Logic

First edition | 9 units | 69 lessons | Half credit | Quick reference

FIRST EDITION

Course guide

Logic is the art of right reasoning. It is the second art of the trivium, after grammar and before rhetoric, and it is the tool every other subject uses; Aristotle's logical works were later gathered under the name *Organon*, "the instrument." This half-credit course follows the order of the three acts of the mind: it moves from **terms** and definitions to **propositions** to **arguments**. Students learn to spot fallacies, to state and test categorical propositions and syllogisms, to use truth tables and short proofs in symbolic logic, and to judge inductive arguments, scientific reasoning and testimony.

The last part of the course puts logic to work. Students learn to see through misleading graphs, averages and risk claims (a unit shared with the School Launch Statistics course), to map arguments and find hidden premises, and to question, refute and debate with courtesy. The year ends with a formal disputatio, in which the student defends a thesis in writing and aloud against objections put in syllogistic form.

Logic serves truth. We reason carefully because God is a God of truth and order, and because the Lord Himself invites us, "Come now, and let us reason together, saith the LORD" (Isaiah 1:18). Christians are also called to "be ready always to give an answer to every man that asketh you a reason of the hope that is in you with meekness and fear" (1 Peter 3:15). This course aims at both: clear thinking and a gracious answer.

> A lesson has a video only when a specific clip has passed vetting: one clip of 10 minutes or less, embedded without tracking cookies, with its captions and transcript checked before use and a short parent-preview note on the lesson page. Each video is chosen one at a time, not by channel. Every lesson without a vetted video has a short

> interactive explainer instead, in which the student sorts, picks and orders examples to practice the lesson's main idea.

Start with Lesson 1.1 →

Who it is for

Students in grades 8–10 (ages 13–16) who have completed Pre-Algebra and read at grade level; no prior philosophy is needed. The course is good preparation for rhetoric and debate, for Statistics in grades 10–12, and for careful reading in every subject.

How to use this course

1. The course fills one school year: 32 weeks, 3 days a week, about 30–45 minutes a day, for 96 school days in all: 69 lessons, 9 review days, 9 unit tests, a midterm review day and midterm after Unit 5, a final review day and final exam, and a 5-day capstone.
2. Each lesson takes one day. Begin with the lesson's daily habit, read the core ideas carefully with a pencil in hand, watch the video or work through the interactive explainer, and then work the worked examples yourself before looking at the steps.
3. Do the practice set, check every answer against the key, and correct every mistake in writing. For classification problems, say both the category and the reason; the reason is where the logic is.
4. Worksheet B gives a second set of problems on the same skills for extra practice or for a lesson that needs another day.
5. Each unit ends with a review day (use the mastery checklist and redo missed problems) and a unit test. The midterm covers Units 1–5, and the cumulative final covers Units 1–9.
6. Unit 8 lessons 8.1–8.7 are the same lessons as Statistics Unit 14 and live on the Statistics course pages; the Logic course links to them. Lesson 8.8 is a Logic-only lesson.
7. Students who need more time may stretch the course over more weeks; steady, careful work matters more than speed.

Scope and sequence

UNIT 1 · WEEKS 1–3

Why Logic?

Students learn what logic is and why we study it, the three acts of the mind, how to find premises and conclusions, the difference between deduction and induction, truth, validity and soundness, the first principles of thought, and a short history of logic.

1.1 What Logic Is and Why We Study It

1.2 The Three Acts of the Mind

1.3 Arguments: Premises and Conclusions

1.4 Deduction and Induction; Truth, Validity, Soundness

1.5 The First Principles of Thought

1.6 A Short History of Logic

UNIT 2 · WEEKS 3–6

Terms and Definitions

Students learn how terms are used and what they mean, Aristotle's categories and Porphyry's predicables, how to write a definition by genus and difference, the rules of good definition, and how to divide a class properly.

2.1 Terms: Univocal, Equivocal, Analogical

2.2 Extension and Intension

2.3 The Categories and the Predicables

2.4 Definition by Genus and Difference

2.5 Good and Bad Definitions

2.6 Division and Classification

UNIT 3 · WEEKS 6–10

Informal Fallacies

Students learn to recognize and name the common informal fallacies of relevance, presumption and ambiguity, including those in advertising, and to point them out fairly and courteously.

3.1 What Is a Fallacy?

3.2 Attacking the Person

3.3 Appeals to Authority, the Many and Tradition

3.4 Appeals to Emotion and Force; Red Herring and Straw Man

3.5 Presumption I: Begging the Question, Complex Question, False Dilemma

3.6 Presumption II: Generalizing Badly

3.7 Ambiguity I: Equivocation, Amphiboly, Accent

3.8 Ambiguity II: Composition and Division

3.9 Fallacies in Advertising

3.10 Naming Fallacies Fairly

UNIT 4 · WEEKS 10–13

Categorical Propositions

Students learn the four categorical propositions (A, E, I, O), how to put everyday sentences in standard form, distribution, the square of opposition, the immediate inferences and Venn diagrams for propositions.

4.1 The Four Forms: A, E, I, O

4.2 Standard Form

4.3 Distribution

4.4 The Square of Opposition

4.5 Conversion, Obversion, Contraposition

4.6 Existential Import and the Modern Square

4.7 Venn Diagrams for Propositions

UNIT 5 · WEEKS 13–17

Categorical Syllogisms

Students learn the categorical syllogism: its terms, mood and figure, the rules of validity, Venn diagram tests, the named valid forms, enthymemes, chains of reasoning and Carroll's puzzles, and the hypothetical and disjunctive syllogisms.

5.1 The Syllogism

5.2 Mood and Figure

5.3 The Rules of Validity

5.4 Testing with Venn Diagrams

5.5 The Named Forms

5.6 Enthymemes

5.7 Chains of Reasoning: Sorites and Carroll's Puzzles

5.8 Hypothetical and Disjunctive Syllogisms

UNIT 6 · WEEKS 17–20

Propositional (Symbolic) Logic

Students learn propositional logic: symbolizing statements, truth tables, conditionals, equivalence, testing validity, the valid forms and formal fallacies, short proofs, and the link between logic and machines.

6.1 Statements and Connectives

6.2 Truth Tables for the Connectives

6.3 Conditionals

6.4 Compound Statements and Equivalence

6.5 Testing Validity with Truth Tables

6.6 The Valid Forms

6.7 The Formal Fallacies

6.8 Short Proofs

6.9 Logic and Machines

UNIT 7 · WEEKS 21–23

Induction and Scientific Reasoning

Students learn to judge inductive reasoning: generalizations, analogies, Mill's methods for finding causes, testing hypotheses, simple probability, the weighing of testimony, and inference to the best explanation.

7.1 Strong and Weak Inductive Arguments

7.2 Argument from Analogy

7.3 Finding Causes: Mill's Methods

7.4 Hypotheses and Testing

7.5 Reasoning about Chance

7.6 Testimony and Evidence

7.7 Best Explanation, and the Reach of Science

UNIT 8 · WEEKS 24–27

Statistical Deception

Students learn to see through misleading graphs, maps, samples, averages, risk claims, correlations and streaks (lessons shared with Statistics Unit 14), and write a short refutation of an advertisement's statistical claim. Lessons 8.1–8.7 are taught from the Statistics course (Unit 14).

8.1 Graphs That Deceive

8.2 Maps and Scales That Deceive

8.3 Who Got Counted?

8.4 Averages and Percents That Mislead

8.5 Risk: Relative, Absolute and Base Rates

8.6 Correlation, Cause and Lurking Variables

8.7 Regression to the Mean and Streaks

8.8 Refuting a Statistical Claim

UNIT 9 · WEEKS 27–30

Logic in Argument and Debate

Students learn to map arguments, find hidden premises, question in the Socratic manner, write a disputed question in Aquinas's form, use logic in rhetoric, find the real point at issue, cross-examine and refute, and debate in a practice round.

9.1 Mapping Arguments

9.2 Hidden Premises and the Strongest Form

9.3 Dialectic: Socratic Questioning

9.4 The Disputed Question

9.5 Logic in Rhetoric

9.6 Finding the Real Question

9.7 Cross-Examination and Refutation

9.8 Debate Formats and a Practice Round

Weekly pacing calendar

WEEK	PLAN
1	• Day 1: Lesson 1.1 What Logic Is and Why We Study It • Day 2: Lesson 1.2 The Three Acts of the Mind • Day 3: Lesson 1.3 Arguments: Premises and Conclusions *Three school days a week. Each lesson takes one day of about 30–45 minutes.*
2	• Day 1: Lesson 1.4 Deduction and Induction; Truth, Validity, Soundness • Day 2: Lesson 1.5 The First Principles of Thought • Day 3: Lesson 1.6 A Short History of Logic
3	• Day 1: Unit 1 review (review sheet and mastery checklist) • Day 2: Unit 1 test • Day 3: Lesson 2.1 Terms: Univocal, Equivocal, Analogical
4	• Day 1: Lesson 2.2 Extension and Intension • Day 2: Lesson 2.3 The Categories and the Predicables • Day 3: Lesson 2.4 Definition by Genus and Difference

WEEK	PLAN
5	• Day 1: Lesson 2.5 Good and Bad Definitions • Day 2: Lesson 2.6 Division and Classification • Day 3: Unit 2 review (review sheet and mastery checklist)
6	• Day 1: Unit 2 test • Day 2: Lesson 3.1 What Is a Fallacy? • Day 3: Lesson 3.2 Attacking the Person
7	• Day 1: Lesson 3.3 Appeals to Authority, the Many and Tradition • Day 2: Lesson 3.4 Appeals to Emotion and Force; Red Herring and Straw Man • Day 3: Lesson 3.5 Presumption I: Begging the Question, Complex Question, False Dilemma
8	• Day 1: Lesson 3.6 Presumption II: Generalizing Badly • Day 2: Lesson 3.7 Ambiguity I: Equivocation, Amphiboly, Accent • Day 3: Lesson 3.8 Ambiguity II: Composition and Division
9	• Day 1: Lesson 3.9 Fallacies in Advertising • Day 2: Lesson 3.10 Naming Fallacies Fairly • Day 3: Unit 3 review (review sheet and mastery checklist)
10	• Day 1: Unit 3 test • Day 2: Lesson 4.1 The Four Forms: A, E, I, O • Day 3: Lesson 4.2 Standard Form
11	• Day 1: Lesson 4.3 Distribution • Day 2: Lesson 4.4 The Square of Opposition • Day 3: Lesson 4.5 Conversion, Obversion, Contraposition
12	• Day 1: Lesson 4.6 Existential Import and the Modern Square • Day 2: Lesson 4.7 Venn Diagrams for Propositions • Day 3: Unit 4 review (review sheet and mastery checklist)
13	• Day 1: Unit 4 test • Day 2: Lesson 5.1 The Syllogism • Day 3: Lesson 5.2 Mood and Figure
14	• Day 1: Lesson 5.3 The Rules of Validity

WEEK	PLAN
	• Day 2: Lesson 5.4 Testing with Venn Diagrams • Day 3: Lesson 5.5 The Named Forms
15	• Day 1: Lesson 5.6 Enthymemes • Day 2: Lesson 5.7 Chains of Reasoning: Sorites and Carroll's Puzzles • Day 3: Lesson 5.8 Hypothetical and Disjunctive Syllogisms
16	• Day 1: Unit 5 review (review sheet and mastery checklist) • Day 2: Unit 5 test • Day 3: Midterm review (Units 1–5)
17	• Day 1: Midterm exam (Units 1–5) • Day 2: Lesson 6.1 Statements and Connectives • Day 3: Lesson 6.2 Truth Tables for the Connectives *Midterm week.*
18	• Day 1: Lesson 6.3 Conditionals • Day 2: Lesson 6.4 Compound Statements and Equivalence • Day 3: Lesson 6.5 Testing Validity with Truth Tables
19	• Day 1: Lesson 6.6 The Valid Forms • Day 2: Lesson 6.7 The Formal Fallacies • Day 3: Lesson 6.8 Short Proofs
20	• Day 1: Lesson 6.9 Logic and Machines • Day 2: Unit 6 review (review sheet and mastery checklist) • Day 3: Unit 6 test
21	• Day 1: Lesson 7.1 Strong and Weak Inductive Arguments • Day 2: Lesson 7.2 Argument from Analogy • Day 3: Lesson 7.3 Finding Causes: Mill's Methods
22	• Day 1: Lesson 7.4 Hypotheses and Testing • Day 2: Lesson 7.5 Reasoning about Chance • Day 3: Lesson 7.6 Testimony and Evidence

WEEK	PLAN
23	• Day 1: Lesson 7.7 Best Explanation, and the Reach of Science • Day 2: Unit 7 review (review sheet and mastery checklist) • Day 3: Unit 7 test
24	• Day 1: Lesson 8.1 Graphs That Deceive (Statistics 14.1, on the Statistics pages) • Day 2: Lesson 8.2 Maps and Scales That Deceive (Statistics 14.2, on the Statistics pages) • Day 3: Lesson 8.3 Who Got Counted? (Statistics 14.3, on the Statistics pages) *Unit 8 lessons 8.1–8.7 are taught from the Statistics course pages (Unit 14), using Practice Set L.*
25	• Day 1: Lesson 8.4 Averages and Percents That Mislead (Statistics 14.4, on the Statistics pages) • Day 2: Lesson 8.5 Risk: Relative, Absolute and Base Rates (Statistics 14.5, on the Statistics pages) • Day 3: Lesson 8.6 Correlation, Cause and Lurking Variables (Statistics 14.6, on the Statistics pages) *Unit 8 lessons 8.1–8.7 are taught from the Statistics course pages (Unit 14), using Practice Set L.*
26	• Day 1: Lesson 8.7 Regression to the Mean and Streaks (Statistics 14.7, on the Statistics pages) • Day 2: Lesson 8.8 Refuting a Statistical Claim • Day 3: Unit 8 review (review sheet and mastery checklist) *Unit 8 lessons 8.1–8.7 are taught from the Statistics course pages (Unit 14), using Practice Set L.*
27	• Day 1: Unit 8 test • Day 2: Lesson 9.1 Mapping Arguments • Day 3: Lesson 9.2 Hidden Premises and the Strongest Form *Unit 8 lessons 8.1–8.7 are taught from the Statistics course pages (Unit 14), using Practice Set L.*
28	• Day 1: Lesson 9.3 Dialectic: Socratic Questioning • Day 2: Lesson 9.4 The Disputed Question • Day 3: Lesson 9.5 Logic in Rhetoric
29	• Day 1: Lesson 9.6 Finding the Real Question • Day 2: Lesson 9.7 Cross-Examination and Refutation • Day 3: Lesson 9.8 Debate Formats and a Practice Round
30	• Day 1: Unit 9 review (review sheet and mastery checklist)

WEEK	PLAN
	• Day 2: Unit 9 test • Day 3: Final review (Units 1–9)
31	• Day 1: Final exam (Units 1–9) • Day 2: Capstone day 1: choose the resolution and write it as a disputed question; define the key terms • Day 3: Capstone day 2: write three objections as valid syllogisms *Final exam week.*
32	• Day 1: Capstone day 3: write the sed contra, respondeo and replies • Day 2: Capstone day 4: write the advertisement critique; rehearse • Day 3: Capstone day 5: oral defense (or Lincoln-Douglas round) and portfolio due *Capstone week: a formal disputatio with a written logic portfolio.*

The Capstone: A Formal Disputatio

The Capstone: A Formal Disputatio. The course ends with five school days in which the student brings together everything learned: clear terms, valid arguments, fair treatment of the other side and honest use of evidence. The capstone has a written portfolio and an oral defense.

Steps

1. Choose a resolution from the Classics Library resolution bank, or a timeless question your parent approves, and write it as a disputed question beginning "Whether ..." (for example, "Whether a promise binds when keeping it costs the one who made it").
2. Define the key terms of the question by genus and difference.
3. Write three objections to your thesis, each put as a valid syllogism, the strongest you can find.
4. Write the *sed contra* (an authority on the other side), your *respondeo* (your own answer, with its reasons) and a reply to each objection.
5. Write a one-page critique of a real advertisement or product claim, naming any fallacies and any misuse of statistics, and saying what evidence would settle the claim.

6. Defend your thesis aloud against a parent or classmate who presses objections in syllogistic form.

The oral defense

- The objector states an argument against the thesis in syllogistic form.
- The defender repeats it fairly, then answers each premise with *concedo* ("I grant it"), *nego* ("I deny it") or *distinguo* ("I distinguish": in one sense it is true, in another it is false).
- The defender gives a reason for every denial and every distinction.
- The round ends when the objector has no more objections or the time (about 20 minutes) is up.

Rubric

- Clarity of terms: definitions are exact and used the same way throughout.
- Validity: the objections and replies are valid arguments, and the defender spots invalid ones.
- Charity: the other side is stated at full strength before it is answered.
- Honesty with evidence: facts and statistics are reported fairly, with their limits.

Alternate for co-ops

Groups may hold a Lincoln-Douglas round on the same resolution in place of the oral defense; the written portfolio is the same.

Cross-links to the Classics Library

Cross-links to the Classics Library. The School Launch Classics Library has a Logic & Debate section (classicslaunch.org/logic-debate). The course gives the method and the drill; the Classics pages give the great-books examples and the debate reference. Units 3 and 9 link to it directly, and students may visit it at any time.

What you will find there

- Aristotle's *Organon* and the three acts of the mind (with Units 1 and 2).
- The square of opposition (with Unit 4).

- Fallacies with examples from the great books, and the Spot the Device quiz of 27 devices (with Unit 3, especially Lesson 3.9).
- Socratic questioning and the disputatio (with Lessons 9.3 and 9.4).
- Debate formats and a bank of 316 resolutions (with Lesson 9.8 and the capstone).

The Statistics crossover works the same way: Lessons 8.1–8.7 are Statistics Lessons 14.1–14.7, and the Logic course links to those pages rather than repeating them.

Materials

Pencil and eraser
Work in pencil so you can correct mistakes neatly.

A notebook
Keep all practice, corrections, definitions and tests in one place; it becomes the start of the capstone portfolio.

Index cards
For daily habits, key terms, the named syllogisms, the truth tables and the names of the fallacies.

A ruler and colored pencils
For Venn diagrams, truth tables and argument maps.

A Bible (King James Version)
For the Scripture passages in the lessons and for examples of reasoning.

Grading

Grades combine daily practice, unit tests, the two exams and the capstone disputatio, with the most weight on the unit tests.

PART	%
Daily practice sets	15
Unit tests	40
Midterm exam	15
Final exam	15

PART	%
Capstone disputatio and portfolio	15

Scale: A 90–100, B 80–89, C 70–79, D 60–69, below 60 redo

- Grade daily practice for completion and for corrections: a set counts as done when every problem is attempted and every mistake has been corrected.
- Reasons count as much as labels: a classification answer needs the correct category and a correct one-sentence reason for full credit.
- A student who scores below 70 on a unit test may study the missed material and take a retake with fresh problems; record the higher score, up to 80.
- Give partial credit for correct reasoning even when a final label is wrong.

A NOTE TO PARENTS

You do not need to have studied logic to teach this course well. Every lesson explains its ideas step by step, every practice problem has a full answer, and every unit test and exam has a complete key. Your role is to keep the work steady, to read along when you can, and to ask good questions.

Help without doing the work for your student. Ask: “What is the conclusion?” “What reasons are given for it?” “If the reasons were true, would the conclusion have to be true?” “Are the reasons actually true?” “Can you state the other side fairly before you answer it?” These five questions carry most of the course.

Logic students love to argue, and that is a gift to be trained, not a fault to be scolded. The course teaches students to argue well: to aim at truth rather than victory, to read others charitably, and to answer a bad argument with courtesy. Model this at home, and expect it from your student at the dinner table as well as on paper.

Logic matters far beyond the classroom. It helps your student read advertising and product claims wisely, follow a hard book, write a clear essay, and give a reasoned account of the faith. Above all, it trains the habit of honest thinking: saying exactly what follows from what, and admitting what does not.

Why Logic?

Logic is the art of right reasoning: the study of how to tell good reasoning from bad, and how to move from what we know to what we do not yet know without going astray. This first unit answers the question every student asks at the start: **why study logic at all?** It shows where logic sits among the arts of the trivium, why Aristotle's logical works were called the *Organon*, "the instrument," and why a Christian, who serves a God of truth and order, has every reason to reason well.

Students learn the **three acts of the mind** (simple apprehension, judgment and reasoning) and their products (the term, the proposition and the argument). They learn to find an argument's premises and conclusion, using indicator words, and to tell an argument from an explanation or a list of assertions. They learn the difference between **deduction and induction**, and they learn to keep truth, validity and soundness apart. They study the **first principles of thought** (identity, non-contradiction and excluded middle) and see why denying them defeats itself. The unit ends with a short history of logic from Aristotle to Frege.

Everything later in the course builds on this unit. Unit 2 studies terms, the product of the first act of the mind; Unit 4 studies propositions, the product of the second; Units 5 and 6 study arguments, the product of the third. The words *valid*, *sound*, *strong* and *cogent*, learned here, will be used in every unit through the capstone disputation.

Review day and test. On the review day, use the mastery checklist and the review sheet (with its answer key). The unit test is below; a printable copy is ready for test day.

PREREQUISITES

- Read a paragraph of grade-level prose and say in your own words what its main point is.
- Tell a statement from a question or a command.
- Use simple variables, as in Pre-Algebra (for example, “let x stand for any number”).
- Place dates on a timeline, including dates before Christ (B.C.).

MASTERY CHECKLIST

- ☐ I can define logic as the art of right reasoning and explain its place in the trivium and in a Christian’s life.
- ☐ I can name the three acts of the mind, the product of each, and the question each one answers.
- ☐ I can tell a term, a proposition and an argument apart, and say which can be true or false and which can be valid or invalid.
- ☐ I can find the premises and conclusion of an argument, with or without indicator words, and write it in standard form.
- ☐ I can tell an argument from an explanation and from a set of mere assertions.
- ☐ I can classify an argument as deductive or inductive and explain why.
- ☐ I can define valid, sound, strong and cogent, and use the four combinations of true or false premises and valid or invalid reasoning.
- ☐ I can state the laws of identity, non-contradiction and excluded middle, and tell contradictories from contraries.
- ☐ I can show why a statement that denies a first principle refutes itself.
- ☐ I can trace the history of logic from Aristotle and the Stoics to Boole,

Carroll and Frege, naming the main contribution of each figure.

LESSONS IN THIS UNIT

1.1	**What Logic Is and Why We Study It**	40 minutes
1.2	**The Three Acts of the Mind**	40 minutes
1.3	**Arguments: Premises and Conclusions**	40 minutes
1.4	**Deduction and Induction; Truth, Validity, Soundness**	45 minutes
1.5	**The First Principles of Thought**	40 minutes
1.6	**A Short History of Logic**	40 minutes

Unit 1 Test

You have 45 minutes. No notes or books. Answer each question in complete sentences unless it asks for a list or a single word. For classification questions, name the category and give a one-sentence reason. Question 14 is a short written answer of four to six sentences. The test is worth 100 points.

1. **6 pts**
   (a) Define logic. (b) Name the three arts of the trivium in order.

2. **6 pts**
   Name the three acts of the mind. For each, give its product (as expressed in words) and say whether that product is clear or unclear, true or false, or valid or invalid.

3. **6 pts**

   Classify each as a term, a proposition, an argument, or none of these.

   1. the red tractor in the machine shed
   2. Feed the calves before school.
   3. The red tractor needs new tires.
   4. The tires are bald and the roads are muddy, so the tractor needs new tires before spring.

4. **8 pts**

   Find the premises and the conclusion, and write the argument in standard form: “Hannah should bring an umbrella to the game, for the forecast calls for rain all afternoon, and the bleachers at Riverside Park have no roof.”

5. **8 pts**

   Is each passage an argument, an explanation or a set of mere assertions? Give a reason for each.

   1. The milk soured because it was left on the porch in the sun all afternoon.
   2. Someone must have left the milk out, since it has soured and it was fresh this morning.
   3. Brookside Dairy delivers on Tuesdays. Its trucks are painted blue. It sells cream in glass bottles.

6. **6 pts**

   Deductive or inductive? Give a reason for each.

   1. Every apple tree we planted on the south slope has borne fruit by its fourth year, so the new tree there will probably bear fruit by its fourth year.
   2. All mammals breathe air. Every horse is a mammal. So every horse breathes air.
   3. The last nine customers who tried Sunburst lemonade liked it, so the next customer will probably like it too.

7. **9 pts**

   For each argument, say whether it is valid or invalid and whether it is sound or unsound, with a reason.

   1. All spiders have eight legs. A tarantula is a spider. So a tarantula has eight legs.
   2. All birds have feathers. All robins have feathers. So all robins are birds.
   3. All metals are liquid at room temperature. Iron is a metal. So iron is liquid at room temperature.

8. **6 pts**

   Strong or weak? Give a reason for each.

   1. Caleb tried one dish at a new restaurant and disliked it, so every dish the restaurant serves is bad.
   2. For thirty years the first frost at the Ortiz farm has come between October 5 and October 25, so this year's first frost will probably come in October.

9. **8 pts**

   (a) Of the four combinations of true or false premises and valid or invalid reasoning, which single outcome is impossible? (b) Write a valid argument with at least one false premise and a true conclusion.

10. **8 pts**

    State the law of identity, the law of non-contradiction and the law of excluded middle.

11. **7 pts**

    Contradictories or contraries? Give a reason for each.

    1. "The gate is open" and "The gate is not open."
    2. "All the seeds sprouted" and "None of the seeds sprouted."
    3. "The bread is fresh" and "The bread is not fresh."

12. **6 pts**

    Explain why the statement "Nobody can know anything for certain" refutes itself.

13. **8 pts**

Match each person or book to its description. People and books: Aristotle; Chrysippus; Boethius; Abelard; Port-Royal *Logic*; Isaac Watts; George Boole; Gottlob Frege. Descriptions:

1. Translated Aristotle's logic into Latin around 500 and wrote *The Consolation of Philosophy*.
2. Published the *Begriffsschrift* ("concept-script") in 1879.
3. Founded logic as a subject; his logical works are called the *Organon*.
4. Wrote *Sic et Non* in the 1100s.
5. Treated logic as algebra in 1847 and 1854.
6. A 1662 textbook arranged by the acts of the mind, by Arnauld and Nicole.
7. Stoic logician who studied propositions joined by "if," "and" and "or."
8. Hymn writer who published a *Logick* in 1724.

14. **8 pts**

Short written answer (four to six sentences): Why should a Christian study logic? Use at least one idea from this unit and at least one passage of Scripture (you may give the reference without quoting it exactly).

ANSWER KEY

1. (a) Logic is the art (and science) of right reasoning: the study of how to tell good reasoning from bad, especially whether a conclusion follows from its premises. (b) Grammar, logic (dialectic), rhetoric.
2. Simple apprehension produces the term, which is clear or unclear (never true or false). Judgment produces the proposition, which is true or false. Reasoning produces the argument, which is valid or invalid (or, if inductive, strong or weak).
3. 1: term (it names a thing and claims nothing). 2: none of these; it is a command, which is neither true nor false. 3: proposition (it claims something that is true or false). 4: argument (two premises offered as reasons, signaled by "so," for the conclusion).
4. 1. The forecast calls for rain all afternoon.
   2. The bleachers at Riverside Park have no roof.
   Therefore, Hannah should bring an umbrella to the game. ("For" introduces the premises; the conclusion comes first.)
5. 1: explanation; the souring is already accepted, and the passage tells its cause. 2: argument; it offers reasons ("since") to prove a claim not yet known, that someone left the milk out. 3: mere assertions; none of the claims is offered as a reason for another.
6. 1: inductive; it reasons from past cases to a future one and claims only probability. 2: deductive; it applies a rule with no exceptions and claims the conclusion follows necessarily. 3: inductive; it reasons from a sample of customers to the next one and claims only probability.
7. 1: valid and sound; the conclusion must follow, and both premises are true. 2: invalid and therefore unsound; even though every statement is true, the premises do not guarantee the conclusion. Having feathers does not by itself make something a bird, so the premises leave open a case in which robins have feathers but are not birds; also, the same form gives "All birds are animals; all dogs are animals; so all dogs are birds," with true premises and a false conclusion. (Accept either reason.) 3: valid but unsound; the conclusion would follow if the premises were true, but the first premise is false (iron, for example, is solid at room temperature).
8. 1: weak; one dish is far too small a sample to support a conclusion about every dish. 2: strong; thirty years of consistent records make the conclusion, which claims only probability, very likely.

9. (a) A valid argument with all true premises and a false conclusion is impossible. (b) Sample: “All fish are mammals. All whales are fish. So all whales are mammals.” Both premises are false, the form is valid, and the conclusion is true. Any correct example earns full credit.
10. Identity: everything is what it is (A is A); a term must keep the same meaning throughout an argument. Non-contradiction: nothing can both be and not be at the same time and in the same respect; a proposition and its denial cannot both be true. Excluded middle: every proposition is either true or false; there is no third possibility between a proposition and its denial.
11. 1: contradictories; one must be true and the other false. 2: contraries; they cannot both be true, but both are false if some seeds sprouted and some did not. 3: contradictories; one must be true and the other false.
12. Whoever asserts it presents it as something known for certain. But if it is true, then nobody can know it for certain either, including the speaker. And if anyone does know it for certain, then something is known for certain, and the statement is false. It cannot be known for certain without being false, so it refutes itself.
13. 1: Boethius. 2: Gottlob Frege. 3: Aristotle. 4: Abelard. 5: George Boole. 6: Port-Royal *Logic*. 7: Chrysippus. 8: Isaac Watts. (1 point each.)
14. Sample answer: A Christian should study logic because God is a God of truth and order who cannot lie and is not the author of confusion (Titus 1:2; 1 Corinthians 14:33). In Isaiah 1:18 the LORD invites us to reason together, and 1 Peter 3:15 commands us to be ready to give a reason for our hope with meekness and fear. Logic helps us do this by teaching us to define our terms, find the premises and conclusion of an argument, and tell a sound argument from an unsound one. It also protects us from being fooled by mere assertions and from fooling ourselves. The laws of thought, such as non-contradiction, reflect the faithfulness of a God who cannot deny himself. Grading: 3 points for a clear reason tied to Scripture, 3 points for a correct use of at least one idea from the unit, 2 points for clear, well-organized sentences.

Printable answer key (PDF, for the parent)

1.1 What Logic Is and Why We Study It

OBJECTIVES

- Define logic as the art of right reasoning and explain what reasoning is.
- Explain the place of logic in the trivium, between grammar and rhetoric, and why Aristotle's logical works came to be called the *Organon*.
- Distinguish the question "Are the claims true?" from the question "Does the conclusion follow?"
- Explain why a Christian studies logic, using Isaiah 1:18 and 1 Peter 3:15.

WATCH

Why Study Traditional Logic in Classical Curriculum?

Memoria Press · 1:32

https://www.youtube.com/watch?v=te0AdG6DWto

> **WHEN TO WATCH**
> Before the core ideas: a short answer to "why study logic?"

Note: Parent preview: Martin Cothran, author of Memoria Press's logic books, explains that logic is one of the seven liberal arts and one of the three arts of language (grammar, logic, rhetoric), the art of rational thought. He mentions Memoria Press's own programs; this course is not connected with them. Nothing objectionable; transcript checked October 18, 2026.

KEY TERMS

Logic

The art and science of right reasoning: the study of the rules that tell good reasoning from bad, especially whether a conclusion follows from the reasons given for it.

Reasoning

The movement of the mind from things already known (or accepted) to something new that follows from them.

Art

In the classical sense, a skill guided by rules and perfected by practice, such as carpentry, baking or logic.

Trivium

Latin for "the place where three roads meet": the three language arts of grammar, logic and rhetoric, studied first in classical and medieval schools.

Quadrivium

The four arts of number that followed the trivium: arithmetic, geometry, music and astronomy.

Dialectic

An older name for logic, especially the art of testing ideas by question and answer.

Organon

Greek for "tool" or "instrument"; the name later given to Aristotle's collected works on logic, because logic is the tool used in every subject.

CORE IDEAS

Everyone reasons; not everyone reasons well. When Hannah sees muddy paw prints on the kitchen floor and the back door standing open, she concludes that Biscuit, the family's beagle, has been out in the garden. She did not see the dog go out. She moved from things she knew (the prints, the open door, the rain last night) to something she did not yet know. That movement of the mind is **reasoning**. We reason all day long: when we plan a route, judge a sale price or decide whether a story makes sense. But reasoning can go wrong. The prints might belong to the neighbor's cat. Logic is the study that teaches us to tell good reasoning from bad.

Logic is the art of right reasoning. An **art**, in the old sense of the word, is a skill guided by rules, like carpentry or baking bread: you learn the rules and practice them until they become habit. Logic is also a science, because its rules are not made up; they are discovered, the way the rules of arithmetic are discovered. Logic does not describe how people usually think (that is the job of psychology). It describes how we ought to think if we want to reach the truth. Above all, logic studies connections: whether a conclusion really follows from the reasons given for it.

Two questions, not one. Every argument can be tested in two ways. First: are the starting claims true? Second: does the conclusion follow from them? Take this argument: "All birds can fly. Penguins are birds. So penguins can fly." The conclusion follows perfectly; if the first two claims were true, the third would have to be true. But the first claim is false, so the argument fails. Now take this one: "Some dogs are brown. Rex is a dog. So Rex is brown." Even if Rex happens to be brown, the conclusion does not follow from the reasons. Logic is chiefly the study of the second question. The first question belongs to whatever field the claims are about: biology, history or farming.

The trivium: three roads. For more than a thousand years, schools in Europe began with three language arts called the **trivium**, Latin for "the place where three roads meet": grammar, logic and rhetoric. Grammar teaches us to read and say things correctly; logic teaches us to reason correctly; rhetoric teaches us to speak and write well and persuasively. After them came the four arts of number, the **quadrivium**: arithmetic, geometry, music and astronomy. Logic was often called **dialectic**, the art of testing ideas by question and answer. It sits in the middle on purpose. Grammar gives logic its words to work with, and rhetoric needs logic so that persuasion rests on truth and not merely on feeling.

The instrument of every subject. After Aristotle's death, his writings on reasoning were gathered together and came to be called the **Organon**, Greek for "tool" or "instrument." The name fits. Logic has no subject matter of its own, the way botany has plants and astronomy has stars; it is the tool you use in every subject. A doctor reasons from symptoms to a diagnosis. A historian reasons from old letters and records to what happened. A farmer reasons from a soil test to what to plant. A Bible reader reasons from the words of a passage to its meaning. In each case the facts come from the field, and the logic is the same.

Why a Christian studies logic. Logic is not a rival to faith. Scripture presents God as a God of truth, who cannot lie (Titus 1:2) and who, as 1 Corinthians 14:33 puts it, is "not the author of confusion." The Gospel of John calls Christ the Word, in Greek the *Logos*, the word from which "logic" comes. In Isaiah 1:18 the LORD himself invites, "Come now, and let us reason together." Peter tells believers to be ready to give "a reason of the hope that is in you" (1 Peter 3:15), and to do it "with meekness and fear." Logic helps us keep that command honestly: clearly, charitably and without tricks.

What logic protects you from. Good reasoning guards you against being fooled. An advertisement for Glowmax vitamins says, "Nine out of ten customers felt more energetic!" Logic teaches you to ask: Which customers? Compared with what? Who asked them, and how? Logic also guards you against fooling yourself. We all like conclusions that flatter us, and we are quick to believe what we already want to believe. Logic makes us slow down and ask whether the reasons really support the claim. And it helps us treat other people fairly, by stating their arguments accurately before we answer them.

What this course will do. The course follows the order of the mind itself: first terms (Unit 2), then propositions (Unit 4), then arguments (Units 5 and 6). Along the way you will learn the common fallacies (Unit 3), the reasoning of science and statistics (Units 7 and 8) and the use of logic in debate (Unit 9). The rest of this unit lays the foundation: the three acts of the mind, how to find the parts of an argument, the difference between deduction and induction, the first principles of thought, and a short history of the subject.

> **From history.** The seven arts of the trivium and quadrivium took shape in the late Roman world. In the fifth century, the North African writer Martianus Capella described all seven in a long allegory that medieval schools read for centuries. A generation or two later, the Christian scholar Boethius (about 480–524) used the word *quadrivium* for the four arts of number. In the cathedral schools of the Middle Ages and in the first universities, such as Paris and Oxford, students studied the trivium first, and logic took up the largest share of their time. A student who could not reason carefully was not thought ready for theology, law or medicine.

> **Practice habit.** When you hear a claim today, ask two questions: Are the reasons true? And does the conclusion really follow from them?

> **Enrichment: Isaac Watts on what logic is for.** Isaac Watts, the pastor who wrote the hymns "Joy to the World" and "When I Survey the Wondrous Cross," also wrote a logic textbook in 1724 that was used for generations in English and American schools. He opens the book by defining logic as "the art of using reason well in our enquiries after truth, and the communication of it to others." Notice the two halves of his definition: logic helps us *find* the truth, and it helps us *share* it clearly. Which half do you think is harder? Why?

Example 1. Which of these involve reasoning, the movement of the mind from what is known to something new? (a) Caleb memorizes his locker combination. (b) Mrs. Ortiz finds the oven light off and the cake batter still runny after an hour, and concludes that the oven was never turned on. (c) Hannah lists her three favorite foods.

▼ **Show solution**

1. Reasoning means drawing a new conclusion from things already known. Look for a move from "what I know" to "so this must be true."
2. (a) Memorizing stores information. Caleb does not conclude anything new, so this is not reasoning.
3. (b) Mrs. Ortiz knows two facts (the light is off; the batter is runny after an hour) and draws a new conclusion from them (the oven was never turned on). This is reasoning.
4. (c) Listing favorites reports preferences. Nothing is concluded from anything else, so this is not reasoning.

Answer: Only (b) involves reasoning: Mrs. Ortiz moves from known facts to a new conclusion.

Example 2. Test each argument with the two questions (Are the claims true? Does the conclusion follow?).

1. All metals conduct electricity. Copper is a metal. Therefore copper conducts electricity.
2. All fish live in water. Whales live in water. Therefore whales are fish.

▼ **Show solution**

1. Argument 1, first question: metals do conduct electricity, and copper is a metal. The claims are true.
2. Argument 1, second question: if every metal conducts and copper is a metal, copper must conduct. The conclusion follows.
3. Argument 2, first question: fish live in water, and whales live in water. Both claims are true.
4. Argument 2, second question: the premises say that fish are among the things that live in water, but other things (whales, frogs, water lilies) live in water too. Living in water does not make something a fish, so the conclusion does not follow. In fact the conclusion is false: whales are mammals.

Answer: Argument 1 passes both tests: its claims are true and its conclusion follows. Argument 2 has true claims, but its conclusion does not follow, so it fails the second test.

PRACTICE

1. Easy Define logic in one sentence.

   ▼ **Show answer**

   Logic is the art (and science) of right reasoning: the study of how to tell good reasoning from bad, especially whether a conclusion follows from the reasons given.

2. Easy Name the three arts of the trivium in order.

   ▼ **Show answer**

   Grammar, logic (also called dialectic) and rhetoric.

3. **Easy** Name the four arts of the quadrivium.

▼ **Show answer**

Arithmetic, geometry, music and astronomy.

4. **Easy** What does the Greek word *Organon* mean, and why is it a fitting name for Aristotle's collected works on logic?

▼ **Show answer**

It means "tool" or "instrument." It fits because logic has no subject matter of its own; it is the tool we use to reason in every subject.

5. **Medium** Which of these is an example of reasoning? Explain. (a) Reciting a psalm from memory. (b) Seeing frost on the windshield and concluding that the temperature fell to freezing overnight. (c) Watching a sunset.

▼ **Show answer**

(b) is reasoning: it moves from a known fact (frost on the windshield) to a new conclusion (it was freezing overnight). Reciting and watching do not draw any conclusion.

6. **Medium** Is logic mainly the study of whether claims are true, or of whether a conclusion follows from the claims? Who settles the other question?

▼ **Show answer**

Logic is mainly the study of whether a conclusion follows from the claims (the reasons given). Whether the claims themselves are true is settled by the field they are about, such as biology, history or farming.

7. **Medium** Test this argument with the two questions: "All squares have four sides. This shape has four sides. So this shape is a square."

▼ **Show answer**

First question: the claims can be true (squares do have four sides, and the shape may have four sides). Second question: the conclusion does not follow, because other shapes, such as rectangles that are not squares, also have four sides. The argument fails the second test.

8. Medium Explain in two sentences why logic is called both an art and a science.

▼ **Show answer**

Logic is an art because it is a skill guided by rules that we learn and practice until it becomes habit, like carpentry or baking. It is a science because its rules are not invented but discovered, the way the rules of arithmetic are discovered.

9. Hard Explain why logic sits between grammar and rhetoric in the trivium.

▼ **Show answer**

Grammar comes first because logic needs words and sentences used correctly before it can reason with them. Rhetoric comes after logic because persuasion should rest on sound reasoning and truth, not merely on feeling; logic is the bridge that makes good speaking honest.

10. Hard A poster in a shoe store says: "TrailPro boots were worn by more hikers on Mount Pell last summer than any other brand. TrailPro: the best boots you can buy." Write three questions that logic would have you ask about this claim.

▼ **Show answer**

Sample questions (any three reasonable ones): How was the number of hikers counted, and by whom? Does being worn by the most hikers show that a boot is the best, or only that it is popular or cheap? Compared with which brands, and on what measures of quality (comfort, durability, grip)? Did TrailPro pay for or run the count?

11. Hard A friend says, "Christians should just trust God. Logic is a cold, worldly thing." Answer your friend kindly in three or four sentences, using this lesson.

▼ **Show answer**

Sample answer: Logic is not a rival to faith, because God is a God of truth who cannot lie and is not the author of confusion. In Isaiah 1:18 the LORD himself says, "Come now, and let us reason together," and 1 Peter 3:15 tells us to be ready to give a reason for our hope. Logic helps us understand Scripture, avoid being fooled, and explain what we believe clearly and gently. Used rightly, it is a way of loving the truth.

12. **Challenge** Write two short arguments of your own: (a) one whose claims are all true but whose conclusion does not follow, and (b) one whose conclusion follows but which has a false claim. Explain which of the two questions each argument fails.

▼ **Show answer**

Sample answer: (a) “All horses have four legs. My dog has four legs. So my dog is a horse.” The claims are true, but the conclusion does not follow (many animals that are not horses have four legs), so it fails the second question. (b) “All vegetables are sweet. Spinach is a vegetable. So spinach is sweet.” The conclusion follows (if both claims were true, spinach would have to be sweet), but the first claim is false, so it fails the first question. Any pair built the same way is correct.

COMMON MISTAKES

- Thinking logic is about winning quarrels. Logic is about finding and sharing the truth, and it requires us to state the other side fairly.
- Thinking an argument is good just because its conclusion is true. A true conclusion can come from reasons that do not support it.
- Expecting logic to supply the facts. Logic tests whether a conclusion follows; the truth of the starting claims comes from the field they are about.
- Thinking faith and reason are enemies. Scripture invites us to reason and to give a reason for our hope.

REFERENCE GUIDE

An eight-page, color-coded Logic guide: what logic is, terms and definitions, the A E I O propositions, the square of opposition and immediate inferences, syllogisms with the six rules and Venn tests, the named forms, informal fallacies of relevance, presumption and ambiguity, propositional logic with truth tables, conditionals, valid forms and short proofs, the classical tradition, dialectic, argument maps and practice. The booklet prints on 11×17 paper and folds to letter size. Prints in color and reads clearly in black and white.

> **Scripture.** “Come now, and let us reason together, saith the LORD” (Isaiah 1:18, KJV)
>
> The course verse: the God of truth invites his people to reason with him, so reasoning well is part of answering his call.

UNIT 1 · WHY LOGIC? · LESSON 1.2 OF 69 · 40 MINUTES

1.2 The Three Acts of the Mind

OBJECTIVES

- Name the three acts of the mind: simple apprehension, judgment and reasoning.
- Name the product of each act as expressed in language: the term, the proposition and the argument.
- Explain which products can be true or false and which can be valid or invalid.
- Classify examples as a term, a proposition, an argument, or none of these.

KEY TERMS

Simple apprehension

The first act of the mind: grasping what a thing is, without affirming or denying anything about it.

Concept

The idea of a thing that the mind holds in simple apprehension, such as the idea of a horse or of courage.

Term

A concept expressed in words, such as “oak tree” or “the oldest barn in the county.” A term is neither true nor false.

Judgment

The second act of the mind: joining or separating two concepts by affirming or denying one of the other.

Proposition

A judgment expressed in words: a statement that is either true or false.

Reasoning

The third act of the mind: moving from judgments already held to a new judgment that follows from them.

Argument

Reasoning expressed in words: a group of propositions in which some (the premises) are offered as reasons for another (the conclusion).

CORE IDEAS

The mind works in three steps. Think about how you come to know that a strange dog might bite. First you grasp what a dog is. Then you join ideas together: "This dog is growling." Then you move from what you know to something new: "Growling dogs may bite, and this dog is growling, so this dog may bite." Logicians in the tradition of Aristotle and Thomas Aquinas call these the **three acts of the mind**: simple apprehension, judgment and reasoning. Each act builds on the one before it, and each has its own product.

First act: simple apprehension. To apprehend is to grasp. In **simple apprehension** the mind grasps what something is, without saying anything about it yet. When you think "oak tree," "triangle" or "courage," you hold an idea, a **concept**, in your mind. You have not affirmed or denied anything. The question simple apprehension answers is "What is it?" A small child who learns what a tractor is, and can pick out every tractor in a picture of a farm, has apprehended the thing, even before saying anything true or false about one.

Its product: the term. A concept expressed in words is a **term**. A term may be one word ("horse") or several ("the tallest horse in the barn," "people who bake bread for a living"). A term by itself is neither true nor false. If someone says only "purple cow," you cannot answer "That's false!" because nothing has been claimed. Terms can be clear or unclear, well defined or badly defined, and Unit 2 will teach you how to define them. But truth and falsity arrive only with the second act.

Second act: judgment. In **judgment** the mind joins or separates two concepts, affirming or denying one of the other. "Maple trees lose their leaves in autumn" joins the concept "maple trees" with the concept "things that lose their leaves in autumn." "No square is round" separates "square" from "round." Judgment answers the question "Is it so?" Here, for the first time, the mind can be right or wrong, because it now claims something about the way things are.

Its product: the proposition. A judgment expressed in words is a **proposition**: a statement that is either true or false. “Water boils at 212 degrees Fahrenheit at sea level” is a true proposition; “Spiders have six legs” is a false one, but it is still a proposition. Not every sentence is a proposition. Questions (“Is it raining?”), commands (“Close the gate.”) and exclamations (“What a catch!”) claim nothing true or false. A good test: can you sensibly reply, “That’s true” or “That’s false”?

Third act: reasoning. In **reasoning** the mind moves from judgments it already holds to a new judgment that follows from them. “Every plant needs light. This fern is a plant. So this fern needs light.” Reasoning answers the question “Why?” or “What follows?” It is the act that lets us know things we have not seen for ourselves, which is why it is the act logic studies most closely.

Its product: the argument. Reasoning expressed in words is an **argument**: a group of propositions in which some, the premises, are offered as reasons for another, the conclusion. An argument as a whole is not called true or false. Each of its propositions is true or false, but the argument itself is called **valid** or **invalid** (or, as you will see in Lesson 1.4, strong or weak), depending on whether the conclusion really follows.

Why the order matters. Each act depends on the one before. You cannot judge well about things you do not understand, and you cannot reason well from false judgments. Many disagreements are really first-act problems: two people use the same word for different things. Others are second-act problems: someone starts from a false claim. Only when these are cleared up can reasoning settle the matter. This course follows the same order: terms (Unit 2), propositions (Unit 4) and arguments (Units 5 and 6).

A summary to memorize.
Simple apprehension → concept, expressed as a *term* → answers “What is it?” → clear or unclear.
Judgment → expressed as a *proposition* → answers “Is it so?” → true or false.
Reasoning → expressed as an *argument* → answers “Why?” or “What follows?” → valid or invalid (strong or weak).

From history. Aristotle's logical works, as later editors arranged them, follow the order of the three acts. The *Categories* deals with terms, *On Interpretation* with propositions, and the *Prior Analytics* with the syllogism, a kind of argument. In the 1200s Thomas Aquinas, at the start of his commentary on Aristotle's *Posterior Analytics*, distinguished three acts of reason: understanding simple things, joining or separating them, and moving from one thing to another. In 1662 the Port-Royal *Logic* was arranged the same way, with parts on conceiving, judging and reasoning, plus a fourth part on method (how to order our thoughts). The pattern has shaped logic textbooks ever since.

Practice habit. When a disagreement starts, ask which act of the mind it is really about: the meaning of a word, the truth of a claim, or whether a conclusion follows.

WORKED EXAMPLES

Example 1. Classify each item as a term, a proposition or an argument.

1. the oldest barn in Millbrook County
2. The oldest barn in Millbrook County was built in 1842.
3. Every barn in Millbrook County with hand-hewn beams was built before 1880. The Hollis barn has hand-hewn beams. So the Hollis barn was built before 1880.

▼ **Show solution**

1. Item 1 names a thing but claims nothing about it. You cannot reply "true" or "false." It is a term (several words can make one term).
2. Item 2 joins the term "the oldest barn in Millbrook County" to "built in 1842." It claims something that could be checked and found true or false. It is a proposition.
3. Item 3 has three propositions, and the word "so" shows that the last is offered as following from the first two. It is an argument.

Answer: 1: term. 2: proposition. 3: argument.

Example 2. Which of these sentences are propositions?

1. Please wipe your boots.
2. Did the hens lay today?
3. Tomatoes need warm soil to grow well.
4. The moon is made of cheese.

▼ **Show solution**

1. Apply the test: can you sensibly reply "That's true" or "That's false"?
2. Sentence 1 is a command. "That's false!" makes no sense as a reply. Not a proposition.
3. Sentence 2 is a question. It asks but does not claim. Not a proposition.
4. Sentence 3 claims something about tomatoes that is true. A proposition.
5. Sentence 4 claims something about the moon that is false. A false statement is still a proposition.

Answer: Sentences 3 and 4 are propositions (3 is true, 4 is false). Sentence 1 is a command and sentence 2 is a question, so neither is a proposition.

PRACTICE

1. Easy Name the three acts of the mind in order.

   ▼ **Show answer**

   Simple apprehension, judgment and reasoning.

2. Easy Name the product (as expressed in words) of each of the three acts of the mind.

   ▼ **Show answer**

   Simple apprehension produces the term; judgment produces the proposition; reasoning produces the argument.

3. Easy Is "a basket of ripe peaches" a term, a proposition or an argument?

   ▼ **Show answer**

   A term: it names something but claims nothing, so it cannot be true or false.

4. **Easy** Is “Peaches grow on trees” a term, a proposition or an argument?

   ▼ **Show answer**

   A proposition: it affirms something about peaches, and it is true.

5. **Medium** Is this a term, a proposition or an argument? “Peaches bruise easily, and these peaches were dropped on the floor, so they are probably bruised.”

   ▼ **Show answer**

   An argument: the first two propositions are offered as reasons (“so”) for the conclusion that the peaches are probably bruised.

6. **Medium** Is “Shut the barn door!” a proposition? Explain.

   ▼ **Show answer**

   No: it is a command, and a command claims nothing that could be true or false.

7. **Medium** Which act of the mind answers the question “Is it so?” What is its product, and what quality can that product have?

   ▼ **Show answer**

   Judgment answers “Is it so?” Its product is the proposition, which is either true or false.

8. **Medium** Caleb says, “The term ‘six-legged spider’ is false, because spiders have eight legs.” What is wrong with what he said?

   ▼ **Show answer**

   A term by itself is neither true nor false, because it claims nothing. Only a proposition, such as “This spider has six legs” or “Spiders have six legs,” can be false.

9. Hard Classify each as a term, a proposition, an argument, or none of these.
   1. Lions are mammals.
   2. the lion in the east enclosure of the zoo
   3. Is the lion asleep?
   4. All lions are mammals, and Leo is a lion; therefore Leo is a mammal.

   ▼ **Show answer**

   1: proposition (it affirms something and is true). 2: term (it names a thing and claims nothing). 3: none of these; it is a question, which claims nothing true or false (though it contains terms). 4: argument (two premises offered as reasons for the conclusion "Leo is a mammal").

10. Hard Two neighbors disagree about whether Mr. Patel's "field" is large. It turns out that one means the small pasture by the road and the other means all 300 acres of the farm. Which act of the mind is the real source of the disagreement? Explain.

    ▼ **Show answer**

    The first act, simple apprehension: they are using the same term, "field," for two different concepts. Until they agree on what the term means, judging and reasoning about it cannot settle anything.

11. Hard Explain why an argument as a whole is called valid or invalid rather than true or false.

    ▼ **Show answer**

    Truth and falsity belong to propositions, which claim something about the way things are. An argument is a group of propositions joined by the claim that one follows from the others, so what we judge about the argument itself is whether that claim holds: whether the conclusion really follows. That is validity, not truth.

12. **Challenge** Write your own example of (a) a term, (b) a proposition that uses that term, and (c) an argument that uses that proposition as a premise. Label each with the act of the mind that produces it.

    ▼ **Show answer**

    Sample answer: (a) “a sourdough loaf” is a term, the product of simple apprehension. (b) “A sourdough loaf needs a long rise” is a proposition, the product of judgment. (c) “A sourdough loaf needs a long rise. Mother has only one hour before dinner. So Mother cannot bake a sourdough loaf from scratch before dinner” is an argument, the product of reasoning. Any example in which the term appears in the proposition and the proposition serves as a premise is correct.

COMMON MISTAKES

- Calling a term true or false. A term only names; it claims nothing until it is used in a proposition.
- Thinking every sentence is a proposition. Questions, commands and exclamations are not true or false.
- Thinking a term must be a single word. “The tallest horse in the barn” is one term.
- Calling an argument true. Its propositions are true or false; the argument itself is valid or invalid (or strong or weak).

> **Scripture.** “Wisdom is the principal thing; therefore get wisdom: and with all thy getting get understanding.” (Proverbs 4:7, KJV)
>
> Understanding begins with the first act of the mind, grasping clearly what things are, before we judge or reason about them.

UNIT 1 · WHY LOGIC? · LESSON 1.3 OF 69 · 40 MINUTES

1.3 Arguments: Premises and Conclusions

OBJECTIVES

- Define argument, premise and conclusion.
- Use premise and conclusion indicator words to find the parts of an argument, and find the conclusion when no indicator word is used.

- Tell an argument from an explanation and from a set of mere assertions.
- Write a short argument in standard form, with the premises first and the conclusion last.

KEY TERMS

Argument

A group of propositions in which one (the conclusion) is claimed to follow from the others (the premises).

Premise

A proposition offered as a reason for accepting the conclusion.

Conclusion

The proposition the premises are meant to support; what the speaker is trying to get you to believe.

Conclusion indicator

A word or phrase that usually comes just before a conclusion: therefore, so, thus, hence, consequently, it follows that.

Premise indicator

A word or phrase that usually comes just before a premise: because, since, for, given that, as shown by, seeing that.

Explanation

A passage that tells why or how something already accepted is true, rather than trying to prove that it is true.

Mere assertion

A claim stated without any reason offered for it. A list of assertions with no claimed support among them is not an argument.

Standard form

An argument written with each premise numbered and listed first, followed by a line and then the conclusion.

CORE IDEAS

An argument in logic is not a quarrel. In everyday speech, an argument is a fight. In logic, an **argument** is a group of propositions in which one, the **conclusion**, is claimed to follow from the others, the **premises**. The premises are the reasons; the conclusion is what they are meant to support. Two farmers can lean on a fence and calmly discuss whether to plant corn early this year, and the whole time they are trading arguments without ever raising their voices.

Indicator words point the way. Certain words signal which part is which. **Conclusion indicators** usually come just before a conclusion: therefore, so, thus, hence, consequently, it follows that, which shows that. **Premise indicators** usually come just before a premise: because, since, for, given that, as shown by, seeing that. In "Caleb should take the early bus, since the roads are icy and the late bus runs behind on icy days," the word "since" introduces the premises, and the conclusion is "Caleb should take the early bus." But an indicator word is a clue, not a proof. "Since" can mean "from the time that" ("Since June it has rained twice"), and "so" can mean "very" ("The pie was so good").

The conclusion can come anywhere. A conclusion is not always the last sentence. It may come first: "We should cover the tomatoes tonight. The forecast calls for frost, and frost kills tomato plants." It may come in the middle or at the end. To find it, ask: What is the speaker trying to get me to believe? Which claim do the others support? A useful check is the *therefore test*: put "therefore" in front of the claim you think is the conclusion and "because" in front of the others. If the passage still makes sense, you have probably found it.

Arguments without indicator words. Many arguments use no indicator words at all. "The Panthers have lost their three best pitchers to injuries. They won't win the championship this year." The second sentence is the conclusion; the first is offered as the reason. Context and the therefore test do the work: "The Panthers have lost their three best pitchers; therefore they won't win the championship" makes sense, while "They won't win the championship; therefore they have lost their pitchers" does not.

Standard form. To study an argument, rewrite it in **standard form**: number the premises and list them first, draw a line, and write the conclusion last. The tomato argument becomes:

1. The forecast calls for frost tonight.
2. Frost kills tomato plants.

Therefore, we should cover the tomatoes tonight.

Drop the indicator words and needless repetition, but do not change the meaning. Standard form lets you see the reasoning at a glance and check each premise on its own.

Explanations are not arguments. Arguments and explanations both use "because," so they are easy to confuse. An argument tries to show *that* something is true; its conclusion is in doubt until the premises support it. An **explanation** tells *why* or *how* something is true that we already accept. "The bread didn't rise because the yeast was old" explains a fact everyone at the table can see. "The yeast must be old, because the package expired last year and the bread didn't rise" argues for a claim we did not know before. Ask: Is the speaker trying to convince me of this claim, or telling me the cause of something we both already accept?

Mere assertions are not arguments. A set of claims with no claimed support among them is not an argument, however confidently they are stated. "Brookside Dairy makes the best cheese in the state. Its cows graze on clover. The company was founded in 1911." These are three assertions, not an argument, because none is offered as a reason for another. Reports, descriptions, lists, warnings, advice and opinions stated without reasons are not arguments. A single if-then sentence by itself ("If it rains, the game will be canceled") is not an argument either: it states a connection but draws no conclusion.

Why this matters. You cannot judge an argument until you know what it claims and on what grounds. Many confusions come from attacking the wrong part, or from demanding proof for an explanation of a fact no one doubts. Advertisers often rely on mere assertion: a confident list of claims dressed up to sound like proof. Learning to say, politely, "That is not an argument; no reason was given" is one of the most useful skills in this course.

> **From history.** The words we use for the parts of an argument come from Latin. "Premise" comes from *praemissa*, "sent ahead," because premises go before the conclusion. "Conclusion" comes from *concludere*, "to shut up" or "to close," because the conclusion closes the argument. The Latin word *ergo*, "therefore," rang through medieval classrooms, where students argued in strict syllogisms. Aristotle, in the *Prior Analytics*, described a syllogism, roughly, as speech in which certain things are laid down and something different from them follows necessarily because they are so. That idea, that a conclusion must follow from what is laid down, is the heart of every argument.

> **Practice habit.** When you read an advertisement or hear a claim, ask: What is the conclusion, and what reasons, if any, are given for it?

WORKED EXAMPLES

Example 1. Find the premises and the conclusion, and write the argument in standard form: "The ice on Miller's Pond is only two inches thick. It follows that no one should skate on the pond today, since ice must be at least four inches thick to be safe for skating."

▼ **Show solution**

1. Look for indicators. "It follows that" introduces the conclusion: no one should skate on the pond today.
2. "Since" introduces a premise: ice must be at least four inches thick to be safe for skating.
3. The first sentence has no indicator, but it is offered as a reason too. Therefore test: "The ice is only two inches thick; therefore no one should skate" makes sense.
4. Write the premises first, then the conclusion.

Answer: 1. The ice on Miller's Pond is only two inches thick.
2. Ice must be at least four inches thick to be safe for skating.
Therefore, no one should skate on Miller's Pond today.

Example 2. Is each passage an argument, an explanation or a set of mere assertions?

1. The cake fell because the oven door was opened too early.
2. Someone must have opened the oven door too early, since the cake fell and nothing else was different from last week.
3. Mrs. Ortiz bakes every Saturday. Her pound cake won a blue ribbon. She uses her grandmother's recipe.

▼ **Show solution**

1. Passage 1: everyone can see that the cake fell; the passage tells why it fell. It is not trying to prove that the cake fell. Explanation.
2. Passage 2: the claim "someone opened the door too early" is not known; the passage offers reasons ("since") to prove it. Argument.
3. Passage 3: three claims, none offered as a reason for another. Mere assertions.

Answer: 1: explanation. 2: argument (conclusion: someone opened the oven door too early). 3: mere assertions.

PRACTICE

1. Easy List three conclusion indicators and three premise indicators.

   ▼ **Show answer**

   Conclusion indicators (any three): therefore, so, thus, hence, consequently, it follows that, which shows that. Premise indicators (any three): because, since, for, given that, as shown by, seeing that.

2. Easy Find the premise and the conclusion: "The soil is dry and cracked, so the garden needs water."

   ▼ **Show answer**

   Premise: the soil is dry and cracked. Conclusion: the garden needs water (introduced by "so").

3. **Easy** Find the premise and the conclusion: “We should leave for the game early, because the bridge on Route 9 is closed.”

   ▼ **Show answer**

   Premise: the bridge on Route 9 is closed (introduced by “because”).
   Conclusion: we should leave for the game early.

4. **Easy** Is this an argument? “Our school was built in 1924. It has twelve classrooms. The gym has a maple floor.”

   ▼ **Show answer**

   No, it is a set of mere assertions: none of the three claims is offered as a reason for another.

5. **Medium** Find the conclusion: “Barley can’t be planted in the north field this week. The ground is still frozen, and seed won’t sprout in frozen ground.”

   ▼ **Show answer**

   Conclusion: barley can’t be planted in the north field this week (it comes first). The other two claims are the premises.

6. **Medium** Argument or explanation? “The tomatoes split because heavy rain came right after a long dry spell.”

   ▼ **Show answer**

   Explanation: the splitting is already accepted, and the passage tells its cause rather than trying to prove that the tomatoes split.

7. **Medium** Argument or explanation? “The tomatoes will probably split this week, because heavy rain is coming after a long dry spell, and that is when tomatoes split.”

   ▼ **Show answer**

   Argument: it offers reasons to convince us of a claim not yet known (that the tomatoes will probably split this week).

8. Medium This argument has no indicator words. Find the premises and the conclusion: “No player may play in a game without a signed physical form. Josiah has not turned in his form. He can’t play in Saturday’s game.”

▼ **Show answer**

Premises: no player may play in a game without a signed physical form; Josiah has not turned in his form. Conclusion: Josiah can’t play in Saturday’s game. (Therefore test: “No one may play without a form, and Josiah has none; therefore he can’t play” makes sense.)

9. Hard Write this argument in standard form: “Given that every bird in the henhouse was counted at dusk, and the count came to 23, it follows that one hen is missing, for we had 24 hens this morning.”

▼ **Show answer**

1. Every bird in the henhouse was counted at dusk.
2. The count came to 23.
3. We had 24 hens this morning.

Therefore, one hen is missing.

10. Hard Is “If the river rises another foot, the lower field will flood” an argument? Explain, and then add one premise that turns it into an argument with a conclusion.

▼ **Show answer**

No: a single if-then sentence states a connection but draws no conclusion. Adding the premise “The river will rise another foot” gives an argument: “If the river rises another foot, the lower field will flood. The river will rise another foot. Therefore the lower field will flood.”

11. Hard Does this passage contain an argument? “Since last spring, Caleb has run two miles every morning, and his mile time has dropped by a full minute.” Explain.

▼ **Show answer**

No: here “since” means “from the time that,” not “because,” and the passage simply reports two facts without offering either as a reason to believe a conclusion.

12. **Challenge** Read the passage: “Brightline paint is the best choice for the barn. The barn faces west, so it takes the full afternoon sun all summer. Brightline is the only paint at the hardware store rated to last ten years in full sun. Also, it comes in a handsome barn red. The store is open until six.” (a) What is the main conclusion? (b) Which claims support it? (c) Which claim is itself supported by another claim? (d) Which sentence is not part of the argument at all? (e) Write the argument in standard form.

▼ **Show answer**

(a) Brightline paint is the best choice for the barn. (b) The barn takes the full afternoon sun all summer; Brightline is the only paint at the store rated to last ten years in full sun; it comes in a handsome barn red. (c) “The barn takes the full afternoon sun all summer” is supported by “the barn faces west” (“so”). (d) “The store is open until six” is a mere assertion that gives no reason for the conclusion. (e) 1. The barn faces west.
2. The barn takes the full afternoon sun all summer (this follows from 1).
3. Brightline is the only paint at the store rated to last ten years in full sun.
4. Brightline comes in a handsome barn red.
Therefore, Brightline paint is the best choice for the barn.

COMMON MISTAKES

- Assuming the conclusion is always the last sentence. Ask what the speaker is trying to get you to believe.
- Treating every “because” as a premise indicator. Explanations use “because” too, to give the cause of something already accepted.
- Treating “since” or “so” as indicators when they mean “from the time that” or “very.”
- Calling a confident list of claims an argument. If no claim is offered as a reason for another, there is no argument.
- Thinking a single if-then sentence is an argument. It states a connection but draws no conclusion.

> **Scripture.** “And Paul, as his manner was, went in unto them, and three sabbath days reasoned with them out of the scriptures” (Acts 17:2, KJV)
>
> Paul did not simply assert his message; he reasoned, taking his premises from the Scriptures his hearers already accepted.

UNIT 1 · WHY LOGIC? · LESSON 1.4 OF 69 · 45 MINUTES

1.4 Deduction and Induction; Truth, Validity, Soundness

OBJECTIVES

- Classify an argument as deductive or inductive by the kind of support it claims.
- Define validity and soundness for deductive arguments, and strength and cogency for inductive arguments.
- Keep truth (a property of propositions) apart from validity (a property of arguments), using the four combinations of true or false premises and valid or invalid reasoning.
- Explain why a valid argument with true premises cannot have a false conclusion.

WATCH

Difference between inductive and deductive reasoning

Khan Academy · 2:00

https://www.youtube.com/watch?v=GEId0GonOZM

> **WHEN TO WATCH**
> After the core ideas on deduction and induction.

Note: Parent preview: a town’s past population figures. Projecting the trend into future years is inductive (likely, not certain); drawing conclusions that must follow from known facts is deductive. Nothing objectionable; transcript checked October 18, 2026.

KEY TERMS

Deductive argument

An argument that claims its conclusion follows necessarily: if the premises are true, the conclusion must be true.

Inductive argument

An argument that claims its conclusion follows probably: if the premises are true, the conclusion is likely to be true.

Valid

Said of a deductive argument in which it is impossible for all the premises to be true and the conclusion false.

Sound

Said of a deductive argument that is valid and has all true premises. A sound argument always has a true conclusion.

Strong

Said of an inductive argument whose conclusion would probably be true if its premises were true. An inductive argument that fails this test is weak.

Cogent

Said of an inductive argument that is strong and has all true premises.

Truth

A property of propositions: a proposition is true when it says of what is that it is, or of what is not that it is not. Arguments are not called true or false.

CORE IDEAS

Two kinds of support. Every argument claims that its premises support its conclusion, but the claim can be stronger or weaker. A **deductive argument** claims that the conclusion follows *necessarily*: if the premises are true, the conclusion must be true. An **inductive argument** claims only that the conclusion follows *probably*: if the premises are true, the conclusion is likely. “Every hen in this coop is a Leghorn, and Pearl is in this coop, so Pearl is a Leghorn” is deductive. “Every Leghorn we have ever owned laid white eggs, so our new Leghorn will lay white eggs” is inductive.

How to tell them apart. Look first at the words. “Must,” “necessarily” and “it is certain that” point to deduction; “probably,” “likely” and “I expect” point to

induction. Then look at the kind of reasoning. Arguments from a sample to a whole group, from the past to the future, from one case to a similar case, or from an effect to its likely cause are usually inductive. Arguments from definitions, from mathematics, or that apply a rule with no exceptions to a particular case are usually deductive. Avoid the old shortcut "deduction goes from general to particular." The argument "All dogs are mammals, and all mammals are animals, so all dogs are animals" is deductive, yet it is general from start to finish.

Truth belongs to propositions; validity belongs to arguments. A proposition is true or false. A deductive argument is valid or invalid. An argument is **valid** when it is impossible for all its premises to be true and its conclusion false: the conclusion really follows. Validity is about the connection, not about whether the premises are actually true. "All fish can fly. A trout is a fish. So a trout can fly" is valid: if the premises were true, the conclusion would have to be true. But the first premise is false. To show that an argument is invalid, describe a possible case in which every premise is true and the conclusion is false.

Soundness: valid plus true. A deductive argument is **sound** when it is valid and all its premises are true. A sound argument always has a true conclusion, which is why soundness is the goal of deductive reasoning. To show that a deductive argument is unsound, you need only show that it is invalid or that at least one premise is false. An argument can be valid but unsound, like the flying trout. It can never be sound but invalid.

The four boxes. Set true or false premises against valid or invalid reasoning, and you get four combinations.
Valid, all premises true: the conclusion must be true. This is a sound argument.
Valid, at least one premise false: the conclusion may be true or false. "All birds are mammals; all bats are birds; so all bats are mammals" has two false premises and a true conclusion. The flying trout has a false conclusion.
Invalid, all premises true: the conclusion may be true or false. "All cows are animals; all horses are animals; so all horses are cows" has true premises and a false conclusion.
Invalid, at least one premise false: again the conclusion may be true or false.
Only one thing is impossible: a valid argument with all true premises and a false conclusion.

Inductive arguments: strong or weak. Inductive arguments are never called valid or invalid, because they never claim necessity. Instead they are **strong** or **weak**. An inductive argument is strong when, if its premises were true, its conclusion would probably be true. "Caleb has been on time for practice every day for two years, so he will probably be on time tomorrow" is strong. "The first two apples I picked from one tree were sweet, so every apple in the whole orchard is sweet" is weak: the sample is tiny and the conclusion reaches much too far. Strength comes in degrees; validity does not.

Cogency: strong plus true. An inductive argument is **cogent** when it is strong and its premises are true. Cogent is to induction what sound is to deduction. Even a cogent argument can have a false conclusion, since induction claims only probability: a rooster that has crowed at dawn every day for three years might be sick tomorrow. That is not a flaw in induction; it is simply what "probably" means. Most of what we know about the world, from farming to medicine, rests on cogent inductive arguments.

The two questions, with names. Lesson 1.1 gave two questions: Are the premises true? Does the conclusion follow? Now they have names. For a deductive argument, "Does it follow?" asks whether it is valid, and both questions together ask whether it is sound. For an inductive argument, "Does it follow?" asks whether it is strong, and both together ask whether it is cogent. Keep each word in its place: propositions are true or false; deductive arguments are valid or invalid, sound or unsound; inductive arguments are strong or weak, cogent or not cogent.

> **From history.** Aristotle studied deduction in the *Prior Analytics*, where he worked out the valid forms of the syllogism. He also named induction (in Greek, *epagōgē*), the movement from particular cases to a general truth, though he treated it more briefly. Nearly two thousand years later, the English writer Francis Bacon published the *Novum Organum* (1620), the "New Instrument," whose title answered Aristotle's *Organon*. Bacon urged careful induction from many observations and warned against leaping to general conclusions from a few cases. In 1843 John Stuart Mill's *System of Logic* gave rules for finding causes by induction, which you will study in Unit 7.

> **Practice habit.** When you meet an argument today, label it deductive or inductive, then ask whether its premises are true and whether its conclusion follows.

WORKED EXAMPLES

Example 1. Classify and evaluate: “All the eggs in this carton are brown. This egg came from this carton. Therefore this egg is brown.”

▼ **Show solution**

1. Kind of support: the conclusion is claimed to follow necessarily from a rule with no exceptions applied to one case. Deductive.
2. Validity: could both premises be true and the conclusion false? If every egg in the carton is brown and this egg came from the carton, it cannot fail to be brown. Valid.
3. Soundness: if the premises are in fact true (we checked the carton), the argument is sound, and the conclusion is true.

Answer: Deductive and valid. It is sound if both premises are true.

Example 2. Classify and evaluate: “All squares are rectangles. This figure is a rectangle. So this figure is a square.”

▼ **Show solution**

1. The argument claims its conclusion follows necessarily from the premises. Deductive.
2. Look for a possible case with true premises and a false conclusion: a long, thin rectangle is a rectangle but not a square. Both premises can be true while the conclusion is false.
3. So the argument is invalid, and an invalid argument is unsound no matter how true its premises are.

Answer: Deductive, invalid (a rectangle that is not a square is a counterexample), and therefore unsound.

Example 3. Classify and evaluate: “Out of 200 tomato seeds from the same packet planted last week, 190 have sprouted. So most of the remaining seeds in the packet will sprout too.”

▼ **Show solution**

1. The conclusion is about seeds not yet planted, reasoning from a sample to the rest. It claims probability, not necessity. Inductive.
2. The sample is large (200 seeds) and comes from the same packet as the rest, and the conclusion claims only “most.” If the premise is true, the conclusion is very likely. Strong.
3. If the count of 190 out of 200 is accurate, the premise is true, so the argument is cogent.

Answer: Inductive and strong; cogent if the count is accurate.

PRACTICE

1. Easy Define a valid argument.

   ▼ **Show answer**

   A valid argument is a deductive argument in which it is impossible for all the premises to be true and the conclusion false.

2. Easy Define a sound argument.

   ▼ **Show answer**

   A sound argument is a deductive argument that is valid and has all true premises (so its conclusion must be true).

3. Easy Deductive or inductive? “Every time we have visited the lake in July, the water was warm enough to swim. So it will probably be warm enough this July.”

   ▼ **Show answer**

   Inductive: it reasons from past visits to the future and claims only that the conclusion is probable.

4. **Easy** Deductive or inductive? "All triangles have three sides. This shape is a triangle. So it has three sides."

   ▼ **Show answer**

   Deductive: it applies a rule with no exceptions to one case and claims the conclusion follows necessarily.

5. **Medium** Valid or invalid? Sound or unsound? "All roses are flowers. All flowers are plants. Therefore all roses are plants."

   ▼ **Show answer**

   Valid and sound: if the premises are true the conclusion must be true, and both premises are in fact true.

6. **Medium** Valid or invalid? Sound or unsound? "All whales are fish. All fish have gills. So all whales have gills."

   ▼ **Show answer**

   Valid but unsound: the conclusion would have to be true if the premises were true, but the first premise is false (whales are mammals, not fish).

