, 185 H, 147 BB). Compute each BA and simple OBP.
3. With a parent, look up three real players from one team's season and copy their AB, H and BB. Compute BA and simple OBP.
4. Rank the players by BA and then by OBP. Did the order change?
5. **Backyard version.** Play a batting game in which the pitcher sometimes throws wide on purpose. A batter who lets four wide pitches go by takes a walk to first. Keep a record for 5 innings.
6. Compute each backyard player's BA and OBP.
7. **Decide.** If you were the manager, who would you want batting first? Use OBP in your reason.

Tips

- Round averages to three decimal places, the baseball way.
- A walk adds to the top and the bottom of OBP, but it does not change batting average at all.
- For Ted Williams, simple OBP comes out a little lower than the official .553, because the official formula also counts the times he was hit by pitches.

Check your work

- ☐ My table has correct BA and OBP for at least six players.
- ☐ I ranked the players both ways and noted any changes.
- ☐ I chose a leadoff hitter and explained why using OBP.

SCOREBOOK

1. Copy your hitters table neatly, with the highest OBP highlighted.
2. Draw a diagram of a baseball field with 27 little circles for outs. Write: “Every out is one of 27. A walk saves one.”
3. Write: “When a pitcher throws a bad pitch, I would decide to... because...”

From history. Before coming to Brooklyn, Branch Rickey ran the St. Louis Cardinals, where he built baseball’s first system of minor-league “farm” teams to train young players. He was elected to the Hall of Fame in 1967.

WORKED EXAMPLES

Example 1. A player has 150 hits in 500 at bats and 70 walks. Find his batting average and simple on-base percentage.

Show solution

1. BA: $150 \div 500 = .300$.

2. OBP: $(150 + 70) \div (500 + 70) = 220 \div 570 \approx .386$.

Answer: Batting average .300 and on-base percentage about .386.

Example 2. How many outs does a team get in a nine-inning game? If a batter reaches base 40% of the time, about how many outs does he make in 10 plate appearances?

Show solution

1. Three outs an inning: $3 \times 9 = 27$ outs.

2. He makes an out about 60% of the time: $0.60 \times 10 = 6$ outs.

Answer: 27 outs per game; about 6 outs in 10 trips to the plate.

COMMON MISTAKES

- Adding walks to batting average. Batting average uses only hits and at bats.

- Thinking a .300 hitter always helps more than a .280 hitter. The one who gets on base more may help more.
- Rounding too early. Divide first, then round to three places.

Make it a game · On-Base Showdown. Make two players on paper with their at-bats, hits and walks. Compute each batting average and on-base percentage against the clock. Your parent calls the winner for each stat before you finish, and you check. Explain why every out matters out of 27. Switch and make new players for your parent to calculate.

Scripture. “The thoughts of the diligent tend only to plenteousness; but of every one that is hasty only to want.” (Proverbs 21:5, KJV)

A patient hitter who waits for a good pitch, and a manager who counts carefully before deciding, are both practicing the diligence Proverbs praises over haste.

5.3 KEEPING SCORE OF A REAL GAME

OBJECTIVES

- Use position numbers and simple symbols to score plays.
- Keep score of at least three innings of a real game.
- Compute statistics from your own scorecard.

Warm-up. Number the nine fielding positions from memory: pitcher, catcher, first base, second base, third base, shortstop, left field, center field, right field. Then check the list in the lesson. Which numbers surprised you?

WATCH

Learn to Keep Baseball Score in 4 Minutes

Baseball 101 • 3:56

https://www.youtube.com/watch?v=Ei6DGH_Y6Q8

WHEN TO WATCH

Before the project. Keep a pencil and a blank scorecard ready and copy the symbols.

BACKUP VIDEO

Using a Baseball Scorecard with Greg Bancroft

Mighty Media Press • 1:39

<https://www.youtube.com/watch?v=xz4RR-UcrNs>

A short introduction to the scorecard from a children's book author.

KEY TERMS

scorecard

A grid for recording every plate appearance in a game, inning by inning.

position numbers

Numbers for the fielders: 1 pitcher, 2 catcher, 3 first base, 4 second base, 5 third base, 6 shortstop, 7 left field, 8 center field, 9 right field.

6-3

Scorer's shorthand for a ground ball fielded by the shortstop (6) and thrown to first base (3) for an out.

K

The scorer's symbol for a strikeout, chosen by Henry Chadwick.

play-by-play

A record of every play in a game, in order.

CORE IDEAS

The story. In 1989 a biology professor named Dave Smith started a project called Retrosheet. He and a team of volunteers began collecting old scorecards and scorebooks from newspapers, teams and fans, and copying every play into a computer, game by game. Today Retrosheet has the play-by-play of most major-league games going back many decades, free for anyone to read. All of it began with ordinary people who sat in the stands or by the radio with a pencil and kept score.

How they figured it out. Over the years scorekeepers settled on a clever shorthand so that a whole play fits in one small box. Every fielder has a number, so "6-3" means the shortstop fielded the ball and threw to first. Hits are drawn as lines along the base paths, with "1B" for a single and "2B" for a double. A strikeout is K, a walk is BB, and a run is shown by coloring in the diamond. With this code a careful scorer can record a whole game on one sheet, and anyone who knows the code can read it years later.

The mathematics. A scorecard is a record of counts, and counts become statistics. From your card you can count each batter's at bats, hits and walks and compute his batting average and on-base percentage for that game. You can also count how many pitches or batters each pitcher faced, how many outs were strikeouts, and how often the first batter of an inning reached base. When you combine several of your own scorecards, your sample grows and your numbers become more trustworthy.

On the field. Coaches of youth teams keep a scorebook at every game to track outs, runs, pitch counts and batting order. Pitch counts matter for safety: young pitchers are limited in how many pitches they may throw, so their arms are not hurt. **The risk side:** a scorebook is only as good as its honesty and care. One missed play can make

the totals wrong. Scorers check that the runs on the card match the scoreboard at the end of every inning.

PICTURE STUDY



Baseball Players Practicing, Thomas Eakins, 1875 · Watercolor on paper · Rhode Island School of Design Museum, Providence

Image: [Wikimedia Commons](#), public domain

Thomas Eakins of Philadelphia was one of America's great painters, and he loved to paint people doing real things well: rowing, sailing, swimming and playing ball. These two players were members of the Philadelphia Athletics. Eakins studied how they stood and moved before he painted them.

Look closely

- Find the batter and the catcher. How is each one standing? What is each ready to do?
- Look at the bat. Is it held high or low? What might the batter be waiting for?
- Look at the empty stands. Why might the players be practicing on a day with no game?

Talk about it

1. If this batter waits patiently for a good pitch, how might that help his on-base percentage?
2. If you were keeping score of this batter, which symbols might you write in his box?

PROJECT

My First Scorecard

You will keep score of at least three innings of a real baseball or softball game: a local youth or high-school game, or a game on television or the radio with a parent. Then you will compute statistics from your own card.

You will need

- A printed scorecard or one you draw: a grid with nine rows (batters) and columns for innings, each box with a small diamond
- A pencil with an eraser
- A real game to watch or listen to, with a parent
- Your scorebook and calculator

Steps

1. Before the game, write the batting order of one team down the left side of your card, with each player's position number.
2. Practice the symbols: K for strikeout, BB for walk, 1B 2B 3B HR for hits, and fielder numbers for outs (6-3, F8 for a fly out to center).
3. Keep score of at least three innings for one team. Write each plate appearance in that batter's box for that inning.
4. At the end of each inning, count the runs and check them against the scoreboard.
5. After the game, count each batter's AB, H and BB from your card.
6. Compute BA and simple OBP for each batter for this game.
7. Count how many outs were strikeouts and how many were balls in play.
8. Write two sentences about something your scorecard shows that you might not have remembered without it.

Tips

- If you miss a play, leave a note and ask your parent or listen for the announcer to repeat it. Do not guess.

- Radio broadcasts are good for scoring because the announcer describes every play.
- A parent supervises any trip to a ballpark.

Check your work

- ☐ My card has at least three complete innings for one team.
- ☐ My run totals match the scoreboard.
- ☐ I computed BA and OBP for each batter from my own card.

SCOREBOOK

1. Paste in or copy your scorecard. Put a star by your favorite play and explain the symbols you used.
2. Make a table from your card: Batter, AB, H, BB, BA, OBP.
3. Write a short report of the game as Henry Chadwick might have written it.

From history. Retrosheet takes its name from “retro,” meaning looking back, and “sheet,” for the scoresheets it collects. Its volunteers have also found mistakes in old official records and corrected them, a good example of careful counting.

WORKED EXAMPLES

Example 1. On your scorecard one batter’s boxes read: K, 6-3, 1B, BB. What are his AB, H and BB? His BA and simple OBP for the game?

Show solution

1. At bats: K, 6-3 and 1B count (3 AB). The walk does not.
2. Hits: 1. Walks: 1.
3. BA: $\frac{1}{3} \approx .333$. OBP: $\frac{1+1}{3+1} = .500$.

Answer: 3 AB, 1 H, 1 BB; BA .333 and OBP .500 for the game.

COMMON MISTAKES

- Waiting until later to fill in the card. Write each play as it happens.
- Mixing up position numbers 4 and 6 (second base and shortstop).
- Not checking the runs after each inning.

Make it a game • Score the Scene. Your parent narrates a pretend inning with plays like a ground ball to short, a strikeout and a fly to center. Write each play on a score sheet using position numbers and K. At the end, explain two plays in plain words. Parents may slip in an impossible play to see if you catch it. Score one point per correct entry.

Scripture. “Divers weights are an abomination unto the LORD; and a false balance is not good.” (Proverbs 20:23, KJV)

A scorebook is a kind of balance. Writing down exactly what happened, not what we wish had happened, honors God.

5.4 TY COBB, BABE RUTH AND THE NUMBERS

OBJECTIVES

- Compare Ty Cobb and Babe Ruth using batting average, on-base percentage and home runs.
- Compute a simple slugging percentage.
- Explain how two different styles of hitting can both be high-percentage play.

Warm-up. Which helps a team more: a batter who gets a single 4 times in 10, or a batter who hits a home run 1 time in 10 and walks 2 times in 10? There is no easy answer. Talk about it and write your first guess.

WATCH

Opening of the Baseball Hall of Fame with Babe Ruth and Cy Young

National Baseball Hall of Fame and Museum • 2:48

<https://www.youtube.com/watch?v=WWEpyXidZ4M>

WHEN TO WATCH

After “The story.” Look for Ruth and Cobb among the first members of the Hall of Fame in 1939.

BACKUP VIDEO

Babe Ruth: His Life and Legend

National Baseball Hall of Fame and Museum • 1:44

<https://www.youtube.com/watch?v=Ci4QHK7fpYs>

A short Hall of Fame film about Babe Ruth’s life.

KEY TERMS

home run (HR)

A hit on which the batter circles all the bases and scores.

total bases

A count of bases from hits: 1 for a single, 2 for a double, 3 for a triple, 4 for a home run.

slugging percentage (SLG)

Total bases divided by at bats. It measures hitting power.

dead-ball era

The years before about 1920, when the ball was softer and home runs were rare.

career statistics

A player's totals and rates over all his seasons, a large sample.

CORE IDEAS

The story. Ty Cobb of the Detroit Tigers was the greatest star of baseball's early years. He hit for a high average, ran the bases fiercely, and stole 897 bases in his career. His career batting average of .366 is the highest in major-league history. Babe Ruth began as a pitcher for the Boston Red Sox and then became a hitter for the New York Yankees. In 1919 he hit 29 home runs, a new record; in 1920 he hit 54; and in 1927 he hit 60. When the first class was elected to the Baseball Hall of Fame in 1936, Cobb received the most votes of all, and Ruth tied for second.

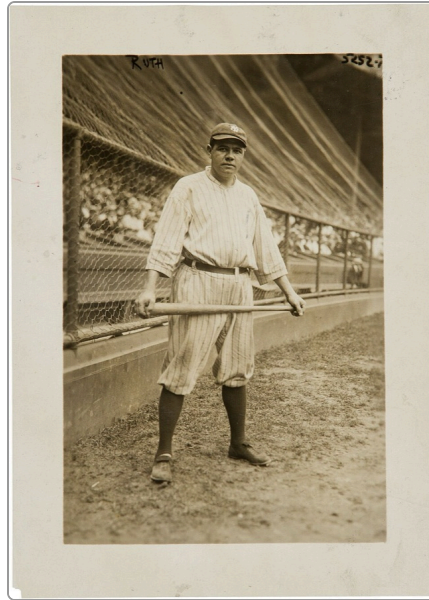
How they figured it out. Cobb played the old way: in the dead-ball era, a soft ball and huge ballparks made home runs rare, so the high-percentage play was to put the ball in play, get on base and run. Ruth saw that with a livelier ball he could swing hard and hit the ball over the fence. Many people thought this was reckless. But his numbers showed otherwise. He did make more strikeouts than Cobb, but he also walked a great deal, because pitchers were afraid of him, and his hits were worth far more bases. Each man found the style that earned the most for his team in his time.

The mathematics. $SLG = \frac{\text{total bases}}{AB}$. A single counts 1, a double 2, a triple 3 and a home run 4. In their careers, Cobb batted .366 with an on-base percentage of .433 and a slugging percentage of .512. Ruth batted .342, a little lower, but his on-base percentage was .474 and his slugging percentage .690, the highest in major-league history. In 1920 Ruth's on-base percentage was .532 and his slugging percentage .847. Batting average favors Cobb; on-base percentage and slugging favor Ruth.

On the field. There is more than one way to play the percentages. A small, fast player may be wise to put the ball in play and use his speed. A strong hitter may be wise to wait for a pitch he can drive. The numbers help each player find his own best style, instead of copying someone whose gifts are different. **The risk side:** a hard swing for

power also brings more strikeouts. Ruth accepted that cost because his walks and extra bases more than made up for it. A hitter who swings hard without Ruth's power gets the strikeouts without the reward.

PICTURE STUDY



Babe Ruth, Bain News Service, 1920 · Photograph · New York; Library of Congress

Image: [Wikimedia Commons](#), public domain

This photograph was taken in 1920, Ruth's first season with the New York Yankees, the year he hit 54 home runs, almost twice his own record. The Yankees then shared the Polo Grounds with the New York Giants, until Yankee Stadium opened in 1923.

Look closely

- Look at how Ruth holds the bat. Is it a light bat or a heavy one?
- Find the stripes on his uniform. The Yankees still wear pinstripes today.
- Look at his stance and his face. What kind of hitter does he look like?

Talk about it

1. Ruth struck out often but also walked and hit many home runs. Why was his style still high-percentage baseball?
2. Cobb and Ruth were very different hitters. What does that teach you about choosing your own best way to play?

PROJECT

Cobb versus Ruth: A Number Duel

You will compare Cobb and Ruth season by season with a parent's help, compute their statistics, and write a short report on which was the more valuable hitter, and why the answer depends on the numbers you choose.

You will need

- Your scorebook and calculator
- With a parent: a baseball statistics website, or the numbers in this lesson

Steps

1. Make a table: Player, AB, H, HR, BA, OBP, SLG.
2. Fill in the career numbers from this lesson: Cobb .366, .433, .512; Ruth .342, .474, .690.
3. With a parent, look up one great season for each: Cobb in 1911 and Ruth in 1920. Copy AB, H, HR, BA, OBP and SLG.
4. For each statistic, circle the higher number.
5. Compute at bats per home run for each player's career (Cobb hit 117 in 11,440 at bats; Ruth hit 714 in 8,399).
6. **Backyard test.** In your bucket game, try two styles for 20 turns each: "contact" (always aim for the near bucket) and "power" (always aim for the far bucket worth more bases). Compute BA and SLG for each style.
7. Write a one-page report: Which hitter was more valuable, and which numbers convinced you? Which style worked better for you in the backyard?

Tips

- No single number tells the whole story. Look at several.
- Remember the times: Cobb played most of his career in the dead-ball era.

Check your work

- ☐ My table compares Cobb and Ruth on at least five numbers.
- ☐ I computed at bats per home run for both.

- My report gives a clear answer with reasons, and includes my own backyard test.

SCOREBOOK

1. Make a side-by-side bar graph of Cobb's and Ruth's career BA, OBP and SLG.
2. Copy your backyard contact-versus-power table.
3. Write: "The style of play that fits my gifts best is... because..."

From history. The Hall of Fame's first class, elected in 1936, was Ty Cobb, Babe Ruth, Honus Wagner, Christy Mathewson and Walter Johnson. The museum in Cooperstown, New York, opened on June 12, 1939.

WORKED EXAMPLES

Example 1. In 10 at bats a hitter gets 2 singles, 1 double and 1 home run. Find his batting average and slugging percentage.

Show solution

1. Hits: $2 + 1 + 1 = 4$. BA: $\frac{4}{10} = .400$.

2. Total bases: $2(1) + 1(2) + 1(4) = 8$. SLG: $\frac{8}{10} = .800$.

Answer: Batting average .400 and slugging percentage .800.

Example 2. Ruth hit 714 home runs in 8,399 career at bats. About how many at bats did he need per home run?

Show solution

1. $8,399 \div 714 \approx 11.8$.

Answer: About one home run every 12 at bats.

COMMON MISTAKES

- Judging hitters by batting average alone.
- Forgetting that a home run counts 4 total bases, not 1.
- Comparing players from different eras without remembering how the game changed.

Make it a game • Number Duel. Set up a number duel between Ty Cobb and Babe Ruth, with a few stats on slips of paper. Each of you argues for one player using batting average, on-base percentage, home runs or slugging percentage. Compute slugging percentage live. After two minutes, swap sides and argue the other player. Winner is the one who used the most correct numbers.

UNIT 5

BASEBALL BY THE NUMBERS

No sport keeps track of itself like baseball. Every pitch, every hit and every out can be written down, and fans have been writing them down for more than 160 years. In this unit you will meet Henry Chadwick, who invented the box score, and learn to read one. You will learn the two most famous numbers about a hitter, the batting average and the on-base percentage, and why Branch Rickey argued that the second one matters more.