7. **Medium** Someone claims this proves its conclusion: "Some students in Mrs. Ortiz's class play the violin. Hannah is in Mrs. Ortiz's class. So Hannah plays the violin." Is it valid or invalid? Explain.

   ▼ **Show answer**

   Invalid: both premises could be true while Hannah is one of the students who do not play the violin, so the conclusion does not follow necessarily (and the argument is therefore unsound).

8. **Medium** Strong or weak? "Elijah tasted one strawberry from a fifty-acre field and found it sour, so the whole crop in the field is sour."

   ▼ **Show answer**

   Weak: a sample of one strawberry is far too small to support a conclusion about a whole fifty-acre crop.

9. Hard “All cats are dogs. All dogs are animals. Therefore all cats are animals.” Which of the four boxes does this argument belong in? What does it show about true conclusions?

▼ **Show answer**

Valid with at least one false premise (“All cats are dogs” is false), and its conclusion is true. It shows that a true conclusion does not prove an argument is sound; a valid argument with a false premise can still reach a true conclusion.

10. Hard Can an argument be valid, have all true premises, and have a false conclusion? Explain.

▼ **Show answer**

No: that is exactly what “valid” rules out. A valid argument is one in which it is impossible for all the premises to be true and the conclusion false, so a valid argument with true premises (a sound argument) always has a true conclusion.

11. Hard Compare these two inductive arguments. Which is stronger, and why?
    1. A survey of 1,000 shoppers chosen at random at grocery stores across the state found that 62% prefer Crispy Valley apples to Redwing apples. So probably most shoppers in the state prefer Crispy Valley apples to Redwing apples.
    2. A survey of 20 people standing in line at a Crispy Valley tasting booth found that 18 prefer Crispy Valley apples to Redwing apples. So probably most shoppers in the state prefer Crispy Valley apples to Redwing apples.

▼ **Show answer**

Argument 1 is stronger: its sample is large and chosen at random from across the state, so it fairly represents the state’s shoppers. Argument 2 is weak: its sample is small and made of people who chose to stand at a Crispy Valley booth, who are likely to favor that apple already. If its survey was done accurately, argument 1 is also cogent.

12. **Challenge** Write your own example of each: (a) a valid argument with a false premise and a false conclusion; (b) an invalid argument with true premises and a true conclusion; (c) a cogent inductive argument. Explain briefly why each fits.

▼ **Show answer**

Sample answers: (a) “All birds can swim. All ostriches are birds. So all ostriches can swim” is valid (the conclusion must follow if the premises are true), its first premise is false, and its conclusion is false. (b) “All dogs are animals. All beagles are animals. So all beagles are dogs” has true premises and a true conclusion, but it is invalid: the same form gives “All dogs are animals; all cats are animals; so all cats are dogs,” with true premises and a false conclusion. (c) “Every one of the 300 Brookside Dairy cows tested this spring was free of the disease, so the herd is probably free of it” is strong (a large sample from the same herd), and it is cogent if the test results are true. Any examples that fit the descriptions are correct.

COMMON MISTAKES

- Calling a premise “valid” or an argument “true.” Propositions are true or false; arguments are valid or invalid (or strong or weak).
- Thinking a valid argument must have a true conclusion. Only a sound argument (valid with all true premises) guarantees a true conclusion.
- Calling an inductive argument valid or invalid. Inductive arguments are strong or weak, cogent or not cogent.
- Deciding deductive or inductive by “general to particular.” Ask instead whether the argument claims necessity or probability.
- Thinking a true conclusion proves an argument is good. True conclusions can come from invalid arguments and from false premises.

> **Scripture.** “searched the scriptures daily, whether those things were so” (Acts 17:11, KJV)
>
> The Bereans did not stop at whether Paul’s conclusions followed; they checked his premises against Scripture, testing for soundness.

1.5 The First Principles of Thought

OBJECTIVES

- State the law of identity, the law of non-contradiction and the law of excluded middle.
- Explain the conditions "at the same time and in the same respect," and tell a real contradiction from an apparent one.
- Tell contradictories from contraries, and apply the law of excluded middle only to contradictories.
- Show how a statement that denies a first principle refutes itself.

KEY TERMS

First principle

A truth so basic that it is known as soon as its terms are understood, and that every proof uses rather than proves.

Law of identity

Everything is what it is: A is A. In reasoning, a term must keep the same meaning throughout an argument.

Law of non-contradiction

Nothing can both be and not be, at the same time and in the same respect. A proposition and its denial cannot both be true.

Law of excluded middle

Every proposition is either true or false; there is no third possibility between a proposition and its denial.

Contradictories

Two propositions that cannot both be true and cannot both be false: one must be true and the other false ("The soup is hot" and "The soup is not hot").

Contraries

Two propositions that cannot both be true but can both be false ("The soup is hot" and "The soup is cold").

Self-refuting statement

A statement that, if it were true, would make itself false, such as "There is no truth."

Some things cannot be proved, and need not be. Every proof starts from premises. If each premise needed its own proof, and each of those another, no proof could ever begin. So reasoning rests on **first principles**: truths so basic that they are known as soon as their terms are understood, and that every proof uses. Aristotle, in Book IV of his *Metaphysics*, said in effect that demanding a proof of everything shows a lack of training, because an educated person knows which things need proof and which do not. Three such principles are often called the laws of thought.

The law of identity. Everything is what it is: A is A. A horse is a horse; a promise is a promise. This sounds too plain to need saying, but it means that things have real natures and that our words, used consistently, can name them. In reasoning, identity requires a term to keep the same meaning all the way through an argument. If "bank" means the edge of a river in one premise and a place to keep money in the next, the term has quietly become two terms, and the argument falls apart.

The law of non-contradiction. Nothing can both be and not be, at the same time and in the same respect. Put in terms of propositions: a statement and its denial cannot both be true. "The gate is locked" and "The gate is not locked" cannot both be true of the same gate at the same moment. Aristotle called this the firmest of all principles, the one about which no one can be mistaken. Without it, saying anything would mean nothing, because every claim would fit just as well with its opposite.

"At the same time and in the same respect." These words matter. "The water is hot" and "The water is not hot" can both be true at different times, before and after the kettle boils. "Grandpa is tall" and "Grandpa is not tall" can both be true in different respects: tall compared with his grandchildren, not tall compared with a basketball center. "The ring is gold and not gold" may mean gold in color but not gold in metal. Before you call something a contradiction, check that the thing, the time, the respect and the meaning of the words are really the same. Many apparent contradictions disappear when you do.

The law of excluded middle. Every proposition is either true or false; there is no third option between a statement and its denial. Either the lamp is on or it is not on. Either Caleb finished his chores or he did not. We may not *know* which is true, but one of them is. Aristotle made the point in *Metaphysics* IV, in paraphrase: there can

be nothing in between two contradictory statements, so about any one subject we must either affirm or deny any one thing.

Contradictories and contraries. Excluded middle applies to **contradictories**, pairs in which one must be true and the other false: "hot" and "not hot," or "all the eggs hatched" and "not all the eggs hatched." It does not apply to **contraries**, pairs that cannot both be true but can both be false: "hot" and "cold" (the bath may be lukewarm), or "all the eggs hatched" and "none of the eggs hatched" (some may have hatched). Many false choices come from treating contraries as contradictories: "Either you love baseball or you hate it." You might simply not care much either way.

Denying the laws defeats itself. A **self-refuting statement** is one that, if true, would make itself false. "There is no such thing as truth" claims to be true. "No one can know anything for certain" claims to be known for certain. "I cannot write a single sentence in English" refutes itself when written in English. Even to argue against non-contradiction, a person must mean one thing by their words and not its opposite; that is, they must use the very law they deny. Aristotle made this point: anyone who says something definite has already granted the principle.

Why these laws matter to a Christian. The laws of thought were not invented by Aristotle; he noticed them and set out two of them, non-contradiction and excluded middle, with great care. They reflect the order of a world made by a God who, in the words of 2 Timothy 2:13, "cannot deny himself," and who, as Titus 1:2 says, "cannot lie." Our Lord taught, "let your communication be, Yea, yea; Nay, nay" (Matthew 5:37): an honest yes means yes and not also no. At bottom, logic is a discipline of honesty. We refuse to say a thing and its opposite at once, and we keep our words meaning what they mean.

> **From history.** Aristotle defended the law of non-contradiction at length in Book IV of the *Metaphysics*. Some thinkers of his day seemed to deny it, and some reported that Heraclitus had said the same thing can be and not be, though Aristotle doubted that anyone could truly believe this. Rather than prove the law from something more basic, Aristotle showed that anyone who denies it must use it. Medieval logicians treated non-contradiction as the first principle of all reasoning. The phrase "laws of thought" became common much later, in the 1800s; George Boole even titled his 1854 book *An Investigation of the Laws of Thought*.

Practice habit. When two claims seem to clash, ask: same thing, same time, same respect, same meaning?

Enrichment: Aristotle and the firmest principle. In Book IV of the *Metaphysics*, Aristotle meets a person who claims that a thing can both be and not be. He does not try to prove the law from something more basic, since nothing is more basic. Instead he asks the person simply to say something that means something, such as the word "man." If the word means one thing and not its opposite, the person has already granted the law. If the person refuses to mean anything at all, Aristotle says, in paraphrase, that such a person is no better off in argument than a plant. Try it: can you deny the law of non-contradiction without using it?

WORKED EXAMPLES

Example 1. A sportswriter says, "The Wildcats are the best team in the league, and the Wildcats are not the best team in the league." Is this a real contradiction?

▼ **Show solution**

1. A real contradiction affirms and denies the same thing of the same subject, at the same time and in the same respect.
2. Ask what the writer means. Suppose the Wildcats have the best win-loss record this season but the weakest record in close games against good teams.
3. Then "best" is used in two respects: best by season record, not best in close games against strong teams.
4. Restate the claim to make the respects plain: "The Wildcats have the best record in the league, but they are not the best team in close games." Nothing is both affirmed and denied in the same respect.

Answer: Not a real contradiction if "best" is meant in two different respects; it only sounds like one. Stated carefully, both parts can be true.

Example 2. Show that the statement "All statements are false" refutes itself.

▼ **Show solution**

1. The statement is itself a statement, so it speaks about itself.
2. Suppose it is true. Then all statements are false, including this one. So it is false. Assuming it true leads to its being false.
3. Therefore it cannot be true. It must be false: at least one statement is true.

Answer: If "All statements are false" were true, it would be false, since it is one of the statements it describes. So it is self-refuting and must be false.

Example 3. Are "Every apple in the basket is ripe" and "No apple in the basket is ripe" contradictories or contraries? What is the contradictory of the first?

▼ **Show solution**

1. Can both be true? No: the apples cannot all be ripe and all be unripe.
2. Can both be false? Yes: if some apples are ripe and some are not, both statements are false.
3. Cannot both be true but can both be false: contraries.
4. The contradictory of "Every apple in the basket is ripe" must be true exactly when the first is false: "Not every apple in the basket is ripe," that is, "At least one apple in the basket is not ripe."

Answer: Contraries (both are false when some apples are ripe and some are not). The contradictory of the first is "Not every apple in the basket is ripe."

PRACTICE

1. Easy State the law of identity.

   ▼ **Show answer**

   Everything is what it is: A is A. (In reasoning, a term must keep the same meaning throughout an argument.)

2. **Easy** State the law of non-contradiction, including its two conditions.

   ▼ **Show answer**

   Nothing can both be and not be at the same time and in the same respect; a proposition and its denial cannot both be true.

3. **Easy** State the law of excluded middle.

   ▼ **Show answer**

   Every proposition is either true or false; there is no third possibility between a proposition and its denial.

4. **Easy** Which law says that "The barn is red" and "The barn is not red" cannot both be true of the same barn at the same time and in the same respect?

   ▼ **Show answer**

   The law of non-contradiction.

5. **Medium** Contradictories or contraries? "The soup is hot" and "The soup is cold."

   ▼ **Show answer**

   Contraries: they cannot both be true, but both can be false if the soup is lukewarm.

6. **Medium** Contradictories or contraries? "The soup is hot" and "The soup is not hot."

   ▼ **Show answer**

   Contradictories: one must be true and the other false, with no third possibility.

7. **Medium** Mr. Lee is 38. His neighbor says, "Mr. Lee is old and not old." Is this a real contradiction? Explain.

   ▼ **Show answer**

   Not necessarily: it is an apparent contradiction if "old" is meant in two respects, for example old for a professional ballplayer but not old for a farmer or a teacher, so nothing is affirmed and denied in the same respect.

8. Medium Is the sentence “I never speak English,” said aloud in English, self-refuting? Explain.

▼ **Show answer**

Yes: the act of saying it in English makes it false, so if it were true it could not be said this way.

9. Hard Explain why the statement “There are no absolute truths” refutes itself.

▼ **Show answer**

The statement presents itself as an absolute truth, true always and for everyone. If it were true, there would be at least one absolute truth (itself), so it would be false. Therefore it cannot be true.

10. Hard Someone says, “Either the new reading plan is perfect or it is useless.” What is wrong with this reasoning? Use the lesson’s terms.

▼ **Show answer**

It treats contraries as contradictories. “Perfect” and “useless” cannot both be true, but both can be false: the plan may be helpful but imperfect. Excluded middle applies only to contradictories, such as “perfect” and “not perfect.”

11. Hard Which law is broken, and how? “A pen is a writing tool. My pig lives in a pen. So my pig lives in a writing tool.”

▼ **Show answer**

The law of identity: the term “pen” does not keep the same meaning (a writing tool in the first premise, an animal enclosure in the second), so the argument really has two different terms and the conclusion does not follow.

12. **Challenge** Proverbs 26:4 tells us not to answer a fool according to his folly, lest we become like him. The very next verse, Proverbs 26:5, tells us to answer a fool according to his folly, lest he be wise in his own conceit. Is this a violation of the law of non-contradiction? Explain carefully.

    ▼ **Show answer**

    No. The law forbids affirming and denying the same thing at the same time and in the same respect. The two verses give different reasons, so they speak in different respects: do not answer a fool in a way that copies his folly (sinking to his level), but do answer him in a way that exposes his folly (so he does not think himself wise). Wisdom is needed to know which situation calls for which, but the verses do not contradict each other.

COMMON MISTAKES

- Calling something a contradiction without checking that the subject, the time, the respect and the meaning of the words are the same.
- Applying excluded middle to contraries. “Hot or cold” leaves room for lukewarm; only “hot or not hot” leaves no middle.
- Thinking first principles are weak because they cannot be proved. They are not proved because every proof depends on them.
- Treating a self-refuting statement as merely an opinion we disagree with. It cannot possibly be true, because its truth would make it false.

> **Scripture.** “If we believe not, yet he abideth faithful: he cannot deny himself.” (2 Timothy 2:13, KJV)
>
> God never contradicts himself, and the law of non-contradiction reflects the faithfulness and order of the God who made our minds.

UNIT 1 · WHY LOGIC? · LESSON 1.6 OF 69 · 40 MINUTES

1.6 A Short History of Logic

OBJECTIVES

- Trace the main line of logic from Aristotle and the Stoics through Boethius and the medieval schoolmen to Port-Royal, Watts, Leibniz, Boole, Carroll and Frege.
- Name the main contribution of each figure and place it in the right century.

- Explain how logic moved from words to symbols in the 1800s.
- Explain why Christian scholars preserved and developed logic.

KEY TERMS

Syllogism

A deductive argument with two premises and a conclusion, built from three terms; Aristotle's central discovery was which forms of it are valid.

Term logic

Aristotle's logic, which studies how terms such as "dogs" and "mammals" are related in propositions and syllogisms.

Propositional logic

The logic of how whole propositions are joined by words such as "if," "and" and "or"; first developed by the Stoics.

Scholastics

The teachers of the medieval schools and universities (also called schoolmen), who made logic the center of their curriculum.

Port-Royal Logic

Logic, or the Art of Thinking (1662), by Antoine Arnauld and Pierre Nicole, a widely used textbook arranged by the acts of the mind.

Symbolic logic

Logic written in symbols, like algebra, so that the form of reasoning can be studied exactly; developed by Boole, Frege and others in the 1800s.

Begriffsschrift

German for "concept-script": Gottlob Frege's 1879 symbolic system, from which modern logic grows.

CORE IDEAS

Aristotle, the founder. Aristotle (384–322 B.C.) was the first to study reasoning as a subject of its own. His six logical works, later gathered as the *Organon*, move from terms (*Categories*) to propositions (*On Interpretation*) to the syllogism (*Prior Analytics*), and then to scientific proof (*Posterior Analytics*), probable reasoning (*Topics*) and fallacies (*Sophistical Refutations*). His great discovery was that validity depends on form: "All M are P; all S are M; so all S are P" is valid whatever S, M and P stand for.

Near the end of the *Sophistical Refutations* he remarks that on reasoning he had no earlier work to build on.

The Stoics and Chrysippus. In the century after Aristotle, Stoic philosophers in Athens built a different kind of logic. Chrysippus (about 279–206 B.C.) studied how whole propositions are joined by "if," "and" and "or." The Stoics listed basic argument forms such as "If the first, then the second; the first; therefore the second," the form later called *modus ponens*. Their logic of propositions is an ancestor of the symbolic logic you will study in Unit 6. Chrysippus is said to have written hundreds of books, but nearly all of them are lost.

Boethius, the bridge. Boethius (about 480–524), a Christian scholar and Roman official, set out to put Aristotle into Latin. He translated several of the logical works, along with Porphyry's short introduction to them, and wrote commentaries. For about six hundred years his versions of the *Categories* and *On Interpretation*, with Porphyry, were the core of logic in Western Europe; later scholars called this body of texts the "old logic." Boethius is best remembered for *The Consolation of Philosophy*, written in prison before his execution.

Abelard and the schoolmen. In the early 1100s Peter Abelard (1079–1142), a brilliant and combative teacher at Paris, made logic exciting. His *Sic et Non* ("Yes and No") gathered statements from respected Christian writers that seemed to disagree, and challenged students to resolve them by careful distinctions. During the 1100s the rest of Aristotle's logic became available in Latin, and this "new logic" joined the old. The teachers of the cathedral schools and new universities, the scholastics or schoolmen, made logic the center of their studies.

Peter of Spain and Thomas Aquinas. In the 1200s Peter of Spain wrote the *Summulae Logicales* ("Little Summaries of Logic"), a textbook so popular that it was used in universities for centuries. Thomas Aquinas (1225–1274) wrote commentaries on Aristotle's logical works and used logic throughout his *Summa Theologiae*, where each article sets out objections, a statement "on the contrary," his own answer and replies to the objections. For the schoolmen, logic served theology: faith seeks understanding, and clear reasoning helps it.

Port-Royal and Watts. In 1662 Antoine Arnauld and Pierre Nicole, of the Port-Royal community near Paris, published *Logic, or the Art of Thinking*. It was arranged by the acts of the mind (conceiving, judging and reasoning), with a fourth part on method,

and it was used widely for two centuries. In 1724 the English pastor and hymn writer Isaac Watts published his *Logick*, subtitled *The Right Use of Reason in the Enquiry after Truth*. It taught generations of students in Britain and America.

Leibniz's dream. Gottfried Wilhelm Leibniz (1646–1716), the German thinker who invented calculus at about the same time as Isaac Newton, dreamed of a universal symbolic language in which reasoning could be done like arithmetic. Then, he hoped, two people who disagreed could settle the matter by sitting down together and saying, "Let us calculate." He worked out pieces of such a logic, but most of his notes on it were not published until long after his death.

Boole, Frege and Carroll. George Boole, a largely self-taught English mathematician, treated logic as a kind of algebra in *The Mathematical Analysis of Logic* (1847) and *An Investigation of the Laws of Thought* (1854); his algebra of true and false later became the basis of computer circuits. In 1879 the German mathematician Gottlob Frege published his *Begriffsschrift* ("concept-script"), a symbolic system strong enough to express the reasoning of mathematics; modern logic grows from it. Charles Dodgson, an Oxford mathematics lecturer better known as Lewis Carroll, wrote *The Game of Logic* (1886) and *Symbolic Logic* (1896), full of cheerfully absurd puzzles for young readers.

One long conversation. Logic did not begin again in each age. Each generation received the work of the last, corrected it and added to it. Christian scholars such as Boethius, Abelard, Aquinas, the Port-Royal authors and Watts preserved Aristotle's logic and built on it, because they believed that truth is one and that all of it belongs to God. When you study logic, you join that conversation.

> **From history.** Two English logicians of the 1800s kept the old logic alive while the new symbolic logic was being born. In 1826 Richard Whately, an Oxford teacher who later became an archbishop, published his *Elements of Logic*, which revived the study of logic at Oxford after a long decline. In 1870 William Stanley Jevons published *Elementary Lessons in Logic*, a clear textbook used in schools for decades. Jevons also built a wooden "logic piano," a machine with keys that could work out the conclusions of simple logical problems mechanically. He showed it to the Royal Society in London in 1870, one of the first machines ever built to reason.

Practice habit. When you learn a new rule of logic this year, ask who first noticed it and how it was handed down to you.

Enrichment: Lewis Carroll's playful syllogisms. Lewis Carroll wrote his logic books in the 1880s and 1890s to make logic fun for young people. His premises are deliberately silly, so that readers must follow the form rather than rely on what they already believe. Here is a puzzle in his spirit (not one of his own): "No kittens that wear hats are shy. All kittens in this basket wear hats." What follows? (No kitten in this basket is shy.) Try writing a silly two-premise puzzle of your own, and trade it with someone at home.

WORKED EXAMPLES

Example 1. Put these in order from earliest to latest, with dates: Boole's *Laws of Thought*; the Port-Royal *Logic*; Boethius's translations; Aristotle's *Organon*; Frege's *Begriffsschrift*; Abelard's *Sic et Non*.

▼ **Show solution**

1. Aristotle lived 384–322 B.C., so his works come first.
2. Boethius lived about 480–524, in the late Roman world.
3. Abelard taught in the early 1100s (he lived 1079–1142).
4. The Port-Royal *Logic* was published in 1662.
5. Boole's *Laws of Thought* appeared in 1854, and Frege's *Begriffsschrift* in 1879.

Answer: Aristotle's *Organon* (300s B.C.), Boethius's translations (about 500), Abelard's *Sic et Non* (1100s), the Port-Royal *Logic* (1662), Boole's *Laws of Thought* (1854), Frege's *Begriffsschrift* (1879).

Example 2. Match each figure to a contribution: Chrysippus, Peter of Spain, Leibniz, Lewis Carroll. Contributions: (a) dreamed of doing reasoning by calculation; (b) wrote *The Game of Logic* and *Symbolic Logic*; (c) developed the logic of propositions joined by "if," "and" and "or"; (d) wrote the *Summulae Logicales*, a textbook used for centuries.

▼ **Show solution**

1. Chrysippus was the great Stoic logician, who studied how whole propositions are joined: (c).
2. Peter of Spain wrote the medieval textbook *Summulae Logicales*: (d).
3. Leibniz hoped that disputes could be settled by saying "Let us calculate": (a).
4. Lewis Carroll (Charles Dodgson) wrote *The Game of Logic* (1886) and *Symbolic Logic* (1896): (b).

Answer: Chrysippus (c); Peter of Spain (d); Leibniz (a); Lewis Carroll (b).

PRACTICE

1. Easy Who was the first to study logic as a subject of its own, and what name was later given to his collected logical works?

   ▼ **Show answer**

   Aristotle (384–322 B.C.); his logical works were later gathered as the *Organon*, "the instrument."

2. Easy Which Stoic logician is especially remembered, and what kind of logic did the Stoics develop?

   ▼ **Show answer**

   Chrysippus; the Stoics developed propositional logic, the logic of whole propositions joined by "if," "and" and "or."

3. Easy Who published a book called *Logick* in 1724, and what else is he known for?

   ▼ **Show answer**

   Isaac Watts, an English pastor who is also famous as a hymn writer (for example, "Joy to the World").

4. **Easy** In what year was the Port-Royal *Logic* published, and who wrote it?

   ▼ **Show answer**

   1662, by Antoine Arnauld and Pierre Nicole of the Port-Royal community near Paris.

5. **Medium** Why is Boethius called a bridge in the history of logic?

   ▼ **Show answer**

   He translated Aristotle's logical works (with Porphyry's introduction) into Latin and wrote commentaries, and for about six hundred years his versions were the core of logic in Western Europe, carrying Aristotle's logic from the ancient world into the Middle Ages.

6. **Medium** What did Peter Abelard's *Sic et Non* contain, and what did it train students to do?

   ▼ **Show answer**

   It gathered statements from respected Christian writers that seemed to disagree, and it trained students to resolve the apparent disagreements by careful distinctions.

7. **Medium** What was the *Summulae Logicales*, and why was it important?

   ▼ **Show answer**

   It was a logic textbook written by Peter of Spain in the 1200s, and it was so popular that universities used it for centuries.

8. **Medium** Name George Boole's two books on logic with their dates, and say why his work matters today.

   ▼ **Show answer**

   The Mathematical Analysis of Logic (1847) and *An Investigation of the Laws of Thought* (1854). He treated logic as algebra, and his algebra of true and false later became the basis of computer circuits.

9. Hard Put these in order from earliest to latest, with a date for each: Frege's *Begriffsschrift*; Carroll's *Symbolic Logic*; Boole's *Mathematical Analysis of Logic*; Leibniz; Watts's *Logick*.

▼ **Show answer**

Leibniz (1646–1716); Watts's *Logick* (1724); Boole's *Mathematical Analysis of Logic* (1847); Frege's *Begriffsschrift* (1879); Carroll's *Symbolic Logic* (1896).

10. Hard Explain the difference between Aristotle's term logic and Stoic propositional logic, with an example of each.

▼ **Show answer**

Aristotle's term logic studies how terms are related inside propositions, for example "All roses are flowers; all flowers are plants; so all roses are plants," where the terms "roses," "flowers" and "plants" carry the reasoning. Stoic propositional logic treats whole propositions as units joined by "if," "and" and "or," for example "If it is day, it is light; it is day; therefore it is light," where the reasoning depends on how the two whole propositions are connected.

11. Hard What was Leibniz's dream for logic, and how did Boole and Frege move toward it?

▼ **Show answer**

Leibniz dreamed of a universal symbolic language in which reasoning could be done like arithmetic, so that people who disagreed could say "Let us calculate." Boole moved toward it by treating logic as algebra (1847, 1854), and Frege by creating a full symbolic "concept-script" (1879) strong enough to express the reasoning of mathematics.

12. **Challenge** A classmate says, “Logic was forgotten in the Middle Ages and only rediscovered in modern times.” Answer your classmate respectfully, using at least three facts from this lesson.

▼ **Show answer**

Sample answer: That is not accurate. Boethius translated Aristotle’s logic into Latin around 500, and his versions were studied for some six hundred years. In the 1100s Abelard made logic exciting at Paris with his *Sic et Non*, and the rest of Aristotle’s logic returned to the West in Latin. In the 1200s Peter of Spain’s *Summulae Logicales* became a standard textbook for centuries, and Aquinas used logic throughout his work. Far from forgetting logic, medieval scholars made it the center of their schools, and modern logic built on what they preserved.

COMMON MISTAKES

- Thinking Aristotle himself named his logical works the *Organon*. The name was given later, by those who gathered and studied his works.
- Mixing up Aristotle’s term logic with the Stoics’ propositional logic. Aristotle studied how terms relate; the Stoics studied how whole propositions are joined.
- Thinking logic was forgotten in the Middle Ages. Medieval schools made logic the center of their studies.
- Mixing up the dates of the symbolic logicians: Boole 1847 and 1854, Frege 1879, Carroll 1886 and 1896.

> **Scripture.** “Prove all things; hold fast that which is good.” (1 Thessalonians 5:21, KJV)
>
> Each generation of logicians tested what it received and kept what was sound, as Paul tells every Christian to do.

UNIT 2

Terms and Definitions

Unit 1 showed that the mind works in three acts, and that the first act, simple apprehension, grasps *what a thing is* and expresses it in a **term**. This unit studies that first act closely. Many arguments go wrong long before anyone reasons badly: they go wrong because a word changes meaning halfway through, or because two people

use the same word for different things, or because a key word was never defined at all. Clear terms come first. As the old saying of the schools put it, before you argue, define your terms.

Students learn to tell when a word is used **univocally**, **equivocally** or **analogically**, and to catch a word that slides from one meaning to another. They learn the Port-Royal distinction between a term's **extension** (what it applies to) and its **intension** (what it means), and why one shrinks as the other grows. They meet Aristotle's ten **categories** and Porphyry's five **predicables**, then use genus and difference to write **classical definitions** and to place a term on the **Tree of Porphyry**. They learn the rules that separate good definitions from bad ones, including how to spot a **persuasive definition**, and the rules for **dividing** a class into parts that do not overlap and leave nothing out.

These tools are used for the rest of the course. Unit 3 names the fallacies of ambiguity (equivocation among them) that this unit first teaches you to notice. Unit 4's propositions and Unit 5's syllogisms depend on terms that keep one meaning and one extent from line to line. And in Unit 9 and the capstone disputation, you will define the key terms of your own resolution by genus and difference, because an argument about undefined words is usually a quarrel about nothing.

Review day and test. On the review day, use the mastery checklist and the review sheet (with its answer key). The unit test is below; a printable copy is ready for test day.

PREREQUISITES

- Name the three acts of the mind and their products: term, proposition and argument (Lesson 1.2).
- Find the premises and conclusion of a short argument and write it in standard form (Lesson 1.3).
- Tell a valid argument from an invalid one in simple cases (Lesson 1.4).
- State the law of identity and explain why a term must keep one meaning throughout an argument (Lesson 1.5).
- Use a dictionary to find the different meanings of a word.

MASTERY CHECKLIST

- ☐ I can tell whether a word is used univocally, equivocally or analogically, and give a reason.
- ☐ I can find a word that changes meaning in the middle of an argument and explain why the argument fails.
- ☐ I can tell a term's extension from its intension and explain why, along a chain of genus and species, one shrinks as the other grows.
- ☐ I can name Aristotle's ten categories and say which category a predicate falls under.
- ☐ I can name Porphyry's five predicables and say how a predicate is related to its subject (genus, species, difference, property or accident).
- ☐ I can write a classical definition by genus and difference and test it by turning it around.
- ☐ I can place a term on the Tree of Porphyry and read a definition from the tree.
- ☐ I can test a definition by the rules (not too broad or too narrow, states the essence, not circular, not negative, not obscure or figurative) and fix a bad one.
- ☐ I can recognize a persuasive definition and explain how it slants an argument.

- ☐ I can divide a class by a single principle into parts that do not overlap and leave nothing out, and find the faults in a bad division.

LESSONS IN THIS UNIT

2.1	**Terms: Univocal, Equivocal, Analogical**	40 minutes
2.2	**Extension and Intension**	40 minutes
2.3	**The Categories and the Predicables**	45 minutes
2.4	**Definition by Genus and Difference**	45 minutes
2.5	**Good and Bad Definitions**	40 minutes
2.6	**Division and Classification**	40 minutes

Unit 2 test

You have 45 minutes. No notes or books. Answer each question in complete sentences unless it asks for a list or a single word. For classification questions, name the category and give a one-sentence reason. Question 14 is a short written answer of four to six sentences. The test is worth 100 points.

1. **6 pts**

   Is the italic word used univocally, equivocally or analogically in each pair? Give a reason for each.

   1. “A *bat* flew out of the cave.” / “Caleb swung the *bat*.”
   2. “a *healthy* colt” / “a *healthy* breakfast”
   3. “Mrs. Ortiz’s *tractor*” / “Mr. Lee’s *tractor*”

2. **7 pts**

   Find the word that shifts meaning, give its two meanings, and explain why the argument fails: “Every good pitcher can throw a strike. The blue pitcher on Grandma’s table is a good pitcher. So the blue pitcher on Grandma’s table can throw a strike.”

3. **6 pts**

   (a) Define extension and intension. (b) Give the extension and the intension of “season of the year.”

4. **6 pts**

   Arrange these terms in order of increasing intension, and say what happens to their extension: parallelogram, plane figure, square, polygon, rectangle.

5. **6 pts**

   (a) Do “unicorn” and “winged horse” have the same extension? The same intension? Explain. (b) Is “unicorn” a meaningless word? Explain.

6. **8 pts**

   Name the category of each predicate about Mrs. Ortiz: (a) is a woman (b) is patient (c) is the mother of three (d) is in the orchard (e) is kneeling (f) is wearing gloves (g) is pruning a pear tree (h) is being stung by a bee

7. **8 pts**

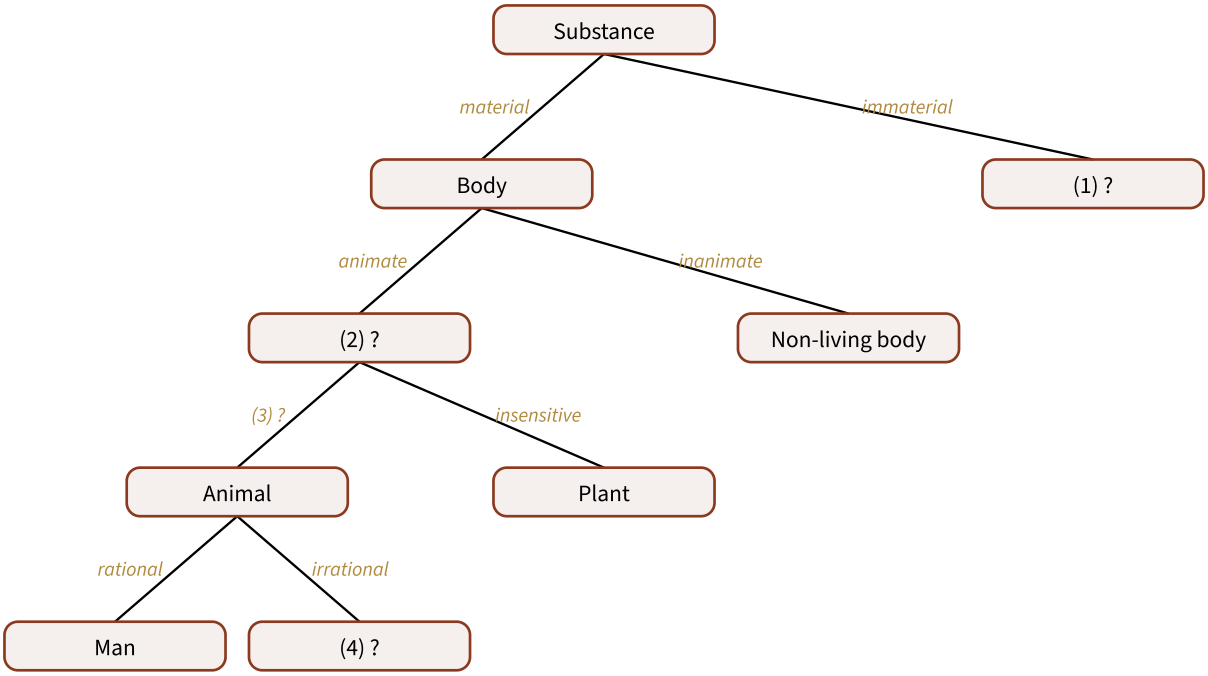
   A rectangle is defined as a parallelogram with four right angles. Name the predicable in each statement, with a short reason: (a) “A rectangle is a parallelogram.” (b) “A rectangle has four right angles.” (c) “This figure is a rectangle.” (d) “This rectangle is drawn in pencil.” (e) “Man is able to laugh.”

8. **8 pts**

   (a) Write a classical definition of “foal” and of “rectangle,” and name the genus and the difference in each. (b) Test your definition of “rectangle” by turning it around.

9. **8 pts**

Here is the Tree of Porphyry with four blanks.



(a) Fill in blanks 1–4. (b) Using the tree, define “man” and “plant.”

10. **9 pts**

Name the fault in each definition and write a better one.

1. “A clock is a machine.”
2. “Happiness is the state of being happy.”
3. “A lie is anything said that is not true.”
4. “A teacher is a lamp in a dark room.”

11. **7 pts**

Which statement is a persuasive definition and which is a stipulative one? Explain the difference. (a) Brightway Tutoring’s poster: “A serious student is one who signs up for tutoring.” (b) The spelling bee rules: “In this contest, a ‘word’ means any entry in the Westfield School Dictionary.”

12. **7 pts**

(a) Is dividing a farm into fields, barn, house and orchard a logical or a physical division? Why? (b) Find every fault in this division: “The vegetables in our garden are root vegetables, leafy vegetables, green vegetables and vegetables picked yesterday.”

13. **8 pts**

Divide the whole numbers from 1 to 20 by dichotomy at two levels: divide them into two parts, then divide one of the parts into two. Name the principle at each step, list the members of every part, and explain why each step is mutually exclusive and jointly exhaustive.

14. **6 pts**

Short written answer (four to six sentences): Why should the two sides of a debate define their key terms before they argue? Use ideas from at least two lessons of this unit.

ANSWER KEY

1. 1: Equivocal; a flying animal and a wooden club are unrelated meanings that share only a spelling. 2: Analogical; the colt has health and the breakfast helps cause it, so the second meaning points back to the first. 3: Univocal; "tractor" means the same kind of machine in both. (2 points each.)
2. "Pitcher" shifts. In the first premise it means a baseball player who throws to the batter; in the second it means a jug for pouring. The two premises are about different things, so they do not connect, and the conclusion does not follow. This shift is called equivocation. (2 points for the word, 2 for the two meanings, 3 for the explanation.)
3. (a) Extension: the range of things a term applies to. Intension: the characteristics a thing must have for the term to apply; the term's meaning. (b) Extension: spring, summer, autumn (fall) and winter. Intension: one of the four parts into which the year is divided, each marked by its own weather and length of day. (Accept any intension that gives the characteristics rather than a list.)
4. Plane figure, polygon, parallelogram, rectangle, square. Each term is a species of the one before, so as the intension grows (a characteristic is added at each step), the extension shrinks: each term applies to fewer figures than the one before.
5. (a) Same extension: both are empty, since nothing that exists is a unicorn or a winged horse. Different intension: one means a horse-like creature with a single horn, the other a horse with wings. (b) No. It has a clear intension (we know exactly what would count as a unicorn); it simply has an empty extension.
6. (a) substance (b) quality (c) relation (d) place (e) position (posture) (f) state (having) (g) action (h) passion (being acted on). (1 point each.)
7. (a) Genus: the part of the essence shared with other parallelograms. (b) Difference: the part of the essence that sets rectangles apart from other parallelograms. (c) Species: the whole of what this figure is. (d) Accident: it could be drawn in ink, or not drawn at all, and still be a rectangle. (e) Property: not part of man's essence, but it follows from being rational and belongs to every man, only to men, and always. (Accept "ability to laugh is a property" with any correct reason.)
8. (a) "A foal is a young horse": genus horse, difference young. "A rectangle is a parallelogram with four right angles": genus parallelogram, difference with four right angles. (Accept "a quadrilateral with four right angles," which uses a more remote but correct genus.) (b) Every rectangle is a parallelogram with four right

angles, and every parallelogram with four right angles is a rectangle: both directions are true, so the definition is convertible.

9. (a) 1: Spirit. 2: Living body. 3: sensitive. 4: Irrational animal. (b) Man is a rational animal. A plant is an insensitive living body. (1 point for each blank, 2 points for each definition.)
10. 1: Too broad: tractors and sewing machines are machines but not clocks. Better: "A clock is an instrument that measures and shows the time." 2: Circular: it uses "happy" to explain "happiness." Better (sample): "Happiness is the lasting enjoyment of what is truly good." 3: Too broad: an honest mistake (Caleb says the game starts at three, believing it) is untrue but not a lie. (Accept also "too narrow": a person who says what he believes is false, in order to deceive, is lying even if the statement turns out by chance to be true.) Better: "A lie is a statement the speaker believes to be false, made in order to deceive." 4: Figurative: it says what a teacher is like, not what a teacher is. Better: "A teacher is a person who helps others learn." (1 point for each fault named, 1 point for each sound repair, and 1 point for a correct counterexample in item 1 or item 3; accept any repair that obeys the rules, and accept "too broad" for item 4 if the student adds that a lamp also fits.)
11. (a) is persuasive: it is slanted to sell tutoring, and it is offered as if it were the real meaning of "serious student," so that anyone who does not buy tutoring seems unserious. (b) is stipulative: it openly announces a special meaning for one stated purpose, the contest, and does not pretend to report what "word" means everywhere. A persuasive definition hides its slant; a stipulative one is announced honestly.
12. (a) Physical: the parts are pieces of one whole, and none of them can be called "a farm." (b) Cross-division: it uses three principles (the part of the plant eaten, the color, and when it was picked). Not mutually exclusive: spinach is both leafy and green, and a carrot picked yesterday is both a root vegetable and picked yesterday. Not jointly exhaustive: a red tomato picked last week fits none of the parts. (2 points for (a); 5 points for (b), naming cross-division, overlap and a gap, each with an example.)
13. Sample: Level 1, by evenness: even (2, 4, 6, 8, 10, 12, 14, 16, 18, 20) and odd (1, 3, 5, 7, 9, 11, 13, 15, 17, 19). Level 2, even numbers by divisibility by 4: divisible by 4 (4, 8, 12, 16, 20) and not divisible by 4 (2, 6, 10, 14, 18). Each step divides by one feature and its opposite, so every number either has the feature or lacks it (nothing left out), and none both has it and lacks it (no overlap). Accept any correct two-level dichotomy with every member listed correctly; for example,

odd numbers divided into those less than 10 (1, 3, 5, 7, 9) and those greater than 10 (11, 13, 15, 17, 19).

14. Sample answer: If a key word is used equivocally or analogically, the two sides may seem to disagree when they are really talking about different things, or an argument may seem to work only because a word shifts meaning. Defining the term by genus and difference fixes its intension, and so fixes its extension: everyone knows what the term covers. The rules of definition guard against definitions that are too broad or too narrow, circular, negative or figurative. They also expose persuasive definitions, which try to win the argument by slanting the meaning of a word. As Isaiah 5:20 warns, calling things by false names leads to judging them falsely. Grading: 2 points for each correct use of an idea from a different lesson (up to 4), 2 points for clear, well-organized sentences.

> Printable answer key (PDF, for the parent)

UNIT 2 · TERMS AND DEFINITIONS · LESSON 2.1 OF 69 · 40 MINUTES

2.1 Terms: Univocal, Equivocal, Analogical

OBJECTIVES

- Explain how one word can express several concepts, and why a reasoner must know which concept a word carries.
- Define univocal, equivocal and analogical uses of a word, and give an example of each.
- Explain analogy using the word “healthy,” and tell analogy by relation to a main meaning from analogy by likeness of relation.
- Catch a word that changes meaning in the middle of an argument and explain why the argument fails.

KEY TERMS

Term

A word or group of words that expresses a concept and can serve as the subject or predicate of a proposition, such as “dog” or “the oldest barn in the county.”

Univocal

Used with exactly the same meaning each time it is applied to different things, as “dog” is said of a beagle and of a collie.

Equivocal

Used with different meanings that have nothing to do with each other, so that the uses share only a spelling or a sound, as “bat” the animal and “bat” the wooden club.

Analogical

Used with meanings that are different but related, so that one meaning is understood by pointing to the other, as “healthy” is said of a horse and of the oats that keep it well.

Context

The surrounding words and situation that show which meaning a word has in a particular sentence.

Ambiguous

Open to more than one meaning in a given sentence, because the context does not settle which meaning is intended.

Equivocation

Shifting from one meaning of a word to another within a single argument, so that the conclusion only seems to follow.

CORE IDEAS

Words are not the same as terms. In Unit 1 you learned that a term is a concept expressed in words. But words and concepts do not line up one for one. One concept can be expressed by several words: “sofa” and “couch” name the same piece of furniture. And one word can carry several concepts. Say “bat” and a listener may picture a small furry animal that flies at dusk or a smooth wooden club. Before you can reason well with a word, you must know which concept it is carrying. Logicians

since Aristotle have sorted the ways one word can be said of different things into three kinds: **univocal**, **equivocal** and **analogical**.

Univocal: one word, one meaning. The Latin *uni-* means "one" and *vox* means "voice." A word is used **univocally** when it has exactly the same meaning each time it is applied. "Dog" said of Hannah's collie and of Caleb's beagle is univocal. The two animals differ in size, color and temper, but the word means the same thing in both cases. "Farmer" said of Mrs. Ortiz and of Mr. Lee is univocal too. Notice that univocal does not mean the things are identical; it means the word keeps one meaning. Careful reasoning depends on univocal words. In "Every dog is a mammal; Rex is a dog; so Rex is a mammal," the argument works only because "dog" means the same thing in both premises.

Equivocal: one word, unrelated meanings. A word is used **equivocally** when it has different meanings that have nothing to do with each other. The *bark* of a dog and the *bark* of an oak tree share a spelling and nothing else. So do the *pitcher* on the baseball mound and the *pitcher* of lemonade on the table, or the *seal* on a rock and the *seal* pressed into wax. Knowing what a dog's bark is tells you nothing about tree bark. Equivocal words are the raw material of puns and riddles, which is harmless fun. They become dangerous only when an argument quietly uses one word in two of its meanings.

Analogical: different meanings that are related. Between these two lies a third kind of use. Consider the word "healthy." A horse is healthy; the oats it eats are healthy; its glossy coat is healthy. The meanings are not the same: oats do not *have* good health, they help *cause* it, and the glossy coat is a *sign* of it. Yet the meanings are not unrelated either, for each one points back to the health of the animal. The horse has health, the oats produce it and the coat shows it. A word used this way is **analogical**: its meanings differ, but you explain the secondary meanings by referring to a main one. Thomas Aquinas often used "healthy" to teach this idea.

Analogy by likeness of relation. A second kind of analogy works by comparing relations. We speak of the *foot* of a mountain because the bottom of the mountain is to the mountain as a foot is to a body: the lowest part, on which the rest stands. We call a student *bright* because a quick mind is to truth somewhat as a lamp is to things in a dark room: it lets them be seen. Many everyday expressions work like this: the *head* of a river, a *sharp* mind, a *sweet* grandmother. Here is a useful test. If you can

explain one meaning by pointing to the other, the use is analogical. If the only link is spelling or sound, it is equivocal.

Context usually settles the meaning. Most common words have several meanings, and that is no defect in a language; context usually tells us which one is meant. "Caleb leaned his bat against the dugout fence" leaves no doubt. But "Hannah saw a bat near the barn" is **ambiguous**: the context does not settle whether she saw an animal or a club. A careful writer removes the doubt by adding a detail ("a bat flying near the barn") or by choosing a different word. A careful reader notices when the doubt is there.

Catching a word that shifts. The real danger comes when an argument uses one word in two senses. "Every bank keeps money in a vault. The Millers ate their picnic on the bank of Elk Creek. So the Millers ate their picnic on something that keeps money in a vault." The argument looks orderly, but "bank" means a business in the first premise and the edge of a creek in the second, so the premises never connect. This error is called **equivocation**, and you will meet it again among the fallacies in Unit 3 and in syllogisms in Unit 5. To catch it, find each word that appears in more than one line, say in your own words what it means in each line, and check whether the meanings match.

Analogical words can slip too. A shift between related meanings is just as fatal as a shift between unrelated ones. "A sharp knife must be kept away from small children. Caleb is sharp. So Caleb must be kept away from small children." "Sharp" is analogical here (a keen edge, a keen mind), but the argument still fails, because the premises are about two different kinds of sharpness. Related meanings are often harder to catch than unrelated ones, precisely because they feel alike. The law of identity from Lesson 1.5 applies: within an argument, a term must mean what it means, every time it appears.

From history. Aristotle's *Categories* begins with this very distinction. Things are named equivocally, he says, when they share only a name but the account of what the name means is different; they are named univocally when they share both the name and its meaning. His own example of the first is the Greek word *zōon*, which could mean a living creature or a painted figure: a man and a picture of a man could both be called by it. Boethius carried these terms into Latin around 500. Medieval thinkers, above all Thomas Aquinas in the 1200s, worked out a middle path between the two, analogy, especially to explain how words such as "good" and "wise" can be said truly both of creatures and of God.

Practice habit. When an argument uses the same word twice, stop and say in your own words what it means each time. If the meanings differ, the argument is broken.

Enrichment: Aquinas on how we speak of God. In the *Summa Theologica* (First Part, Question 13, Article 5), Thomas Aquinas asks whether words like "wise" and "good" mean the same thing when said of God and of creatures. His answer, put roughly in our own words: not univocally, because God is not wise in the limited way a person is wise; but not purely equivocally either, because then we could learn nothing about God from his works, as Scripture says we can (Romans 1:20). Such words are said analogically. Can you explain, in a sentence or two, why "Father" said of God and of an earthly father is neither univocal nor merely equivocal?

WORKED EXAMPLES

Example 1. Is the italic word used univocally, equivocally or analogically in each pair? (a) "The *pitcher* struck out three batters." / "Mom filled the *pitcher* with lemonade." (b) "Hannah's collie is a *dog*." / "Caleb's beagle is a *dog*." (c) "Fresh spinach is *healthy*." / "Mrs. Ortiz's calves are *healthy*."

▼ **Show solution**

1. For each pair, say in your own words what the word means in each sentence, then ask: same meaning, related meanings or unrelated meanings?
2. (a) A baseball player who throws to the batter; a jug for pouring. Knowing one tells you nothing about the other. Unrelated meanings: equivocal.
3. (b) In both sentences "dog" means the same kind of animal. The two dogs differ, but the word does not. Same meaning: univocal.
4. (c) Spinach helps cause good health; the calves have good health. The meanings differ, but the first is explained by pointing to the second. Related meanings: analogical.

Answer: (a) Equivocal. (b) Univocal. (c) Analogical: spinach causes health, and the calves have it.

Example 2. Find the shift in meaning: "The sign at the lot says, 'Fine for parking here.' Whatever is fine is acceptable. So parking here is acceptable."

▼ **Show solution**

1. Find the word that appears in more than one line: "fine."
2. In the sign, "fine" is a noun meaning a money penalty: you will be charged for parking here.
3. In the second premise, "fine" is an adjective meaning all right or acceptable.
4. The meanings are unrelated, so "fine" is used equivocally. The premises do not connect, and the conclusion does not follow. In fact, the sign says the opposite of the conclusion.

Answer: "Fine" shifts from "a penalty" (on the sign) to "all right" (in the second premise). Because of this equivocation the argument fails; the sign actually warns against parking there.

PRACTICE

1. Easy In your own words, define a univocal, an equivocal and an analogical use of a word.

   ▼ **Show answer**

   Univocal: the word has exactly the same meaning each time it is used. Equivocal: the word has different meanings that are unrelated, sharing only spelling or sound. Analogical: the word has different but related meanings, so that one is explained by pointing to the other. Accept any wording that keeps these three ideas: same, unrelated, related.

2. Easy Univocal, equivocal or analogical? Give a reason. "The *bark* of the old oak is rough." / "The dog's *bark* woke the baby."

   ▼ **Show answer**

   Equivocal: the outer covering of a tree and the sound a dog makes are unrelated meanings that only share a spelling.

3. Easy Univocal, equivocal or analogical? Give a reason. "Mrs. Ortiz is a *farmer*." / "Mr. Lee is a *farmer*."

   ▼ **Show answer**

   Univocal: in both sentences "farmer" means the same thing, a person who works the land and raises crops or animals for a living.

4. Easy Aquinas used the word "healthy" to explain analogy. A horse, its oats and its glossy coat can all be called healthy. How is each one related to health?

   ▼ **Show answer**

   The horse *has* health; the oats *cause* or help produce health; the glossy coat is a *sign* of health. (Accept "shows" or "is evidence of" for the coat, and "keeps it healthy" for the oats.)

5. **Medium** Classify the italic word in each pair as univocal, equivocal or analogical, with a reason.
   1. "We camped at the *foot* of the hill." / "Caleb hurt his left *foot*."
   2. "A *seal* lay sunning on the rock." / "The letter was closed with a wax *seal*."
   3. "This peach is *sweet*." / "Grandma has a *sweet* way with toddlers."

   ▼ Show answer

   1: Analogical; the bottom of the hill is to the hill as a foot is to a body, the lowest part on which the rest stands. 2: Equivocal; a sea animal and a stamped piece of wax have nothing to do with each other. 3: Analogical; a pleasant taste and a pleasant, gentle manner are different but related meanings (the second is named from the first).

6. **Medium** Is "bright" used univocally in "a bright lamp" and "a bright student"? Explain.

   ▼ Show answer

   No. It is used analogically. A bright lamp gives off much light; a bright student has a quick mind. The meanings differ, but they are related by likeness: a quick mind "sees" and makes things clear somewhat as a lamp lets the eye see.

7. **Medium** The sentence "Hannah went to the bank" is ambiguous. Give its two meanings, and rewrite the sentence twice so that each version has only one meaning.

   ▼ Show answer

   Meaning 1: she went to a business that keeps and lends money. Meaning 2: she went to the edge of a river or creek. Sample rewrites: "Hannah went to the bank to deposit her birthday money." "Hannah went down to the bank of the creek to fish." Accept any rewrites whose added details settle each meaning.

8. Medium Find the word that shifts meaning, and give its two meanings: “All stars shine in the night sky. Ellen Brooks is the star of the Riverside softball team. So Ellen Brooks shines in the night sky.”

▼ Show answer

“Star” shifts. In the first premise it means a heavenly body that gives light; in the second it means the most outstanding player on a team. Because the meaning changes, the premises do not connect and the conclusion does not follow. (The question does not ask for a classification. If a student adds one, analogical is best, since the player is named by likeness to a bright star; do not take off points for calling it equivocal, since the answer to the question asked is still right.)

9. Hard Find the shift and explain why the argument fails: “Nothing is better than a warm supper. A cold sandwich is better than nothing. So a cold sandwich is better than a warm supper.”

▼ Show answer

“Nothing” shifts. In the first premise, “nothing is better than a warm supper” means “there is no thing that is better than a warm supper.” In the second premise, “better than nothing” means “better than having no food at all.” The two premises are about different things, so they cannot be joined, and the conclusion does not follow.

10. Hard Why must the key words in a careful argument be used univocally? Use an argument of your own to show what goes wrong when they are not.

▼ Show answer

An argument connects its premises through the words they share; if a shared word changes meaning, the premises are really about different things and the link between them breaks, even though the words look the same. Sample: “Every pen should be cleaned out each week. This ballpoint is a pen. So this ballpoint should be cleaned out each week.” “Pen” means an animal enclosure in the first premise and a writing tool in the second, so the conclusion does not follow. Accept any example in which a shared word changes meaning and the student explains the break.

11. Hard Classify each use as univocal, equivocal or analogical, and explain the relation where there is one.
    1. "the *key* to the barn door" / "the *key* to the riddle"
    2. "a *cold* glass of milk" / "a *cold* stare"
    3. "a *triangle* drawn on the chalkboard" / "a *triangle* traced in the sand"

    ▼ **Show answer**

    1: Analogical; a key opens a lock, and the answer "opens" the riddle, so the second meaning is named by likeness to the first. 2: Analogical; a cold stare is unfriendly and chilling, named by likeness to something low in temperature. 3: Univocal; both are three-sided plane figures, and the meaning of "triangle" is the same though the materials differ.

12. Challenge Write a short argument of your own in which an *analogical* word changes meaning, so that the argument fails. Then explain the two meanings, why they are related, and how to rewrite the argument so that the word keeps one meaning.

    ▼ **Show answer**

    Sample: "A strong rope will hold a heavy load. Caleb made a strong argument. So Caleb's argument will hold a heavy load." "Strong" means able to bear weight without breaking (the rope) and hard to refute (the argument); they are related because an argument that "holds up" under objections is like a rope that holds up under weight. Rewrite with one meaning: "A strong rope will hold a heavy load. This new rope is strong. So this rope will hold a heavy load." Grading: accept any example in which the word's two meanings are related by likeness or by reference to a main meaning, the student names both meanings, and the rewrite keeps one meaning throughout.

COMMON MISTAKES

- Calling every pair of different meanings equivocal. If one meaning can be explained by pointing to the other, the use is analogical.
- Thinking a word is univocal only when the things are identical. A beagle and a collie differ, but "dog" means the same thing of both.
- Thinking a word with several meanings is a defect. Most words have several meanings; the trouble comes only when the meaning shifts inside one argument.

- Missing a shift because the two meanings feel close. Analogical words (“sharp,” “strong,” “bright”) slip more easily than equivocal ones, because the change is less obvious.

> **Scripture.** “But as God is true, our word toward you was not yea and nay.” (2 Corinthians 1:18, KJV)
>
> Paul’s word did not mean one thing and then another. Because God is true, a Christian’s words should keep their meaning, not slide between senses to win a point.

UNIT 2 · TERMS AND DEFINITIONS · LESSON 2.2 OF 69 · 40 MINUTES

2.2 Extension and Intension

OBJECTIVES

- Define the extension and the intension (comprehension) of a term, and give both for a simple term.
- Tell what a term applies to from what it means, and recognize a term that means something but applies to nothing.
- Explain why, along a chain of genus and species, extension shrinks as intension grows, and state the limits of that rule.
- Arrange terms in order of increasing intension or increasing extension.

KEY TERMS

Extension

The whole range of things a term rightly applies to. The extension of “Gospel” (as a book of the New Testament) is Matthew, Mark, Luke and John.

Intension

The set of characteristics a thing must have for the term to apply to it; the meaning of the term. The intension of “square” is: a plane figure with four equal straight sides and four right angles.

Comprehension

The Port-Royal *Logic*'s name for intension: the characteristics an idea contains, which cannot be removed without changing the idea.

Denotation and connotation

John Stuart Mill's names for extension and intension: a term *denotes* the things it applies to and *connotes* the characteristics it implies. (In logic, "connotation" does not mean an emotional overtone.)

Genus and species

A genus is a wider class that contains several narrower classes; each narrower class is a species of it. "Tree" is a genus of which "oak" is a species.

Empty term

A term that has a meaning but applies to nothing that actually exists, such as "unicorn" or "a horse with wings."

Inverse relation

The rule that, along a chain of genus and species, as intension increases, extension decreases (or at least does not increase).

CORE IDEAS

Two questions about any term. Take the term "oak." You can ask two quite different questions about it. First: *What does it apply to?* The white oak in the Ortizes' pasture, the oaks along Elk Creek, every oak that has ever grown. Second: *What does it mean?* A tree, of a certain kind, that bears acorns. The answer to the first question is the term's **extension**; the answer to the second is its **intension**. The Port-Royal *Logic* of 1662 drew this distinction sharply, calling the two sides of an idea its extension and its **comprehension**. Most later logicians say "intension."

Extension: how far a term reaches. The extension of a term is the whole range of things it rightly applies to. Sometimes the extension is short enough to list. The extension of "Gospel" (as a book of the New Testament) is Matthew, Mark, Luke and John. The extension of "even number between 1 and 9" is 2, 4, 6 and 8. Usually it is far too large to list: no one could name every dog or every triangle. But the extension is still definite. Every dog is inside it and everything else is outside. An extension can

also be described by the smaller kinds it contains: the extension of “tree” takes in oaks, maples, pines and every other kind of tree.

Intension: what a term means. The intension of a term is the set of characteristics a thing must have for the term to apply to it. The intension of “triangle” is: a plane figure, bounded by straight lines, with three sides. Anything with all of these characteristics is a triangle; anything lacking even one is not. The Port-Royal logicians described comprehension as the characteristics that an idea contains and that cannot be taken away without destroying the idea. So not every true fact about the members belongs to the intension. Most dogs have names, and many triangles are drawn in chalk, but having a name is not part of what “dog” means, and being drawn in chalk is not part of what “triangle” means.

Meaning decides membership. We usually find out what a term applies to by knowing what it means. A “calf” is a young bovine, an animal of the cattle kind. Knowing that, you can sort the animals on the Ortiz farm without a list: the young bull born in March is a calf; the grown cow is not, because she is not young; the young goat is not, because it is not of the cattle kind. The intension works like a gate, and the extension is everything that can pass through it. That is why defining a term (Lessons 2.4 and 2.5) matters so much: a vague meaning leaves the gate wobbling, and no one can say for sure what the term covers.

Meaning and reach are not the same thing. Two facts prove it. First, some terms have a meaning but no extension at all. “Unicorn” and “horse with wings” are perfectly meaningful: you know exactly what would count as one. But nothing in the world does count. Such terms are **empty**, and they are not meaningless, for “unicorn” and “winged horse” mean different things even though both apply to nothing. Second, two terms can have the same extension but different intensions. “Equilateral triangle” (equal sides) and “equiangular triangle” (equal angles) apply to exactly the same triangles, yet one term speaks of sides and the other of angles. Same reach, different meaning.

The inverse relation. Now follow a chain of terms, each one a narrower kind, or **species**, of the one before: living thing, animal, mammal, dog, beagle. Each step down adds a characteristic to the intension (an animal is a living thing that senses and moves; a mammal is an animal that nurses its young; and so on). And each step leaves some things out of the extension (plants drop out at “animal,” birds at

"mammal," cats at "dog," collies at "beagle"). So as intension grows, extension shrinks. Climb back up the chain and the reverse happens: take away characteristics, and the term reaches more things. Logicians after Port-Royal stated this inverse relation as a rule, and it has been taught ever since.

The rule, stated carefully. The inverse relation is not a law of arithmetic that works for any two terms you like. It holds along a chain of genus and species, where each term falls wholly inside the one before. You cannot compare "dog" and "tree" this way, since neither is a species of the other. Also, adding a characteristic that every member already has does not shrink the extension: "rectangle" and "four-sided rectangle" apply to exactly the same figures. So careful logicians say: as intension increases, extension decreases *or at least does not increase*.

Two words to watch. In 1843 John Stuart Mill called a term's extension its **denotation** and its intension its **connotation**. Be careful: in everyday speech, "connotation" means the feeling a word stirs up ("stingy" and "thrifty" have different connotations). In logic it means the characteristics the term implies. Notice, too, that extension has nothing to do with the size of the things. "Elephant" does not have a larger extension than "ant"; in fact, there are far more ants than elephants, so "ant" has the larger extension.

> **From history.** The Port-Royal *Logic* was written in 1662 by Antoine Arnauld and Pierre Nicole, two teachers connected with the abbey of Port-Royal near Paris. Its title in English is *Logic, or the Art of Thinking*, and it was arranged by the acts of the mind. In its first part, on ideas, it distinguished an idea's comprehension (the characteristics it contains) from its extension (the subjects it applies to). It used the pair to explain how a general idea, such as "triangle," can stand for many particular things at once. The book was read in schools for two centuries. In 1843 John Stuart Mill, in his *System of Logic*, gave the same pair the names "connotation" and "denotation."

> **Practice habit.** When you meet a key term in an argument, ask two questions: What does it mean? And what, exactly, does it apply to?

WORKED EXAMPLES

Example 1. Give the extension and the intension of (a) “even number between 1 and 9” and (b) “prime number less than 12.”

▼ **Show solution**

1. Extension: list every thing the term applies to. Intension: list the characteristics a thing must have to fit.
2. (a) Extension: 2, 4, 6, 8. Intension: a whole number that is divisible by 2, greater than 1 and less than 9.
3. (b) A prime number is a whole number greater than 1 whose only whole-number divisors are 1 and itself. Check each number from 2 to 11: 2, 3, 5, 7 and 11 are prime; 4, 6, 8, 9 and 10 are not.
4. (b) Extension: 2, 3, 5, 7, 11. Intension: a whole number greater than 1, whose only divisors are 1 and itself, and which is less than 12.

Answer: (a) Extension 2, 4, 6, 8; intension: a whole number divisible by 2, greater than 1 and less than 9. (b) Extension 2, 3, 5, 7, 11; intension: a whole number greater than 1, divisible only by 1 and itself, and less than 12.

Example 2. Arrange these terms in order of increasing intension, and say what happens to the extension at each step: beagle, animal, living thing, dog, mammal.

▼ **Show solution**

1. Find the widest term: the one every other term falls under. Everything named here is a living thing.
2. Then find the next widest: every animal is a living thing, every mammal is an animal, every dog is a mammal, every beagle is a dog.
3. Order: living thing, animal, mammal, dog, beagle. Each term is a species of the one before, so the inverse relation applies.
4. At each step a characteristic is added (senses and moves; nurses its young; of the dog kind; of the beagle breed), and things drop out of the extension (plants, then birds and fish, then cats and horses, then collies and every other breed).

Answer: Living thing, animal, mammal, dog, beagle. Intension increases at each step, and extension decreases: each term applies to fewer things than the one before.

PRACTICE

1. Easy Define extension and intension in your own words.

   ▼ **Show answer**

   Extension: the whole range of things a term applies to. Intension: the characteristics a thing must have for the term to apply; the term's meaning. (Accept "comprehension" for intension, or "denotation" and "connotation" used in Mill's sense.)

2. Easy List the extension of "Gospel" (as a book of the New Testament).

   ▼ **Show answer**

   Matthew, Mark, Luke and John.

3. **Easy** Give the intension of "square."

▼ Show answer

A plane figure with four straight sides, all equal, and four right angles. Accept "a rectangle with four equal sides" or "an equilateral rectangle," which give the same intension in fewer words.

4. **Easy** Which term has the greater extension, "bird" or "robin"? Which has the greater intension?

▼ Show answer

"Bird" has the greater extension, since every robin is a bird but many birds are not robins. "Robin" has the greater intension, since it includes every characteristic of a bird plus those that mark a robin.

5. **Medium** Arrange in order of increasing extension (narrowest first): polygon, square, plane figure, rectangle, quadrilateral.

▼ Show answer

Square, rectangle, quadrilateral, polygon, plane figure. Every square is a rectangle, every rectangle is a quadrilateral, every quadrilateral is a polygon, and every polygon is a plane figure, so each term reaches more figures than the one before.

6. **Medium** Give the extension and the intension of "prime number less than 20."

▼ Show answer

Extension: 2, 3, 5, 7, 11, 13, 17, 19. Intension: a whole number greater than 1, whose only whole-number divisors are 1 and itself, and which is less than 20.

7. **Medium** A calf is a young animal of the cattle kind. Which of these characteristics belong to the intension of "calf"? (a) young (b) of the cattle kind (c) brown (d) born on the Ortiz farm (e) named Daisy

▼ Show answer

Only (a) and (b). Being brown, being born on the Ortiz farm or being named Daisy may be true of a particular calf, but none is part of what "calf" means: a black calf born elsewhere is still a calf.

8. Medium Give two empty terms (terms that apply to nothing that exists) that have different intensions. What does your example show?

▼ **Show answer**

Sample: “unicorn” and “dragon.” Both apply to nothing, so their extensions are the same (empty), but they mean different things: one is a horse-like creature with a single horn, the other a winged, fire-breathing reptile. This shows that a term’s meaning is not the same as what it applies to, and that an empty term is not meaningless. Accept any two meaningful terms that apply to nothing.

9. Hard Do “equilateral triangle” and “equiangular triangle” have the same extension? The same intension? Explain.

▼ **Show answer**

Same extension: every triangle with three equal sides also has three equal angles, and every triangle with three equal angles also has three equal sides, so the two terms apply to exactly the same triangles. Different intension: one term means a triangle with equal sides, the other a triangle with equal angles. Two terms can reach the same things and still mean different things.

10. Hard Hannah says, “The more you say about a thing, the fewer things you are talking about.” Is she right? Use a chain of terms to explain, and give one limit of her rule.

▼ **Show answer**

She is right along a chain of genus and species, so accept “yes, within limits” or “partly right”: plant, tree, oak, white oak (accept any correct chain of genus and species). Each term adds a characteristic and applies to fewer things. Limits (accept either): (1) adding a characteristic that every member already has does not shrink the extension (“triangle” and “three-sided triangle” apply to the same figures), so the extension decreases or at least does not increase; (2) the rule compares only terms in one chain, where each falls inside the one before; it cannot compare unrelated terms such as “oak” and “dog.”

11. Hard Caleb says, “In logic, ‘thrifty’ and ‘stingy’ have different connotations, because one sounds kind and the other sounds rude.” Is that what Mill meant by connotation? Explain.

▼ **Show answer**

No. Caleb is using the everyday meaning of connotation, the feeling a word stirs up. For Mill, the connotation of a term is the set of characteristics it implies, its intension. (The two words may also differ in their logical connotation, since “stingy” implies being unwilling to spend even when one should; but that is a difference in meaning, not in feeling.)

12. Challenge Build a chain of five terms of your own, from widest to narrowest, each a species of the one before. For each step, name the characteristic that is added and one kind of thing that drops out of the extension.

▼ **Show answer**

Sample: plane figure → polygon (adds: bounded by straight lines; circles drop out) → quadrilateral (adds: four sides; triangles drop out) → parallelogram (adds: two pairs of parallel sides; quadrilaterals with only one pair of parallel sides drop out) → rectangle (adds: four right angles; parallelograms with slanted corners drop out). Accept any chain in which each term falls wholly inside the one before, each step names a real added characteristic, and the thing that drops out really lacks it.

COMMON MISTAKES

- Confusing extension with the size of the things. “Elephant” does not have a larger extension than “ant”; extension is about how many things a term applies to.
- Putting accidental facts into the intension. Most dogs have names, but having a name is not part of what “dog” means.
- Thinking an empty term is meaningless. “Unicorn” has a clear meaning; it simply applies to nothing that exists.
- Using “connotation” in its everyday sense of feeling. In logic, a term’s connotation is the set of characteristics it implies.
- Applying the inverse relation to unrelated terms. It holds only along a chain in which each term falls wholly inside the one before.

> **Scripture.** “He telleth the number of the stars; he calleth them all by their names.” (Psalms 147:4, KJV)
>
> We know the intension of “star,” but no one could list its extension. God knows every member of it, each by name.

UNIT 2 · TERMS AND DEFINITIONS · LESSON 2.3 OF 69 · 45 MINUTES

2.3 The Categories and the Predicables

OBJECTIVES

- Name Aristotle’s ten categories and the question each one answers.
- Tell substance from accident, and sort predicates into the ten categories.
- Name Porphyry’s five predicables (genus, species, difference, property, accident) and define each.
- Say how a predicate is related to its subject, and tell a property from an accident.

KEY TERMS

Category

One of Aristotle’s ten highest kinds of being, under which every simple term falls: substance, quantity, quality, relation, place, time, position, state, action and passion.

Substance

A thing that exists in itself and not in another, such as a horse, an oak tree or Caleb; the first of the categories.

Accident

(1) Among the categories: anything that exists only in a substance, such as a color, a size or a place (the nine categories other than substance). (2) Among the predicables: a characteristic that may belong to a subject or not without changing what the subject is.

Predicable

One of Porphyry’s five ways in which a predicate can be related to its subject: genus, species, difference, property or accident.

Genus

The part of a thing's essence that it shares with other kinds: "animal" is the genus of man, and "polygon" is the genus of triangle.

Species

The whole essence of a thing, said of the individuals that have it: "Socrates is a man"; "this figure is a triangle."

Difference

The part of a thing's essence that marks it off from other species in the same genus: "rational" for man, "three-sided" for triangle.

Property

A characteristic that is not part of the essence but follows from it, belonging to every member of the species, only to them, and always: man's ability to laugh; a triangle's angles adding up to 180 degrees.

CORE IDEAS

Saying something about something. Every proposition says something about something: a predicate about a subject. "Daisy is brown." "Daisy is a cow." "Daisy is in the barn." Aristotle noticed that predicates like these are not all the same kind of thing. Some tell what the subject is, some tell what it is like, some tell where it is. Logic sorts predicates in two ways. The **categories** ask, "What kind of being does this predicate name?" The **predicables** ask, "How is this predicate related to this subject?" Both help us think clearly about terms and, as you will see in Lesson 2.4, to define them.

The ten categories. In his short book the *Categories*, Aristotle listed ten highest kinds under which every simple term falls. Each answers a different question:

Category	Question	Examples
Substance	What is it?	a horse, an oak, Caleb
Quantity	How much? How big?	two pounds, six feet tall
Quality	What sort?	red, sour, honest, skillful
Relation	Related to what?	older than, father of, half
Place	Where?	in the barn, at the market
Time	When?	yesterday, at dawn
Position (posture)	In what posture?	sitting, lying down

Category	Question	Examples
State (having)	Having what on or about it?	wearing boots, saddled
Action	Doing what?	plowing, cutting
Passion (being acted on)	Undergoing what?	being plowed, being cut

Substance and the nine accidents. The first category is different from the other nine. A **substance** exists in itself: a horse, an oak tree, a man. The other nine exist only *in* a substance, and so they are called **accidents**. You never meet brownness walking around by itself, or "in the barn" without something that is in the barn. The color, the weight, the place and the posture all belong to some substance. Aristotle also distinguished the individual substance (this horse, Duke) from the kinds it belongs to (horse, animal), which he called substance in a secondary sense.

Using the categories. Here is a cow described in all ten: Daisy (substance) weighs 1,200 pounds (quantity), is brown (quality), is older than her calf (relation), is in the barn (place) this morning (time), is standing (position), wears a bell (state), is chewing her cud (action) and is being milked (passion). The categories train you to notice what kind of thing is being said. They also expose confused questions. "How much does honesty weigh?" asks a question of quantity about a quality, and "What color is Tuesday?" asks a question of quality about a time. Such questions have no answer, not because the answer is hidden, but because the question mixes categories.

The five predicables. About six hundred years after Aristotle, the Greek philosopher Porphyry wrote a short *Introduction* (in Greek, *Isagoge*) to the *Categories*. In it he explained five ways a predicate can be related to its subject: **genus**, **species**, **difference**, **property** and **accident**. These are the **predicables**. The first three concern the *essence* of a thing, what it is; the last two concern what belongs to a thing beyond its essence.

Genus, species and difference. Take the classical example, man. The **species** is the whole of what a thing is, said of the individuals that have it: "Socrates is a man." The **genus** is the part of the essence shared with other kinds: "Man is an animal," and so are dogs and horses. The **difference** is the part of the essence that marks the species off from the others in its genus: "Man is rational." Genus plus difference gives the whole species: a man is a rational animal. The same pattern works in geometry.

“This figure is a triangle” (species); “A triangle is a polygon” (genus); “A triangle has three sides” (difference).

Property and accident. A **property** is not part of the essence, but it follows from the essence, so it belongs to every member of the species, only to them, and always. Porphyry’s example was man’s ability to laugh: only a rational creature can see that something is funny. In geometry, the angles of a triangle always add up to 180 degrees. That is not what a triangle *is* (it is not in the definition), but it follows from what a triangle is, and it is true of every triangle. An **accident** may be present or absent without the subject ceasing to be what it is: Socrates may be sitting or standing, a triangle may be drawn in chalk or in ink, Daisy may be brown or black, and each remains what it is.

How to tell which predicable. Ask these questions in order. Is the predicate the whole of what the subject is? Then it is the species. Is it part of what the subject is, shared with other kinds? Genus. Part of what it is, marking it off from those other kinds? Difference. Not part of what it is, but following from it in every member and only in them? Property. Able to come and go, or to be absent, without the subject changing its kind? Accident. Notice that the property must be the *ability* or the necessary consequence: man is always *able* to laugh, but actually laughing right now is an accident.

Two meanings of “accident.” Be careful, because “accident” is used in both lists. Among the categories, an accident is anything that exists in a substance (all nine categories after substance). Among the predicables, an accident is a characteristic that need not belong to the subject. So a property, such as the ability to laugh, is an accident in the first sense (it is a quality, not a substance) but not in the second sense (it can never be missing from a man). The context tells you which list is meant.

From history. Aristotle's *Categories* stands first among his logical works. Porphyry (about 234–305), a Greek philosopher born in Tyre, wrote the *Isagoge* as a beginner's introduction to it. Aristotle himself, in the *Topics*, had sorted predicates into four kinds (definition, property, genus and accident); Porphyry's list of five, with species and difference, became the standard one. Porphyry also wrote against the Christians, yet Christian teachers kept his little logic book, because its reasoning was sound whoever wrote it. In the early 500s Boethius translated the *Isagoge* into Latin and wrote commentaries on it, and for about a thousand years it was the first logic book that students in Europe read.

Practice habit. When you read a description today, sort a few of its predicates: what category is each, and is it essential to the thing or only an accident?

WORKED EXAMPLES

Example 1. Name the category of each predicate about Duke, the Ortizes' horse. (a) is a horse (b) is gray (c) is sixteen hands high (d) is in the north pasture (e) is lying down (f) is wearing a saddle (g) is pulling a wagon (h) is being washed (i) is stronger than the pony (j) was born last spring

▼ **Show solution**

1. For each predicate, ask which of the ten questions it answers.
2. (a) What is it? Substance. (b) What sort? Quality. (c) How big? Quantity.
3. (d) Where? Place. (e) In what posture? Position. (f) Having what on? State.
4. (g) Doing what? Action. (h) Undergoing what? Passion: Hannah does the washing, and Duke is acted on. (i) Related to what? Relation: "stronger than" compares Duke to the pony. (j) When? Time.

Answer: (a) substance (b) quality (c) quantity (d) place (e) position (f) state (g) action (h) passion (i) relation (j) time

Example 2. Name the predicable in each statement, given that a triangle is defined as a polygon with three sides. (a) "This figure is a triangle." (b) "A triangle is a polygon." (c) "A triangle has three sides." (d) "The angles of a triangle add up to 180 degrees." (e) "This triangle is drawn in blue chalk."

▼ **Show solution**

1. Start from the definition: triangle = polygon (genus) + three-sided (difference).
2. (a) The predicate "triangle" gives the whole of what this figure is: species.
3. (b) "Polygon" is part of the essence, shared with squares and hexagons: genus. (c) "Three sides" is the part of the essence that marks triangles off from other polygons: difference.
4. (d) The angle sum is not in the definition, but it follows from it and is true of every triangle and only of triangles among polygons: property. (e) A triangle could be drawn in any color, or not drawn at all, and remain a triangle: accident.

Answer: (a) species (b) genus (c) difference (d) property (e) accident

PRACTICE

1. Easy Name Aristotle's ten categories.

   ▼ **Show answer**

   Substance, quantity, quality, relation, place, time, position (posture), state (having), action and passion (being acted on). Accept the names in any order, and accept "posture" for position and "having" or "habit" for state.

2. Easy Name Porphyry's five predicables.

   ▼ **Show answer**

   Genus, species, difference, property and accident.

3. Easy Which category answers each question? (a) Where? (b) When? (c) How much? (d) What sort? (e) Doing what?

   ▼ **Show answer**

   (a) place (b) time (c) quantity (d) quality (e) action

4. **Easy** Substance or accident? Give the category of each accident. (a) an oak tree (b) green (c) in the meadow (d) Mrs. Ortiz (e) last Tuesday

▼ Show answer

(a) substance (b) accident: quality (c) accident: place (d) substance (e) accident: time

5. **Medium** Name the category of each predicate about Biscuit the beagle. (a) weighs twenty-two pounds (b) is brown and white (c) is lying down (d) on the porch (e) this afternoon (f) is wearing a red collar (g) is chewing a bone (h) is being brushed by Hannah (i) is smaller than the collie

▼ Show answer

(a) quantity (b) quality (c) position (d) place (e) time (f) state (g) action (h) passion (i) relation, because "smaller than" compares Biscuit to another dog. (If a student answers quantity for (i) and explains that it concerns size, give partial credit and point out that the comparison is what makes it a relation.)

6. **Medium** Name the predicable in each statement, with a reason: (a) "Man is an animal." (b) "Man is rational." (c) "Socrates is a man." (d) "Socrates is snub-nosed." (e) "Man is able to learn grammar."

▼ Show answer

(a) genus: animal is the part of man's essence shared with other animals. (b) difference: rational is what marks man off from other animals. (c) species: man is the whole of what Socrates is. (d) accident: Socrates would still be a man with a different nose. (e) property: the ability to learn grammar follows from being rational, and belongs to every man, only to men, and always.

7. **Medium** Why is "able to laugh" called a property of man rather than his difference?

▼ Show answer

The difference is part of what man is (rational). The ability to laugh is not part of the essence; it follows from it, since only a rational being can see that something is funny. So it belongs to every man, only to men, and always, but as a result of his essence rather than as part of it.

8. Medium A square is defined as a rectangle with four equal sides. Name the predicable in each statement: (a) “A square is a rectangle.” (b) “A square has four equal sides.” (c) “This figure is a square.” (d) “This square is painted on the gym floor.”

▼ **Show answer**

(a) genus (b) difference (c) species (d) accident: a square could be painted anywhere, or drawn on paper, and still be a square.

9. Hard Caleb says, “Being able to laugh can’t be a property of man, because most of the time people aren’t laughing.” Answer him.

▼ **Show answer**

The property is the *ability* to laugh, not the act. Every human being always has the ability, even when not laughing at the moment (as Caleb has the ability to read while he is asleep). Actually laughing right now is an accident; being able to laugh is a property.

10. Hard Explain how the ability to laugh can be an accident in the list of categories but not an accident in the list of predicables.

▼ **Show answer**

In the categories, an accident is anything that exists in a substance rather than in itself; the ability to laugh is a quality of a man, so it is an accident in that sense. In the predicables, an accident is something that may or may not belong to the subject; the ability to laugh always belongs to every man, so it is a property, not a predicable accident. The word “accident” has a different meaning in each list.

11. Hard Use the categories to explain what is wrong with each question: (a) “How much does honesty weigh?” (b) “What color is next Tuesday?”

▼ **Show answer**

(a) Honesty is a quality, but weight is a quantity that belongs only to bodies, so the question asks for a quantity of something that has none. (b) Next Tuesday is a time, but color is a quality of bodies, so the question asks for a quality that a time cannot have. Both questions mix categories, so they have no answer.

12. **Challenge** Choose a substance (for example, a horse, a tree or a person) and write ten short predicates about it, one in each category. Then choose one predicate that is a property of its species and one that is an accident, and explain the difference.

▼ **Show answer**

Sample about Mr. Lee: is a man (substance); is six feet tall (quantity); is patient (quality); is the uncle of Caleb (relation); is in the workshop (place); at noon (time); is sitting (position); is wearing an apron (state); is sanding a chair (action); is being called to lunch (passion). Property: he is able to learn a language, which follows from being rational and belongs to every man always. Accident: he is wearing an apron, which he could take off and still be a man. Accept any set that places one predicate correctly in each category and gives a true property (belongs to all, only, always, following from the essence) and a true accident.

COMMON MISTAKES

- Confusing the two lists. The categories sort kinds of being (substance, quality, place …); the predicables sort how a predicate relates to a subject (genus, species …).
- Calling a property part of the definition. A property follows from the essence but is not part of it; the definition gives only genus and difference.
- Mixing up the act and the ability. Man is always able to laugh (property); laughing right now is an accident.
- Treating position and place as the same. “Lying down” is position (posture); “on the porch” is place.
- Calling a comparison a quantity. “Taller than the pony” is a relation, because it relates one thing to another.

> **Scripture.** “O LORD, how manifold are thy works! in wisdom hast thou made them all: the earth is full of thy riches.” (Psalms 104:24, KJV)
>
> The categories and predicables are ways of noticing the order in a world that God made “manifold” and yet made “in wisdom.”

2.4 Definition by Genus and Difference

OBJECTIVES

- Explain what a definition does, and why the classical definition gives a genus and a difference.
- Write a classical definition by finding the nearest genus and the difference that sets the term apart from its neighbors.
- Test a definition by turning it around to see whether it is convertible with the term.
- Place a term on the Tree of Porphyry, and read definitions from that tree or from a tree of your own.

KEY TERMS

Definition

A statement of what a term means or what a thing is, which sets the boundary between what the term covers and what it does not.

Classical definition

A definition by genus and difference: it names the wider class the thing belongs to and the feature that sets it apart within that class, as in "A puppy is a young dog."

Nearest genus

The narrowest class that still contains the term being defined; also called the proximate genus. The nearest genus of "square" is "rectangle."

Specific difference

The feature that sets a species apart from the other species in its genus, such as "with four equal sides" for a square among rectangles.

Convertible

Able to be turned around: a definition is convertible with its term when "Every S is D" and "Every D is S" are both true.

Tree of Porphyry

A diagram that divides substance, step by step, by a difference and its opposite, down to man; it shows how each species is its genus plus a difference.

Highest genus and lowest species

The highest genus (in Latin, *summum genus*) has no genus above it, like substance; the lowest species (*infima species*) has no species below it, only individuals, like man.

CORE IDEAS

What a definition does. A definition says what a term means, and in the best case what a thing is. It works like the gate described in Lesson 2.2: it sets the boundary between what the term covers and what it does not. There are many ways to explain a word. You can point ("That is a heron"), give examples ("Fruits are things like apples and pears"), give a synonym ("Glad means happy") or tell where the word came from. These all help, but none of them says what the thing *is*. The classical definition does.

Genus plus difference. A **classical definition** has two parts. The **genus** names the wider class the thing belongs to; the **difference** names the feature that sets it apart within that class. "A puppy is a young dog": genus "dog," difference "young." "A triangle is a polygon with three sides." "Man is a rational animal." Genus plus difference equals the species, the thing defined. Notice that in logic, "genus" and "species" are relative words: any wider class is a genus to the narrower classes inside it. They do not mean what they mean in a biology book. For a logician, "dog" is a genus with "puppy" as one of its species.

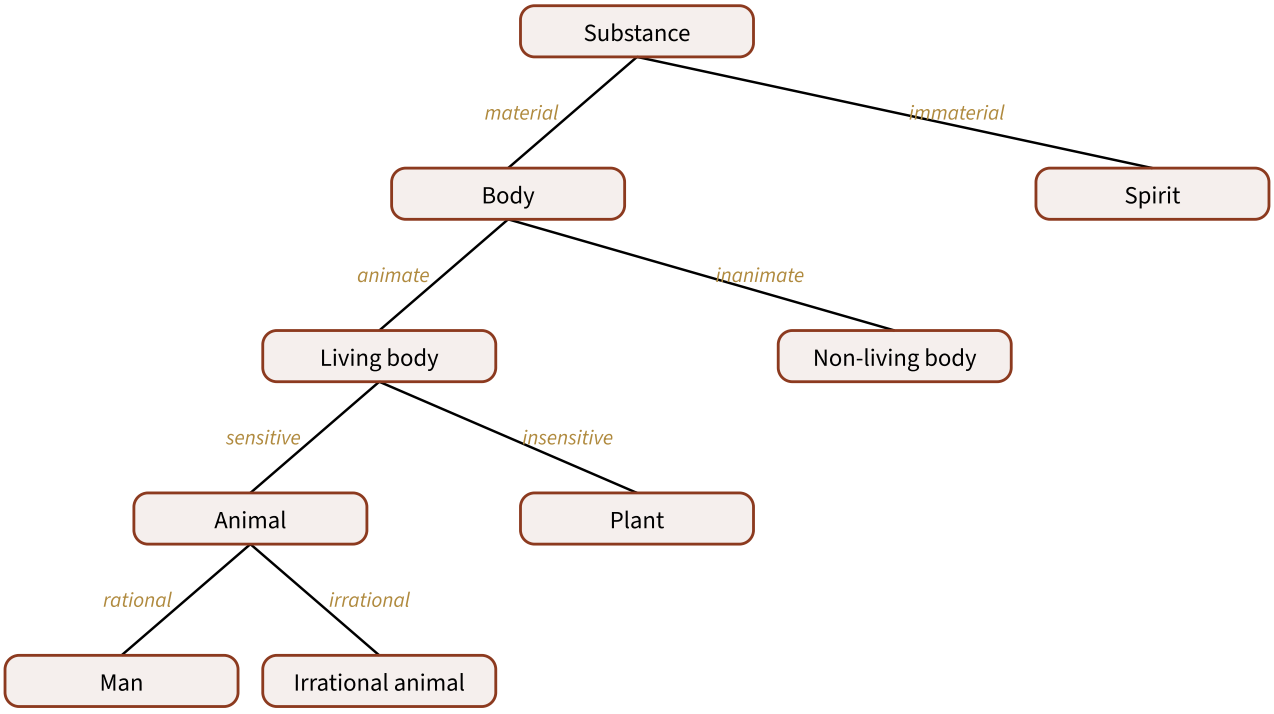
Step one: find the nearest genus. Choose the narrowest class that still contains the thing. A square is a plane figure, a polygon, a quadrilateral and a rectangle. All are true, but "rectangle" is the **nearest genus**, and it does the most work, because it already carries every characteristic of the wider classes. If you start from "plane figure," you must add a long string of differences (straight sides, four of them, right angles, equal sides) to get back to a square. That definition is not wrong, but it is clumsy, and it hides where the square sits among its neighbors.

Step two: find the specific difference. Now ask: what sets this thing apart from the other species in the same genus? List the neighbors. Among rectangles, some have four equal sides and some do not; so a square is "a rectangle with four equal sides."

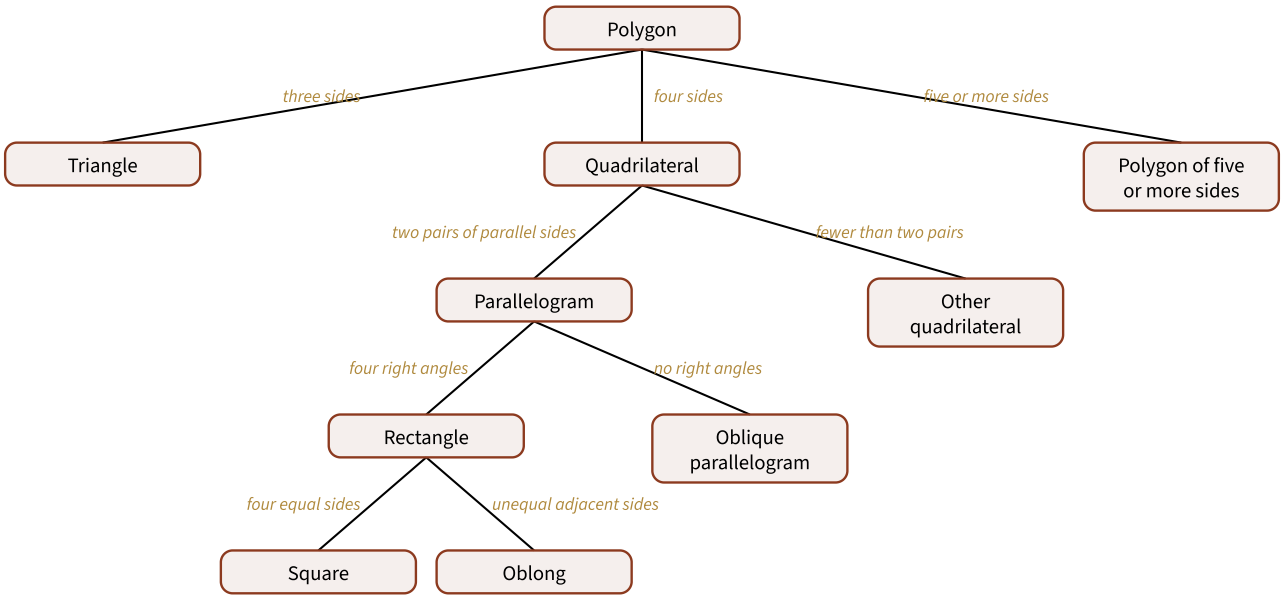
Among dogs, some are young and some are grown; so a puppy is “a young dog.” For things people make, the difference is often their purpose: a hammer is “a hand tool with a heavy head set across a handle, used for pounding.” The difference must belong to every member of the species and to no member of the other species in that genus.

Step three: turn it around. A good definition is **convertible** with the term: you can swap subject and predicate and both statements stay true. “Every square is a rectangle with four equal sides” and “Every rectangle with four equal sides is a square”: both true. Now try “A square is a figure with four sides.” The first direction is true, but “Every figure with four sides is a square” is false, since a kite has four sides. The test shows that the definition lets in too much. Lesson 2.5 will study such faults in detail.

The Tree of Porphyry. Porphyry’s *Isagoge* traced a ladder from the highest genus, substance, down to man, and medieval teachers drew it as a branching tree. At each step the genus is divided by a difference and its opposite. Substance is either material (body) or immaterial (spirit). Body is either animate (living) or inanimate. Living body is either sensitive (animal) or insensitive (plant). Animal is either rational (man) or irrational. Below man come only individuals, such as Socrates and Plato. Substance is the **highest genus**, man the **lowest species**, and the genera in between are both species of the one above and genera of the one below.



Reading definitions from a tree. To define any term on the tree, take the genus just above it and the difference written on the branch that leads down to it. Body is a material substance. A living body is an animate body. An animal is a sensitive living body. Man is a rational animal. Notice the link to Lesson 2.2: as you go down the tree, the intension grows (man is rational, sensitive, animate, material substance) and the extension shrinks. The same method works for any field. Here is a tree of polygons, built the same way:



What cannot be defined this way. The method has limits. The highest genera, such as substance, quantity and quality, have no genus above them, so they can only be explained by examples and by contrast. Individuals, such as Caleb or the white oak in the pasture, sit below the lowest species; they can be described and pointed to, but not defined. And for many natural things we know the genus but not the true essential difference, so in practice we name a property or a clear outward mark instead. A classical definition is a goal to aim at, and a well-chosen property is often the best we can do on the way.

> **From history.** Aristotle taught, in the *Topics* and the *Posterior Analytics*, that a definition should state what a thing is, by placing it in its genus and adding its differences. Porphyry's *Isagoge* (late 200s) illustrated genus and species with the ladder that runs from substance through body, animate body and animal down to man, and then to individuals such as Socrates and Plato. Porphyry described this ladder in words. Medieval teachers, who learned logic from Boethius's Latin translation of the *Isagoge*, drew it as a branching diagram, and it became known as the Tree of Porphyry (in Latin, *arbor porphyriana*). It appeared in logic books for centuries as the standard picture of how genus, difference and species fit together.

> **Practice habit.** Before you argue about an important word, try to name its genus and its difference, then turn your definition around to test it.

Example 1. Write a classical definition of (a) "puppy" and (b) "hammer," and test each by turning it around.

▼ **Show solution**

1. (a) Nearest genus: a puppy is a dog. Neighbors in that genus: grown dogs. What sets a puppy apart? It is young. Definition: "A puppy is a young dog."
2. Turn it around: "Every puppy is a young dog" (true) and "Every young dog is a puppy" (true). The definition is convertible.
3. (b) Nearest genus: a hammer is a hand tool. Neighbors: saws, screwdrivers, pliers, chisels. What sets a hammer apart? Its heavy head set across a handle, and its purpose, pounding (driving nails, flattening metal, knocking parts into place). Definition: "A hammer is a hand tool with a heavy head set across a handle, used for pounding."
4. Turn it around: every hammer fits the definition, and every hand tool with a heavy head set across a handle, used for pounding, is some kind of hammer (a mallet and a sledgehammer are kinds of hammer; an axe has a similar shape but is used for cutting, so it does not fit). The definition is convertible.

Answer: (a) A puppy is a young dog. (b) A hammer is a hand tool with a heavy head set across a handle, used for pounding. Both pass the turn-around test.

Example 2. Use the Tree of Porphyry. (a) Define “animal” and “man.” (b) Where on the tree do a rose bush, a granite boulder and a horse belong?

▼ **Show solution**

1. To define a term on the tree, take the genus just above it and the difference on the branch leading down to it.
2. (a) Above “animal” is “living body,” and the branch says “sensitive”: an animal is a sensitive living body. Above “man” is “animal,” and the branch says “rational”: man is a rational animal.
3. (b) A rose bush is a body, living, but without sensation: plant. A granite boulder is a body but not living: non-living body. A horse is a living body with sensation, but not rational: irrational animal.

Answer: (a) An animal is a sensitive living body; man is a rational animal. (b) Rose bush: plant. Granite boulder: non-living body. Horse: irrational animal.

Example 3. Use the tree of polygons to define “rectangle” and “square,” and test the definition of “square.”

▼ **Show solution**

1. Above “rectangle” is “parallelogram,” and the branch says “four right angles”: a rectangle is a parallelogram with four right angles.
2. Above “square” is “rectangle,” and the branch says “four equal sides”: a square is a rectangle with four equal sides.
3. Turn it around: every square is a rectangle with four equal sides, and every rectangle with four equal sides is a square. Convertible.

Answer: A rectangle is a parallelogram with four right angles. A square is a rectangle with four equal sides; the definition is convertible.

PRACTICE

1. **Easy** What are the two parts of a classical definition? What does each part do?

   ▼ **Show answer**

   The genus, which names the wider class the thing belongs to, and the difference, which names the feature that sets it apart from the other members of that class.

2. **Easy** Name the genus and the difference in each definition: (a) "A ewe is a female sheep." (b) "A triangle is a polygon with three sides."

   ▼ **Show answer**

   (a) Genus: sheep. Difference: female. (b) Genus: polygon. Difference: with three sides.

3. **Easy** On the Tree of Porphyry, what is the highest genus, and what is the lowest species?

   ▼ **Show answer**

   The highest genus is substance; the lowest species is man (below man come only individuals, such as Socrates).

4. **Easy** On the Tree of Porphyry, what is the genus of "animal," and what is its difference?

   ▼ **Show answer**

   Genus: living body. Difference: sensitive (able to sense). So an animal is a sensitive living body.

5. **Medium** Write a classical definition of each: (a) kitten (b) lamb (c) circle.

   ▼ **Show answer**

   (a) A kitten is a young cat. (b) A lamb is a young sheep. (c) A circle is a plane figure bounded by one curved line, every point of which is the same distance from a point inside it (the center). Accept any definition with a correct genus and a difference that sets the term apart, such as "a closed curve whose points are all the same distance from a center" for (c).

6. **Medium** Test each definition by turning it around. Is it convertible? (a) “A puppy is a young dog.” (b) “A square is a figure with four sides.”

▼ **Show answer**

(a) Convertible: every puppy is a young dog, and every young dog is a puppy. (b) Not convertible: every square has four sides, but not every figure with four sides is a square (a kite or an oblong rectangle has four sides too). The definition lets in too much.

7. **Medium** Where on the Tree of Porphyry does each belong? (a) an oak tree (b) a granite boulder (c) a horse (d) Caleb (e) an angel

▼ **Show answer**

(a) plant (b) non-living body (c) irrational animal (d) man; Caleb is an individual under the lowest species, man (e) spirit (an immaterial substance).

8. **Medium** Hannah defines a square as “a plane figure with four straight, equal sides and four right angles.” Caleb defines it as “a rectangle with four equal sides.” Are both correct? Which uses the nearest genus, and why does that matter?

▼ **Show answer**

Both are correct and convertible. Caleb’s uses the nearest genus, “rectangle,” which already carries the characteristics of the wider classes (plane figure, straight sides, four of them, right angles), so it needs only one difference. Hannah’s uses a remote genus, “plane figure,” and must pile up several differences. The nearest genus is shorter and shows where the square sits among its neighbors.

9. **Hard** Using the Tree of Porphyry, write definitions of “body,” “living body” and “plant.”

▼ **Show answer**

A body is a material substance. A living body is an animate body. A plant is an insensitive living body (a living body without sensation).

10. Hard Using the tree of polygons, define “parallelogram” and “oblong,” and list every characteristic in the intension of “square,” from the top of the tree down.

▼ **Show answer**

A parallelogram is a quadrilateral with two pairs of parallel sides. An oblong is a rectangle whose adjacent sides are unequal. Intension of “square”: a polygon (a plane figure bounded by straight lines), with four sides, two pairs of parallel sides, four right angles and four equal sides.

11. Hard Why can neither “substance” nor “Caleb” be given a classical definition?

▼ **Show answer**

Substance is the highest genus; nothing stands above it to serve as its genus, so it can only be explained by examples and contrast. Caleb is an individual, below the lowest species; a definition by genus and difference gives the species (man), which fits every man, not Caleb alone, so Caleb can be described or pointed out but not defined.

12. Challenge Build a tree of your own with at least four levels, in which each genus is divided by a difference written on each branch. Then read two classical definitions from your tree and test one by turning it around.

▼ **Show answer**

Sample: Plane figure → Polygon (bounded by straight lines) / Figure with a curved side. Polygon → Triangle (three sides) / Polygon of four or more sides. Triangle → Right triangle (has a right angle) / Oblique triangle (no right angle). Right triangle → Isosceles right triangle (two equal legs) / Scalene right triangle (unequal legs). Definitions: “A right triangle is a triangle with a right angle.” “An isosceles right triangle is a right triangle with two equal legs.” Test: every right triangle is a triangle with a right angle, and every triangle with a right angle is a right triangle: convertible. Accept any tree whose branches at each level are divided by one clear difference and its opposite (or by one principle), with definitions read correctly from it.

COMMON MISTAKES

- Choosing a genus that is too remote (“a square is a thing that …”). Use the nearest class that still contains the term.

- Giving a difference that other members of the genus also have ("a puppy is a dog that barks"). The difference must set the species apart from its neighbors.
- Forgetting the turn-around test. A definition that is true in only one direction lets in too much.
- Confusing the logician's genus and species with the biologist's. In logic they are relative: any wider class is a genus to the narrower classes in it.
- Trying to define an individual or a highest genus. Individuals can be described, and highest genera explained by examples, but neither has a genus and difference.

REFERENCE GUIDE

An eight-page, color-coded Logic guide: what logic is, terms and definitions, the A E I O propositions, the square of opposition and immediate inferences, syllogisms with the six rules and Venn tests, the named forms, informal fallacies of relevance, presumption and ambiguity, propositional logic with truth tables, conditionals, valid forms and short proofs, the classical tradition, dialectic, argument maps and practice. The booklet prints on 11×17 paper and folds to letter size. Prints in color and reads clearly in black and white.

> **Scripture.** "and whatsoever Adam called every living creature, that was the name thereof." (Genesis 2:19, KJV)
>
> In Scripture, the first human work with words is naming the creatures. Defining carries on that work: saying clearly what each thing is.

UNIT 2 · TERMS AND DEFINITIONS · LESSON 2.5 OF 69 · 40 MINUTES

2.5 Good and Bad Definitions

OBJECTIVES

- State the five rules of a good definition.
- Test a definition with counterexamples to show that it is too broad, too narrow or both.
- Recognize definitions that state only accidents or properties, or that are circular, negative, obscure or figurative, and repair them.

- Recognize a persuasive definition, tell it from an honest stipulative definition, and explain how it slants an argument.

KEY TERMS

Too broad

Said of a definition that lets in things the term does not cover: "A chair is a seat with legs" lets in benches and stools.

Too narrow

Said of a definition that leaves out things the term does cover: "A tool is something used to drive nails" leaves out saws and wrenches.

Circular definition

A definition that uses the term being defined (or a mere synonym of it) to explain it, as in "A farmer is a person who farms."

Negative definition

A definition that says what a thing is not instead of what it is, as in "A square is a rectangle that is not an oblong."

Figurative definition

A definition given in a metaphor or other figure of speech, which tells what a thing is like rather than what it is: "Bread is the staff of life."

Persuasive definition

A definition slanted to win an argument or stir a feeling, offered as if it were the ordinary meaning of the word.

Stipulative definition

An openly announced special meaning given to a word for a particular purpose, as in "For this contest, a 'giant' pumpkin means one over 200 pounds."

CORE IDEAS

Why definitions need rules. A definition is a tool, and a bent tool spoils everything it touches. If a key word is badly defined, every argument that uses it inherits the fault. Since Aristotle, who devoted a whole book of his *Topics* to testing definitions, logicians have gathered a short set of rules. A good definition (1) is convertible with the term, neither too broad nor too narrow; (2) states the essence, by genus and

difference; (3) is not circular; (4) is not negative when it can be positive; and (5) is not obscure or figurative.

Rule 1: not too broad, not too narrow. Test a definition with **counterexamples**. If you can name something that fits the definition but is not the thing, the definition is **too broad**: "A chair is a seat with legs" lets in benches and stools. If you can name something that is the thing but does not fit, it is **too narrow**: "A tool is something used to drive nails" leaves out saws and wrenches. A definition can fail both ways at once. "A bird is an animal that flies" lets in bats and bees and leaves out penguins and ostriches. Choose counterexamples that everyone will grant at once; a doubtful counterexample proves nothing.

Rule 2: state the essence. A definition should say what a thing is, by genus and difference, not merely list accidents. "A dog is an animal that sleeps on porches" names an accident. Even a property, which fits exactly, is second best. "A triangle is a polygon whose angles add up to 180 degrees" fits all triangles and only triangles, yet it puts the result before the cause: the angles add up to 180 degrees *because* the figure has three straight sides. "A triangle is a polygon with three sides" states what the thing is, and the angle sum can be proved from it.

Rule 3: not circular. A **circular definition** uses the term itself, or a mere synonym, to explain the term. "A farmer is a person who farms." "Gratitude is the feeling of being grateful." Anyone who did not already know the word learns nothing. Circles can be wider, too: a careless dictionary might define "big" as "large" and "large" as "big." Be fair, though. "A beekeeper is a person who keeps bees" repeats parts of the word, but it is not circular, because "bee" and "keep" are already understood. A definition is circular only when the part you did not understand comes back unexplained.

Rule 4: not negative when it can be positive. "A square is a rectangle that is not an oblong" tells us what a square is not, but there are endless things it is not. Better: "A square is a rectangle with four equal sides." There is an important exception. Some things are themselves an absence or a lack, and for them a negative definition is exactly right: "Darkness is the absence of light." "Blindness is the lack of sight in a creature that would naturally see." "Silence is the absence of sound." Here the negative words state the essence.

Rule 5: not obscure or figurative. A definition should use words plainer than the word being defined. Samuel Johnson's 1755 dictionary famously defined "network"

in words such as “reticulated” and “decussated,” which are harder than “network” itself. A **figurative definition** uses a metaphor: “Bread is the staff of life,” “A friend is a sunbeam on a cloudy day.” Such sayings may be true and beautiful, and poetry and sermons rightly use them. But they tell us what a thing is *like*, not what it *is*, so they cannot settle an argument about the word.

Persuasive definitions. Some definitions are bent on purpose. A **persuasive definition** is slanted to win an argument or stir a feeling, and it is offered as if it were simply what the word means. “A real athlete trains before sunrise every day” is meant to shut out Caleb, who practices after school. “Homework is busywork teachers assign to fill time” builds a complaint into the definition. An advertisement that says “At Grainhaven, ‘natural’ means made with our golden oat clusters” defines a pleasant word so that only its own product fits. The trick is that once you accept the definition, you have already accepted the conclusion. The answer is to state the ordinary meaning fairly and point out what the slanted definition smuggled in.

Persuasive is not the same as stipulative. It is not wrong to give a word a special meaning for a purpose, if you say so openly. “For this contest, a ‘giant’ pumpkin means one that weighs over 200 pounds” is a **stipulative definition**: everyone knows it is a rule for the contest, not a claim about what “giant” means everywhere. A persuasive definition hides what it is doing. It pretends to report the usual meaning while it quietly changes it.

A checklist. To test any definition, ask in order: Can I find something that fits but is not the thing (too broad)? Something that is the thing but does not fit (too narrow)? Does it give the genus and difference, or only accidents? Does it use the term to explain the term? Does it say only what the thing is not? Does it use hard words or figures of speech? Is it slanted to win a point? Then repair the faults you find, and test the new definition the same way.

From history. Book VI of Aristotle's *Topics* is a manual for attacking and defending definitions. It warns, among other things, against definitions that are more obscure than the thing defined, that use metaphors, or that explain a term through something not better known than the term itself. Many centuries later, in 1755, Samuel Johnson published his great *Dictionary of the English Language*, nearly single-handed. It was a monument of English learning, but it had its slips. According to his friend James Boswell, when a lady asked Johnson why he had defined "pastern" (part of a horse's leg near the hoof) as the knee of a horse, he answered at once: "Ignorance, madam, pure ignorance." Even a great definer must test his definitions.

Practice habit. When someone says "a real … is …," stop and ask whether the definition is fair, or whether it was built to win the point.

Enrichment: Diogenes and the plucked chicken. Diogenes Laertius, an ancient writer of philosophers' lives, tells this story. Plato had defined man as a two-footed animal without feathers, and his students admired the definition. Diogenes the Cynic plucked a chicken, carried it into Plato's school and announced that here was Plato's man. According to the story, the words "with broad nails" were then added to the definition. Which rule of definition did Plato's definition break? (Think about counterexamples, and about whether "featherless" and "broad-nailed" say what man is.)

WORKED EXAMPLES

Example 1. Test the definition "A bird is an animal that flies," and repair it.

▼ **Show solution**

1. Too broad? Look for things that fit the definition but are not birds. Bats fly and are animals; so do bees and butterflies. Yes, too broad.
2. Too narrow? Look for birds that do not fit. Penguins and ostriches are birds but cannot fly. Yes, too narrow.
3. Repair: find a characteristic that every bird has and nothing else has. Every bird has feathers, wings and lays eggs, and no other animal has feathers.
4. New definition: "A bird is an egg-laying animal with feathers and wings." Test: every bird fits, including penguins and ostriches; nothing that is not a bird fits. Convertible.

Answer: The definition is both too broad (bats, bees) and too narrow (penguins, ostriches). A sound repair: "A bird is an egg-laying animal with feathers and wings."

Example 2. Name the fault in each definition and repair it. (a) “A farmer is a person who farms.” (b) “A square is a rectangle that is not an oblong.” (c) “Courage is the backbone of character.” (d) “A true friend is someone who always lets you copy the homework.”

▼ **Show solution**

1. (a) The term “farm” explains “farmer”: circular. Repair: “A farmer is a person who raises crops or livestock on land, usually for a living.”
2. (b) It says what a square is not, though a positive definition is easy: negative. Repair: “A square is a rectangle with four equal sides.”
3. (c) “Backbone” is a metaphor: figurative. Repair: “Courage is the virtue by which a person faces danger or hardship firmly for the sake of what is good.”
4. (d) It is slanted to excuse cheating and is offered as the real meaning of “friend”: persuasive. Repair: “A true friend is someone who loves you faithfully and seeks your good,” which might well mean refusing to let you copy.

Answer: (a) circular (b) negative (c) figurative (d) persuasive, each repaired as shown. Accept any repair that obeys all five rules.

PRACTICE

1. Easy List the five rules of a good definition.

   ▼ **Show answer**

   A good definition (1) is convertible with the term, neither too broad nor too narrow; (2) states the essence, by genus and difference; (3) is not circular; (4) is not negative when it can be positive; (5) is not obscure or figurative. Accept the rules in any order and in the student’s own words.

2. Easy Too broad or too narrow? Give a counterexample. “A chair is a piece of furniture for sitting.”

   ▼ **Show answer**

   Too broad: benches, sofas and stools are furniture for sitting but are not chairs.

3. **Easy** Too broad or too narrow? Give a counterexample. “A parent is a person’s mother.”

   ▼ **Show answer**

   Too narrow: fathers are parents too, but the definition leaves them out.

4. **Easy** Which rule does each definition break? (a) “An author is a person who authors books.” (b) “Music is the language of the soul.”

   ▼ **Show answer**

   (a) It is circular: it uses “author” to explain “author.” (b) It is figurative: “language of the soul” is a metaphor that says what music is like, not what it is. (Accept “obscure” with the same reason; also accept a note that it fails to state the genus and difference.)

5. **Medium** Name the fault in each definition and write a better one.
   1. “A triangle is a polygon whose angles add up to 180 degrees.”
   2. “Night is the time that is not daytime.”
   3. “Compost is the gardener’s black gold.”

   ▼ **Show answer**

   1: It states a property, not the essence (the angle sum follows from having three sides). Better: “A triangle is a polygon with three sides.” 2: Negative when a positive definition is available (accept also “nearly circular,” since it explains night only by its opposite). Better: “Night is the time between sunset and sunrise.” 3: Figurative. Better: “Compost is decayed plant and food matter used to enrich soil.” Accept any repair that obeys the rules.

6. **Medium** Is “Darkness is the absence of light” a bad definition because it is negative? Explain.

   ▼ **Show answer**

   No. Darkness is itself an absence, so the only way to say what it is, is to say what is missing. The rule forbids negative definitions only when a positive one is possible.

7. Medium Is each definition too broad, too narrow, both, or convertible? Give a counterexample for each fault.
   1. "A fish is an animal that lives in water."
   2. "A sibling is a brother."
   3. "A calf is a young bovine."
   4. "A tree is a tall plant."

   ▼ **Show answer**

   1: Too broad: whales, lobsters and water beetles live in water but are not fish. (It is not too narrow, since every fish lives in water.) 2: Too narrow: sisters are siblings too. 3: Convertible: every calf is a young bovine and every young bovine is a calf. 4: Both: too broad, because tall plants such as sunflowers and cornstalks are not trees; too narrow, because a young sapling or a dwarf tree is a tree but is not tall. (Give full credit for "both" with a counterexample each way; partial credit for one fault with a correct counterexample.)

8. Medium Coach Hale says, "A team player is someone who never questions the coach." Why is this a persuasive definition? Give a fairer one.

   ▼ **Show answer**

   It is slanted to win a point (that players should never question him), and it is offered as if it were the ordinary meaning of "team player." A player could ask a respectful question and still put the team first. Fairer: "A team player is a player who puts the good of the team ahead of personal glory." Accept any fair, unslanted definition.

9. Hard A carton from Brookside Dairy says, "Fresh milk means Brookside milk." What kind of definition is this? What does it smuggle in, and what would a fair definition of "fresh milk" be?

   ▼ **Show answer**

   It is a persuasive definition: it defines a pleasant word so that only Brookside's product fits, and it presents this as the meaning of the word. It smuggles in the conclusion that other dairies' milk is not fresh, without any evidence about how old the milk is. Fair: "Fresh milk is milk that was recently produced and has been kept cold, well before its use-by date." Accept any fair definition based on age and keeping, not on brand.

10. Hard Compare these two statements. Which is a stipulative definition and which is a persuasive one? Explain the difference. (a) “For the county fair contest, a ‘giant’ pumpkin means one that weighs over 200 pounds.” (b) “A pumpkin under 200 pounds isn’t a real pumpkin.”

▼ **Show answer**

(a) is stipulative: it openly gives a special meaning for a stated purpose (the contest), and no one is misled about the ordinary meaning of “giant.” (b) is persuasive: it pretends to state what a “real” pumpkin is, while it actually changes the meaning to belittle smaller pumpkins.

11. Hard For the word “window,” write (a) a definition that is too broad, (b) one that is too narrow, and (c) a good one. Give a counterexample for (a) and (b).

▼ **Show answer**

Sample: (a) “A window is an opening in a wall.” Too broad: a doorway is an opening in a wall but not a window. (b) “A window is a glass pane in the wall of a house.” Too narrow: a car or a barn has windows too. (c) “A window is an opening in the wall or side of a building or vehicle, usually fitted with glass, made to let in light and air.” Accept any set in which (a) and (b) fail as stated and (c) has no obvious counterexample.

12. **Challenge** The Hollyhock Seed Company catalog has this glossary. Evaluate each definition by the rules, and repair each faulty one.
    1. "Perennial: a plant that is not an annual."
    2. "Compost: the secret of every great garden."
    3. "Real gardener: someone who grows everything from seed."
    4. "Mulch: material spread over soil."

    ▼ **Show answer**

    1: Negative, and too broad as a result: biennials (plants that live two years) are not annuals but are not perennials either. Repair: "A perennial is a plant that lives more than two years." 2: Figurative or vague (it praises compost but does not say what it is). Repair: "Compost is decayed plant and food matter used to enrich soil." 3: Persuasive: a seed company defines "real gardener" so that buying seeds becomes part of the meaning; many gardeners rightly use transplants. Repair: "A gardener is a person who tends a garden." 4: Too broad: fertilizer and lime are spread over soil but are not mulch. Repair: "Mulch is a layer of material spread over the surface of soil to hold in moisture and keep down weeds." Accept other sound repairs, and accept "too broad" or "negative" for item 1 if the reason is right.

COMMON MISTAKES

- Using a counterexample that is not plainly true. A counterexample works only if everyone will grant it at once.
- Calling every negative definition bad. When a thing is itself a lack (darkness, silence, blindness), a negative definition is correct.
- Calling a definition circular because it shares a familiar word part ("A beekeeper is a person who keeps bees"). It is circular only if the unknown part comes back unexplained.
- Accepting a definition just because it fits exactly. It must also state the essence, not only a property or an accident.
- Confusing a stipulative definition, which openly announces a special meaning, with a persuasive one, which hides a slanted meaning.

> **Scripture.** "Woe unto them that call evil good, and good evil; that put darkness for light, and light for darkness; that put bitter for sweet, and sweet for bitter!" (Isaiah 5:20, KJV)
>
> Calling a thing by a false name in order to judge it falsely is the root of the persuasive definition, and Scripture warns sternly against it.

UNIT 2 · TERMS AND DEFINITIONS · LESSON 2.6 OF 69 · 40 MINUTES

2.6 Division and Classification

OBJECTIVES

- Tell logical division (a class into its kinds) from physical division (a whole into its parts).
- State the rules of division: one principle at each step, parts that do not overlap, parts that leave nothing out, and no leaps.
- Divide a class correctly, and find and repair the faults in a bad division, including cross-division.
- Explain dichotomy, and tell division from classification.

KEY TERMS

Logical division

Breaking a class (a genus) into the smaller classes (species) it contains, so that each part can be called by the name of the whole: trees into oaks, maples, pines and so on.

Physical division

Breaking a single whole into its parts, which cannot be called by the name of the whole: a tree into roots, trunk, branches and leaves. Also called partition.

Principle of division

The single feature by which a class is divided, such as the number of equal sides when dividing triangles.

Mutually exclusive

Not overlapping: no member of the class falls into more than one of the parts.

Jointly exhaustive

Leaving nothing out: every member of the class falls into at least one of the parts.

Cross-division

The fault of dividing by two or more principles at the same step, which usually makes the parts overlap.

Dichotomy

Division into exactly two parts, a class and its opposite (X and not-X), such as whole numbers into even and odd.

Classification

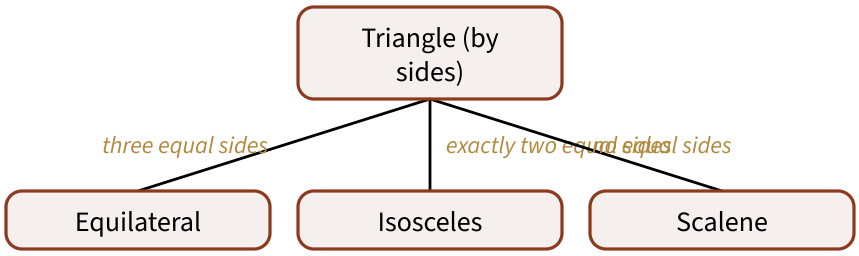
Grouping individual things into classes by shared characteristics, and those classes into wider classes; division worked from the bottom up.

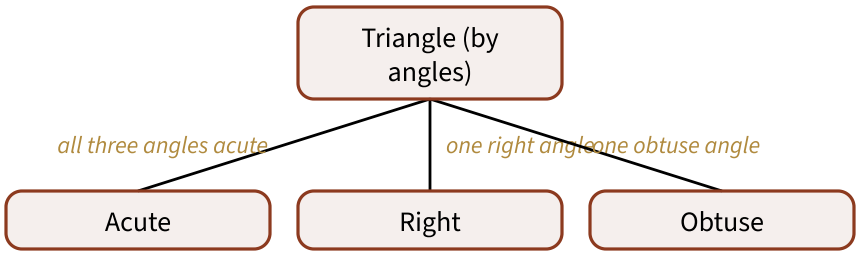
CORE IDEAS

Definition goes up; division goes down. To define a term, you climb to its genus and add a difference. To **divide** a term, you go the other way: you break the class into the smaller kinds it contains. “Triangles are either equilateral, isosceles or scalene.” Definition unfolds a term’s intension (what it means); division unfolds its extension (what it covers). The two work together. A Tree of Porphyry, read upward, gives definitions; read downward, it is a division.

Logical division and physical division. Be careful to divide the right kind of thing. **Logical division** breaks a class into its kinds: trees into oaks, maples, pines and the rest. **Physical division**, or partition, breaks one whole into its parts: a tree into roots, trunk, branches and leaves. Here is the test. In a logical division, each part can be called by the name of the whole: “An oak is a tree” is true. In a physical division it cannot: “A branch is a tree” is false. Both kinds are useful, but the rules of this lesson are rules for logical division.

Rule 1: one principle at each step. The **principle of division** is the single feature by which you divide. Triangles can be divided by their sides: three equal sides (equilateral), exactly two equal sides (isosceles) or no equal sides (scalene). They can also be divided by their angles: all angles acute, one right angle, or one obtuse angle. Both divisions are good, because each uses one principle.





But “Triangles are acute, right, isosceles or scalene” mixes the two principles. That fault is called **cross-division**, and it nearly always causes the next fault.

Rule 2: the parts must not overlap. The parts of a division must be **mutually exclusive**: no member may fall into two of them. In the cross-division above, a triangle with a right angle and two equal legs is both “right” and “isosceles,” so it is counted twice. Overlap also creeps in through careless definitions. If “isosceles” meant “at least two equal sides,” every equilateral triangle would also be isosceles, so the division by sides would overlap. That is why the division above says “exactly two.” Good division depends on good definition.

Rule 3: the parts must leave nothing out. The parts must also be **jointly exhaustive**: together they must take in every member of the class. “The days of March were rainy or snowy” leaves out the dry days. “The Ortiz farm keeps cows, horses and chickens” is an incomplete division if the farm also keeps sheep. To check, try to think of a member that fits none of the parts. If you find one, add a part or change the principle.

Rule 4: go step by step. A division should move to the nearest kinds first, then subdivide. “Living bodies are plants, dogs, cats, horses and robins” leaps over the step “animal” and, as a result, also leaves out countless animals. Better: living bodies are plants or animals; then divide animals. When you subdivide, you may use a new principle at each new level. The tree of polygons in Lesson 2.4 divided first by the number of sides, then by parallel sides, then by angles, then by equal sides. One principle per step, not one principle for the whole tree.

Dichotomy. The safest division of all is **dichotomy**, cutting a class in two: X and not-X. Whole numbers are even or odd. Animals are rational or irrational. Since every member either has the feature or lacks it, a dichotomy can never overlap and never

leave anything out. That is why the Tree of Porphyry divides by a difference and its opposite at every step. Its weakness is that the "not-X" side tells you little: "quadrilaterals that are not parallelograms" is a true part, but a vague one. Use dichotomy when you cannot list all the kinds, and list the kinds when you can.

Classification: building from the bottom up. Division starts with a class and breaks it down. **Classification** starts with individual things and gathers them into classes by shared characteristics, then gathers those classes into wider ones. A librarian classifies books by subject; a gardener sorts seeds by when they should be planted. Which principle to use depends on the purpose: a cook and a botanist may sort the same vegetables in different ways, and both may be right. But within one classification the rules still hold: one principle at a time, no overlap, nothing left out.

> **From history.** Division is one of the oldest tools of logic. In Plato's *Sophist*, a visitor from Elea tracks down what an angler is by dividing the arts again and again, keeping one half each time. In the *Phaedrus*, Socrates says that a good thinker divides a subject at its natural joints, the way a good carver cuts meat, without splintering any part. Aristotle later warned that division alone cannot prove a definition, because at each step the divider simply assumes which side the thing falls on. In the 1500s the French teacher Peter Ramus (1515–1572) became famous for charts that split every subject into two, then two again. In 1876 Melvil Dewey published a library system that classifies all books into ten main classes, each numbered in hundreds.

> **Practice habit.** When you see things sorted into groups (a menu, a store, a website), ask: what is the principle, do the groups overlap, and does anything fit nowhere?

WORKED EXAMPLES

Example 1. Caleb sorts the coins in his jar into four piles: pennies, quarters, shiny coins and coins made before 1950. Find the faults in his division and repair it.

▼ **Show solution**

1. Principle: he has used three (the coin's value, its condition and its date). That is cross-division.
2. Overlap: a shiny penny made in 1945 belongs in three piles at once. The parts are not mutually exclusive.
3. Gaps: a dull nickel made in 1990 fits no pile at all. The parts are not jointly exhaustive.
4. Repair: divide by one principle, value: pennies, nickels, dimes, quarters (and half-dollars or dollar coins, if he has any). Then, if he wishes, subdivide one pile by a new principle at the next level: pennies made before 1950 and pennies made in 1950 or later.

Answer: The division mixes three principles, so the parts overlap (a shiny old penny) and leave coins out (a dull new nickel). Repair: divide by value (pennies, nickels, dimes, quarters and any larger coins), then subdivide a pile by date if wanted.

Example 2. Logical or physical division? (a) A bicycle into frame, wheels, handlebars, seat and pedals. (b) The books of the Bible into the Old Testament and the New Testament. (c) A year into four seasons.

▼ **Show solution**

1. Use the test: can each part be called by the name of the whole?
2. (a) "A wheel is a bicycle" is false. The parts are pieces of one whole: physical division.
3. (b) "Genesis is a book of the Bible" and "Romans is a book of the Bible" are true; each group is a kind of book of the Bible. Logical division.
4. (c) "Spring is a year" is false. The seasons are parts of one whole year: physical division.

Answer: (a) physical (b) logical (c) physical

PRACTICE

1. Easy What is logical division? State its four rules.

   ▼ **Show answer**

   Logical division breaks a class (genus) into the smaller classes (species) it contains. Rules: (1) use one principle at each step; (2) the parts must be mutually exclusive (no overlap); (3) the parts must be jointly exhaustive (nothing left out); (4) go step by step, to the nearest kinds first, without leaps. Accept the rules in any order and in the student's own words.

2. Easy Logical or physical division? (a) A house into kitchen, bedrooms and bathroom. (b) Triangles into acute, right and obtuse. (c) A day into morning, afternoon, evening and night. (d) Even numbers into those divisible by 4 and those not divisible by 4.

   ▼ **Show answer**

   (a) physical: a kitchen is not a house. (b) logical: a right triangle is a triangle. (c) physical: a morning is not a day. (d) logical: each part is a kind of even number (8, divisible by 4, is an even number; so is 6, which is not).

3. **Easy** What is the principle of division in "Triangles are equilateral, isosceles or scalene"?

   ▼ **Show answer**

   The number of equal sides (accept "the sides" or "side lengths").

4. **Easy** In your own words, what does it mean for the parts of a division to be mutually exclusive? Jointly exhaustive?

   ▼ **Show answer**

   Mutually exclusive: no member belongs to more than one part (no overlap). Jointly exhaustive: every member belongs to some part (nothing left out).

5. **Medium** Find the faults: "The students at Brookside High School are freshmen, sophomores, juniors, seniors and athletes."

   ▼ **Show answer**

   Cross-division: it mixes two principles (year in school and taking part in sports). As a result the parts overlap: a junior who plays soccer is in two parts. Repair: divide by year only (freshmen, sophomores, juniors, seniors), or by sports only (athletes and non-athletes).

6. **Medium** Find the faults and repair: "The days in March were sunny days, rainy days and snowy days."

   ▼ **Show answer**

   Not mutually exclusive: a day can be sunny in the morning and rainy later, or have both rain and snow. Probably not jointly exhaustive: a cloudy, dry day fits none of the parts. Repair (sample): "Days with some rain or snow, and days with none," a dichotomy that cannot overlap or leave a day out. Accept any repair that uses one principle with parts that do not overlap and cover every day.

7. Medium Divide triangles by their angles. Is your division mutually exclusive and jointly exhaustive? Explain.

▼ **Show answer**

Acute triangles (all three angles acute), right triangles (one right angle) and obtuse triangles (one obtuse angle). Yes: since a triangle's angles add up to 180 degrees, it can have at most one angle of 90 degrees or more, so no triangle is in two parts; and every triangle either has no such angle (acute), one right angle or one obtuse angle, so none is left out.

8. Medium Find the faults: "Living bodies are plants, dogs, cats and robins."

▼ **Show answer**

It leaps over a step (it skips "animals" and jumps to particular kinds of animal), and so it is not jointly exhaustive: horses, fish, insects and countless other animals are left out. Repair: "Living bodies are plants or animals," then subdivide animals.

9. Hard Divide the whole numbers by dichotomy at two levels (divide, then divide one of the parts again). Explain why each step is mutually exclusive and jointly exhaustive.

▼ **Show answer**

Sample: Whole numbers → even / odd. Even numbers → divisible by 4 / not divisible by 4. Each step divides by one feature and its opposite: every number either has the feature or lacks it (nothing left out), and none can both have it and lack it (no overlap). Accept any correct two-level dichotomy, such as odd numbers divided into those ending in 5 and those not ending in 5.

10. Hard Hannah divides the books on the family shelf into hardbacks, paperbacks, storybooks and books about horses. Name every fault and rewrite her division as a correct two-level division.

▼ **Show answer**

Faults: cross-division, since she uses three principles (binding, kind of writing and subject); overlap, since a paperback storybook about horses falls into three parts. Rewrite (sample): Level 1, by binding: hardbacks and paperbacks (adding a part for any other binding the shelf holds). Level 2, by kind of writing, dividing each: storybooks and books that are not storybooks. Accept any rewrite that uses one principle per level, with parts that do not overlap and leave nothing out.

11. Hard Explain the difference between division and classification, with an example of each.

▼ **Show answer**

Division starts with a class and breaks it down into its kinds: “Triangles are equilateral, isosceles or scalene.” Classification starts with individual things and gathers them into classes by shared characteristics, then into wider classes: a gardener sorts a pile of seed packets into “plant in spring” and “plant in fall,” and then groups both under “garden seeds.” Accept any pair of examples that shows top-down and bottom-up sorting.

12. Challenge The Ortiz farm keeps cows, horses, sheep, chickens, ducks and one barn cat. Divide “the animals on the Ortiz farm” into a tree of at least two levels, naming the principle at each step, and show that each step is mutually exclusive and jointly exhaustive.

▼ **Show answer**

Sample: Level 1, by number of legs: two-legged (chickens, ducks) and four-legged (cows, horses, sheep, the cat). Level 2a, two-legged animals by feet: webbed feet (ducks) and feet without webs (chickens). Level 2b, four-legged animals by feet: hoofed (cows, horses, sheep) and clawed (the cat). Each step uses one principle; every animal on the farm lands in exactly one part at each step, so the parts neither overlap nor leave anything out. Accept any tree that uses one principle per step and places each of the six kinds in exactly one part.

COMMON MISTAKES

- Confusing logical division with physical division. Check whether each part can be called by the name of the whole.
- Mixing principles at one step (cross-division), which usually makes the parts overlap.
- Forgetting a part, so that some members fit nowhere. Always try to think of a member that fits none of the parts.
- Leaping to small kinds before dividing into the nearest ones, which usually leaves members out.
- Thinking one principle must be used for the whole tree. A new principle may be used at each new level, but only one at each step.

> **Scripture.** "All flesh is not the same flesh: but there is one kind of flesh of men, another flesh of beasts, another of fishes, and another of birds." (1 Corinthians 15:39, KJV)
>
> Paul divides "flesh" into its kinds to make a point about the resurrection body; clear division helps make clear teaching.

UNIT 3

Informal Fallacies

A **fallacy** is a mistake in reasoning: an argument that seems to support its conclusion but does not. Some fallacies are mistakes of form, which later units test with rules and diagrams. This unit studies the **informal fallacies**, the ones you can catch only by looking at what an argument actually says: whether its reasons are relevant, what it quietly takes for granted, and whether its words keep the same meaning from start to finish. These are the fallacies you meet every day in conversation, in sports talk, in family arguments and in advertising.

Students learn to sort fallacies into three families. **Fallacies of relevance** offer reasons that are beside the point: attacking the person, bad appeals to authority, popularity and tradition, appeals to pity, fear and force, the red herring and the straw man. **Fallacies of presumption** smuggle in an unproved assumption: begging the question, the complex question, the false dilemma, hasty generalization, accident, the slippery slope and false cause. **Fallacies of ambiguity** turn on a shifting word or sentence: equivocation, amphiboly, accent, composition and division. Each

fallacy is given its common English name and its traditional Latin name where one exists, and each is set beside the legitimate move it imitates, because a fair question about a witness or a real expert's judgment is not a fallacy at all.

The unit ends with two lessons of application. Lesson 3.9 turns the whole toolkit on advertising (weasel words, testimonials and "studies show" lines) and sends students to the Classics Library's Spot the Device quiz. Lesson 3.10 teaches students to name fallacies fairly: to read an argument charitably, to remember that a bad argument does not make its conclusion false, and to answer "with meekness and fear" (1 Peter 3:15). The fallacies learned here return in Unit 8, where statistics are used to deceive, and in Unit 9, where students map, question and debate real arguments.

Review day and test. On the review day, use the mastery checklist and the review sheet (with its answer key). The unit test is below; a printable copy is ready for test day.

PREREQUISITES

- Find the premises and conclusion of an argument and write it in standard form (Lesson 1.3).
- Tell deductive from inductive arguments, and use the words valid, sound, strong and cogent correctly (Lesson 1.4).
- Tell univocal, equivocal and analogical uses of a term, and catch a word that changes meaning (Lesson 2.1).
- Recognize a good definition and a persuasive definition (Lesson 2.5).
- Read a short paragraph and say in one sentence what it is trying to prove.

MASTERY CHECKLIST

- ☐ I can define a fallacy, tell a formal fallacy from an informal one, and sort informal fallacies into relevance, presumption and ambiguity.
- ☐ I can spot abusive and circumstantial ad hominem, tu quoque and the genetic fallacy, and tell them from fair questions about a witness's reliability.
- ☐ I can tell a legitimate appeal to authority from an illegitimate one, and spot appeals to popularity, tradition and novelty.
- ☐ I can spot appeals to pity, fear and force, the red herring and the straw man.
- ☐ I can spot begging the question, the complex question and the false dilemma, and answer them fairly.
- ☐ I can spot hasty generalization, accident, the slippery slope and false cause, and say what evidence would repair each.
- ☐ I can find the shifted word, loose sentence or misplaced emphasis in equivocation, amphiboly and accent.
- ☐ I can tell when what is true of the parts is true of the whole, and the reverse, and name composition and division.
- ☐ I can analyze an advertisement's weasel words, testimonials and "studies show" claims.

- [ ] I can read an argument charitably, avoid the fallacy fallacy, and answer a fallacy courteously.

LESSONS IN THIS UNIT

3.1	**What Is a Fallacy?**	40 minutes
3.2	**Attacking the Person**	40 minutes
3.3	**Appeals to Authority, the Many and Tradition**	40 minutes
3.4	**Appeals to Emotion and Force; Red Herring and Straw Man**	40 minutes
3.5	**Presumption I: Begging the Question, Complex Question, False Dilemma**	40 minutes
3.6	**Presumption II: Generalizing Badly**	45 minutes
3.7	**Ambiguity I: Equivocation, Amphiboly, Accent**	40 minutes
3.8	**Ambiguity II: Composition and Division**	35 minutes
3.9	**Fallacies in Advertising**	45 minutes
3.10	**Naming Fallacies Fairly**	40 minutes

Unit 3 test

You have 45 minutes. No notes or books. When you name a fallacy, give its common English name (the Latin name is welcome but not required) and a one-sentence reason. Some items are not fallacies; say so and explain why. Question 14 is a short written answer of four to six sentences. The test is worth 100 points.

1. **6 pts**

   (a) Define a fallacy. (b) What is the difference between a formal and an informal fallacy? (c) Name the three families of informal fallacies.

2. **6 pts**

   Sort each argument into its family (relevance, presumption or ambiguity) and give a one-sentence reason.

   1. “Don’t trust Ellie’s plan for the class trip; she is always late to everything.”
   2. “Either we buy the Thunder 500 mower or our lawn will be a jungle all summer.”
   3. “The band director said our concert was a big hit. Hitting people is wrong. So our concert was wrong.”

3. **8 pts**

   Name the fallacy in each, with a one-sentence reason.

   1. “Mr. Dunn’s advice about pruning apple trees is worthless. He is the grumpiest man in town.”
   2. “Of course the owner of Hilltop Feed says hens lay more eggs on good feed. He sells feed.”
   3. Mom asks Sam to put his phone away at dinner. Sam answers, “You check your phone at dinner too!”
   4. “That way of saving money came from a cartoon, so it can’t be a good idea.”

4. **6 pts**

   One judge of the county fair’s quilt contest is the aunt of an entrant. (a) The other entrants ask that a different judge score her niece’s quilt. Is this an ad hominem fallacy? Explain. (b) Another entrant, Mrs. Polk, points out that her own quilt has tighter, more even stitches than the others, and anyone can look closely and see it. The aunt replies, “She just wants a ribbon.” Is this a fallacy? Explain.

5. **8 pts**

Is each a legitimate appeal or a fallacy? If a fallacy, name it. Give a one-sentence reason for each.

1. “Dr. Hart, a dentist who examined Ava’s teeth this morning, says she needs a filling.”
2. “Champion skier Mara Voss says Sunburst orange juice has more vitamins than any other brand.”
3. “Grandpa always sharpened his saw with a file, so a file is the best way to sharpen a saw.”
4. “Over a million people bought Zoomer scooters last year, so Zoomer must make the safest scooter.”

6. **8 pts**

Name the fallacy in each, with a one-sentence reason.

1. “Coach, please let me start on Saturday. My parents will be so disappointed if I sit on the bench.”
2. “Say my story is the best one in the class, or I won’t sit with you at lunch anymore.”
3. Mom asks Nate why the garden wasn’t weeded. Nate answers, “Did you know the Beckers got a new puppy?”
4. Mia says the team should practice on Saturday mornings during the season. Leo replies, “So Mia thinks we should never have a free weekend again for the rest of our lives.”

7. **8 pts**

Name the fallacy in each, or say that it is not a fallacy, with a one-sentence reason.

1. “Ridgecrest milk is the freshest milk you can buy, because no milk you can buy is fresher than Ridgecrest.”
2. “When did you start forgetting to feed the cat?”
3. “Either you are a genius at math or you will fail algebra.”
4. “The pasture gate is either latched or not latched.”

8. **8 pts**

   Name the fallacy in each, with a one-sentence reason.

   1. “My cousin’s two Kestrel trucks both broke down, so Kestrel trucks are unreliable.”
   2. “The rule says to stay in your seat during class, so Ben should stay seated even though the fire alarm is ringing.”
   3. “If we let the chickens into the yard once, they’ll want to be there every day, then they’ll dig up the whole garden, and we’ll never grow anything again.”
   4. “I started eating Crunchos cereal last week, and then I got an A on my history test. Crunchos made me smarter.”

9. **6 pts**

   “Children who grow up in homes with many books tend to do better in school. So buying more books will make any child do better in school.” Name the fallacy and give an alternative explanation for the fact in the first sentence.

10. **8 pts**

    Name the fallacy in each.

    1. “Mr. Hale says his new truck is a beast. Beasts belong in the barn. So Mr. Hale’s truck belongs in the barn.” Also show the substitution test.
    2. A notice on the church bulletin board reads: “Wanted: a sitter for our three boys who can cook.” What are the two readings?
    3. A review says, “This soup would be delicious if it had any flavor at all.” The restaurant’s sign reads, “‘Delicious!’ (Valley News).”

11. **6 pts**

    Composition, division, or no fallacy? Give a one-sentence reason for each.

    1. “Every part of this birdhouse is made of cedar, so the birdhouse is made of cedar.”
    2. “Each grain of rice in this sack weighs almost nothing, so the sack weighs almost nothing.”
    3. “The Fairview High band is the best band in the state, so each member must be the best player of his or her instrument in the state.”

12. **8 pts**

Name four devices in this advertisement and explain each in a sentence: “ALL-NEW Zestwell Juice! Up to 30% more vitamins!* Doctor recommended. Join the millions of families who have already switched!” In small print: “*Compared with our old recipe.”

13. **6 pts**

(a) Kara argues, “We should floss every day, because a famous actor says he does.” Her brother replies, “That’s a bad appeal to authority, so flossing is useless.” Name the brother’s mistake and explain it. (b) A neighbor says, “Everybody knows you need a heavy coat in January around here.” Give a charitable reading of the claim.

14. **8 pts**

A friend says: “You can’t believe anything Mr. Price says about growing pumpkins. He grew up in the city. Anyway, either you use the expensive Megagrow fertilizer or your pumpkins will be tiny.” (a) Name the two fallacies. (b) Write a courteous reply of four to six sentences, following the four steps (understand, test, separate, answer).

ANSWER KEY

1. (a) A fallacy is a mistake in reasoning: an argument that seems to support its conclusion but does not. (b) A formal fallacy lies in the form (shape) of the argument and can be caught without knowing what the terms mean; an informal fallacy can be caught only by looking at its content: the meaning of the words, the relevance of the reasons, or what is assumed. (c) Relevance, presumption and ambiguity.
2. 1: Relevance: Ellie's lateness has nothing to do with whether her plan is good (ad hominem). (If a student argues that her lateness matters to whether she can be trusted to carry out the plan, accept it as a fair question about reliability, provided the student notes that the plan itself must still be judged on its merits.) 2: Presumption: it assumes without proof that there are only two options (false dilemma); we could buy another mower, borrow one or fix the old one. 3: Ambiguity: "hit" means "a success" in the first premise and "a blow" in the second (equivocation).
3. 1: Abusive ad hominem: his grumpiness says nothing about whether his pruning advice is right. 2: Circumstantial ad hominem: his interest in selling feed is a reason to check his claim, not proof it is false. 3: Tu quoque: Mom's habit does not make her request wrong. 4: Genetic fallacy: an idea must be judged by whether it works, not by where it came from. (2 points each.)
4. (a) No. A judge's scores rest on her fair judgment (like testimony), so her interest in her niece is relevant to whether she can be trusted to judge that quilt; asking for an impartial judge is a fair question. (b) Yes, a circumstantial ad hominem. Mrs. Polk gave a reason that anyone can check by looking at the stitches, and the aunt dismisses it by pointing to Mrs. Polk's motive instead of examining the stitches.
5. 1: Legitimate: a real expert, speaking within her field, about a patient she examined. 2: Fallacy, bad appeal to authority (ad verecundiam): a skier is not an expert on vitamins, and she is probably paid. 3: Fallacy, appeal to tradition: that Grandpa always did it does not by itself show it is best (though his method may carry real experience worth weighing). 4: Fallacy, appeal to the many (ad populum, bandwagon): popularity is not evidence of safety, which is settled by safety tests. (2 points each.)
6. 1: Appeal to pity: the parents' disappointment does not bear on who should start, which depends on skill and the team's needs. 2: Appeal to force (ad baculum): the threat gives a reason to say the story is best, not to believe it. 3: Red herring:

Nate changes the subject, and the question about the weeding is dropped. 4: Straw man: Leo exaggerates Mia's actual proposal (Saturday mornings during the season) into an extreme one she never made. (2 points each.)

7. 1: Begging the question: the premise only restates the conclusion. 2: Complex question: it assumes, without proof, that the person has been forgetting to feed the cat; answer by dividing the question. 3: False dilemma: most students are neither geniuses nor failures; there are many possibilities in between. 4: Not a fallacy: "latched" and "not latched" are contradictories, so one of them must be true (a true dilemma). (2 points each.)
8. 1: Hasty generalization: two trucks are too few to judge the whole brand. 2: Accident: the rule is meant for ordinary times, not for an emergency it was never meant to cover. 3: Slippery slope: no evidence is given that any link in the chain is likely; letting them out once does not mean every day, and the family can fence the garden. 4: False cause (post hoc ergo propter hoc): the A came after the cereal, but studying is a far better explanation. (2 points each.)
9. False cause (mistaking things that go together for cause and effect). Alternative: a common cause. Families that own many books are often families that read together, talk about ideas and help with schoolwork, and those habits may explain both the books and the good grades. Buying books without the reading may change little. Accept any reasonable common-cause or other alternative explanation (3 points for the fallacy, 3 for the explanation).
10. 1: Equivocation (3 points). Substitution: "Mr. Hale's truck is powerful and tough. Animals belong in the barn. So his truck belongs in the barn." With the meanings written out, the premises do not connect. 2: Amphiboly (3 points). Reading 1: the family wants a sitter who can cook. Reading 2: the three boys can cook, and the family wants a sitter for them. The loosely placed phrase "who can cook" allows both. 3: Accent, by quoting out of context (2 points): dropping the "if" clause turns criticism into praise.
11. 1: No fallacy: material carries over from the parts to the whole. 2: Composition: weight adds up, so a sack of many light grains can be heavy. 3: Division: the band's excellence belongs to the group playing together, not necessarily to each member. (2 points each.)
12. Any four (2 points each): (1) "ALL-NEW": appeal to novelty; new is not proof of better. (2) "Up to": weasel words; 30% is the most, not the usual amount. (3) "30% more vitamins" is an unfinished comparison, finished only in the fine print with a weak standard (the old recipe), a form of accent. (4) "Doctor recommended": a vague appeal to authority; how many doctors, which ones, and were they paid?

(5) “Join the millions”: appeal to the many (bandwagon). Accept other correct labels with a correct explanation.

13. (a) The fallacy fallacy: Kara’s reason is a bad appeal to authority, but a bad argument leaves its conclusion unsupported, not disproved; there are good reasons to floss (dentists recommend it to clean between the teeth) that she did not give. (4 points.) (b) Charitably, the neighbor means that January here is usually very cold, so a heavy coat is wise, not that every single person has studied the matter. (2 points; accept any reading that treats “everybody knows” as loose everyday speech for a common, well-founded belief.)
14. (a) Ad hominem (circumstantial or genetic: dismissing Mr. Price because of where he grew up) and false dilemma (Megagrow or tiny pumpkins; there are other fertilizers, compost, good soil and watering). (2 points each.) (b) Sample answer: “I think you’re saying you don’t trust Mr. Price’s advice, and you want my pumpkins to grow well, which I appreciate. Still, where he grew up doesn’t tell us whether his advice is right; we’d have to look at what he actually says and how his own pumpkins turn out. And I don’t think Megagrow and tiny pumpkins are the only choices; compost and steady watering work for a lot of growers. You might be right that his advice is wrong, but those reasons don’t show it. Would you want to compare a few of our plants with and without his fertilizer this summer?” (4 points: understands charitably, names or describes both problems, avoids the fallacy fallacy, and answers courteously.)

Printable answer key (PDF, for the parent)

UNIT 3 · INFORMAL FALLACIES · LESSON 3.1 OF 69 · 40 MINUTES

3.1 What Is a Fallacy?

OBJECTIVES

- Define a fallacy as a mistake in reasoning that makes an argument seem to support its conclusion when it does not.
- Tell a formal fallacy from an informal fallacy.
- Sort informal fallacies into the three families of relevance, presumption and ambiguity, using a test question for each.
- Describe Aristotle’s *Sophistical Refutations* and his division of fallacies into those that depend on language and those that do not.

WATCH

Logical Fallacies

Texas A&M University Writing Center · 3:51

https://www.youtube.com/watch?v=HpP71KmmlYQ

> **WHEN TO WATCH**
> After the definition of a fallacy: five common examples before we sort them into families.

Note: Parent preview: A university writing center defines a fallacy as an argument that doesn't work and gives five examples: hasty generalization, non sequitur, slippery slope, bandwagon and red herring. Examples are everyday (a college rivalry, skipping a party, a made-up "PiPhone," a school board meeting). It does not use Aristotle's three families; the lesson does that. Nothing objectionable; transcript checked October 18, 2026.

KEY TERMS

Fallacy

A mistake in reasoning: an argument that seems to support its conclusion but does not. The word comes from the Latin *fallere*, "to deceive."

Formal fallacy

A fallacy that lies in the form (the shape) of an argument, so that it can be caught without knowing what the terms mean.

Informal fallacy

A fallacy that can be caught only by looking at the content of an argument: what its words mean, whether its reasons are relevant, and what it assumes.

Fallacy of relevance

A fallacy whose premises, even if true, do not bear on the conclusion; they are beside the point.

Fallacy of presumption

A fallacy whose premises quietly assume something unproved or false that the conclusion needs.

Fallacy of ambiguity

A fallacy that depends on a word, phrase or sentence that can be taken in more than one way.

Sophist

In ancient Greece, a paid teacher of speaking and arguing; the word came to mean someone who argues cleverly to win rather than to find the truth.

CORE IDEAS

An argument that looks better than it is. After practice, Caleb tells his teammates, "Everybody on the varsity team wears Swiftline cleats, so Swiftline must make the best cleats." The argument sounds reasonable, and plenty of players would nod along. But a moment's thought shows the gap: the varsity players might wear Swiftline because the team got a discount, or because the cleats look sharp, or because the captain wears them. A **fallacy** is a mistake in reasoning of exactly this kind: an argument that seems to support its conclusion but does not. The word comes from the Latin *fallere*, "to deceive." A fallacy deceives because it has the look of good reasoning without the substance.

A fallacy is a mistake in reasoning, not just a false claim. "The Nile is the shortest river in Africa" is false, but it is not a fallacy; it is simply a mistaken statement. A fallacy is a fault in the connection between reasons and conclusion. That is why an argument can commit a fallacy and still have a true conclusion. Swiftline cleats might really be the best. If so, Caleb is right by luck, not by reasoning, and the next time he reasons the same way he may be wrong. Logic cares about the road as well as the destination.

Why fallacies persuade. If fallacies were obviously bad, nobody would fall for them. They work because they imitate good arguments. Asking a real expert is sound practice, so a bad appeal to authority borrows its look. Feelings often track something real, so an appeal to pity can feel like a reason. Words usually keep their meaning, so we do not notice when one slips. Fallacies also feed on hurry and on wishes: we accept weak reasons quickly when we like the conclusion. Most fallacies are not lies told on purpose. More often we fool ourselves, which is why learning them is a kind of self-defense.