Then you will keep score of a real game yourself with a simple scorecard, the way fans and volunteers have done for generations, and use your own records to compute real statistics. Finally you will compare two of the greatest players who ever lived, Ty Cobb and Babe Ruth, and see how their numbers tell the story of two very different ways to play the odds at the plate.

PREREQUISITES

- Can find a rate as a fraction, a decimal and a percent (Units 1–4)
- Can divide with a calculator and round to three decimal places
- Knows the basic rules of baseball or softball: outs, bases, hits and walks

MASTERY CHECKLIST

- ☐ I can read a simple box score and tell what each column means.
- ☐ I can compute a batting average and an on-base percentage and write them the baseball way (such as .300).
- ☐ I can explain why getting on base matters so much when each team has only 27 outs.
- ☐ I can keep score of a real game using position numbers and simple symbols.
- ☐ I can compare Ty Cobb and Babe Ruth using their real numbers.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Baseball-Reference:** Free statistics for every major-league player and season, used in Lessons 5.2 and 5.4. Contains advertising; a parent should look at it with the student.
- **Retrosheet:** Free. Volunteer-built play-by-play records and box scores of past major-league games, mentioned in Lesson 5.3. Best for ages 11 and up with a parent.
- **National Baseball Hall of Fame and Museum:** Free online biographies of Henry Chadwick, Ty Cobb, Babe Ruth and Branch Rickey, which fit Lessons 5.1, 5.2 and 5.4.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

5.1	Henry Chadwick's Box Score	2 sessions of 45 minutes
5.2	Batting Average and On-Base Percentage	2 sessions of 45 minutes
5.3	Keeping Score of a Real Game	2 sessions of 45 minutes
5.4	Ty Cobb, Babe Ruth and the Numbers	2 sessions of 45 minutes

UNIT 5 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. 4 pts

Who was Henry Chadwick, and what did he give baseball?

2. 5 pts

A batter has 4 hits in 12 at bats and 3 walks. Find his batting average and simple on-base percentage.

3. 4 pts

Why does a walk help a team even though it does not count as a hit?

4. (4 pts)

What did Branch Rickey argue in his 1954 *Life* article, and whose numbers did he use?

5. (4 pts)

Write the position numbers for the pitcher, catcher, shortstop and center fielder. What does “4-3” mean?

6. (5 pts)

A batter’s scorecard boxes read: 1B, K, BB, HR, 6-3. Find AB, H, BB, BA and simple OBP.

7. (5 pts)

In 20 at bats a hitter has 3 singles, 2 doubles and 1 home run. Find his slugging percentage.

8. (4 pts)

Cobb’s career BA was .366 and Ruth’s .342. Does that prove Cobb was the better hitter? Explain using OBP and SLG.

9. (5 pts)

What would happen to a young player’s statistics if he swung hard at every pitch, like Ruth, without Ruth’s strength?

10. (4 pts)

Why is a statistic from one game less trustworthy than one from a whole season?

11. (6 pts)

Short answer: In three or four sentences, explain how keeping an honest scorebook connects to Proverbs 20:23.

ANSWER KEY

1. An English-born New York sportswriter (1824–1908) who created the baseball box score around 1859, gave us K for a strikeout, wrote guides and rules, and was elected to the Hall of Fame in 1938.
2. $BA\ 4 \div 12 \approx .333$. $OBP\ (4 + 3) \div (12 + 3) = 7 \div 15 \approx .467$.
3. It puts a runner on base and avoids an out. A team has only 27 outs, so every out saved keeps the inning alive.
4. That getting on base matters more than batting average; he used the numbers of Allan Roth, the Dodgers' statistician.
5. Pitcher 1, catcher 2, shortstop 6, center fielder 8. A 4-3 is a ground ball fielded by the second baseman and thrown to first for an out.
6. AB 4 (1B, K, HR, 6-3), H 2, BB 1. $BA\ 2 \div 4 = .500$; $OBP\ 3 \div 5 = .600$.
7. Total bases $3 + 4 + 4 = 11$; $SLG\ 11 \div 20 = .550$.
8. No. Ruth's OBP (.474 to .433) and SLG (.690 to .512) were both higher: he reached base more often and his hits were worth far more bases.
9. He would make many strikeouts without hitting many home runs or drawing many walks, so his BA, OBP and SLG would likely all fall. A style must fit the player's gifts.
10. One game is a very small sample; chance can make a good hitter go 0 for 4. Many games give a larger sample and a more trustworthy rate.
11. Proverbs 20:23 says false weights and balances are not good in God's sight. A scorebook is a kind of balance that measures what players did. Writing down exactly what happened, even when it is not what we hoped, is honest measuring, and every statistic built from it can be trusted.

PORTFOLIO PIECE: MY SEASON SCOREBOOK

Keep score of three games (real games watched with a parent, or radio or television broadcasts, or your own backyard games played with full rules). For each game include your complete scorecard for at least three innings. Then make a “season” table for one team or for your backyard players that adds up all three games: AB, H, BB, HR, BA, OBP and SLG for each player. Finish with a one-page report that names the team’s most valuable hitter and explains why, using at least two statistics, and that says how a one-game number differed from the three-game number for at least one player.

LOOK FOR	WHAT IT MEANS
Scorecards	Three games scored neatly with correct symbols and runs that match the final scores.
Season table	Totals added correctly across games, with BA, OBP and SLG computed and rounded to three places.
Reasoning	The most valuable hitter is chosen with at least two statistics, and the report explains why a larger sample is more trustworthy.
Honesty and neatness	Missed plays are marked honestly, not guessed, and the work is easy to read.

UNIT 6

THE HIGH-PERCENTAGE SHOT

In Unit 4 you learned to judge a shot by its points per try. In this unit you will see the same idea in four more sports. In soccer and hockey, where every goal counts the same, the high-percentage shot is usually the one taken closest to the goal and straight in front of it. In golf, it is often the shot aimed at the safe middle of the green instead of straight at the flag. In tennis, it is the second serve hit with care, so that a missed first serve does not become a lost point.

You will chart shots in your yard or living room, color them by how often they score, and learn why the best players are so patient about waiting for a good chance. In every lesson you will look at both sides: the percentage, or how often a choice pays off, and the risk, or what happens when it does not.

PREREQUISITES

- Can find a make rate and points per try (Unit 4)
- Can make a tally chart and a bar graph (Unit 2)
- Knows that larger samples are more trustworthy (Units 2 and 5)

MASTERY CHECKLIST

- ☐ I can explain why shots close to the goal and in front of it score more often in soccer and hockey.
- ☐ I can make a shot chart, color it by scoring rate and use it to plan.
- ☐ I can explain why a golfer often aims at the middle of the green, and test the idea at home.
- ☐ I can compute a tennis player's chance of winning a point with a hard or a safe second serve.
- ☐ I can name the percentage and the risk for each high-percentage choice.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Hockey-Reference**: Free statistics for every NHL season, including shots and shooting percentages, used in Lesson 6.2. Contains advertising; a parent should look at it with the student.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

6.1 Close to the Goal: Shots in Soccer

2 sessions of 45 minutes

6.2 The Shot Chart: Hockey's Slot

2 sessions of 45 minutes

6.3 Aim for the Middle of the Green

2 sessions of 45 minutes

6.4 First Serve, Second Serve

2 sessions of 45 minutes

UNIT 6 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. **4 pts**

Why do shots from close in front of the goal score more often than long shots or shots from a sharp angle?

2. **5 pts**

What was Charles Reep's base-rate mistake? How should he have counted?

3. **5 pts**

A team's shots are worth 0.30, 0.20, 0.05 and 0.05 xG. What are its expected goals? If it scored 2 goals, was it lucky or unlucky?

4. **4 pts**

Wayne Gretzky scored 92 goals on 370 shots in 1981–82. Find his shooting percentage.

5. **4 pts**

What is the slot in hockey, and why do defenders try to keep attackers out of it?

6. **5 pts**

Aiming at the flag, a golfer averages $0.8 \times 2 + 0.2 \times 4$ strokes. Aiming at the middle, he averages 2.2. Which plan is better, and by how much per hole?

7. (4 pts)

What would happen if a golfer far behind on the last hole always aimed at the safe middle? Explain.

8. (5 pts)

A server's first serve goes in 60% of the time and wins 70% of points when in. Her second serve goes in 90% and wins 50% when in. Find her chance of winning a point.

9. (4 pts)

In question 8, how often does she double-fault?

10. (4 pts)

Why do most tennis players hit a safer serve on the second try?

11. (6 pts)

Short answer: Choose two of the four sports in this unit. For each, name the high-percentage choice and its risk side in one or two sentences.

ANSWER KEY

1. The goal looks bigger and the goalkeeper has less time to react; from far away or a sharp angle, the goal looks narrow and the keeper can see the shot coming.
2. He counted that most goals came from short moves, but most moves of all kinds were short. He should have compared the fraction of short moves that scored with the fraction of long moves that scored.
3. $0.30 + 0.20 + 0.05 + 0.05 = 0.60$ xG. Scoring 2 is more than its chances were worth, so it was lucky.
4. $92 \div 370 \approx 0.249$, about 24.9%.
5. The area straight in front of the goal between the face-off circles. Shots from there score far more often than shots from outside it.
6. Flag: $1.6 + 0.8 = 2.4$. Middle: 2.2. The middle is better by 0.2 strokes per hole.
7. He would usually make a safe score but would almost never make the low score he needs to catch up. When far behind, a bolder aim may give a better chance to win.
8. $0.60 \times 0.70 + 0.40 \times 0.90 \times 0.50 = 0.42 + 0.18 = 0.60$, so 60%.
9. $0.40 \times 0.10 = 0.04$, so 4% of points.
10. A missed second serve loses the point at once (a double fault), so a safer serve that goes in more often usually wins more points overall.
11. Sample: Soccer: shoot from close and central; risk: waiting too long may mean no shot. Hockey: shoot from the slot; risk: it is the most guarded area. Golf: aim at the safe middle; risk: too safe when you must catch up. Tennis: safe second serve; risk: a strong returner can attack it.

PORTFOLIO PIECE: THE HIGH-PERCENTAGE PLAYBOOK

Make a four-page “playbook” with one page for each sport in this unit: soccer, hockey (or basketball), golf and tennis. Each page has your own data from the lesson project (a shot map, a shot chart, your putting averages or your serve rates), a colored diagram, the high-percentage choice in one bold sentence, and a short “risk side” box that says when the choice might be wrong. On the cover, write a one-paragraph explanation of what all four sports have in common.

LOOK FOR	WHAT IT MEANS
Own data	Each page uses the student’s own tallies, with rates correctly computed.
Diagrams	Shot maps and charts are neat, labeled and colored by rate.
Percentage and risk	Each page names the high-percentage choice and a sensible risk side.
Big idea	The cover paragraph clearly explains the shared idea of choosing the option that pays off most over many tries.

6.1 CLOSE TO THE GOAL: SHOTS IN SOCCER

OBJECTIVES

- Explain why shots from close in front of the goal score more often than long or wide shots.
- Tell the story of Charles Reep's notebooks and the mistake later analysts found in his reasoning.
- Measure your own scoring rate from different spots and add up expected goals.

Warm-up. Set up a small goal with two shoes or cones about 1 meter apart. Kick a soft ball at it 5 times from 2 meters straight in front, then 5 times from 2 meters off to the side at a sharp angle. Which spot scored more? Why?

WATCH

What is xG? | By The Numbers

Tifo Football by The Athletic · 4:08

<https://www.youtube.com/watch?v=zSaeaFcm1SY>

WHEN TO WATCH

After “How they figured it out.” Watch how each shot is given a number from 0 to 1 by where it was taken.

KEY TERMS

shot

An attempt to score a goal.

angle

How far to the side a shot is taken.
From a sharp angle, the goal looks narrow.

expected goals (xG)

The chance that a shot from a certain place becomes a goal, found by studying many thousands of shots. Adding a team's xG gives the number of goals its chances were worth on average.

base rate

How common something is overall. Forgetting the base rate is a common counting mistake.

CORE IDEAS

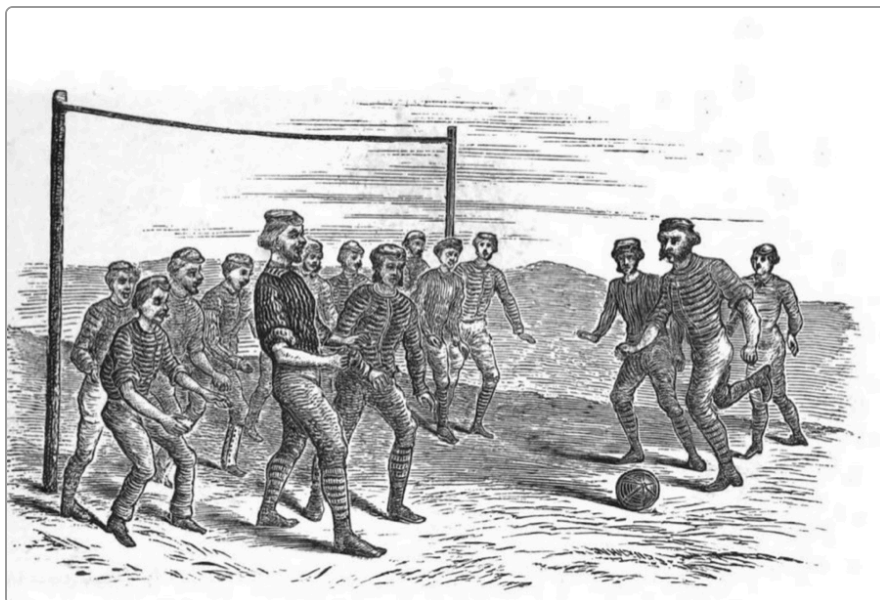
The story. On March 18, 1950, at a match between Swindon Town and Bristol Rovers in England, an accountant and former Royal Air Force officer named Charles Reep took out a notebook and began writing down every pass and every shot. He kept doing it for years, recording thousands of matches by hand. Reep noticed that most goals came from moves of only a few passes. He decided that teams should kick the ball forward quickly, and his ideas shaped English coaching for a generation.

How they figured it out. Later analysts found a mistake in Reep's reasoning. Most moves in soccer are short, because the ball is usually lost after a few passes. So of course most goals came from short moves; most moves of every kind were short! That is a base-rate mistake. The better question is: out of all the short moves, and out of all the long ones, which kind scores more often? Modern analysts asked a different question too. They recorded *where* hundreds of thousands of shots were taken and how often each spot scored. They found that the place a shot is taken from matters enormously. They call the scoring chance of a shot its **expected goals**, or xG.

The mathematics. Every shot has a scoring chance between 0 and 1. In many published models, a penalty kick scores about 3 times in 4, worth about 0.76 xG. A shot from close range straight in front of goal is often worth a few tenths. A shot from 25 meters out is worth only a few hundredths, about 1 or 2 times in 50. Adding up a team's shots gives its expected goals: three shots worth 0.40, 0.10 and 0.05 add to $0.40 + 0.10 + 0.05 = 0.55$ xG. One great chance is often worth more than several poor ones: a single 0.40 chance is worth more than ten 0.03 shots, which add to only 0.30.

On the field. Good teams try to work the ball close to goal and into the middle before shooting: a pass across the front of the goal, a cut-back to a teammate near the penalty spot. Defenders try the opposite: they force attackers wide and far out. **The percentage:** patient play that creates close, central shots. **The risk side:** waiting too long for a perfect chance may mean no shot at all, and a team that never shoots from distance lets defenders crowd the goal. And remember Reep's lesson: when you count, ask how often each kind of thing happens before you draw a conclusion.

PICTURE STUDY



An Early Association Football Match, Unknown artist, 1874 · Wood engraving · England

Image: [Wikimedia Commons](#), public domain

Association football, the game Americans call soccer, got its first written rules in England in 1863. This engraving from 1874 shows an early match. In those days the top of the goal was a tape stretched between the posts; the solid crossbar came a few years later.

Look closely

- Find the goal. How is it different from a modern goal with a crossbar and a net?
- Find the ball. How close to the goal are the players who are trying to score?
- Look at how many players crowd around the goal. Why would both teams gather there?

Talk about it

1. Without a net, how could the referee tell whether a shot had gone in? Why might shots from close range be easier to judge?
2. Why do both the attackers and the defenders crowd the space right in front of the goal?

Safety first. Use a soft ball, kick only toward the goal, and keep younger children and pets away from the shooting area. Do not practice heading; young players should not head the ball.

PROJECT

Backyard Expected Goals

You will find your own scoring rate from five spots around a homemade goal, turn each rate into an expected-goals value, and draw a shot map.

You will need

- A soft ball (a foam or light playground ball)
- Two cones, shoes or buckets for goalposts, about 1.5 meters apart
- A family member as goalkeeper (optional, sitting and using hands only)
- Painter's tape or chalk, a tape measure, graph paper and colored pencils

Steps

1. Mark five spots: close and straight in front (2 m), far and straight in front (5 m), close at a sharp angle, far at a sharp angle, and a “penalty” spot (3 m, straight in front).
2. **Predict.** Rank the five spots from best to worst before you shoot.
3. Take 15 shots from each spot. Tally goals and misses.
4. Find each spot's scoring rate as a decimal. That is your own xG for a shot from there.
5. Draw a bird's-eye shot map of your goal and spots. Write each spot's xG beside it and color the spots from green (best) to red (worst).
6. **Game test.** Play a short game with a family member: each player gets 10 shots and may choose any spot. Before shooting, add up the xG of the spots you choose. After shooting, compare your expected goals with your real goals.
7. Write which spots you would choose in a real game and why.

Tips

- Shoot with the same foot and the same kind of kick from every spot.
- If the goalkeeper is too good, use an empty goal. The point is to compare the spots.
- If two spots are very close in rate, shoot 15 more from each before deciding.

Check your work

- ☐ I took 15 shots from each of five spots and found each scoring rate.
- ☐ My shot map shows each spot's xG and is colored by rate.
- ☐ I compared expected goals with real goals in the game test.