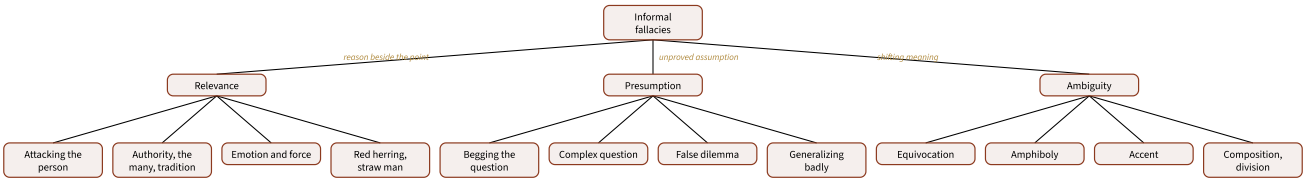
Formal and informal fallacies. Some fallacies lie in the **form** of an argument. "All roses are plants. All tulips are plants. So all tulips are roses." You can see the flaw

without knowing anything about flowers: put “dogs,” “cats” and “animals” into the same pattern and you get “All dogs are animals. All cats are animals. So all cats are dogs,” which is plainly absurd. That is a **formal fallacy**, and Units 5 and 6 will teach rules that catch such errors by their shape alone. An **informal fallacy** cannot be caught that way. Its form may look perfectly fine; the trouble is in its content: what the words mean, whether the reasons are relevant, and what is being taken for granted. This unit is about informal fallacies.

The first family: fallacies of relevance. In a **fallacy of relevance** the premises may be true, but they do not bear on the conclusion. “Coach Reyes never played college basketball, so his advice about free throws must be wrong” attacks the coach instead of examining the advice. Appeals to pity, fear, force, popularity and tradition belong here, along with the red herring (changing the subject) and the straw man (attacking a distorted version of the other side). The test question is: *Even if this reason is true, does it give me any reason to believe that conclusion?*

The second family: fallacies of presumption. In a **fallacy of presumption** the reasons are relevant, but the argument quietly assumes something it has not earned. “Have you stopped leaving your bike out in the rain?” assumes you used to. “Either we paint the barn this summer or it will fall down” assumes there are only two choices. Begging the question, the complex question, the false dilemma, hasty generalization, accident, the slippery slope and false cause belong here. The test question is: *What is this argument taking for granted, and has it shown that it is true?*

The third family: fallacies of ambiguity. A **fallacy of ambiguity** turns on a word, phrase or sentence that can be taken in two ways. “A feather is light. Whatever is light is not dark. So a feather is not dark” slides from “light” meaning “not heavy” to “light” meaning “bright.” Equivocation, amphiboly, accent, composition and division belong here. The test question is: *Does any word or phrase change its meaning between the reasons and the conclusion?* The diagram shows the three families and the fallacies this unit will study in each.



Aristotle's guide to the tricks of the sophists. In Athens, teachers called **sophists** charged fees to teach young men to win arguments, and some of them cared more about winning than about truth. Aristotle answered them in a short book, the *Sophistical Refutations*, which lists thirteen ways an argument can seem to refute an opponent without really doing so. He divided them into two groups: six that depend on language (such as a word used in two senses) and seven that do not (such as assuming the point at issue). The three families used in this course are a later teaching arrangement, but they grow from his division: ambiguity matches his first group, and relevance and presumption split his second.

Names are tools, not weapons. Learning the names of fallacies helps you see clearly and explain exactly what went wrong. But the names are not a scorecard. One argument can commit two fallacies at once, and the borders between families are not walls. Some arguments that look like fallacies are perfectly fair, such as trusting a veterinarian about a lame horse. The goal is to understand an argument, not to stick a label on it. Scripture tells us to "Prove all things; hold fast that which is good" (1 Thessalonians 5:21): testing arguments is so that we can keep what is good. For great-books examples of each fallacy, visit the Classics Library Logic & Debate page; after Lesson 3.9 you will take its Spot the Device quiz.

> **From history.** Aristotle (384–322 B.C.) wrote the *Sophistical Refutations* as the last book of the collection later called the *Organon*. It is the first known attempt to list and explain the ways arguments go wrong. Aristotle names thirteen fallacies: six "in the language," such as equivocation and amphiboly, and seven "outside the language," such as begging the question, false cause and many questions asked as one. Medieval teachers studied the book closely, and many of the Latin names you will learn in this unit come from their discussions of it. More than two thousand years later, most lists of fallacies still draw on Aristotle's thirteen.

> **Practice habit.** When an argument sounds convincing but leaves you uneasy, ask the three test questions: Is the reason beside the point? What is being assumed? Has any word changed its meaning?

Enrichment: Aristotle on seeming and being. Aristotle opens the *Sophistical Refutations* by pointing out that some things only seem to be what they are not. Some people really are in good condition, while others only look that way by puffing themselves up; some things are truly beautiful, while others only seem so by decoration. In the same way, he says, roughly, some reasonings are real and others only seem to be. Can you think of something in everyday life (a coin, a fruit, a painting) that can look genuine without being genuine? How do you test it? How is that like testing an argument?

WORKED EXAMPLES

Example 1. Sort each argument into a family (relevance, presumption or ambiguity) and say why. (a) "Coach Reyes never played college basketball, so his advice about free throws is worthless." (b) "Have you stopped copying your brother's homework?" (c) "Every brick in this wall is light, so the whole wall is light."

▼ **Show solution**

1. Ask the three test questions of each: Is the reason beside the point? Is something assumed without proof? Does a word shift its meaning?
2. (a) Whether the coach played in college does not show whether his free-throw advice is right or wrong; the advice must be judged on its merits. The reason is beside the point: relevance.
3. (b) The question assumes, without proof, that you used to copy your brother's homework. Either answer, "yes" or "no," admits it. That is a hidden assumption: presumption.
4. (c) "Light" is true of each brick taken one at a time, but the conclusion applies it to the whole wall, which is very heavy. The reasoning slides from the parts to the whole (composition): ambiguity.

Answer: (a) Relevance (attacking the person instead of the advice). (b) Presumption (a complex question that assumes you used to copy). (c) Ambiguity (composition: what is true of each brick is not true of the wall).

Example 2. Is each fallacy formal or informal? (a) “All roses are plants. All tulips are plants. So all tulips are roses.” (b) “A feather is light. Whatever is light is not dark. So a feather is not dark.”

▼ **Show solution**

1. A formal fallacy can be seen in the shape alone. Test by keeping the shape and changing the words.
2. (a) The shape is: All A are C; all B are C; so all B are A. Fill it with “dogs,” “cats” and “animals”: “All dogs are animals. All cats are animals. So all cats are dogs.” True premises, false conclusion, same shape. The shape itself is bad, so the fallacy is formal.
3. (b) The shape looks like a good one: A is B; whatever is B is not C; so A is not C. That shape is valid when “B” keeps one meaning. The trouble is that “light” means “not heavy” in the first premise and “bright” in the second. You can only see this by looking at what the word means, so the fallacy is informal (equivocation).

Answer: (a) Formal: the pattern itself lets true premises lead to a false conclusion. (b) Informal: the form is fine, but the word “light” changes meaning (equivocation, a fallacy of ambiguity).

PRACTICE

1. Easy Define a fallacy in one sentence.

   ▼ **Show answer**

   A fallacy is a mistake in reasoning: an argument that seems to support its conclusion but does not.

2. Easy What is the difference between a formal fallacy and an informal fallacy?

   ▼ **Show answer**

   A formal fallacy lies in the form (shape) of the argument and can be caught without knowing what the terms mean; an informal fallacy can be caught only by looking at the content: what the words mean, whether the reasons are relevant, or what is assumed.

3. **Easy** Name the three families of informal fallacies, and give the test question for each.

▼ Show answer

Relevance: Even if this reason is true, does it give any reason to believe the conclusion? Presumption: What is the argument taking for granted, and has it shown that it is true? Ambiguity: Does any word or phrase change its meaning between the reasons and the conclusion? (Accept the test questions in the student's own words.)

4. **Easy** Who wrote the *Sophistical Refutations*, and into what two groups did he divide the fallacies?

▼ Show answer

Aristotle. He divided his thirteen fallacies into those that depend on language (six, such as equivocation) and those that do not (seven, such as begging the question or false cause).

5. **Medium** Hannah says, "The Nile is the shortest river in Africa." Has she committed a fallacy? Explain.

▼ Show answer

No. She has made a false statement, but she has not given an argument, and a fallacy is a mistake in reasoning (in the link between reasons and conclusion), not merely a false claim.

6. **Medium** Which family does this fallacy belong to? Give a one-sentence reason. "Either we paint the barn this summer or it will fall down."

▼ Show answer

Presumption: it assumes without proof that there are only two possibilities, when there are others (paint it next summer, repair only the bad boards, put on new siding). (This is a false dilemma.)

7. Medium Which family does this fallacy belong to? Give a one-sentence reason. “The sign in the lot says ‘Fine for parking here,’ so it must be fine for me to park here.”

▼ Show answer

Ambiguity: the word “fine” means “a money penalty” on the sign but “all right” in the conclusion. (This is an equivocation.)

8. Medium Which family does this fallacy belong to? Give a one-sentence reason. “You can’t trust Mr. Dunn’s plan for the vegetable garden. He drives a rusty old truck.”

▼ Show answer

Relevance: the condition of Mr. Dunn’s truck has nothing to do with whether his garden plan is good; the plan must be judged on its merits. (This attacks the person.)

9. Hard Is this fallacy formal or informal? Explain how you can tell. “If it rained last night, the field is wet. The field is wet. So it rained last night.”

▼ Show answer

Formal. The flaw is in the shape (If A, then B; B; so A), and any argument with that shape can have true premises and a false conclusion: “If it is a dog, it is an animal. It is an animal. So it is a dog” fails when the animal is a cat. Here the field could be wet from sprinklers or dew. (This is affirming the consequent, studied in Unit 6.)

10. Hard Can an argument commit a fallacy and still have a true conclusion? Explain with an example.

▼ Show answer

Yes. A fallacy is a fault in the reasoning, not in the conclusion. Example: “Everybody at school says Mount Everest is the tallest mountain on earth, so it must be.” The conclusion is true, but popularity is not good evidence for it; the reasoner is right by luck. Accept any example where a true conclusion is reached by a faulty reason.

11. Hard Is this a fallacy? Explain. “Dr. Patel, a veterinarian who has treated horses for twenty years, examined our mare and says she has a stone bruise. So she probably has a stone bruise.”

▼ **Show answer**

No. This is a legitimate appeal to authority: Dr. Patel is a real expert speaking within her own field, about a case she examined, so her judgment is good evidence. The argument is inductive and strong (it says “probably”), though an expert can still be mistaken.

12. Challenge Write one short example of your own for each family of fallacies (relevance, presumption, ambiguity), and explain in one sentence what goes wrong in each.

▼ **Show answer**

Sample answer: Relevance: “Don’t listen to Jonah’s idea for the science fair; he’s only in seventh grade.” His grade does not show that the idea is bad. Presumption: “Either you join the soccer team or you will never make any friends.” It assumes there are only two possibilities. Ambiguity: “The coach told us to run the bases, so we should be in charge of the bases.” “Run” shifts from “run around” to “manage.” Accept any examples that fit each family, with a correct explanation.

COMMON MISTAKES

- Calling any false statement a fallacy. A fallacy is a mistake in reasoning; a false claim with no argument is simply false.
- Thinking that a fallacy proves its conclusion false. A fallacious argument fails to support its conclusion, but the conclusion may still be true.
- Treating every appeal to an expert or to feelings as a fallacy. Trusting a real expert in her own field is reasonable, and some feelings rest on real reasons.
- Forcing every bad argument into exactly one box. One argument can commit more than one fallacy; what matters is explaining what goes wrong.

REFERENCE GUIDE

An eight-page, color-coded Logic guide: what logic is, terms and definitions, the A E I O propositions, the square of opposition and immediate inferences, syllogisms with the six rules and Venn tests, the named forms, informal fallacies of relevance,

presumption and ambiguity, propositional logic with truth tables, conditionals, valid forms and short proofs, the classical tradition, dialectic, argument maps and practice. The booklet prints on 11×17 paper and folds to letter size. Prints in color and reads clearly in black and white.

> **Scripture.** “Prove all things; hold fast that which is good.” (1 Thessalonians 5:21, KJV)
>
> To “prove” here means to test. Learning the fallacies is one way of testing what we hear, so that we can keep what is good and set aside what only looks good.

UNIT 3 · INFORMAL FALLACIES · LESSON 3.2 OF 69 · 40 MINUTES

3.2 Attacking the Person

OBJECTIVES

- Spot abusive and circumstantial ad hominem, poisoning the well, tu quoque and the genetic fallacy.
- Explain why an argument must be judged by its reasons, not by the person who gives it or the place it came from.
- Tell an attack on the person from a fair question about a witness’s honesty, knowledge or interest.
- Answer a personal attack courteously by returning to the reasons.

KEY TERMS

Ad hominem

Latin for “to the man”: the fallacy of attacking the person who gives an argument instead of the argument itself.

Abusive ad hominem

An attack on a person’s character, intelligence, looks, age or habits, offered as a reason to reject what he or she says.

Circumstantial ad hominem

Rejecting a claim because of the speaker’s circumstances, such as a job, an interest or a group, by suggesting that he or she says it only because of them.

Poisoning the well

Discrediting a person in advance, before he or she has spoken, so that listeners will dismiss whatever is said.

Tu quoque

Latin for "you too": rejecting a criticism or a piece of advice because the person giving it does not follow it either.

Testimony

A claim we accept on someone's word rather than on reasons we can check for ourselves; its value depends on the witness's honesty, knowledge and interest.

Genetic fallacy

Judging an idea, claim or practice good or bad by where it came from (its origin) instead of by its present merits.

CORE IDEAS

An argument stands or falls on its reasons. At a school meeting, Mrs. Ortiz proposes moving the vegetable garden to the south bed, because it gets six hours of sun a day and drains well after rain. Mr. Hale answers, "Mrs. Ortiz can't even keep her own lawn green." Notice what has happened. Mrs. Ortiz gave two reasons, sunlight and drainage, and Mr. Hale has answered neither of them. He has turned from the argument to the person. This is the fallacy called **ad hominem**, Latin for "to the man." Even if every word about her lawn is true, the south bed gets just as much sun as it did before he spoke.

The abusive ad hominem. The bluntest form is the **abusive ad hominem**: an insult offered as a reason. "Don't listen to Jonah's plan for the car wash; he's lazy." "Why would anyone take spelling advice from a sixth grader?" The insult may even be accurate. Jonah may be lazy. But a lazy person can still have a good plan, and a sixth grader can still spell "necessary" correctly. The plan must be judged on its merits. Notice that an insult by itself is rude, not a fallacy; it becomes a fallacy when it is offered as a reason to reject what someone says.

The circumstantial ad hominem and poisoning the well. A quieter form points to a person's situation instead of their character. "Of course Mr. Becker says the apple festival should move to October. He owns an orchard." This is the **circumstantial ad hominem**: it suggests that the speaker says something only because of a job, an interest or a group. Mr. Becker may indeed benefit, but if he argues that October is

drier and the apples are ripe then, those reasons still need an answer. A related trick is **poisoning the well**: discrediting someone in advance, before he or she speaks. "Before Ava gives her report, remember that she always exaggerates." Now whatever Ava says will be heard through suspicion, whether it deserves it or not.

Tu quoque: "You too." Dad tells Caleb that eating sweets late at night will spoil his sleep before the big game. Caleb answers, "You eat ice cream every night!" Caleb may be right about Dad, but Dad's habit does not make his advice wrong. Sugar at bedtime affects Caleb the same way whether or not his father practices what he preaches. This reply is called **tu quoque**, Latin for "you too." Hypocrisy is a fault in the person, not in the argument. A doctor who smokes can still be right that smoking harms the lungs.

The genetic fallacy: judging by origin. The **genetic fallacy** judges an idea by where it came from instead of by what it is now. "That bread recipe came from a gas-station cookbook, so it can't be any good." "The custom of shaking hands began as a way to show you held no weapon, so it's pointless today." A recipe is good or bad by how the bread tastes, and a custom may have gained new meaning since it began. The same mistake runs the other way: "It came from a famous chef, so it must be good" also skips the question of merit. When Philip told Nathanael about Jesus, Nathanael answered, "Can there any good thing come out of Nazareth?" Philip did not argue; he said, "Come and see" (John 1:46). Look at the thing itself.

When the person does matter: testimony. Not every question about a person is a fallacy. Much of what we know we accept on someone's word: a witness to an accident, a neighbor reporting that the creek has flooded the road, a salesperson saying a truck has never been in a wreck. This is **testimony**, and its value depends on the witness: Is he honest? Was she in a position to know? Does he stand to gain? If a pie-contest judge has a daughter in the contest, the other families are right to ask for a different judge, because the contest rests on the judge's fair word. Asking about a witness's reliability is good sense, not an attack.

Arguments and testimony are judged differently. Here is the key distinction. When people ask you to take their word, their honesty and interest are relevant. When people give reasons you can check for yourself, check the reasons. If a tent salesman says, "This tent is waterproof," you may fairly want more than his word. But if he sprays the tent with a hose for ten minutes and lets you look inside, the dry floor is

evidence that does not depend on his honesty, and "You'd say anything to sell a tent" is now a fallacy. An interest is a reason to check a claim, not a proof that it is false. Biased people are sometimes right.

Answering an attack on the person. When someone attacks you instead of your argument, the best answer is calm and short: bring the conversation back to the reasons. "Maybe I am only a seventh grader, but what do you think of the plan itself?" "You may be right that I eat too many sweets myself. Is the advice still true?" Do not return the insult, which only trades one fallacy for another. And watch your own speech. It is tempting to dismiss an idea because we dislike the person who holds it. Fairness means letting even an opponent's argument be heard on its merits.

> **From history.** The English philosopher John Locke, in *An Essay Concerning Human Understanding* (1690), described several kinds of arguments people use to win agreement, and he gave them Latin names, among them the *argumentum ad hominem*. For Locke this meant pressing a man with the consequences of his own principles or admissions, which can be a fair way to argue: "By your own rule, you must also accept this." Later writers used the phrase more and more for arguments aimed at a speaker's character or circumstances instead of at the question, and that is the meaning it carries today. The idea of the fault, of course, is much older than the name.

> **Practice habit.** When someone answers an argument by talking about the person who gave it, ask yourself: "And what about the reasons?"

Example 1. Mrs. Ortiz argues that the school should buy a second set of soccer goals so that two teams can practice at once. Name the fallacy in each reply. (a) “Mrs. Ortiz doesn’t know anything; she can’t even kick a ball.” (b) “Of course she wants more goals. Her son is on the team.” (c) “Last year you voted against new bleachers, and now you want to spend money?”

▼ **Show solution**

1. First find the argument: two sets of goals would let two teams practice at once. Then ask whether each reply addresses that reason.
2. (a) It insults her ability and offers the insult as a reason to reject her proposal. Her kicking has nothing to do with whether two practice spaces are needed: abusive ad hominem.
3. (b) It points to her circumstances (her son on the team) and suggests that is the only reason she says it. Even if she benefits, the reason about practice space still needs an answer: circumstantial ad hominem.
4. (c) It accuses her of being inconsistent with her own past position. Even if she was inconsistent, that does not show the goals are not needed: tu quoque.

Answer: (a) Abusive ad hominem. (b) Circumstantial ad hominem. (c) Tu quoque. None of the replies answers her reason about practice space.

Example 2. Fallacy or fair question? (a) A used-truck salesman says, "This truck has never been in an accident," and the buyer asks to see the vehicle's repair history before believing it. (b) The salesman shows the window sticker and a printed road test showing 25 miles per gallon, and the buyer says, "You're a salesman, so I won't believe any of it."

▼ **Show solution**

1. Ask: Is the buyer being asked to take the salesman's word (testimony), or is he being shown evidence he can check?
2. (a) "Never been in an accident" is testimony. The salesman stands to gain if it is believed, so his interest is relevant. Asking for a record that does not depend on his word is a fair question.
3. (b) The sticker and road test are evidence that can be checked independently of the salesman. Dismissing them because of his job ignores the evidence: circumstantial ad hominem.

Answer: (a) A fair question: the salesman's interest is relevant to testimony, so the buyer asks for independent evidence. (b) A fallacy (circumstantial ad hominem): the buyer rejects checkable evidence only because of who presented it.

PRACTICE

1. Easy What does *ad hominem* mean in Latin, and what is the ad hominem fallacy?

   ▼ **Show answer**

   It means "to the man." The ad hominem fallacy attacks the person who gives an argument (his or her character or circumstances) instead of answering the argument itself.

2. Easy What does *tu quoque* mean, and what does this fallacy do?

   ▼ **Show answer**

   It means "you too." It rejects a criticism or a piece of advice because the person giving it does not follow it either, as if hypocrisy made the advice false.

3. **Easy** Define the genetic fallacy and give a one-sentence example.

▼ Show answer

The genetic fallacy judges an idea, claim or practice by where it came from instead of by its present merits. Example: “That song was first written as a commercial jingle, so it can’t be good music.” Accept any example that judges something only by its origin (good or bad origin).

4. **Easy** Name the fallacy and give a one-sentence reason: “Mr. Lee’s plan for the bake sale is foolish. He’s the most boring teacher in the school.”

▼ Show answer

Abusive ad hominem: it insults Mr. Lee’s personality and offers that as a reason to reject his plan, without saying anything about the plan itself.

5. **Medium** Name the fallacy and give a one-sentence reason: “Of course the manager of Brookside Dairy says milk is good for growing children. She sells milk.”

▼ Show answer

Circumstantial ad hominem: it dismisses the claim because of the speaker’s interest instead of looking at the evidence. Her interest is a fair reason to look for evidence from someone else, but it does not show that the claim is false.

6. **Medium** Caleb’s older sister tells him not to stay up late the night before his game. He answers, “You stayed up till midnight before your piano recital!” Name the fallacy and explain what is wrong with Caleb’s reply.

▼ Show answer

Tu quoque. Even if his sister did the same thing, that does not make her advice wrong; staying up late will still tire Caleb before his game. Her inconsistency is a fault in her, not in the advice.

7. Medium Name the fallacy and give a one-sentence reason: “The idea of rotating crops came from a book written by a man who lived in the city, so it can’t work on a real farm.”

▼ **Show answer**

Genetic fallacy: it judges crop rotation by where the idea came from instead of by whether it works on a farm. Also accept circumstantial ad hominem (dismissing the author because of his circumstances); either way, the idea must be tested on its merits.

8. Medium At a spelling bee, the mother of one contestant is also serving as a judge, and the other families ask for a different judge. Is this an ad hominem fallacy? Explain.

▼ **Show answer**

No. The judge’s decisions rest on her fair word (testimony about what each speller said), so her interest in one contestant is relevant to whether she can be trusted to judge. Asking for an impartial judge is a fair question about reliability, not an attack on her argument.

9. Hard Explain in two or three sentences why it is fair to question a witness’s interest but unfair to reject an argument because of the arguer’s interest.

▼ **Show answer**

When we accept testimony, we rely on the witness’s honesty and knowledge, so anything that might bend the witness’s word, such as an interest in the outcome, is relevant. An argument gives reasons that can be checked on their own, so its strength does not depend on who offers it. The arguer’s interest is a reason to check the reasons carefully, not a reason to reject them.

10. Hard Before Ava gives her report on the class trip, Leo says to the group, “Remember, Ava always exaggerates everything.” Name what Leo is doing and explain why it is unfair.

▼ **Show answer**

Poisoning the well (a kind of ad hominem): he discredits Ava before she speaks, so that the group will distrust whatever she says, whether it is accurate or not. Accept “abusive ad hominem” as the family name. If Ava’s report is testimony, the listeners can fairly check her facts, but they should judge what she actually says rather than reject it in advance.

11. Hard A clerk at Peak Outfitters says a tent is waterproof. (a) Why is it reasonable for the customer to want more than the clerk’s word? (b) The clerk then sprays the tent with a hose for ten minutes and invites the customer to look inside; the floor is dry. The customer says, “You’d say anything to sell a tent.” What has the customer done wrong now?

▼ **Show answer**

(a) The clerk’s claim is testimony, and he stands to gain from the sale, so his interest is a fair reason to want independent evidence. (b) The hose test is evidence the customer can see for himself; it does not depend on the clerk’s honesty. Rejecting it because of the clerk’s job is a circumstantial ad hominem. (The customer could fairly ask whether ten minutes of spray is the same as a night of rain, but that answers the evidence, not the clerk.)

12. **Challenge** A coach who rarely stretches tells his players to stretch before every practice, and a player says, “Why should we? You never do.” (a) Write a courteous reply the coach could give. (b) Explain when a person’s failure to follow his own advice *is* relevant, and when it is not.

▼ **Show answer**

Sample answer: (a) “You’re right that I should stretch more, and I’ll try. But the reason for stretching is to prevent pulled muscles, and that is true for you whether I do it or not.” (b) It is relevant when the question is about the person: whether to trust his promises, follow him as an example, or believe his testimony about what he does. It is not relevant to whether the advice itself is true, which depends on the reasons for it. Accept any reply that is courteous and returns to the reasons, and any explanation that separates the person from the truth of the claim.

COMMON MISTAKES

- Calling every question about a person an ad hominem. When we rely on someone’s word, questions about honesty, knowledge and interest are fair.
- Thinking an insult is always a fallacy. An insult is rude, but it is an ad hominem only when it is offered as a reason to reject what someone says.
- Thinking hypocrisy refutes advice. A person who does not follow good advice can still be right about it.
- Treating a speaker’s interest as proof that he is wrong. Interest is a reason to check the claim, not a disproof of it.
- Answering an ad hominem with another one. The best reply returns calmly to the reasons.

> **Scripture.** “Can there any good thing come out of Nazareth? Philip saith unto him, Come and see.” (John 1:46, KJV)
>
> Nathanael judged by origin, and Philip answered without argument: look for yourself. The way to judge a claim is to examine it, not to dismiss it because of where it comes from.

3.3 Appeals to Authority, the Many and Tradition

OBJECTIVES

- Tell a legitimate appeal to authority from an illegitimate one by applying a set of tests.
- Spot the appeal to the many (ad populum), including the bandwagon and snob appeal.
- Spot the appeal to tradition and the appeal to novelty.
- Explain when what many people think, or what has long been done, is real evidence and when it is not.

KEY TERMS

Appeal to authority

Supporting a claim by the word of someone who is said to know. It is legitimate when the authority is a real expert speaking within his or her field; otherwise it is a fallacy, traditionally called *ad verecundiam*.

Ad verecundiam

Latin for "to modesty" or "to reverence": the fallacy of resting a claim on an authority that is not a real authority on the question.

Appeal to the many (ad populum)

Arguing that a claim is true, or a thing is good, because many people believe it or choose it. *Ad populum* is Latin for "to the people."

Bandwagon

The form of the appeal to the many that says, in effect, "Everyone is doing it, so you should too."

Snob appeal

The form of the appeal to the many that says, in effect, "The best people, the select few, choose this, so you should too."

Appeal to tradition

Arguing that something is right or good only because it is old or has always been done (sometimes called *ad antiquitatem*).

Appeal to novelty

Arguing that something is better only because it is new (sometimes called *ad novitatem*).

CORE IDEAS

We all depend on authorities. You have never measured the distance to the moon, tested the purity of your town's water, or read the original records of the Battle of Hastings. You accept these things on the word of people who have. That is not a weakness; it is how most human knowledge works. A student who refused to believe anything he had not checked himself would know very little. So an **appeal to authority** is not a fallacy in itself. The real question is always: *Is this a good authority on this question?*

Tests of a good authority. Ask five questions. (1) Is the person a real expert, with training or long experience in the field? (2) Is the person speaking within that field? (3) Do other experts in the field generally agree, or is the question still disputed among them? (4) Does the person have an interest that might bend his or her judgment, such as being paid to say it? (5) Is the authority named and quoted accurately, or only hinted at? When Dr. Lin, an eye doctor, examines Caleb and says he needs glasses, she passes every test. Her word is strong evidence, though even good experts can be mistaken.

Bad appeals to authority. The fallacy, traditionally called **ad verecundiam** ("to modesty," because it makes us feel too modest to question), fails one or more of the tests. A champion racecar driver says Greensward fertilizer grows the greenest lawns; he is an expert in driving, not in grass, and he is being paid. An advertisement says "Experts agree that Crunchos are the healthiest cereal," but which experts, and agree on what? A brilliant physicist offers an opinion about how to train a puppy; she is speaking outside her field. A respected astronomer declares that there must be life on other planets, but on an unsettled question, no one's say-so can settle it.

The highest authority. Christians accept many truths on the authority of God speaking in Scripture, and this is not a fallacy. A human authority can be mistaken or dishonest; God can be neither. That is why Thomas Aquinas could say that an argument from human authority is the weakest kind, while an argument from divine revelation is the strongest. Notice that this keeps the tests in place: the question is still whether the authority is real and is being quoted accurately. Reading a verse out of context, or claiming that Scripture says what it does not, is still a bad appeal.

The appeal to the many. “Millions of families can’t be wrong!” “Everybody at school has a Stridewell backpack.” The **appeal to the many**, or **ad populum** (“to the people”), argues that a claim is true or a thing is good because many people believe it or choose it. The **bandwagon** says, “Everyone is doing it; don’t be left out.” **Snob appeal** flatters instead: “Hartwell watches, for the few who demand the finest.” Both work on our wish to belong, not on our judgment. Long ago, most people believed that the sun went around the earth; their number did not make it so. Scripture warns, “Thou shalt not follow a multitude to do evil” (Exodus 23:2).

When numbers do count. Some questions are about what people think or like. If the question is “What is our class’s favorite lunch?”, a class vote is exactly the right evidence. The meaning of a word is set largely by how most speakers use it. And when many people independently try a product and report the same result, their experiences are real, though weak, inductive evidence. The fallacy is not counting heads; it is counting heads when the question is about something else, such as whether a claim is true or a product is safe or well made.

The appeal to tradition. “We have always planted potatoes on Good Friday, so that is the right time.” The **appeal to tradition** argues that something is right only because it is old. Age alone proves nothing: a practice can last for generations simply because nobody questioned it. Yet tradition deserves respect. A custom that has lasted often carries tested experience: Grandpa’s planting date may track the last frost in your valley very well. The fair approach is to treat tradition as a reason to look for the wisdom behind it, and then to weigh that wisdom on its merits. And some traditions, such as a family’s holiday meal, are valued partly because they are traditions, which is a perfectly good reason to keep them.

The appeal to novelty. The opposite mistake says that something is better simply because it is new. “The all-new Brightway lamp: this year’s model!” “That’s an old-fashioned idea.” The **appeal to novelty** treats the date as if it were the measure of quality. New things are sometimes better and sometimes worse; an old hammer may outlast a new one. Both the appeal to tradition and the appeal to novelty make the same error: they judge by when instead of by what. The right question is, “What are the reasons this is good or true?”

From history. In the first question of his *Summa Theologica*, Thomas Aquinas (about 1225–1274) asked whether sacred teaching may argue from authority. He answered, roughly, that an argument from human authority is the weakest of arguments, but an argument from the authority of God's revelation is the strongest, because it rests on God himself. Centuries later, John Locke (1690) gave the name *argumentum ad verecundiam* to the habit of pressing an opinion on others by citing the learned men who held it, so that anyone who disagreed would seem immodest. The Latin *verecundia* means modesty or shame.

Practice habit. When a claim leans on a name, a crowd or a date, ask: "Is this a real authority on this question, and what are the reasons themselves?"

WORKED EXAMPLES

Example 1. Apply the tests of a good authority. Is each appeal legitimate? (a) Dr. Lin, an eye doctor, examines Caleb and says he needs glasses. (b) Rex Malloy, a champion racecar driver, says in a paid advertisement that Greensward fertilizer grows the greenest lawns. (c) A cereal box says, "Experts agree: Crunchos are the healthiest breakfast."

▼ **Show solution**

1. Ask the five questions: real expert? speaking within the field? do experts agree? any bending interest? named and quoted accurately?
2. (a) Dr. Lin is a trained eye doctor, speaking about eyes, after examining the patient herself. Her judgment is the kind other eye doctors could check. Legitimate.
3. (b) Malloy is an expert at driving, not at lawn care, so he is outside his field; and he is paid. Illegitimate (ad verecundiam).
4. (c) The "experts" are not named, so we cannot check who they are, what field they are in, or what they actually said. "Healthiest" is vague as well. Illegitimate as it stands.

Answer: (a) Legitimate: a real expert, in her field, on a case she examined. (b) Illegitimate: an expert outside his field, and paid. (c) Illegitimate as stated: unnamed experts and a vague claim that cannot be checked.

Example 2. Fallacy or not? (a) "Twenty of the twenty-four students in our class voted for pizza, so pizza is our class's favorite lunch." (b) "Most people believe that a goldfish remembers things for only three seconds, so it must be true." (c) "Grandpa has always planted potatoes on Good Friday, so that is the right time to plant them."

▼ **Show solution**

1. Ask what the question is about. If it is about what people think or like, counting people is the right evidence. If it is about something else, it is not.
2. (a) The question is about the class's preference, and a vote measures exactly that. Not a fallacy.
3. (b) The question is about goldfish memory, which is settled by testing goldfish, not by counting believers (and tests show goldfish remember far longer). Appeal to the many.
4. (c) "Always done" is not by itself a reason. Grandpa's date may carry real experience about frost in the area, so the fair move is to look for that reason (for example, the usual last frost date) and weigh it. As stated, it is an appeal to tradition.

Answer: (a) Not a fallacy: the question is about the class's preference. (b) Appeal to the many (ad populum). (c) Appeal to tradition as stated, though the tradition may contain wisdom worth checking.

PRACTICE

1. Easy Is every appeal to authority a fallacy? Explain in one or two sentences.

▼ **Show answer**

No. Relying on a real expert speaking within his or her field is reasonable, and most of what we know comes this way. It becomes a fallacy (ad verecundiam) when the authority is not a real authority on the question.

2. **Easy** List four of the five tests of a good authority.

   ▼ **Show answer**

   Any four: (1) a real expert with training or experience; (2) speaking within his or her field; (3) other experts generally agree (the question is not still disputed); (4) no interest that might bend the judgment, such as being paid; (5) named and quoted accurately.

3. **Easy** What is the difference between the bandwagon and snob appeal?

   ▼ **Show answer**

   Both are appeals to the many. The bandwagon says everyone is doing it, so you should too; snob appeal says the best or select few choose it, so you should too. One plays on the wish to fit in, the other on the wish to be special.

4. **Easy** Name the fallacy and give a one-sentence reason: "The Brightway 9 is this year's newest lamp, so it must be better than the one you have."

   ▼ **Show answer**

   Appeal to novelty: being new does not by itself make the lamp better; it must be judged by its features and quality.

5. **Medium** Is this a good appeal to authority? Explain. "Tanner Cole, three-time national swimming champion, says Vexo batteries last longer than any other brand."

   ▼ **Show answer**

   No (ad verecundiam). Cole is an expert in swimming, not in batteries, so he is speaking outside his field; he is probably also paid for the advertisement. Battery life is settled by testing batteries.

6. **Medium** Is this a good appeal to authority? Explain. "My uncle has been a plumber for thirty years, and after looking under the sink he says the leak is coming from a cracked joint."

   ▼ **Show answer**

   Yes. The uncle has long experience in the field, he is speaking within it, and he examined the leak himself. His judgment is good (inductive) evidence, though it could still be checked.

7. Medium Name the fallacy and give a one-sentence reason: “Everybody on our street uses Tidewell lawn service, so you should too.”

▼ **Show answer**

Appeal to the many (bandwagon): the number of neighbors using Tidewell does not show that it is the best service or the right one for you. (Accept a note that many satisfied neighbors are weak evidence the service is acceptable, but not proof it is best.)

8. Medium Name the fallacy and give a one-sentence reason: “Hartwell watches: chosen by those who will accept nothing less than the finest.”

▼ **Show answer**

Snob appeal (a form of the appeal to the many): it flatters the buyer’s wish to belong to a select group and gives no evidence about the watch’s quality.

9. Hard “Dr. Reed, a famous astronomer, says that there must be living creatures on other planets, so there are.” Which test of a good authority does this fail? Explain.

▼ **Show answer**

It fails the test of agreement among experts: this is an open question that no one has settled by observation, and astronomers disagree about it, so no single expert’s word can settle it. Accept also that it claims more than any expert can know, or that it may go beyond his field (astronomy observes planets but has not found life). The argument treats an opinion on an unsettled question as if it were expert knowledge.

10. Hard Mom says, “Our family has eaten turkey at Thanksgiving for as long as anyone can remember, so we’ll have turkey again this year.” Is this a fallacious appeal to tradition? Explain.

▼ **Show answer**

Not necessarily. For a family custom, the fact that it is a tradition is itself a fair reason to keep it, because part of the custom’s value is its continuity. It would be an appeal to tradition if it were offered as proof of something tradition cannot show, such as “turkey is the healthiest meat” or “turkey is the only proper holiday food.” Accept either answer if the student explains the difference.

11. Hard Jonah tells his mother, “Dr. Ames, the best dentist in town, says whitening strips are completely harmless, so I can use them every day.” What Dr. Ames actually said was that the strips are safe when used as directed, for no more than two weeks at a time. Which test of a good authority does Jonah’s appeal fail, and why does it matter?

▼ **Show answer**

It fails the test of being quoted accurately. Dr. Ames is a real expert speaking within his field, but Jonah has stretched what he said into a claim he did not make (harmless every day). An appeal to authority is only as good as the report of what the authority said, so Jonah’s conclusion has no expert support.

12. Challenge Write a short advertisement for a made-up product that commits three fallacies from this lesson (label each). Then rewrite one of the claims so that it gives a legitimate reason.

▼ **Show answer**

Sample answer: “Glimmer toothpaste, NEW for this year (novelty)! Famous figure skater Lila Brandt uses it every day (authority outside her field). Join the millions who have switched (bandwagon)!” Legitimate rewrite: “In a test run by the Hollis Dental School, patients who brushed with Glimmer for six months had fewer cavities than those who brushed with a plain paste.” Accept any made-up product with three correctly labeled fallacies and a rewrite that gives checkable evidence or a real expert within the field.

COMMON MISTAKES

- Thinking every appeal to authority is a fallacy. Trusting a real expert within his or her field is reasonable.
- Accepting a famous person as an expert on everything. Fame or skill in one field gives no authority in another.
- Thinking that popularity never matters. When the question is about what people prefer or how they use a word, counting people is the right evidence.
- Treating tradition as either always right or always wrong. A lasting custom deserves a hearing, and its reasons should be weighed.
- Assuming new means better, or old means wiser. Judge by the reasons, not by the date.

> **Scripture.** "Thou shalt not follow a multitude to do evil" (Exodus 23:2, KJV)
>
> The law of Moses warned judges and witnesses not to bend the truth to go along with the crowd. What is right and true does not depend on how many people are on its side.

UNIT 3 · INFORMAL FALLACIES · LESSON 3.4 OF 69 · 40 MINUTES

3.4 Appeals to Emotion and Force; Red Herring and Straw Man

OBJECTIVES

- Spot appeals to pity, fear and force, and tell them from fair appeals to feelings that rest on true, relevant facts.
- Spot the red herring, which changes the subject.
- Spot the straw man, which attacks a distorted version of the other side.
- Answer each of these fallacies by returning to the real question or the real position.

KEY TERMS

Appeal to pity (ad misericordiam)

Trying to win agreement by stirring up sympathy when the sympathy has no bearing on whether the conclusion is true.

Appeal to fear (ad metum)

Trying to win agreement by stirring up fear, without good evidence that the danger is real or that the conclusion would remove it.

Appeal to force (ad baculum)

Latin for "to the stick": using a threat, open or hidden, in place of reasons to get someone to accept a conclusion.

Red herring

Changing the subject to something else, so that the real question is dropped without being answered.

Straw man

Replacing an opponent's real position with a weaker or more extreme version of it, then refuting that version instead.

Irrelevant conclusion (ignoratio elenchi)

Aristotle's name for arguing for something other than the point at issue; the red herring and the straw man are both kinds of it.

Feelings are not the enemy of reason. God made us to feel as well as to think. Pity, fear and indignation are fitting responses to real things: it is right to feel pity for a family whose barn has burned and fear at the sight of thin ice. A **fair appeal to emotion** presents true, relevant facts that rightly move us toward a conclusion those facts support. An appeal to emotion becomes a fallacy when feeling is used in place of reasons, to push a conclusion that the feeling has nothing to do with. The test is the same as for every fallacy of relevance: even if I feel this, does the feeling give me a reason to believe that?

The appeal to pity. "Please give me an A on this essay. I worked on it all weekend, and if my grade drops, I can't play in the tournament." The **appeal to pity**, or **ad misericordiam**, asks for agreement on the grounds of sympathy. But a grade is supposed to measure the quality of the essay, and the student's weekend and tournament do not change what is on the page. Compare a neighbor who asks for help rebuilding a burned barn. There pity is exactly relevant, because the question is whether to help someone in need. The difference is not whether the appeal moves us, but whether the feeling bears on the question.

The appeal to fear. "Without a Nightwarden alarm, your family is in danger every single night!" The **appeal to fear** (**ad metum**) paints a frightening picture instead of giving evidence: no numbers, no comparison, no reason to think this alarm would help more than any other. A fair warning is different. When a forest ranger says, "Stay off the pond; I drilled the ice this morning and it is only an inch thick," fear is the right response to a real, measured danger. Ask two questions: Is there evidence that the danger is real? And would the recommended action actually reduce it?

The appeal to force. "Agree that my plan is the best, or you're not in the club." The **appeal to force**, **ad baculum** ("to the stick"), replaces reasons with a threat. A threat may give someone a reason to keep quiet or to go along, but it gives no reason at all to *believe* that the plan is best. Be careful not to confuse this with a rule that has a stated consequence. When Dad says, "If you leave the gate open again, you lose your bike for a week," he is not arguing that some claim is true; he is setting a rule, which is a parent's rightful job. The fallacy is using a threat as if it were evidence.

The red herring: changing the subject. Mom asks Caleb why the dog has not been fed. Caleb answers, "Well, Sarah never took out the trash." Sarah's chores may be

worth discussing, but they are not the question, and if Mom follows Caleb's lead, the dog stays hungry. A **red herring** drags in a different subject so that the real question is dropped. The name comes from a strong-smelling smoked fish that, according to an old story, could be dragged across a trail to throw hounds off the scent. Red herrings often work because the new subject is interesting or emotional. The answer is simple: "That may be worth talking about later. But back to the question: why hasn't the dog been fed?"

The straw man: attacking a distortion. Hannah says, "I think we should have fewer hours of screen time on school nights." Her brother replies, "Hannah wants to throw every computer out of the house and go back to candles." He has replaced her real position with a ridiculous one that is easy to knock down. This is the **straw man**: like a figure stuffed with straw, it looks like an opponent but cannot fight back. A straw man may exaggerate ("fewer" becomes "none"), leave out key qualifications, or pick the weakest thing someone said and ignore the rest. Proverbs warns against exactly this habit: "He that answereth a matter before he heareth it, it is folly and shame unto him" (Proverbs 18:13).

Two ways to miss the point. Aristotle grouped these mistakes under one heading, later called in Latin **ignoratio elenchi**, the "ignorance of refutation" or **irrelevant conclusion**: proving something other than what is at issue. The red herring moves to a different question. The straw man keeps the topic but changes the position. Both can be caught by asking, "What exactly is the question, and what exactly did the other person say?" A good test for a straw man is whether the other person would agree that you have stated his or her view fairly.

Answering fairly, and arguing fairly. When someone uses a red herring on you, name the real question politely and return to it. When someone attacks a straw man, restate your actual view: "That's not quite what I said. I said *fewer* hours, not none." When someone presses with pity or fear, ask what the facts are. And guard your own arguments. Before you answer someone, restate their position in words they would accept. If you cannot do that, you are not yet ready to answer. This is simply the Golden Rule applied to arguing: treat other people's views as you would want your own to be treated.

From history. The phrase "red herring" became popular through a story by the English writer William Cobbett in 1807. He wrote that as a boy he had dragged a red herring, a herring smoked until it turns reddish and smells strongly, across a hare's trail to draw the hunting hounds away. Cobbett used the story to scold newspapers that had been led off the important questions of the day by a false report. Much earlier, Aristotle had described the general fault in his *Sophistical Refutations*: an argument that seems to refute an opponent but actually proves a different point. Medieval writers called it *ignoratio elenchi*.

Practice habit. Before you answer someone who disagrees with you, restate his or her view in a sentence that person would accept as fair.

WORKED EXAMPLES

Example 1. Is each a fallacy? If so, name it. (a) Jacob asks the coach to start him in Saturday's game because his grandparents are driving four hours to watch. (b) A tree service tells Mr. Becker, "That dead oak leans over your roof. Our arborist found a deep crack in the trunk, and it could fall in the next big storm." (c) An older boy tells his cousin, "Say my drawing is the best one, or you can't play with my trains."

▼ **Show solution**

1. For each, ask: what is the question, and does the feeling or threat bear on it?
2. (a) The question is who should start, which depends on skill, effort and the team's needs. The grandparents' trip is touching, but it does not bear on that. Appeal to pity.
3. (b) The fear is tied to evidence (a dead, leaning tree with a deep crack) and to a real action (removing it) that would reduce the danger. The tree service has an interest, so Mr. Becker might ask for a second look, but the warning itself is fair.
4. (c) The threat gives the cousin a reason to *say* the drawing is best, but no reason to believe it. Appeal to force.

Answer: (a) Appeal to pity. (b) Not a fallacy: a fair appeal to fear, resting on evidence of real danger. (c) Appeal to force (ad baculum).

Example 2. Caleb says to the team, "We should practice free throws for twenty minutes every day." Name the fallacy in each reply. (a) Jonah: "So you think games don't matter and we should just shoot free throws all season?" (b) Ella: "Speaking of practice, did you see the new scoreboard they put in the gym?"

▼ **Show solution**

1. Caleb's actual position: twenty minutes of free-throw practice each day. That is all he claimed.
2. (a) Jonah replaces Caleb's modest proposal with an extreme one (only free throws, games don't matter) that Caleb never said, and then mocks it. Caleb would not agree that this is his view. Straw man.
3. (b) Ella keeps the conversation on "practice" for a moment but moves to a different subject (the scoreboard), and Caleb's proposal is dropped. Red herring.

Answer: (a) Straw man: it distorts Caleb's position into something extreme. (b) Red herring: it changes the subject and drops the question.

PRACTICE

1. Easy Match each Latin name with its fallacy: *ad misericordiam*, *ad baculum*, *ad metum*. (Choices: appeal to fear, appeal to force, appeal to pity.)

   ▼ **Show answer**

   Ad misericordiam: appeal to pity. *Ad baculum*: appeal to force ("to the stick"). *Ad metum*: appeal to fear.

2. Easy Define the red herring in one sentence.

   ▼ **Show answer**

   A red herring changes the subject to something else so that the real question is dropped without being answered.

3. Easy Define the straw man in one sentence.

   ▼ **Show answer**

   A straw man replaces an opponent's real position with a weaker or more extreme version of it and then refutes that version instead.

4. **Easy** Is every appeal to emotion a fallacy? Explain in one or two sentences.

   ▼ **Show answer**

   No. An appeal to emotion is fair when it presents true, relevant facts that rightly move us toward a conclusion the facts support (such as pity moving us to help someone in real need). It is a fallacy when the feeling is used in place of reasons for a conclusion it does not bear on.

5. **Medium** Name the fallacy and give a one-sentence reason: "Please give my science project first prize. I stayed up all night working on it, and I'll be heartbroken if I lose."

   ▼ **Show answer**

   Appeal to pity: the prize should go to the best project, and the student's lost sleep and disappointment do not show that the project is the best.

6. **Medium** Name the fallacy and give a one-sentence reason: "Buy a Fortwell smoke detector today, or your house could burn down tonight with your family inside!"

   ▼ **Show answer**

   Appeal to fear: it uses a frightening picture instead of evidence that Fortwell detectors work well or better than others. Accept the note that owning a working smoke detector is genuinely wise; the fallacy is pressing fear in place of reasons to buy this brand today.

7. **Medium** Mom asks Ben why the chickens were not let out this morning. Ben answers, "Well, Sarah never put the bikes away last night." Name the fallacy and say what Mom should do.

   ▼ **Show answer**

   Red herring: Ben changes the subject to Sarah's chores, and the question about the chickens is dropped. Mom should return to the real question ("We can talk about the bikes later; why weren't the chickens let out?"). Accept also the note that pointing to someone else's failure is like a tu quoque, but the main fault is that the question is dropped.

8. Medium Coach Reyes says, “This old basketball has lost its grip; we should buy a new one for games.” A player answers, “So you think everything old is useless? Should we tear down the gym, too?” Name the fallacy and give a one-sentence reason.

▼ **Show answer**

Straw man: the coach said only that this worn ball should be replaced, and the player distorts that into the extreme claim that everything old is useless.

9. Hard Dad says, “If you leave the gate open again and the goats get out, you will lose your bike for a week.” Is this an appeal to force? Explain.

▼ **Show answer**

No. Dad is not using a threat as evidence that some claim is true; he is setting a rule with a stated consequence, which a parent has the right to do. It would be an appeal to force if a threat were used to make someone believe a claim, for example, “Agree that leaving the gate open is harmless, or you’ll lose your bike.”

10. Hard A store manager tells the workers, “Anyone who doesn’t agree that the new schedule is the best one can start looking for another job.” Name the fallacy and explain why the threat proves nothing about the schedule.

▼ **Show answer**

Appeal to force (ad baculum). The threat may give the workers a reason to stay quiet or go along, but it gives no evidence at all about whether the schedule is the best one; that depends on things like hours, fairness and customer needs.

11. Hard Lucy says, “Our chess club should meet every other week instead of every week, so that members have more time for homework.” Write (a) a straw-man reply to Lucy and (b) a fair reply that disagrees with her.

▼ **Show answer**

Sample answer: (a) Straw man: “Lucy doesn’t care about chess at all; she wants to shut the club down.” (It turns “meet less often” into “shut it down.”) (b) Fair reply: “Lucy is right that homework matters, but meeting every other week would make it hard to keep our tournament practice going. Maybe we could shorten each weekly meeting instead.” Accept any (a) that distorts or exaggerates her actual position and any (b) that states her view accurately before disagreeing.

12. Challenge Compare these two appeals. Which is a fallacy, and why? (a) “Since the flood, forty families in Millbrook have been sleeping in the church gym. A gift of twenty dollars buys one family a blanket and a hot meal.” (b) “The Millbrook flood was heartbreaking. So buy Millbrook Mugs: they are the finest mugs made.”

▼ **Show answer**

(b) is the fallacy (appeal to pity). In (a), the facts are specific and the pity they stir bears directly on the conclusion, which is to give help to people in need; that is a fair appeal to emotion. In (b), pity for the flood has nothing to do with whether the mugs are the finest; the feeling is used in place of evidence about quality. (If (b) said that profits go to flood relief, that would be a fair reason to buy, though still not evidence that the mugs are the finest.)

COMMON MISTAKES

- Calling every emotional appeal a fallacy. Feelings that rest on true, relevant facts are a fair part of a good argument.
- Confusing a stated rule with an appeal to force. A parent or coach setting consequences is not arguing that a claim is true.
- Missing a red herring because the new subject is interesting. Keep track of the question that was actually asked.
- Answering what you think someone said instead of what he or she actually said. Restate the other view in words the other person would accept.

- Fighting a straw man with another straw man. Correct the distortion calmly and restate your real position.

> **Scripture.** "He that answereth a matter before he heareth it, it is folly and shame unto him." (Proverbs 18:13, KJV)
>
> The straw man answers before it has heard. Listening carefully, until you can state the other side fairly, is both good logic and a matter of wisdom and humility.

UNIT 3 · INFORMAL FALLACIES · LESSON 3.5 OF 69 · 40 MINUTES

3.5 Presumption I: Begging the Question, Complex Question, False Dilemma

OBJECTIVES

- Spot begging the question (circular reasoning and question-begging words) and explain why such an argument proves nothing even when it is valid.
- Spot the complex question and answer it by dividing it.
- Spot the false dilemma, and tell a false dilemma from a true one by asking whether the options are really the only ones.
- Explain why all three are fallacies of presumption.

KEY TERMS

Begging the question (petitio principii)

Assuming in the premises the very conclusion that is supposed to be proved. The Latin means "asking for the starting point."

Circular reasoning

A form of begging the question in which the conclusion, or something that depends on it, is offered as a premise for itself.

Question-begging word

A loaded word that assumes the point at issue, such as calling a rule "useless" while arguing that it should be dropped.

Complex question

A question that hides an unproved assumption, so that any direct answer seems to admit it (Aristotle's "many questions as one").

False dilemma

Presenting two options as the only possible ones when there are others (also called the either-or fallacy or false dichotomy).

Contradictories

Two claims that cannot both be true and cannot both be false, such as "the light is on" and "the light is not on." A choice between contradictories is a true dilemma.

CORE IDEAS

Arguments that take too much for granted. Every argument starts somewhere; it has to assume its premises. That is fine when the premises are true and the listener has good reason to accept them. A **fallacy of presumption** starts from something it has not earned: an assumption that is doubtful, unproved, or simply the conclusion itself in disguise. The reasons in such arguments are often relevant, and the arguments may even be valid. The trouble is that they quietly ask the listener to grant what is really in question. This lesson studies three of them.

Begging the question. "Thunderbolt sneakers are the fastest shoes because no shoe is faster than a Thunderbolt." The premise and the conclusion say the same thing in different words. This is **begging the question**, in Latin **petitio principii**, "asking for the starting point": the arguer asks you to grant at the start the very thing he is supposed to prove. Often the circle is wider. "Mr. Grant is honest. How do I know? He told me he never lies." Believing Mr. Grant's statement requires already trusting his honesty, which is the point in question. This is **circular reasoning**: the conclusion is propped up by something that leans on the conclusion.

Valid, but useless. Here is something surprising. "Thunderbolts are the fastest; therefore Thunderbolts are the fastest" is *valid*: if the premise is true, the conclusion cannot be false. So what is wrong? An argument is meant to give someone who doubts the conclusion a reason to accept it. Anyone who doubts the conclusion of a circular argument has exactly the same reason to doubt its premise, so the argument moves him not one step. Begging the question fails not in its form but in its purpose. (In everyday speech, people often say "that begs the question" when they mean "that raises the question." In logic the phrase keeps its old meaning.)

Question-begging words. Sometimes a single word does the begging. "We should drop this useless homework assignment." Whether the assignment is useless is the

very thing in dispute, and the word "useless" assumes it. "No sensible family would buy a minivan" assumes that buying one is not sensible. A **question-begging word** slips the conclusion into the description. When you notice a strong adjective doing all the work, ask, "Has that been shown, or only said?"

The complex question. "Have you stopped leaving your bike in the driveway?" If you answer "yes," you admit you used to; if you answer "no," you admit you still do. A **complex question** hides an assumption inside a question, so that any direct answer seems to grant it. Aristotle called this asking "many questions as one." Others are less obvious: "Why did you break the vase?" "Where did you hide the cookies?" "How long have you been cheating at checkers?" The fair answer is to *divide the question*: "I didn't break the vase, so I can't tell you why." Note that a question like this is perfectly fair when its assumption has already been established. If Mom watched you knock the vase over, "Why did you do it?" is an honest question.

The false dilemma. "Either we cancel the picnic or everybody gets soaked." A **false dilemma** presents two options as the only ones when there are others: move the picnic to the pavilion, start earlier, or bring tarps. It is also called the either-or fallacy or false dichotomy. It often rides on a pair of extremes: "Either you practice three hours a day or you will never play well." "You either love baseball or you hate it." Most of life is lived between the extremes, and most choices have more than two options.

True dilemmas and false ones. Not every either-or is a fallacy. "Either the light is on or it is not on" covers every possibility, because the two options are **contradictories**: one of them must be true. "Either the light is bright or it is dim" does not, because the light could be off, or somewhere in between; bright and dim are contraries, which cannot both be true but can both be false. Sometimes, too, the situation really does leave only two choices worth weighing: if the island ferry runs once a day, we either take it this morning or wait until tomorrow. To test an either-or, ask: Could both options be false? Is there a third way?

A trap refused. When some of Jesus' opponents wanted to trap him, they asked whether it was lawful to give tribute to Caesar. They assumed that any answer would ruin him: "yes" would seem to betray God, and "no" would seem to defy Rome. Jesus asked for a coin, pointed to Caesar's image on it, and answered: "Render therefore unto Caesar the things which are Caesar's; and unto God the things that are God's"

(Matthew 22:21). He refused the hidden assumption that honoring the one ruled out honoring the other. When you meet a loaded question or a forced choice, you too may decline the assumption and say what is actually true.

> **From history.** Aristotle discussed begging the question in his *Prior Analytics* and *Topics*. His Greek phrase meant "asking for the thing at the beginning," that is, asking the other side to grant the original point in dispute. Medieval writers translated it into Latin as *petitio principii*, "asking for the starting point," and the English phrase "begging the question" comes from that Latin. In the *Sophistical Refutations*, Aristotle also described the trick of combining several questions into one, so that a single answer seems to concede them all; later logicians called it the fallacy of many questions, or the complex question.

> **Practice habit.** When you hear "either … or …," ask yourself, "Could both of these be false? Is there a third way?"

Example 1. Which of these arguments beg the question? (a) “Brookside honey is the purest honey because no honey is more pure than Brookside’s.” (b) “Mr. Grant is honest, because he told me himself that he never lies.” (c) “Our dog must have dug the hole by the fence, because there is fresh dirt on his paws and nose.”

▼ **Show solution**

1. For each, ask: would a person who doubts the conclusion have any independent reason to accept the premise?
2. (a) “No honey is more pure than Brookside’s” is just another way of saying “Brookside’s is the purest.” Someone who doubts the conclusion doubts the premise equally. Begs the question.
3. (b) The premise is Mr. Grant’s own word, which we can trust only if he is already known to be honest. The argument leans on its own conclusion. Begs the question (circular).
4. (c) The dirt on the dog is evidence we can see for ourselves, apart from the conclusion. It is a reasonable inductive argument, not circular, though another animal could have dug the hole.

Answer: (a) and (b) beg the question; (c) does not. It gives independent evidence.

Example 2. Name the fallacy and give a fair answer or repair. (a) "Why do you always forget your lunch?" (asked of a student who forgot it once). (b) "Either we cancel the picnic or everybody gets soaked."

▼ **Show solution**

1. (a) The question assumes that the student "always" forgets, which has not been established. Any answer to "why" would grant it. This is a complex question. Divide it: "I forgot it today, but I don't always forget it. Today I was rushing to catch the bus."
2. (b) The speaker offers two options as if they were the only ones. Could both be false? Yes: the picnic could move to the pavilion, start earlier, or go on with tarps and umbrellas. This is a false dilemma. Repair: "If it rains, we'll need to cancel, move under cover or bring tarps. Which is best?"

Answer: (a) Complex question; answer by dividing it and denying the hidden assumption. (b) False dilemma; repair by naming the other options.

PRACTICE

1. Easy Define begging the question and give its Latin name.

   ▼ **Show answer**

   Begging the question is assuming in the premises the very conclusion that is supposed to be proved. Its Latin name is *petitio principii* ("asking for the starting point").

2. Easy What is a complex question, and what is the fair way to answer one?

   ▼ **Show answer**

   A complex question hides an unproved assumption, so that any direct answer seems to admit it (for example, "Have you stopped ...?"). The fair way to answer is to divide the question: deny or address the hidden assumption first, then answer what is left.

3. Easy Define the false dilemma.

   ▼ **Show answer**

   A false dilemma presents two options as the only possible ones when there are others. It is also called the either-or fallacy or false dichotomy.

4. **Easy** In everyday speech people say, "That begs the question of where we will park." Is this the same as the fallacy of begging the question? Explain.

   ▼ **Show answer**

   No. In everyday speech it means "that raises the question." In logic, begging the question means assuming the conclusion in the premises.

5. **Medium** Name the fallacy and give a one-sentence reason: "The Kestrel is the most reliable truck on the road, because no truck on the road is as reliable as a Kestrel."

   ▼ **Show answer**

   Begging the question: the premise only restates the conclusion in other words, so it gives no independent reason.

6. **Medium** Name the fallacy and write a fair answer: "Did you ever fix that sloppy handwriting of yours?"

   ▼ **Show answer**

   Complex question: it assumes that the person's handwriting was sloppy. A fair answer divides it: "I don't think my handwriting was sloppy, but I have been working on writing more neatly." Accept any answer that addresses the hidden assumption before answering. (Also accept noting the question-begging word "sloppy.")

7. **Medium** Name the fallacy and list two options the speaker left out: "Either you practice the piano three hours a day or you will never play well."

   ▼ **Show answer**

   False dilemma. Left-out options include practicing thirty minutes or an hour a day, practicing carefully with a teacher, or practicing more on weekends. Accept any two reasonable options between the extremes.

8. **Medium** Which word begs the question, and why? "The school should get rid of this pointless dress code."

   ▼ **Show answer**

   "Pointless." Whether the dress code has a point is exactly what is in dispute when we ask whether to get rid of it, so the word assumes the conclusion instead of proving it.

9. Hard Is either of these a false dilemma? Explain. (a) “Either the porch light is on or it is not on.” (b) “Either the porch light is bright or it is dim.”

▼ **Show answer**

(a) is not a false dilemma: “on” and “not on” are contradictories, so one of them must be true. (b) is a false dilemma: bright and dim are contraries, which can both be false, because the light could be off or in between.

10. Hard “Pinewood soap is the gentlest soap; therefore Pinewood soap is the gentlest soap.” Is this argument valid? Explain why it still fails as an argument.

▼ **Show answer**

It is valid: if the premise is true, the conclusion cannot be false, because they are the same claim. It still fails because it begs the question: anyone who doubts the conclusion has exactly the same reason to doubt the premise, so it gives no reason to believe anything new.

11. Hard Mr. Becker watched his son leave the barn door open, and the calves got out. He asks, “Why did you leave the barn door open?” Is this a complex question? Would your answer change if Mr. Becker had not seen who left it open? Explain.

▼ **Show answer**

Not a fallacy as asked: the assumption (the son left the door open) has been established, because Mr. Becker saw it, so the question is honest. If he had not seen who did it, the same question would be a complex question, because it would assume without proof that the son did it; the son could fairly answer by dividing it (“I didn’t leave it open”).

12. **Challenge** Aunt Ruth says to Ben: “Are you still wasting your allowance on comic books? Either you start saving every penny or you’ll be broke all your life. Saving everything is wise because it’s the smart thing to do.” Name the three fallacies in her speech, and write a courteous reply that answers each.

▼ **Show answer**

(1) Complex question: “Are you still wasting …?” assumes that buying comics is a waste and that he has been doing it. (2) False dilemma: saving every penny and being broke for life are not the only options; he could save part of his allowance and spend part. (3) Begging the question: “Saving everything is wise because it’s the smart thing to do” only restates the conclusion. Sample reply: “I do buy some comic books, though I don’t think of it as wasting. I’m saving half of my allowance and spending the rest, which is a third choice. Could you tell me why you think saving all of it is better?” Accept any courteous reply that addresses each fallacy. (Accept also noting the question-begging word “wasting.”)

COMMON MISTAKES

- Thinking a circular argument must be invalid. It is usually valid; it fails because it gives no independent reason.
- Answering a complex question with a plain “yes” or “no.” Divide the question and deal with its hidden assumption first.
- Calling every either-or a false dilemma. A choice between contradictories, or a situation that really allows only two options, is a true dilemma.
- Missing the single loaded word that begs the question. Watch for strong adjectives that assume the point in dispute.

> **Scripture.** “Render therefore unto Caesar the things which are Caesar’s; and unto God the things that are God’s.” (Matthew 22:21, KJV)
>
> Jesus’ questioners offered what they thought was a no-win choice. He answered by refusing their hidden assumption, a model for answering a loaded question with truth instead of being trapped by it.

3.6 Presumption II: Generalizing Badly

OBJECTIVES

- Spot hasty generalization from too few or unrepresentative cases.
- Spot the fallacy of accident, which applies a general rule to a case it was not meant to cover.
- Spot the slippery slope, and tell it from a chain of consequences that is actually likely.
- Spot false cause (including post hoc and mistaking things that go together for cause and effect), and say what evidence would repair each of these arguments.

KEY TERMS

Hasty generalization

Drawing a general conclusion from too few cases, or from cases that are not representative of the whole group (older books also call it converse accident).

Accident

Applying a general rule to a special case that the rule was not meant to cover, because of some unusual feature of that case.

Slippery slope

Claiming that one step will lead through a chain of further steps to some bad result, without good evidence that each link in the chain is likely.

False cause (non causa pro causa)

Treating something as the cause of an event when there is not good evidence that it is. The Latin means "a non-cause for a cause."

Post hoc ergo propter hoc

Latin for "after this, therefore because of this": concluding that A caused B only because B came after A.

Common cause

A third thing that causes two others, so that they go together even though neither causes the other.

CORE IDEAS

Generalizing is necessary. We could not live without general conclusions. A farmer learns that frost kills tomato plants; a cook learns that bread dough rises in a warm kitchen; a child learns that the stove is hot. Each lesson comes from particular cases and is applied to new ones. Unit 7 will study good inductive reasoning in detail. This

lesson studies four common ways of generalizing badly: drawing a rule from too little, applying a rule where it does not fit, predicting a slide that is not likely, and finding a cause that is not there.

Hasty generalization. "I bought two Brightway light bulbs, and both burned out in a week. Brightway bulbs are junk." Two bulbs out of the many thousands Brightway makes are far too few to judge them all; perhaps these came from one bad box, or the lamp itself is faulty. A **hasty generalization** draws a general conclusion from too few cases or from cases that do not fairly represent the group. A survey of the chess club cannot tell you what the whole school likes to do on weekends, however many club members you ask. The repair is more cases, and more varied ones, chosen fairly from the whole group.

Accident: a rule where it does not fit. The library rule says, "Quiet at all times." Should a student who sees smoke pouring from a vent therefore keep quiet? Obviously not. The rule was made for ordinary times, and a fire is not an ordinary time. The fallacy of **accident** applies a general rule to a special case that it was not meant to cover, ignoring some unusual feature (the "accident") of that case. Most good rules hold generally, not without exception: "No running in the halls" does not forbid the school nurse from running to a student who has fainted. Hasty generalization and accident are mirror images. The first jumps from a few special cases to a rule; the second jumps from a rule to a special case.

The slippery slope. "If we let Sam stay up fifteen minutes later tonight, next he will want an hour, then he will stay up every night, then he will fall asleep in class and fail the year." A **slippery slope** argument claims that one small step will lead, link by link, to disaster. The fallacy is not in describing a chain of consequences; it is in giving no evidence that each link is likely. Parents can say no to the second request. Real chains exist, though. "If you leave the hose running into the garden all night, the bed will flood and the seedlings will rot" describes steps that each follow very likely from the one before. The test is to check each link: how likely is *this* step to lead to the next one?

False cause: after, therefore because. "I wore my lucky socks, and we won. The socks did it." The fallacy of **false cause** (in Latin **non causa pro causa**, "a non-cause for a cause") treats something as a cause without good evidence. Its most common form is **post hoc ergo propter hoc**, "after this, therefore because of this": B came after A, so

A caused B. But many things come after many other things. The rooster crows before sunrise every morning, yet no one thinks the rooster brings up the sun. Coming first is needed for a cause, but it is not enough to prove one.

Things that go together. Another form of false cause notices that two things go together and concludes that one causes the other. “On days when the town sells the most ice cream, the most people get sunburned. Ice cream must cause sunburn.” In fact a **common cause**, hot sunny weather, brings both. Sometimes the cause runs the other way: “Players who drink the most water sweat the most, so water makes you sweat,” when sweating makes you thirsty. And sometimes two things line up by sheer coincidence. To show a cause, we need comparisons that rule out the alternatives, such as testing the lucky socks over many games, on days with and without them.

A false cause in Scripture. When Jesus’ disciples saw a man blind from birth, they asked him (John 9:2), “Master, who did sin, this man, or his parents, that he was born blind?” Their question assumed that the blindness must have been caused by someone’s sin, and it offered only two choices. Jesus rejected both assumptions: “Neither hath this man sinned, nor his parents: but that the works of God should be made manifest in him” (John 9:3). The disciples had generalized from the truth that sin brings suffering to the false rule that every suffering is caused by a particular sin.

Repairing a bad generalization. Each of these fallacies points to the evidence that would fix it. For a hasty generalization, gather more cases and make sure they represent the whole group. For accident, ask what the rule is for and whether this case has some unusual feature the rulemaker did not intend to cover. For a slippery slope, ask for evidence that each link is likely. For false cause, look for a fair comparison and for other explanations: a common cause, a reversed cause or coincidence. Careful reasoning about causes is slower, but it is how farmers, doctors and engineers learn what actually works.

From history. Francis Bacon, in his *Novum Organum* (1620), warned that the mind notices the cases that agree with its beliefs and neglects those that do not. He told an old story to make the point. A man was shown a temple wall hung with pictures of sailors who had made vows to the gods and then escaped shipwreck, and he was asked whether this did not prove the power of the gods. He replied, roughly, "But where are the pictures of those who made their vows and drowned anyway?" Bacon said that most superstitions work this way: people remember the hits and forget the misses. The story is a warning against hasty generalization and false cause alike.

Practice habit. When someone says "this caused that," ask: "Could something else have caused both, or could it have happened anyway?"

Enrichment: Bacon on the pictures in the temple. Francis Bacon's *Novum Organum* (1620) is written in short numbered paragraphs called aphorisms. In one of them, Bacon tells the story of the man shown the pictures of sailors who had made vows and been saved from shipwreck. As Bacon put it, roughly, the man asked where the pictures were of those who had made their vows and drowned. Think of a modern example: a team that "always wins when it wears its blue uniforms," or a cereal box covered with happy customers' letters. Whose pictures, or letters, are missing? Why would they be missing?

Example 1. Name the fallacy in each, and say what is wrong. (a) “I bought two Brightway light bulbs, and both burned out within a week. Brightway bulbs are junk.” (b) “The rule says no running in the halls, so the school nurse shouldn’t run to help a student who has fainted.”

▼ **Show solution**

1. (a) The conclusion is about all Brightway bulbs, but the evidence is two bulbs, which may have come from one bad box or a faulty lamp. Too few cases for a general conclusion: hasty generalization.
2. (b) The rule is meant to keep hallways safe in ordinary times. A fainting student is an emergency, a special case the rule was not meant to cover. Applying the rule to it anyway is the fallacy of accident.
3. Notice the mirror image: (a) goes from a few cases to a rule; (b) goes from a rule to a special case.

Answer: (a) Hasty generalization: two bulbs are too few to judge the whole brand. (b) Accident: the general rule is applied to an emergency it was never meant to cover.

Example 2. Name the fallacy and suggest a better explanation. (a) “I wore my lucky socks on Friday, and we won. The socks did it.” (b) “On the days the snack stand sells the most ice cream, the lifeguards treat the most sunburns. Ice cream causes sunburn.”

▼ **Show solution**

1. (a) The only evidence is that the win came after the socks. That is the post hoc form of false cause. Better explanations: the team played well, the other team was weaker, or both. A fair test would compare many games with and without the socks.
2. (b) Two things go together, and the argument jumps to “one causes the other.” Ask whether a third thing could cause both. Hot, sunny days bring more swimmers, more ice cream sales and more sunburns. This is false cause, missing a common cause.

Answer: (a) False cause (post hoc ergo propter hoc): the win followed the socks but was caused by the play. (b) False cause: hot, sunny weather is a common cause of both ice cream sales and sunburns.

Example 3. Which is a slippery slope fallacy, and which is a fair warning? (a) “If you skip one piano lesson, you’ll skip another, then you’ll quit, and you’ll never learn any instrument at all.” (b) “If you leave the hose running into the garden bed all night, the bed will flood and the seedlings will rot.”

▼ **Show solution**

1. Check each link: how likely is each step to lead to the next?
2. (a) Skipping one lesson does not make skipping another likely, let alone quitting, let alone never learning any instrument. No evidence is given for any link. Slippery slope.
3. (b) A hose running for hours will very likely flood a small bed, and seedlings standing in water for a long time very likely rot. Each link is likely, so this is a fair warning.

Answer: (a) is a slippery slope: no link is shown to be likely. (b) is a fair warning: each step very likely leads to the next.

PRACTICE

1. **Easy** Define hasty generalization.

   ▼ **Show answer**

   Hasty generalization is drawing a general conclusion from too few cases, or from cases that are not representative of the whole group.

2. **Easy** What does *post hoc ergo propter hoc* mean, and what mistake does it name?

   ▼ **Show answer**

   It means "after this, therefore because of this." It names the mistake of concluding that A caused B only because B came after A.

3. **Easy** What is the fallacy of accident, and how is it the mirror image of hasty generalization?

   ▼ **Show answer**

   Accident applies a general rule to a special case it was not meant to cover. Hasty generalization goes from a few special cases to a general rule; accident goes from a general rule to a special case.

4. **Easy** Define the slippery slope fallacy.

   ▼ **Show answer**

   A slippery slope claims that one step will lead through a chain of further steps to some bad result, without good evidence that each link in the chain is likely.

5. **Medium** Name the fallacy and give a one-sentence reason: "The two Ridgeview students I met at the tournament were rude, so Ridgeview students are rude."

   ▼ **Show answer**

   Hasty generalization: two students are far too few to judge a whole school, and two met at one tournament may not represent the rest.

6. **Medium** Name the fallacy and give a one-sentence reason: "The pool rule says no one may enter the water after the whistle blows, so the lifeguard should not jump in to save a struggling swimmer."

   ▼ **Show answer**

   Accident: the rule is meant for ordinary swimmers in ordinary times, and a rescue is an emergency the rule was never meant to cover.

7. Medium Name the fallacy and give a one-sentence reason: “Ever since we put up the new weather vane, it has rained more often. The vane is bringing the rain.”

▼ **Show answer**

False cause (post hoc ergo propter hoc): the rain came after the weather vane went up, but that does not show the vane caused it; a rainy season is a far better explanation.

8. Medium Name the fallacy and point to a weak link: “If we let the dog sleep on the couch once, he’ll want to sleep there every night, then he’ll chew the cushions, and soon the whole living room will be ruined.”

▼ **Show answer**

Slippery slope. Any link could be named as weak: one night does not make every night likely (the family can say no), and sleeping on the couch does not make chewing cushions likely, let alone ruining the room.

9. Hard “Towns with more fire trucks have more fires each year. So fire trucks cause fires.” Name the fallacy and give a better explanation.

▼ **Show answer**

False cause. A common cause explains both: larger towns have more buildings, so they have more fires and need more fire trucks. Also accept a reversed cause: towns with many fires buy more fire trucks. Either way, the trucks do not cause the fires.

10. Hard Is this a slippery slope fallacy? Explain. “If you leave the freezer door open all night, the ice cream will melt, then it will drip onto the floor, and in the morning you’ll have a sticky mess to clean.”

▼ **Show answer**

No. Each link is very likely: an open freezer warms up overnight, melted ice cream drips, and dripped ice cream leaves a sticky floor. A chain of consequences is a fallacy only when its links are not shown to be likely.

11. Hard Mrs. Ortiz asked all twelve members of the chess club how they like to spend Saturdays, and ten said “quiet indoor games.” She concludes that most students in the school prefer quiet indoor games. What is wrong, and how should she repair her survey?

▼ **Show answer**

Hasty generalization from an unrepresentative sample: chess club members are especially likely to enjoy quiet indoor games, so they do not represent the whole school (and twelve is a small number). She should ask a larger group chosen fairly from the whole school, for example a few students picked at random from every class.

12. Challenge Caleb says his lucky socks make his basketball team win. Describe a fair way to test his claim, and say what result would count against it.

▼ **Show answer**

Sample answer: Over a whole season, Caleb could keep a record of every game, noting whether he wore the socks and whether the team won, and decide in advance (for example, by a coin flip) which games he will wear them, so that he does not pick easy games for the socks. If the team wins about as often without the socks as with them, the claim fails; even a small difference over a few games could be coincidence. Accept any test that compares games with and without the socks over many games and treats similar win rates as evidence against the claim.

COMMON MISTAKES

- Thinking that every generalization is hasty. A conclusion drawn from many, varied, fairly chosen cases can be strong.
- Treating a good general rule as if it had no exceptions. Ask what the rule is for.
- Calling every chain of consequences a slippery slope. When each link is likely, the warning is fair.
- Concluding that one thing causes another because it came first or because the two go together. Look for a common cause, a reversed cause or coincidence.
- Remembering the hits and forgetting the misses. Count the cases that do not fit as well as those that do.

> **Scripture.** “Jesus answered, Neither hath this man sinned, nor his parents: but that the works of God should be made manifest in him.” (John 9:3, KJV)
>
> The disciples assumed a cause for the man’s blindness and offered only two choices. Jesus corrected both mistakes, showing that suffering cannot always be traced to a particular sin.

UNIT 3 · INFORMAL FALLACIES · LESSON 3.7 OF 69 · 40 MINUTES

3.7 Ambiguity I: Equivocation, Amphiboly, Accent

OBJECTIVES

- Spot equivocation, in which a word shifts its meaning within an argument, and test for it by substituting definitions.
- Spot amphiboly, in which a loosely built sentence can be read two ways.
- Spot the fallacy of accent, including misplaced emphasis and quoting out of context.
- Repair an ambiguous argument by defining terms and rewriting sentences so they can be read only one way.

KEY TERMS

Ambiguous

Able to be understood in more than one way. A word, phrase or sentence can be ambiguous.

Equivocation

The fallacy of using a word or phrase in two different senses within one argument, so that the argument seems to work only because the meaning shifts.

Relative term

A word such as “small,” “tall” or “expensive” whose meaning depends on what it is compared with; a small elephant is not a small animal.

Amphiboly

The fallacy of drawing a conclusion from a sentence whose grammar is loose, so that the whole sentence can be read in two ways.

Accent

The fallacy of changing a statement's meaning by stressing certain words, by the way it is printed, or by quoting part of it out of context.

Quoting out of context

Repeating part of what someone said or wrote in a way that changes its meaning, by leaving out the words around it.

CORE IDEAS

When words play tricks. Most words have more than one meaning, and that is not a defect of language; it is what lets a small number of words do a great deal of work. Usually the context settles which meaning is intended, and nobody is confused. A **fallacy of ambiguity** happens when an argument depends on two meanings at once: it seems to work only because a word, a sentence or an emphasis shifts between the premises and the conclusion. In Unit 2 you learned to tell univocal from equivocal terms. Now you will see what happens when an equivocal term slips into an argument unnoticed.

Equivocation: one word, two meanings. "Coach says Mia's serve is a real weapon. Weapons are not allowed at school. So Mia's serve is not allowed at school." The word "weapon" means "a strong advantage" in the first premise and "an instrument for injuring people" in the second. This is **equivocation**: using a word in two senses in one argument. Silly examples are easy to catch; real ones are subtler. "Ben has a right to spend his allowance as he likes. So whatever he buys with it is right." Having a right to do something (no one may stop you) is not the same as its being right (good to do). Ben may have a right to spend all his money on candy, but that does not make it a wise choice.

The substitution test. To catch an equivocation, write the argument out and replace the suspicious word with its meaning in each line. "Mia's serve is a strong advantage. Instruments for injuring people are not allowed at school. So Mia's serve is not allowed." Now the premises plainly have nothing to do with each other. If an argument works only when one word is allowed to mean two things, it equivocates. In Unit 5 you will see that an equivocating syllogism really has four terms instead of three, which is why it fails.

Relative terms. Some words shift meaning because they quietly compare. "Small," "tall," "fast," "cheap" and "old" are **relative terms**: they mean "small for its kind," "tall

for its kind," and so on. "A Great Dane puppy is a small Great Dane, so it is a small dog" equivocates, because the puppy may already weigh more than most full-grown dogs. A "cheap" house still costs far more than an expensive bicycle. When an argument moves a relative term from one comparison group to another, check whether it still means the same thing.

Amphiboly: a sentence with two readings. Sometimes no single word is ambiguous, but the way the sentence is built is. "Mrs. Ortiz saw the deer with binoculars." Did she use binoculars, or did the deer have them? "Grandma told Mrs. Lee that she would call when she got home." Whose home? This is **amphiboly** (from Greek words meaning "thrown both ways"): loose grammar, a misplaced phrase or an unclear pronoun lets the whole sentence be read two ways. It becomes a fallacy when someone draws a conclusion from the wrong reading, or deliberately writes a sentence so that it can be taken in whichever way suits him later. A warranty that promises protection "for life" may mean your life or only the life of the product.

Accent: the power of emphasis. Read this sentence aloud five times, stressing a different word each time: "Mom said *we* could have one cookie." "Mom said we could have *one* cookie." "*Mom* said we could have one cookie." Each version suggests something different: that others could not, that one is the limit, that someone else said otherwise. The fallacy of **accent** changes a statement's meaning by its emphasis. In print, the same trick is done with size: "SAVE 50%" in huge letters, and "on selected items, while supplies last" in tiny ones. A headline can do it too, by stressing one part of a story and burying the rest.

Quoting out of context. The most common form of accent is **quoting out of context**. A reviewer writes, "The new Kestrel truck is a marvel of discomfort." An advertisement quotes her: "A marvel!" A teacher says, "This essay would be excellent if it had any evidence." The student tells his parents the teacher called it "excellent." Even Scripture can be misquoted this way. Psalm 14:1 contains the words "There is no God," but the whole verse begins, "The fool hath said in his heart, There is no God." Removing the surrounding words reverses the meaning. Whenever you see a short quotation, especially one in an advertisement, ask what the rest of the sentence said.

Clearing up ambiguity. The cure for every fallacy of ambiguity is the same: say exactly what you mean. Define your key words and keep them fixed through the

whole argument. Rewrite loose sentences so that they can be read only one way: "Using binoculars, Mrs. Ortiz saw the deer." Quote enough of what others said that its meaning is not changed. When someone else's argument turns on a slippery word, ask kindly, "What do you mean by that word here?" Jesus' hearers once misunderstood him when he spoke of a temple; John explains, "But he spake of the temple of his body" (John 2:21). Careful readers, and careful listeners, ask which meaning is meant.

> **From history.** Aristotle put equivocation and amphiboly first in his list of fallacies that depend on language, and the Greek historian Herodotus told a famous story of amphiboly. Croesus, the wealthy king of Lydia, asked the oracle at Delphi whether he should make war on the Persians. He was told that if he did, he would destroy a great empire. Croesus attacked, confident of victory, and a great empire was destroyed: his own. The answer could be read two ways, and Croesus chose the reading he wanted. Aristotle's fallacy of "accent" first concerned the marks that change the sound and meaning of written Greek words; later logicians widened it to include emphasis and quoting out of context.

> **Practice habit.** When an argument turns on a key word, ask, "What does that word mean here, and does it mean the same thing every time it appears?"

> **Enrichment: Lewis Carroll's Nobody.** Lewis Carroll loved jokes that turn on words. In *Through the Looking-Glass* (1871), Alice tells the White King that she sees nobody on the road. The King answers, roughly, that he only wishes he had such eyes, to be able to see Nobody, and at that distance too. A little later, a messenger says that nobody walks much faster than he does, and the King replies that in that case Nobody would have arrived first. What word is the King treating as a name? Why is the joke a kind of equivocation?

Example 1. Use the substitution test on this argument. “Ben has a right to spend his allowance as he likes. Whatever a person has a right to do is right. So whatever Ben buys with his allowance is right.”

▼ **Show solution**

1. Find the suspicious word: “right.” It appears as “has a right to” and as “is right.”
2. Meaning 1: “has a right to” means “is allowed to; no one may stop him.” Meaning 2: “is right” means “is good or wise to do.”
3. Substitute: “Ben is allowed to spend his allowance as he likes. Whatever a person is allowed to do is good to do. So whatever Ben buys is good to do.” Now the second premise is plainly false: people are allowed to do many foolish things, such as spending all their money on candy.
4. Kept to one meaning, the argument either has a false premise or does not reach its conclusion. It worked only by letting “right” mean two things.

Answer: The argument equivocates on “right” (“allowed” versus “good to do”). With the meanings written out, the second premise is false, so the conclusion is not supported.

Example 2. Name the fallacy in each and explain the two readings or the trick. (a) "The farmer told his neighbor that his cow was sick, so the neighbor called the vet for his own cow." (b) A review says, "Brookside's new cheese is anything but delicious." Brookside's advertisement quotes it: "delicious."

▼ **Show solution**

1. (a) The pronoun "his" could refer to the farmer or to the neighbor. Reading 1: the farmer's cow was sick. Reading 2: the neighbor's cow was sick. The neighbor acted on one reading without checking. The ambiguity is in the sentence's structure (an unclear pronoun), not in a single word with two meanings: amphiboly.
2. (b) The reviewer said the cheese is not delicious. By quoting one word without "anything but," the advertisement reverses the meaning. This is accent, in the form of quoting out of context.

Answer: (a) Amphiboly: "his cow" can mean the farmer's or the neighbor's. (b) Accent (quoting out of context): dropping "anything but" turns a criticism into praise.

PRACTICE

1. Easy Define equivocation.

   ▼ **Show answer**

   Equivocation is using a word or phrase in two different senses within one argument, so that the argument seems to work only because the meaning shifts.

2. Easy How is amphiboly different from equivocation?

   ▼ **Show answer**

   In equivocation a single word or phrase has two meanings; in amphiboly the words may each be clear, but the grammar of the whole sentence (a misplaced phrase or an unclear pronoun, for example) lets it be read two ways.

3. **Easy** Define the fallacy of accent and name two forms it can take.

▼ Show answer

Accent changes a statement's meaning by its emphasis. Forms include stressing certain words when speaking, printing some words large and others small, misleading headlines, and quoting out of context. (Any two.)

4. **Easy** Which word shifts its meaning? "My brother is a real bear in the morning. Bears sleep all winter. So my brother sleeps all winter."

▼ Show answer

"Bear": in the first premise it means "a grumpy person"; in the second it means the animal.

5. **Medium** Name the fallacy and explain: "A Great Dane puppy is a small Great Dane. So a Great Dane puppy is a small dog."

▼ Show answer

Equivocation on a relative term: "small" means "small for a Great Dane" in the premise but "small for a dog" in the conclusion. A Great Dane puppy can be larger than most grown dogs of other breeds.

6. **Medium** Give the two readings of this sentence and name the kind of ambiguity: "Grandma told Mrs. Lee that she would call when she got home."

▼ Show answer

Reading 1: Grandma would call when Grandma got home. Reading 2: Mrs. Lee would call when Mrs. Lee got home. (Accept other combinations of the two pronouns, such as Grandma calling when Mrs. Lee got home.) The ambiguity is amphiboly, from unclear pronouns.

7. **Medium** A newspaper review says, "Mrs. Hale's pie was not the best at the fair, but it was an honest pie." Her bakery sign now reads: "'The best at the fair!' (County Gazette)." Name the fallacy and explain.

▼ Show answer

Accent, in the form of quoting out of context: by dropping "was not," the sign turns the reviewer's denial into praise, reversing the meaning.

8. Medium Rewrite this sentence two ways so that each version can be read only one way: “Mrs. Ortiz saw the deer with binoculars.”

▼ Show answer

Sample answers: (1) “Using binoculars, Mrs. Ortiz saw the deer.” (2) “Mrs. Ortiz saw the deer that was carrying binoculars” (the silly reading, now unmistakable). Accept any two rewrites that each allow only one reading of who had the binoculars.

9. Hard According to Herodotus, the oracle told Croesus that if he made war on the Persians, he would destroy a great empire. Name the fallacy Croesus fell into and explain.

▼ Show answer

Amphiboly: the answer could be read two ways, since “a great empire” might mean the Persians’ empire or his own. Croesus drew his conclusion from the reading he wanted, and the empire destroyed was his own. (Accept the explanation that the phrase “a great empire” is ambiguous, if the student shows the two readings.)

10. Hard Someone says, “The Bible itself says ‘There is no God.’” What is wrong with this claim? Name the fallacy.

▼ Show answer

It quotes Psalm 14:1 out of context (the fallacy of accent). The full verse begins, “The fool hath said in his heart, There is no God,” so the Bible reports the words as what a fool says, which is the opposite of teaching them.

11. Hard A store window reads “SAVE 50%” in letters a foot high, with “on selected items, this Saturday only” in tiny print at the bottom. A shopper says, “Everything in the store is half price!” What fallacy does the sign invite, and what should the shopper have done?

▼ Show answer

Accent: the sign’s printing emphasizes “SAVE 50%” and buries the limits, inviting the false reading that everything is half off. The shopper should have read the whole sign (and asked which items are on sale). Also accept noting that “selected items” is vague, which Lesson 3.9 calls a weasel phrase.

12. **Challenge** Write your own short example of each: (a) an argument that equivocates (then show with the substitution test why it fails); (b) a sentence that is amphibolous, with its two readings; (c) a quotation that changes meaning when taken out of context.

▼ **Show answer**

Sample answer: (a) “Hannah is a sharp student. Sharp things should be kept away from small children. So Hannah should be kept away from small children.” Substituting: “Hannah is a quick-minded student. Things with cutting edges should be kept away from small children…” The premises no longer connect. (b) “The coach told the captain that he would lead the warm-ups.” Reading 1: the coach would lead them; reading 2: the captain would lead them. (c) Original: “This would be a fine plan if it cost half as much.” Out of context: “a fine plan.” Accept any examples that fit each fallacy, with a correct explanation.

COMMON MISTAKES

- Thinking that any word with two meanings makes an argument fallacious. The fallacy happens only when the argument depends on the meaning shifting.
- Missing relative terms. “Small,” “cheap” and “old” quietly compare, and the comparison can change.
- Confusing amphiboly with equivocation. Amphiboly is about the structure of the whole sentence, not one word.
- Trusting a short quotation without its context. Ask what the full sentence said.
- Thinking that clearing up ambiguity is merely fussy. Defining terms is how careful people keep from talking past each other.

> **Scripture.** “But he spake of the temple of his body.” (John 2:21, KJV)
>
> When Jesus said “Destroy this temple,” his hearers took “temple” to mean the building in Jerusalem. John explains which meaning Jesus intended, a reminder to ask what a speaker means before drawing conclusions from his words.

3.8 Ambiguity II: Composition and Division

OBJECTIVES

- Spot the fallacy of composition, which argues that what is true of the parts must be true of the whole.
- Spot the fallacy of division, which argues that what is true of the whole must be true of the parts.
- Tell the distributive use of a term (each member) from its collective use (the group as a whole).
- Decide whether a given property really does pass from parts to whole, or from whole to parts.

KEY TERMS

Composition

The fallacy of arguing that because each part (or member) has a property, the whole (or group) must have it too, when the property does not carry over.

Division

The fallacy of arguing that because a whole (or group) has a property, each of its parts (or members) must have it too, when the property does not carry over.

Distributive use

Using a term of a group to say something about each member taken one at a time: “The players on our team are tall” (each one is tall).

Collective use

Using a term of a group to say something about the group taken as a whole: “The players on our team ate forty pounds of pasta this season” (all together).

Whole and part

A whole is something made up of parts arranged in a certain way; its properties sometimes come from the parts and sometimes from the way they are put together.

Parts and wholes. If every board in a table is made of oak, the table is made of oak. If every brick in a wall is red, the wall is red. Some properties pass from the parts to the whole without any trouble. But if every brick in the wall is light enough to lift with one hand, the wall is not light enough to lift with one hand. Weight adds up. The fallacies of this lesson treat a property as if it passed between parts and whole when it does not. Like equivocation, they trade on a shift in meaning: here the shift is between speaking of things one at a time and speaking of them all together.

Composition: from the parts to the whole. The fallacy of **composition** argues that what is true of each part must be true of the whole. "Every player on the all-star team is excellent, so the team will be excellent." Perhaps; but a team is more than its players. Five excellent players who have never practiced together may lose to five good ones who have. "Every ingredient in this dish tastes good, so the dish will taste good." Pickles, chocolate sauce and garlic each taste good, but nobody wants them in one bowl. The whole depends not only on its parts but on how they are put together.

Composition with groups. The same mistake happens with groups of people or things. "Each family on our street produces only a small bag of trash a week, so our street produces only a little trash." Fifty small bags make a large pile. "Each of these cereals has only a little sugar per spoonful, so a whole bowl has only a little sugar." Small amounts per member can add up to a large amount for the group. When a property is about size, amount, cost or weight, be especially careful: such properties usually add up rather than carry over.

Division: from the whole to the parts. The fallacy of **division** runs the other way, arguing that what is true of the whole must be true of each part. "The Riverside Orchestra is the finest in the state, so its tuba player must be the finest tuba player in the state." An orchestra can be excellent as a whole while one player is merely good. "Brookside School is a large school, so its classrooms must be large." A large school may have many small classrooms. In chemistry, table salt is safe to eat, but one of its two elements, pure sodium, is a metal that reacts violently with water. The whole has properties the parts lack.

Each, or all together? Many cases of composition and division turn on two ways of using a word for a group. In the **distributive use**, we speak of each member one at a time: "The cows on the Becker farm are healthy" means each cow is healthy. In the

collective use, we speak of the group as a whole: "The cows on the Becker farm give two thousand gallons of milk a week" means all of them together. To argue "The farm's cows give two thousand gallons a week, and Daisy is one of the farm's cows, so Daisy gives two thousand gallons a week" is division: it treats a collective claim as if it were distributive.

When it is not a fallacy. Arguing from parts to whole, or whole to parts, is not always wrong. If every part of a chair is made of oak, the chair is oak; if a whole loaf is made with whole-wheat flour, every slice is whole wheat. The question is what kind of property is involved. Properties such as material and color usually carry over. Properties that add up (weight, cost, size), properties that depend on arrangement (being well designed, being a good team, tasting good together), and abilities (a choir can sing a four-part song; no single singer can) usually do not. There is no mechanical rule. You have to think about the property itself.

The body and its members. In 1 Corinthians 12:14, Paul uses the human body to teach the church about wholes and parts: "For the body is not one member, but many." The eye cannot do the work of the hand, and no single member does what the whole body does. Paul's point warns against both fallacies. A church, a family or a team has strengths that no single member has (composition goes wrong when we forget this), and the whole may have a reputation or a gift that not every member shares (division goes wrong when we assume it). Each member matters because the whole is more than any one part.

Testing composition and division. To test such an argument, first ask whether a claim is about each member or about the group as a whole. Then ask whether the property is one that carries over or one that adds up, depends on arrangement, or belongs only to the whole. A quick check is to find an argument of the same shape with an obviously false conclusion: "Each grain of sand is tiny, so the beach is tiny." If the same move fails there, it needs a better reason here.

From history. The names "composition" and "division" come from Aristotle, but he used them a little differently. His examples concern how the words of a sentence are grouped. One of them is roughly this: "A man who is sitting can walk." Read with the words combined, it says he can walk-while-sitting, which is false. Read with the words divided, it says that a man now sitting has the power to walk later, which is true. Medieval logicians called these the "composite sense" and the "divided sense." Later logicians extended the names to arguments that move between parts and wholes, the meaning they usually carry today.

Practice habit. When you hear a claim about a group, ask: "Is this true of each one, or of all of them together?"

WORKED EXAMPLES

Example 1. Is each argument a fallacy of composition? Explain. (a) "Every part of this rocking chair is made of oak, so the chair is made of oak." (b) "Every player on the all-star team is excellent, so the team will be excellent." (c) "Each brick weighs four pounds, so the wall weighs four pounds."

▼ **Show solution**

1. Ask what kind of property is involved: does it carry over from parts to whole, or does it add up or depend on arrangement?
2. (a) Material carries over: a chair made entirely of oak parts is an oak chair. Not a fallacy.
3. (b) Being an excellent team depends on how the players work together, not only on each player. Composition.
4. (c) Weight adds up: a wall of a thousand four-pound bricks weighs about four thousand pounds. Composition.

Answer: (a) Not a fallacy: material carries over. (b) Composition: team quality depends on arrangement and teamwork. (c) Composition: weight adds up instead of carrying over.

Example 2. Name the fallacy and explain. (a) "Brookside School is a large school, so its classrooms must be large." (b) "The cows on the Becker farm give two thousand gallons of milk a week. Daisy is one of the farm's cows. So Daisy gives two thousand gallons a week."

▼ **Show solution**

1. Both arguments move from a whole or group to a part or member, so check for division.
2. (a) "Large" is true of the school as a whole (many students and rooms), but that does not make each room large. Division.
3. (b) The first premise uses "the cows" collectively (all together). The conclusion treats it as if it were distributive (each cow). Division, resting on the shift from collective to distributive.

Answer: (a) Division: a large school can have small classrooms. (b) Division: the milk total belongs to the herd collectively, not to each cow.

PRACTICE

1. Easy Define the fallacy of composition.

   ▼ **Show answer**

   Composition argues that because each part (or member) has a property, the whole (or group) must have it too, when the property does not carry over.

2. Easy Define the fallacy of division.

   ▼ **Show answer**

   Division argues that because a whole (or group) has a property, each of its parts (or members) must have it too, when the property does not carry over.

3. Easy What is the difference between the distributive and the collective use of a term? Give an example of each.

   ▼ **Show answer**

   The distributive use speaks of each member one at a time ("The players are tall": each one is tall); the collective use speaks of the group as a whole ("The players ate forty pounds of pasta this season": all together). Accept any correct pair of examples.

4. **Easy** Composition or division? “Each page of this dictionary is thin, so the dictionary is thin.”

   ▼ **Show answer**

   Composition: it argues from the parts (pages) to the whole (the book), but thickness adds up, so a book of many thin pages can be very thick.

5. **Medium** Name the fallacy and give a one-sentence reason: “The Riverside Orchestra is the finest in the state, so its tuba player must be the finest tuba player in the state.”

   ▼ **Show answer**

   Division: the orchestra’s excellence belongs to the whole group playing together, and an excellent orchestra can have a player who is not the best in the state.

6. **Medium** Is this a fallacy? Explain. “Every brick in this wall is red, so the wall is red.”

   ▼ **Show answer**

   No. Color carries over from the parts to the whole: a wall made entirely of red bricks is red (allowing for the mortar between them).

7. **Medium** Name the fallacy and explain: “Water puts out fires. Water is made of hydrogen and oxygen. So oxygen puts out fires.”

   ▼ **Show answer**

   Division: it argues that a property of the whole (water puts out fires) belongs to each part. In fact oxygen feeds fires; the whole has a property its parts lack.

8. **Medium** Name the fallacy and give a one-sentence reason: “Each hymn in our hymnal takes only three minutes to sing, so singing the whole hymnal would take only a few minutes.”

   ▼ **Show answer**

   Composition: time adds up, so hundreds of three-minute hymns would take many hours to sing.

9. Hard “The families on Maple Street give five hundred dollars a year to the town library. The Ortiz family lives on Maple Street. So the Ortiz family gives five hundred dollars a year to the library.” Name the fallacy and explain which use of the term causes it.

▼ **Show answer**

Division. The first premise uses “the families on Maple Street” collectively (all of them together give five hundred dollars), but the conclusion treats it distributively (each family gives that much). The Ortiz family may give only a small part of the total. (Accept also that the premise could be read distributively, in which case the argument is valid; the point is that the conclusion follows only on that reading.)

10. Hard “Every part of this tractor engine is light enough for one person to lift, so the whole engine is light enough for one person to lift.” Name the fallacy and explain why this property does not carry over.

▼ **Show answer**

Composition. Weight adds up: many parts that are each liftable can together weigh hundreds of pounds, far more than one person can lift.

11. Hard For each property, say whether it carries over from the parts to the whole (if every part has it, must the whole have it?): (a) made of steel; (b) weighs less than a pound; (c) painted blue; (d) able to play a four-part hymn; (e) waterproof (thinking of a tent made of waterproof panels sewn together).

▼ **Show answer**

(a) Yes: material carries over. (b) No: weight adds up. (c) Yes: if every part is painted blue, the whole is blue. (d) No: an ability depends on how the parts work together, so the whole need not have an ability just because each part does (and, the other way around, a choir can sing a four-part hymn that no single singer can). (e) Not necessarily: the tent can leak at the seams, so being waterproof depends on how the panels are joined. Accept “no” for (e) with that reason.

12. **Challenge** Using Paul's picture of the body in 1 Corinthians 12, write one example of composition and one of division about a church, a family or a team, and explain what goes wrong in each.

▼ **Show answer**

Sample answer: Composition: "Each person on our church's choir is a quiet, shy singer, so the choir will sound quiet and shy." Thirty singers together can fill a building; the whole has a power no single member has. Division: "Our family is known for its fine garden, so my little brother must be a fine gardener." The family's reputation belongs to the whole, perhaps thanks to one or two members, not to every member. Paul's point is that the body is "not one member, but many": the whole and its parts each have gifts the other lacks. Accept any correct pair with clear explanations.

COMMON MISTAKES

- Thinking that every move from parts to whole is a fallacy. Some properties, such as material and color, really do carry over.
- Forgetting that size, weight, cost and time add up.
- Missing the shift from collective to distributive. Ask, "Is this about each one, or about all of them together?"
- Confusing composition with hasty generalization. Hasty generalization goes from some members to all members; composition goes from the members to the group as a whole.

> **Scripture.** "For the body is not one member, but many." (1 Corinthians 12:14, KJV)
>
> Paul teaches that the church, like a body, is a whole made of many members with different gifts. Neither the whole nor any one member can be judged as if it were simply the other.

UNIT 3 · INFORMAL FALLACIES · LESSON 3.9 OF 69 · 45 MINUTES

3.9 Fallacies in Advertising

OBJECTIVES

- Treat an advertisement as an argument and separate its real information from its persuasive devices.

- Spot weasel words, unfinished comparisons, puffery and fine-print tricks.
- Evaluate testimonials and "studies show" claims by asking who, how many, compared with what, and who paid.
- Name the fallacies from earlier lessons (authority, the many, novelty, emotion, accent) when they appear in advertising, and take the Spot the Device quiz.

KEY TERMS

Weasel word

A word that quietly empties a claim while seeming to strengthen it, such as "helps," "up to," "may," "virtually" or "fights."

Unfinished comparison

A comparison that never says what it is comparing with, such as "30% more cleaning power" or "now with less fat."

Puffery

Vague, glowing praise that cannot be tested, such as "the world's finest coffee" or "simply the best."

Testimonial

A statement from a customer or a famous person praising a product, offered as evidence that it works.

"Studies show" claim

An appeal to research that does not say who did the research, what was measured, how many were studied, or what the results were.

Fine print

Small or hidden words in an advertisement that limit or change the meaning of the large print.

CORE IDEAS

Every advertisement is an argument. An advertisement almost never states its conclusion, but everyone knows what it is: "Buy this." The premises are the claims, pictures and feelings the ad offers. Not every advertisement is deceptive. An honest one gives real information: the price, the size, what the product does, how it compares with others on a fair test. The trouble is that advertisements are written to persuade, and the quickest persuasion often uses the devices you have studied in this unit. Your job as a reader is to separate the information from the devices.

Weasel words. "Helps fight cavities." "Up to 50% off." "May reduce frizz." "Virtually spotless." A **weasel word** quietly empties a claim while seeming to fill it. The name comes from the old belief that a weasel could suck the inside out of an egg and leave

the shell looking whole. “Helps fight cavities” promises nothing about how much, and even a glass of water “helps.” “Up to 50% off” is true if one item is half price and everything else is 5% off. “May” means it also may not. When you see such a word, read the claim again without it and see what is left.

Unfinished comparisons. “Sparkle detergent: 30% more cleaning power!” More than what? Than the leading brand? Than last year’s Sparkle? Than plain water? An **unfinished comparison** sounds like a measurement but never names the other side. “Now with less sugar” may mean less than an extremely sugary recipe. “Better taste” may mean better than nothing. This is a cousin of the fallacy of accent: the big number is stressed, and the missing part is hidden by never being said at all. The question to ask is always, “Compared with what?”

Testimonials. “I lost my fear of math in just two weeks! — Kayla, age 12.” A **testimonial** lets one customer’s story stand in for evidence. Even if Kayla is real and sincere, one story is a hasty generalization. And testimonials are chosen: a company prints the happy letters and throws away the others, just as Bacon’s temple showed only the sailors who were saved. When the testimonial comes from a famous athlete or actor, it is usually also a bad appeal to authority: fame in one field gives no expertise in mattresses, batteries or cereal, and the star is paid. A fair testimonial comes from someone who really knows the field, or from many customers whose reviews were not hand-picked.

“Studies show.” “Studies show that students who use Mathwise improve their grades.” “Clinically proven.” “Doctor recommended.” These phrases borrow the authority of research without giving any of its details. Ask: Who did the study, and did the company pay for it? How many people were studied? Improved by how much, and compared with whom? Was it published where other experts could check it? Notice also the difference between “clinically *tested*,” which only means a test was done, and “clinically *proven*,” which claims the test succeeded. “Recommended by doctors” could mean three doctors out of thirty thousand.

Puffery and the old fallacies in new clothes. “The world’s finest coffee.” “Pure.” “Premium.” “Simply the best.” **Puffery** is glowing praise so vague that it cannot be tested, and so it gives no information. Around it, look for the fallacies of earlier lessons. “America’s favorite!” is the appeal to the many; “for those who accept only the finest” is snob appeal; “all-new!” is the appeal to novelty; “grandmother’s recipe

since 1910” is the appeal to tradition; a frightened child in a dark house is the appeal to fear. Each one sets a feeling where a reason should be.

Fine print and price tricks. In print, the **fine print** often carries the real message. “FREE carrying case!*” *with purchase of two. “Starting at $9.99” means the cheapest model, which may be the one nobody wants. “Was $80, now $39.99!” is a real bargain only if the item ever actually sold for $80. These are all forms of accent: the eye is drawn to the big words, and the meaning hides in the small ones. Read the whole ad, especially the parts printed small.

Five questions for any advertisement. (1) What exactly is being claimed, once the weasel words are removed? (2) Compared with what? (3) Who says so, and how do they know? (4) What is left out: the fine print, the unhappy customers, the price of the alternative? (5) Even if the claims are true, do they show that *I* should buy this? Proverbs observes that buyers and sellers both bend the truth in the marketplace: “It is naught, it is naught, saith the buyer: but when he is gone his way, then he boasteth” (Proverbs 20:14). Now visit the Classics Library Logic & Debate page and take the Spot the Device quiz, which asks you to identify 27 persuasive devices. Keep a list of the ones you missed.

> **From history.** In 1842 the showman P. T. Barnum advertised a “Feejee Mermaid” in New York. His posters and pamphlets showed a beautiful mermaid, and crowds paid to see it. What they found was a small, shriveled object made from the upper half of a monkey sewn onto the tail of a fish. Barnum’s advertising let the pictures and the excitement do the work. Barnum became famous for this kind of promotion, and his career is still a lesson in how a confident claim and a striking picture can carry people past their own good sense.

> **Practice habit.** Before you believe a product claim, read it again without its weasel words and ask, “Compared with what?”

Example 1. Analyze this advertisement: “NEW Glimmer toothpaste! Helps fight up to 40% more plaque!* Recommended by dentists.” In tiny print: “*Compared with not brushing.”

▼ **Show solution**

1. Find the conclusion: buy Glimmer toothpaste.
2. “NEW!” is an appeal to novelty: being new is not a reason to think it is better.
3. “Helps fight up to 40% more plaque” stacks three weasel words: “helps” (how much?), “fight” (fight is not remove), “up to” (40% is the most, not the usual).
4. The comparison is finished only in the fine print, and the finish gives the game away: compared with not brushing, any toothpaste, or a plain toothbrush, removes more plaque. This is accent: the big claim hides its small meaning.
5. “Recommended by dentists” does not say how many dentists, which ones, or whether they were paid: a vague appeal to authority.

Answer: Devices: appeal to novelty (“NEW”); weasel words (“helps,” “fight,” “up to”); an unfinished comparison completed in fine print with a meaningless standard (not brushing), a form of accent; and a vague appeal to authority (unnamed dentists). The ad gives no real evidence that Glimmer is better than other toothpastes.

Example 2. Mathwise Tutoring's flyer says, "'I lost my fear of math in two weeks!' — Kayla, age 12. Studies show Mathwise students improve!" Write the questions a careful parent should ask.

▼ **Show solution**

1. The testimonial is one story. Ask: How many students use Mathwise, and how did most of them do? Were only happy families quoted?
2. "Studies show" gives no details. Ask: Who did the studies, and did Mathwise pay for them? How many students were studied?
3. "Improve" is an unfinished claim. Ask: Improve by how much, and compared with whom? Would the same students have improved anyway with ordinary practice or another tutor?
4. Finally ask: Even if the claims are true, is Mathwise right for my child, at this price?

Answer: Sample questions: How typical is Kayla's experience, and how were testimonials chosen? Who ran the studies, how many students were in them, and did Mathwise pay? How much did students improve, and compared with what? What does it cost, and is it a better choice than the alternatives for my child?

PRACTICE

1. Easy What is a weasel word? Give three examples.

   ▼ **Show answer**

   A weasel word quietly empties a claim while seeming to strengthen it. Examples: "helps," "up to," "may," "virtually," "fights," "as much as," "starting at." (Any three.)

2. Easy What is an unfinished comparison? Give an example of your own.

   ▼ **Show answer**

   A comparison that never says what it is comparing with. Example: "Zestwell juice: 20% more flavor!" (more than what?). Accept any comparison that leaves out its other side.

3. **Easy** Give two reasons why a single customer's testimonial is weak evidence that a product works.

▼ Show answer

Any two: one story is a hasty generalization from a single case; testimonials are hand-picked, so the unhappy customers are left out; the customer may be paid or may be mistaken about the cause of the improvement (false cause); and a famous person's testimonial is usually an appeal to authority outside his or her field.

4. **Easy** List four questions to ask about a "studies show" claim.

▼ Show answer

Any four: Who did the study? Did the company pay for it? How many people (or things) were studied? What exactly was measured, and by how much did it change? Compared with what or whom? Was it published where other experts could check it?

5. **Medium** Analyze this claim: "Crispwell crackers: now 25% less salt!"

▼ Show answer

It is an unfinished comparison: less than what? Probably less than the old Crispwell recipe, which may have been very salty, so the new crackers could still be saltier than other brands. The claim does not show that Crispwell crackers are low in salt.

6. **Medium** A package says, "Brightway bulbs last up to ten years!" What is the weasel word, and what does the claim actually promise?

▼ Show answer

"Up to." It promises only that ten years is the most a bulb might last (perhaps under ideal conditions); it promises nothing about how long a typical bulb lasts. Accept also noting that ten years of what use (hours per day) is not stated.

7. **Medium** In a commercial, a famous quarterback says, "I sleep on a Snoozewell mattress, and it's the best mattress made." Name the fallacy and give a one-sentence reason.

▼ **Show answer**

An illegitimate appeal to authority (ad verecundiam) in the form of a celebrity testimonial: the quarterback is an expert in football, not mattresses, and he is paid to say it. Accept "testimonial" with that reason; "the best mattress made" is also puffery.

8. **Medium** What is the difference between "clinically tested" and "clinically proven"? Why might an advertiser choose "tested"?

▼ **Show answer**

"Clinically tested" means only that a test was done; it says nothing about the result. "Clinically proven" claims that a test showed the product works (and even then, we should ask what was proven, how much, and who did the test). An advertiser may choose "tested" because it sounds scientific without claiming that the product passed.

9. **Hard** A cereal commercial says, "Sunrise Puffs: part of a good breakfast," while showing a bowl of the cereal beside eggs, fruit, toast and a glass of milk. What device is being used, and what does the picture suggest that the words do not say?

▼ **Show answer**

The words are a weasel claim ("part of") combined with accent through the picture: the healthful foods around the bowl make the breakfast good, but the picture suggests that the cereal itself is nutritious. The ad never says that Sunrise Puffs are good for you. Accept "weasel words" or "accent" (or both) with this explanation.

10. Hard “Nine out of ten customers surveyed would recommend Tidewell lawn service!” Write three questions a careful reader should ask, and name the fallacy the ad may be committing.

▼ **Show answer**

Sample questions: How many customers were surveyed, and how were they chosen? Were only current customers asked, leaving out those who quit? Who ran the survey, and was it Tidewell itself? What exactly were they asked? The ad may commit hasty generalization from an unrepresentative sample (and it uses the appeal to the many). Accept any three fair questions.

11. Hard A sign reads: “Was $80, NOW ONLY $39.99! FREE carrying case!” In small print: “Case free with purchase of two.” Name the devices and say what a buyer should check.

▼ **Show answer**

Fine print and accent: the large words suggest a free case with any purchase, but the small print requires buying two. The “was $80” price may be a price the item never really sold for, so the bargain may be inflated. The buyer should check what the item sells for elsewhere and whether he or she actually wants two.

12. Challenge Invent a product. Write (a) a short, honest advertisement for it that makes only claims a buyer could check, and (b) a manipulative version that uses at least four devices from this lesson. Label each device in (b).

▼ **Show answer**

Sample answer: Product: Trailmate water bottle. (a) “Trailmate holds 24 ounces, keeps water cold for 12 hours in our 75-degree test, weighs 10 ounces, and costs $18. Lifetime replacement of the lid.” (b) “ALL-NEW Trailmate (novelty)! Helps keep you up to 50% more refreshed (weasel words; unfinished comparison)! Olympic sprinter Dana Fox swears by it (celebrity testimonial, authority outside her field). Studies show hikers love it (vague “studies show”). The world’s finest bottle (puffery).” Accept any honest version with checkable claims and any manipulative version with four correctly labeled devices.

COMMON MISTAKES

- Treating every advertisement as a lie. Many ads give real information; the skill is telling information from devices.
- Reading only the large print. The fine print often changes the meaning.
- Accepting "studies show" or "clinically tested" without asking who, how many, what was measured and compared with what.
- Trusting a testimonial because the person seems sincere. Sincere people can be mistaken, and testimonials are hand-picked.
- Forgetting the last question: even if the claims are true, do they show that I should buy it?

> **Scripture.** "It is naught, it is naught, saith the buyer: but when he is gone his way, then he boasteth." (Proverbs 20:14, KJV)
>
> "Naught" means worthless. The buyer runs down the goods to lower the price, then boasts of his bargain. Proverbs shows that the marketplace tempts both sides to bend the truth, so a wise person listens carefully to every sales pitch.

UNIT 3 · INFORMAL FALLACIES · LESSON 3.10 OF 69 · 40 MINUTES

3.10 Naming Fallacies Fairly

OBJECTIVES

- Read an argument charitably, in its most reasonable sense, before judging it.
- Avoid the fallacy fallacy: explain why a bad argument does not prove its conclusion false.
- Check for the legitimate move a suspected fallacy may be imitating, and repair a weak argument into its strongest form.
- Answer a fallacy courteously, as 1 Peter 3:15 says, "with meekness and fear," using plain words and questions.

KEY TERMS

Principle of charity

The rule of reading another person's argument in its most reasonable sense, the sense he or she most likely meant, before judging it.

Strongest form

The best version of an argument: the one that states the other side's position accurately and gives its best reasons, which is the version we should answer.

Fallacy hunting

The bad habit of searching arguments for something to label, instead of trying to understand them and find the truth.

Fallacy fallacy

Concluding that a claim is false because an argument for it is fallacious (also called the argument from fallacy). A bad argument leaves its conclusion unsupported, not disproved.

Repair

Improving a weak argument by replacing a fallacious reason with a good one, or by stating a hidden assumption and supporting it.

CORE IDEAS

A new tool, and a new temptation. You now know the names of more than twenty fallacies. That is a real gain: you can see what goes wrong in an argument and explain it exactly. But every tool can be misused. A student who has just learned the fallacies may start **fallacy hunting**, combing every conversation for something to label and calling out "Ad hominem!" or "Straw man!" as if scoring points in a game. That habit wins quarrels and loses friends, and it often gets the logic wrong. The purpose of logic is to find and share the truth, and this lesson is about using the names fairly.

The fallacy fallacy. Suppose Caleb says, "The earth goes around the sun, because my favorite pitcher said so in an interview." That is a bad appeal to authority. Does it follow that the earth does *not* go around the sun? Of course not. A fallacious argument fails to support its conclusion; it does not disprove it. The conclusion may be true for other reasons that Caleb did not give. To conclude that a claim is false because someone argued for it badly is the **fallacy fallacy**, also called the argument

from fallacy. The right response to a bad argument is, "That reason doesn't show it," not, "So the opposite is true."

The principle of charity. Before you judge an argument, make sure you understand it. The **principle of charity** says: read the other person's words in their most reasonable sense, the sense he or she most likely meant. When Grandpa says, "Nobody plants tomatoes before May around here," he does not mean that not a single person in the county has ever done it; he means that local gardeners wait until the danger of frost is past. Treating his words as a literal claim and calling it a hasty generalization would be unfair, and it would miss his real point, which is about frost. Charity is not agreeing. It is making sure that, when you disagree, you disagree with what was actually meant.

Look for the legitimate move first. Nearly every fallacy imitates a good kind of reasoning, so before you name one, ask whether the good version is what is really going on. Is this an appeal to a real expert in his field? Is the warning backed by evidence? Is this either-or a true dilemma? Is the question about a witness's reliability fair? Is the generalization drawn from many varied cases? Is the chain of consequences actually likely? Many arguments that sound like fallacies at first turn out to be reasonable once you see the context, and those that remain fallacies can then be named with confidence.

Repair the argument. Often a person has a good point and has simply made it badly. "Everybody on the team uses Swiftline cleats, so you should buy them" is a bandwagon as it stands. But perhaps the real reason is that the coach tested five brands on our muddy field and Swiftline gripped best. You can help: "Is the reason that they grip well on our field? If so, that's a good reason." To **repair** an argument is to replace the fallacious reason with a good one, or to state a hidden assumption and see whether it can be supported. Then answer the **strongest form** of the argument. If your view can stand only against weak versions of the other side, it is not yet worth holding.

Answer in plain words and questions. When an argument really is fallacious, you rarely need the Latin name. Describe the problem in plain words, and ask questions that let the other person see it: "I agree the coach didn't play in college. But does that show his free-throw advice is wrong?" "Are those really the only two choices? What about painting it next summer?" "When you say 'right,' do you mean we're allowed

to, or that it's a good idea?" Questions invite the other person to think rather than to defend himself. Grant what is true in his argument before you point out what is weak. In Proverbs 15:1 we read, "A soft answer turneth away wrath: but grievous words stir up anger."

With meekness and fear. In 1 Peter 3:15, Christians are told to "be ready always to give an answer to every man that asketh you a reason of the hope that is in you with meekness and fear." Notice both halves. We must be ready with reasons, which is why we study logic. And we must give them with meekness (gentleness, not harshness) and fear (reverence toward God and respect toward the other person). An answer that is logically correct but proud or mocking fails the second half of the command. Jesus also tells us to take the beam out of our own eye first: the arguments we should test most strictly are our own.

Four steps for naming a fallacy fairly. (1) *Understand*: restate the argument in its most reasonable sense, in words the other person would accept. (2) *Test*: ask the three questions of relevance, presumption and ambiguity, and check whether a legitimate move is being made. (3) *Separate*: remember that a bad argument leaves its conclusion unproved, not disproved. (4) *Answer*: explain the problem in plain words, ask honest questions, grant what is true, and offer better reasons if you have them. Used this way, the names of the fallacies serve their real purpose: helping us and our neighbors reason toward the truth.

> **From history.** Thomas Aquinas (about 1225–1274) built his great *Summa Theologica* out of short sections called articles, and each one models fair argument. An article begins with a question, then states the objections to Aquinas's own position, often at their full strength. Next comes "On the contrary" (the *sed contra*), usually a brief appeal to an authority, and then "I answer that" (the *respondeo*), where he gives his own view. He then replies to each objection in turn, frequently granting what is true in it before showing where it goes wrong. Aquinas did not answer weak versions of the other side; he answered the strongest ones he knew. Unit 9 will return to this form, called the disputed question, and the capstone will ask you to use it.

> **Practice habit.** Before you name a fallacy in someone else's argument, restate the argument in its strongest form and ask yourself whether you have made the same mistake.

Enrichment: Watts on judging fairly. Isaac Watts's *Logick* (1724) gives many rules for guarding our judgment against prejudice. He warns, roughly, that our affection or dislike for particular persons can bend our judgment of what they say, and that we should weigh a claim by its own evidence rather than by the person who makes it. Think of a time when you were quick to reject something because of who said it, or quick to accept something because you liked the person. Which fallacies from this unit were at work?

WORKED EXAMPLES

Example 1. Ben argues, "We should wear helmets when we ride, because everybody does." His sister answers, "That's the bandwagon fallacy, so helmets are pointless." What has his sister done wrong, and what should she have said?

▼ **Show solution**

1. Ben's reason (everybody does it) is indeed a bandwagon appeal, so his argument does not support its conclusion.
2. His sister concludes that the conclusion is false (helmets are pointless). That does not follow: a bad argument leaves its conclusion unproved, not disproved. This is the fallacy fallacy.
3. There are good reasons for helmets that Ben did not give: they protect the head in a fall.
4. A fair answer would point out the weak reason and supply a better one.

Answer: She committed the fallacy fallacy: Ben's bad reason does not make his conclusion false. A fair answer: "'Everybody does' isn't really the reason. The real reason is that a helmet protects your head if you fall, and that's a good one."

Example 2. Grandpa says, “Young people today don’t know how to work. When I was a boy, we all had chores before school.” Use the four steps to answer him fairly.

▼ **Show solution**

1. Understand: charitably, Grandpa is not claiming that every young person is lazy; he is concerned that many young people have fewer chores than he did and may not learn to work hard. That concern deserves respect.
2. Test: as stated, “young people today don’t know how to work” is a hasty generalization (it covers millions from a few cases), but his point that daily chores build work habits is reasonable.
3. Separate: even if the generalization is too broad, the conclusion that chores are valuable may still be true.
4. Answer: grant what is true, ask an honest question, and avoid mocking him.

Answer: Sample answer: “I think you’re right that chores teach people to work, and I’m glad I have some. I do know a lot of kids who work hard, though, on farms, in sports and in part-time jobs. Do you think the problem is that some kids never get the chance to learn?” This grants the true point, questions the broad generalization gently, and keeps the conversation going.

PRACTICE

1. Easy Define the fallacy fallacy.

   ▼ **Show answer**

   The fallacy fallacy is concluding that a claim is false because an argument for it is fallacious. A bad argument leaves its conclusion unsupported, not disproved.

2. Easy State the principle of charity in your own words.

   ▼ **Show answer**

   Read another person’s argument in its most reasonable sense, the sense he or she most likely meant, before you judge it. Accept any wording with this meaning.

3. **Easy** Why is calling out the Latin name of a fallacy ("Ad hominem!") usually a poor way to answer someone?

   ▼ **Show answer**

   It treats the conversation as a contest instead of a search for truth, it often makes the other person defensive, and it explains nothing; a plain description of the problem, or an honest question, helps the other person see it. (It may also be wrong, if the argument was a legitimate move.) Accept any two of these reasons.

4. **Easy** According to 1 Peter 3:15, how should a Christian give a reason for the hope that is in him or her?

   ▼ **Show answer**

   "With meekness and fear": gently, not harshly, and with reverence toward God and respect toward the other person, while being ready with real reasons.

5. **Medium** "Hannah said we need a good night's sleep before the game because she'd feel awful if we lost. That's an appeal to pity, so we don't need sleep before the game." What is wrong with this reasoning?

   ▼ **Show answer**

   It commits the fallacy fallacy. Hannah's reason may be a weak appeal to pity, but that does not show the conclusion is false; there are good reasons for sleeping well before a game (rest improves energy and focus) that she did not give.

6. **Medium** Grandpa says, "Nobody around here plants tomatoes before May." Give the charitable reading of his claim, and say what the real question is.

   ▼ **Show answer**

   Charitable reading: gardeners in this area generally wait until May to plant tomatoes, not that no one has ever done it. The real question is about frost: whether the danger of frost has passed before May. Accept any reading that treats "nobody" as "hardly anyone" and identifies the reason behind the custom.

7. Medium Fallacy or legitimate move? Explain. “The lifeguard says the current is too strong for swimming today, so we should stay out of the water.”

▼ **Show answer**

A legitimate move. The lifeguard is a trained authority speaking within his or her field about present conditions, and the warning is a fair appeal to caution about a real danger. Calling it an appeal to authority or to fear would be unfair.

8. Medium Lucy proposed shorter club meetings, and Max replied, “Lucy wants to shut the club down.” Write a courteous reply Lucy could give that corrects the straw man without attacking Max.

▼ **Show answer**

Sample answer: “I think I didn’t explain myself well. I don’t want to shut the club down; I love the club. I’m only suggesting that we meet for forty-five minutes instead of an hour and a half. What do you think of that?” Accept any reply that restates her real position calmly, does not insult Max, and invites a response.

9. Hard Repair this argument by replacing the fallacious reason with a good one: “Everybody on the team uses Swiftline cleats, so you should buy Swiftline cleats too.”

▼ **Show answer**

Sample repair: “The coach tested five brands of cleats on our muddy field, and Swiftline gave the best grip and lasted longest, so Swiftline is a good choice for our field.” Accept any repair that replaces the appeal to popularity with relevant evidence about the cleats’ quality or fit for the player.

10. Hard A parent says, “Coach Reyes never played college basketball, so I’m not sure his advice about how college recruiting works is reliable.” Is this an ad hominem? Explain, reading the parent charitably.

▼ **Show answer**

Not necessarily. Read charitably, the parent is asking whether the coach knows the recruiting process firsthand, which is a fair question about his knowledge when we rely on his word (testimony). It would become an ad hominem if the parent dismissed reasons or evidence the coach actually gave just because he did not play in college. Accept either judgment if the student explains this distinction.

11. Hard Explain why reading an argument charitably is not the same as agreeing with it.

▼ **Show answer**

Charity is about understanding: it means taking the other person’s words in their most reasonable sense before judging them. After that, you may still find the argument weak or the conclusion false. Charity simply ensures that, when you disagree, you disagree with what the person actually meant and with the strongest form of the argument, not with a distortion.

12. Challenge A friend says, “There’s no point reading Aesop’s fables. They were written thousands of years ago, before people knew anything.” Answer your friend in four to six sentences, following the four steps (understand, test, separate, answer).

▼ **Show answer**

Sample answer: “I think you mean that old stories can’t teach us much about life today (understand). But being old doesn’t make a story wrong or useless; that’s judging by the date, not by what the story says (test: appeal to novelty, or a genetic fallacy). You might still be right that some old books aren’t worth our time, but their age alone doesn’t show it (separate). Have you read ‘The Tortoise and the Hare’? I think its lesson about steady work is just as true today. Which fable would you want to test?” Accept any answer that restates the friend’s view fairly, names or describes the problem (appeal to novelty or genetic fallacy, judging by date or origin), avoids the fallacy fallacy, and answers courteously.

COMMON MISTAKES

- Concluding that a claim is false because someone argued for it badly. That is the fallacy fallacy.
- Taking someone's loose everyday wording literally in order to call it a fallacy. Read for the most reasonable meaning first.
- Naming a fallacy before checking for the legitimate move it imitates.
- Answering only the weakest version of the other side. Answer the strongest form.
- Winning the argument and losing the person. A correct answer given proudly fails the command to answer "with meekness and fear."

> **Scripture.** "be ready always to give an answer to every man that asketh you a reason of the hope that is in you with meekness and fear" (1 Peter 3:15, KJV)
>
> Peter joins readiness with gentleness. Logic makes us ready with reasons; charity and courtesy make sure we give them "with meekness and fear," as people who respect both God and our neighbor.

UNIT 4

Categorical Propositions

In Unit 1 you met the three acts of the mind, and in Unit 2 you studied the product of the first act, the term. This unit studies the product of the second act, the judgment, expressed in words as a **proposition**. It concentrates on one especially useful kind, the **categorical proposition**, which says that one class of things is wholly or partly included in, or excluded from, another: "All robins are birds," "No snakes on the farm are poisonous animals," "Some apples in the bin are bruised fruit." Logicians from Aristotle to Lewis Carroll used these four shapes of sentence as the building blocks of reasoning.

Students learn to name the four forms (**A, E, I, O**) by their quantity and quality, and to translate everyday sentences, including tricky ones with "only," "none but" and "not all," into standard form. They learn which terms each form **distributes.** They learn the traditional **square of opposition**, which tells what follows about three propositions once the truth or falsity of a fourth is known, and how a single counterexample can bring down a universal claim. They practice the immediate inferences of **conversion, obversion and contraposition** and learn when each is

valid. Then they see how George Boole's reading of universal propositions changes the square, and they learn to draw each form as a two-circle **Venn diagram.**

This unit is the foundation for Unit 5. A categorical syllogism is built from three categorical propositions, and every tool learned here (the form letters, distribution, the reading of "some," the question of existential import, and Venn diagrams) is used to test syllogisms there. Take the time to make these skills automatic now; the next unit will move much faster if they are.

Review day and test. On the review day, use the mastery checklist and the review sheet (with its answer key). The unit test is below; a printable copy is ready for test day.

PREREQUISITES

- Tell a term from a proposition and an argument, and say which can be true or false (Unit 1).
- Define a term and name a class by its members, for example “dogs” or “tools in the shed” (Unit 2).
- Tell contradictory statements (one must be true, the other false) from contrary ones (both can be false), as in Unit 1.
- Read a simple diagram of overlapping circles, as in a basic sorting exercise.

MASTERY CHECKLIST

- ☐ I can find the quantifier, subject term, copula and predicate term of a categorical proposition.
- ☐ I can name a proposition’s quantity and quality and label it A, E, I or O.
- ☐ I can explain that “some” means “at least one” and does not mean “not all.”
- ☐ I can translate everyday sentences, including ones with “only,” “none but,” “not all” and “all … not,” into standard form.
- ☐ I can say which terms each of the four forms distributes and explain why.
- ☐ I can use the traditional square of opposition to find what follows from the truth or falsity of any one corner.
- ☐ I can refute a universal claim with a single, plainly true counterexample.
- ☐ I can perform conversion, obversion and contraposition and say when each is valid.
- ☐ I can explain how Boole’s reading of universal propositions changes the square of opposition, using an example with an empty class.
- ☐ I can draw a two-circle Venn diagram for each of the four forms and read a diagram back into a proposition.

LESSONS IN THIS UNIT

4.1	**The Four Forms: A, E, I, O**	40 minutes
4.2	**Standard Form**	45 minutes
4.3	**Distribution**	40 minutes
4.4	**The Square of Opposition**	45 minutes
4.5	**Conversion, Obversion, Contraposition**	45 minutes
4.6	**Existential Import and the Modern Square**	40 minutes
4.7	**Venn Diagrams for Propositions**	40 minutes

Unit 4 test

You have 45 minutes. No notes or books. Answer each question in complete sentences unless it asks for a list, a letter or a diagram. When you translate, write the full standard-form proposition. Unless a question says otherwise, use the traditional square (assume the subject class has members) for Questions 7–8. Draw Venn diagrams with two overlapping circles, the subject on the left, and label both circles. The test is worth 100 points.

1. **6 pts**

   Give the quantity, quality and letter of each.

   1. Some fences on the farm are not painted fences.
   2. All wrens are birds.
   3. No trout in the pond are pike.
   4. Some loaves in the case are rye loaves.

2. **6 pts**

   Caleb checked every sheep in the flock, and all of them had been sheared. (a) Is “Some sheep in the flock are sheared sheep” true or false? Explain. (b) Does “Some S are P” ever, by itself, tell you that “Some S are not P”?

3. **8 pts**

   Translate each into standard form and name its form.

   1. Only seniors may park in the north lot.
   2. Not every bus was on time.
   3. Nobody in the choir sings bass.
   4. The only fish in the tank are guppies.

4. **5 pts**

   Explain why “All the lights in the gym are not on” is ambiguous, and give both standard-form readings with their letters.

5. **8 pts**

   For each, name the form and list the distributed terms (or write “none”).

   1. All carrots are root vegetables.
   2. No sparrows are hawks.
   3. Some students are musicians.
   4. Some tools in the shed are not sharp tools.

6. **5 pts**

   Explain why the predicate of an A proposition is undistributed, but the predicate of an O proposition is distributed.

7. **8 pts**

   Use the traditional square. (a) Given that “All books on the shelf are hardcover books” is *false*, give the truth value of the E, I and O. (b) Given that “Some players on the team are goalkeepers” is *true*, give the truth value of the A, E and O.

8. **6 pts**

   Use the traditional square. Given that “Some horses in the stable are not saddled horses” is *false*, give the truth value of the A, E and I, and name the relation you used for each.

9. **6 pts**

   Refute “All mammals live on land” with a counterexample. Write the proposition your counterexample makes true, name its form, and name its relation to the original claim.

10. **8 pts**

Perform each inference and say whether it is valid.

1. Convert “Some bakers are farmers.”
2. Obvert “No pencils in the box are sharp pencils.”
3. Contrapose “All hens in the coop are egg layers.”
4. Convert “All tulips are flowers.”

11. **6 pts**

Show, with an example whose premise is plainly true and whose conclusion is plainly false, that conversion of an O proposition is invalid.

12. **8 pts**

There are no ghosts in the attic. (a) On the Boolean reading, give the truth value of: “All ghosts in the attic are noisy things,” “No ghosts in the attic are noisy things,” “Some ghosts in the attic are noisy things,” “Some ghosts in the attic are not noisy things.” (b) Use your answers to name the relations of the traditional square that fail, and the one that survives.

13. **6 pts**

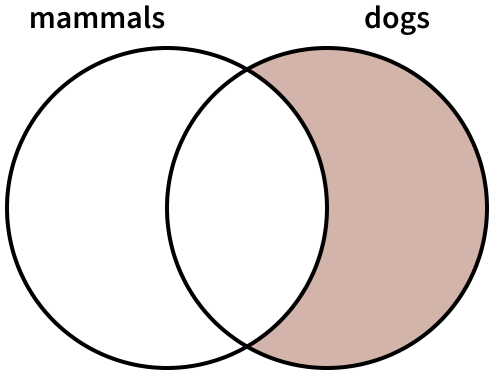
“All runners who finish the course in under four minutes are runners who win a gold ribbon. So some runners who finish the course in under four minutes are runners who win a gold ribbon.” Is this inference valid on the traditional reading? On the Boolean reading? What premise would make it valid on the Boolean reading?

14. **8 pts**

Draw a labeled Venn diagram for each: (a) “No cats are reptiles.” (b) “Some flowers in the vase are not roses.”

15. **6 pts**

Write the standard-form proposition this diagram shows, and name its form.



ANSWER KEY

1. 1: particular negative, O. 2: universal affirmative, A. 3: universal negative, E. 4: particular affirmative, I.
2. (a) True. In logic "some" means "at least one, and possibly all," so it is true even though every sheep was sheared. (b) No. "Some S are P" is true even when all S are P, so it never by itself tells us that some S are not P.
3. 1: "All people who may park in the north lot are seniors" (A). 2: "Some buses are not buses that were on time" (O). 3: "No members of the choir are people who sing bass" (E). 4: "All fish in the tank are guppies" (A). Accept any class terms with the same meaning (for example, "on-time buses," "bass singers").
4. "All … are not" can mean "none" or "not all." Reading 1: "No lights in the gym are lights that are on" (E). Reading 2: "Some lights in the gym are not lights that are on" (O), that is, not all of them are on. Without context we cannot tell which is meant.
5. 1: A; "carrots" only. 2: E; both "sparrows" and "hawks." 3: I; none. 4: O; "sharp tools" only.
6. "All S are P" places the S's somewhere inside P but says nothing about every P (it does not say all P are S), so P is undistributed. "Some S are not P" says that some S is outside the whole class P, not any member of it at all; to shut something out of a class, the proposition must speak of every member, so P is distributed.
7. (a) O true (contradictory); E and I undetermined. (b) E false (contradictory); A and O undetermined.
8. A true (contradictory of the false O). E false (contrary of the true A; or, falsity flows up from O to E). I true (subaltern of the true A; or, subcontraries cannot both be false). Accept any correct relation for E and for I (for example, E false as the contradictory of the true I).
9. Whales are mammals that do not live on land (accept dolphins, porpoises or any other mammal that plainly lives in water). This makes true "Some mammals are not land-dwelling animals" (accept any wording with the same terms), an O, the contradictory of the original A, so the A is false.
10. 1: "Some farmers are bakers"; valid (I converts). 2: "All pencils in the box are non-sharp pencils"; valid (obversion is always valid). 3: "All non-egg-layers are things that are not hens in the coop" (in plain English, nothing that fails to lay eggs is a hen in the coop); valid (A contraposes). 4: "All flowers are tulips";

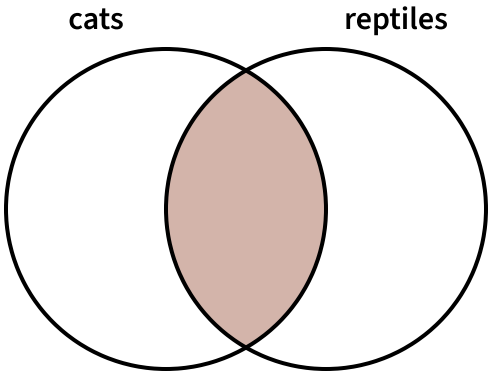
invalid (A does not convert simply). Full credit also for noting that by limitation, on the traditional reading, "Some flowers are tulips" follows.

11. "Some animals are not dogs" is true (a cat is an animal that is not a dog), but its converse, "Some dogs are not animals," is false. Accept any example with a plainly true O and a plainly false converse, such as "Some fruits are not apples" and "Some apples are not fruits."

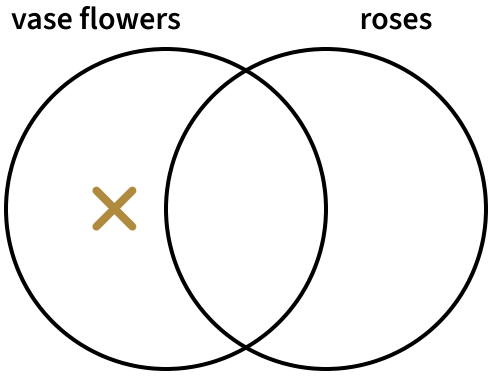
12. (a) A true (vacuously), E true (vacuously), I false, O false. (b) Contraries fail (A and E are both true), subcontraries fail (I and O are both false), and subalternation fails (A true but I false; E true but O false). Contradictories survive: A and O, and E and I, still have exactly one true each.

13. Valid on the traditional reading (subalternation, A to I). Invalid on the Boolean reading, because the A makes no claim that anyone finishes in under four minutes; if no one does, the premise is true and the conclusion false. Adding "There is at least one runner who finishes the course in under four minutes" makes it valid on the Boolean reading.

14. (a) Circles "cats" (left) and "reptiles" (right); shade the overlap.



(b) Circles "vase flowers" (left) and "roses" (right); an X in the part of "vase flowers" outside "roses"; nothing shaded.



Deduct for an X in the E diagram or shading in the O diagram.

15. “All dogs are mammals” (A). The shaded region is dogs outside mammals, so no dog is outside the class of mammals. (It is not “All mammals are dogs,” which would shade the mammals-only region. Accept the equivalent obverse, “No dogs are non-mammals,” if the student names it as an E.)

Printable answer key (PDF, for the parent)

UNIT 4 · CATEGORICAL PROPOSITIONS · LESSON 4.1 OF 69 · 40 MINUTES

4.1 The Four Forms: A, E, I, O

OBJECTIVES

- Find the quantifier, subject term, copula and predicate term of a categorical proposition.
- Tell the quantity (universal or particular) and the quality (affirmative or negative) of a categorical proposition.
- Name any standard-form categorical proposition as an A, E, I or O proposition.
- Explain that in logic “some” means “at least one,” and that it does not mean “not all.”

KEY TERMS

Categorical proposition

A proposition that says, without any “if,” that one class of things is wholly or partly included in, or excluded from, another class.

Subject term

The term that names the class the proposition is about; in standard form it comes right after the quantifier (S).

Predicate term

The term that names the class the subject is said to be in or not in; in standard form it comes after the copula (P).

Copula

The linking verb that joins subject and predicate: “are” or “are not” (in the singular, “is” or “is not”).

Quantifier

The word that tells how much of the subject class is meant: “all,” “no” or “some.”

Quantity

Whether a proposition is universal (about every member of the subject class) or particular (about at least one member).

Quality

Whether a proposition is affirmative (it includes the subject in the predicate) or negative (it excludes the subject from the predicate).

A, E, I, O propositions

The four standard forms: A, “All S are P” (universal affirmative); E, “No S are P” (universal negative); I, “Some S are P” (particular affirmative); O, “Some S are not P” (particular negative).

CORE IDEAS

From terms to propositions. In Unit 2 you studied terms, the product of the first act of the mind. A term such as “robins” or “tools in the shed” names something but claims nothing. The second act of the mind, judgment, joins two terms by affirming or denying one of the other, and the result, expressed in words, is a proposition: a statement that is true or false. This unit studies one very useful kind, the **categorical proposition**. It speaks of two classes (groups of things) and says, plainly and without any “if,” whether the first is included in the second or excluded from it, wholly or in part.

Four parts in a fixed order. A categorical proposition in standard form has four parts. First comes the **quantifier**: “all,” “no” or “some.” Next comes the **subject term** (S), the class the sentence is about. Then comes the **copula**, the linking verb “are” or “are not.” Last comes the **predicate term** (P), the class the subject is placed in or kept out of. In “All robins are birds,” the quantifier is “all,” the subject term is “robins,” the copula is “are” and the predicate term is “birds.” A term may be a whole phrase. In “Some tools in the shed are things that need oiling,” the subject term is “tools in the shed” and the predicate term is “things that need oiling.”

Quantity: all of the class, or some of it. The **quantity** of a proposition tells how much of the subject class it speaks about. A **universal** proposition speaks about every member of the subject class: “All robins are birds,” “No snakes are mammals.” A **particular** proposition speaks about at least one member, without saying which or

how many: “Some dogs are hunters,” “Some apples are not ripe.” The quantifier shows the quantity. “All” and “no” mark a universal; “some” marks a particular.

Quality: affirming or denying. The **quality** of a proposition tells whether it affirms or denies. An **affirmative** proposition puts the subject class (all of it or part of it) inside the predicate class: “Some dogs are hunters.” A **negative** proposition shuts the subject class (all of it or part of it) out of the predicate class: “No snakes are mammals,” “Some apples are not ripe things.” In standard form a negative proposition always carries “no” at the front or “not” right after the copula. Notice that the little word “no” does two jobs at once: it makes the proposition universal and also negative.

The four forms. Two qualities times two quantities give exactly four forms, and logicians have named them with four vowels for many centuries.

Letter	Form	Quantity and quality	Example
A	All S are P.	universal affirmative	All robins are birds.
E	No S are P.	universal negative	No snakes are mammals.
I	Some S are P.	particular affirmative	Some dogs are hunters.
O	Some S are not P.	particular negative	Some apples are not ripe things.

The letters come from two Latin verbs: *AffIrmo*, “I affirm,” gives A and I, the affirmatives; *nEgO*, “I deny,” gives E and O, the negatives. In each word the first vowel names the universal and the second the particular.

“Some” means “at least one.” In logic, “some” means “at least one, and possibly all.” This is narrower than everyday speech, where “Some of the eggs are cracked” often hints that the rest are fine. Logic does not count hints. If Caleb says, “Some of the pies at the fair were blueberry,” and in fact every pie was blueberry, his statement is still true; it simply tells less than he could have told. So “Some S are P” never by itself tells you that “Some S are not P.” If you want to say “some but not all,” you must say two things: some are, and some are not.

Where the “not” belongs. A proposition is negative only when the denial belongs to the copula, the joining itself. “Some apples are not ripe things” is negative: it keeps part of the apples out of the class of ripe things. But “Some apples are unripe things” is affirmative: it puts part of the apples into the class of unripe things. The “un-” belongs to the predicate term, not to the copula. In the same way, “All non-members

are guests" is an A proposition about the class of non-members. Always ask, "Is the subject being put in, or shut out?"

Why four little forms matter. It may seem odd to spend a whole unit on four shapes of sentence. But a great deal of what we say can be put into one of these shapes, and once it is, its logical behavior becomes clear. The rest of this unit shows how to put everyday sentences into standard form, which terms each form covers completely, what each form tells you about the other three, and how to draw each one with circles. In Unit 5 you will join three such propositions into a syllogism and test whether the conclusion follows.

> **From history.** Aristotle laid the groundwork for this lesson in his short work *On Interpretation*. There he sorted statements into affirmations and denials and showed that a statement may be about a class universally ("every man") or not universally ("some man"). In the early sixth century the Roman Christian scholar Boethius translated the work into Latin and wrote long commentaries on it, and through him medieval schools learned it. Medieval teachers then gave the four forms their vowel names, taken from the Latin verbs *affirmo* and *nego*, so that a student could name a form in a single letter. Students have used those letters ever since.

> **Practice habit.** When you hear a general claim today, ask two questions: Does it speak of all, none, or some? And does it put the subject in, or shut it out?

> **Enrichment: Aristotle on affirming and denying.** In *On Interpretation*, Aristotle explains, roughly, that a simple statement either affirms something of something or denies something of something, and that every affirmation has a denial that is its exact opposite. He also notices that "every man is white" and "no man is white" are opposed in a different way from "every man is white" and "not every man is white." Before reading Lesson 4.4, try to say in your own words what the difference is.

WORKED EXAMPLES

Example 1. Name the four parts of this proposition, give its quantity and quality, and name its form: “No poison ivy plants are safe things to touch.”

▼ **Show solution**

1. Find the quantifier at the front: “No.”
2. Find the subject term, the class the sentence is about: “poison ivy plants.”
3. Find the copula: “are.” (In an E proposition the denial is carried by “no” at the front, so the copula is simply “are.”)
4. Find the predicate term, everything after the copula: “safe things to touch.”
5. Quantity: “no” speaks of every poison ivy plant, so the proposition is universal. Quality: it shuts poison ivy plants out of the class of safe things to touch, so it is negative.
6. A universal negative is an E proposition.

Answer: Quantifier “no,” subject term “poison ivy plants,” copula “are,” predicate term “safe things to touch.” It is universal and negative, an E proposition.

Example 2. Label each proposition A, E, I or O.

1. Some calves in the north pasture are not weaned animals.
2. All tulips in the front bed are red flowers.
3. Some wrens are birds that nest in boxes.
4. No players on the bench are injured players.

▼ **Show solution**

1. For each sentence, ask two questions: Is the quantifier "all/no" (universal) or "some" (particular)? Is the subject put in (affirmative) or shut out (negative)?
2. 1. "Some" is particular; "are not" shuts the calves out. Particular negative: O.
3. 2. "All" is universal; the tulips are put into the class of red flowers. Universal affirmative: A.
4. 3. "Some" is particular; the wrens are put in. Particular affirmative: I.
5. 4. "No" is universal and negative. Universal negative: E.

Answer: 1: O. 2: A. 3: I. 4: E.

Example 3. Hannah counted the eggs in the basket and every one was brown. Her brother says, "Some of the eggs in the basket are brown eggs." Is his statement true or false in logic?

▼ **Show solution**

1. In logic, "some" means "at least one, and possibly all."
2. Every egg was brown, so at least one egg was brown.
3. The statement is therefore true. It tells less than the whole truth (he could have said "All"), but saying less is not saying something false.

Answer: True. "Some" means "at least one," so it is true even when all of the eggs are brown.

PRACTICE

1. Easy Name the four parts of a standard-form categorical proposition, in order.

▼ **Show answer**

Quantifier, subject term, copula, predicate term.

2. **Easy** Write the standard form for each letter and give its quantity and quality: A, E, I, O.

▼ Show answer

A: All S are P (universal affirmative). E: No S are P (universal negative). I: Some S are P (particular affirmative). O: Some S are not P (particular negative).

3. **Easy** Give the subject term and the predicate term: “All maple trees in the park are trees that turn red in the fall.”

▼ Show answer

Subject term: “maple trees in the park.” Predicate term: “trees that turn red in the fall.”

4. **Easy** What does the word “some” mean in logic?

▼ Show answer

“At least one” (possibly all). It does not mean “not all.”

5. **Medium** Give the quantity, quality and letter of each.
   1. No sheepdogs on the Lindqvist farm are lazy animals.
   2. Some loaves in the bakery case are rye loaves.
   3. Some players on the soccer team are not seventh graders.
   4. All hammers on the pegboard are tools.

▼ Show answer

1: universal negative, E. 2: particular affirmative, I. 3: particular negative, O. 4: universal affirmative, A.

6. **Medium** Using the subject term “books on Grandpa’s shelf” and the predicate term “books about birds,” write one proposition of each form: A, E, I and O.

▼ Show answer

A: All books on Grandpa’s shelf are books about birds. E: No books on Grandpa’s shelf are books about birds. I: Some books on Grandpa’s shelf are books about birds. O: Some books on Grandpa’s shelf are not books about birds.

7. **Medium** Mrs. Ortiz says, “Some of the tomatoes on the vine are ripe tomatoes.” Caleb says, “So some of them are not ripe.” Does Caleb’s statement follow from hers? Explain.

▼ Show answer

No. In logic “some” means “at least one, and possibly all,” so her statement would still be true if every tomato were ripe. It does not tell us that any tomato is unripe. (In conversation her words may hint that not all are ripe, but a hint is not a statement.)

8. **Medium** Explain why the word “no” in an E proposition does two jobs at once.

▼ Show answer

It shows the quantity, universal (it speaks of every member of the subject class), and it shows the quality, negative (it shuts the subject class out of the predicate class).

9. **Hard** Give the letter of each, and explain the difference: (a) “Some kittens in the litter are not black animals.” (b) “Some kittens in the litter are non-black animals.”