SCOREBOOK

1. Draw your shot map neatly with a key.
2. Make a table: Spot, Distance, Goals, Shots, Scoring rate (xG).
3. Write: "Reep's mistake was... I would avoid it by..."

From history. Charles Reep (1904–2002) kept notebooks on matches for more than fifty years and advised several English clubs. Expected-goals models became common in the 2010s, when companies began recording the exact location of every shot in professional matches.

WORKED EXAMPLES

Example 1. A team takes four shots in a game: a penalty kick (0.76 xG), a close shot (0.35), and two long shots (0.03 each). What are its expected goals? It scored 0 goals. Did it play badly?

Show solution

1. Add the xG: $0.76 + 0.35 + 0.03 + 0.03 = 1.17$.
2. The team's chances were worth more than one goal on average; scoring 0 was unlucky.

Answer: 1.17 expected goals. The team made good chances and was unlucky to score none. Over many games, luck like this tends to even out.

COMMON MISTAKES

- Counting where goals came from without counting where all the shots came from (forgetting the base rate).
- Thinking more shots always means more goals. Shot quality matters as much as shot quantity.
- Judging a team by one game's score instead of by the chances it made.

Make it a game · Backyard xG Chart. Mark five spots on the floor at different distances and angles from a goal made of two shoes. Predict which spot scores most often. Take ten kicks from each with a paper ball and tally the makes with beans. Rank the spots. Say what Charles Reep missed in his reasoning, or why base rates matter.

6.2 THE SHOT CHART: HOCKEY'S SLOT

OBJECTIVES

- Explain what the slot is in hockey and why shots from there score more often.
- Compute shooting percentage and compare players and spots.
- Make and read a shot chart for a game of your own.

Warm-up. Lay a small box on its side as a goal. With a broom or a hockey stick and a soft ball, take 5 shots from straight in front at 2 meters and 5 shots from far off to the side. Which spot was better? What would a goalie do differently at each spot?

WATCH

Hockey Scoring-Slot Area Opportunities

Score100goals · 3:42

<https://www.youtube.com/watch?v=mPt-lpX8-zw>

WHEN TO WATCH

After “The mathematics,” before the project.

KEY TERMS

slot

The area straight in front of a hockey goal, between the face-off circles. Shots from there score most often.

shooting percentage

Goals divided by shots, written as a percent.

shot chart

A drawing of the playing area that shows where each shot was taken and whether it scored.

screen

A player standing in front of the goalie so he cannot see the shot.

CORE IDEAS

The story. In the 1981–82 season, Wayne Gretzky of the Edmonton Oilers scored 92 goals in the National Hockey League, a record that still stands. He took 370 shots that season and scored on 24.9% of them, about one goal for every four shots. Gretzky was not the hardest shooter in the league. His secret was where and when he shot. He often set up behind the opponent’s net, in a spot people called his “office,” and passed to teammates in front, or slipped into open space in front of the goal himself.

How they figured it out. For years coaches told players to “get to the net,” but they had only their eyes to go on. When leagues and analysts began recording the location of every shot, they could draw shot charts for whole seasons. The pattern was clear: shots from the **slot**, the area straight in front of the goal, go in far more often than shots from the blue line or from sharp angles near the boards. Rebounds, deflections and shots taken while a teammate screens the goalie are especially dangerous. In recent NHL seasons the average shot has gone in only about one time in ten, but shots from close in front go in much more often.

The mathematics. shooting percentage = $\frac{\text{goals}}{\text{shots}} \times 100\%$. Gretzky:

$\frac{92}{370} \approx 0.249 = 24.9\%$. A shot chart sorts shots by place. Suppose a player takes 20 shots from the slot and scores 4, and 30 from the blue line and scores 1. Slot: $\frac{4}{20} = 20\%$; blue line: $\frac{1}{30} \approx 3\%$. The slot shot is about six times as likely to score.

On the field. Coaches design plays to get the puck to a player in the slot: a quick pass from behind the net, a shot from the point that a teammate tips, a rebound. Defenders fight to keep opponents out of the slot and let them shoot from the outside. **The percentage:** shots from the slot. **The risk side:** the slot is also the most crowded, guarded part of the ice, and a long shot that reaches the net can still create a rebound. Good teams mix outside shots with patient passing to the slot, so defenders cannot simply wait in front.

PICTURE STUDY



A Hockey Game in High Park, Toronto, John Boyd, 1927 · Photograph · High Park, Toronto, Ontario, Canada

Image: [Wikimedia Commons](#), public domain

John Boyd was a Canadian news photographer who took thousands of pictures of life in Toronto. Here he shows an outdoor game on a frozen rink in High Park, one of the city's largest parks, in the 1920s. Outdoor rinks like this are where generations of Canadian players learned the game.

Look closely

- Find the players with their sticks on the ice. Where is the puck likely to be?
- Can you find a goal? If not, where do you think it might be?
- Look at the players gathered in a group. In hockey, where do players crowd together most?

Talk about it

1. If you were playing in this game, where would you try to get to before you shot? Why?
2. How would you keep a shot chart of an outdoor game like this one?

Safety first. Use a soft ball or a sock ball, not a puck, indoors. Keep sticks below the waist, keep others well out of the way, and shoot only toward the goal.

PROJECT

My Hockey (or Basketball) Shot Chart

You will play a short indoor or driveway game, record the place and result of every shot, and turn your record into a colored shot chart. Choose floor hockey with a soft ball, or basketball with your basket from Unit 4.

You will need

- A soft ball and a broom or plastic stick, with a box or small net for a goal (or your basket and soft ball)
- Painter's tape to mark zones: the slot, the sides and the far "point"
- A family member to play or to keep the chart
- Graph paper and colored pencils

Steps

1. Draw a map of your playing area on graph paper and divide it into zones: the slot (close and in front), the left side, the right side and the far point.
2. Play a short game, or a shooting drill of at least 40 shots, moving around the zones.
3. For every shot, the chart keeper puts a mark in the right zone: an O for a goal and an X for a miss.
4. Count the shots and goals in each zone and find the shooting percentage.
5. Color the zones: green for the highest percentage, yellow for the middle, red for the lowest.
6. Write three sentences: Which zone was best? Did you take most of your shots from the best zone? What would you change?
7. Play once more, trying to take more shots from your best zone, and see whether your total goals go up.

Tips

- The chart keeper should mark each shot immediately.
- Keep the zones the same size and shape in both games so the charts can be compared.
- A goalie (sitting, using hands only) makes the chart more realistic.

Check your work

- ☐ My chart shows at least 40 shots, each in the right zone.
- ☐ I found the shooting percentage for each zone and colored the chart.
- ☐ I played a second time and compared the results.

SCOREBOOK

1. Draw your colored shot chart with a key and the percentage in each zone.
2. Make a table: Zone, Goals, Shots, Shooting %.
3. Write: “If I were the coach, I would tell my players to... because...”

From history. Gretzky also had 120 assists in 1981–82 and 212 points, both records at the time. By the end of his career he held more than sixty NHL records, and his number, 99, was retired by every team in the league.

WORKED EXAMPLES

Example 1. Player A scores 12 goals on 150 shots. Player B scores 9 goals on 60 shots. Find each shooting percentage. Who shoots better on average?

Show solution

1. A: $12 \div 150 = 0.08 = 8\%$.
2. B: $9 \div 60 = 0.15 = 15\%$.

Answer: Player B shoots better per shot (15% against 8%), even though A scored more goals. A takes more shots, perhaps from farther out.

COMMON MISTAKES

- Judging a shooter only by goals, without counting shots.
- Mixing up the zones in the middle of the game, so the chart becomes unreliable.
- Thinking a long shot is always a bad choice. It can still be right when nothing better is open.

Make it a game • Slot Shooter. Mark a slot area on a large paper sheet in front of a bucket. Take ten tosses from the slot and ten from a wide angle. Compute your shooting percentage for each. Your parent predicts first. Say what a screen does in hockey. The spot with the higher percentage wins the round, and you chart both spots.

6.3 AIM FOR THE MIDDLE OF THE GREEN

OBJECTIVES

- Explain why a golfer often aims at the middle of the green instead of straight at the flag.
- Tell how Mark Broadie measured golf shots with strokes gained.
- Test aiming at the target versus aiming at the safe middle with a home putting course.

Warm-up. Put a coaster on the floor as a “hole” and lay a towel just beyond it as a “pond.” Roll a small ball at the coaster 10 times. Any ball that stops on the towel costs you an extra point. Then roll 10 more aiming a little short of the coaster, away from the towel. Which way gave you fewer points?

WATCH

The Clever Aiming Strategy Tour Pros Actually Use | The Game Plan | Golf Digest

Golf Digest · 9:25

<https://www.youtube.com/watch?v=6HUyoAvHL88>

WHEN TO WATCH

After “How they figured it out.” Watch how the coach chooses a target that leaves room for a miss.

KEY TERMS

green

The short, smooth grass around the hole in golf, where players putt.

hazard

A place on a golf course, such as water or sand, that makes the next shot much harder.

course management

Planning each golf shot so that a miss leaves an easy next shot, not a disaster.

strokes gained

A way to measure a golf shot by how many strokes it saved compared with an average player.

dispersion

How much a player's shots spread out around where he aimed.

CORE IDEAS

The story. Ben Hogan was one of the greatest golfers of the twentieth century, famous for practicing for hours and for planning every shot. Most golfers aim straight at the flag. Hogan often aimed at the safe side of the green, so that a shot that drifted would still leave an easy putt or chip, instead of landing in water or sand. Many years later a professor at Columbia University, Mark Broadie, turned that wisdom into numbers. His way of measuring shots, called **strokes gained**, was adopted by the PGA TOUR in 2011.

How they figured it out. Broadie studied the location of hundreds of thousands of shots by professional golfers. From every spot on a course, he found how many more strokes an average tour player needs to finish the hole. A shot is good if it moves the ball to a spot that saves strokes. The data also showed that every golfer's shots spread out around the target, even the best players'. So the best place to aim is the one that gives the best result for the whole spread of shots, not just for a perfect shot. When the flag is right next to water, aiming at the middle of the green gives up a little when the shot is perfect, and saves a great deal when it is not.

The mathematics. Compare two plans with simple, made-up numbers. **Aim at the flag:** 8 times in 10 the ball lands on the green and the player finishes in 2 more strokes; 2 times in 10 it goes in the water and he needs 4 more. Average: $0.8 \times 2 + 0.2 \times 4 = 1.6 + 0.8 = 2.4$ strokes. **Aim at the middle:** 10 times in 10 the ball lands on the green, but farther from the hole, so he finishes in 2.2 strokes on average. Average: 2.2. Aiming at the middle saves 0.2 strokes a hole, which over 18 holes is about 3.6 strokes.

On the field. Wise golfers ask, "Where is the trouble, and where is the safe miss?" before every shot. **The percentage:** the target that gives the fewest strokes on average over all the ways the shot might go. **The risk side:** playing safe is not always right. A golfer far behind on the last holes may need to attack the flag, because a safe score will not catch up. Self-control is part of course management: "every man that striveth for the mastery is temperate in all things" (1 Corinthians 9:25).

PICTURE STUDY



Harry Vardon, Unknown photographer, 1899 · Photograph · England

Image: [Wikimedia Commons](#), public domain

Harry Vardon, from the island of Jersey, won the Open Championship six times between 1896 and 1914, more than anyone else in history. He was famous for hitting the ball straight and steady, again and again, and the overlapping grip he made popular is still called the Vardon grip.

Look closely

- Look at how Vardon stands. Does he look calm or tense?
- Look at his clothes. How are they different from what golfers wear today?
- Find the golf club. Golf clubs of his day had wooden shafts.

Talk about it

1. Vardon was known for hitting straight shots time after time. How does a steady, small spread of shots change where a golfer should aim?
2. Why would a calm, patient person make a good golfer?

Safety first. Use a soft or plastic golf ball and a putter or a broom handle indoors, with short gentle strokes only. Full swings with real clubs are only for outdoors with an adult watching and a wide clear circle.

PROJECT

A Home Putting Course: Flag or Middle?

You will build a small putting course with a “hole” next to a “hazard,” test two aiming plans, and find which one gives fewer strokes on average.

You will need

- A plastic or soft golf ball (or a tennis ball), and a putter, broom handle or yardstick to push it
- A coaster or paper plate for the hole, a towel for the hazard
- Painter’s tape for the starting line, and a tape measure
- Your scorebook

Steps

1. Place the “hole” about 3 meters from the starting line. Lay the towel just past and to one side of the hole.
2. Make the rules: you finish when the ball stops touching the hole. A ball that stops on the towel adds 2 penalty strokes and is played from the start again.
3. **Plan A, aim at the hole.** Play 15 holes aiming straight at the hole. Record the strokes for each.
4. **Plan B, aim at the safe side.** Play 15 holes aiming a little short and away from the towel. Record the strokes.
5. Find the average strokes per hole for each plan.
6. Move the towel farther away and test again with 10 holes each. Does the better plan change?
7. **Decide.** Write a rule for yourself: “When trouble is close to the hole, I will aim... When trouble is far away, I will aim...”

Tips

- Use the same ball and the same kind of stroke for both plans.
- Alternate plans every 5 holes so both get the same warm-up.
- Record every stroke, including penalties. Honest scores make honest averages.

Check your work

- ☐ I played at least 15 holes with each plan and found each average.
- ☐ I tested a second setup with the hazard moved.
- ☐ I wrote an aiming rule based on my data.

SCOREBOOK

1. Draw your putting course from above, with the hole, the hazard and both aiming lines.
2. Make a table: Plan, Holes, Total strokes, Average strokes per hole.
3. Write: “A safe aim is wise when... but a bold aim is wise when...”

From history. Mark Broadie explained strokes gained for ordinary golfers in his 2014 book *Every Shot Counts*. The PGA TOUR first used it for putting in 2011 and later for every kind of shot.

WORKED EXAMPLES

Example 1. Aiming at the flag, a golfer finishes in 2 strokes 70% of the time and in 4 strokes 30% of the time. Aiming at the middle, he always finishes in 2.3 strokes on average. Which plan is better?

Show solution

1. Flag: $0.7 \times 2 + 0.3 \times 4 = 1.4 + 1.2 = 2.6$.
2. Middle: 2.3.

Answer: Aiming at the middle is better: 2.3 strokes against 2.6 on average.

COMMON MISTAKES

- Judging a plan by its best shot instead of its average over many shots.
- Forgetting to count penalty strokes.
- Thinking safe is always best. When you must catch up, a bolder aim may be right.

Make it a game • Flag or Middle?. Place a cup as a flag at the edge of a rug with a hazard line near it. Putt ten balls aiming at the flag and ten at the middle of the rug, counting how many finish on the green. Predict first. Explain course management to your parent. Which aim scored more? Say why Mark Broadie measured shots.

Scripture. “And every man that striveth for the mastery is temperate in all things.” (1 Corinthians 9:25, KJV)

Aiming at the safe middle when the flag is tempting takes self-control. Paul says every athlete who wants to master his sport must learn to be temperate.

6.4 FIRST SERVE, SECOND SERVE

OBJECTIVES

- Explain the tennis rule of two serves and what a double fault is.
- Compute a server's chance of winning a point with a safe or a hard second serve.
- Test a hard serve and a safe serve with a soft ball and decide which second serve is better for you.

Warm-up. Stand 4 meters from a wall with a soft ball and mark a box on the wall with tape. Throw 10 hard throws into the box and 10 soft, careful throws. Which went in more often? If you only had two chances to hit the box, how would you use them?

WATCH

Get Rid of Double Faults: Serve Lesson with Patrick Mouratoglou

Patrick Mouratoglou • 11:27

https://www.youtube.com/watch?v=6ko_0cjfP-0

WHEN TO WATCH

After “The mathematics.” Listen for why the second serve uses more spin.

BACKUP VIDEO

How To Teach Kids Tennis (Ep 1) - The Serve

The Tennis Mentor • 5:58

<https://www.youtube.com/watch?v=hXLeq30tGAc>

For younger players: how to learn a simple, safe serve.

KEY TERMS

serve

The shot that starts each point in tennis. It must land in the service box diagonally across the net.

fault

A serve that misses the service box or hits the net and falls on the server's side.

double fault

Missing both the first and the second serve. The server loses the point.

spin

Turning of the ball as it flies. Spin helps a serve curve down into the box, so it can be hit with more margin over the net.

CORE IDEAS

The story. The first Wimbledon Championships were played in the summer of 1877 at the All England Croquet and Lawn Tennis Club in London. Twenty-two men entered. In those early days many players served underhand, just to get the ball safely into play. As players grew stronger, they learned to serve overhand and hit the ball hard. But the rules gave every server two tries. If the first serve missed, the server could serve again. Only if the second one missed too, a **double fault**, did he lose the point.

How they figured it out. Players and coaches noticed that a hard, flat serve wins many points when it goes in, but it misses often. A careful serve with spin goes in much more often, but the opponent can return it more easily. The two-serve rule means you can take a risk on the first serve, because a miss still leaves you a second chance. On the second serve, there is no chance left, so most players switch to a safer serve with spin. Analysts who count serve results for professional players have shown why this plan usually wins the most points.