▼ Show answer

(a) is O: the “not” belongs to the copula, so it shuts part of the kittens out of the class of black animals. (b) is I: the copula is plain “are,” and the “non-” belongs to the predicate term, so it puts part of the kittens into the class of non-black animals. (The two say much the same thing about the world, but they have different forms.)

10. **Hard** Is “All non-members of the club are guests at the picnic” affirmative or negative? Universal or particular? Name its form and explain.

▼ Show answer

It is universal and affirmative, an A proposition. The quantifier is “all,” and the copula “are” has no “not”; the “non-” belongs to the subject term “non-members of the club,” so the sentence puts every member of that class into the class of guests.

11. Hard Mrs. Ortiz claims, “All jars of jam in the pantry are labeled jars.” Caleb finds one unlabeled jar of jam in the pantry. (a) What form is her claim? (b) Write, in standard form, the proposition Caleb’s discovery shows to be true, and name its form.

▼ **Show answer**

(a) A (universal affirmative). (b) “Some jars of jam in the pantry are not labeled jars,” an O proposition. (Accept “are unlabeled jars,” an I proposition with the predicate “unlabeled jars,” if the student labels it correctly as I.) One unlabeled jar is enough, because “some” means at least one.

12. Challenge (a) Can an I proposition and an O proposition with the same subject and predicate both be true? (b) Can an A and an E with the same subject and predicate both be true, if the subject class has members? Give an example using the cows in a herd for each part.

▼ **Show answer**

(a) Yes. If the herd has some brown cows and some cows of other colors, then “Some cows in the herd are brown animals” and “Some cows in the herd are not brown animals” are both true. (b) No. If there is at least one cow in the herd, “All cows in the herd are brown animals” and “No cows in the herd are brown animals” cannot both be true, since that cow would have to be both brown and not brown. (Both can be false: in a mixed herd, neither “all” nor “no” is true.) Lesson 4.4 names these relations.

COMMON MISTAKES

- Reading “some” as “some but not all.” In logic “some” means “at least one,” and it is still true if all are.
- Taking only one word as the predicate term. The predicate term is everything after the copula, such as “things that need oiling.”
- Calling a proposition negative because a negative word appears somewhere in it. It is negative only when the denial belongs to the copula (“are not”) or the quantifier (“no”); “Some apples are unripe things” is affirmative.
- Mixing up the letters. Remember the Latin: *AffIrmo* gives the affirmatives A and I, *nEgO* the negatives E and O.

> **Scripture.** “But let your communication be, Yea, yea; Nay, nay” (Matthew 5:37, KJV)
>
> Every categorical proposition either affirms or denies. Our Lord asks for speech that affirms plainly and denies plainly, and logic trains us to know exactly what we are affirming or denying.

UNIT 4 · CATEGORICAL PROPOSITIONS · LESSON 4.2 OF 69 · 45 MINUTES

4.2 Standard Form

OBJECTIVES

- State the rules for writing a categorical proposition in standard form.
- Translate everyday sentences with missing or unusual quantifiers and with verb or adjective predicates into standard form.
- Translate “only,” “none but,” “the only” and “not all” correctly, and explain why “All S are not P” is ambiguous.
- Check a translation by asking whether it is true in exactly the same cases as the original.

KEY TERMS

Standard form

The fixed pattern of a categorical proposition: quantifier (all, no, some), subject term, copula (are, are not), predicate term, with both terms written as names of classes.

Translation

Rewriting an everyday sentence in standard form so that it says the same thing, no more and no less.

Class term

A noun or noun phrase that names a class, such as “birds” or “things that need oiling,” rather than an adjective or a verb.

Exclusive proposition

A proposition with “only” or “none but,” which says that the predicate belongs to the subject class alone. “Only S are P” is translated “All P are S.”

Singular proposition

A proposition whose subject is one named individual, such as “Mount Pell is snowy.” Traditional logic treats it as universal, because it speaks of the whole of its subject.

Ambiguous sentence

A sentence that can fairly be read in more than one way, such as “All the cookies are not burned,” which may mean an E or an O proposition.

CORE IDEAS

Why translate? People rarely speak in standard form. We say “Robins sing,” “Nobody missed practice” and “Only members may use the pool.” Each of these makes a categorical claim, but its logical shape is hidden under the variety of English. Translating a sentence into standard form brings the shape into the light, so that we can name its form, see which terms it distributes and test any argument it belongs to. A good translation says exactly what the original says, no more and no less.

The pattern to aim for. A standard-form proposition has a quantifier (“all,” “no” or “some”), a subject term, a copula (“are” or “are not”) and a predicate term. Both terms must be **class terms**, nouns or noun phrases that name a group of things. So when the predicate is an adjective or a verb, add a noun that names a class. “Some dogs are loud” becomes “Some dogs are loud animals.” “Robins sing” becomes “All robins are birds that sing.” “Some hens laid eggs today” becomes “Some hens are hens that laid eggs today.” Repeating the noun may sound stiff, but it keeps both terms clear.

Unusual quantifiers. Many words besides “all,” “no” and “some” carry quantity. “Every,” “each,” “any” and “whoever” usually mean “all”: “Every trumpet player in the band owns a mute” becomes “All trumpet players in the band are people who own a mute.” A sentence with no quantifier at all, such as “Snakes are reptiles” or “A whale is a mammal,” usually means “all,” but read it with care, since “Dogs are in the yard” plainly means only some. “Nobody,” “none,” “nothing” and “not a single” mean “no.” “A few,” “several,” “many,” “most” and “there are” become “some.” Translating “most” as “some” loses information, but standard form has no word for “most,” and “some” is the part that is surely claimed.

Singular propositions. A sentence about one named thing, such as “Mount Pell is covered with snow,” has no quantifier. Traditional logic treats such a **singular**

proposition as universal, because it speaks of the whole of its subject: there is only one Mount Pell, and the sentence speaks of all of it. You may write it as "All places identical to Mount Pell are snow-covered places," or simply recognize "Mount Pell is a snow-covered place" as having the force of an A proposition. A negative singular, such as "Caleb is not a goalkeeper," has the force of an E proposition.

"Only" and "none but" turn the terms around. "Only members may use the pool" does not say that every member uses the pool. It says that anyone who uses the pool is a member. So "Only S are P" is translated **"All P are S"**: the term after "only" becomes the predicate. "None but" works the same way: "None but ticket holders are admitted" becomes "All persons admitted are ticket holders." Be careful with "the only," which does *not* turn the terms around: "The only animals in the barn are goats" becomes "All animals in the barn are goats."

"Not all" and the ambiguous "all ... not." "Not all the peaches are ripe" denies the A proposition "All the peaches are ripe," and so says "Some peaches are not ripe peaches," an **O**. "Not every" works the same way. But a sentence of the shape "All S are not P" is **ambiguous**. "All the cookies are not burned" might mean that none of them is burned ("No cookies are burned cookies," an E), or that not all of them are ("Some cookies are not burned cookies," an O). When the context settles the meaning, follow it. When it does not, say that the sentence is ambiguous and give both readings, or ask the speaker.

Time and place words. Words such as "always," "whenever," "never," "wherever" and "sometimes" quantify over times or places. Use "times" or "places" as the class term. "Whenever the creek floods, the lower field is muddy" becomes "All times the creek floods are times the lower field is muddy." "Hannah never forgets her lunch" becomes "No days are days Hannah forgets her lunch," and "Sometimes the bus is late" becomes "Some days are days the bus is late."

Check the translation. After translating, test your work. Ask: Is there any situation in which the original is true and my version false, or the other way round? Suppose someone translates "Only seniors are on the yearbook staff" as "All seniors are on the yearbook staff." Picture a senior who is not on the staff. The original can still be true, but the translation is false, so the translation is wrong. The right one is "All members of the yearbook staff are seniors." Checking cases like this is the surest way to catch a reversed "only."

From history. Logicians have long known that ordinary speech hides the four forms. The *Port-Royal Logic* (1662), written by Antoine Arnauld and Pierre Nicole for the students of the Port-Royal schools in France, gave careful attention to sentences that look simple but say more than one thing, among them "exclusive" sentences built with "only" and "exceptive" sentences built with "except." It became one of the most widely used logic books in Europe and was put into English more than once, including Thomas Spencer Baynes's translation of 1850. Its authors wanted logic to help people think clearly about real speech, not only about textbook examples.

Practice habit. When you hear "only" today, pause and ask which way the sentence really runs: "Only S are P" means "All P are S."

WORKED EXAMPLES

Example 1. Translate into standard form and name the form: "Only registered players may enter the tournament."

▼ **Show solution**

1. "Only" marks an exclusive proposition: "Only S are P" becomes "All P are S."
2. The term after "only" is "registered players." It becomes the predicate.
3. The other term is "people who may enter the tournament." It becomes the subject.
4. Write the A proposition: "All people who may enter the tournament are registered players."
5. Check: if some registered player chooses not to enter, the original is still true, and so is the translation. Good.

Answer: "All people who may enter the tournament are registered players." This is an A proposition.

Example 2. Translate into standard form and name the form: “Not all the peaches in the crate are ripe.”

▼ **Show solution**

1. “Not all” denies the whole A proposition “All the peaches in the crate are ripe.”
2. To deny that all are ripe is to say that at least one is not ripe.
3. Make the predicate a class term: “ripe peaches.”
4. Write the O proposition: “Some peaches in the crate are not ripe peaches.”

Answer: “Some peaches in the crate are not ripe peaches.” This is an O proposition. (It is not the E “No peaches in the crate are ripe peaches,” which says far more.)

Example 3. Translate into standard form: “Every cat in the barn hunts mice.”

▼ **Show solution**

1. “Every” means “all.”
2. The subject term is “cats in the barn.”
3. The predicate is a verb phrase, “hunts mice.” Turn it into a class term: “animals that hunt mice.”
4. Write the A proposition.

Answer: “All cats in the barn are animals that hunt mice.” (Accept any class term with the same meaning, such as “mouse hunters.”) A proposition.

PRACTICE

1. Easy Translate into standard form and name the form: “Every trumpet player in the band owns a mute.”

   ▼ **Show answer**

   “All trumpet players in the band are people who own a mute.” A. (Accept any class term with the same meaning, such as “mute owners.”)

2. **Easy** Translate into standard form and name the form: “No one on the team missed practice.”

   ▼ **Show answer**

   “No members of the team are people who missed practice.” E. (Accept any wording with the same terms, such as “No players on the team are players who missed practice.”)

3. **Easy** Translate into standard form and name the form: “A few of the hens laid eggs today.”

   ▼ **Show answer**

   “Some hens are hens that laid eggs today.” I. (Accept “Some hens are egg layers today” or any class term with the same meaning.)

4. **Easy** Translate into standard form and name the form: “Snakes are reptiles.”

   ▼ **Show answer**

   “All snakes are reptiles.” A. (With no quantifier, the sentence plainly means every snake.)

5. **Medium** Translate into standard form and name the form: “Only ticket holders are admitted to the concert.”

   ▼ **Show answer**

   “All people admitted to the concert are ticket holders.” A. The term after “only” becomes the predicate. (Accept “persons” or “those” for “people.”)

6. **Medium** Translate into standard form and name the form: “Not all the pumpkins in the patch are ready to pick.”

   ▼ **Show answer**

   “Some pumpkins in the patch are not pumpkins ready to pick.” O. (Accept any class term with the same meaning, such as “ripe pumpkins.”) It is not an E.

7. **Medium** Translate into standard form and name the form: “None but members may use the pool.”

   ▼ **Show answer**

   “All people who may use the pool are members.” A. “None but S are P” is translated “All P are S.” (Accept “persons” or “those” for “people.”)

8. **Medium** Translate into standard form and name the form: “Mount Pell is covered with snow.”

   ▼ **Show answer**

   It is a singular proposition, treated as universal: “All places identical to Mount Pell are snow-covered places,” an A. (Accept “Mount Pell is a snow-covered place” if the student names it as having the force of an A.)

9. **Hard** “All the cookies in the jar are not burned.” Explain why this sentence is ambiguous, and give both standard-form readings with their letters.

   ▼ **Show answer**

   “All … are not” can mean “none” or “not all.” Reading 1: “No cookies in the jar are burned cookies” (E). Reading 2: “Some cookies in the jar are not burned cookies” (O), that is, not all of them are burned. Without more context we cannot tell which the speaker meant.

10. **Hard** Translate both sentences into standard form, and explain why they say different things: (a) “The only animals in the barn are goats.” (b) “Only animals in the barn are goats.”

    ▼ **Show answer**

    (a) “All animals in the barn are goats”: there is nothing in the barn but goats. (b) “All goats are animals in the barn”: there are no goats anywhere else. They differ: (a) is true if the barn holds only goats while other goats graze in the field, but then (b) is false. “The only” keeps the terms in order; “only” reverses them.

11. **Hard** Translate into standard form and name the form: “Whenever the creek floods, the lower field is muddy.”

    ▼ **Show answer**

    “All times the creek floods are times the lower field is muddy.” A. (Accept “occasions” or “days” in place of “times.”)

12. **Challenge** Caleb translates "Only seniors are on the yearbook staff" as "All seniors are on the yearbook staff." Describe a situation that shows his translation is wrong, and give the correct translation.

▼ **Show answer**

Suppose every member of the yearbook staff is a senior, but some seniors are not on the staff. Then the original sentence is true and Caleb's translation is false, so they do not say the same thing. The correct translation is "All members of the yearbook staff are seniors." (Any situation with a senior not on the staff, while the staff has only seniors, works.)

COMMON MISTAKES

- Translating "Only S are P" as "All S are P." The term after "only" becomes the predicate: "All P are S."
- Reversing the terms after "the only." "The only S are P" means "All S are P."
- Translating "not all" as an E. "Not all S are P" denies the A and means "Some S are not P."
- Leaving an adjective or verb as the predicate. Add a class noun: "loud animals," "birds that sing."
- Treating "All S are not P" as clearly E or clearly O. It is ambiguous; use the context, or give both readings.

> **Scripture.** "A word fitly spoken is like apples of gold in pictures of silver." (Proverbs 25:11, KJV)
>
> A word fitted to its meaning is a treasure. Translating into standard form is practice in saying exactly what we mean, no more and no less.

UNIT 4 · CATEGORICAL PROPOSITIONS · LESSON 4.3 OF 69 · 40 MINUTES

4.3 Distribution

OBJECTIVES

- Define what it means for a term to be distributed in a proposition.
- State which terms each of the four forms distributes: A the subject, E both, I neither, O the predicate.
- Explain why each form distributes the terms it does, using examples.

- Explain that distribution depends on the form of a proposition, not on facts about the world.

KEY TERMS

Distributed term

A term is distributed in a proposition when the proposition says something about every member of the class the term names.

Undistributed term

A term about whose whole class the proposition says nothing; it speaks of only part of the class, or leaves the rest unknown.

Extension

The whole set of things a term names, all of its members (from Unit 2). A distributed term is taken in its full extension.

Universal subject

The subject of an A or E proposition. Because the proposition speaks of all of the subject class, its subject is distributed.

Negative predicate

The predicate of an E or O proposition. Because the subject is shut out of the whole predicate class, the predicate is distributed.

CORE IDEAS

Does the proposition cover the whole class? Every categorical proposition has two terms, and each term names a class. For each term we can ask a simple question: Does this proposition tell us something about *every* member of that class? If it does, the term is **distributed**. If it speaks of only part of the class, or says nothing about the class as a whole, the term is **undistributed**. In Unit 2 you met the extension of a term, the whole set of things it names. A distributed term is used in its full extension.

A distributes its subject only. "All robins are birds" tells us something about every robin: each one is a bird. So "robins" is distributed. But it tells us nothing about every bird. It says that robins are somewhere among the birds; it does not say that all birds are robins, or anything about sparrows and owls. So "birds" is

undistributed. Picture the robins as a small circle drawn inside a large circle of birds. The sentence fills in the whole robin circle but only part of the bird circle.

E distributes both terms. "No snakes are mammals" says of every snake that it is outside the class of mammals, so "snakes" is distributed. It also says, of every mammal, that it is outside the class of snakes: if even one mammal were a snake, the sentence would be false. So "mammals" is distributed too. A universal negative builds a wall between two whole classes, and a wall between two whole classes says something about every member of each.

I distributes neither term. "Some dogs are hunters" says that at least one dog is a hunter. It speaks of only part of the class of dogs, so "dogs" is undistributed. And it says that at least one hunter is a dog, which is again only part of the class of hunters, so "hunters" is undistributed. An I proposition reports a meeting place between two classes and nothing about either class as a whole.

O distributes its predicate only. This is the case students find hardest. "Some dogs are not retrievers" speaks of only part of the class of dogs, so "dogs" is undistributed. But look at what it says about retrievers. It picks out at least one dog and says that this dog is not any retriever at all: not this retriever, not that one, not one in the whole class. To shut something out of a class, you must shut it out of the whole class. So the predicate "retrievers" is distributed. Think of a guest list: to say that Caleb is not on the list, you must have checked every name on it.

A short rule to remember. The four results fit one sentence: *universals distribute their subjects; negatives distribute their predicates.*

Form	Subject	Predicate
A: All S are P	distributed	undistributed
E: No S are P	distributed	distributed
I: Some S are P	undistributed	undistributed
O: Some S are not P	undistributed	distributed

A and E are universal, so their subjects are distributed. E and O are negative, so their predicates are distributed. I is neither universal nor negative, so it distributes nothing.

Form, not facts. Distribution depends on what a proposition *says*, not on what happens to be true in the world. “All equilateral triangles are equiangular triangles” is an A proposition, so its predicate is undistributed, even though geometry teaches that every equiangular triangle is in fact equilateral. The sentence itself makes no claim about every equiangular triangle; that further truth would need a second proposition. When you name distribution, look only at the form: A, E, I or O.

Why distribution matters. Distribution is a tool for catching reasoning that claims more than it has earned. If Caleb argues, “All robins are birds, so all birds are robins,” the term “birds” is undistributed in his premise but distributed in his conclusion. His conclusion speaks of every bird, though his premise spoke of only some. In the next lessons you will see that this is why an A proposition cannot simply be converted. In Unit 5 two of the rules for syllogisms are rules about distribution: the middle term must be distributed at least once, and no term may be distributed in the conclusion unless it was distributed in a premise.

> **From history.** The idea behind distribution grew out of the medieval study of *supposition*, that is, of how a term stands for things in a sentence. Thirteenth-century logic textbooks, such as the widely copied *Summaries of Logic* of Peter of Spain, explained that in “every man runs” the word “man” stands for each and every man, one by one, while in “some man runs” it does not. Later teachers boiled this down into the simpler language of “distributed” and “undistributed” terms. In English, Richard Whately’s *Elements of Logic* (1826) used these words to state the rules of the syllogism, and generations of students learned them that way.

> **Practice habit.** When you hear a sweeping claim today, ask: Which class is this really speaking about as a whole, and which only in part?

WORKED EXAMPLES

Example 1. Which terms are distributed in "No owls are daytime hunters"? Explain.

▼ **Show solution**

1. Name the form: "No S are P" is E.
2. Subject: the sentence speaks of every owl (each one is outside the class of daytime hunters), so "owls" is distributed.
3. Predicate: it also speaks of every daytime hunter (none of them is an owl), so "daytime hunters" is distributed.
4. Check with the rule: E is universal (subject distributed) and negative (predicate distributed).

Answer: Both terms are distributed: "owls" and "daytime hunters."

Example 2. Which terms are distributed in "Some fences on the farm are not painted fences"? Explain.

▼ **Show solution**

1. Name the form: "Some S are not P" is O.
2. Subject: the sentence speaks of only some fences on the farm, so "fences on the farm" is undistributed.
3. Predicate: it says that the fences it picks out are outside the whole class of painted fences; none of them is any painted fence. So "painted fences" is distributed.
4. Check with the rule: O is particular (subject undistributed) and negative (predicate distributed).

Answer: Only the predicate, "painted fences," is distributed.

Example 3. Hannah says, "In 'All squares are rectangles,' the predicate must be distributed, because the sentence is about all squares." Is she right?

▼ **Show solution**

1. The form is A, which distributes its subject only.
2. "About all squares" shows that the subject, "squares," is distributed.
3. The sentence says nothing about every rectangle; it does not say that all rectangles are squares (and they are not). So "rectangles" is undistributed.

Answer: No. "Squares" (the subject) is distributed, but "rectangles" (the predicate) is undistributed.

PRACTICE

1. Easy What does it mean for a term to be distributed in a proposition?

   ▼ **Show answer**

   The proposition says something about every member of the class the term names (it takes the term in its full extension).

2. Easy For each form, say which terms are distributed: A, E, I, O.

   ▼ **Show answer**

   A: the subject only. E: both subject and predicate. I: neither. O: the predicate only.

3. Easy Which terms are distributed in "All carrots are root vegetables"?

   ▼ **Show answer**

   Only the subject, "carrots." (A distributes its subject; "root vegetables" is undistributed.)

4. Easy Which terms are distributed in "Some players on the team are goalkeepers"?

   ▼ **Show answer**

   Neither term. It is an I proposition, which distributes neither subject nor predicate.

5. **Medium** Which terms are distributed in “No sparrows in the barn are hawks”? Explain.

▼ Show answer

Both. It is an E proposition: it shuts every sparrow in the barn out of the class of hawks, and every hawk out of the class of sparrows in the barn.

6. **Medium** Which terms are distributed in “Some books in the library are not novels”? Explain.

▼ Show answer

Only the predicate, “novels.” It is an O proposition: it speaks of only some books in the library, but says that those books are outside the whole class of novels.

7. **Medium** Translate “Only members may vote at the meeting” into standard form, then say which terms are distributed.

▼ Show answer

“All people who may vote at the meeting are members.” It is an A proposition, so the subject, “people who may vote at the meeting,” is distributed and “members” is undistributed.

8. **Medium** Explain in your own words why an O proposition distributes its predicate.

▼ Show answer

An O proposition says that some S is not a P at all, that is, not any member of the P class. To shut something out of a class, the proposition must speak of the whole class, so the predicate is distributed. (Accept any clear explanation, such as the guest-list example.)

9. **Hard** In fact, every equiangular triangle is equilateral. Is the predicate distributed in “All equilateral triangles are equiangular triangles”? Explain.

▼ Show answer

No. It is an A proposition, and an A does not distribute its predicate. Distribution depends on what the proposition says, not on what is true in the world; this sentence says nothing about every equiangular triangle.

10. Hard Give the form and the distributed terms of each. (a) “Some apples in the bin are unripe things.” (b) “No non-members are voters.”

▼ **Show answer**

(a) I (the copula is affirmative; “unripe things” is the predicate term), so neither term is distributed. (b) E, so both “non-members” and “voters” are distributed.

11. Hard Caleb argues, “All robins are birds, so all birds are robins.” Use distribution to explain what went wrong.

▼ **Show answer**

In the premise (an A) “birds” is the predicate and is undistributed: it speaks of only part of the class of birds. In the conclusion “birds” is the subject of an A and is distributed: it speaks of every bird. The conclusion claims something about all birds that the premise never said, so it does not follow.

12. Challenge Using distribution, explain why “Some S are P” can be turned into “Some P are S” without saying more than the original, but “All S are P” cannot be turned into “All P are S.”

▼ **Show answer**

In “Some S are P” neither term is distributed, and in “Some P are S” neither is distributed either, so the new sentence claims nothing about a whole class that the old one did not. In “All S are P,” P is undistributed; in “All P are S,” P becomes the subject of an A and is distributed. The new sentence would speak of every P when the old one did not, so it says more than the original and does not follow.

COMMON MISTAKES

- Thinking that the predicate of an A proposition is distributed because the sentence begins with “all.” “All” covers the subject only.
- Forgetting that O distributes its predicate. To shut something out of a class, you must shut it out of the whole class.
- Deciding distribution by facts about the world. Distribution depends only on the form of the proposition.
- Mixing up the subject and predicate after translating. Translate first (for example, reverse an “only”), then decide distribution.

REFERENCE GUIDE

An eight-page, color-coded Logic guide: what logic is, terms and definitions, the A E I O propositions, the square of opposition and immediate inferences, syllogisms with the six rules and Venn tests, the named forms, informal fallacies of relevance, presumption and ambiguity, propositional logic with truth tables, conditionals, valid forms and short proofs, the classical tradition, dialectic, argument maps and practice. The booklet prints on 11×17 paper and folds to letter size. Prints in color and reads clearly in black and white.

> **Scripture.** "For all have sinned, and come short of the glory of God" (Romans 3:23, KJV)
>
> Here "all" is a universal: it speaks of every one of us and leaves no one out. Paul's point depends on that word, and it is why the next verse speaks of grace offered freely.

UNIT 4 · CATEGORICAL PROPOSITIONS · LESSON 4.4 OF 69 · 45 MINUTES

4.4 The Square of Opposition

OBJECTIVES

- Name the four relations of the traditional square: contradictories, contraries, subcontraries and subalterns.
- State the rule for each relation, assuming that the subject class has members.
- Given the truth or falsity of one corner, find which of the other three are true, false or undetermined.
- Refute a universal claim with a single, plainly true counterexample, and explain why examples cannot prove a universal.

KEY TERMS

Opposition

The relation between two categorical propositions that have the same subject and the same predicate but differ in quantity, in quality, or in both.

Contradictories

A and O, and E and I: pairs that differ in both quantity and quality. They cannot both be true and cannot both be false; exactly one is true.

Contraries

A and E: they cannot both be true, but they can both be false.

Subcontraries

I and O: they cannot both be false, but they can both be true.

Subalternation

The relation of a universal to the particular of the same quality (A to I, E to O). Truth flows down from the universal to the particular; falsity flows up from the particular to the universal.

Undetermined

Said of a proposition whose truth value does not follow from what is given; it might be true or false.

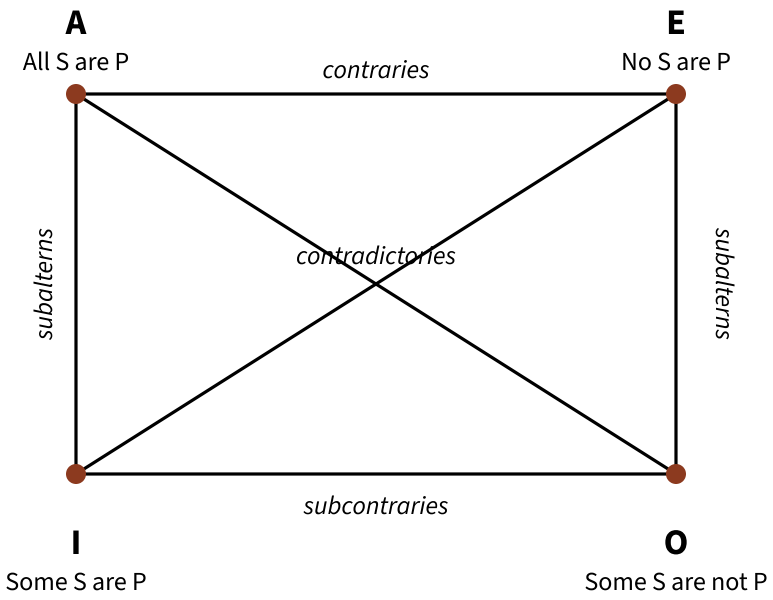
Counterexample

A single case that shows a universal claim to be false, such as one S that is not P against "All S are P."

CORE IDEAS

Four propositions about the same two classes. Take one subject and one predicate, say "cows in the herd" and "brown animals." From them we can make four propositions: all are, none are, some are, some are not. These four are not independent of one another. If we learn that one of them is true, or false, we can often tell at once about some of the others. Propositions with the same subject and predicate that differ in quantity, quality or both are said to stand in **opposition**, and the traditional diagram that shows their relations is called the **square of opposition.**

The square. The universals stand on top, A at the left and E at the right; the particulars stand below, I under A and O under E.



In this lesson we follow the traditional rule and **assume that the subject class has at least one member.** (There are cows in the herd.) Lesson 4.6 shows what changes if we drop that assumption.

Contradictories: exactly one is true. The diagonals join A with O and E with I. These pairs differ in both quantity and quality, and they are **contradictories**: they cannot both be true, and they cannot both be false. "All cows in the herd are brown" is false exactly when at least one cow in the herd is not brown, that is, exactly when "Some cows in the herd are not brown" is true. To deny one contradictory is to affirm the other. This is the strongest relation on the square, and, as Lesson 4.6 shows, the only one every logician accepts.

Contraries and subcontraries. Across the top, A and E are **contraries**: they cannot both be true, but they can both be false. "All cows in the herd are brown" and "No cows in the herd are brown" cannot both be true (a cow cannot be brown and not brown), but in a herd of mixed colors both are false. Across the bottom, I and O are **subcontraries**: they cannot both be false, but they can both be true. In the mixed herd, "Some are brown" and "Some are not brown" are both true. They cannot both be false, because if no cow were brown and no cow failed to be brown, the herd would have no cows, and we assumed it has.

Subalternation: truth flows down, falsity flows up. Down each side, a universal is joined to the particular of the same quality: A to I, E to O. If the universal is true, the particular below it is true: if all the cows are brown, then (since there are cows) some of them are. If the particular is false, the universal above it is false: if not even one cow is brown, it cannot be that all of them are. But the other directions tell us nothing. A false A leaves the I undetermined, and a true I leaves the A undetermined. Remember: *truth flows down, falsity flows up*.

Working the square. Given one corner, start with its contradictory, which is always settled; then use the other relations. Suppose "All tomatoes in the crate are ripe" is *true*. Its contradictory O is false. Its contrary E cannot also be true, so E is false. Truth flows down, so I is true. Now suppose instead that the A is *false*. Then the O is true, but the E and I are **undetermined**: perhaps no tomatoes are ripe, perhaps some are. "Undetermined" is a real answer; it means the information given does not settle the question.

Given	A	E	I	O
A true	T	F	T	F
E true	F	T	F	T
I true	?	F	T	?
O true	F	?	?	T
A false	F	?	?	T
E false	?	F	T	?
I false	F	T	F	T
O false	T	F	T	F

Do not memorize the table; learn to rebuild it from the four rules.

One counterexample brings down a universal. Because A and O are contradictories, a universal affirmative is refuted by a single case. "All fruits are sweet" is false if one fruit is not sweet, and lemons are fruits that are plainly not sweet. In the same way, "No birds are swimmers" is refuted by one bird that swims, such as a duck. A good counterexample must be plainly true to everyone, so choose a case no one can dispute. And notice the other side of the lesson: no number of examples can prove a universal. Caleb may have watched hundreds of birds fly, but the penguin is still a bird that cannot fly.

From history. Aristotle described the key relations in *On Interpretation*. He saw that "every man is white" and "not every man is white" cannot be true together or false together, and that "every man is white" and "no man is white" cannot both be true, though both may be false. The diagram came later. A square of these relations appears in a Latin handbook on logic traditionally credited to Apuleius in the second century, and in the sixth century Boethius drew it in his commentary on Aristotle. From there it passed into medieval schools, where it became one of the most familiar pictures in all of learning, copied into logic books for well over a thousand years.

Practice habit. When you hear "all" or "never" today, look for one plain counterexample before you accept the claim.

WORKED EXAMPLES

Example 1. Given that "Some players on the team are left-handed players" is *false*, what can you say about the A, E and O propositions with the same terms?

▼ **Show solution**

1. The given proposition is an I, and it is false.
2. Contradictory first: the E, "No players on the team are left-handed players," must be true.
3. Falsity flows up: if not even one player is left-handed, the A, "All players on the team are left-handed players," is false.
4. Truth flows down from the true E to its subaltern: the O, "Some players on the team are not left-handed players," is true. (Also, I and O cannot both be false.)

Answer: E true, A false, O true.

Example 2. Given that "No apples in the bin are bruised apples" is *false*, what can you say about the others?

▼ **Show solution**

1. The given proposition is an E, and it is false.
2. Contradictory: the I, "Some apples in the bin are bruised apples," is true.
3. Contrary: A and E can both be false, so a false E tells us nothing about the A. Perhaps all the apples are bruised, perhaps only some. The A is undetermined.
4. Subaltern: falsity does not flow down, so the O, "Some apples in the bin are not bruised apples," is undetermined. (It is true if only some are bruised, false if all are.)

Answer: I true; A and O undetermined.

Example 3. Refute the claim "All metals are solid at room temperature" with a counterexample, and write the counterexample as a standard-form proposition.

▼ **Show solution**

1. To refute an A, find one S that is not P: one metal that is not solid at room temperature.
2. Mercury is a metal, and it is a liquid at room temperature; this is plain and well known (it was used in old thermometers).
3. The counterexample makes the contradictory O true, so the A is false.

Answer: Mercury is a metal that is liquid at room temperature, so "Some metals are not things solid at room temperature" (O) is true, and the A is false.

PRACTICE

1. Easy On the traditional square, name the relation between (a) A and O, (b) A and E, (c) I and O, (d) E and O.

   ▼ **Show answer**

   (a) Contradictories. (b) Contraries. (c) Subcontraries. (d) Subalternation (E is the superaltern, O the subaltern).

2. **Easy** State the rule for contraries and the rule for subcontraries.

   ▼ **Show answer**

   Contraries (A and E) cannot both be true but can both be false. Subcontraries (I and O) cannot both be false but can both be true.

3. **Easy** If an A proposition is true, what is the truth value of the I with the same terms (on the traditional square)?

   ▼ **Show answer**

   True. Truth flows down from A to its subaltern I.

4. **Easy** Write the contradictory of "Some dogs at the shelter are not puppies."

   ▼ **Show answer**

   "All dogs at the shelter are puppies."

5. **Medium** Given that "All students in the choir are sopranos" is *false*, give the truth value of the E, I and O with the same terms.

   ▼ **Show answer**

   O ("Some students in the choir are not sopranos") is true, as the contradictory of a false A. E and I are undetermined: a false A could go with a true E or with a false E.

6. **Medium** Given that "Some birds at the feeder are finches" is *true*, give the truth value of the A, E and O.

   ▼ **Show answer**

   E is false (contradictory). A and O are undetermined: some finches at the feeder fits both "all are finches" and "some are not."

7. **Medium** Given that "No horses in the stable are saddled horses" is *true*, give the truth value of the A, I and O.

   ▼ **Show answer**

   I is false (contradictory). A is false (contrary of a true E). O is true (subaltern of a true E; truth flows down).

8. Medium Refute "All fruits are sweet" with a counterexample, and write the proposition your counterexample makes true.

▼ Show answer

Lemons are fruits that are not sweet. (Accept limes, cranberries or any plainly sour fruit.) This makes true the O, "Some fruits are not sweet things," the contradictory of the A, so the A is false.

9. Hard Given that "Some cakes on the table are not chocolate cakes" is *false*, give the truth value of the A, E and I.

▼ Show answer

A is true (contradictory of a false O). E is false (contrary of a true A). I is true (subaltern of a true A). (Or: falsity flows up from O to E, and I and O cannot both be false.)

10. Hard Hannah says, "It is possible that both 'Some eggs in the basket are cracked eggs' and 'Some eggs in the basket are not cracked eggs' are false." On the traditional square, is she right? Explain.

▼ Show answer

No. They are subcontraries, which cannot both be false. If both were false, then no egg would be cracked and no egg would fail to be cracked, so the basket would hold no eggs, but the traditional square assumes the subject class has members. Every egg is either cracked or not, so at least one statement is true.

11. Hard Caleb says, "Every bird can fly. I have seen hundreds of birds fly, and never one that couldn't." Refute his claim, and explain why his hundreds of examples do not prove it.

▼ Show answer

Penguins are birds that cannot fly (accept ostriches, emus or kiwis), so "Some birds are not flying things" (accept "not things that can fly") is true and his A is false. Examples cannot prove a universal, because a universal speaks of every member of the class, including ones he has not seen; a single exception is enough to make it false.

12. **Challenge** Suppose that both the A and the E with the same terms are false. What must be true of the I and the O? Explain using the square, and give an example with real classes.

▼ **Show answer**

Both I and O are true. The E is false, so its contradictory I is true; the A is false, so its contradictory O is true. Example: “All dogs at the park are black dogs” and “No dogs at the park are black dogs” are both false when the park has black dogs and dogs of other colors, and then “Some dogs at the park are black dogs” and “Some dogs at the park are not black dogs” are both true. (Accept any example with a mixed class.)

COMMON MISTAKES

- Treating contraries as contradictories. A and E can both be false; only A and O (or E and I) are exact opposites.
- Letting falsity flow down or truth flow up. A false A does not make the I false, and a true I does not make the A true.
- Refusing to say “undetermined.” When the given facts do not settle a corner, “undetermined” is the correct answer.
- Choosing a counterexample that is not plainly true. Pick a case no one can dispute, such as lemons against “All fruits are sweet.”
- Thinking many examples prove a universal. They can support it, but only a counterexample settles the question, and it settles it against.

> **Scripture.** “Prove all things; hold fast that which is good.” (1 Thessalonians 5:21, KJV)
>
> To “prove” here means to test. A universal claim is tested by looking for a single case that breaks it; what survives honest testing is worth holding fast.

UNIT 4 · CATEGORICAL PROPOSITIONS · LESSON 4.5 OF 69 · 45 MINUTES

4.5 Conversion, Obversion, Contraposition

OBJECTIVES

- Define an immediate inference and form the complement of a term.
- Perform conversion, obversion and contraposition on any standard-form proposition.

- State which of these inferences are valid for each form, including conversion and contraposition by limitation.
- Show that an invalid immediate inference is invalid with a plain example of a true premise and a false conclusion.

KEY TERMS

Immediate inference

An inference from a single proposition to another, with no second premise and no middle term.

Complement

The class of everything that is not in a given class; the complement of "voters" is "non-voters." The complement of "non-voters" is "voters."

Conversion

Swapping the subject and predicate terms. Valid for E and I; valid for A only by limitation (All S are P, so Some P are S); never valid for O.

Obversion

Changing the quality (affirmative to negative or back) and replacing the predicate with its complement, keeping the subject and quantity. Valid for all four forms.

Contraposition

Replacing the subject with the complement of the predicate and the predicate with the complement of the subject. Valid for A and O; valid for E only by limitation; never valid for I.

By limitation

Drawing a particular conclusion where a universal one would not follow; valid on the traditional reading, which assumes the subject class has members.

Equivalent propositions

Propositions that are true in exactly the same cases, so that each can be inferred from the other.

CORE IDEAS

Inferring from a single proposition. An **immediate inference** moves from one proposition to another with no second premise. The square of opposition gave us some. This lesson adds three more, which rearrange or change the terms of a

proposition: conversion, obversion and contraposition. For each, the question is whether the new proposition *must* be true whenever the old one is. If so, the inference is valid; if one plain case makes the old proposition true and the new one false, it is invalid.

Conversion: swap the terms. To **convert** a proposition, exchange its subject and predicate. For E and I this is valid, and the result is equivalent to the original. "No reptiles are warm-blooded animals" converts to "No warm-blooded animals are reptiles," and "Some farmers are beekeepers" to "Some beekeepers are farmers." For A, simple conversion is invalid: "All dogs are mammals" is true, but "All mammals are dogs" is false. Traditional logic allows conversion of A **by limitation**: from "All dogs are mammals" infer the weaker "Some mammals are dogs." For O, conversion is never valid: "Some animals are not dogs" is true, but "Some dogs are not animals" is false.

Distribution explains the pattern. Conversion works when it does not make any term cover more than it did before. In E both terms are distributed, and in I neither is, so swapping them changes nothing. In A the predicate is undistributed, but after simple conversion it would become the distributed subject of a new A, a claim about every mammal that the original never made. In O the subject is undistributed, but after conversion it would become the distributed predicate of a new O. Each failure is a case of saying more than you were given.

Complements. The other two inferences use the **complement** of a term: the class of everything outside it. The complement of "voters" is "non-voters," and the complement of "non-swimmers" is "swimmers." Be careful: a complement is not the same as an opposite. The complement of "hot soups" is "non-hot soups," which includes warm and cool ones; "cold soups" is only part of that class. Likewise the complement of "fresh loaves" is "non-fresh loaves," not necessarily "stale loaves." Use "non-" whenever you are unsure.

Obversion: change the quality, complement the predicate. To **obvert**, keep the subject and the quantity, change the quality, and replace the predicate with its complement. Obversion is valid for all four forms, and the result is always equivalent.

Form	Original	Obverse
A	All S are P.	No S are non-P.
E	No S are P.	All S are non-P.
I	Some S are P.	Some S are not non-P.
O	Some S are not P.	Some S are non-P.

“All the roses in the garden are red flowers” becomes “No roses in the garden are non-red flowers.” Saying a thing is in a class is the same as saying it is not outside that class.

Contraposition: swap and complement both terms. To **contrapose**, replace the subject with the complement of the predicate and the predicate with the complement of the subject. For A it is valid: “All squares are rectangles” becomes “All non-rectangles are non-squares,” and if every square is a rectangle, anything that is not a rectangle cannot be a square. For O it is valid: “Some S are not P” becomes “Some non-P are not non-S.” For E it is valid only by limitation: “No S are P” gives “Some non-P are not non-S.” For I it is never valid: “Some animals are non-dogs” is true, but its contrapositive, “Some dogs are non-animals,” is false.

A shortcut that explains contraposition. Contraposition is simply obversion, then conversion, then obversion again. Start with “All S are P.” Obvert: “No S are non-P.” Convert (valid for E): “No non-P are S.” Obvert: “All non-P are non-S.” Now try it with an I, “Some S are P.” Obvert: “Some S are not non-P.” The next step would convert an O, which is never valid, so the chain breaks. That is why I has no contrapositive, and why E, whose obverse is an A, can be contraposed only by limitation.

The whole picture. Conversion: E and I valid; A by limitation; O never. Obversion: all four. Contraposition: A and O valid; E by limitation; I never. The inferences “by limitation” depend on the traditional assumption that the subject class has members, and Lesson 4.6 shows that they fail without it. When an inference is invalid, prove it the way a logician does: find one plain case where the premise is true and the conclusion false.

From history. Conversion is as old as logic itself. In the *Prior Analytics*, Aristotle showed that a universal negative and a particular affirmative convert, that a universal affirmative converts only into a particular, and that a particular negative does not convert at all, and he used these conversions to prove which syllogisms are valid. Medieval logicians called conversion of the A by limitation *conversio per accidens*. Obversion and contraposition were worked out more fully by later logicians, and their present names became standard in English logic textbooks of the nineteenth century. The rules themselves, however, are simple consequences of what the four forms say.

Practice habit. Before you turn someone's sentence around today ("So you're saying that all…"), check whether it really converts.

WORKED EXAMPLES

Example 1. Convert "No reptiles are warm-blooded animals," and say whether the conversion is valid.

▼ **Show solution**

1. Name the form: E.
2. Swap the subject and predicate: "No warm-blooded animals are reptiles."
3. E converts validly: both terms are distributed before and after, so nothing new is claimed. A wall between two classes stands from both sides.

Answer: "No warm-blooded animals are reptiles." Valid; the two propositions are equivalent.

Example 2. Obvert "Some seeds in the tray are not sprouted seeds."

▼ **Show solution**

1. Name the form: O. Keep the subject ("seeds in the tray") and the quantity ("some").
2. Change the quality: negative becomes affirmative, so "are not" becomes "are."
3. Replace the predicate with its complement: "sprouted seeds" becomes "non-sprouted seeds."

Answer: "Some seeds in the tray are non-sprouted seeds." Obversion is valid for every form, so the two are equivalent.

Example 3. Contrapose "All members of the chess club are students," and say whether the contrapositive is valid. Then explain why contraposition fails for "Some animals are non-dogs."

▼ **Show solution**

1. Form: A. Replace the subject with the complement of the predicate ("non-students") and the predicate with the complement of the subject ("non-members of the chess club").
2. Result: "All non-students are non-members of the chess club." Valid for A: anyone who is not a student cannot be a member, since all members are students.
3. Now "Some animals are non-dogs" (an I, and true: cats are animals that are not dogs). Its contrapositive is "Some non-non-dogs are non-animals," that is, "Some dogs are non-animals."
4. That is plainly false. A true premise with a false conclusion shows the inference is invalid.

Answer: "All non-students are non-members of the chess club" (valid). Contraposition of an I is invalid: "Some animals are non-dogs" is true, but "Some dogs are non-animals" is false.

PRACTICE

1. Easy In one sentence each, say how to perform conversion, obversion and contraposition.

   ▼ **Show answer**

   Conversion: swap the subject and predicate. Obversion: keep the subject and quantity, change the quality, and replace the predicate with its complement. Contraposition: replace the subject with the complement of the predicate and the predicate with the complement of the subject.

2. Easy Give the complement of (a) “voters,” (b) “non-swimmers,” (c) “ripe pears.”

   ▼ **Show answer**

   (a) “non-voters.” (b) “swimmers.” (c) “non-ripe pears,” that is, things that are not ripe pears (accept either wording; when the talk is only about pears, “unripe pears” may also be accepted, since it then names every pear that is not ripe).

3. Easy Convert “No cats are reptiles.” Is the conversion valid?

   ▼ **Show answer**

   “No reptiles are cats.” Valid; E converts simply.

4. Easy Obvert “All roses in the garden are red flowers.”

   ▼ **Show answer**

   “No roses in the garden are non-red flowers.” (Accept “No roses in the garden are flowers that are not red.”)

5. Medium Convert “Some farmers in the county are beekeepers.” Is the conversion valid?

   ▼ **Show answer**

   “Some beekeepers are farmers in the county.” Valid; I converts simply.

6. Medium Obvert “Some loaves on the rack are not fresh loaves.”

   ▼ **Show answer**

   “Some loaves on the rack are non-fresh loaves.” (Do not write “stale loaves” unless every loaf that is not fresh counts as stale; “non-fresh” is the exact complement.)

7. **Medium** Contrapose "All squares are rectangles." Is the contrapositive valid?

▼ **Show answer**

"All non-rectangles are non-squares." Valid; A contraposes. (In plain English: anything that is not a rectangle is not a square.)

8. **Medium** Hannah converts "All dogs are mammals" into "All mammals are dogs." What is wrong, and what may she validly infer by converting on the traditional reading?

▼ **Show answer**

An A does not convert simply: "mammals" is undistributed in the premise but would be distributed in the conclusion, and cats show the result is false. On the traditional reading she may convert by limitation: "Some mammals are dogs."

9. **Hard** Show with an example that the conversion of an O proposition is invalid.

▼ **Show answer**

"Some animals are not dogs" is true (a cat is an animal that is not a dog), but its converse, "Some dogs are not animals," is false. A true premise with a false conclusion shows the inference is invalid. (Accept any example with a plainly true premise and a plainly false converse.)

10. **Hard** Contrapose "Some students in the class are not athletes," say whether it is valid, and put the result in plain English.

▼ **Show answer**

"Some non-athletes are not non-students in the class." Valid; O contraposes. In plain English: some people who are not athletes are students in the class.

11. **Hard** Caleb obverts "No soups on the menu are hot soups" as "All soups on the menu are cold soups." What mistake did he make? Give the correct obverse.

▼ **Show answer**

He used an opposite ("cold") instead of the complement ("non-hot"). A soup can be neither hot nor cold (warm or cool), so his version claims more than the original. The correct obverse is "All soups on the menu are non-hot soups."

12. **Challenge** Show step by step that the contrapositive of “All S are P” can be reached by obverting, converting and obverting again. Then show why the same three steps fail for “Some S are P.”

    ▼ **Show answer**

    A: “All S are P.” Obvert: “No S are non-P.” Convert (valid for E): “No non-P are S.” Obvert: “All non-P are non-S,” the contrapositive. I: “Some S are P.” Obvert: “Some S are not non-P,” an O. The next step would convert an O, which is never valid, so the chain breaks and I has no valid contrapositive.

COMMON MISTAKES

- Converting an A simply. “All dogs are mammals” does not give “All mammals are dogs”; it gives only “Some mammals are dogs” (by limitation).
- Converting an O. “Some animals are not dogs” does not give “Some dogs are not animals.”
- Using an opposite instead of a complement. The complement of “hot soups” is “non-hot soups,” not “cold soups.”
- Forgetting to change the quality in obversion, or changing the quantity by mistake. Obversion keeps “all” or “some” and switches only affirmative and negative.
- Contraposing an I, or contraposing an E without limitation.

> **Scripture.** “Seest thou a man that is hasty in his words? there is more hope of a fool than of him.” (Proverbs 29:20, KJV)
>
> Hasty words often come from turning a statement around without thinking, as when “All dogs are mammals” becomes “All mammals are dogs.” The rules of immediate inference teach us to slow down and say only what follows.

UNIT 4 · CATEGORICAL PROPOSITIONS · LESSON 4.6 OF 69 · 40 MINUTES

4.6 Existential Import and the Modern Square

OBJECTIVES

- Define existential import and say which forms have it on the traditional reading and on Boole’s reading.

- Explain, with an example of an empty class, why contraries, subcontraries and subalternation fail on Boole's reading while contradictories survive.
- Name the immediate inferences that fail without existential import (conversion and contraposition by limitation).
- Decide when the difference matters, and repair an inference by adding a premise that the subject class has members.

KEY TERMS

Existential import

A proposition has existential import if it asserts that at least one member of its subject class exists.

Empty class

A class with no members, such as "unicorns in the barn" or "students who have finished next year's homework."

Traditional reading

The reading of the four forms, used in Lessons 4.4 and 4.5, that assumes the subject class has at least one member, so that all four forms carry existential import.

Boolean (modern) reading

The reading, following George Boole, on which only I and O assert existence; "All S are P" means only that nothing is an S that is not a P, and "No S are P" that nothing is both.

Modern square of opposition

The square as it stands on the Boolean reading: only the contradictory diagonals (A–O and E–I) remain.

Vacuously true

True because there is nothing to be a counterexample; on the Boolean reading every universal proposition about an empty class is vacuously true.

CORE IDEAS

A hidden assumption. The traditional square in Lesson 4.4 rested on one assumption: the subject class has at least one member. We assumed there were cows in the herd. Most of the time that assumption is harmless, because we usually talk about things that exist. But not always. Consider "All unicorns in the barn are white." There are no unicorns in the barn. Is the sentence true, false or neither? The answer depends on whether a universal proposition claims that its subject exists.

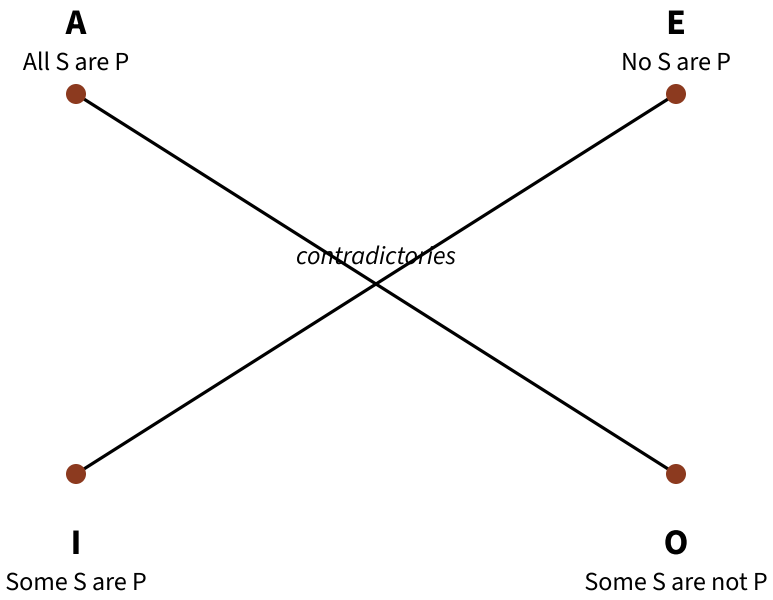
Existential import. A proposition has **existential import** if it asserts that its subject class has at least one member. Particular propositions plainly do. “Some apples in the bin are bruised apples” says that there is at least one apple in the bin and it is bruised; if the bin is empty, the sentence is false. The question is about the universals. Does “All S are P” assert that there are S’s, or does it only say that no S fails to be P?

Boole’s answer. The English mathematician George Boole, building an algebra of classes in 1854, read universal propositions as making *no* claim about existence. On this **Boolean** or modern reading, “All S are P” means “Nothing is an S that is not a P,” and “No S are P” means “Nothing is both an S and a P.” Each forbids a certain kind of thing without saying that anything exists. This fits many universals well. A sign that says “All trespassers will be fined” is not made false by a summer in which no one trespasses. A rule may stand even if no one ever breaks it.

What happens with an empty class. Apply the Boolean reading to the unicorns. “All unicorns in the barn are white” forbids a unicorn in the barn that is not white, and there is none, so the sentence is true. “No unicorns in the barn are white” forbids a white unicorn in the barn, and there is none, so it is true as well. Such universals are **vacuously true**: true because there is nothing to be a counterexample. Meanwhile “Some unicorns in the barn are white” and “Some unicorns in the barn are not white” both claim that a unicorn is in the barn, so both are false.

Three relations fall. Look at what the unicorns have done to the square. A and E are both true, so they are no longer contraries (contraries cannot both be true). I and O are both false, so they are no longer subcontraries (subcontraries cannot both be false). A is true while I is false, so truth no longer flows down by subalternation, and the same holds for E and O. On the Boolean reading, then, contraries, subcontraries and subalternation all fail, because each depended on the subject class having members.

The contradictories stand. Only the diagonals survive. “All S are P” says there is no S outside P; “Some S are not P” says there is at least one S outside P. Whether or not any S’s exist, exactly one of these is true. The same holds for E and I. The result is the **modern square**, which keeps only the contradictory diagonals.



Immediate inferences on the modern reading. The inferences that work "by limitation" also fall. "All trespassers on this field will be fined" does not give "Some people who will be fined are trespassers," since there may be no trespassers at all; so conversion of A by limitation fails, and so does contraposition of E by limitation. The others stand on either reading: conversion of E and I, obversion of all four forms, and contraposition of A and O. None of these ever needed the subject class to have members.

When it matters. The two readings agree whenever we know the subject class has members. If Caleb knows there are cows in his herd, then from "All the cows in our herd are ear-tagged cows" he may safely infer "Some cows in our herd are ear-tagged cows." On the Boolean reading he simply needs one more premise, "There is at least one cow in our herd." The difference matters when the class may be empty: rules, laws, warnings, promises about the future and imaginary things. Good practice is to read universals in Boole's way and to add an existence premise openly whenever an argument needs one. In Unit 5 you will meet syllogisms that are valid on one reading and not the other, and you will be asked to say which reading you are using.

From history. George Boole (1815–1864), a largely self-taught schoolmaster who became a professor of mathematics in Cork, Ireland, published *An Investigation of the Laws of Thought* in 1854. In it he treated classes like quantities in algebra and wrote universal propositions as equations saying that a certain combination of classes is empty. On that treatment a universal says what does not exist, not what does. John Venn, a Cambridge logician, made Boole's reading visible with his diagrams of overlapping circles, published in 1880 and in his *Symbolic Logic* of 1881. The Boolean reading became the standard one in the logic of the twentieth century.

Practice habit. When you hear "all" today, ask: Is this a claim about things that exist, or a rule that would hold even if there were none?

Enrichment: Lewis Carroll takes the other side. Lewis Carroll, who taught mathematics at Oxford, wrote his *Symbolic Logic* (1896) for young readers, and on this question he did not follow Boole. As he put it, roughly, a proposition beginning with "All" is really two propositions in one: it says that some of the things exist, and that none of them lacks the predicate. Carroll knew that other logicians read "All" differently, and he chose the traditional reading on purpose. Which reading do you think fits the sentence "All the cookies on the plate are oatmeal cookies"? Which fits "All visitors must sign in"?

Example 1. On the Boolean reading, give the truth value of each, and explain: (a) All unicorns in the barn are white things. (b) No unicorns in the barn are white things. (c) Some unicorns in the barn are white things. (d) Some unicorns in the barn are not white things.

▼ **Show solution**

1. The class “unicorns in the barn” is empty.
2. (a) The A forbids a unicorn in the barn that is not white. There is none, so (a) is vacuously true.
3. (b) The E forbids a unicorn in the barn that is white. There is none, so (b) is vacuously true.
4. (c) The I claims at least one unicorn in the barn that is white. There is no unicorn in the barn, so (c) is false.
5. (d) The O claims at least one unicorn in the barn that is not white. Again there is none, so (d) is false.
6. Check the contradictories: (a) true and (d) false; (b) true and (c) false. Exactly one of each pair is true, as it must be.

Answer: (a) true, (b) true, (c) false, (d) false. Contraries are both true, subcontraries both false, and A is true while its subaltern I is false; only the contradictories behave as before.

Example 2. Is this inference valid on the Boolean reading? "All students who finish the test early are students who may read quietly. So some students who finish the test early are students who may read quietly." If not, repair it.

▼ **Show solution**

1. The premise is an A; the conclusion is the I below it on the square. This is subalternation.
2. On the Boolean reading the A does not say that any student finishes early. Perhaps no one does, and then the premise is true but the conclusion false.
3. So the inference is invalid on the Boolean reading (though valid on the traditional reading).
4. Repair: add the premise "There is at least one student who finishes the test early." Now the premise forbids that student from failing to be allowed to read, so the conclusion must be true.

Answer: Invalid on the Boolean reading, because the class may be empty. It becomes valid when we add "At least one student finishes the test early."

PRACTICE

1. Easy What is existential import?

   ▼ **Show answer**

   A proposition has existential import if it asserts that at least one member of its subject class exists.

2. Easy On Boole's reading, which of the four forms have existential import?

   ▼ **Show answer**

   Only the particulars, I and O. The universals A and E make no claim that anything exists.

3. Easy Which relation of the traditional square survives on the modern square?

   ▼ **Show answer**

   Contradictories only (A with O, and E with I).

4. **Easy** On the Boolean reading, what does “All S are P” say? What does “No S are P” say?

   ▼ **Show answer**

   “All S are P” says that nothing is an S that is not a P. “No S are P” says that nothing is both an S and a P. Neither says that any S exists.

5. **Medium** There are no dragons in the garden. On the Boolean reading, give the truth value of: (a) All dragons in the garden are green things. (b) No dragons in the garden are green things. (c) Some dragons in the garden are green things. (d) Some dragons in the garden are not green things.

   ▼ **Show answer**

   (a) true (vacuously). (b) true (vacuously). (c) false. (d) false. The universals forbid something that does not exist; the particulars claim a dragon that does not exist.

6. **Medium** Use your answers to question 5 to show which three relations of the traditional square fail on the Boolean reading.

   ▼ **Show answer**

   Contraries fail, because A and E are both true. Subcontraries fail, because I and O are both false. Subalternation fails, because A is true while I is false (and E is true while O is false). Only the contradictories, (a) with (d) and (b) with (c), still have exactly one true.

7. **Medium** On the Boolean reading, an E proposition is given as true. What can you say about the A, I and O with the same terms?

   ▼ **Show answer**

   I is false (contradictory). A and O are undetermined: if the subject class is empty, A is true and O false; if it has members, A is false and O true. (On the traditional reading, A would be false and O true.)

8. Medium Is this inference valid on the Boolean reading? “All trespassers on this field are people who will be fined. So some people who will be fined are trespassers on this field.” Explain.

▼ **Show answer**

No. It is conversion of an A by limitation, which needs the subject class to have members. If no one trespasses, the premise is still true (vacuously) but the conclusion is false. (It is valid on the traditional reading.)

9. Hard A sign on a bus says, “All passengers who lose their tickets must pay the full fare.” On which reading is it best understood, and why? Does the sign say that any passenger has lost a ticket?

▼ **Show answer**

The Boolean reading fits best. The sign states a rule that forbids a passenger who loses a ticket and does not pay; it is meant to hold even if no one ever loses a ticket. So it does not say that any passenger has lost a ticket.

10. Hard Given that an I proposition is false, give the truth values of the A, E and O, first on the traditional reading and then on the Boolean reading.

▼ **Show answer**

Traditional: E true (contradictory), A false (falsity flows up from I), O true (subaltern of the true E). Boolean: E true (contradictory); A and O undetermined, since if the subject class is empty A is true and O false, and if the class has members (none of them P) A is false and O true.

11. Hard Caleb reasons, “All the cows in our herd are ear-tagged cows, so some cows in our herd are ear-tagged cows.” Is his reasoning safe? On the Boolean reading, what premise must he add to make it valid?

▼ **Show answer**

It is safe, because he knows the herd has cows; the two readings agree whenever the subject class is known to have members. On the Boolean reading he must add the premise “There is at least one cow in our herd” (accept any wording that asserts the herd has a cow).

12. **Challenge** Lewis Carroll held that a proposition beginning with "All" does assert that its subject exists. Give one advantage of Boole's reading and one advantage of the traditional reading.

    ▼ **Show answer**

    Sample answer: Boole's reading handles rules, laws and warnings about classes that may be empty ("All trespassers will be fined" is not made false because no one trespasses), and its square has one simple relation that never fails. The traditional reading matches much ordinary speech, where we speak of things we know exist, and it keeps more inferences (the whole square and the inferences by limitation). Accept any reasonable advantage for each side.

COMMON MISTAKES

- Thinking a universal about an empty class must be false. On the Boolean reading it is vacuously true.
- Thinking the contradictories fail too. A and O, and E and I, are exact opposites on both readings.
- Using subalternation or conversion by limitation in an argument about a class that may be empty.
- Forgetting to say which reading you are using when the answer depends on it.

> **Scripture.** "for he that cometh to God must believe that he is, and that he is a rewarder of them that diligently seek him." (Hebrews 11:6, KJV)
>
> The writer states two things to be believed: first that God is (a claim of existence), and then what he is like. Logic also keeps the question "Does it exist?" separate from the question "What is it like?"

UNIT 4 · CATEGORICAL PROPOSITIONS · LESSON 4.7 OF 69 · 40 MINUTES

4.7 Venn Diagrams for Propositions

OBJECTIVES

- Name the four regions of a two-circle Venn diagram and explain what shading, an X and a blank region mean.
- Draw a Venn diagram for each of the four forms: A shades S-only, E shades the overlap, I puts an X in the overlap, O puts an X in S-only.

- Read a Venn diagram back into a standard-form proposition.
- Use diagrams to check conversion, obversion and contradiction, and to see why universals make no existence claim on the Boolean reading.

KEY TERMS

Venn diagram
A drawing of overlapping circles, one for each class, in which shading and X marks show what a proposition says about the regions.

Region
One of the four areas of a two-circle diagram: inside S only, inside both S and P (the overlap), inside P only, and outside both.

Shading
A shaded region is empty: nothing is in it.

X mark
An X in a region means that at least one thing is in it.

Blank region
A region with neither shading nor an X; the proposition says nothing about whether anything is there.

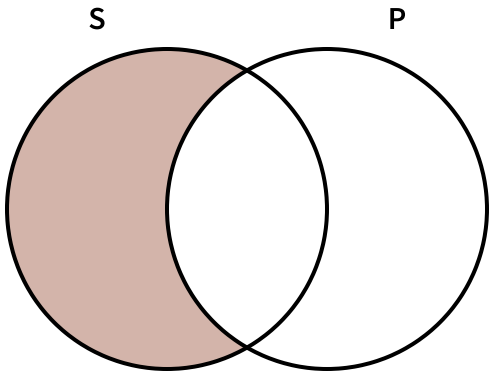
Universe of discourse
Everything being talked about, pictured as the space around the circles; the region outside both circles is part of it.

CORE IDEAS

Pictures of classes. In 1880 John Venn proposed a simple way to draw what a categorical proposition says. Draw two overlapping circles, the left one for the subject class S and the right one for the predicate class P. The circles divide the page into four **regions**: things that are S but not P (the left part of the S circle), things that are both S and P (the overlap, SP), things that are P but not S (the right part of the P circle), and things that are neither (outside both). Every thing in the universe of discourse belongs to exactly one of these regions.

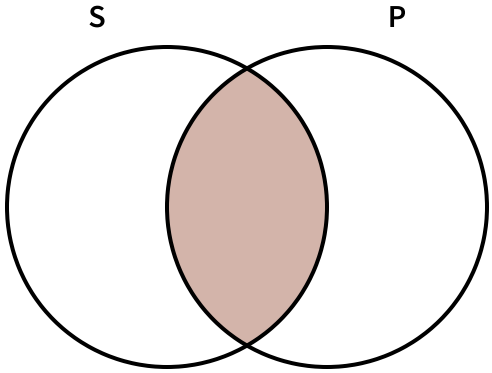
Two marks, and silence. A Venn diagram uses only two marks. **Shading** a region means it is empty: nothing is there. An **X** in a region means at least one thing is there. A region left **blank** means the proposition says nothing about it; there may be something there or nothing. Learn this well, because the most common mistake is to read a blank region as empty. Blank means "no information," not "nothing."

A: shade S-only. “All S are P” says that no S lies outside P. So the part of S that is outside P must be empty: shade the S-only region.



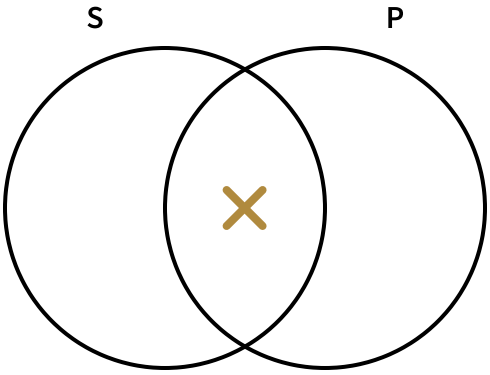
Notice that nothing is marked in the overlap. The diagram does not say that any S exists; it says only that whatever S’s there are, they are inside P.

E: shade the overlap. “No S are P” says that nothing is both S and P. So shade the overlap, the SP region.



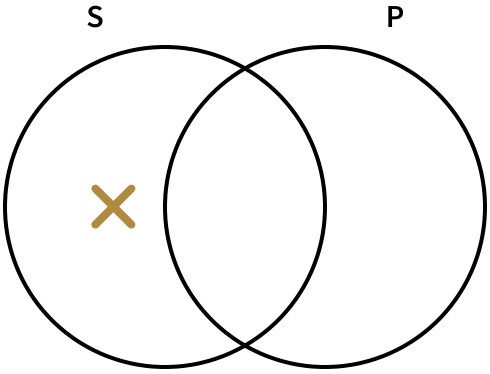
Because the overlap belongs to both circles, the same picture serves for “No P are S.” That is why an E proposition converts: its converse has exactly the same diagram.

I: an X in the overlap. “Some S are P” says that at least one thing is both S and P. So put an X in the overlap.



Again the picture is the same for "Some P are S," so I converts. Nothing is shaded, because an I proposition does not say that any region is empty.

O: an X in S-only. "Some S are not P" says that at least one thing is in S but outside P. So put an X in the S-only region.



Compare it with the diagram for A. The A diagram says the S-only region is empty; the O diagram says something is there. Exactly one of these can be right, and that is what it means for A and O to be contradictories. In the same way, E shades the overlap and I puts an X in it.

The diagrams follow Boole. The diagrams for A and E contain only shading, no X. They say what is empty, not what exists, which is just Boole's reading from Lesson 4.6. That is why you cannot get from the A diagram to the I diagram: nothing in the A diagram puts an X in the overlap. If an argument needs the traditional assumption that some S exists, state it as a separate premise ("There is at least one S") and then mark it. In this course, draw every universal with shading alone.

Checking inferences with pictures. Diagrams make immediate inferences easy to check: two propositions are equivalent exactly when they have the same diagram.

“All dogs are mammals” shades the dogs-only region, but “All mammals are dogs” shades the mammals-only region. Different diagrams, different claims, so the A does not convert simply. “Some animals are not dogs” puts an X in animals-only; “Some dogs are not animals” would put it in dogs-only. Different again, so the O does not convert. When you diagram, label each circle with its term, keep the subject on the left, and check every region before you finish.

> **From history.** Drawing classes as circles is older than Venn. The Swiss mathematician Leonhard Euler used circles to explain the forms of reasoning in a series of letters on science and philosophy written for a German princess in the 1760s. In Euler’s drawings, though, the placing of the circles itself carried the claim, so a drawing could seem to say more than the proposition did. John Venn (1834–1923), a clergyman and logician at Cambridge, improved on this in an 1880 paper and in his *Symbolic Logic* of 1881. He always drew every region and marked the empty ones, so that one diagram could show exactly what a proposition says and leaves unsaid.

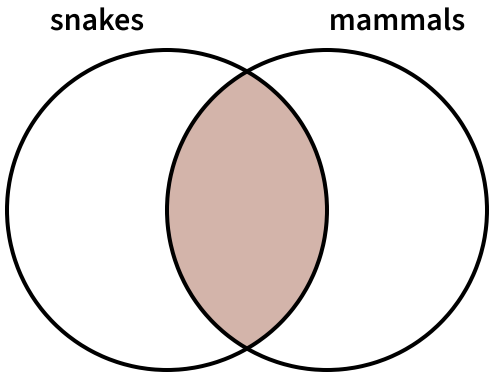
> **Practice habit.** When a claim today seems confusing, sketch two circles on scrap paper and ask: Which region does it say is empty, and which does it say has something in it?

Example 1. Draw a Venn diagram for "No snakes are mammals."

▼ **Show solution**

1. Name the form: E.
2. Draw two overlapping circles. Label the left circle "snakes" (the subject) and the right circle "mammals" (the predicate).
3. An E says nothing is both, so shade the overlap.
4. Check: snakes-only, mammals-only and the outside are left blank, because the proposition says nothing about them.

Answer: Shade the overlap of "snakes" and "mammals."

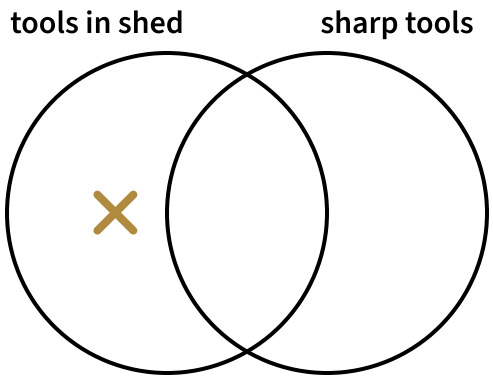


Example 2. Draw a Venn diagram for "Some tools in the shed are not sharp tools."

▼ **Show solution**

1. Name the form: O.
2. Label the left circle "tools in shed" (subject) and the right circle "sharp tools" (predicate).
3. An O says at least one S is outside P, so put an X in the S-only region.
4. Shade nothing: an O says no region is empty.

Answer: An X in the part of "tools in shed" outside "sharp tools."

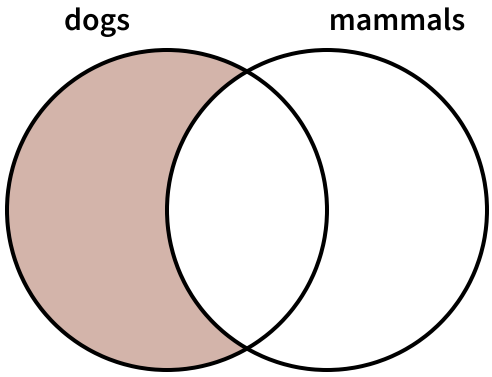


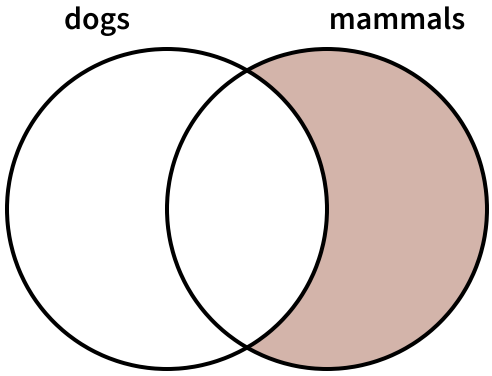
Example 3. Use Venn diagrams to show that "All dogs are mammals" and "All mammals are dogs" do not say the same thing.

▼ **Show solution**

1. Draw both on circles labeled "dogs" (left) and "mammals" (right), so they can be compared.
2. "All dogs are mammals" says no dog is outside the mammals: shade dogs-only.
3. "All mammals are dogs" says no mammal is outside the dogs: shade mammals-only.
4. The shaded regions differ, so the propositions make different claims, and the first does not give the second.

Answer: The first shades dogs-only; the second shades mammals-only. Different diagrams show that simple conversion of an A is invalid.





PRACTICE

1. Easy In a Venn diagram, what does a shaded region mean? An X? A blank region?

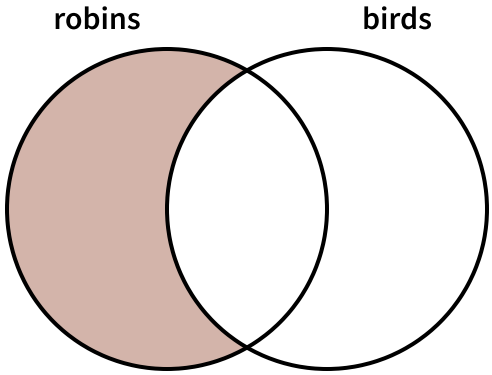
▼ **Show answer**

Shaded: the region is empty. X: at least one thing is in the region. Blank: the proposition says nothing about that region (it may or may not have members).

2. Easy Draw a Venn diagram for "All robins are birds."

▼ **Show answer**

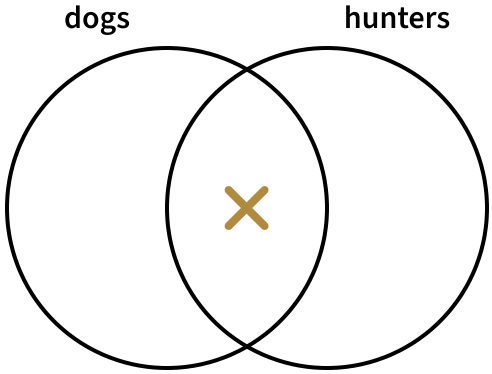
Label the left circle "robins" and the right "birds." Shade the part of "robins" outside "birds."



3. Easy Draw a Venn diagram for "Some dogs are hunters."

▼ **Show answer**

Label the left circle "dogs" and the right "hunters." Put an X in the overlap; shade nothing.



4. **Easy** For each form, say which region is shaded or where the X goes: A, E, I, O.

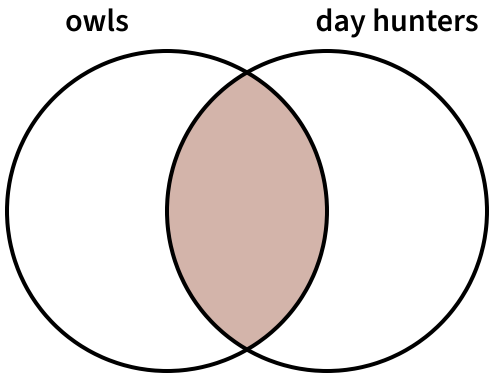
▼ **Show answer**

A: shade S-only. E: shade the overlap (SP). I: X in the overlap. O: X in S-only.

5. **Medium** Draw a Venn diagram for "No owls are daytime hunters."

▼ **Show answer**

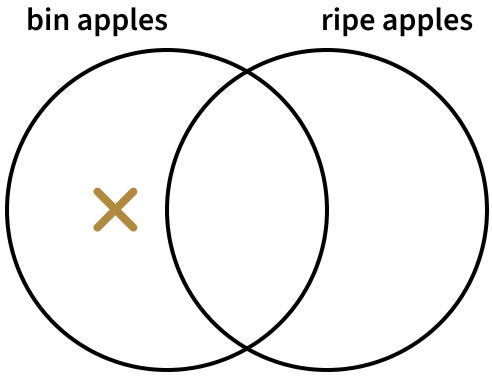
Label the left circle "owls" and the right "day hunters." Shade the overlap.



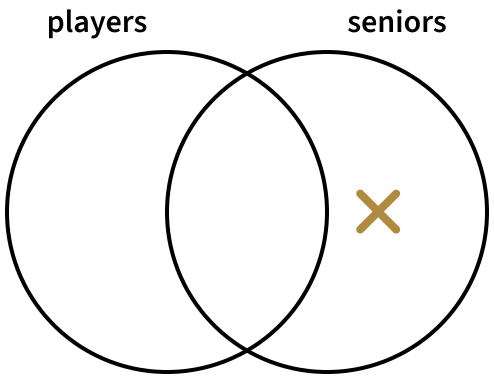
6. **Medium** Draw a Venn diagram for "Some apples in the bin are not ripe apples."

▼ **Show answer**

Label the left circle "bin apples" and the right "ripe apples." Put an X in the part of "bin apples" outside "ripe apples"; shade nothing.



7. **Medium** Write the standard-form proposition this diagram shows.



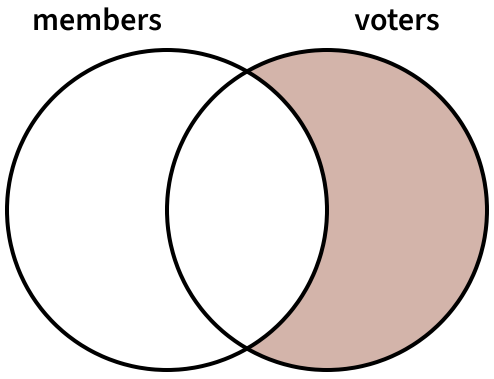
▼ **Show answer**

"Some seniors are not players" (an O). The X is in the seniors-only region, so at least one senior is outside the class of players. (It is not "Some players are not seniors," which would put the X in players-only.)

8. **Medium** Translate "Only members may vote" into standard form, then diagram it on circles labeled "members" (left) and "voters" (right).

▼ **Show answer**

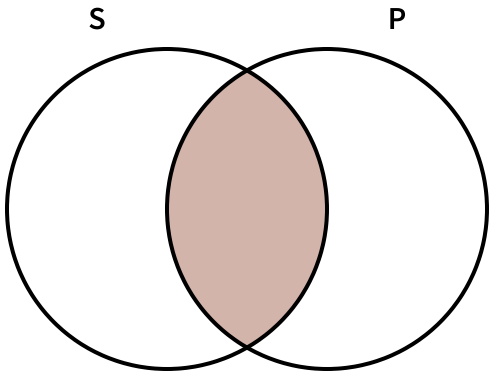
"All voters are members." Shade the part of "voters" outside "members" (the right-hand region).



9. Hard Use a Venn diagram to explain why “No S are P” converts validly to “No P are S.”

▼ **Show answer**

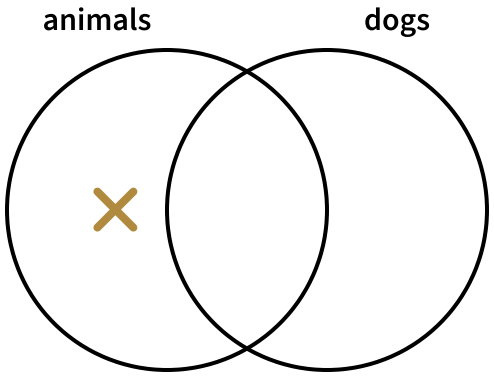
Both propositions say the same region, the overlap of S and P, is empty, so both have exactly the same diagram: the overlap shaded. Propositions with the same diagram are equivalent, so the conversion is valid.

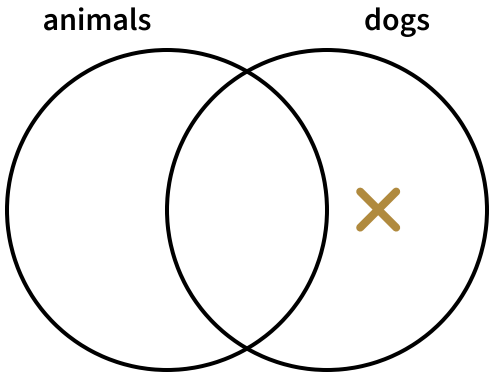


10. Hard Diagram “Some animals are not dogs” and “Some dogs are not animals” on the same pair of labels, and explain what the diagrams show about converting an O.

▼ **Show answer**

With “animals” on the left and “dogs” on the right: the first puts an X in animals-only; the second puts an X in dogs-only. The diagrams differ, so the propositions say different things, and the first does not give the second; conversion of an O is invalid. (The first is true, since cats are animals that are not dogs, and the second is false.)

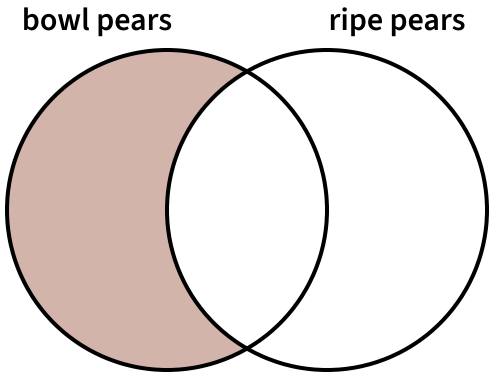


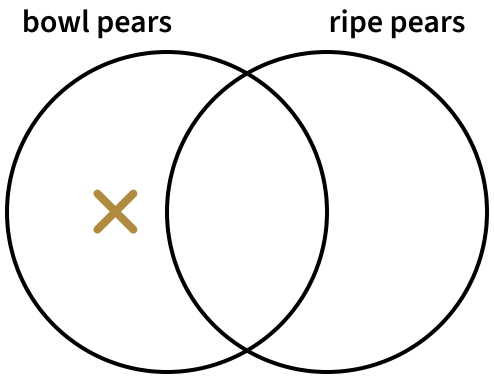


11. Hard Diagram "All pears in the bowl are ripe pears" and "Some pears in the bowl are not ripe pears." Use the diagrams to explain why exactly one of them is true.

▼ **Show answer**

The A shades the region of "bowl pears" outside "ripe pears" (empty); the O puts an X in that same region (at least one thing there). A region is either empty or not, so exactly one diagram is correct: the two are contradictories.

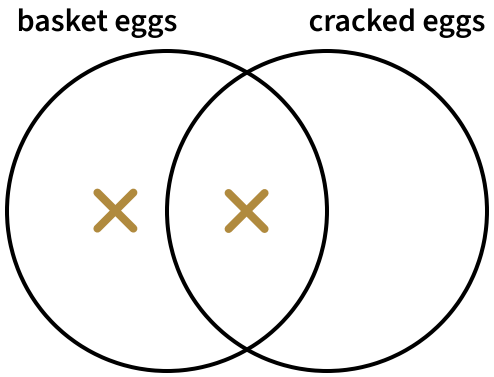




12. **Challenge** Hannah says, “Some, but not all, of the eggs in the basket are cracked.” Translate her statement into two standard-form propositions, and draw one diagram that shows both.

▼ **Show answer**

“Some eggs in the basket are cracked eggs” (I) and “Some eggs in the basket are not cracked eggs” (O). On circles labeled “basket eggs” (left) and “cracked eggs” (right), put one X in the overlap and another X in basket-eggs-only; shade nothing.



COMMON MISTAKES

- Reading a blank region as empty. Only shading means empty; blank means the proposition is silent.
- Putting an X into the A or E diagram. Universals are drawn with shading only; an existence claim must be stated as a separate premise.
- Shading the overlap for an A. For “All S are P,” shade the part of S *outside* P; the overlap is where the S’s are allowed to be.
- Putting the O’s X in the wrong circle. “Some S are not P” puts the X in S-only, not in P-only.
- Forgetting to label the circles, or swapping them between two diagrams you mean to compare.

REFERENCE GUIDE

An eight-page, color-coded Logic guide: what logic is, terms and definitions, the A E I O propositions, the square of opposition and immediate inferences, syllogisms with the six rules and Venn tests, the named forms, informal fallacies of relevance,

presumption and ambiguity, propositional logic with truth tables, conditionals, valid forms and short proofs, the classical tradition, dialectic, argument maps and practice. The booklet prints on 11×17 paper and folds to letter size. Prints in color and reads clearly in black and white.

> **Scripture.** "Write the vision, and make it plain upon tables, that he may run that readeth it." (Habakkuk 2:2, KJV)
>
> The prophet was told to write plainly so that a reader could take it in at a glance. A Venn diagram does the same for a proposition: it makes plain exactly what is claimed and what is left unsaid.

UNIT 5

Categorical Syllogisms

In Unit 4 you learned to read and judge single categorical propositions: the A, E, I and O statements, their distribution, the square of opposition and the immediate inferences. This unit puts those propositions to work. A **categorical syllogism** joins two categorical propositions to draw a third, and for more than two thousand years it was the heart of logic as it was taught in schools from Athens to Oxford. Aristotle studied it first, the medieval schoolmen gave its valid forms names and a memory verse, and Lewis Carroll turned it into puzzles for children.

Students learn to find a syllogism's **major, minor and middle terms**, to put it in standard order, and to name its **mood and figure** (for example, AAA-1). They test syllogisms in two independent ways, with the **six rules of validity** and with **three-circle Venn diagrams**, and learn to name the fallacy when a rule is broken: four terms, undistributed middle, illicit major, illicit minor, exclusive premises and the existential fallacy. They learn the fifteen forms that are valid on any reading, with their medieval names (Barbara, Celarent, Darii, Ferio and the rest), and the extra forms that the traditional reading accepts. Then they carry the syllogism into ordinary life: supplying the missing premise of an **enthymeme**, untangling chains of premises in **sorites** and Carroll-style puzzles, and using the **hypothetical and disjunctive syllogisms** while catching their invalid look-alikes.

This unit completes the study of the third act of the mind in its traditional form, and it closes the first half of the course; the midterm covering Units 1–5 follows it. Unit 6 will take up the hypothetical and disjunctive arguments of Lesson 5.8 again in symbols, with truth tables and proofs, and Unit 9 will use enthymemes and hidden premises in the analysis of real debates. The habits built here (find the conclusion first, keep every term in the same sense, check what each premise really guarantees) will be used in every unit that follows.

Review day and test. On the review day, use the mastery checklist and the review sheet (with its answer key). The unit test is below; a printable copy is ready for test day.

PREREQUISITES

- Identify the four standard-form categorical propositions (A, E, I, O) and their quantity and quality (Lesson 4.1).
- Translate ordinary sentences, including “only,” “none but” and “not all,” into standard form (Lesson 4.2).
- State which terms each of the four propositions distributes (Lesson 4.3).
- Use conversion, obversion and contraposition, and know when each is valid (Lessons 4.5–4.6).
- Draw two-circle Venn diagrams for A, E, I and O propositions (Lesson 4.7).

MASTERY CHECKLIST

- ☐ I can identify the major, minor and middle terms and the major and minor premises of a syllogism, and write it in standard order.
- ☐ I can name the mood and figure of any standard-form syllogism, and build a syllogism of a given mood and figure.
- ☐ I can test a syllogism with the six rules and name the fallacy committed when a rule is broken.
- ☐ I can explain when the traditional and Boolean readings give different verdicts, as with Darapti (AAI-3).
- ☐ I can test a syllogism with a three-circle Venn diagram, drawing universal premises first and placing each X correctly.
- ☐ I can recognize the fifteen unconditionally valid forms by name and recite the first lines of the medieval memory verse.
- ☐ I can supply the missing premise of an enthymeme charitably and test the completed syllogism.
- ☐ I can solve a sorites or Carroll-style puzzle by translating, chaining and drawing the conclusion.
- ☐ I can use modus ponens, modus tollens and the disjunctive syllogism, and catch affirming the consequent, denying the antecedent and affirming a disjunct.

LESSONS IN THIS UNIT

5.1	**The Syllogism**	40 minutes
5.2	**Mood and Figure**	40 minutes
5.3	**The Rules of Validity**	45 minutes
5.4	**Testing with Venn Diagrams**	45 minutes
5.5	**The Named Forms**	40 minutes
5.6	**Enthymemes**	40 minutes
5.7	**Chains of Reasoning: Sorites and Carroll's Puzzles**	45 minutes
5.8	**Hypothetical and Disjunctive Syllogisms**	40 minutes

Unit 5 test

You have 45 minutes. No notes or books. Put every syllogism in standard form and standard order before you judge it. When a question asks you to test a syllogism, say "valid" or "invalid" and give the reason: the form's name, or the rule broken and the fallacy's name. Draw three-circle Venn diagrams where asked, with S at the top left, P at the top right and M at the bottom. Unless a question says otherwise, judge validity on the Boolean reading, but say so when the traditional reading would differ. Question 15 is a short written answer of four to six sentences. The test is worth 100 points.

1. **6 pts**

   Put in standard form and standard order, and name the major, minor and middle terms: "Some members of the youth orchestra practice every day, since every student of Mr. Pell practices every day, and some members of the youth orchestra are students of Mr. Pell."

2. **6 pts**

   Name the mood and figure of each.

   1. All squares are rectangles. No circles are rectangles. Therefore no circles are squares.
   2. Some herbs in Grandma's garden are perennials. All herbs in Grandma's garden are fragrant plants. Therefore some fragrant plants are perennials.
   3. No cats are reptiles. Some pets are cats. Therefore some pets are not reptiles.

3. **4 pts**

   Build a syllogism of the form AEE-4, using terms of your choice.

4. **8 pts**

   Test each with the rules. If it is invalid, name the fallacy.

   1. Some vegetables are things grown in Hannah's garden. All carrots are vegetables. Therefore some carrots are things grown in Hannah's garden.
   2. All trout are fish. All trout are swimmers. Therefore all swimmers are fish.
   3. No birds are mammals. Some pets are not birds. Therefore some pets are not mammals.
   4. All poodles are dogs. No cats are poodles. Therefore no cats are dogs.

5. **6 pts**

   (a) What is wrong with this syllogism? "Every pitcher on the team throws hard. The thing on the picnic table holding the lemonade is a pitcher. So the thing on the picnic table throws hard." (b) Is AAI-3 (Darapti) valid? Answer for both the traditional and the Boolean reading.

6. **8 pts**

   Test with a three-circle Venn diagram: "All the hens in the yard are laying hens. Some hens in the yard are brown hens. Therefore some brown hens are laying hens."

7. **8 pts**

   Test with a three-circle Venn diagram, then check your verdict with the rules: "Some cooks are early risers. All bakers are early risers. Therefore some bakers are cooks."

8. **6 pts**

   (a) Give the mood and figure of Celarent, Festino, Bocardo and Camenes. (b) Which one of Darii, Darapti and Datisi is valid only on the traditional reading, and which class must have members?

9. **6 pts**

   (a) Write the first two lines of the medieval memory verse. (b) Use the letters of Cesare to reduce this syllogism to the first figure: “No insects are eight-legged animals. All spiders are eight-legged animals. Therefore no spiders are insects.”

10. **8 pts**

    An advertisement says, “The new Gallowind kite will fly in any wind, because it is made of ripstop nylon.” (a) Supply the missing premise that makes the argument valid, and name the order of the enthymeme and the form of the completed syllogism. (b) Is the argument sound? Explain.

11. **6 pts**

    Supply the missing premise charitably and name the form: “Hannah can’t enter the junior baking contest; she is over fourteen.”

12. **8 pts**

    Solve, showing your translations: (1) Only goats that are welcome at the tea party wear bow ties. (2) All the goats on Willow Hill eat the laundry. (3) No goat that eats the laundry is welcome at the tea party.

13. **6 pts**

    Is this chain valid? If not, find the broken link and name the fallacy. “All the books on the top shelf are dictionaries. No dictionaries are novels. Some novels are mysteries. So no books on the top shelf are mysteries.”

14. **8 pts**

Name the form of each and say whether it is valid.

1. If the paint is dry, we can hang the picture. We can hang the picture. So the paint is dry.
2. If the well is dry, the pump will not work. The pump works. So the well is not dry.
3. Either the recipe calls for baking soda or it calls for baking powder. It calls for baking soda. So it does not call for baking powder.
4. If the frost comes early, the pumpkins will be small. The frost did not come early. So the pumpkins will not be small.

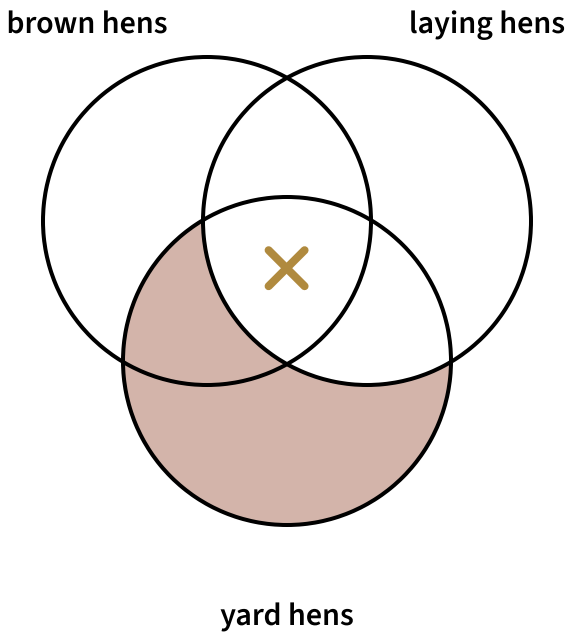
15. **6 pts**

In four to six sentences, explain the difference between the traditional and the Boolean readings of the syllogism, using Darapti (AAI-3) as your example. Say how each reading treats Rule 6 and the Venn diagram.

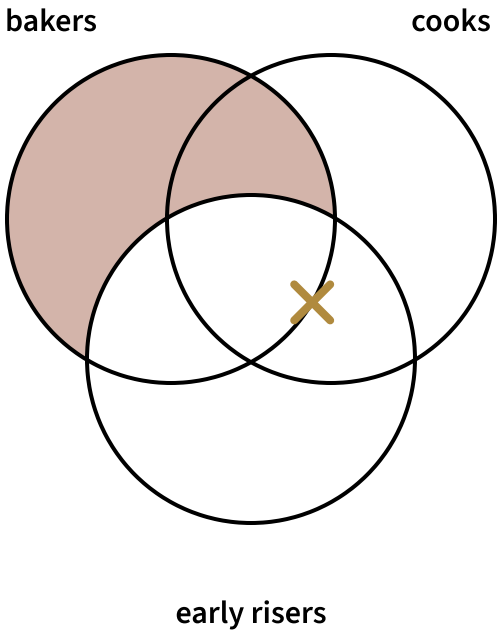
ANSWER KEY

1. 1. All students of Mr. Pell are people who practice every day. (major premise)
   2. Some members of the youth orchestra are students of Mr. Pell. (minor premise)
   Therefore, some members of the youth orchestra are people who practice every day.
   Major term (P) = people who practice every day; minor term (S) = members of the youth orchestra; middle term (M) = students of Mr. Pell. (It is Darii, AII-1.)
2. 1: AEE-2 (S = circles, P = squares, M = rectangles; the middle term is the predicate of both premises). 2: IAI-3 (S = fragrant plants, P = perennials, M = herbs in Grandma's garden; the middle term is the subject of both premises). 3: EIO-1 (S = pets, P = reptiles, M = cats; the middle term is the subject of the major premise and the predicate of the minor). (All three are valid: Camestres, Disamis and Ferio.)
3. Pattern: All P are M. No M are S. Therefore no S are P. Sample: "All robins are birds. No birds are mammals. Therefore no mammals are robins." Accept any argument that follows the pattern exactly (major premise A with P as subject and M as predicate; minor premise E with M as subject and S as predicate; conclusion No S are P).
4. 1: Invalid, undistributed middle: "vegetables" is the subject of an I and the predicate of an A, distributed in neither. 2: Invalid, illicit minor: "swimmers" is distributed in the conclusion (subject of an A) but not in the minor premise (predicate of an A). 3: Invalid, exclusive premises: both premises are negative. 4: Invalid, illicit major: "dogs" is distributed in the conclusion (predicate of an E) but not in the major premise (predicate of an A). (Its conclusion is true, but it does not follow.)
5. (a) Fallacy of four terms (equivocation on the middle term): "pitcher" means a baseball player in the first premise and a jug in the second, so there are four terms. (Accept also the answer that, read with "pitcher on the team" as the middle term, the second premise does not contain it, so nothing links the terms.) (b) Traditional reading: valid, assuming the middle term's class has members. Boolean reading: invalid, the existential fallacy (a particular conclusion from two universal premises).
6. S = brown hens, P = laying hens, M = hens in the yard. Universal first: "All M are P" shades M and SM. "Some M are S" points to SM and SPM; SM is shaded, so the

X goes in SPM. The conclusion "Some S are P" needs an X in SP or SPM: there is one in SPM. Valid (AII-3, Datisi).



7. S = bakers, P = cooks, M = early risers. Universal first: "All S are M" shades S and SP. "Some P are M" points to PM and SPM; neither is shaded, so the X goes on the line between them. The conclusion "Some S are P" needs an X in SP or SPM; the X might be in PM (cooks who rise early but are not bakers). Invalid. The rules agree: undistributed middle ("early risers" is the predicate of an I and of an A). The form is IAI-2.



8. (a) Celarent: EAE-1. Festino: EIO-2. Bocardo: OAO-3. Camenes: AEE-4. (b) Darapti (AAI-3); the middle term's class must have members.

9. (a) *Barbara, Celarent, Darii, Ferioque prioris; Cesare, Camestres, Festino, Baroco secundae.* (b) The C says it reduces to Celarent; the *s* after the first *e* says to convert the major premise simply. "No eight-legged animals are insects. All spiders are eight-legged animals. Therefore no spiders are insects." (Celarent, EAE-1.)

10. (a) Missing premise: "All kites made of ripstop nylon are kites that will fly in any wind." It is the major premise (it contains P), so this is a first-order enthymeme. Completed: All kites made of ripstop nylon fly in any wind; the Gallowind kite is a kite made of ripstop nylon; so the Gallowind kite flies in any wind: Barbara (AAA-1), valid. (b) Not sound: the hidden premise is false. The material is only one thing that matters; a kite's shape, balance and the strength of the wind matter too, and no kite flies in every wind, from the lightest breeze to a storm. Accept any reasonable explanation of why the hidden premise is false or doubtful.

11. Missing major premise: "No people over fourteen are people who can enter the junior baking contest." Completed: No people over fourteen can enter; Hannah is a person over fourteen; so Hannah is not a person who can enter. Celarent (EAE-1), valid. Accept also "All people who can enter the junior baking contest are people fourteen or under," with the minor premise "Hannah is not a person fourteen or under," which gives Camestres (AEE-2), also valid.

12. Classes: W = goats on Willow Hill, L = goats that eat the laundry, T = goats welcome at the tea party, B = goats that wear bow ties. (2) All W are L. (3) No L are T; by obversion, All L are non-T. (1) "Only T are B" means All B are T; by contraposition, All non-T are non-B. Chain: W → L → non-T → non-B. Conclusion: no goat on Willow Hill wears a bow tie.

13. Invalid. The first link is good: no dictionaries are novels and all the top-shelf books are dictionaries, so no books on the top shelf are novels (Celarent). The second link fails: from "Some novels are mysteries" and "No books on the top shelf are novels," the conclusion "No books on the top shelf are mysteries" commits an illicit major, because "mysteries" is distributed in the E conclusion but not in the I premise. (Accept also: a universal conclusion cannot follow when a premise is particular.)

14. 1: Affirming the consequent, invalid. 2: Modus tollens (denying the consequent "the pump will not work"), valid. 3: Affirming a disjunct, invalid, because the

"or" is inclusive and many recipes call for both. 4: Denying the antecedent, invalid.

15. Sample answer: The traditional reading assumes that every class named in a syllogism has members, while the Boolean reading treats universal propositions as making no claim that anything exists. Darapti (All M are P; All M are S; so Some S are P) has two universal premises and a particular conclusion. On the Boolean reading, it breaks Rule 6 (the existential fallacy), and its Venn diagram shows only shading, with no X in SP or SPM, so the conclusion is not shown. On the traditional reading, Rule 6 is not used, and we may put an X in the only unshaded part of M, the center region SPM, which shows the conclusion. So Darapti is valid traditionally, if there are M's, but invalid on the Boolean reading. (Award full credit for a correct, clear explanation that covers both readings, the rule and the diagram.)

Printable answer key (PDF, for the parent)

UNIT 5 · CATEGORICAL SYLLOGISMS · LESSON 5.1 OF 69 · 40 MINUTES

5.1 The Syllogism

OBJECTIVES

- Define a categorical syllogism and tell one from other kinds of argument.
- Identify the major, minor and middle terms, starting from the conclusion.
- Identify the major and minor premises by the terms they contain, not by their position.
- Translate a syllogism from ordinary language into standard form and standard order.

KEY TERMS

Categorical syllogism

A deductive argument with exactly two premises and a conclusion, all categorical propositions, containing exactly three terms, each of which appears in two of the propositions. It may be valid or invalid.

Major term (P)

The predicate term of the conclusion.

Minor term (S)

The subject term of the conclusion.

Middle term (M)

The term that appears in both premises but not in the conclusion; it links the other two terms.

Major premise

The premise that contains the major term.

Minor premise

The premise that contains the minor term.

Standard order

The arrangement of a syllogism with the major premise first, the minor premise second and the conclusion last.

CORE IDEAS

From propositions to arguments. In Unit 4 you studied the categorical proposition, the product of the second act of the mind. Now we join two of them and draw a third: "All hounds are dogs that follow a scent. All beagles are hounds. Therefore all beagles are dogs that follow a scent." This is a **categorical syllogism**, the most carefully studied kind of argument in the history of logic. The Greek word *syllogismos* means a "reckoning together": two claims are counted up together, and a third comes out of them.

What a categorical syllogism is. A categorical syllogism is a deductive argument with exactly two premises and a conclusion, all three of them categorical propositions (A, E, I or O). It contains exactly three terms, and each term appears twice, in two different propositions. Notice what the definition does not say: it does not say the argument is valid. "All sparrows are birds. Some birds are singers. So some sparrows are singers" is a categorical syllogism, but a faulty one, as Lessons 5.3 and 5.4 will show. A syllogism is a shape of argument; whether a given syllogism is good is a separate question.

The three terms. Always begin with the conclusion. Its predicate is the **major term**, written P. Its subject is the **minor term**, written S. The term that appears in both premises but not in the conclusion is the **middle term**, written M. In the beagle

argument the conclusion is “All beagles are dogs that follow a scent,” so S is “beagles,” P is “dogs that follow a scent,” and M is “hounds.” The middle term is the link. Like a go-between who introduces two neighbors and then steps aside, it connects S with P in the premises and then drops out of the conclusion.

The two premises. The premise that contains the major term is the **major premise**; the premise that contains the minor term is the **minor premise**. Each premise therefore holds the middle term together with one of the other two. The names come from the terms a premise contains, not from where it is written: the premise you read first is not the major premise unless it contains P. Why “major” and “minor”? In the commonest kind of syllogism the major term names the widest class and the minor term the narrowest, with the middle between them: beagles inside hounds, hounds inside scent-followers. The names stuck even for syllogisms where that picture does not fit.

Standard order. A syllogism is in **standard order** when the major premise comes first, the minor premise second and the conclusion last. People rarely talk this way. They often state the conclusion first and give the reasons after “since” or “because,” or they tuck the conclusion in the middle. Rewriting an argument in standard order is like setting up an equation before you solve it: nothing about the argument changes, but now you can see it plainly. Lessons 5.2 to 5.4 all assume that the syllogism has been put in standard order first.

Standard-form propositions. Each of the three propositions must also be in standard form: a quantifier, a subject term, a copula (“are” or “are not”) and a predicate term, with both terms naming classes. “Every beagle hunts by scent” becomes “All beagles are dogs that hunt by scent.” Use the tools from Unit 4: “Only S are P” becomes “All P are S,” and “Not all S are P” becomes “Some S are not P.” Traditional logicians treat a singular proposition such as “Rex is a hound” as a universal (an A, or an E if it denies), because its subject names the whole of a class with one member. Finally, word each term the same way every time it appears. If one premise says “hounds” and the other says “hunting dogs,” you may have hidden a fourth term.

A recipe for standard form. (1) Find the conclusion, using indicator words such as “therefore,” “so” and “since” to help. (2) Translate all three propositions into standard form. (3) Name S and P from the conclusion; M is the term left over in the premises. (4) Find the major premise (the one with P) and the minor premise (the

one with S). (5) Write them in order, major premise, minor premise, then "Therefore" and the conclusion. The numbered premises carry no indicator words. Then check: there should be exactly three terms, each used exactly twice. If you count four, either a term is worded two ways (fix the wording if the meaning is the same) or the argument is not a categorical syllogism at all.

Why the form matters. Standard form strips an argument down to its skeleton. Once the skeleton shows, you can see that an argument about beagles and an argument about planets, or about tomatoes, or about saints, may have exactly the same shape, and if the shape is good for one it is good for all. That is why Aristotle, the first to study the syllogism, replaced the terms with letters. In the next lesson you will learn to name each shape with three letters and a number, the way a botanist names a plant.

> **From history.** Aristotle (384–322 B.C.) worked out the theory of the syllogism in the *Prior Analytics*, the first systematic study of the forms of reasoning that has come down to us. He described a syllogism, roughly, as speech in which certain things are laid down and something other than them follows of necessity because they are so. He also did something new: he used letters in place of terms, so that he could speak about every argument of a given shape at once. For Aristotle the word named only reasoning that succeeds. Later logicians, and this course, use "syllogism" for any argument of this shape, good or bad, and then ask whether it is valid. For nearly two thousand years the syllogism was the center of logic in the schools.

> **Practice habit.** When you hear an argument with two reasons, find the conclusion first, then ask which term in the reasons is doing the linking.

> **Enrichment: Aristotle's letters.** In the *Prior Analytics* Aristotle does not write about beagles or barns. He states his first pattern of argument with letters, saying, roughly, that if A belongs to every B, and B belongs to every C, then A must belong to every C. (Notice that he names the predicate first: "A belongs to every B" means "All B are A.") Rewrite the beagle argument from this lesson with Aristotle's letters in place of the terms. Then write a new argument about anything you like that fits the same letters. Why do you think letters helped Aristotle see what makes an argument good?

Example 1. Put this syllogism in standard order and name its terms: “Every beagle is a hound, and no hound is a quiet dog; so no beagle is a quiet dog.”

▼ **Show solution**

1. Find the conclusion. “So” introduces it: “No beagle is a quiet dog,” in standard form “No beagles are quiet dogs.”
2. Name the terms from the conclusion. The subject, “beagles,” is the minor term S. The predicate, “quiet dogs,” is the major term P. The term left over in the premises, “hounds,” is the middle term M.
3. Translate the premises: “All beagles are hounds” and “No hounds are quiet dogs.”
4. The premise with P (“quiet dogs”) is the major premise: “No hounds are quiet dogs.” The premise with S (“beagles”) is the minor premise: “All beagles are hounds.” As written, the minor premise came first.
5. Write the major premise, then the minor premise, then the conclusion. Check: three terms, each used twice.

Answer: 1. No hounds are quiet dogs. (major premise)
2. All beagles are hounds. (minor premise)
Therefore, no beagles are quiet dogs.
S = beagles, P = quiet dogs, M = hounds.

Example 2. Put this argument in standard form and standard order: "Some vegetables in Mrs. Ortiz's garden are not hardy plants, because every hardy plant survives frost, and some vegetables in Mrs. Ortiz's garden do not survive frost."

▼ **Show solution**

1. Find the conclusion. "Because" introduces the premises, so the conclusion is the first clause: "Some vegetables in Mrs. Ortiz's garden are not hardy plants." This is an O proposition.
2. S = vegetables in Mrs. Ortiz's garden; P = hardy plants. The remaining term is "things that survive frost," so we word it as a class: M = plants that survive frost.
3. Translate the premises: "Every hardy plant survives frost" becomes "All hardy plants are plants that survive frost." "Some vegetables in Mrs. Ortiz's garden do not survive frost" becomes "Some vegetables in Mrs. Ortiz's garden are not plants that survive frost."
4. The premise with P ("hardy plants") is the major premise; the premise with S is the minor premise. Here they happen to be written in standard order already, but the conclusion must move to the end, and "because" is dropped.

Answer: 1. All hardy plants are plants that survive frost.
2. Some vegetables in Mrs. Ortiz's garden are not plants that survive frost.
Therefore, some vegetables in Mrs. Ortiz's garden are not hardy plants.
S = vegetables in Mrs. Ortiz's garden, P = hardy plants, M = plants that survive frost.

PRACTICE

1. Easy Define a categorical syllogism in one or two sentences.

▼ **Show answer**

A categorical syllogism is a deductive argument with exactly two premises and a conclusion, all categorical propositions (A, E, I or O), containing exactly three terms, each used twice. (It may be valid or invalid; accept any definition that gives the two premises, the categorical propositions and the three terms.)

2. **Easy** Name the major, minor and middle terms: “All birds are animals with feathers. All robins are birds. Therefore all robins are animals with feathers.”

   ▼ **Show answer**

   Major term (P, predicate of the conclusion): animals with feathers. Minor term (S, subject of the conclusion): robins. Middle term (M, in both premises only): birds.

3. **Easy** What is the standard order of a syllogism, and how do you tell which premise is the major premise?

   ▼ **Show answer**

   Major premise, then minor premise, then conclusion. The major premise is the one that contains the major term, the predicate of the conclusion (not simply the premise written first).

4. **Easy** Which of the three terms never appears in the conclusion, and what is its job?

   ▼ **Show answer**

   The middle term. It appears once in each premise and links the minor term to the major term, then drops out of the conclusion.

5. **Medium** Put in standard order and name S, P and M: “All pitchers are members of the Riverside team. Some pitchers are left-handed players. So some members of the Riverside team are left-handed players.”

   ▼ **Show answer**

   1. Some pitchers are left-handed players. (major premise, contains P)
   2. All pitchers are members of the Riverside team. (minor premise, contains S)
   Therefore, some members of the Riverside team are left-handed players.
   S = members of the Riverside team, P = left-handed players, M = pitchers.

6. **Medium** Put in standard form and standard order: “Since every cast-iron pan is heavy and no heavy thing belongs in Caleb’s backpack, no cast-iron pan belongs in Caleb’s backpack.”

▼ Show answer

1. No heavy things are things that belong in Caleb’s backpack.
2. All cast-iron pans are heavy things.

Therefore, no cast-iron pans are things that belong in Caleb’s backpack.

S = cast-iron pans, P = things that belong in Caleb’s backpack, M = heavy things. (Accept equivalent class wording, such as “items that belong in Caleb’s backpack,” if it is the same in the premise and the conclusion.)

7. **Medium** Is each a categorical syllogism? Give a reason. (a) “If it rains, the game is off. It is raining. So the game is off.” (b) “All sparrows are birds. Some birds are singers. So some sparrows are singers.”

▼ Show answer

(a) No. Its first premise is a conditional (“if … then”), not a categorical proposition, so it is a hypothetical syllogism (Lesson 5.8). (b) Yes. It has two premises and a conclusion, all categorical, with exactly three terms (sparrows, birds, singers), each used twice. It is a categorical syllogism even though, as later lessons show, it is invalid.

8. **Medium** Put in standard form and standard order: “Every apple in this crate is bruised, and nothing bruised will be sold at the market stand, so none of the apples in this crate will be sold at the market stand.”

▼ Show answer

1. No bruised things are things that will be sold at the market stand.
2. All apples in this crate are bruised things.

Therefore, no apples in this crate are things that will be sold at the market stand.

S = apples in this crate, P = things that will be sold at the market stand, M = bruised things. (Accept “bruised apples” or “bruised fruit” for M if used the same way in both premises.)

9. Hard Put in standard form and standard order, and name S, P and M: “Only registered members may borrow the canoes at Pine Lake Camp, and none of the visitors from Elm Street is a registered member. So none of the visitors from Elm Street may borrow the canoes.”

▼ **Show answer**

“Only registered members may borrow the canoes” becomes “All people who may borrow the canoes are registered members.”
1. All people who may borrow the canoes are registered members. (major premise)
2. No visitors from Elm Street are registered members. (minor premise)
Therefore, no visitors from Elm Street are people who may borrow the canoes.
S = visitors from Elm Street, P = people who may borrow the canoes, M = registered members.

10. Hard A student says, “The major premise is always the first premise.” Use this argument to show the mistake: “All finches are seed eaters. All seed eaters are birds that visit Grandpa’s feeder. So all finches are birds that visit Grandpa’s feeder.”

▼ **Show answer**

The major term is the predicate of the conclusion, “birds that visit Grandpa’s feeder,” and it appears in the second premise as written. So the major premise is “All seed eaters are birds that visit Grandpa’s feeder,” even though it comes second; the argument as written is simply not in standard order. Which premise is major depends on the terms it contains, not on where it is written. In standard order: All seed eaters are birds that visit Grandpa’s feeder. All finches are seed eaters. Therefore all finches are birds that visit Grandpa’s feeder.

11. Hard Write a syllogism in standard order with S = maple trees in the park, P = trees that lose their leaves in autumn, and M = deciduous trees, whose conclusion is “All maple trees in the park are trees that lose their leaves in autumn.”

▼ **Show answer**

1. All deciduous trees are trees that lose their leaves in autumn.
2. All maple trees in the park are deciduous trees.
Therefore, all maple trees in the park are trees that lose their leaves in autumn.
(The major premise must join M and P, and the minor premise S and M. With these terms, only this arrangement, Barbara (AAA-1), gives the universal affirmative conclusion validly. Accept any wording that keeps the same three terms.)

12. Challenge Put in standard form and standard order, name S, P and M, and say which premise is the major premise and why: “None but those who have finished the safety course may use the lathe. Some of the eighth graders have not finished the safety course. Hence some of the eighth graders may not use the lathe.”

▼ **Show answer**

“None but X are Y” means “All Y are X,” so the first premise becomes “All people who may use the lathe are people who have finished the safety course.”
1. All people who may use the lathe are people who have finished the safety course.
2. Some eighth graders are not people who have finished the safety course.
Therefore, some eighth graders are not people who may use the lathe.
S = eighth graders, P = people who may use the lathe, M = people who have finished the safety course. Premise 1 is the major premise because it contains the major term, “people who may use the lathe,” the predicate of the conclusion.

COMMON MISTAKES

- Calling the first premise written the major premise. The major premise is the one that contains the major term, the predicate of the conclusion.
- Looking for the terms before finding the conclusion. Find the conclusion first; its subject and predicate tell you S and P.

- Letting a term change its wording (“hounds” in one premise, “hunting dogs” in the other). Use one wording for one meaning.
- Thinking that “syllogism” means “valid argument.” Invalid syllogisms exist; the word names the shape, not the verdict.
- Leaving “since” or “because” in the numbered premises. Standard form drops the indicator words.

REFERENCE GUIDE

An eight-page, color-coded Logic guide: what logic is, terms and definitions, the A E I O propositions, the square of opposition and immediate inferences, syllogisms with the six rules and Venn tests, the named forms, informal fallacies of relevance, presumption and ambiguity, propositional logic with truth tables, conditionals, valid forms and short proofs, the classical tradition, dialectic, argument maps and practice. The booklet prints on 11×17 paper and folds to letter size. Prints in color and reads clearly in black and white.

> **Scripture.** “Wisdom is the principal thing; therefore get wisdom: and with all thy getting get understanding.” (Proverbs 4:7, KJV)
>
> The proverb itself reasons: a premise, then “therefore,” then what follows. Learning to see how one truth follows from another is part of getting understanding.

UNIT 5 · CATEGORICAL SYLLOGISMS · LESSON 5.2 OF 69 · 40 MINUTES

5.2 Mood and Figure

OBJECTIVES

- Name the mood of a standard-form syllogism from the letters of its three propositions.
- Name the figure of a syllogism from the position of its middle term.
- Give the full form of a syllogism (for example, AAA-1) and explain why form decides validity.
- Build a syllogism of a given mood and figure.

KEY TERMS

Mood

The three letters (A, E, I or O) of a syllogism's major premise, minor premise and conclusion, in that order, as in AAA or EIO.

Figure

The arrangement of the middle term in the two premises; there are four figures, numbered 1 to 4.

Form

The mood and figure together, such as AAA-1 or EIO-2. Every syllogism of the same form is valid, or every one is invalid.

First figure

M–P, S–M: the middle term is the subject of the major premise and the predicate of the minor premise.

Second figure

P–M, S–M: the middle term is the predicate of both premises.

Third figure

M–P, M–S: the middle term is the subject of both premises.

Fourth figure

P–M, M–S: the middle term is the predicate of the major premise and the subject of the minor premise.

CORE IDEAS

Every syllogism has a name. A botanist can name a plant by its leaves and flowers without caring which garden it grows in. In the same way, a logician can name a syllogism by its shape without caring whether it is about beagles or barns. The name has two parts: the **mood**, three letters, and the **figure**, a number from 1 to 4. "All hounds are dogs that follow a scent. All beagles are hounds. Therefore all beagles are dogs that follow a scent" is named AAA-1. This lesson teaches you to read both parts of the name.

The mood: three letters. The mood lists the type of each proposition, A, E, I or O, in standard order: first the major premise, then the minor premise, then the conclusion. Take "No hounds are quiet dogs. All beagles are hounds. Therefore no beagles are quiet dogs." The major premise is an E, the minor premise an A and the conclusion an E, so the mood is EAE. If you need a reminder: A is universal

affirmative (All S are P), E universal negative (No S are P), I particular affirmative (Some S are P) and O particular negative (Some S are not P).

Standard order first. The letters only mean something when the syllogism is in standard order and every proposition is in standard form. Suppose the beagle argument were written with the minor premise first. A careless reader would call it AEE, which is the name of a different form. Always find the conclusion, name S and P, put the major premise first, translate everything into standard form, and only then write the letters. A sentence such as "Not every hound is quiet" must be translated (here, to an O) before it gets its letter.

The figure: where the middle term stands. Each premise has two places, subject and predicate, and the middle term must stand in one of them in each premise. That gives four possible arrangements, the four **figures**. Writing the subject of each premise first:
Figure 1: M–P, S–M (middle term is the subject, then the predicate)
Figure 2: P–M, S–M (middle term is the predicate both times)
Figure 3: M–P, M–S (middle term is the subject both times)
Figure 4: P–M, M–S (middle term is the predicate, then the subject)

A way to remember the figures. Say the position of the middle term in the major premise and then in the minor premise: figure 1 is "subject, predicate," figure 2 is "predicate, predicate," figure 3 is "subject, subject," and figure 4 is "predicate, subject." Notice that figures 2 and 3 are the matched pairs, and figures 1 and 4 are mirror images. The beagle argument puts "hounds" first in the major premise (No hounds are quiet dogs) and last in the minor premise (All beagles are hounds): subject, predicate, so it is figure 1. Its full name is EAE-1.

Mood and figure together make the form. There are four choices of letter for each of three propositions, so there are $4 \times 4 \times 4 = 64$ moods. Each mood can come in any of the four figures, so there are $64 \times 4 = 256$ forms of categorical syllogism. Only a small number of them are valid, and you will learn them in Lesson 5.5. The mood alone does not decide validity: AAA-1 is valid, but AAA-2 ("All cats are mammals. All dogs are mammals. So all dogs are cats") is not. It takes both parts of the name to fix the form.

Why the form decides. Validity depends on form alone, not on what the argument is about. If one syllogism of the form EAE-1 is valid, every syllogism of the form EAE-1

is valid, whether its terms are hounds, planets or loaves of bread. That is why naming the form is so useful. Once you have tested a form, you never need to test it again; you only need to recognize it. It also means that a single clear example of a form with true premises and a false conclusion shows that the whole form is invalid.

Building a syllogism from its name. You can also work backward. To build an IAI-3, write the figure-3 skeleton (M–P, M–S, S–P) and fill in the letters: "Some M are P. All M are S. Therefore some S are P." Then choose terms: "Some pumpkins are prizewinners at the county fair. All pumpkins are squashes. Therefore some squashes are prizewinners at the county fair." Building examples like this is the best way to learn the figures, and later it will help you invent counterexamples to show that a form is invalid.

> **From history.** Aristotle described three figures in the *Prior Analytics*, telling them apart by whether the middle term is the subject of one premise and the predicate of the other, the predicate of both, or the subject of both. Arguments of the shape now called the fourth figure were not treated as a separate figure by him. His pupil Theophrastus handled them as extra, "indirect" moods of the first figure. Later writers credited the physician Galen, of the second century A.D., with adding a fourth figure, but historians doubt the story, and its origin is uncertain. By the later Middle Ages, and in the logic books of the following centuries, four figures had become the usual teaching.

> **Practice habit.** Before you judge a syllogism, name it: put it in standard order, read the three letters, then find where the middle term stands.

Example 1. Name the mood and figure: "No hounds are quiet dogs. All beagles are hounds. Therefore no beagles are quiet dogs."

▼ **Show solution**

1. Check standard order. The conclusion's predicate, "quiet dogs," is P; it appears in the first premise, so the major premise is first. The order is standard.
2. Mood: major premise "No hounds are quiet dogs" is E; minor premise "All beagles are hounds" is A; conclusion "No beagles are quiet dogs" is E. Mood: EAE.
3. Figure: the middle term "hounds" is the subject of the major premise and the predicate of the minor premise. Subject, predicate: figure 1.

Answer: EAE-1.

Example 2. Name the mood and figure: "Some members of the choir are not altos, since all altos read music, and some members of the choir do not read music."

▼ **Show solution**

1. Find the conclusion: "since" introduces the premises, so the conclusion is "Some members of the choir are not altos," an O. S = members of the choir, P = altos, M = people who read music.
2. Translate the premises: "All altos are people who read music" (A) and "Some members of the choir are not people who read music" (O).
3. Standard order: the premise with P ("altos") is the major premise, so it goes first: All altos are people who read music. Some members of the choir are not people who read music. Therefore some members of the choir are not altos.
4. Mood: A, O, O, so AOO.
5. Figure: "people who read music" is the predicate of the major premise and the predicate of the minor premise. Predicate, predicate: figure 2.

Answer: AOO-2.

Example 3. Build a syllogism of the form IAI-3.

▼ **Show solution**

1. Figure 3 puts the middle term first in both premises: M–P, M–S, and the conclusion is always S–P.
2. Fill in the mood: the major premise is an I (Some M are P), the minor premise an A (All M are S), the conclusion an I (Some S are P).
3. Choose terms: M = pumpkins, P = prizewinners at the county fair, S = squashes.

Answer: Some pumpkins are prizewinners at the county fair. All pumpkins are squashes. Therefore some squashes are prizewinners at the county fair. (Any terms will do, as long as the pattern is Some M are P, All M are S, Some S are P.)

PRACTICE

1. Easy What is the mood of a syllogism, and in what order are its letters written?

   ▼ **Show answer**

   The mood is the list of the three propositions' types (A, E, I or O), written in standard order: major premise, minor premise, conclusion.

2. Easy Write the pattern of the middle term for each of the four figures.

   ▼ **Show answer**

   Figure 1: M–P, S–M. Figure 2: P–M, S–M. Figure 3: M–P, M–S. Figure 4: P–M, M–S. (Accept the "subject, predicate" wording: 1 subject–predicate, 2 predicate–predicate, 3 subject–subject, 4 predicate–subject.)

3. Easy Name the mood and figure: "No reptiles are mammals. All snakes are reptiles. Therefore no snakes are mammals."

   ▼ **Show answer**

   EAE-1. Major premise E, minor premise A, conclusion E; the middle term "reptiles" is the subject of the major premise and the predicate of the minor premise.

4. **Easy** How many forms of categorical syllogism are possible? Show how you get the number.

   ▼ **Show answer**

   256. Each of the three propositions can be A, E, I or O, giving $4 \times 4 \times 4 = 64$ moods, and each mood can be in any of 4 figures: $64 \times 4 = 256$.

5. **Medium** Name the mood and figure: "All gardeners are early risers. Some gardeners are beekeepers. Therefore some beekeepers are early risers."

   ▼ **Show answer**

   AII-3. S = beekeepers, P = early risers, M = gardeners. Major premise "All gardeners are early risers" (A), minor premise "Some gardeners are beekeepers" (I), conclusion (I); the middle term is the subject of both premises.

6. **Medium** Name the mood and figure: "No fish are mammals. All whales are mammals. Therefore no whales are fish."

   ▼ **Show answer**

   EAE-2. S = whales, P = fish, M = mammals. Major premise "No fish are mammals" (E), minor premise "All whales are mammals" (A), conclusion (E); the middle term is the predicate of both premises.

7. **Medium** Put in standard order and name the mood and figure: "Some loaves at the bake sale are not sourdough loaves, because no sourdough loaves are made with packet yeast, and some loaves at the bake sale are made with packet yeast."

   ▼ **Show answer**

   1. No sourdough loaves are loaves made with packet yeast. (E)
   2. Some loaves at the bake sale are loaves made with packet yeast. (I)

   Therefore, some loaves at the bake sale are not sourdough loaves. (O)

   The middle term is the predicate of both premises, so the form is EIO-2.

8. Medium Name the mood and figure: “All collies are herding dogs. No herding dogs are lapdogs. Therefore no lapdogs are collies.”

▼ **Show answer**

AEE-4. S = lapdogs, P = collies, M = herding dogs. The premise with P, “All collies are herding dogs” (A), is the major premise; “No herding dogs are lapdogs” (E) is the minor; the conclusion is E. The middle term is the predicate of the major premise and the subject of the minor premise.

9. Hard Build a syllogism of the form EIO-3, using terms of your choice.

▼ **Show answer**

Pattern: No M are P. Some M are S. Therefore some S are not P. Sample: “No cacti are water-loving plants. Some cacti are flowering plants. Therefore some flowering plants are not water-loving plants.” Accept any argument that follows the pattern exactly, with the middle term as the subject of both premises.

10. Hard Caleb says, “If I know the mood, I know whether a syllogism is valid.” Use the mood AAA in figures 1 and 2 to show he is wrong.

▼ **Show answer**

AAA-1 (All M are P; All S are M; so All S are P) is valid: “All birds are animals; all robins are birds; so all robins are animals.” AAA-2 (All P are M; All S are M; so All S are P) is invalid: “All cats are mammals; all dogs are mammals; so all dogs are cats” has true premises and a false conclusion. Same mood, different figures, different verdicts, so the figure matters too. (Accept any pair of examples that shows this.)

11. Hard Translate, put in standard order, and name the mood and figure: “Only mammals nurse their young, and every bat nurses its young; it follows that every bat is a mammal.”

▼ **Show answer**

“Only mammals nurse their young” becomes “All animals that nurse their young are mammals.”
1. All animals that nurse their young are mammals. (A, major premise: it contains P, “mammals”)
2. All bats are animals that nurse their young. (A)
Therefore, all bats are mammals. (A)
The middle term is the subject of the major premise and the predicate of the minor: AAA-1.

12. Challenge Translate, put in standard order, and name the mood and figure: “Not all the tomato plants in the greenhouse have spotted leaves, and every plant with blight has spotted leaves. So not all the tomato plants in the greenhouse have blight.”

▼ **Show answer**

“Not all S are P” is an O. S = tomato plants in the greenhouse, P = plants with blight, M = plants with spotted leaves.
1. All plants with blight are plants with spotted leaves. (A, major premise: it contains P)
2. Some tomato plants in the greenhouse are not plants with spotted leaves. (O)
Therefore, some tomato plants in the greenhouse are not plants with blight. (O)
The middle term is the predicate of both premises: AOO-2. (A student who writes OAO has forgotten to put the major premise first.)

COMMON MISTAKES

- Writing the letters before putting the syllogism in standard order. The mood must read major premise, minor premise, conclusion.
- Looking at the conclusion to find the figure. The figure depends only on where the middle term stands in the premises.

- Mixing up figures 2 and 3. In figure 2 the middle term is the predicate twice; in figure 3 it is the subject twice.
- Giving a letter to an untranslated sentence. “Not every hound is quiet” is an O, not an A.
- Thinking the mood alone settles validity. AAA-1 is valid; AAA-2 is not.

> **Scripture.** “Let all things be done decently and in order.” (1 Corinthians 14:40, KJV)
>
> Paul wrote these words about worship, but the principle reaches further: order helps understanding. Putting an argument in order before judging it is a small way of honoring the God of order.

UNIT 5 · CATEGORICAL SYLLOGISMS · LESSON 5.3 OF 69 · 45 MINUTES

5.3 The Rules of Validity

OBJECTIVES

- State the six rules of the categorical syllogism.
- Mark which terms are distributed in each premise and in the conclusion.
- Test a syllogism against the rules and name the fallacy when a rule is broken.
- Explain how the traditional and Boolean readings differ on forms such as AAI-3.

KEY TERMS

Fallacy of four terms

Breaking Rule 1: using four terms, often by using one word in two senses (Latin *quaternio terminorum*).

Undistributed middle

Breaking Rule 2: the middle term is not distributed in either premise.

Illicit major

Breaking Rule 3 with the major term: it is distributed in the conclusion but not in the major premise.

Illicit minor

Breaking Rule 3 with the minor term: it is distributed in the conclusion but not in the minor premise.

Exclusive premises

Breaking Rule 4: both premises are negative.

Existential fallacy

Breaking Rule 6: drawing a particular conclusion from two universal premises, which (on the Boolean reading) do not say that anything exists.

CORE IDEAS

Two independent tests. A syllogism is valid when its premises guarantee its conclusion. You can test that in two ways: with rules, which look at the terms and the propositions, and with Venn diagrams, which picture the classes (Lesson 5.4). The two tests always agree, so each can check the other. This lesson gives six rules. Each one guards against a particular way of going wrong, and each broken rule has a name, so that you can say not only "invalid" but exactly why.

First, distribution. Most of the rules depend on distribution, which you learned in Lesson 4.3. A term is distributed when the proposition says something about every member of its class. A distributes its subject; E distributes both terms; I distributes neither; O distributes its predicate only. A short way to remember: universals distribute the subject, and negatives distribute the predicate. Before you apply the rules, mark each term in each proposition as distributed (d) or undistributed (u).

Rule 1: exactly three terms, each used in the same sense. A syllogism works by linking two terms through a third. If there are four terms, nothing links them, and the argument commits the **fallacy of four terms**. Usually the fourth term hides inside one word used in two senses: "All cranes are birds. Some machines at the dock are cranes. So some machines at the dock are birds." The middle term means a bird in one premise and a lifting machine in the other. This is equivocation, which you met in Unit 3, in its syllogism form.

Rule 2: the middle term must be distributed at least once. If neither premise speaks of all of the middle class, S and P may be linked to different parts of it. "All cats are mammals. All dogs are mammals. So all dogs are cats." Cats are somewhere among the mammals, and dogs are somewhere among them too, but nothing says they are in the same part. This is the **undistributed middle**, probably the most common fault in everyday reasoning.

Rule 3: no term may be distributed in the conclusion unless it is distributed in its premise. A conclusion may not say more than its premises. If the conclusion speaks of every member of a class, the premise must have spoken of every member too. Breaking the rule with the major term is the **illicit major**: "All horses are animals. No cows are horses. So no cows are animals." The conclusion (an E) distributes "animals," but the major premise (an A) does not. Breaking it with the minor term is the **illicit minor**: "All bakers are early risers. All bakers are cooks. So all cooks are early risers." The conclusion distributes "cooks," but the minor premise does not. The rule works in one direction only: a term distributed in a premise need not be distributed in the conclusion.

Rule 4: no two negative premises. If both premises deny, they say only that S and P are each shut out from M, and that tells us nothing about S and P. "No dogs are cats. No cats are poodles. So no poodles are dogs." Both premises are true and the conclusion is false. Two negative premises commit the fallacy of **exclusive premises**.

Rule 5: a negative premise requires a negative conclusion, and a negative conclusion requires a negative premise. If one premise separates a term from M, the conclusion can only separate, not join. "No reptiles are mammals. Some pets are reptiles. So some pets are mammals" has true premises and a true conclusion, but the conclusion does not follow; it should have been "Some pets are not mammals." In the other direction, two affirmative premises only join classes, so they can never prove a negative conclusion. The fallacies are drawing an affirmative conclusion from a negative premise and drawing a negative conclusion from affirmative premises.

Rule 6: no particular conclusion from two universal premises. On the Boolean reading (Lesson 4.6) universal propositions do not say that anything exists, but particular propositions do. So two universal premises cannot prove a particular conclusion; trying is the **existential fallacy**. "All students who scored 101 on the 100-point quiz are geniuses. All students who scored 101 on the 100-point quiz are in Mrs. Ortiz's class. So some students in Mrs. Ortiz's class are geniuses." No one scored 101, so the premises claim nothing, yet the conclusion claims someone exists. The traditional reading assumes that every term names something that exists; on that reading this form (AAI-3, called Darapti) is valid, and Rule 6 is not used. When the rules give different verdicts, always say which reading you are using.

Using the rules. Put the syllogism in standard form and order, mark the distribution of every term, then go through the rules one by one. A syllogism that breaks none of the six rules is valid on the Boolean reading; one that breaks none of the first five is valid on the traditional reading. A syllogism that breaks any rule is invalid, and it may break more than one; name every fault you find.

> **From history.** Aristotle proved which syllogisms are valid by reducing them to the clearest forms of the first figure, but the short list of general rules came later. Medieval logicians collected rules about the middle term, negative premises and particular premises, and gave Latin names to the faults; the fallacy of four terms is still called *quaternio terminorum*. The Port-Royal *Logic* (1662) set out general rules for all syllogisms, such as that the middle term must be taken universally at least once and that nothing may be concluded from two negative premises. The sixth rule is younger. It comes from the Boolean reading of universal propositions, which treats them as making no claim that anything exists.

> **Practice habit.** When an argument links two things through a third, ask: did either premise speak about all of that third thing?

Example 1. Test with the rules: "All tulips are flowers that grow from bulbs. All tulips are spring flowers. Therefore all spring flowers are flowers that grow from bulbs."

▼ **Show solution**

1. Terms: S = spring flowers, P = flowers that grow from bulbs, M = tulips. Rule 1: three terms, used in the same sense. Passes.
2. Distribution: major premise (A) distributes "tulips" only; minor premise (A) distributes "tulips" only; conclusion (A) distributes "spring flowers" only.
3. Rule 2: the middle term "tulips" is distributed (in both premises). Passes.
4. Rule 3: the minor term "spring flowers" is distributed in the conclusion but not in the minor premise, where it is the predicate of an A. Fails: illicit minor. (The major term is undistributed in the conclusion, so it causes no trouble.)
5. Rules 4–6: no negative premises, an affirmative conclusion, and a universal conclusion. These pass.

Answer: Invalid: illicit minor. (The form is AAA-3.) The premises speak of the spring flowers that are tulips, but the conclusion speaks of all spring flowers.

Example 2. Test with the rules: “No reptiles are warm-blooded animals. Some pets are warm-blooded animals. Therefore some pets are not reptiles.”

▼ **Show solution**

1. Terms: S = pets, P = reptiles, M = warm-blooded animals. Rule 1 passes.
2. Distribution: major premise (E) distributes “reptiles” and “warm-blooded animals”; minor premise (I) distributes nothing; conclusion (O) distributes “reptiles.”
3. Rule 2: the middle term is distributed in the major premise. Passes.
4. Rule 3: “reptiles” is distributed in the conclusion and also in the major premise. “Pets” is not distributed in the conclusion. Passes.
5. Rule 4: only one negative premise. Rule 5: one negative premise and a negative conclusion. Rule 6: the minor premise is particular, so a particular conclusion is allowed. All pass.

Answer: Valid on either reading; it breaks none of the six rules. (The form is EIO-2.)

Example 3. Test with the rules: “No cats are dogs. Some pets are not cats. Therefore some pets are not dogs.”

▼ **Show solution**

1. Terms: S = pets, P = dogs, M = cats. Rule 1 passes.
2. Rule 2: “cats” is distributed in the major premise (E). Passes.
3. Rule 3: “dogs,” distributed in the conclusion (O predicate), is distributed in the major premise (E). Passes.
4. Rule 4: both premises are negative. Fails: exclusive premises.
5. Notice that the premises and the conclusion are all true. That does not rescue the argument: knowing that pets are partly outside the cats, and that cats are outside the dogs, tells us nothing about where the pets stand with respect to dogs.

Answer: Invalid: exclusive premises (two negative premises). The form is EOO-1.

PRACTICE

1. **Easy** State the six rules of the categorical syllogism.

   ▼ **Show answer**

   (1) Exactly three terms, each used in the same sense. (2) The middle term must be distributed at least once. (3) No term may be distributed in the conclusion unless it is distributed in its premise. (4) No two negative premises. (5) A negative premise requires a negative conclusion, and a negative conclusion requires a negative premise. (6) (Boolean) No particular conclusion from two universal premises.

2. **Easy** Which terms does each kind of proposition distribute: A, E, I and O?

   ▼ **Show answer**

   A distributes the subject only; E distributes both subject and predicate; I distributes neither; O distributes the predicate only.

3. **Easy** Name the fallacy: "All dogs are mammals. All cats are mammals. Therefore all cats are dogs."

   ▼ **Show answer**

   Undistributed middle. The middle term "mammals" is the predicate of an A proposition in both premises, so it is never distributed.

4. **Easy** What is the fallacy called when both premises are negative, and why can no conclusion follow?

   ▼ **Show answer**

   Exclusive premises. Two negative premises only say that S and P are each shut out (wholly or partly) from M, which tells us nothing about how S and P are related to each other.

5. **Medium** Test with the rules and name the fallacy: "All horses are animals. No cows are horses. Therefore no cows are animals."

   ▼ **Show answer**

   Illicit major. The major term "animals" is distributed in the conclusion (the predicate of an E) but not in the major premise "All horses are animals" (the predicate of an A). The other rules pass: "horses" is distributed, there is one negative premise and the conclusion is negative.

6. **Medium** Test with the rules and name the fallacy: “All bakers are early risers. All bakers are cooks. Therefore all cooks are early risers.”

   ▼ **Show answer**

   Illicit minor. The minor term “cooks” is distributed in the conclusion (the subject of an A) but not in the minor premise “All bakers are cooks” (the predicate of an A). The middle term “bakers” is distributed, and the other rules pass.

7. **Medium** What is wrong with this syllogism? “All bats are mammals. Some pieces of sporting equipment are bats. Therefore some pieces of sporting equipment are mammals.”

   ▼ **Show answer**

   Fallacy of four terms (equivocation). “Bats” means flying animals in the first premise and baseball or cricket bats in the second, so there are really four terms and no middle term links the other two. (Accept “equivocation on the middle term.” A student who notes that the shape would otherwise be the valid AII-1 deserves extra credit.)

8. **Medium** Test with the rules and name the fallacy: “No vegetables in the cellar are frozen things. Some vegetables in the cellar are carrots. Therefore some carrots are frozen things.”

   ▼ **Show answer**

   Drawing an affirmative conclusion from a negative premise (Rule 5). The major premise is negative, so only a negative conclusion (“Some carrots are not frozen things”) could follow. The other rules pass: the middle term is distributed in the E premise, the I conclusion distributes nothing, and the minor premise is particular. (The form is EII-3.)

9. Hard Is this valid? Answer on both the traditional and the Boolean reading. “All the quilts in the contest are handmade things. All the quilts in the contest are entries from Brookside County. Therefore some entries from Brookside County are handmade things.”

   ▼ **Show answer**

   The form is AAI-3 (Darapti). It passes Rules 1–5: “quilts in the contest” is distributed in both premises, the I conclusion distributes nothing, and nothing is negative. Traditional reading: valid, because it assumes there are quilts in the contest. Boolean reading: invalid, the existential fallacy (Rule 6), because two universal premises do not say that any quilts in the contest exist, but the particular conclusion says that something exists.

10. Hard Name every rule this syllogism breaks: “All lemons are citrus fruits. All citrus fruits are fruits rich in vitamin C. Therefore some fruits rich in vitamin C are not lemons.”

    ▼ **Show answer**

    S = fruits rich in vitamin C, P = lemons, M = citrus fruits; the form is AAO-4. It breaks Rule 5 (a negative conclusion from two affirmative premises) and Rule 6 (a particular conclusion from two universal premises: the existential fallacy). It passes Rules 1–3: “citrus fruits” is distributed in the minor premise, and “lemons,” distributed in the conclusion, is distributed in the major premise. Because it breaks Rule 5 it is invalid on the traditional reading as well. (Its conclusion happens to be true, which shows that a true conclusion does not prove an argument valid.)

11. Hard Test with all six rules: “No people who skipped the safety course are people allowed to use the lathe. Some students in the shop class are people who skipped the safety course. Therefore some students in the shop class are not people allowed to use the lathe.”

    ▼ **Show answer**

    Valid on either reading (EIO-1, Ferio). Rule 1: three terms in one sense. Rule 2: the middle term “people who skipped the safety course” is distributed in the E premise. Rule 3: “people allowed to use the lathe” is distributed in the conclusion and in the major premise (E); “students in the shop class” is undistributed in the conclusion. Rule 4: one negative premise. Rule 5: one negative premise and a negative conclusion. Rule 6: the minor premise is particular.

12. Challenge Use the rules to show that no valid syllogism has two particular premises.

    ▼ **Show answer**

    Consider the cases. Two I premises: I distributes nothing, so the middle term is undistributed (Rule 2). Two O premises: both negative (Rule 4). One I and one O: there is a negative premise, so the conclusion must be negative (Rule 5), and a negative conclusion distributes P. Then the premises must distribute both M (Rule 2) and P (Rule 3). But the I distributes nothing and the O distributes only one term, its predicate, so only one term is distributed in the premises, and one of the two rules must be broken. In every case some rule fails, so two particular premises never give a valid syllogism. (Accept any complete case-by-case argument.)

COMMON MISTAKES

- Reading Rule 3 backward. A term distributed in a premise need not be distributed in the conclusion; only the reverse is forbidden.
- Calling an argument valid because its premises and conclusion are all true. The rules test the form, not the facts.
- Forgetting that an O distributes its predicate. “Some pets are not dogs” says something about every dog: none of them is among those pets.

- Calling AAI-3 simply valid or simply invalid. Say which reading you use: valid traditionally, invalid on the Boolean reading.
- Stopping at the first broken rule when the question asks for every fault.

REFERENCE GUIDE

An eight-page, color-coded Logic guide: what logic is, terms and definitions, the A E I O propositions, the square of opposition and immediate inferences, syllogisms with the six rules and Venn tests, the named forms, informal fallacies of relevance, presumption and ambiguity, propositional logic with truth tables, conditionals, valid forms and short proofs, the classical tradition, dialectic, argument maps and practice. The booklet prints on 11×17 paper and folds to letter size. Prints in color and reads clearly in black and white.

> **Scripture.** “Prove all things; hold fast that which is good.” (1 Thessalonians 5:21, KJV)
>
> To “prove” here means to test. The rules of the syllogism are tools for testing arguments, so that we hold fast only to what really follows.

UNIT 5 · CATEGORICAL SYLLOGISMS · LESSON 5.4 OF 69 · 45 MINUTES

5.4 Testing with Venn Diagrams

OBJECTIVES

- Draw and label a three-circle Venn diagram and name its regions.
- Diagram the two premises of a syllogism, universal premises first, placing each X correctly.
- Read the finished diagram to decide whether the conclusion is shown, and so whether the syllogism is valid.
- Check a Venn verdict against the rules, and show how the traditional reading changes the diagram for forms such as Darapti.

KEY TERMS

Three-circle Venn diagram

Three overlapping circles for the minor term (S, top left), the major term (P, top right) and the middle term (M, bottom), dividing the page into eight regions.

Region

One of the areas of the diagram, named by the circles it lies inside: S, P or M alone; SP, SM or PM (exactly those two); SPM (all three); and the outside.

Shading

A mark that a region is empty: nothing exists there. Universal premises are drawn by shading.

X

A mark that at least one thing exists in a region. Particular premises are drawn with an X.

X on a line

An X placed on the boundary between two regions, meaning that something exists in one or the other, but the premises do not say which.

Venn test

Diagram the premises only; the syllogism is valid if and only if the diagram then already shows the conclusion.

CORE IDEAS

A picture of three classes. In Lesson 4.7 you drew one proposition with two circles. A syllogism has three terms, so it needs three circles: S, the minor term, at the top left; P, the major term, at the top right; and M, the middle term, at the bottom. The circles overlap so that every possible combination has its own region. A region is named by the circles it lies inside: “S” means inside S only, “SM” means inside S and M but not P, “SPM” means inside all three, and so on. With the outside, there are eight regions in all.

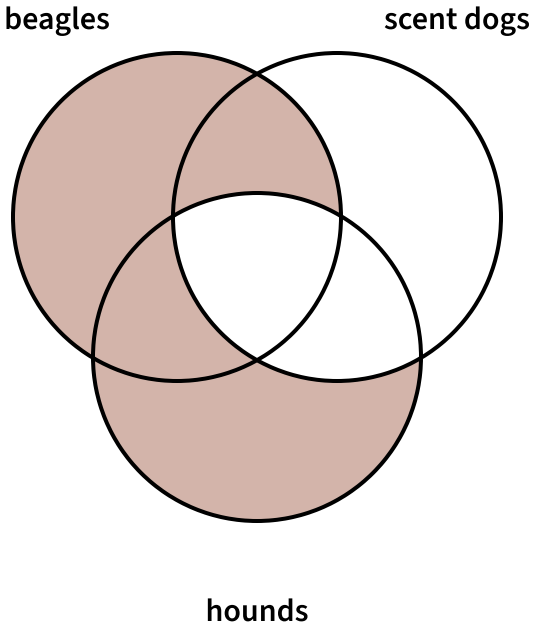
Three marks. Shading a region means it is empty: nothing exists there. An X in a region means at least one thing exists there. A blank region means we know nothing about it; it might be empty, or it might not. Universal propositions are drawn by shading: “All M are P” shades every part of M that lies outside P (the regions M and SM). Particular propositions are drawn with an X: “Some S are M” puts an X somewhere in the part of S that lies inside M (the regions SM and SPM).

The Venn test. Draw the two premises, and only the premises. Never draw the conclusion. Then look at the finished diagram and ask: does it already show the conclusion? If it does, the premises have forced the conclusion, and the syllogism is valid. If it does not, the premises leave room for the conclusion to be false, and the syllogism is invalid. To read a conclusion: "All S are P" needs the regions S and SM both shaded; "No S are P" needs SP and SPM both shaded; "Some S are P" needs an X inside SP or SPM; "Some S are not P" needs an X inside S or SM.

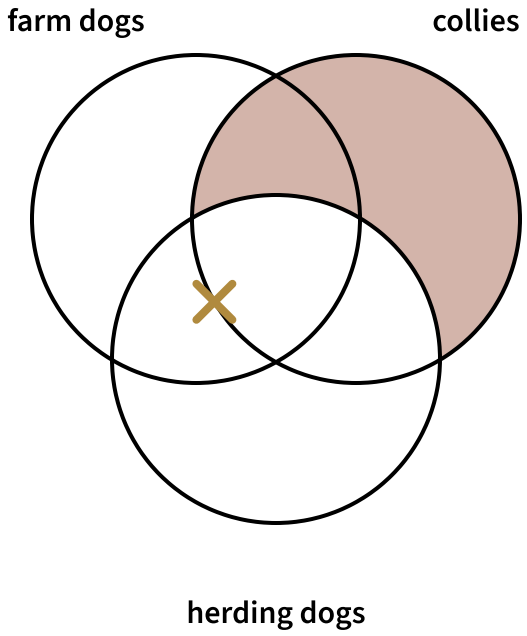
Draw universals first. If one premise is universal and one particular, draw the universal premise first, whichever it is. Shading first can settle where the X must go. Suppose a particular premise points to two regions and the universal premise has already shaded one of them. Then the X must go in the other one, because nothing can exist in an empty region. If you drew the X first, you would not know where to put it, and you might put it in a region that the other premise later shades.

When an X goes on the line. A particular premise always points to two regions, because its two terms overlap in two places (one inside the third circle and one outside it). If neither of those regions is shaded, the premises do not tell you which one holds the thing, so you put the X on the line between them. An X on a line means "in one of these two, we don't know which." If the conclusion needs the X to be in one particular region, an X on the line is not enough, and the syllogism is invalid.

A valid example. Barbara: "All hounds are scent dogs. All beagles are hounds. Therefore all beagles are scent dogs." Here S = beagles, P = scent dogs, M = hounds. The major premise shades the parts of M outside P (M and SM). The minor premise shades the parts of S outside M (S and SP). Now look at S: every part of it outside P (the regions S and SM) is shaded. The conclusion "All beagles are scent dogs" is already drawn, so the syllogism is valid.



An invalid example. "All collies are herding dogs. Some farm dogs are herding dogs. Therefore some farm dogs are collies." S = farm dogs, P = collies, M = herding dogs. Draw the universal first: it shades the parts of P outside M (P and SP). The particular premise points to SM and SPM; neither is shaded, so the X goes on the line between them. The conclusion needs an X inside SP or SPM. The X might be in SPM, but it might be in SM, among the herding dogs that are not collies. The diagram does not show the conclusion, so the syllogism is invalid.



Check against the rules. The Venn test and the six rules always agree, so use each to check the other. The collie syllogism has an undistributed middle: "herding dogs" is the predicate of an A and of an I, distributed in neither. The X on the line is the picture of that fault. If your diagram and the rules disagree, you have made a mistake somewhere; look first at whether you drew the universal premise first and whether an X belongs on a line.

The traditional reading. Venn diagrams, as drawn here, test the Boolean reading: a universal premise only shades and never puts an X anywhere. That is why AAI-3 (Darapti) fails the Venn test, just as it breaks Rule 6. On the traditional reading we assume that the class named by the term in question has members. To test that reading, add an X in the only unshaded region left inside that circle, if there is just one. If the conclusion now shows, the syllogism is valid on the traditional reading, on that assumption.

> **From history.** Circles had been used to picture propositions before. The Swiss mathematician Leonhard Euler drew them in his *Letters to a German Princess*, written in the 1760s to teach a young student. In 1880 the Cambridge logician John Venn published a new method, and he explained it again in his *Symbolic Logic* (1881). Venn's idea was to draw every possible region first, whether or not anything was in it, and then mark the regions the premises declare empty. That made the diagrams a reliable mechanical test. Lewis Carroll, a few years later, designed his own diagram of squares for *The Game of Logic* (1886), with counters for the marks.