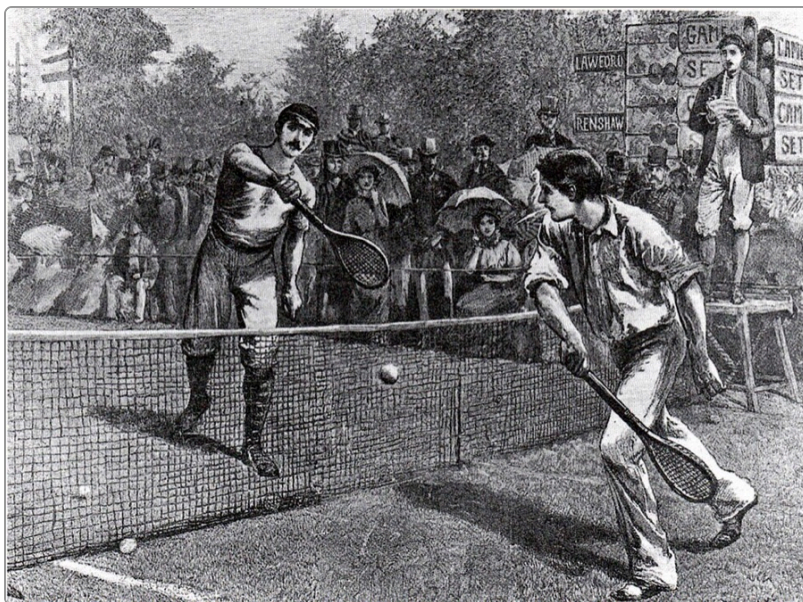
The mathematics. Here are illustrative numbers. A hard serve goes in 60% of the time and wins 75% of the points when it does. A safe serve goes in 90% of the time and wins 55% of the points when it does. **Plan 1, hard then safe:**

$0.60 \times 0.75 + 0.40 \times 0.90 \times 0.55 = 0.45 + 0.198 \approx 0.65$. **Plan 2, hard then hard:**

$0.60 \times 0.75 + 0.40 \times 0.60 \times 0.75 = 0.45 + 0.18 = 0.63$. The safe second serve wins a little more often, mainly because it double-faults far less: Plan 2 double-faults $0.40 \times 0.40 = 16\%$ of the time, Plan 1 only $0.40 \times 0.10 = 4\%$.

On the field. Good servers take a bigger risk on the first serve and a smaller one on the second. **The percentage:** the plan that wins the most points overall. **The risk side:** a second serve that is too safe can be attacked by a strong returner, so the numbers can change against a great opponent. And a double fault is not always a mistake; sometimes it is the price of a reasonable risk. Judge a serving plan over many points, not one.

PICTURE STUDY



Lawford and Renshaw at Wimbledon, Arthur Hopkins, 1881 · Wood engraving from a London illustrated newspaper · Wimbledon, London

Image: [Wikimedia Commons](#), public domain

This picture was printed in a London illustrated newspaper in August 1881. It shows William Renshaw and Herbert Lawford playing at the Wimbledon Championships that summer. Renshaw won Wimbledon seven times, and he and Lawford were two of the great players of the early game.

Look closely

- Find the two players. Which one is reaching high, and which one is bending low?
- Look at the rackets. How are their shapes different from rackets today?
- Find the score boards and the crowd. How did spectators keep track of the score?

Talk about it

1. In the early days, many players served underhand. Why might a safe underhand serve have made sense then?
2. If you were serving in this match and had already missed your first serve, what kind of second serve would you hit? Why?

Safety first. Use a soft ball or balloon indoors and swing only a light plastic racket or your hand, with plenty of space. Keep clear of windows, lamps and people.

PROJECT

Hard Serve, Safe Serve

You will measure your own hard and safe “serves” with a soft ball against a wall, compute your chance of winning a point with each plan, and choose your best second serve.

You will need

- A soft ball or a tennis ball (outdoors) and a wall or garage door
- Painter’s tape or chalk to mark a “service box” on the wall and a line on the ground for the net height
- A family member to act as the returner (optional)
- Your scorebook and calculator

Steps

1. Mark a box on the wall about 1 meter square, with its bottom edge about 1 meter above the ground (the “net”). Mark a serving line 4 to 5 meters away.
2. **Hard serves.** Throw or hit 20 hard serves. Tally how many land in the box.
3. **Safe serves.** Throw or hit 20 careful serves with a high arc. Tally how many land in the box.
4. Find each “in” rate as a decimal.
5. With a returner, play 10 points with hard serves that go in and 10 with safe serves that go in. Tally how many points you win each way. (Or use the illustrative 75% and 55%.)
6. Use the formula from the lesson to find your chance of winning a point with Plan 1 (hard then safe) and Plan 2 (hard then hard).
7. **Decide.** Which plan wins more points for you? How often does each plan double-fault?

Tips

- Warm up your arm with easy throws before the hard serves.
- Keep the serving line and box the same for every try.
- If you play outdoors with a real tennis ball, a parent should watch, and nobody should stand near the wall.

Check your work

- ☐ I measured my hard and safe in rates with 20 tries each.
- ☐ I computed my chance of winning a point with both plans.
- ☐ I chose a second serve and explained the double-fault risk.

SCOREBOOK

1. Draw a tree diagram for a service point: first serve in or out, then second serve in or out, then point won or lost. Write your rates on the branches.
2. Make a table: Plan, Chance of winning a point, Chance of a double fault.
3. Write: “On a second serve I would decide to... because...”

From history. The Wimbledon Championships began in 1877 with only men’s singles; women’s singles were added in 1884. Wimbledon is still played on grass, the surface the game was invented on.

WORKED EXAMPLES

Example 1. A young player’s first serve goes in 50% of the time and wins 70% of points when in. Her safe second serve goes in 80% of the time and wins 50% of points when in. Find her chance of winning a point on serve.

Show solution

1. First serve in and won: $0.50 \times 0.70 = 0.35$.
2. First serve missed (0.50), second in (0.80) and won (0.50):
 $0.50 \times 0.80 \times 0.50 = 0.20$.
3. Add: $0.35 + 0.20 = 0.55$.

Answer: She wins about 55% of points on her serve. Her double faults happen $0.50 \times 0.20 = 10\%$ of the time.

COMMON MISTAKES

- Thinking the hardest serve is always the best serve.
- Forgetting that a missed second serve loses the point immediately.
- Multiplying when you should add. Multiply along one branch of the tree, then add the branches that win.

Make it a game • Safe Serve Test. Toss a ball underhand into a box on the floor for a first serve, with a hard throw and a gentle throw. Take ten hard throws and ten gentle ones, counting each as a bean. Predict which gives the better chance of winning the point. Explain what a double fault is. Calculate which second serve is smarter.

UNIT 6

THE HIGH-PERCENTAGE SHOT

In Unit 4 you learned to judge a shot by its points per try. In this unit you will see the same idea in four more sports. In soccer and hockey, where every goal counts the same, the high-percentage shot is usually the one taken closest to the goal and straight in front of it. In golf, it is often the shot aimed at the safe middle of the green instead of straight at the flag. In tennis, it is the second serve hit with care, so that a missed first serve does not become a lost point.

You will chart shots in your yard or living room, color them by how often they score, and learn why the best players are so patient about waiting for a good chance. In every lesson you will look at both sides: the percentage, or how often a choice pays off, and the risk, or what happens when it does not.

PREREQUISITES

- Can find a make rate and points per try (Unit 4)
- Can make a tally chart and a bar graph (Unit 2)
- Knows that larger samples are more trustworthy (Units 2 and 5)

MASTERY CHECKLIST

- ☐ I can explain why shots close to the goal and in front of it score more often in soccer and hockey.
- ☐ I can make a shot chart, color it by scoring rate and use it to plan.
- ☐ I can explain why a golfer often aims at the middle of the green, and test the idea at home.
- ☐ I can compute a tennis player's chance of winning a point with a hard or a safe second serve.
- ☐ I can name the percentage and the risk for each high-percentage choice.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Hockey-Reference:** Free statistics for every NHL season, including shots and shooting percentages, used in Lesson 6.2. Contains advertising; a parent should look at it with the student.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

6.1	Close to the Goal: Shots in Soccer	2 sessions of 45 minutes
6.2	The Shot Chart: Hockey's Slot	2 sessions of 45 minutes
6.3	Aim for the Middle of the Green	2 sessions of 45 minutes
6.4	First Serve, Second Serve	2 sessions of 45 minutes

UNIT 6 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. 4 pts

Why do shots from close in front of the goal score more often than long shots or shots from a sharp angle?

2. 5 pts

What was Charles Reep's base-rate mistake? How should he have counted?

3. 5 pts

A team's shots are worth 0.30, 0.20, 0.05 and 0.05 xG. What are its expected goals? If it scored 2 goals, was it lucky or unlucky?

4. 4 pts

Wayne Gretzky scored 92 goals on 370 shots in 1981–82. Find his shooting percentage.

5. 4 pts

What is the slot in hockey, and why do defenders try to keep attackers out of it?

6. 5 pts

Aiming at the flag, a golfer averages $0.8 \times 2 + 0.2 \times 4$ strokes. Aiming at the middle, he averages 2.2. Which plan is better, and by how much per hole?

7. (4 pts)

What would happen if a golfer far behind on the last hole always aimed at the safe middle? Explain.

8. (5 pts)

A server's first serve goes in 60% of the time and wins 70% of points when in. Her second serve goes in 90% and wins 50% when in. Find her chance of winning a point.

9. (4 pts)

In question 8, how often does she double-fault?

10. (4 pts)

Why do most tennis players hit a safer serve on the second try?

11. (6 pts)

Short answer: Choose two of the four sports in this unit. For each, name the high-percentage choice and its risk side in one or two sentences.

ANSWER KEY

1. The goal looks bigger and the goalkeeper has less time to react; from far away or a sharp angle, the goal looks narrow and the keeper can see the shot coming.
2. He counted that most goals came from short moves, but most moves of all kinds were short. He should have compared the fraction of short moves that scored with the fraction of long moves that scored.
3. $0.30 + 0.20 + 0.05 + 0.05 = 0.60$ xG. Scoring 2 is more than its chances were worth, so it was lucky.
4. $92 \div 370 \approx 0.249$, about 24.9%.
5. The area straight in front of the goal between the face-off circles. Shots from there score far more often than shots from outside it.
6. Flag: $1.6 + 0.8 = 2.4$. Middle: 2.2. The middle is better by 0.2 strokes per hole.
7. He would usually make a safe score but would almost never make the low score he needs to catch up. When far behind, a bolder aim may give a better chance to win.
8. $0.60 \times 0.70 + 0.40 \times 0.90 \times 0.50 = 0.42 + 0.18 = 0.60$, so 60%.
9. $0.40 \times 0.10 = 0.04$, so 4% of points.
10. A missed second serve loses the point at once (a double fault), so a safer serve that goes in more often usually wins more points overall.
11. Sample: Soccer: shoot from close and central; risk: waiting too long may mean no shot. Hockey: shoot from the slot; risk: it is the most guarded area. Golf: aim at the safe middle; risk: too safe when you must catch up. Tennis: safe second serve; risk: a strong returner can attack it.

PORTFOLIO PIECE: THE HIGH-PERCENTAGE PLAYBOOK

Make a four-page “playbook” with one page for each sport in this unit: soccer, hockey (or basketball), golf and tennis. Each page has your own data from the lesson project (a shot map, a shot chart, your putting averages or your serve rates), a colored diagram, the high-percentage choice in one bold sentence, and a short “risk side” box that says when the choice might be wrong. On the cover, write a one-paragraph explanation of what all four sports have in common.

LOOK FOR	WHAT IT MEANS
Own data	Each page uses the student’s own tallies, with rates correctly computed.
Diagrams	Shot maps and charts are neat, labeled and colored by rate.
Percentage and risk	Each page names the high-percentage choice and a sensible risk side.
Big idea	The cover paragraph clearly explains the shared idea of choosing the option that pays off most over many tries.

ODDS AT THE GAME TABLE

Family board games are wonderful places to practice playing the odds, because every turn is a small decision under uncertainty. Should you keep those three 5s and roll for more, or take the sure points? Where should you fire your next shot in *Battleship*? Which squares on a *Monopoly* board get landed on most? When should you stop rolling and keep what you have?

In this unit you will use everything you have learned (counting outcomes, expected points, tallies and fair tests) to become a wiser game player. You will keep a scorebook beside the board, test strategies over many games, and learn that a good strategy is judged over many turns, not by one lucky roll. As always, every game keeps score in points only.

PREREQUISITES

- Can count the 36 outcomes of two dice and find the probability of any total (Unit 2)
- Can find expected points per round (Lesson 3.4)
- Knows that dice have no memory (Lesson 2.4)

MASTERY CHECKLIST

- ☐ I can find the chance of getting a needed number when rerolling dice in Yahtzee, and decide whether to keep or reroll.
- ☐ I can explain the hunt-and-target method in Battleship and why a checkerboard pattern of shots works.
- ☐ I can explain why Jail and the orange properties are landed on so often in Monopoly.
- ☐ I can use a stopping rule in a push-your-luck game and explain why it works.
- ☐ I can test two game strategies fairly over many games and compare their average scores.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Yahtzee (Hasbro):** The publisher's page for the classic five-dice game in Lesson 7.1. Ages 8 and up. Five ordinary dice and a paper score sheet work just as well.
- **Can't Stop (Eagle-Gryphon Games):** The current publisher's page for Sid Sackson's push-your-luck dice game in Lesson 7.4. Ages 8 and up. The lesson includes a paper-and-dice version.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

7.1	Yahtzee: Keep or Reroll?	2 sessions of 45 minutes
7.2	Battleship: Hunt and Target	2 sessions of 45 minutes
7.3	Monopoly's Most-Landed Squares	2 sessions of 45 minutes
7.4	Can't Stop: Knowing When to Stop	2 sessions of 45 minutes

UNIT 7 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. 5 pts

In Yahtzee you have four 3s and two rolls left. What is the chance you get the fifth 3? Show your work.

2. 4 pts

Why is a one-roll Yahtzee so rare, and how do rerolls change that?

3. 4 pts

Late in a Yahtzee game you can take a sure 20 points or reroll for a 50-point Yahtzee with an 11-in-36 chance, scoring 0 if you miss. Which has more expected points?

4. (4 pts)

In Battleship, the ships cover 17 of 100 squares. What is the chance a random first shot hits?

5. (5 pts)

Explain hunt and target, and why hunting on a checkerboard pattern is enough to find every ship.

6. (4 pts)

Name three ways a Monopoly player can be sent to Jail.

7. (5 pts)

From Jail (square 10), what is the probability of landing on square 19 (New York Avenue)? On any orange square (16, 18 or 19)?

8. (5 pts)

In Pig your turn total is 15. Should you roll again? Show the expected gain and loss.

9. (4 pts)

What would happen if a Pig player never stopped rolling until he rolled a 1?

10. (4 pts)

In Can't Stop, why is it safer to push your luck when your markers are in columns 6, 7 and 8 than in columns 2, 3 and 12?

11. (6 pts)

Short answer: In three or four sentences, explain why a wise player sets a stopping rule before the turn begins, and connect it to Ecclesiastes 3:6.

ANSWER KEY

1. Missing twice: $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$. Getting it: $1 - \frac{25}{36} = \frac{11}{36} \approx 31\%$.
2. Only 6 of the 7,776 rolls of five dice are all the same (1 in 1,296). Keeping matching dice and rerolling the rest gives several chances, so a Yahtzee comes about 1 turn in 22 with good keeping.
3. Reroll: $\frac{11}{36} \times 50 \approx 15.3$ points on average, less than the sure 20. Take the sure 20.
4. $\frac{17}{100} = 17\%$.
5. Hunt: search for a ship; target: after a hit, shoot the neighboring squares to finish it. The smallest ship is two squares long, so it always covers one square of each checkerboard color; shooting only one color will find every ship.
6. Landing on the “Go to Jail” square, drawing a card that sends you there, and rolling doubles three times in a row.
7. Square 19 needs a 9: $\frac{4}{36} = \frac{1}{9}$. Any orange: $\frac{5+5+4}{36} = \frac{14}{36} \approx 39\%$.
8. Gain about $\frac{20}{6} \approx 3.3$; loss $\frac{15}{6} = 2.5$. Gain is bigger, so roll again.
9. Every turn would end in a bust, so he would never keep any points.
10. Totals near 7 can be made in many more ways, so a roll of four dice can move you about 92% of the time in 6, 7 and 8 but only about 44% of the time in 2, 3 and 12.
11. Feelings in the moment push people to stop too soon after a scare or go too far after a lucky streak. A rule made calmly, using counting, does better over many turns. Ecclesiastes 3:6 says there is “a time to keep, and a time to cast away”; a stopping rule is a way of knowing which time it is.

PORTFOLIO PIECE: A STRATEGY GUIDE FOR A FAMILY GAME

Choose one family game from this unit (Yahtzee, Battleship, Monopoly or Can’t Stop) or another dice or board game your family plays. Write and illustrate a three- or four-page strategy guide for a younger player. Include: the basic rules in a few sentences; at least two pieces of advice backed by counting (for example, a reroll chance, a hunting pattern or a busy square); data from your own tests in this unit, with a table or bar graph; and a “risk side” section that tells when the advice might be wrong. Then teach the game and your advice to a younger child or a grandparent and play three games together, recording the scores in points.

LOOK FOR	WHAT IT MEANS
Counting	At least two pieces of advice are supported by correct probabilities or counts.
Own data	Tables or graphs from the student’s own tests are included and explained.
Risk side	The guide honestly explains when each piece of advice might fail.
Teaching	The guide is clear for a younger reader, and three games with a learner are recorded.

7.1 YAHTZEE: KEEP OR REROLL?

OBJECTIVES

- Find the chance of rolling a needed number on one or two rerolls.
- Decide whether to keep dice and reroll, or take a sure score.
- Test two Yahtzee strategies over many turns and compare their average scores.

Warm-up. Roll five dice. Keep all the dice that match your most common number and reroll the rest, up to two more times. Did you get five of a kind? Try it five times and count your successes.

WATCH

How to Play Yahtzee

[Triple S Games](#) · 2:08

<https://www.youtube.com/watch?v=AHDgpuEzopc>

WHEN TO WATCH

Before the core ideas, if you do not already know the rules of Yahtzee.

BACKUP VIDEO

Yahtzee Probability

[Craig Rutherford](#) · 2:56

<https://www.youtube.com/watch?v=h5ytIVCyrUc>

For older students: counting the chances of a Yahtzee.

KEY TERMS

reroll

Rolling some of your dice again while keeping the others.

sure score

Points you can take now without any more risk.

Yahtzee

Five dice all showing the same number, worth 50 points in the game of the same name.

strategy

A plan for making choices, used turn after turn.

CORE IDEAS

The story. Yahtzee began with a Canadian couple who invented a dice game to play with friends on their yacht, which is how it got its name. In 1956 a New York toy maker, Edwin Lowe, bought the rights and began selling it. Since then it has become one of the best-known family games in the world. Each turn, a player rolls five dice up to three times, keeping any dice he likes between rolls, and then must write a score in one of thirteen boxes on the score card.

How they figured it out. Players soon learned that Yahtzee is full of choices. With three 5s after the first roll, should you go for more 5s, or try for a straight?

Mathematicians and computer programmers later worked out the very best choice for every situation, and the best plan turned out to be complicated. But the basic idea is simple enough for anyone: before you reroll, count your chances, and compare what you might win with what you might lose. The chance of rolling a Yahtzee all at once is only 6 out of $6^5 = 7,776$, or 1 in 1,296, but keeping matching dice and rerolling the others makes it far more likely, about 1 turn in 22.

The mathematics. Suppose you have four 5s and two rolls left to get the fifth. On one roll of one die, the chance of a 5 is $\frac{1}{6}$. The chance of missing is $\frac{5}{6}$. Missing twice in a row has chance $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$, so the chance of getting the 5 within two rolls is $1 - \frac{25}{36} = \frac{11}{36} \approx 31\%$. With three 5s and two rolls left, the chance of finishing a Yahtzee drops to about 9%. Counting “the chance of missing every time” and subtracting from 1 is a useful trick whenever you have several tries.

On the field. Every turn in Yahtzee asks: keep or reroll? **The percentage:** go for the big score when your chances are good and a miss still leaves a useful score. **The risk side:** if you reroll and miss, you may have to write a zero in a box. A wise player keeps the end of the game in mind: early on, there is room to take chances; late in the game, with only a few boxes left, a sure score may be worth more than a long shot. Scripture says there is “a time to keep, and a time to cast away,” and a good Yahtzee player learns to tell which time it is.

PICTURE STUDY



The Game of Knucklebones, Jean-Siméon Chardin, About 1734 · Oil on canvas · Baltimore Museum of Art, Baltimore, Maryland

Image: [Wikimedia Commons](#), public domain

Knucklebones, also called jacks, is one of the oldest games in the world. A player tosses a ball or one bone into the air, picks up other pieces from the table, and catches the ball before it falls. Chardin painted this young woman in the middle of her toss, with the ball still in the air.

Look closely

- Find the ball in the air. What must the player do before it comes down?
- Find the pieces on the table. How many can she safely try to pick up this time?
- Look at her eyes. Where is she looking, and why?

Talk about it

1. In knucklebones, a player who tries to grab too many pieces at once may drop the ball and lose her turn. How is that like deciding whether to reroll in Yahtzee?
2. When would it be wise to try for more pieces, and when would it be wise to take only what is safe?

PROJECT

Two Yahtzee Strategies

You will test two simple strategies for one Yahtzee-style goal, “get as many of a kind as you can,” over 20 turns each, and compare the average results.

You will need

- Five dice and a cup
- Your scorebook and a calculator
- A Yahtzee score sheet, or a copy drawn by hand (optional, for a full game at the end)

Steps

1. **Strategy A, keep the most.** After each roll, keep all the dice showing your most common number and reroll the rest. Stop after three rolls. Record how many of a kind you ended with.
2. **Strategy B, keep the highest pair only.** After the first roll, keep only your highest pair (or highest single die) and never add more keepers. Reroll the others twice. Record how many of a kind you ended with.
3. **Predict.** Which strategy will give more of a kind on average? How many Yahtzees do you expect in 20 turns?
4. Play 20 turns with each strategy. Make a tally of the results: 1, 2, 3, 4 or 5 of a kind.
5. Find the average number of a kind for each strategy, and count the Yahtzees.
6. **Rerolling test.** Set four dice to the same number. Roll the fifth die up to two times, 36 times in all, and count how often you finish the Yahtzee. Compare with $\frac{11}{36}$.
7. Play one full game of Yahtzee with your family. Before each reroll, say your chance out loud. Record your final score.
8. **Decide.** Write the keep-or-reroll rule you will use from now on.

Tips

- Keep the strategies strict during the test, even when you are tempted to change. That is what makes it a fair test.

- Twenty turns is a small sample. If the averages are close, play 20 more turns of each.

Check your work

- ☐ I played 20 turns with each strategy and found the averages.
- ☐ I tested the four-of-a-kind reroll 36 times and compared it with eleven thirty-sixths.
- ☐ I wrote a keep-or-reroll rule based on my data.

SCOREBOOK

1. Make a tally table: Number of a kind (1–5), Strategy A, Strategy B.
2. Draw a bar graph comparing the two strategies.
3. Copy Ecclesiastes 3:6 and write about a choice in a game when you kept what you had, and one when you took a chance.

From history. A Yahtzee score card has thirteen boxes, so every game lasts exactly thirteen turns. That makes it easy to compare two strategies fairly: play the same number of full games with each and compare the average final scores.

WORKED EXAMPLES

Example 1. You need a 6 for a large score and have one die to reroll, with two rolls left. What is your chance of getting the 6?

Show solution

1. Chance of missing on one roll: $\frac{5}{6}$.
2. Missing both rolls: $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$.
3. Getting it at least once: $1 - \frac{25}{36} = \frac{11}{36}$.

Answer: $\frac{11}{36}$, about 31%, a little less than one chance in three.

Example 2. Your score card has an empty Chance box (score the total of the dice) and an empty Yahtzee box (50 points). You have 6, 6, 6, 6, 2 with no rolls left. What should you do?

Show solution

1. The Yahtzee box needs five of a kind; you have only four, so it would score 0.
2. Chance scores the total: $6 + 6 + 6 + 6 + 2 = 26$. (Four of a Kind, if open, would also score 26.)

Answer: Take the sure 26 points rather than writing a 0 in the Yahtzee box, which you may still fill later in the game.

COMMON MISTAKES

- Adding chances instead of using “1 minus the chance of missing every time.” Two rolls at one sixth each is not two sixths.
- Chasing a Yahtzee late in the game when a sure score is worth more.
- Judging a strategy by one lucky or unlucky turn.

Make it a game • Keep or Reroll. Roll five dice or one die five times, aiming for a goal number. Before each reroll, say whether you keep the dice or try again, and give the chance of getting what you need. Your parent keeps your record in beans. After ten rounds, compare sure scores with rerolled scores. Which strategy earned more?

Scripture. “A time to get, and a time to lose; a time to keep, and a time to cast away;” (Ecclesiastes 3:6, KJV)

Solomon wrote that wisdom is knowing the right time for each thing. In a game, part of wisdom is knowing when to keep what you have and when to try for more.

7.2 BATTLESHIP: HUNT AND TARGET

OBJECTIVES

- Find the chance that a random shot hits a ship in Battleship.
- Explain the hunt-and-target method and why a checkerboard pattern of shots works.
- Test a random strategy and a hunt-and-target strategy by counting shots needed.

Warm-up. Draw a 5-by-5 grid. Have someone secretly shade a ship two squares long, across or down. Find it by calling squares like “B3.” How many shots did you need? After your first hit, how did you choose your next shot?

WATCH

The Battleship Algorithm

[Vsauce2](#) · 7:32

<https://www.youtube.com/watch?v=LbALFZoRrw8>

WHEN TO WATCH

After “How they figured it out.” Compare the hunting patterns in the video with your checkerboard.

BACKUP VIDEO

How to 'always' win at Battleship?

[Digital Genius](#) · 4:05

<https://www.youtube.com/watch?v=8FctDuTfcO8>

A shorter explanation of the probability method for choosing the next shot.

KEY TERMS

grid

A board of squares named by a letter and a number, such as B7.

target

After a hit, firing at the squares next to it to finish the ship.

hunt

Searching the board for any ship before you have found one.

checkerboard pattern

Shooting only at every other square, like the dark squares of a checkerboard.

CORE IDEAS

The story. Long before it was a board game, Battleship was played with pencil and paper: two players drew grids, hid their ships and took turns calling shots. In 1967 the Milton Bradley Company (the same company you met in Lesson 3.2) made it a plastic board game with pegs, and it became a family favorite. In the standard game, each player hides five ships on a 10-by-10 grid of 100 squares: a carrier of 5 squares, a battleship of 4, two ships of 3 and a small ship of 2.

How they figured it out. Many players fire at random until they hit something. Thoughtful players noticed two better ideas. First, after a hit, the ship must continue in one of only four directions, so the next shots should go right next to the hit. That is **target** mode. Second, while **hunting**, there is no need to shoot every square. The smallest ship is two squares long, so it must cover at least one square of each color on a checkerboard. Shooting only the “dark” squares will find every ship with at most 50 shots. People who later tested strategies with computers found that hunt-and-target players finish games in far fewer shots than random shooters.

The mathematics. The five ships cover $5 + 4 + 3 + 3 + 2 = 17$ of the 100 squares. So a random first shot hits with probability $\frac{17}{100} = 17\%$. After a hit, at least one of the four neighboring squares must also be part of that ship, so a shot next to the hit has a much better chance, often 1 in 4 or better. As you gather misses, the chances on the squares nearby change too: a square surrounded by misses cannot hold a long ship. Every shot gives information, and wise players use all of it.

On the field. Searching wisely is a skill used far beyond games: rescuers search the most likely places first, and a coach watching film looks hardest where the other team usually attacks. **The percentage:** hunt on a pattern, then target around every hit. **The risk side:** a smart opponent who knows your pattern may hide his ships on the squares you check last, or along the edges where fewer people look. A good player mixes his hunting pattern and does not place his own ships the same way every game.

PICTURE STUDY



The Hunters in the Snow, Pieter Bruegel the Elder, 1565 · Oil on panel · Kunsthistorisches Museum, Vienna

Image: [Wikimedia Commons](#), public domain

Pieter Bruegel the Elder painted this famous winter scene in 1565 as part of a series about the months of the year. Three tired hunters return to their village after a long search, while families below skate and play on the ice.

Look closely

- Find the three hunters and their dogs. Do they look as if the hunt went well? What did they bring home?
- Look down the hill at the people skating and playing on the frozen ponds. How many games can you find?
- Find the birds in the trees and the one flying across the sky. Your eye travels across the picture as if searching it.

Talk about it

1. The hunters searched for a long time and found little. How could a better plan for searching have helped them?
2. When you search a Battleship grid, how is it like hunting across a wide landscape? Where would you look first?

PROJECT

Random Shots versus Hunt and Target

You will play a simplified Battleship on paper and count how many shots each strategy needs to sink all the ships.

You will need

- Graph paper and pencils
- A partner to hide ships (or hide them yourself on a covered sheet and ask a parent to call hits)
- Two dice (for choosing random squares) and your scorebook

Steps

1. Draw two 6-by-6 grids, labeled A to F across and 1 to 6 down.
2. Your partner hides three ships: lengths 3, 2 and 2, across or down, not touching.
3. **Random strategy.** Roll two dice for each shot: the first die picks the column (1 = A ... 6 = F) and the second the row. Skip squares already shot. Count shots until all ships are sunk.
4. **Hunt-and-target strategy.** Hunt only on a checkerboard pattern (A1, C1, E1, B2, D2, F2, and so on). After a hit, shoot the four neighboring squares until the ship is sunk, then return to hunting.
5. Play 5 games with each strategy, with the ships hidden fresh each time.
6. Record the number of shots for every game. Find the average for each strategy.
7. **Decide.** How many shots, on average, did hunt and target save? Write your Battleship plan.
8. Bonus: if your family owns Battleship, play three games using your plan and record the shots needed.

Tips

- Mark hits with X and misses with a dot so you can see the pattern.
- In target mode, once you have two hits in a line, keep shooting along that line.

- Hide ships in many different places so the test is fair.

Check your work

- ☐ I played 5 games with each strategy and counted every shot.
- ☐ I found and compared the average shots needed.
- ☐ I wrote a Battleship plan with hunting and targeting steps.

SCOREBOOK

1. Draw one finished grid from each strategy with hits and misses marked.
2. Make a table: Game, Random shots, Hunt-and-target shots. Add the averages.
3. Write: “When I search for something I have lost, I would decide to look... first, because...”

From history. The pencil-and-paper game was printed as pads of grids under names such as *Salvo* before the plastic version appeared. Computer scientists still use Battleship to teach how a program can search wisely.

WORKED EXAMPLES

Example 1. In a game with only one ship of 3 squares on a 6-by-6 grid, what is the chance that a random first shot hits it?

Show solution

1. Squares on the grid: $6 \times 6 = 36$.
2. Squares covered by the ship: 3.
3. Probability: $\frac{3}{36} = \frac{1}{12}$, about 8%.

Answer: $\frac{1}{12}$, about 8%.

COMMON MISTAKES

- Firing at random again after a hit, instead of targeting the neighbors.
- Shooting squares that cannot hold a ship, such as a single square surrounded by misses.
- Placing your own ships in the same spots every game.

Make it a game • Checkerboard Hunt. Draw a 6-by-6 grid and hide three ships, each two squares long, with paper chips. Your parent hunts using a checkerboard pattern, calling shots. Count how many shots it takes to find all three. Then swap, and hunt with random shots. Compare the counts, and say why checkerboard shots find ships faster.

7.3 MONOPOLY'S MOST-LANDED SQUARES

OBJECTIVES

- Explain why Jail is the most-visited square on a Monopoly board.
- Use two-dice counting to show why the orange properties are landed on so often.
- Tally a real game, or a simulated one, and compare the results with the counting.

Warm-up. Imagine a game piece sitting on square 10 of a 40-square board. You roll two dice. Which square are you most likely to land on next? Which squares are you least likely to land on?

WATCH

How to use math to win at Monopoly

Business Insider • 3:13

https://www.youtube.com/watch?v=7_SXFtdf65s

WHEN TO WATCH

After “How they figured it out.”

Note: Parents: the video talks about rent money inside the game of Monopoly; no real money is involved.

KEY TERMS

board square

One of the 40 spaces around a Monopoly board, numbered from Go (0) to 39.

Jail

Square 10 on a Monopoly board. Players are sent there by a square, by cards and by rolling three doubles in a row.

color group

A set of two or three properties of the same color, such as the three orange properties.

simulation

Playing out many turns, by hand or by computer, to see what usually happens.

CORE IDEAS

The story. In 1904 an American woman named Lizzie Magie received a patent for a board game called *The Landlord's Game*. Players traveled around a square board

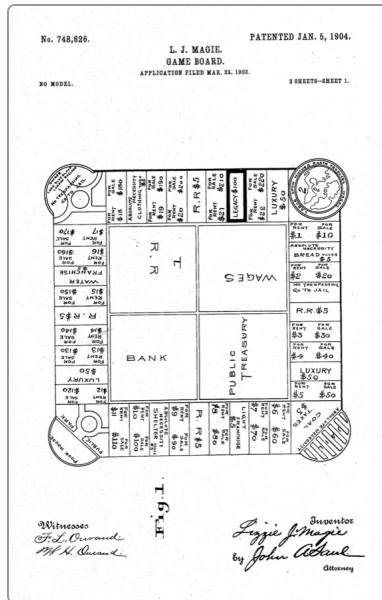
buying properties and paying rent. Over the next thirty years many people made homemade versions, and in 1935 the company Parker Brothers began selling one of them as *Monopoly*. It became one of the best-selling board games in history. Its board has 40 squares, and players move by rolling two dice.

How they figured it out. Players often feel that some squares get landed on more than others, and mathematicians proved it. Many people have counted with computers, playing millions of turns, and some have used a kind of mathematics called **Markov chains**, named after a Russian mathematician, Andrey Markov, who studied chains of chance events in the early 1900s. They all found the same thing. The most-visited square is Jail, because players are sent there in several ways: the “Go to Jail” square, cards that send you there, and rolling doubles three times in a row. Among the properties, Illinois Avenue is landed on most, partly because a card sends players straight to it, and the orange group is visited more than any other color group.

The mathematics. Here is why the orange properties are so busy. Players leave Jail from square 10 by rolling two dice. The most common totals are 6, 7, 8 and 9. From square 10, a roll of 6 lands on square 16, St. James Place; 8 lands on 18, Tennessee Avenue; and 9 lands on 19, New York Avenue, all three orange. (A 7 lands on 17, Community Chest.) The ways to roll 6, 8 and 9 are 5, 5 and 4, so a player leaving Jail lands on orange with probability $\frac{5+5+4}{36} = \frac{14}{36} \approx 39\%$. Since Jail is visited so often, those three squares collect many visitors.

On the field. Knowing where the dice send people most often lets a player choose properties wisely, the way a Catan player chooses spots with more chances. **The percentage:** properties that are landed on often earn rent more often. **The risk side:** landing frequency is only part of the story; the price of the properties and houses, and how much money you keep in reserve, matter too. A player who spends everything on one group may be ruined by one unlucky trip around the board. Even in a game, wise players never risk everything on one hope.

PICTURE STUDY



Patent Drawing for The Landlord's Game, Elizabeth J. Magie, 1904 · Patent drawing, U.S. Patent 748,626 · United States Patent Office, Washington, D.C.

Image: [Wikimedia Commons](#), public domain

Lizzie Magie drew this board for her patent in 1904. Like a Monopoly board, it has spaces all the way around the edge and special squares in the corners. Inventors send careful drawings like this to the patent office to show exactly what they have invented.

Look closely

- Find the four corners of the board. What kinds of squares do you think they are?
- Count the spaces along one side. How does this board compare with a Monopoly board?
- Look for any writing on the squares. What do the words tell you about the game?

Talk about it

1. On any board where players move by dice, why would some squares be visited more than others?
2. If you were designing a board game, how could you use two-dice counting to decide where to put the most important squares?

PROJECT

Where Do the Dice Send Us?

You will use two-dice counting to predict where players land after leaving Jail, test it with 72 rolls, and then tally landings in a real or simulated game.

You will need

- Two dice
- A Monopoly board, or a 40-square loop drawn on paper with squares numbered 0 to 39 and square 10 marked “Jail”
- Your scorebook

Steps

1. **Predict.** For a player starting on square 10, write which square each total from 2 to 12 lands on, and how many ways out of 36 each total can happen.
2. Find the probability of landing on each of squares 16, 17, 18 and 19.
3. Roll two dice 72 times, starting from square 10 each time. Tally which square you land on.
4. Compare your tallies with the expected counts. (Square 16 and square 18: $\frac{5}{36} \times 72 = 10$ each.)
5. **Game tally.** Play a real game of Monopoly for an hour, or move one piece around your paper board for 100 turns (send it to Jail when it lands on square 30). Each time a piece lands on a square, put a tally mark on that square.
6. Find the five squares with the most tallies. Is Jail one of them? Are any orange squares?
7. **Decide.** If you could choose one color group, which would you choose and why? Include the risk side.

Tips

- A long game is the best sample, so combine tallies from several players or several games.
- In the paper version, you can ignore cards and doubles, but then Jail will be visited less often than in the real game. Explain why.

Check your work

- ☐ I listed where each total lands from square 10 and found the probabilities.
- ☐ I rolled 72 times from Jail and compared the tallies with the expected counts.
- ☐ I tallied at least 100 landings in a game and named the busiest squares.

SCOREBOOK

1. Draw the part of the board from square 10 to square 22, with the number of ways (out of 36) to land on each square written underneath.
2. Make a table of your five busiest squares from the game tally.
3. Write: “In a game I would never risk... because...”

From history. Andrey Markov first used his chains in 1913 to study the order of letters in a Russian poem. Today the same idea helps phone keyboards guess the next word you will type.

WORKED EXAMPLES

Example 1. A player on square 10 rolls two dice. What is the probability of landing on square 16 or square 18?

Show solution

1. Square 16 needs a total of 6: 5 ways. Square 18 needs a total of 8: 5 ways.
2. Add the ways: $5 + 5 = 10$.
3. Probability: $\frac{10}{36} = \frac{5}{18} \approx 28\%$.

Answer: About 28%, more than one chance in four.

COMMON MISTAKES

- Thinking every square is landed on equally often. The dice, the cards and Jail all change the chances.
- Forgetting that a total of 7 is the most common roll when counting where players land.
- Choosing properties by landing chance alone and forgetting cost and risk.

Make it a game • Where Do We Land. Draw a ten-square loop board with Jail at one corner. Roll two dice and move a button around, tallying each square you land on across forty rolls. Predict the most-landed square first. Compare your tally to the answer. Explain to your parent why sums near seven send you to certain squares, including orange.

7.4 CAN'T STOP: KNOWING WHEN TO STOP

OBJECTIVES

- Explain how push-your-luck games reward and punish risk.
- Find a good stopping rule for the dice game Pig by comparing expected gain with expected loss.
- Test two stopping rules over many turns and choose the better one.

Warm-up. Play a quick round of Pig: roll one die as many times as you like, adding up the numbers, but if you roll a 1, you lose everything from this turn. Stop whenever you want and keep your total. Play five turns. When did you stop, and why?

WATCH

Sid Sackson's Can't Stop - NEW Edition!

Eagle-Gryphon Games · 2:47

<https://www.youtube.com/watch?v=rldIGQ708ks>

WHEN TO WATCH

Before the project, to learn how Can't Stop is played.

BACKUP VIDEO

How to play Can't Stop

Triple S Games · 3:36

<https://www.youtube.com/watch?v=VUGvOQatVDc>

Another short rules video for Can't Stop.

KEY TERMS

push your luck

A kind of game in which you keep taking chances to gain more, but one bad roll can wipe out your gains for the turn.

bust

To lose all the gains of a turn because of a bad roll.

turn total

The points you have gained so far this turn but have not yet kept.

stopping rule

A rule you decide in advance for when to stop and keep your points, such as “stop at 20.”

CORE IDEAS

The story. Sid Sackson was one of the great American game designers, and in 1980 Parker Brothers published his game *Can't Stop*. Each turn, a player rolls four dice, splits them into two pairs, and moves markers up the columns for the totals of the pairs. He may roll again as often as he likes. But if a roll gives no total he can use, he **busts** and loses all his progress for that turn. The name says it all: the hard part of the game is making yourself stop. The simpler old dice game called **Pig** works the same way with one die.

How they figured it out. In Pig, each roll of 2, 3, 4, 5 or 6 adds to your turn total, but a 1 wipes it out. Should you roll again? Compare what you expect to gain with what you expect to lose. Five times in six you add something, on average 4 points (the average of 2 through 6). One time in six you lose your whole turn total. When your turn total is small, there is little to lose, so rolling again is smart. When it is big, one bad roll costs a lot. In 2004 two computer scientists, Todd Neller and Clifton Presser, worked out the perfect strategy for Pig and found that “hold at 20” is a very good simple rule for most of the game.

The mathematics. If your turn total is t , rolling again gains on average $\frac{5}{6} \times 4 = \frac{20}{6}$ points and loses on average $\frac{1}{6} \times t$ points. Rolling is worth it while $\frac{20}{6} > \frac{t}{6}$, that is, while $t < 20$. So to score the most points per turn, keep rolling until your turn total reaches about 20, then stop. In *Can't Stop*, the chances depend on your columns: counting all $6^4 = 1,296$ rolls of four dice shows that if your markers are in columns 6, 7 and 8, a roll can move you about 92% of the time, but in columns 2, 3 and 12, only about 44% of the time. The middle columns are safer to push in.

On the field. The wise player sets a stopping rule *before* the turn begins and keeps to it, instead of deciding by feel, because feelings make people stop too soon after a scare and push too far after a lucky streak. **The risk side:** the more you have gained in a turn, the more one bad roll costs, so the danger grows with success. Near the end of a game, the rule changes: a player far behind must push harder, and a player

close to winning should stop as soon as he has enough. Knowing when to stop is a skill for life, not just for games.

PICTURE STUDY



Soap Bubbles, Jean-Siméon Chardin, Probably 1733–1734 · Oil on canvas · National Gallery of Art, Washington, D.C.

Image: [Wikimedia Commons](#), public domain

Chardin painted several versions of this quiet scene. A young man leans out of a window and blows a soap bubble through a straw while a small child watches. The bubble is already large. Will he blow a little more, or let it go?

Look closely

- Find the bubble at the end of the straw. How big is it?
- Look at the young man's face. How carefully is he working?
- Find the small child. What is he waiting to see?
- Find the glass of soapy water on the window ledge.

Talk about it

1. Blowing more air makes the bubble bigger, but too much makes it pop. How is that like rolling again in Pig or Can't Stop?
2. How could the young man decide in advance when to stop blowing?

PROJECT

Find the Best Stopping Rule

You will test three stopping rules for Pig, each for 25 turns, and find which one scores the most points per turn on average. Then you will play Can't Stop, or a paper version, with your family.

You will need

- One die (and four dice for Can't Stop)
- Your scorebook and a calculator
- Can't Stop, or graph paper to draw eleven columns numbered 2 to 12

Steps

1. **Rules to test.** "Stop at 10," "Stop at 20" and "Stop at 35." With each rule, keep rolling until your turn total reaches the number or you roll a 1.
2. **Predict.** Which rule will score the most points per turn on average?
3. Play 25 turns with each rule. Record the points kept on each turn (0 if you rolled a 1).
4. Find the average points per turn for each rule. Also count how many turns ended in a bust.
5. Compare the three rules in a table and a bar graph.
6. **Can't Stop.** Play a game with your family (with the real game or a paper version: columns 2 to 12, with shorter columns for the rare totals). Before each turn, write the columns you are in and whether you will be bold or careful.
7. **Decide.** Write your stopping rule for Pig and one for Can't Stop, and say when you would break your rule near the end of a game.

Tips

- Keep each rule strictly during the test, even when you are tempted to keep going.
- Twenty-five turns per rule is a small sample, so if two rules are close, play more turns of each.
- The busts are part of the data. Write down every zero.

Check your work

- ☐ I played 25 turns with each of three rules and found each average.
- ☐ I counted busts for each rule.
- ☐ I wrote stopping rules for Pig and Can't Stop and explained when I would change them.

SCOREBOOK

1. Make a table: Rule, Total points, Turns, Average per turn, Busts.
2. Draw a bar graph of the three averages.
3. Write about a time outside of games when knowing when to stop was wise.

From history. Sid Sackson designed many games and collected thousands of them in his home in New York. Pig is a folk game whose rules were written down by the game writer John Scarne in 1945, and computer-science teachers still use it to teach about decisions.

WORKED EXAMPLES

Example 1. In Pig your turn total is 12. Should you roll again? What if it is 26?

Show solution

1. Expected gain from one more roll: $\frac{20}{6} \approx 3.3$ points.
2. Expected loss at 12: $\frac{12}{6} = 2$. Gain is bigger, so roll.
3. Expected loss at 26: $\frac{26}{6} \approx 4.3$. Loss is bigger, so stop.

Answer: Roll again at 12; stop at 26. The break-even is a turn total of 20.

COMMON MISTAKES

- Deciding when to stop by how you feel in the moment instead of by a rule made in advance.
- Thinking a lucky streak means the next roll is safer. The die has no memory.
- Thinking the boldest rule must be best because it sometimes scores very high. Count the busts too.

Make it a game • Stop at Twenty. Play a pig-style game with one die: roll as many times as you like to add to your turn total, but a one wipes it out. Choose a stopping rule beforehand, such as stop at 15. Play five turns each. Whoever scores more with their rule wins. Then try a new rule and explain why.

ODDS AT THE GAME TABLE

Family board games are wonderful places to practice playing the odds, because every turn is a small decision under uncertainty. Should you keep those three 5s and roll for more, or take the sure points? Where should you fire your next shot in *Battleship*? Which squares on a *Monopoly* board get landed on most? When should you stop rolling and keep what you have?

In this unit you will use everything you have learned (counting outcomes, expected points, tallies and fair tests) to become a wiser game player. You will keep a scorebook beside the board, test strategies over many games, and learn that a good strategy is judged over many turns, not by one lucky roll. As always, every game keeps score in points only.

PREREQUISITES

- Can count the 36 outcomes of two dice and find the probability of any total (Unit 2)
- Can find expected points per round (Lesson 3.4)
- Knows that dice have no memory (Lesson 2.4)

MASTERY CHECKLIST

- ☐ I can find the chance of getting a needed number when rerolling dice in Yahtzee, and decide whether to keep or reroll.
- ☐ I can explain the hunt-and-target method in Battleship and why a checkerboard pattern of shots works.
- ☐ I can explain why Jail and the orange properties are landed on so often in Monopoly.
- ☐ I can use a stopping rule in a push-your-luck game and explain why it works.
- ☐ I can test two game strategies fairly over many games and compare their average scores.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **Yahtzee (Hasbro):** The publisher's page for the classic five-dice game in Lesson 7.1. Ages 8 and up. Five ordinary dice and a paper score sheet work just as well.
- **Can't Stop (Eagle-Gryphon Games):** The current publisher's page for Sid Sackson's push-your-luck dice game in Lesson 7.4. Ages 8 and up. The lesson includes a paper-and-dice version.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

7.1	Yahtzee: Keep or Reroll?	2 sessions of 45 minutes
7.2	Battleship: Hunt and Target	2 sessions of 45 minutes
7.3	Monopoly's Most-Landed Squares	2 sessions of 45 minutes
7.4	Can't Stop: Knowing When to Stop	2 sessions of 45 minutes

UNIT 7 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. 5 pts

In Yahtzee you have four 3s and two rolls left. What is the chance you get the fifth 3? Show your work.

2. 4 pts

Why is a one-roll Yahtzee so rare, and how do rerolls change that?

3. 4 pts

Late in a Yahtzee game you can take a sure 20 points or reroll for a 50-point Yahtzee with an 11-in-36 chance, scoring 0 if you miss. Which has more expected points?

4. (4 pts)

In Battleship, the ships cover 17 of 100 squares. What is the chance a random first shot hits?

5. (5 pts)

Explain hunt and target, and why hunting on a checkerboard pattern is enough to find every ship.

6. (4 pts)

Name three ways a Monopoly player can be sent to Jail.

7. (5 pts)

From Jail (square 10), what is the probability of landing on square 19 (New York Avenue)? On any orange square (16, 18 or 19)?

8. (5 pts)

In Pig your turn total is 15. Should you roll again? Show the expected gain and loss.

9. (4 pts)

What would happen if a Pig player never stopped rolling until he rolled a 1?

10. (4 pts)

In Can't Stop, why is it safer to push your luck when your markers are in columns 6, 7 and 8 than in columns 2, 3 and 12?

11. (6 pts)

Short answer: In three or four sentences, explain why a wise player sets a stopping rule before the turn begins, and connect it to Ecclesiastes 3:6.

ANSWER KEY

1. Missing twice: $\frac{5}{6} \times \frac{5}{6} = \frac{25}{36}$. Getting it: $1 - \frac{25}{36} = \frac{11}{36} \approx 31\%$.
2. Only 6 of the 7,776 rolls of five dice are all the same (1 in 1,296). Keeping matching dice and rerolling the rest gives several chances, so a Yahtzee comes about 1 turn in 22 with good keeping.
3. Reroll: $\frac{11}{36} \times 50 \approx 15.3$ points on average, less than the sure 20. Take the sure 20.
4. $\frac{17}{100} = 17\%$.
5. Hunt: search for a ship; target: after a hit, shoot the neighboring squares to finish it. The smallest ship is two squares long, so it always covers one square of each checkerboard color; shooting only one color will find every ship.
6. Landing on the “Go to Jail” square, drawing a card that sends you there, and rolling doubles three times in a row.
7. Square 19 needs a 9: $\frac{4}{36} = \frac{1}{9}$. Any orange: $\frac{5+5+4}{36} = \frac{14}{36} \approx 39\%$.
8. Gain about $\frac{20}{6} \approx 3.3$; loss $\frac{15}{6} = 2.5$. Gain is bigger, so roll again.
9. Every turn would end in a bust, so he would never keep any points.
10. Totals near 7 can be made in many more ways, so a roll of four dice can move you about 92% of the time in 6, 7 and 8 but only about 44% of the time in 2, 3 and 12.
11. Feelings in the moment push people to stop too soon after a scare or go too far after a lucky streak. A rule made calmly, using counting, does better over many turns. Ecclesiastes 3:6 says there is “a time to keep, and a time to cast away”; a stopping rule is a way of knowing which time it is.

PORTFOLIO PIECE: A STRATEGY GUIDE FOR A FAMILY GAME

Choose one family game from this unit (Yahtzee, Battleship, Monopoly or Can't Stop) or another dice or board game your family plays. Write and illustrate a three- or four-page strategy guide for a younger player. Include: the basic rules in a few sentences; at least two pieces of advice backed by counting (for example, a reroll chance, a hunting pattern or a busy square); data from your own tests in this unit, with a table or bar graph; and a “risk side” section that tells when the advice might be wrong. Then teach the game and your advice to a younger child or a grandparent and play three games together, recording the scores in points.

LOOK FOR	WHAT IT MEANS
Counting	At least two pieces of advice are supported by correct probabilities or counts.
Own data	Tables or graphs from the student’s own tests are included and explained.
Risk side	The guide honestly explains when each piece of advice might fail.
Teaching	The guide is clear for a younger reader, and three games with a learner are recorded.

WISE CHOICES

All year you have learned to count chances, compare points per try and test strategies over many games. In this last unit you will carry those habits off the field and into life. You will see how small, sensible habits like wearing a helmet or buckling a seat belt are the high-percentage play of everyday living. You will learn to judge a decision by how it was made, not only by how it turned out, because even a wise choice sometimes loses.

You will hear the story of Eric Liddell, the Scottish runner who would not run on a Sunday, and think about what Scripture teaches about time and chance. Then you will plan and run a family game day, a small tournament in which every player keeps a scorebook, and you will practice being both a good winner and a good loser.

PREREQUISITES

- Can find the chance of something happening at least once in several tries (Lesson 7.1)
- Can compare two choices by their percentages and their risks (Units 4–7)
- Has kept a scorebook all year

MASTERY CHECKLIST

- ☐ I can explain why a small risk repeated many times becomes a big risk, and how safety habits lower it.
- ☐ I can sort decisions into the four boxes: good decision and good result, good decision and bad result, bad decision and good result, bad decision and bad result.
- ☐ I can tell the story of Eric Liddell and explain what Ecclesiastes 9:11 and Proverbs 16:9 teach about chance and choice.
- ☐ I can plan and run a fair round-robin tournament and keep a scorebook of my decisions.
- ☐ I can win and lose with grace.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **The Eric Liddell Community: Eric's story**: Free. The Edinburgh charity named for Liddell tells his life story, from the 1924 Olympics to his years in China, which fits Lesson 8.3.
- **Lichess**: Free, with no advertising. A parent can set up a private round-robin chess tournament for family members, a possible format for the game day in Lesson 8.4. A parent should create and supervise any account.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

8.1 Safe and Risky Choices: Goalie Masks, Helmets and Seat Belts

2 sessions of 45 minutes

8.2 A Good Decision Can Still Lose

2 sessions of 45 minutes

8.3 Time and Chance: Eric Liddell

2 sessions of 45 minutes

8.4 Family Game Day

2 sessions of 45 minutes

UNIT 8 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. **4 pts**

Who was Jacques Plante, and what did he change in 1959?

2. **5 pts**

A mishap has a 1-in-10 chance each day. What is the chance of at least one mishap in 5 days?

3. **4 pts**

Why does a safety habit matter more when it is repeated every day?

4. (4 pts)

Name the four boxes of decision and result. Which box is “dumb luck”?

5. (5 pts)

What would happen if a coach judged every decision only by its result? Give an example.

6. (4 pts)

Why did Eric Liddell not run the 100 meters at the 1924 Olympics, and what did he do instead?

7. (5 pts)

A team wins 3 of every 4 games. What is the chance it wins two games in a row?

8. (4 pts)

How many games are played in a round robin with 6 players? How many does each player play?

9. (4 pts)

Why do round robins tell more about skill than single-elimination tournaments?

10. (5 pts)

Copy Proverbs 16:9 and explain in two sentences how it applies to judging decisions.

11. (6 pts)

Short answer: In four or five sentences, describe what a good winner and a good loser do at a family game day, using Proverbs 16:32.

ANSWER KEY

1. The Montreal Canadiens goalie who, after being hit in the face by a shot on November 1, 1959, returned wearing a mask in a game; goalie masks soon became standard.
2. No mishap in 5 days: $0.9^5 \approx 0.59$. At least one: $1 - 0.59 = 0.41$, about 41%.
3. A small risk repeated many times becomes a large one. A habit that lowers the risk every time lowers it many times over, which adds up to a big drop in the chance of harm.
4. Good decision/good result, good decision/bad result, bad decision/good result, bad decision/bad result. Bad decision with a good result is dumb luck.
5. He would copy lucky mistakes and abandon wise plans after one loss. For example, he might stop taking his team's best shot after one miss, or keep a reckless play that happened to work once.
6. The first rounds were on a Sunday, and he would not run on the Lord's Day. He trained for the 400 meters and won it in a world-record 47.6 seconds.
7. $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16} \approx 56\%$.
8. $\frac{6 \times 5}{2} = 15$ games; each player plays 5.
9. Each player plays several games, a larger sample, so one unlucky game or pairing cannot decide everything.
10. "A man's heart deviseth his way: but the LORD directeth his steps." We are responsible for planning wisely, but the outcome is in God's hands, so we judge our choices by how well we planned, not only by how they turned out.
11. A good winner thanks his opponent, does not boast and remembers that chance played a part. A good loser congratulates the winner, keeps his temper and looks for what he can learn. Proverbs 16:32 says ruling your own spirit is better than taking a city, so self-control in winning and losing is a greater victory than the game itself.

PORTFOLIO PIECE: FAMILY GAME DAY REPORT

Assemble a final report on your family game day. Include: the plan (players, games, any handicaps and the reasons for them, worked out with expected points); the round-robin schedule and the final standings; your own scorebook page with decisions sorted into the four boxes; a photograph or drawing of your display table with one best project from each unit; and a one-page reflection on what you learned this year about playing the odds, naming at least three ideas from different units and one verse of Scripture that you will remember.

LOOK FOR	WHAT IT MEANS
Planning	A correct round-robin schedule, fair games or sensible handicaps, and a clear standings chart.
Scorebook	Honest scores and decisions recorded and sorted into the four boxes.
Display	One best project from each unit shown and explained to guests.
Reflection	At least three ideas from different units explained correctly, with a fitting verse of Scripture.
Sportsmanship	The day ended with handshakes, and the student named good decisions by opponents.

8.1 SAFE AND RISKY CHOICES: GOALIE MASKS, HELMETS AND SEAT BELTS

OBJECTIVES

- Tell the story of Jacques Plante and the goalie mask.
- Find the chance of at least one mishap in many tries, and see how a safety habit lowers it.
- Explain why safety habits are the high-percentage play in sport and in life.

Warm-up. Roll one die 20 times. Every 1 counts as a “mishap.” How many mishaps did you get? Now roll 20 more times, but this time only a 1 rolled twice in a row (roll again after any 1) counts as a mishap. How many now? Which way is safer?

WATCH

Jacques Plante changed game when he donned mask

NHL • 3:34

<https://www.youtube.com/watch?v=DUGAghriljM>

WHEN TO WATCH

After “The story.”

BACKUP VIDEO

The Physics of Seat Belts

Smithsonian Channel • 2:18

<https://www.youtube.com/watch?v=04gD8dCVbys>

How seat belts work, from the Smithsonian Channel.

KEY TERMS

safety habit

Something you do every time to lower a risk, like buckling a seat belt.

repeated risk

A small risk taken again and again. Over many tries it adds up to a large chance that something goes wrong at least once.

prudent

Wise and careful about what lies ahead.

goalie mask

A protective mask worn by hockey goaltenders, now required in every league.

CORE IDEAS

The story. On November 1, 1959, in a game at Madison Square Garden in New York, Jacques Plante, the goalie of the Montreal Canadiens, was hit in the face by a hard shot from Andy Bathgate of the New York Rangers. He was badly cut and needed stitches. Plante had been wearing a mask in practice, but coaches thought a goalie who wore one in games looked afraid. That night Plante told his coach he would not go back on the ice without it. He returned wearing his homemade mask, and the Canadiens won the game. Other goalies soon followed, and today every goalie wears one. A few goalies had tried masks before, like the Czech goalie in this lesson's picture, but Plante made them part of the game.

How they figured it out. For years many players thought of injuries as just bad luck. People who studied injuries counted instead. They found that the same kinds of injuries happened again and again, and that simple gear prevented many of them. The same thing happened on the roads. When crash records were studied, the case for seat belts became overwhelming: the U.S. National Highway Traffic Safety Administration reports that seat belts lower the risk of death for people in the front seat of a car by 45%. A helmet, a mask or a belt does not stop every accident. It makes a bad accident much less likely to become a disaster.

The mathematics. A small risk repeated many times becomes a big one. Suppose some mishap has a 1-in-20 chance on any one day. The chance of avoiding it for one day is $\frac{19}{20} = 0.95$. The chance of avoiding it for 20 days in a row is $0.95^{20} \approx 0.36$, so the chance of at least one mishap is about $1 - 0.36 = 0.64$, almost two chances in three! A safety habit that cuts the daily risk in half, to 1 in 40, gives $0.975^{20} \approx 0.60$, so the chance of a mishap in 20 days drops to about 40%. Habits matter because they are repeated.

On the field. Helmets in football, hockey, baseball and cycling, masks for goalies and catchers, warm-ups before sprinting, water on hot days, and the rule “when thunder roars, go indoors” all remove a little risk from every game and practice. **The percentage:** a small cost (a minute to buckle up, a little less comfort) for a large drop

in the chance of serious harm. **The risk side:** people tend to fear rare, dramatic dangers and ignore ordinary, repeated ones. Scripture says, “A prudent man foreseeth the evil, and hideth himself.” Prudence is not fear. It is looking ahead and taking sensible steps.

PICTURE STUDY



A Goalkeeper in a Mask, Unknown photographer, 1929 · Photograph from the magazine *Pestrý týden* · Czechoslovakia

Image: [Wikimedia Commons](#), public domain

This photograph appeared in a Czech picture magazine in 1929, thirty years before Jacques Plante wore his mask in the NHL. A few goalies in Europe and North America tried masks long before they became normal, but most players and coaches thought they were unnecessary.

Look closely

- Find the goaltender with the mask. What is the mask made of, and how much of his face does it cover?
- Look at the other players' faces and heads. Is anyone else wearing protection?
- Look at the goalie's leg pads. What other parts of his body did goalies protect?

Talk about it

1. Why do you think it took so many years for goalie masks to become normal, even after some goalies had tried them?
2. What safety habit in your own life do you sometimes skip? What would it take to make it an "always" habit?

Safety first. This lesson's project uses only dice and paper. Talk with a parent about the safety rules your family follows for bicycles, cars and sports.

PROJECT

The Repeated-Risk Experiment

You will use dice to see how a small risk adds up over many tries, and how a safety habit changes the result. Then you will make a family safety checklist.

You will need

- Two dice
- Your scorebook and a calculator

Steps

1. **Without the habit.** One “day” is one roll of two dice. A total of 2, 3 or 12 (4 chances in 36, one in nine) is a mishap. Play a “season” of 20 days and record whether at least one mishap happened.
2. Play 15 seasons and count how many had at least one mishap.
3. **With the habit.** Now a mishap is only a total of 2 or 12 (2 chances in 36, one in eighteen). Play 15 more seasons of 20 days and count the seasons with a mishap.
4. **Predict and compare.** Compute the chances: without the habit, $1 - \left(\frac{8}{9}\right)^{20} \approx 0.91$; with it, $1 - \left(\frac{17}{18}\right)^{20} \approx 0.68$. Compare with your counts.
5. Make a bar graph of the two results.
6. **Family checklist.** With a parent, list five safety habits your family uses (seat belts, bike helmets, sunscreen, water on hot days, thunder rule). For each, write the small cost and the big risk it lowers.
7. **Decide.** Write one habit you will make an “always” habit, and why.

Tips

- A season counts as one “mishap season” no matter whether it had one mishap or several.
- Fifteen seasons is a small sample. If you have time, play more.

Check your work

- ☐ I played 15 seasons without the habit and 15 with it, and counted the mishap seasons.

- ☐ I compared my counts with the computed chances.
- ☐ I made a family safety checklist with costs and risks.

SCOREBOOK

1. Make a table: Rule, Seasons played, Seasons with a mishap, Fraction.
2. Copy your family safety checklist neatly.
3. Copy Proverbs 22:3 and write one sentence about how it applies to a safety habit.

From history. Jacques Plante won the Vezina Trophy as the NHL's best goaltender seven times. His first mask was made of fiberglass and shaped to his face. Catchers in baseball had begun wearing masks back in the 1870s.

WORKED EXAMPLES

Example 1. A rider forgets his helmet 1 time in 10 rides. Over 10 rides, what is the chance that he forgets it at least once?

Show solution

1. Chance he remembers on one ride: 0.9.
2. Chance he remembers all 10 rides: $0.9^{10} \approx 0.35$.
3. At least one forgotten ride: $1 - 0.35 = 0.65$.

Answer: About 65%. A “usually” habit is not enough; an “always” habit is what keeps the risk low.

COMMON MISTAKES

- Thinking a small risk does not matter because it is small. Repeated many times, it becomes large.
- Thinking a safety habit is useless because nothing bad happened last time.
- Confusing prudence with fear. A prudent person still plays hard; he just plays wisely.

Make it a game • Mask Probability Trial. Roll a die twenty times where a six counts as a mishap. Mark how many rounds it takes to get at least one six. Then change the rule so a mishap needs a double six, and repeat. Compare. Tell the story of Jacques Plante, and explain how a safety habit lowers the chance of mishap over many tries.

Scripture. “A prudent man foreseeth the evil, and hideth himself: but the simple pass on, and are punished.” (Proverbs 22:3, KJV)

The prudent person looks ahead at what could go wrong and takes sensible steps to avoid it. A helmet, a mask and a seat belt are ways of hiding ourselves from harm.

8.2 A GOOD DECISION CAN STILL LOSE

OBJECTIVES

- Explain the difference between a decision and its result.
- Sort decisions into the four boxes of decision and result.
- Keep a decision journal and judge your own choices fairly.

Warm-up. You must choose: win a point if a die shows 2 through 6, or win a point if it shows only 6. You choose 2 through 6, of course, and roll a 1. Did you make a bad choice? Talk about it.

WATCH

How to make smart decisions more easily

TED-Ed • 5:16

<https://www.youtube.com/watch?v=X7j8F16eSqs>

WHEN TO WATCH

After “The mathematics.”

BACKUP VIDEO

When good decisions have bad outcomes

Cassie Kozyrkov • 2:00

<https://www.youtube.com/watch?v=x84RsnUzNtE>

A two-minute explanation of why good decisions can have bad outcomes.

KEY TERMS

decision

A choice made with the information you have at the time.

result

What actually happened afterward, which depends on the choice and on chance.

four boxes

A way to sort choices: good decision and good result, good decision and bad result, bad decision and good result, bad decision and bad result.

decision journal

A notebook where you write your choices, your reasons and your chances before you know the result.

CORE IDEAS

The story. On February 1, 2015, in Glendale, Arizona, the Seattle Seahawks trailed the New England Patriots 28–24 in Super Bowl XLIX. With 26 seconds left, Seattle had the ball on the Patriots’ 1-yard line with one timeout left. Nearly everyone expected Seattle to run the ball. Instead, the quarterback threw a quick pass, and a rookie cornerback, Malcolm Butler, stepped in front of the receiver and intercepted it. New England won. Within minutes, many people were calling it the worst play call in Super Bowl history.

How they figured it out. Seattle’s coach, Pete Carroll, explained his reasoning. With only one timeout, a pass that fell incomplete would stop the clock, so Seattle could get three tries to score instead of two. Interceptions from the 1-yard line were very rare. Several people who studied the call afterward decided it was a reasonable choice that turned out badly. Carroll later called it “the worst result of a call ever,” choosing his words carefully: the *result* was terrible, but that did not prove the *decision* was. Whether or not you agree with the call, the lesson is that the result alone cannot tell you.

The mathematics. A decision chooses among chances; the result is one roll of the dice. In the warm-up, choosing “2 through 6” gives a $\frac{5}{6}$ chance of winning, and choosing “only 6” gives $\frac{1}{6}$. The first is the good decision every time, even though it loses one time in six. To judge a decision, ask: what were the chances *before* the result was known, and did the choice give the best chances? Over many decisions, good choices lead to good results more often. In any one case, chance may decide.

On the field. Wise coaches review games by asking “Was it the right call?” before “Did it work?” A **decision journal** helps: before a big choice, write the options, your guess at the chances and your reasons. Afterward, sort the choice into one of the four boxes. A good decision with a bad result was bad luck; keep making that choice. A bad decision with a good result was dumb luck; be careful, because luck is hiding a bad habit. **The risk side:** judging only by results makes people copy lucky mistakes and give up wise plans after one loss.

PICTURE STUDY



Breezing Up (A Fair Wind), Winslow Homer, 1873–1876 · Oil on canvas · National Gallery of Art, Washington, D.C.

Image: [Wikimedia Commons](#), public domain

Winslow Homer painted this scene of a man and three boys sailing home in a fresh wind off the coast of New England. It became one of the most loved American paintings of its time.

Look closely

- Find the man at the tiller. What decisions is he making as he steers?
- Look at where the boys are sitting. Why are they on the high side of the boat?
- Look at the clouds and the waves. Is the weather safe, risky, or something in between?

Talk about it

1. A sailor decides when to go out, how much sail to raise and when to head home, but the wind is not in his control. How would you judge whether the sailor made good decisions if a sudden storm came?
2. Where in this picture can you see both wise preparation and things left to chance?

PROJECT

My Decision Journal

For one week you will keep a decision journal for the games you play and the small choices you make, and then sort every decision into the four boxes.

You will need

- Your scorebook
- Family games or sports you play during the week (any game from this year works)

Steps

1. Make a page with four columns: Decision, Options and chances, My reason, Result.
2. During one week, record at least ten decisions *before* you know the result. They can be game decisions (reroll or keep, which shot to take, stop or roll again) or everyday ones (bring a jacket or not).
3. After each result, write what happened.
4. Draw a large square divided into four boxes: good decision/good result, good decision/bad result, bad decision/good result, bad decision/bad result.
5. Write each decision in the box where it belongs. Be honest; judge the decision by the chances you knew at the time.
6. Count how many decisions fell in each box.
7. **Decide.** Write one habit you will keep because it was a good decision even when it lost, and one habit you will change because it was a bad decision even when it won.

Tips

- Writing before the result is the key. After the result, it is too easy to say you knew all along.
- A loss is not the same as a mistake. A win is not the same as a good choice.

Check your work

- ☐ I recorded at least ten decisions before knowing their results.

- ☐ I sorted every decision into the four boxes honestly.
- ☐ I wrote one habit to keep and one to change.

SCOREBOOK

1. Draw your four-box square neatly with your decisions written in.
2. Write about the Super Bowl XLIX pass: which box do you think it belongs in, and why?
3. Copy Proverbs 16:9 and write what it means for how we judge our own choices.

From history. Malcolm Butler had not been chosen by any team in the draft, and he was playing his first season. He had practiced against that very play the week before, which helped him read it, a reminder that good preparation is a decision too.

WORKED EXAMPLES

Example 1. A player chooses to shoot from her best spot (60% make rate) instead of her worst (20%). She misses. Which of the four boxes is this?

Show solution

1. The decision gave the best chance: good decision.
2. The shot missed: bad result.

Answer: Good decision, bad result: bad luck. She should keep choosing that shot.

COMMON MISTAKES

- Judging a decision only by its result.
- Writing down your chances after you already know what happened.
- Copying a choice that worked once by luck.

Make it a game • Four Boxes Sort. Draw four boxes: good decision with good result, good decision with bad result, bad decision with good result, bad decision with bad result. Your parent calls out a small story, and you place it in a box and explain why. Include a coin-flip story. Score a bean for each correct sort and explanation.

Scripture. “A man’s heart deviseth his way: but the LORD directeth his steps.” (Proverbs 16:9, KJV)

We are responsible for planning wisely, and the outcome belongs to God. That frees us to decide well, accept results with humility and not boast when luck goes our way.

8.3 TIME AND CHANCE: ERIC LIDDELL

OBJECTIVES

- Tell the story of Eric Liddell at the 1924 Olympic Games.
- Explain why even a strong favorite sometimes loses, using simple probability.
- Explain what Scripture teaches about chance, diligence and trusting God.

Warm-up. A team wins 3 out of every 4 games against a rival. They play one game. Is the stronger team sure to win? If they play a best-of-three series instead, would that make the stronger team more likely or less likely to win? Make a guess.

WATCH

The Torchlighters: The Eric Liddell Story (2007) | Episode 4 | Kenny Blyth | Robert Fernandez

Vision Video · 30:55

<https://www.youtube.com/watch?v=-VtOpo0QgBg>

WHEN TO WATCH

After the core ideas, as a family. It tells Liddell's whole life, including the Olympics and his years in China.

Note: Parents: an animated Christian biography from the Torchlighters series. It is about 31 minutes and can be watched in two sittings; the internment camp years are told gently.

KEY TERMS

favorite

The player or team more likely to win.

underdog

The player or team less likely to win.

conviction

A firm belief about what is right, which a person will not give up even at a cost.

diligence

Steady, careful hard work.

CORE IDEAS

The story. Eric Liddell was a Scottish runner and a devoted Christian, born in China to missionary parents. He was one of the fastest sprinters in Britain, and he was chosen for the 100 meters at the 1924 Olympic Games in Paris. But when the schedule was announced, the first rounds of the 100 meters were set for a Sunday.

Liddell kept the Lord's Day and would not run. He did not weigh his chances; he had already decided. Instead he trained for the 400 meters, a longer event in which he was not expected to win. Before the final, an American team masseur handed him a note with words from 1 Samuel 2:30: "Them that honour me I will honour." On July 11, 1924, Liddell won the 400 meters in a world-record time of 47.6 seconds. He also won a bronze medal in the 200 meters.

How they figured it out. After the Olympics, Liddell went to China as a missionary teacher in 1925. During the Second World War he was held, with many others, in the Weihsien internment camp, where he taught science to the children, organized games for them and was much loved. He died there in 1945. Liddell's choice teaches that some decisions are not settled by percentages at all. They are matters of right and wrong. But his story also shows that a person who decides by conviction can still work with all his might in everything else.

The mathematics. Even a strong favorite sometimes loses. A team that wins 3 games in 4 loses one game in 4. In a best-of-three series, the stronger team wins if it wins the first two games, $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$, or splits the first two and wins the third, $2 \times \frac{3}{4} \times \frac{1}{4} \times \frac{3}{4} = \frac{18}{64} = \frac{9}{32}$. Together that is $\frac{18}{32} + \frac{9}{32} = \frac{27}{32}$, about 84%. A longer series helps the favorite, but the underdog still wins about 16% of the time. As Solomon wrote, "time and chance happeneth to them all."

On the field. This year you have learned to do the percentages: take the high-percentage shot, avoid needless risks, judge your decisions honestly. That is diligence, and Scripture praises it. But James warns us not to boast about tomorrow, because we do not know what it will bring; we should say, "If the Lord will, we shall live, and do this, or that" (James 4:15). Liddell did both: he trained hard and ran to win, and he set the limits of his ambition by obedience to God. **The risk side:** probability is a tool, not a master. A wise person does the percentages wherever they belong, keeps his convictions where percentages do not belong, and trusts God with the outcome.

PICTURE STUDY



The 400 Meters Final, Paris 1924, Le Miroir des sports (photographer unknown), July 1924 · Newspaper photograph · Stade de Colombes, Paris
Image: [Wikimedia Commons](#), public domain

This photograph from a French sports newspaper shows the finish of the 400 meters at the 1924 Olympic Games. Eric Liddell is winning ahead of Horatio Fitch of the United States, who took the silver medal.

Look closely

- Find Eric Liddell and Horatio Fitch. Who is ahead?
- Look at the crowd in the grandstand. How many people do you think were watching?
- Liddell was known for running with his head thrown back. Can you see how each runner holds his head and arms?

Talk about it

1. Liddell was not the favorite in the 400 meters. What do you think made the difference?
2. Liddell gave up his best event because of his conviction. What does his story teach about choices that should not be made by percentages?

Safety first. If you run in the project, warm up first, run on a safe, flat surface in good shoes, and drink water. Stop if anything hurts, and tell a parent.

PROJECT

Favorites and Underdogs

You will test how often a favorite wins a single game and a best-of-three series, using dice, and then write about Eric Liddell's choice.

You will need

- One die, or a spinner from Lesson 3.2
- Your scorebook and a calculator

Steps

1. **The favorite.** Roll one die for each game: 1 through 4 means the favorite wins (two chances in three), 5 or 6 means the underdog wins.
2. **Predict.** In 30 single games, how many will the underdog win? In 20 best-of-three series, how many series will the underdog win?
3. Play 30 single games and tally the winners.
4. Play 20 best-of-three series (stop each series when one side has won 2 games) and tally the series winners.
5. Compare: did the underdog win a larger or smaller share of series than of single games? (Single games: one in three. Series: about one in four.)
6. **Write.** In one page, tell Eric Liddell's story in your own words. Explain why his decision about Sunday was not a matter of percentages, and how he still worked with all his might.
7. If you like, time yourself running a short distance (such as 50 meters) three times with a parent and record your times. Did they vary, even though you tried your hardest each time?

Tips

- Stop each best-of-three series as soon as one side reaches 2 wins.
- Write the story before looking back at the lesson, then check your facts.

Check your work

- ☐ I played 30 single games and 20 series and compared the underdog's share.
- ☐ I wrote Eric Liddell's story with correct dates and events.

- I explained the difference between a percentage decision and a conviction.

SCOREBOOK

1. Make a table: Format, Games or series played, Underdog wins, Fraction.
2. Draw a timeline of Eric Liddell's life: 1902 born in China; 1924 Paris Olympics; 1925 returns to China; 1945 dies in the Weih sien camp.
3. Copy Ecclesiastes 9:11 and James 4:15, and write what they teach together.

From history. Liddell was born in Tianjin, China, in 1902. Before the Olympics he also played rugby for Scotland. A memorial stone stands today at the site of the Weih sien camp, now in the city of Weifang.

WORKED EXAMPLES

Example 1. A player wins 2 of every 3 games against her brother. What is the chance she wins two games in a row?

Show solution

1. Each game is independent: multiply.
2. $\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$, about 44%.

Answer: About 44%. Even though she is the favorite in every single game, she is more likely than not to lose at least one of the two.

COMMON MISTAKES

- Thinking a favorite is sure to win. Favorites lose often enough that wise people are never proud.
- Thinking every choice should be made by percentages. Some choices are about right and wrong.
- Thinking trust in God means not working hard. Liddell trusted God and trained diligently.

Make it a game • Favorite Falls. Make a race with two paper chips: the favorite needs a 1, 2, 3, 4 or 5 on a die to move, and the underdog needs a 6. Run twenty rounds. Count how often the underdog wins. Tell your parent Eric Liddell's story. Explain why a favorite can still lose, and why conviction matters more than winning.

Scripture. “but time and chance happeneth to them all.” (Ecclesiastes 9:11, KJV)

Solomon saw that the swift and the strong do not always win. Probability explains why favorites sometimes lose; Scripture reminds us that every outcome is still under God’s hand.

8.4 FAMILY GAME DAY

OBJECTIVES

- Plan a fair round-robin family tournament.
- Keep a scorebook of game results and decisions during the tournament.
- Practice winning and losing with grace.

Warm-up. Four players want to hold a tournament. In a single-elimination tournament, a player who loses once is out. In a round robin, everyone plays everyone. How many games does each format need? Which one gives a stronger player a fairer chance to show it?

WATCH

Create a Round Robin Tournament

MatherPE · 5:39

https://www.youtube.com/watch?v=bP7v_EYWnUY

WHEN TO WATCH

Before the project, to see how to set up a round-robin schedule.

BACKUP VIDEO

Round Robin | Chess Terms | ChessKid.com

ChessKid · 1:34

<https://www.youtube.com/watch?v=rKYsPvnm4eU>

A short ChessKid explanation of round-robin tournaments.

KEY TERMS

round robin

A tournament in which every player plays every other player.

single elimination

A tournament in which a player who loses is out.

standings

A table that shows each player's points in a tournament.

sportsmanship

Playing fairly and treating opponents, teammates and officials with respect, whether you win or lose.

CORE IDEAS

The story. In 1851, during the Great Exhibition in London, the English chess master Howard Staunton organized the first international chess tournament. It was a single-elimination event: players were paired, and a player who lost his match was out. The German player Adolf Anderssen won. But some strong players were knocked out early after an unlucky pairing against another strong player. Organizers learned from this, and soon most chess tournaments became round robins, in which every player meets every other player. That way one bad pairing or one bad day cannot decide everything.

How they figured it out. Tournament organizers saw the same thing you saw with coins and dice: a single game is a small sample. A good player can lose one game by chance. A round robin gives each player several games, so the standings at the end tell more about skill and less about luck. It also keeps everyone playing all day, which is more fun at a family game day than sitting out after one loss.

The mathematics. In a round robin with n players, each player plays $n - 1$ games, and the total number of games is $\frac{n \times (n - 1)}{2}$. With 4 players, that is $\frac{4 \times 3}{2} = 6$ games; with 6 players, $\frac{6 \times 5}{2} = 15$ games. A single-elimination event with 4 players needs only 3 games, but two players play just once. Standings give points for each game: for example, 2 points for a win, 1 for a tie and 0 for a loss.

On the field. A family game day is the perfect place to use everything from this year: fair rules (Unit 3), good choices (Units 4–7), honest scorekeeping (Unit 5) and judging decisions, not just results (Lesson 8.2). **The percentage:** play each game wisely and keep an honest scorebook. **The risk side:** someone will lose, maybe you. Scripture says, “He that is slow to anger is better than the mighty; and he that ruleth his spirit than he that taketh a city” (Proverbs 16:32). A good winner thanks his opponent and does not boast; a good loser congratulates the winner and looks for what he can learn.

PICTURE STUDY



The Chess Game (Portrait of the Artist's Sisters Playing Chess), Sofonisba Anguissola, 1555 · Oil on canvas · National Museum, Poznań, Poland
Image: [Wikimedia Commons](#), public domain

Sofonisba Anguissola was one of the first women in Italy to become a famous painter. In 1555 she painted three of her younger sisters, Lucia, Minerva and Europa, playing chess, with a servant looking on. It is a picture of a family game day more than 450 years ago.

Look closely

- Find the chessboard. Who has just moved, and who is about to answer?
- Look at the face of each sister. Who looks confident? Who looks surprised? Who is enjoying the game?
- Find the servant behind them. What does her face tell you about the game?

Talk about it

1. One sister seems to be winning. How should she act toward her sister when the game is over?
2. What would you include in a family game day so that everyone, young and old, enjoys it like the people in this painting?

PROJECT

Our Family Game Day

You will plan and run a family game day: a round-robin tournament of one or more games from this year, with a scorebook for every player and a display of your best work. This is your final portfolio project.

You will need

- Games from this year: Pig, your own fair game from Unit 3, Yahtzee, Battleship, Can't Stop, a shooting contest with a soft ball, or others
- A large sheet of paper for the standings
- A small scorebook page for each player
- Your best projects from each unit for a display table

Steps

1. **Plan.** Choose the players and one or two games. Short games work best. Check that your games are fair, or give handicaps (extra points) so younger players have a real chance.
2. Make the round-robin schedule: list every pair of players. Check that the number of games equals $\frac{n(n-1)}{2}$.
3. Make a standings chart with each player's name and a column for points.
4. Make a scorebook page for each player with spaces to write each game's score and one decision they made in each game (for example, "I stopped at 20 in Pig").
5. **Play.** Run the tournament. Keep the standings up to date after every game.
6. Ask every player to write one decision in his scorebook after each game and sort it into the four boxes from Lesson 8.2.
7. Set out your display table with your best project from each unit, and explain one of them to the guests.
8. **Finish well.** Announce the standings. Every player shakes hands with every other player and names one good decision he saw an opponent make.

Tips

- Give younger players a head start in points, worked out with expected points, so the games are fair and fun.
- Appoint an honest scorekeeper for each game, and take turns.
- Keep the day light and cheerful. The goal is to enjoy playing well together.
- If any game uses a ball, follow the usual safety rules: soft balls indoors and a clear space.

Check your work

- ☐ Our schedule had every pair of players, and the number of games was correct.
- ☐ Every player kept a scorebook page with scores and decisions.
- ☐ We finished with handshakes, and I named a good decision by an opponent.

SCOREBOOK

1. Copy the final standings and the round-robin schedule.
2. Paste in your own game-day scorebook page, with your decisions sorted into the four boxes.
3. Write a paragraph: “The most important thing I learned about playing the odds this year is...”

From history. Howard Staunton also gave his name to the design of chess pieces first sold in 1849 and still used in tournaments around the world, the Staunton pattern. Adolf Anderssen, the winner of 1851, was a mathematics teacher.

WORKED EXAMPLES

Example 1. Five family members hold a round robin. How many games will be played? How many games does each person play?

Show solution

1. Each person plays $5 - 1 = 4$ games.
2. Total games: $\frac{5 \times 4}{2} = 10$.

Answer: 10 games in all, 4 for each person. (Divide by 2 because each game has two players.)

COMMON MISTAKES

- Forgetting to divide by 2 when counting round-robin games.

- Letting the score matter more than the people. A game day is for enjoying one another.
- Judging the day only by who won. Look back at the decisions too.

Make it a game • Round-Robin Draw. Pretend four players are in a tournament. Draw the full round-robin schedule from memory in two minutes, showing who plays whom. Count the games. Your parent challenges you with a fairness question: who gets the bye? Then fill in a scorebook and standings table for a made-up result.

Scripture. “He that is slow to anger is better than the mighty; and he that ruleth his spirit than he that taketh a city.” (Proverbs 16:32, KJV)

Ruling your own spirit, whether you win or lose, is a greater victory than any tournament. A good game day shows it.

WISE CHOICES

All year you have learned to count chances, compare points per try and test strategies over many games. In this last unit you will carry those habits off the field and into life. You will see how small, sensible habits like wearing a helmet or buckling a seat belt are the high-percentage play of everyday living. You will learn to judge a decision by how it was made, not only by how it turned out, because even a wise choice sometimes loses.

You will hear the story of Eric Liddell, the Scottish runner who would not run on a Sunday, and think about what Scripture teaches about time and chance. Then you will plan and run a family game day, a small tournament in which every player keeps a scorebook, and you will practice being both a good winner and a good loser.

PREREQUISITES

- Can find the chance of something happening at least once in several tries (Lesson 7.1)
- Can compare two choices by their percentages and their risks (Units 4–7)
- Has kept a scorebook all year

MASTERY CHECKLIST

- ☐ I can explain why a small risk repeated many times becomes a big risk, and how safety habits lower it.
- ☐ I can sort decisions into the four boxes: good decision and good result, good decision and bad result, bad decision and good result, bad decision and bad result.
- ☐ I can tell the story of Eric Liddell and explain what Ecclesiastes 9:11 and Proverbs 16:9 teach about chance and choice.
- ☐ I can plan and run a fair round-robin tournament and keep a scorebook of my decisions.
- ☐ I can win and lose with grace.

OPTIONAL BOOKS AND GAMES FOR THIS UNIT

- **The Eric Liddell Community: Eric's story**: Free. The Edinburgh charity named for Liddell tells his life story, from the 1924 Olympics to his years in China, which fits Lesson 8.3.
- **Lichess**: Free, with no advertising. A parent can set up a private round-robin chess tournament for family members, a possible format for the game day in Lesson 8.4. A parent should create and supervise any account.

Not required. Every project in this unit works without them. School Launch has no ties to these authors, publishers or companies.

LESSONS IN THIS UNIT

8.1 Safe and Risky Choices: Goalie Masks, Helmets and Seat Belts

2 sessions of 45 minutes

8.2 A Good Decision Can Still Lose

2 sessions of 45 minutes

8.3 Time and Chance: Eric Liddell

2 sessions of 45 minutes

8.4 Family Game Day

2 sessions of 45 minutes

UNIT 8 REVIEW

Answer each question. Show your work for calculations and use complete sentences for explanations. The review is worth 50 points.

1. **4 pts**

Who was Jacques Plante, and what did he change in 1959?

2. **5 pts**

A mishap has a 1-in-10 chance each day. What is the chance of at least one mishap in 5 days?

3. **4 pts**

Why does a safety habit matter more when it is repeated every day?

4. (4 pts)

Name the four boxes of decision and result. Which box is “dumb luck”?

5. (5 pts)

What would happen if a coach judged every decision only by its result? Give an example.

6. (4 pts)

Why did Eric Liddell not run the 100 meters at the 1924 Olympics, and what did he do instead?

7. (5 pts)

A team wins 3 of every 4 games. What is the chance it wins two games in a row?

8. (4 pts)

How many games are played in a round robin with 6 players? How many does each player play?

9. (4 pts)

Why do round robins tell more about skill than single-elimination tournaments?

10. (5 pts)

Copy Proverbs 16:9 and explain in two sentences how it applies to judging decisions.

11. (6 pts)

Short answer: In four or five sentences, describe what a good winner and a good loser do at a family game day, using Proverbs 16:32.

ANSWER KEY

1. The Montreal Canadiens goalie who, after being hit in the face by a shot on November 1, 1959, returned wearing a mask in a game; goalie masks soon became standard.
2. No mishap in 5 days: $0.9^5 \approx 0.59$. At least one: $1 - 0.59 = 0.41$, about 41%.
3. A small risk repeated many times becomes a large one. A habit that lowers the risk every time lowers it many times over, which adds up to a big drop in the chance of harm.
4. Good decision/good result, good decision/bad result, bad decision/good result, bad decision/bad result. Bad decision with a good result is dumb luck.
5. He would copy lucky mistakes and abandon wise plans after one loss. For example, he might stop taking his team's best shot after one miss, or keep a reckless play that happened to work once.
6. The first rounds were on a Sunday, and he would not run on the Lord's Day. He trained for the 400 meters and won it in a world-record 47.6 seconds.
7. $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16} \approx 56\%$.
8. $\frac{6 \times 5}{2} = 15$ games; each player plays 5.
9. Each player plays several games, a larger sample, so one unlucky game or pairing cannot decide everything.
10. "A man's heart deviseth his way: but the LORD directeth his steps." We are responsible for planning wisely, but the outcome is in God's hands, so we judge our choices by how well we planned, not only by how they turned out.
11. A good winner thanks his opponent, does not boast and remembers that chance played a part. A good loser congratulates the winner, keeps his temper and looks for what he can learn. Proverbs 16:32 says ruling your own spirit is better than taking a city, so self-control in winning and losing is a greater victory than the game itself.

PORTFOLIO PIECE: FAMILY GAME DAY REPORT

Assemble a final report on your family game day. Include: the plan (players, games, any handicaps and the reasons for them, worked out with expected points); the round-robin schedule and the final standings; your own scorebook page with decisions sorted into the four boxes; a photograph or drawing of your display table with one best project from each unit; and a one-page reflection on what you learned this year about playing the odds, naming at least three ideas from different units and one verse of Scripture that you will remember.

LOOK FOR	WHAT IT MEANS
Planning	A correct round-robin schedule, fair games or sensible handicaps, and a clear standings chart.
Scorebook	Honest scores and decisions recorded and sorted into the four boxes.
Display	One best project from each unit shown and explained to guests.
Reflection	At least three ideas from different units explained correctly, with a fitting verse of Scripture.
Sportsmanship	The day ended with handshakes, and the student named good decisions by opponents.