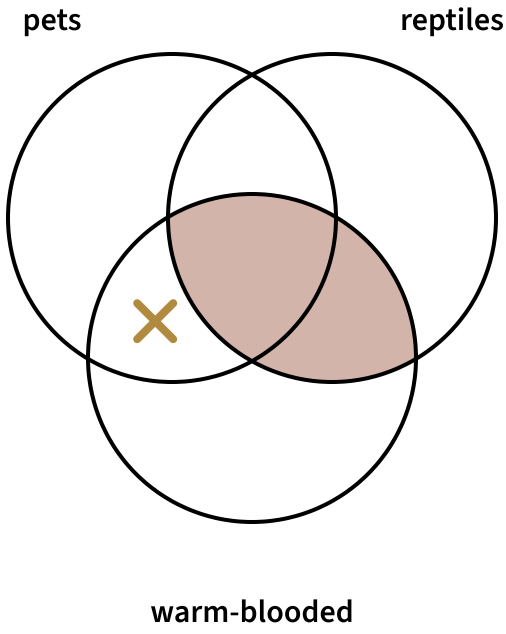
> **Practice habit.** Before you say "It follows," picture the classes: is there any room left for the conclusion to be false?

Example 1. Test with a Venn diagram: "No reptiles are warm-blooded animals. Some pets are warm-blooded animals. Therefore some pets are not reptiles."

▼ **Show solution**

1. Terms: S = pets, P = reptiles, M = warm-blooded animals.
2. Universal first. "No reptiles are warm-blooded animals" (No P are M) shades where P and M overlap: the regions PM and SPM.
3. Particular next. "Some pets are warm-blooded animals" (Some S are M) points to SM and SPM. SPM is already shaded, so the X goes in SM.
4. Read the conclusion. "Some pets are not reptiles" needs an X inside S but outside P: in S or SM. There is an X in SM. The conclusion is shown.
5. Check with the rules: EIO-2 breaks none of the six rules. The verdicts agree.

Answer: Valid.

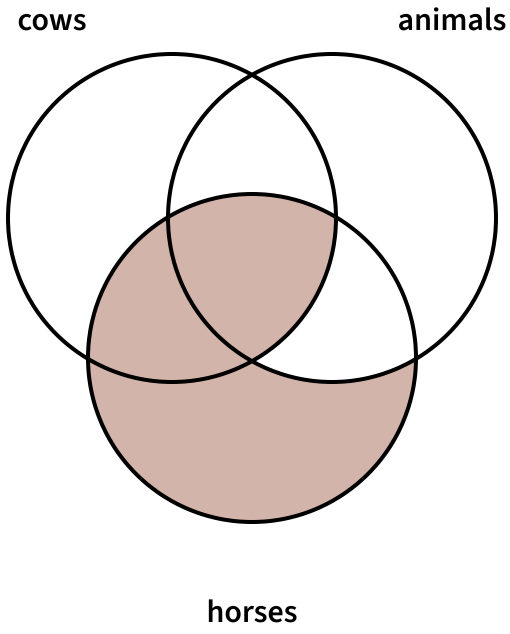


Example 2. Test with a Venn diagram: “All horses are animals. No cows are horses. Therefore no cows are animals.”

▼ **Show solution**

1. Terms: S = cows, P = animals, M = horses.
2. “All horses are animals” (All M are P) shades the parts of M outside P: M and SM.
3. “No cows are horses” (No S are M) shades where S and M overlap: SM (already shaded) and SPM.
4. Read the conclusion. “No cows are animals” needs both SP and SPM shaded. SPM is shaded, but SP is not. The diagram leaves room for cows that are animals, so the conclusion is not shown.
5. Check with the rules: illicit major (“animals” is distributed in the conclusion but not in the major premise). The verdicts agree.

Answer: Invalid.

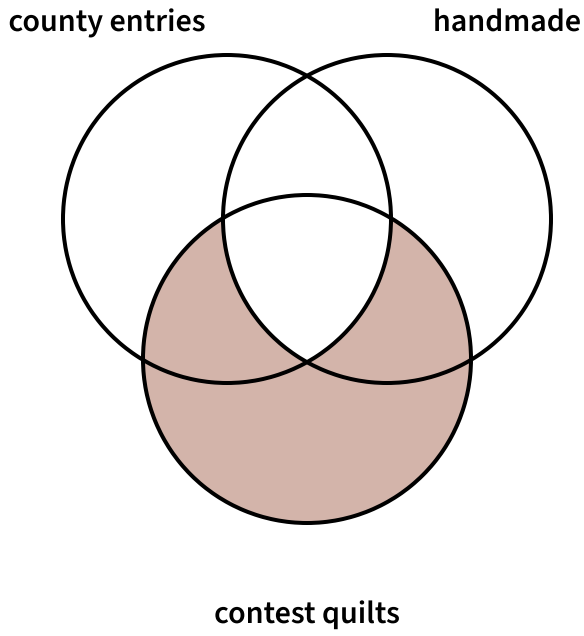


Example 3. Test Darapti (AAI-3) with a Venn diagram on both readings: "All the quilts in the contest are handmade things. All the quilts in the contest are county entries. Therefore some county entries are handmade things."

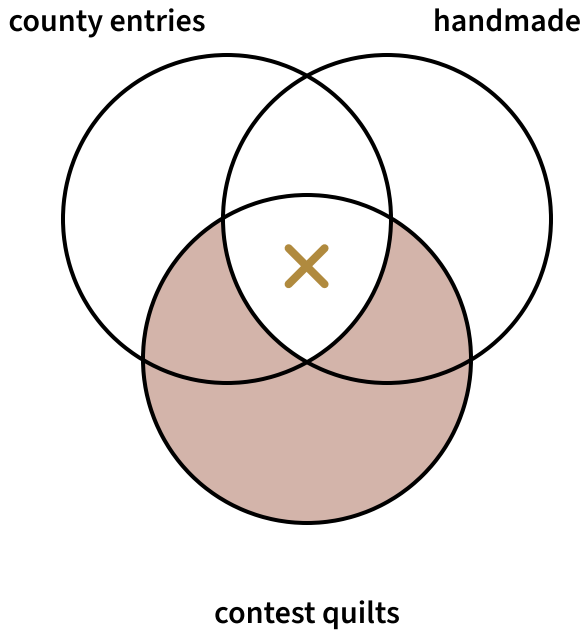
▼ **Show solution**

1. Terms: S = county entries, P = handmade things, M = contest quilts.
2. "All M are P" shades the parts of M outside P: M and SM. "All M are S" shades the parts of M outside S: M and PM.
3. Boolean reading: there are no particular premises, so there is no X. "Some county entries are handmade things" needs an X in SP or SPM. There is none, so the diagram does not show the conclusion. Invalid (the existential fallacy, Rule 6).
4. Traditional reading: we assume there are quilts in the contest, so the M circle has something in it. The only part of M not shaded is SPM, so the X goes there. Now there is an X in SPM, which is inside both S and P. The conclusion is shown.

Answer: Boolean reading: invalid.



Traditional reading (assuming there are quilts in the contest): valid.



PRACTICE

1. Easy In a Venn diagram, what does a shaded region mean? An X? A blank region?

   ▼ **Show answer**

   Shaded: the region is empty (nothing exists there). X: at least one thing exists there. Blank: nothing is known about the region; it may or may not have members.

2. Easy In a three-circle diagram, which regions do you shade for "All M are P"? For "No S are M"?

   ▼ **Show answer**

   "All M are P": shade the parts of M outside P, the regions M (only) and SM. "No S are M": shade where S and M overlap, the regions SM and SPM.

3. Easy Why should you draw a universal premise before a particular premise?

   ▼ **Show answer**

   Because the shading may settle where the X must go: if one of the two regions the particular premise points to is already shaded (empty), the X must go in the other one. Drawing the X first could put it in a region that turns out to be empty, or force it onto a line when it did not belong there.

4. **Easy** When does an X go on a line, and what does an X on a line mean?

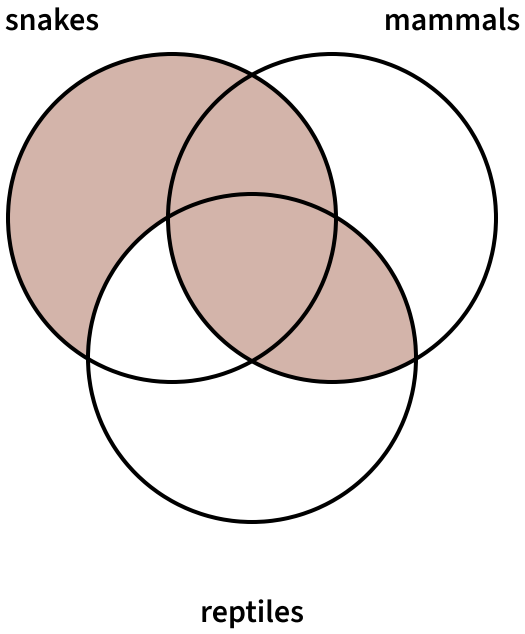
▼ **Show answer**

When neither of the two regions the particular premise points to is shaded. It means that something exists in one of the two regions, but the premises do not say which.

5. **Medium** Test with a Venn diagram: “No reptiles are mammals. All snakes are reptiles. Therefore no snakes are mammals.”

▼ **Show answer**

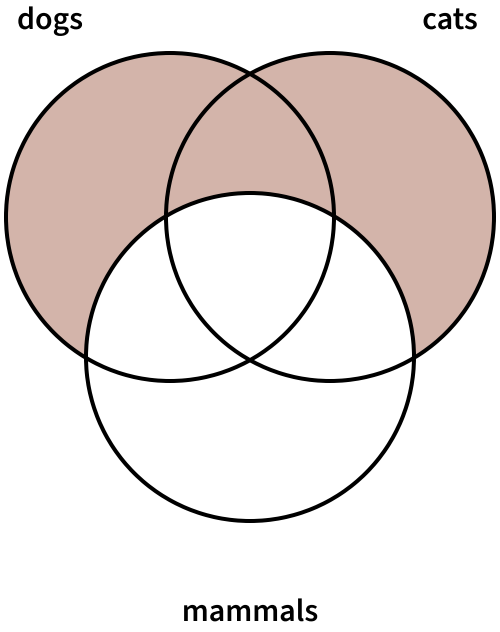
S = snakes, P = mammals, M = reptiles. “No M are P” shades PM and SPM; “All S are M” shades S and SP. The conclusion “No S are P” needs SP and SPM shaded, and both are. Valid (EAE-1, Celarent).



6. **Medium** Test with a Venn diagram: “All cats are mammals. All dogs are mammals. Therefore all dogs are cats.”

▼ **Show answer**

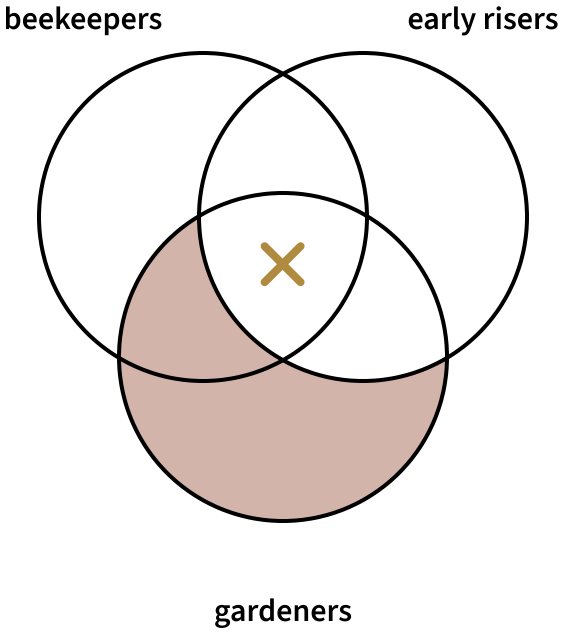
S = dogs, P = cats, M = mammals. “All P are M” shades P and SP; “All S are M” shades S and SP. The conclusion “All S are P” needs S and SM shaded, but SM (dogs that are mammals but not cats) is blank. Invalid; the rules agree (undistributed middle).



7. **Medium** Test with a Venn diagram: “All gardeners are early risers. Some beekeepers are gardeners. Therefore some beekeepers are early risers.”

▼ **Show answer**

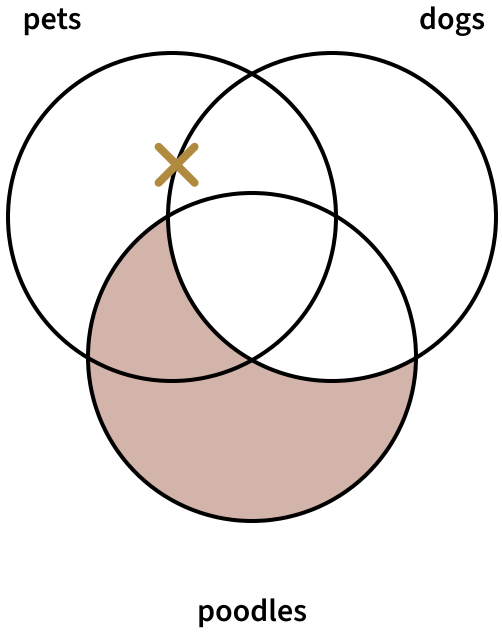
S = beekeepers, P = early risers, M = gardeners. Universal first: “All M are P” shades M and SM. “Some S are M” points to SM and SPM; SM is shaded, so the X goes in SPM. The conclusion “Some S are P” needs an X in SP or SPM: there is one in SPM. Valid (AII-1, Darii).



8. **Medium** Test with a Venn diagram: “All poodles are dogs. Some pets are not poodles. Therefore some pets are not dogs.”

▼ **Show answer**

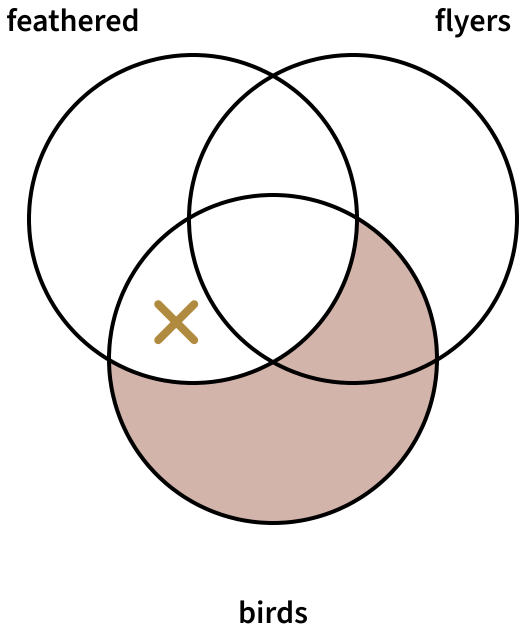
S = pets, P = dogs, M = poodles. “All M are P” shades M and SM. “Some S are not M” points to S and SP; neither is shaded, so the X goes on the line between them. The conclusion “Some S are not P” needs an X in S or SM; the X might be in SP (pets that are dogs but not poodles). Invalid; the rules agree (illicit major: “dogs” is distributed in the conclusion but not in the major premise).



9. Hard Test with a Venn diagram: “Some birds are not flyers. All birds are feathered animals. Therefore some feathered animals are not flyers.”

▼ **Show answer**

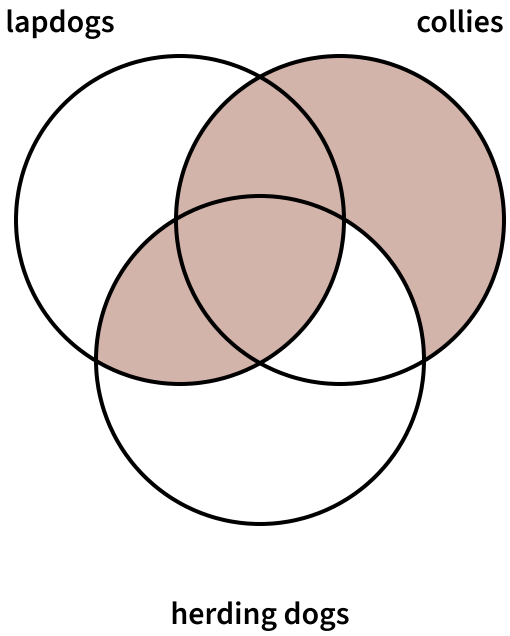
S = feathered animals, P = flyers, M = birds. Universal first: “All M are S” shades the parts of M outside S, the regions M and PM. “Some M are not P” points to M and SM; M is shaded, so the X goes in SM. The conclusion “Some S are not P” needs an X in S or SM: there is one in SM. Valid (OAO-3, Bocardo). Notice that the universal premise is the minor premise here, and it is still drawn first.



10. Hard Test with a Venn diagram: “All collies are herding dogs. No herding dogs are lapdogs. Therefore no lapdogs are collies.”

▼ **Show answer**

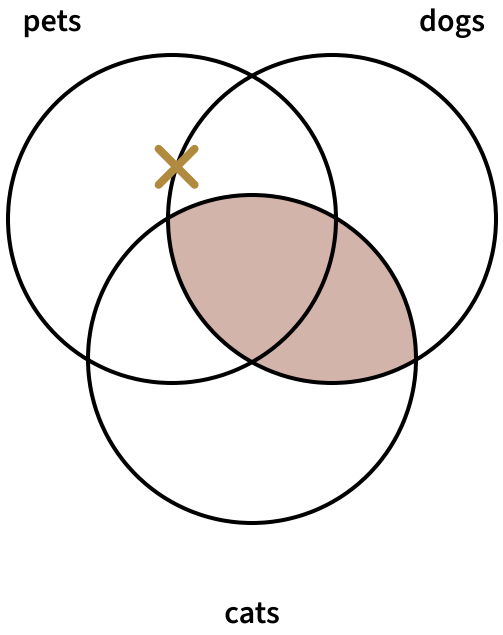
S = lapdogs, P = collies, M = herding dogs. “All P are M” shades P and SP. “No M are S” shades SM and SPM. The conclusion “No S are P” needs SP and SPM shaded: both are. Valid (AEE-4, Camenes).



11. Hard Test with a Venn diagram, then check your verdict with the rules: “No cats are dogs. Some pets are not cats. Therefore some pets are not dogs.”

▼ **Show answer**

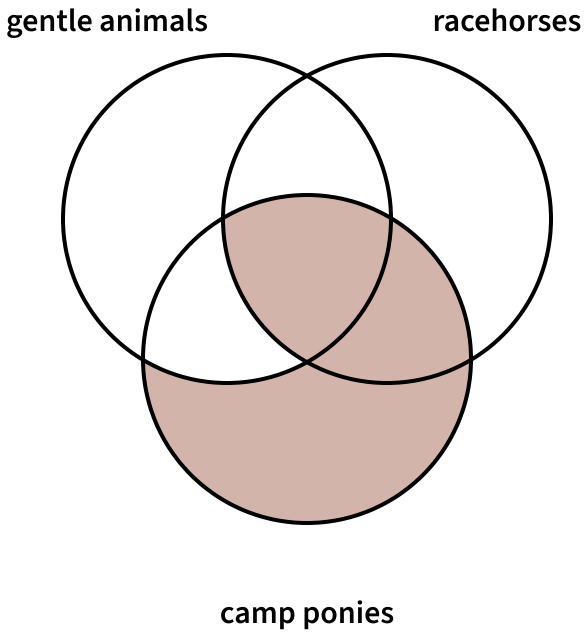
S = pets, P = dogs, M = cats. “No M are P” shades PM and SPM. “Some S are not M” points to S and SP; neither is shaded, so the X goes on the line between them. The conclusion “Some S are not P” needs an X in S or SM; the X might be in SP (pets that are dogs). Invalid. The rules agree: both premises are negative (exclusive premises).



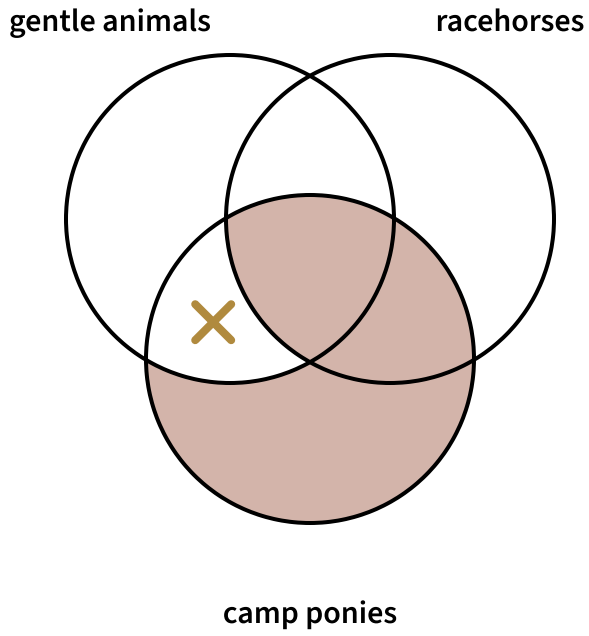
12. **Challenge** Test on both the Boolean and the traditional reading, with a diagram for each: “No ponies at Pine Lake Camp are racehorses. All ponies at Pine Lake Camp are gentle animals. Therefore some gentle animals are not racehorses.”

▼ **Show answer**

S = gentle animals, P = racehorses, M = camp ponies (form EAO-3, Felapton). “No M are P” shades PM and SPM; “All M are S” shades M and PM. Boolean reading: there is no X, and “Some S are not P” needs an X in S or SM, so the conclusion is not shown. Invalid (existential fallacy).



Traditional reading: assume there are ponies at the camp. The only unshaded part of the ponies circle is SM, so an X goes there. Now there is an X in S outside P, and the conclusion is shown. Valid on that assumption. The rules agree: Felapton passes Rules 1–5 and breaks only Rule 6.



COMMON MISTAKES

- Drawing the conclusion on the diagram. Draw only the premises, then look to see whether the conclusion is already there.
- Drawing the particular premise first and putting its X in a region the universal premise later shades.
- Putting an X inside one region when the premises allow either of two. If neither region is shaded, the X goes on the line.
- Reading an X on a line as if it proved the conclusion. It shows only that something is in one of two regions.
- Shading for a particular premise or putting an X for a universal one. Universals shade; particulars put an X.

> **Scripture.** “The heart of the prudent getteth knowledge; and the ear of the wise seeketh knowledge.” (Proverbs 18:15, KJV)
>
> The wise do not take a conclusion on trust; they look for themselves. A Venn diagram lets you see with your own eyes whether a conclusion has been earned.

5.5 The Named Forms

OBJECTIVES

- Read the mood of a named form from the vowels of its name.
- List the fifteen forms that are valid on either reading, by figure, with their names.
- Recite the first lines of the medieval memory verse and explain what the consonants in the names stand for.
- Name the forms that are valid only on the traditional reading, and say which class each assumes has members.

KEY TERMS

Unconditionally valid form

A form that is valid on both the traditional and the Boolean reading; there are fifteen.

Conditionally valid form

A form that is valid only if a certain class has members, as the traditional reading assumes; there are nine, such as Darapti (AAI-3).

Memory verse

The medieval Latin verse that lists the valid forms by name, figure by figure, beginning "Barbara, Celarent, Darii, Ferioque prioris."

Reduction

Showing that a form is valid by changing it, through conversion or other valid steps, into one of the four first-figure forms.

Perfect figure

The first figure, whose forms Aristotle treated as plainly valid, needing no proof.

Weakened form

A conditionally valid form that draws a particular conclusion where the corresponding universal could have been drawn, such as Barbari (AAI-1).

CORE IDEAS

Names for the valid forms. Of the 256 forms of syllogism, only a handful are valid. Medieval teachers gave each valid form a name, made up to sound like a Latin word, so that students could learn the list by heart. The trick is in the vowels: the three vowels of each name, in order, are its mood. **B*a*rb*a*r*a*** is AAA; **C*e*l*a*r*e*nt** is EAE; **D*a*r*ii*** is AII; **F*e*r*io*** is EIO. Once you know which figure a name belongs to, you know its whole form.

The fifteen forms valid on any reading. These forms are valid whether or not we assume that the classes have members, so they are called **unconditionally valid**:
Figure 1: Barbara (AAA), Celarent (EAE), Darii (AII), Ferio (EIO)
Figure 2: Cesare (EAE), Camestres (AEE), Festino (EIO), Baroco (AOO)
Figure 3: Disamis (IAI), Datisi (AII), Bocardo (OAO), Ferison (EIO)
Figure 4: Camenes (AEE), Dimaris (IAI), Fresison (EIO)
Some books spell Camenes as Calemes; it is the same form.

Patterns in the list. Look closely and you will see patterns. In figure 1, the major premise is always universal and the minor premise always affirmative. Every valid conclusion in figure 2 is negative. Every valid conclusion in figure 3 is particular. EIO is valid in all four figures, and all four of its names (Ferio, Festino, Ferison, Fresison) begin with F. These patterns are not accidents; each one follows from the rules of Lesson 5.3, and you can prove them. Aristotle called the syllogisms of the first figure perfect, because they are valid so plainly that they need no proof; later logicians therefore called it the **perfect figure**.

The memory verse. Students in the Middle Ages learned the names in a verse. Its first two lines are:
Barbara, Celarent, Darii, Ferioque prioris;
Cesare, Camestres, Festino, Baroco secundae.
Prioris means "of the first" and *secundae* "of the second"; the ending *-que* means "and." So the lines say: "Barbara, Celarent, Darii and Ferio are of the first figure; Cesare, Camestres, Festino and Baroco of the second." The verse goes on to the third figure, listing Darapti, Disamis, Datisi, Felapton, Bocardo and Ferison, and later versions add the fourth: Bramantip, Camenes, Dimaris, Fesapo and Fresison.

The consonants have jobs too. The names were built to show how each form can be **reduced** to the first figure, which proves it valid. The first letter tells which first-figure form it reduces to: names beginning with C reduce to Celarent, D to Darii, F to Ferio and B to Barbara. An *s* after a vowel says "convert this proposition simply"; a *p* says "convert it by limitation"; an *m* says "swap the premises"; and a *c* (in Baroco and Bocardo) says "prove it indirectly, by contradiction." Take Cesare: "No fish are mammals. All whales are mammals. So no whales are fish." The *s* after its first *e* says to convert the major premise: "No mammals are fish." Now the argument reads "No mammals are fish. All whales are mammals. So no whales are fish," which is Celarent.

Nine more on the traditional reading. If we assume, with the traditional reading, that every class has members, nine more forms become valid. Each one needs a particular class to have at least one member. Four of them are in the verse: Darapti (AAI-3) and Felapton (EAO-3), which need the middle term's class to have members; Bramantip (AAI-4), which needs the major term's class; and Fesapo (EAO-4), which needs the middle term's class. The other five are **weakened forms**, which draw a particular conclusion where the universal could have been drawn: Barbari (AAI-1), Celaront (EAO-1), Cesaro (EAO-2), Camestros (AEO-2) and Calemos (AEO-4). These need the minor term's class to have members. With them, the traditional list has twenty-four valid forms.

Using the names. When you meet a syllogism, put it in standard order and name its form. If the form is one of the fifteen, it is valid. If it is one of the nine, it is valid only on the traditional reading, and you should say which class must have members. If it is neither, it is invalid, and you should find the rule it breaks. Knowing the names is like knowing the multiplication table: you can still work things out the long way, but you will not have to.

Why bother memorizing? It is fair to ask whether a list of strange names is worth learning when the rules and the diagrams can test any syllogism. But the names do more than save time. They connect you with centuries of students who learned the same verse, and they let you speak about forms the way a musician speaks about chords. When you read in an old book that an argument is "in Darii" or "a syllogism in Barbara," you will know exactly what is meant.

> **From history.** The names and the memory verse come from the logic teachers of the Middle Ages. The verse appears in thirteenth-century handbooks, including the *Summulae Logicales* of Peter of Spain, one of the most widely used logic textbooks of the later Middle Ages. The medieval versions treated the fourth-figure forms as extra forms of the first figure, with different names; the names Bramantip, Camenes, Dimaris, Fesapo and Fresison belong to later versions. For centuries students in the universities of Europe chanted the verse the way younger children chant the alphabet. The method of reduction that the consonants record goes back to Aristotle himself, who proved the forms of the other figures by turning them into forms of the first.

Practice habit. When a syllogism looks good, try to name it. If you can name it, you know it is valid; if you cannot, look for the broken rule.

Enrichment: decoding the memory verse. Here again are the opening lines of the verse that students chanted for centuries:
Barbara, Celarent, Darii, Ferioque prioris;
Cesare, Camestres, Festino, Baroco secundae.
Write the mood and figure of each of the eight names. Then use the first letters to say which first-figure form each second-figure form reduces to. One of them, Baroco, begins with B but contains a *c*: it cannot be reduced by conversion alone and is proved by contradiction instead. Can you see why? (Hint: try converting its O premise.)

WORKED EXAMPLES

Example 1. Name the form, using its medieval name: "No fish are mammals. All whales are mammals. Therefore no whales are fish."

▼ **Show solution**

1. S = whales, P = fish, M = mammals. The major premise (with P) comes first, so the order is standard.
2. Mood: E, A, E. Figure: the middle term is the predicate of both premises, so figure 2.
3. EAE-2 is on the list of fifteen. In the second figure, the name with the vowels E, A, E is Cesare.

Answer: EAE-2, Cesare: valid on either reading.

Example 2. Use the letters of Festino to reduce this syllogism to the first figure: "No owls are daytime hunters. Some birds in the barn are daytime hunters. Therefore some birds in the barn are not owls."

▼ **Show solution**

1. The form is EIO-2 (Festino). Its first letter, F, says that it reduces to Ferio.
2. The *s* after the first vowel, *e*, says to convert the major premise simply. An E proposition converts simply: "No owls are daytime hunters" becomes "No daytime hunters are owls."
3. The new syllogism: "No daytime hunters are owls. Some birds in the barn are daytime hunters. Therefore some birds in the barn are not owls."
4. Check: the middle term is now the subject of the major premise and the predicate of the minor premise, so this is EIO-1, Ferio.

Answer: No daytime hunters are owls. Some birds in the barn are daytime hunters. Therefore some birds in the barn are not owls. (Ferio)

Example 3. Is this valid? "All plow horses at the Pell farm are draft horses. All draft horses are strong animals. Therefore some strong animals are plow horses at the Pell farm."

▼ **Show solution**

1. S = strong animals, P = plow horses at the Pell farm, M = draft horses. The major premise (with P) is first.
2. Mood: A, A, I. Figure: the middle term is the predicate of the major premise and the subject of the minor, so figure 4. The form is AAI-4.
3. AAI-4 is not among the fifteen. It is Bramantip, one of the nine forms valid only on the traditional reading, and it needs the major term's class (plow horses at the Pell farm) to have members.
4. If the Pell farm has at least one plow horse, that horse is a draft horse and so a strong animal, and the conclusion follows. If it has none, the premises can be true while the conclusion is false.

Answer: Bramantip (AAI-4): valid on the traditional reading, assuming there are plow horses at the Pell farm; invalid on the Boolean reading (existential fallacy).

PRACTICE

1. **Easy** What do the vowels in the name of a valid form tell you? Give the mood of Barbara and of Celarent.

   ▼ **Show answer**

   The three vowels, in order, give the mood (the letters of the major premise, minor premise and conclusion). Barbara is AAA; Celarent is EAE.

2. **Easy** Name the four valid forms of the first figure and give the mood of each.

   ▼ **Show answer**

   Barbara (AAA), Celarent (EAE), Darii (AII) and Ferio (EIO).

3. **Easy** Give the mood and figure of Ferio, Camestres, Baroco and Disamis.

   ▼ **Show answer**

   Ferio: EIO-1. Camestres: AEE-2. Baroco: AOO-2. Disamis: IAI-3.

4. **Easy** Write the first two lines of the memory verse. What do *prioris* and *secundae* mean?

   ▼ **Show answer**

   Barbara, Celarent, Darii, Ferioque prioris; Cesare, Camestres, Festino, Baroco secundae. Prioris means "of the first" (figure) and *secundae* "of the second" (figure).

5. **Medium** Give the form and its name: "All violins are four-stringed instruments. Some instruments in Grandpa's attic are not four-stringed instruments. Therefore some instruments in Grandpa's attic are not violins."

   ▼ **Show answer**

   AOO-2, Baroco (valid). S = instruments in Grandpa's attic, P = violins, M = four-stringed instruments; the middle term is the predicate of both premises.

6. **Medium** Give the form and its name: "Some pumpkins are prizewinners at the county fair. All pumpkins are squashes. Therefore some squashes are prizewinners at the county fair."

   ▼ **Show answer**

   IAI-3, Disamis (valid). S = squashes, P = prizewinners at the county fair, M = pumpkins; the middle term is the subject of both premises.

7. Medium Give the form and its name: “Some ribbon winners at the fair are pies. All pies are baked goods. Therefore some baked goods are ribbon winners at the fair.”

▼ **Show answer**

IAI-4, Dimaris (valid). S = baked goods, P = ribbon winners at the fair, M = pies; the middle term is the predicate of the major premise and the subject of the minor premise.

8. Medium Look at the list of fifteen forms. What do all the valid conclusions of figure 2 have in common? Of figure 3?

▼ **Show answer**

Every valid second-figure conclusion is negative (E or O). Every valid third-figure conclusion is particular (I or O). (This also holds for the conditionally valid forms of those figures.)

9. Hard Use the rules to explain why every valid syllogism in the second figure has a negative conclusion.

▼ **Show answer**

In figure 2 the middle term is the predicate of both premises. Only negative propositions (E and O) distribute their predicates, so for the middle term to be distributed at least once (Rule 2), one premise must be negative. By Rule 5, a negative premise requires a negative conclusion.

10. Hard Use the letters of Cesare to reduce this syllogism to the first figure, and name the result: “No rabbits are meat eaters. All foxes are meat eaters. Therefore no foxes are rabbits.”

▼ **Show answer**

Cesare begins with C, so it reduces to Celarent, and the *s* after its first *e* says to convert the major premise simply: “No meat eaters are rabbits.” The result: “No meat eaters are rabbits. All foxes are meat eaters. Therefore no foxes are rabbits.” This is EAE-1, Celarent.

11. Hard For each form, say whether it is valid on either reading, valid only on the traditional reading (and which class must have members), or invalid (and why): (a) AAI-1 (b) EAO-3 (c) AEE-1 (d) AEE-2 (e) AAI-2.

▼ **Show answer**

(a) AAI-1 (Barbari): valid only on the traditional reading; the minor term's class must have members. (b) EAO-3 (Felapton): valid only on the traditional reading; the middle term's class must have members. (c) AEE-1: invalid, illicit major (All M are P does not distribute P, but the E conclusion does). (d) AEE-2 (Camestres): valid on either reading. (e) AAI-2: invalid, undistributed middle (M is the predicate of two A premises). (On the Boolean reading, (a) and (b) are invalid by the existential fallacy.)

12. Challenge Use the six rules to show that EIO is valid in all four figures. What do the names of the four EIO forms have in common, and why does that fit?

▼ **Show answer**

Major premise E (No M are P or No P are M), minor premise I, conclusion O (Some S are not P). In every figure the E premise contains M and P and distributes both, so the middle term is distributed (Rule 2), and the major term, distributed in the O conclusion, is distributed in its premise (Rule 3). The minor term is undistributed in the O conclusion, so where it stands in the I premise does not matter. There is one negative premise and the conclusion is negative (Rules 4 and 5), and a premise is particular, so a particular conclusion is allowed (Rule 6). Nothing depends on where M stands, so EIO is valid in every figure. The four names, Ferio, Festino, Ferison and Fresison, all begin with F, because each reduces to Ferio (by simply converting the E premise, the I premise or both, as the *s* letters in each name show; Ferio itself needs no change).

COMMON MISTAKES

- Reading the consonants as part of the mood. Only the three vowels give the mood.
- Forgetting the figure. The vowels of Celarent and Cesare are the same (EAE); the names tell you the figure, and the figure matters.
- Treating Darapti, Felapton, Bramantip and Fesapo as valid on every reading. They need a class to have members.

- Assuming any form not on the list is "probably fine." If it is not one of the fifteen (or the nine, on the traditional reading), it is invalid; find the rule it breaks.

> **Scripture.** "Thy word have I hid in mine heart, that I might not sin against thee." (Psalms 119:11, KJV)
>
> Christians have always memorized what matters most, above all Scripture. Medieval students also memorized the forms of good reasoning, so that they would have them ready when they needed them.

UNIT 5 · CATEGORICAL SYLLOGISMS · LESSON 5.6 OF 69 · 40 MINUTES

5.6 Enthymemes

OBJECTIVES

- Define an enthymeme and name the three orders of enthymeme.
- Supply the missing premise or conclusion so that the completed syllogism is valid.
- Choose the missing premise charitably, and test the completed syllogism with the rules.
- Show how a hidden premise, once stated, can reveal the weakness of an everyday argument or advertisement.

KEY TERMS

Enthymeme

An argument with one of its parts (a premise or the conclusion) left unstated but meant to be understood.

First-order enthymeme

An enthymeme whose major premise is unstated.

Second-order enthymeme

An enthymeme whose minor premise is unstated.

Third-order enthymeme

An enthymeme whose conclusion is unstated.

Hidden (missing) premise

The unstated premise of an enthymeme, which must be supplied before the argument can be fully tested.

Principle of charity

When supplying what was left unsaid, choose the reading that the speaker most likely meant and that gives the argument its best chance, not the weakest one.

CORE IDEAS

Arguments with a piece missing. Few people talk in full syllogisms. Caleb says, "Biscuit must be a mammal; he's a dog." He has given a conclusion (Biscuit is a mammal) and one premise (Biscuit is a dog), and he expects you to supply the other premise yourself: all dogs are mammals. An argument with one of its parts left unstated is called an **enthymeme**. Most of the syllogisms you meet in conversation, in sermons, in newspapers and in advertising are enthymemes.

Three orders. Traditional logic names an enthymeme by the part that is missing. If the major premise is unstated, it is a **first-order** enthymeme: "Biscuit is a mammal, since he is a dog." If the minor premise is unstated, it is **second-order**: "All dogs are mammals, so Biscuit is a mammal." If the conclusion is unstated, it is **third-order**: "All dogs are mammals, and Biscuit is a dog." The speaker leaves you to draw the conclusion yourself, which can be very persuasive.

Why things get left out. Usually the missing part is too obvious to say. Saying it would sound tedious, like explaining a joke. But sometimes the missing part is left out because it would not survive being said aloud. An advertisement that says "Tramblewick boots are made with real leather, so they will keep your feet dry" hopes that you will not stop to state the hidden premise, "All boots made with real leather keep your feet dry," because once it is stated, you can see that it is doubtful. Stating the hidden premise is one of the most useful habits in all of logic.

Finding the missing premise. Start, as always, with the conclusion. Its subject is S and its predicate is P. Now look at the stated premise. It contains one of those terms and a new term, which must be the middle term M. The missing premise must join M with whichever of S or P the stated premise lacks. If the stated premise contains S, the missing one is the major premise and must contain M and P. If the stated

premise contains P, the missing one is the minor premise and must contain S and M. Then decide its quantity and quality and its order of terms.

Make it valid if you fairly can. There are usually several ways to fill the gap, and some make the argument valid while others do not. Take "Hannah's quilt can be entered in the contest, because it is a handmade quilt." The missing premise could be "All handmade quilts can be entered in the contest," which gives Barbara, a valid form. Or it could be "All quilts that can be entered in the contest are handmade quilts," which gives an undistributed middle. The **principle of charity** says to choose the reading the speaker most likely meant and that gives the argument its best chance. Here that is plainly the first. Test the completed syllogism with the rules or a Venn diagram.

Then judge the premise. Making an enthymeme valid is only half the work. Once the missing premise is written out, ask whether it is true. Often the premise that makes the argument valid is the very claim that is in doubt. In the boot advertisement, the argument becomes valid as soon as we add "All boots made with real leather keep your feet dry," but that premise is false, so the argument is unsound. A fair critic says exactly that: "Your argument needs this premise, and this premise is not true."

When the only valid premise is false. Sometimes no true premise will make an enthymeme valid. Suppose someone says, "The plant by the fence must be poison ivy, because it has leaves in groups of three." To make it valid we must add "All plants with leaves in groups of three are poison ivy," which is false: strawberry plants have leaves in threes. The true premise, "All poison ivy has leaves in groups of three," makes the argument invalid. Either way the argument fails. And if the speaker really meant "most" or "usually," the argument is not deductive at all; it is an inductive argument, to be judged strong or weak.

Supplying the conclusion. A third-order enthymeme leaves the conclusion for you to draw. "No one who missed the bus can be at the opening ceremony on time, and Caleb missed the bus." Here the conclusion is plain: Caleb cannot be at the opening ceremony on time. But check that the conclusion you draw really follows. Speakers sometimes hint at a conclusion that their premises do not support, and the listener who draws it for them has been led rather than persuaded. Draw only the conclusion that a valid form allows.

From history. Aristotle discussed the enthymeme in his *Rhetoric*, calling it the rhetorical syllogism, the argument suited to speeches. He noted that a speaker need not state what everyone already knows. His example runs, roughly, like this: to show that Dorieus won a crown, it is enough to say that he won at Olympia, since everyone knows that the Olympic prize is a crown. Aristotle's idea of the enthymeme was wider than "a syllogism with a missing part"; he included arguments from likelihoods and signs. Later logic books narrowed the word to the meaning used in this lesson, and they divided enthymemes into orders by the part left out.

Practice habit. When an argument seems to leap, ask: what would have to be true for this to follow? Say the hidden premise out loud, then ask whether it is true.

WORKED EXAMPLES

Example 1. Supply the missing premise and test: "Hannah's quilt can be entered in the contest, because it is a handmade quilt."

▼ **Show solution**

1. Conclusion: "Hannah's quilt is a thing that can be entered in the contest." S = Hannah's quilt, P = things that can be entered in the contest. (A singular proposition is treated as an A.)
2. Stated premise: "Hannah's quilt is a handmade quilt." It contains S, so it is the minor premise, and M = handmade quilts. The missing premise is the major premise and must join M and P: this is a first-order enthymeme.
3. Try "All handmade quilts are things that can be entered in the contest." Completed: All M are P; All S are M; so All S are P. This is Barbara, valid.
4. Compare "All things that can be entered in the contest are handmade quilts." That gives All P are M; All S are M, an undistributed middle. Charity chooses the first reading, which is also what the speaker plainly meant.

Answer: Missing major premise: "All handmade quilts are things that can be entered in the contest." The completed argument is Barbara (AAA-1), valid. It is sound if that rule is true of this contest.

Example 2. State the hidden premise and judge the argument: “Tramblewick boots are made with real leather, so they will keep your feet dry.”

▼ **Show solution**

1. Conclusion: “All Tramblewick boots are boots that will keep your feet dry.” S = Tramblewick boots, P = boots that will keep your feet dry.
2. Stated premise: “All Tramblewick boots are boots made with real leather.” It contains S, so M = boots made with real leather, and the major premise is missing.
3. The premise that makes it valid: “All boots made with real leather are boots that will keep your feet dry.” The completed argument is Barbara.
4. Is the hidden premise true? No. Leather can soak through unless it is treated, and water can come in at the seams or over the top. The argument is valid but unsound.

Answer: Hidden premise: “All boots made with real leather are boots that will keep your feet dry.” With it the argument is valid (Barbara), but the premise is false, so the argument is unsound. The advertisement hopes you will not state it.

Example 3. Supply the conclusion and test: “No one who missed the bus can be at the opening ceremony on time, and Caleb missed the bus.”

▼ **Show solution**

1. Both premises are given, so this is a third-order enthymeme. The middle term is the one in both premises: “people who missed the bus.”
2. Major premise: “No people who missed the bus are people who can be at the opening ceremony on time” (E). Minor premise: “Caleb is a person who missed the bus” (A, singular).
3. The conclusion joins the other two terms: “Caleb is not a person who can be at the opening ceremony on time” (E).
4. Form: EAE-1, Celarent. Valid.

Answer: Conclusion: Caleb cannot be at the opening ceremony on time. The completed syllogism is Celarent (EAE-1), valid.

PRACTICE

1. **Easy** What is an enthymeme?

   ▼ **Show answer**

   An argument with one of its parts (a premise or the conclusion) left unstated but meant to be understood.

2. **Easy** Name the part that is missing in a first-order, a second-order and a third-order enthymeme.

   ▼ **Show answer**

   First-order: the major premise. Second-order: the minor premise. Third-order: the conclusion.

3. **Easy** Supply the missing premise and name the order: "Rex is a mammal, because he is a dog."

   ▼ **Show answer**

   Missing premise: "All dogs are mammals." It is the major premise (it contains P, "mammals"), so this is a first-order enthymeme. The completed argument is Barbara.

4. **Easy** Supply the missing conclusion: "All the tomatoes in this basket are ripe, and this tomato is one of the tomatoes in this basket."

   ▼ **Show answer**

   "This tomato is ripe." (Third-order enthymeme; the completed argument is Barbara.)

5. **Medium** Supply the missing premise and name the order: "Hannah must attend Friday's meeting, because all members of the robotics club must attend it."

   ▼ **Show answer**

   Missing premise: "Hannah is a member of the robotics club." It contains S (Hannah), so it is the minor premise: a second-order enthymeme. The completed argument is Barbara, valid.

6. **Medium** Supply the missing premise and test: “No reptiles have fur, so no snakes have fur.”

   ▼ **Show answer**

   Missing minor premise: “All snakes are reptiles.” Completed: No reptiles are furry animals; all snakes are reptiles; so no snakes are furry animals. Celarent (EAE-1), valid, and both premises are true, so it is sound. (Second-order enthymeme.)

7. **Medium** Supply the missing premise that makes this valid, and name the form: “Some of the puppies in the litter will make good herding dogs, since some of them are border collies.”

   ▼ **Show answer**

   Missing major premise: “All border collies are dogs that will make good herding dogs.” Completed: All border collies are good herding dogs; some puppies in the litter are border collies; so some puppies in the litter are good herding dogs. Darii (AII-1), valid. (A careful student may note that the hidden premise is doubtful, since not every border collie makes a good herding dog; then the argument is valid but may be unsound. If the speaker meant only “most border collies,” the argument is inductive.)

8. **Medium** Mr. Pell says, “Spindlewood bicycles have lightweight aluminum frames, so a Spindlewood is the right bike for your family.” State the hidden premise. Is it believable?

   ▼ **Show answer**

   Hidden premise: “All bicycles with lightweight aluminum frames are the right bike for your family.” With it the argument is valid (Barbara). But the premise is not believable: a light frame is one good feature, but the right bike also depends on size, price, comfort and how the family will ride it. Stated plainly, the hidden premise shows how weak the advertisement is.

9. Hard “The plant by the fence must be poison ivy, because it has leaves in groups of three.” Supply two different missing premises, one true and one false. Test the argument with each.

▼ **Show answer**

False premise: “All plants with leaves in groups of three are poison ivy.” This gives Barbara, valid, but the premise is false (strawberry plants also have leaves in threes), so the argument is unsound. True premise: “All poison ivy plants are plants with leaves in groups of three.” This gives All P are M; All S are M; an undistributed middle, so the argument is invalid. Either way the argument fails as a proof (though the old warning about leaves of three is still a good reason to be careful).

10. Hard Supply the missing premise charitably and name the form: “Caleb isn’t on the varsity swim team; he hasn’t passed the lifesaving test.”

▼ **Show answer**

Conclusion: “Caleb is not a member of the varsity swim team” (E, singular). Stated premise: “Caleb is not a person who has passed the lifesaving test” (E, minor premise). Missing major premise: “All members of the varsity swim team are people who have passed the lifesaving test.” Form: All P are M; No S are M; so No S are P: Camestres (AEE-2), valid. (The other way round, “All people who have passed the lifesaving test are members of the team,” would give an illicit major, so charity chooses the first. Accept also “No people who have not passed the lifesaving test are members of the varsity swim team,” with the minor premise “Caleb is a person who has not passed the lifesaving test,” which gives Celarent (EAE-1), also valid.)

11. Hard What conclusion follows, if any? “All the hens in the north coop are Rhode Island Reds, and none of the hens in the north coop is a young bird.” Name the form of the completed syllogism and explain any reading it depends on.

▼ **Show answer**

The middle term is “hens in the north coop,” the subject of both premises (figure 3). Two universal premises with a negative premise allow only a negative conclusion, and in figure 3 only a particular one: “Some Rhode Island Reds are not young birds.” The form is EAO-3, Felapton (major premise: No hens in the north coop are young birds; minor premise: All hens in the north coop are Rhode Island Reds). It is valid on the traditional reading, which assumes there are hens in the north coop; on the Boolean reading no conclusion follows (existential fallacy). “No Rhode Island Reds are young birds” does not follow at all (illicit minor), and neither does “Some young birds are not Rhode Island Reds” (illicit major).

12. Challenge A farmer says, “Some of my hens must be sick, because some of them have stopped laying.” (a) Supply the premise that makes the argument valid, and name the form. (b) Is that premise true? (c) What weaker premise might the farmer really believe, and what kind of argument does it give?

▼ **Show answer**

(a) “All hens that have stopped laying are sick hens.” Completed: All hens that have stopped laying are sick; some of my hens are hens that have stopped laying; so some of my hens are sick. Darii (AII-1), valid. (b) No: healthy hens stop laying for other reasons, such as molting, the short days of winter or age. So the argument is valid but unsound. (c) Perhaps “Hens that stop laying are often sick.” Then the argument is inductive, not deductive: it gives some reason to check the hens, but it is not a proof, and its strength depends on how often stopping is really caused by sickness. (Accept any reasonable weaker premise with a matching judgment.)

COMMON MISTAKES

- Supplying a premise that makes the argument invalid when a fair, valid reading was plainly meant. Use the principle of charity.

- Stopping once the argument is valid. Then ask whether the supplied premise is true.
- Supplying a premise far stronger than the speaker needs or believes. Choose the premise the speaker most likely meant.
- Drawing a hinted conclusion that the premises do not support. Draw only the conclusion that a valid form allows.

> **Scripture.** "It is the glory of God to conceal a thing: but the honour of kings is to search out a matter." (Proverbs 25:2, KJV)
>
> Searching out what is hidden is honorable work. In an enthymeme the hidden thing is a premise, and bringing it into the light is how we judge the argument fairly.

UNIT 5 · CATEGORICAL SYLLOGISMS · LESSON 5.7 OF 69 · 45 MINUTES

5.7 Chains of Reasoning: Sorites and Carroll's Puzzles

OBJECTIVES

- Define a sorites and break it into the chain of syllogisms it contains.
- Rewrite premises with obversion and contraposition so that their terms link up.
- Solve a Carroll-style puzzle by naming the classes, translating, chaining and drawing the conclusion.
- Find the broken link in an invalid chain.

KEY TERMS

Sorites

A chain argument with three or more premises, in which the predicate of each premise is the subject of the next (in some order), and the conclusion joins the first subject to the last predicate. It is a chain of syllogisms with the in-between conclusions left unstated.

Intermediate conclusion

A conclusion drawn partway along a chain and used as a premise for the next link; in a sorites it is left unstated.

Aristotelian sorites

A sorites written so that the first premise contains the subject of the conclusion: All A are B, All B are C, All C are D, so All A are D.

Goclenian sorites

A sorites written in the reverse order, beginning with the premise that contains the predicate of the conclusion; named after Rudolph Goclenius (1547–1628).

Complement

The class of everything not in a given class: the complement of "fierce creatures" is "creatures that are not fierce" (written non-fierce).

Link term

In a chain, a term that appears in two premises and joins them; every term except the two in the conclusion is a link term.

CORE IDEAS

Longer chains. Sometimes one middle term is not enough, and reasoning runs through a chain: "Every student in the chess club plays in the Saturday tournament. Everyone who plays in the tournament gets a scorebook. Everyone who gets a scorebook must return it by Monday. So every student in the chess club must return a scorebook by Monday." An argument like this, with three or more premises linked end to end, is called a **sorites** (sor-EYE-teez), from the Greek word for a heap. It is really a chain of syllogisms with the in-between conclusions left out.

Breaking a chain into links. To test a sorites, supply the unstated **intermediate conclusions** and test each syllogism. In the chess-club chain, the first two premises give "Every student in the chess club gets a scorebook" (Barbara). That conclusion and the third premise give "Every student in the chess club must return a scorebook by Monday" (Barbara again). The chain is valid if and only if every link is valid. One bad link breaks the whole chain, just as one broken link lets the bucket fall into the well.

Two orders. When each premise's predicate is the next premise's subject (All A are B; All B are C; All C are D; so All A are D), the chain is called an **Aristotelian sorites**, though Aristotle did not use the name. When the same premises are written in reverse order (All C are D; All B are C; All A are B; so All A are D), it is called a **Goclenian sorites**, after Rudolph Goclenius, a German professor of the 1500s. The order of writing makes no difference to validity. What matters is that the terms link:

every term except the first subject and the last predicate must appear in exactly two premises.

Carroll's puzzles. Lewis Carroll, the Oxford mathematician who wrote *Alice's Adventures in Wonderland*, loved sorites. In his *Symbolic Logic* (1896) he set dozens of them as puzzles, with the premises scrambled, worded in odd ways and filled with absurd creatures. The absurdity has a purpose. When the premises are about dragons that collect teacups, you cannot answer by what you already believe about the world; you have to follow the logic. Carroll-style puzzles train exactly the skill this unit has been building: keeping your eye on the form.

Step 1: name the classes. Read all the premises and list the classes they talk about. Give each class a short name or letter, and use one wording for one class. If one premise says "fierce dragons" and another "dragons that are not fierce," those are a class and its **complement**, not two unrelated classes; write them as "fierce" and "non-fierce." A puzzle with three premises usually has four classes; the two that appear only once will be the terms of the conclusion.

Step 2: put every premise in the form "All x are y." Chains are easiest to follow when every link is an A proposition. Two tools from Unit 4 do the work. **Obversion** turns an E into an A: "No x are y" becomes "All x are non-y." **Contraposition** turns an A around: "All x are y" becomes "All non-y are non-x." Use contraposition when a term appears in the wrong place or with "non-" on the wrong side. Remember also from Lesson 4.2 that "Only x are y" means "All y are x."

Step 3: chain and conclude. Start with the class that appears only once as a subject, and follow the links: from x to y, from y to the premise that begins with y, and so on to the end. The conclusion joins the first subject to the last predicate: "All [first] are [last]." Finally, put the conclusion back into plain English. Sometimes it reads best as an E: "All owls are non-dull" is better said "No owls are dull."

A worked chain. Here is an original puzzle in Carroll's style. (1) Every owl in this wood can recite poetry. (2) Only creatures that are dull at dinner are left off the badger's guest list. (3) No creature that can recite poetry is dull at dinner. Name the classes: owls in this wood, poetry reciters, dull diners, creatures left off the list. Translate: (1) All owls are reciters. (3) No reciters are dull, so all reciters are non-dull. (2) All creatures left off the list are dull; contraposed, all non-dull creatures are

not left off the list. Chain: owls → reciters → non-dull → not left off. Conclusion: no owl in this wood is left off the badger's guest list.

From history. The word *sorites* first named an ancient puzzle, the paradox of the heap: if one grain of sand is not a heap, and adding one grain never turns a non-heap into a heap, when does a heap appear? Later logicians used the name for chains of syllogisms. Rudolph Goclenius (1547–1628), a professor at Marburg in Germany, gave his name to the reversed form. Lewis Carroll was the pen name of Charles Lutwidge Dodgson (1832–1898), who taught mathematics at Christ Church, Oxford. Besides the Alice books, he wrote *The Game of Logic* (1886) and *Symbolic Logic* (1896), which was meant to make logic a pleasure for young people. He filled it with puzzles about babies, crocodiles, kittens and pigs.

Practice habit. When a conclusion is reached by a long chain, test it one link at a time.

Enrichment: Lewis Carroll's *Symbolic Logic*. Lewis Carroll's *Symbolic Logic* (1896) is in the public domain, and its puzzles are still some of the best practice in the world for this lesson. Ask a parent to help you find a copy, and try a few of his sorites puzzles with three or four premises, using the steps in this lesson: name the classes, rewrite each premise as "All x are y," chain and conclude. Then write a puzzle of your own with four premises about imaginary creatures, scramble the order and give it to someone at home to solve.

Example 1. Supply the intermediate conclusion and test each link: "Every student in the chess club plays in the Saturday tournament. Everyone who plays in the Saturday tournament gets a scorebook. Everyone who gets a scorebook must return it by Monday. So every student in the chess club must return a scorebook by Monday."

▼ **Show solution**

1. Name the classes: C = students in the chess club, T = people who play in the Saturday tournament, B = people who get a scorebook, R = people who must return a scorebook by Monday.
2. Premises: All C are T. All T are B. All B are R.
3. First link: All T are B; All C are T; so All C are B (Barbara). This is the unstated intermediate conclusion: every student in the chess club gets a scorebook.
4. Second link: All B are R; All C are B; so All C are R (Barbara).

Answer: Intermediate conclusion: every student in the chess club gets a scorebook. Both links are Barbara, so the sorites is valid.

Example 2. Solve: (1) Everyone who is never late on Sundays is given a hymnbook of their own. (2) All the Hartwell children sing in the choir. (3) No choir singer is ever late on Sundays.

▼ **Show solution**

1. Name the classes: H = Hartwell children, C = choir singers, L = people late on Sundays (so non-L = people never late), B = people given a hymnbook of their own.
2. Translate: (1) All non-L are B. (2) All H are C. (3) No C are L; by obversion, All C are non-L.
3. Find the start: H appears only once, as a subject. B appears only once, as a predicate. So the conclusion will be "All H are B."
4. Chain: (2) All H are C; (3) All C are non-L; (1) All non-L are B. So All H are B.

Answer: All the Hartwell children are given hymnbooks of their own.

Example 3. Solve: (1) No frog in the lily pond is kept out of the garden party. (2) Every creature that is not polite is kept out of the garden party. (3) Every polite creature bows to the queen.

▼ **Show solution**

1. Name the classes: F = frogs in the lily pond, K = creatures kept out of the party, P = polite creatures, Q = creatures that bow to the queen.
2. Translate: (1) No F are K; obverted, All F are non-K. (2) All non-P are K. (3) All P are Q.
3. The chain needs a premise beginning with non-K. Contrapose (2): All non-P are K becomes All non-K are P.
4. Chain: All F are non-K; All non-K are P; All P are Q. So All F are Q.

Answer: All the frogs in the lily pond bow to the queen.

PRACTICE

1. Easy What is a sorites?

   ▼ **Show answer**

   A chain argument with three or more premises in which the terms link from premise to premise, and the conclusion joins the first subject to the last predicate. It is a chain of syllogisms with the in-between conclusions left unstated.

2. Easy What conclusion follows? "All A are B. All B are C. All C are D."

   ▼ **Show answer**

   All A are D.

3. Easy Obvert to make an A proposition: "No hedgehogs are punctual creatures."

   ▼ **Show answer**

   "All hedgehogs are non-punctual creatures" (creatures that are not punctual).

4. Easy Contrapose: "All creatures that sing at breakfast are cheerful creatures."

   ▼ **Show answer**

   "All creatures that are not cheerful are creatures that do not sing at breakfast" (All non-cheerful creatures are non-singers at breakfast).

5. **Medium** What conclusion follows? “All the geese on the farm are noisy. No noisy creature is welcome in the library.”

   ▼ **Show answer**

   “No geese on the farm are welcome in the library.” (Celarent: No noisy creatures are welcome; all the geese are noisy.)

6. **Medium** Put the premises in chain order and draw the conclusion: (1) All bakers in our town rise before dawn. (2) Everyone who rises before dawn is sleepy by nine o’clock at night. (3) All of Mrs. Ortiz’s brothers are bakers in our town.

   ▼ **Show answer**

   Order: (3), (1), (2). All of Mrs. Ortiz’s brothers are bakers in our town; all bakers in our town rise before dawn; everyone who rises before dawn is sleepy by nine. Conclusion: all of Mrs. Ortiz’s brothers are sleepy by nine o’clock at night.

7. **Medium** Give the unstated intermediate conclusion and the final conclusion: (1) All the apples in the bin are Winesaps. (2) All Winesaps are tart apples. (3) No tart apples are used in Grandma’s applesauce.

   ▼ **Show answer**

   Intermediate conclusion: all the apples in the bin are tart apples (Barbara). Final conclusion: no apples in the bin are used in Grandma’s applesauce (Celarent).

8. **Medium** Solve: (1) No dragon that collects teacups is fierce. (2) All the dragons in this valley collect teacups. (3) Every dragon that is not fierce is invited to the village fair.

   ▼ **Show answer**

   Classes: V = dragons in this valley, T = teacup collectors, F = fierce dragons, I = dragons invited to the fair. (2) All V are T. (1) No T are F, so All T are non-F. (3) All non-F are I. Chain: V → T → non-F → I. Conclusion: all the dragons in this valley are invited to the village fair.

9. Hard Solve: (1) Every hedgehog at Bramble Hall reads the newspaper. (2) Only animals that have been to school can read the newspaper. (3) No animal that has been to school goes without spectacles.

▼ Show answer

Classes: H = hedgehogs at Bramble Hall, N = newspaper readers, S = animals that have been to school, W = animals that wear spectacles. (1) All H are N. (2) "Only S are N" means All N are S. (3) No S are non-W (animals without spectacles); by obversion, All S are W. Chain: H → N → S → W. Conclusion: all the hedgehogs at Bramble Hall wear spectacles.

10. Hard Solve: (1) Every badger that is not cross is allowed to play the tuba. (2) All the badgers in the brass band are badgers that sleep until noon. (3) No badger that sleeps until noon is cross.

▼ Show answer

Classes: B = badgers in the brass band, N = badgers that sleep until noon, C = cross badgers, T = badgers allowed to play the tuba. (2) All B are N. (3) No N are C, so All N are non-C. (1) All non-C are T. Chain: B → N → non-C → T. Conclusion: all the badgers in the brass band are allowed to play the tuba.

11. Hard Is this chain valid? If not, find the broken link. "All the cats in the barn are mousers. Some mousers are lazy. All lazy creatures sleep in the hayloft. So all the cats in the barn sleep in the hayloft."

▼ Show answer

Invalid. The first link fails: from "All the cats in the barn are mousers" and "Some mousers are lazy" nothing follows, because the middle term "mousers" is undistributed in both premises (the predicate of an A and the subject of an I). The chain breaks there, so the conclusion is not proved. (It is also too strong: even a valid chain with a particular premise could give only a particular conclusion.)

12. **Challenge** Solve, showing every translation: (1) Only creatures that catch cold are given hot cocoa by the owl. (2) Every creature that is not given hot cocoa by the owl sings in the snow. (3) No creature that wears mittens catches cold. (4) Every kitten in the basket wears mittens.

    ▼ **Show answer**

    Classes: K = kittens in the basket, M = creatures that wear mittens, C = creatures that catch cold, H = creatures given hot cocoa by the owl, S = creatures that sing in the snow. (4) All K are M. (3) No M are C; by obversion, All M are non-C. (1) "Only C are H" means All H are C; by contraposition, All non-C are non-H. (2) All non-H are S. Chain: K → M → non-C → non-H → S. Conclusion: all the kittens in the basket sing in the snow.

COMMON MISTAKES

- Treating a class and its complement as two unrelated classes. "Fierce" and "not fierce" are one class and its complement; link them with obversion or contraposition.
- Contraposing without switching the terms. "All x are y" becomes "All non-y are non-x," not "All non-x are non-y."
- Reading "Only x are y" as "All x are y." It means "All y are x."
- Letting the silliness of the premises distract you. Judge the chain by its form, not by whether dragons collect teacups.
- Ignoring a particular or doubtful link. One bad link breaks the whole chain.

> **Scripture.** "Behold, this have I found, saith the preacher, counting one by one, to find out the account" (Ecclesiastes 7:27, KJV)
>
> The Preacher reached his conclusion by counting one by one. A long chain of reasoning is trustworthy in the same way: one link at a time, each one checked.

UNIT 5 · CATEGORICAL SYLLOGISMS · LESSON 5.8 OF 69 · 40 MINUTES

5.8 Hypothetical and Disjunctive Syllogisms

OBJECTIVES

- Identify the antecedent and consequent of a conditional, including one stated with "only if."

- Use modus ponens, modus tollens, the pure hypothetical syllogism and the disjunctive syllogism.
- Recognize and explain the fallacies of affirming the consequent and denying the antecedent.
- Tell inclusive from exclusive "or," and explain why affirming a disjunct is usually invalid.

KEY TERMS

Conditional (hypothetical) proposition

A proposition of the form "If p, then q." The "if" part, p, is the antecedent; the "then" part, q, is the consequent.

Modus ponens

Affirming the antecedent, a valid form: If p, then q; p; therefore q.

Modus tollens

Denying the consequent, a valid form: If p, then q; not q; therefore not p.

Affirming the consequent

An invalid form: If p, then q; q; therefore p.

Denying the antecedent

An invalid form: If p, then q; not p; therefore not q.

Disjunctive syllogism

A valid form: Either p or q; not p; therefore q (and likewise with q denied).

Inclusive and exclusive "or"

Inclusive "or" means at least one, perhaps both; exclusive "or" means exactly one, not both. In logic, "or" is inclusive unless the meaning plainly rules out both.

Affirming a disjunct

Reasoning "Either p or q; p; therefore not q." Invalid for inclusive "or"; acceptable only when the "or" is exclusive.

CORE IDEAS

Syllogisms with "if" and "or." Not every syllogism is made of A, E, I and O propositions. Many everyday arguments turn on an "if" or an "or": "If the gate is open, the cows are out. The gate is open. So the cows are out." Traditional logic calls these the **hypothetical** and **disjunctive** syllogisms. A **mixed hypothetical syllogism** has one conditional premise ("If p, then q") and one plain premise that affirms or

denies part of it. In the conditional, p is the **antecedent** (what comes before) and q is the **consequent** (what follows).

What a conditional promises. "If the gate is open, the cows are out" promises one thing only: whenever the antecedent is true, the consequent is true too. It does not say the gate is open. It does not say that an open gate is the only way the cows could get out; they might have broken the fence. Keeping in mind exactly what the conditional promises, and no more, is the key to the four forms that follow. Two of them are valid and two are not.

Two valid forms. Modus ponens (Latin for "the way that affirms") affirms the antecedent: "If the gate is open, the cows are out. The gate is open. So the cows are out." **Modus tollens** ("the way that denies") denies the consequent: "If the gate is open, the cows are out. The cows are not out. So the gate is not open." Modus tollens is valid because, if the gate were open, the cows would be out; they are not out, so the gate cannot be open. Both forms are valid whatever p and q are about.

Two invalid look-alikes. Affirming the consequent: "If the gate is open, the cows are out. The cows are out. So the gate is open." Invalid: the cows may have broken through the fence. **Denying the antecedent**: "If the gate is open, the cows are out. The gate is not open. So the cows are not out." Invalid for the same reason. Both mistakes treat "if" as though it meant "if and only if." A quick test: ask whether q could have some other cause besides p. If it could, knowing q tells you nothing certain about p, and knowing that p is false tells you nothing certain about q.

Watch the negatives and the wording. The forms are named by what is affirmed or denied, not by whether the sentence contains "not." "If the pond is frozen solid, the ducks are not swimming. The ducks are swimming. So the pond is not frozen solid" is modus tollens: the consequent is "the ducks are not swimming," and the second premise denies it. Watch also for "only if." "The dough rises only if the yeast is alive" means "If the dough rises, the yeast is alive." The words after "only if" are the consequent.

The pure hypothetical syllogism. When both premises are conditionals, they can be chained: "If the creek floods, the lower field will be under water. If the lower field is under water, we cannot plant corn there this week. So if the creek floods, we cannot plant corn there this week." The form is "If p, then q; if q, then r; therefore if p, then

r." It works like a sorites: the consequent of one premise must be the antecedent of the next, worded the same way.

The disjunctive syllogism. A disjunction says "Either p or q." The **disjunctive syllogism** denies one part and concludes the other: "Either the fuse is blown or the lamp is unplugged. The lamp is not unplugged. So the fuse is blown." This is valid. (Whether the first premise is true, whether those are really the only two possibilities, is a separate question.) In logic, "or" is **inclusive**: it means at least one is true, and perhaps both.

Affirming a disjunct. Because "or" allows both, you cannot affirm one part and deny the other. "Either Hannah will practice piano this evening or she will do her math homework. She practiced piano. So she did not do her math homework." Invalid: she may have done both. Only when the "or" is **exclusive**, when the two parts cannot both be true, does the inference work: "Either the light is on or it is off. It is on. So it is not off." Before you affirm a disjunct, ask: could both be true? If they could, the argument fails.

> **From history.** Aristotle studied the categorical syllogism; the logic of "if" and "or" was worked out above all by the Stoics. Chrysippus (third century B.C.) listed five basic forms that he called indemonstrable because they need no proof. They include what we call modus ponens, modus tollens and the disjunctive syllogism. In the early sixth century Boethius wrote a treatise on hypothetical syllogisms that carried this teaching into the Latin schools of the Middle Ages. Medieval logicians gave the two valid conditional forms their Latin names: *modus ponendo ponens*, "the way that affirms by affirming," and *modus tollendo tollens*, "the way that denies by denying," shortened today to modus ponens and modus tollens.

> **Practice habit.** When you hear "if," ask: could the result have another cause? When you hear "or," ask: could both be true?

WORKED EXAMPLES

Example 1. Name the form and say whether it is valid: “If the bread has risen, the yeast was alive. The bread has not risen. So the yeast was not alive.”

▼ **Show solution**

1. Antecedent: the bread has risen. Consequent: the yeast was alive.
2. The second premise denies the antecedent. The conclusion denies the consequent.
3. Could the consequent be true while the antecedent is false? Yes: the yeast may have been alive, but the kitchen was too cold or the dough was not given enough time.

Answer: Denying the antecedent: invalid.

Example 2. Name the form and say whether it is valid: “If the pond is frozen solid, the ducks are not swimming on it. The ducks are swimming on it. So the pond is not frozen solid.”

▼ **Show solution**

1. Antecedent: the pond is frozen solid. Consequent: the ducks are not swimming on it.
2. The second premise, “The ducks are swimming on it,” denies the consequent.
3. The conclusion denies the antecedent. If p, then q; not q; therefore not p.

Answer: Modus tollens (denying the consequent): valid. The “not” inside the consequent does not change the form.

Example 3. Test both arguments: (a) "Either Caleb left his glove at the ball field or he left it on the bus. He did not leave it on the bus. So he left it at the ball field." (b) "Either Hannah will practice piano this evening or she will do her math homework. She practiced piano. So she did not do her math homework."

▼ **Show solution**

1. (a) One part of the disjunction is denied and the other is concluded. This is a disjunctive syllogism: valid. (It is sound only if those really are the only two places the glove could be.)
2. (b) One part is affirmed and the other denied: affirming a disjunct.
3. Ask whether both parts of (b) could be true. Yes: Hannah could practice piano and also do her homework. So the "or" is inclusive and the inference fails.

Answer: (a) Disjunctive syllogism, valid. (b) Affirming a disjunct, invalid, because both parts could be true.

PRACTICE

1. Easy Name the antecedent and the consequent: "If the cows are out, the gate is open."

   ▼ **Show answer**

   Antecedent: the cows are out. Consequent: the gate is open.

2. Easy Name the form and say whether it is valid: "If p, then q. p. Therefore q."

   ▼ **Show answer**

   Modus ponens (affirming the antecedent): valid.

3. Easy Of the four mixed hypothetical forms (affirming or denying the antecedent, affirming or denying the consequent), which two are valid?

   ▼ **Show answer**

   Affirming the antecedent (modus ponens) and denying the consequent (modus tollens). Affirming the consequent and denying the antecedent are invalid.

4. **Easy** What does “or” mean in logic, unless the meaning plainly says otherwise? What is the other kind of “or” called?

   ▼ **Show answer**

   Inclusive “or”: at least one of the two is true, and perhaps both. The other kind is exclusive “or”: exactly one is true, not both.

5. **Medium** Name the form and say whether it is valid: “If it snowed overnight, school is closed. School is closed. So it snowed overnight.”

   ▼ **Show answer**

   Affirming the consequent: invalid. School could be closed for another reason, such as a broken furnace or a holiday.

6. **Medium** Name the form and say whether it is valid: “If Biscuit ate the cake, there are crumbs on his nose. There are no crumbs on his nose. So Biscuit did not eat the cake.”

   ▼ **Show answer**

   Modus tollens (denying the consequent): valid. (Whether it is sound depends on whether the first premise is true; Biscuit might have licked his nose clean.)

7. **Medium** Name the form and say whether it is valid: “Either the fuse is blown or the lamp is unplugged. The lamp is not unplugged. So the fuse is blown.”

   ▼ **Show answer**

   Disjunctive syllogism: valid. (If the bulb could be burned out instead, the first premise is false and the argument is unsound, but the form is still valid.)

8. **Medium** Name the form and say whether it is valid: “If the tomatoes get enough sun, they will ripen. They did not get enough sun. So they will not ripen.”

   ▼ **Show answer**

   Denying the antecedent: invalid. The conditional says only what happens if they get enough sun; they might still ripen another way, for example indoors on a windowsill.

9. Hard Mrs. Ortiz says, “For dessert you may have pie or ice cream.” Caleb had pie. Hannah concludes, “So Caleb did not have ice cream.” Is Hannah’s reasoning valid?

▼ Show answer

It depends on what Mrs. Ortiz’s “or” meant. If she meant “one or the other, but not both” (exclusive “or”), and Caleb kept to what he was allowed, Hannah’s inference is valid. If she meant “either is allowed, and perhaps both” (inclusive “or”), Hannah is affirming a disjunct, and the inference is invalid. Accept either answer if the student explains which reading is used; the point is that the inference works only with an exclusive “or.”

10. Hard (a) What conclusion follows? “If the creek floods, the lower field will be under water. If the lower field is under water, we cannot plant corn there this week.” (b) Add the premise “We can plant corn in the lower field this week.” What follows now, and by what forms?

▼ Show answer

(a) “If the creek floods, we cannot plant corn in the lower field this week” (pure hypothetical syllogism). (b) By modus tollens on the second premise, the lower field will not be under water; then by modus tollens on the first premise, the creek will not flood. (Or, by modus tollens on the conclusion of (a): the creek will not flood.) Accept either route.

11. Hard Name the form and say whether it is valid: “The dough rises only if the yeast is alive. This dough rose. So the yeast in it was alive.”

▼ Show answer

Valid: modus ponens. “p only if q” means “If p, then q,” so the first premise is “If the dough rises, the yeast is alive.” The second premise affirms the antecedent (the dough rose), and the conclusion affirms the consequent. (A student who calls this affirming the consequent has read “only if” as “if.”)

12. **Challenge** Either the fox or the raccoon got into the henhouse (perhaps both). If the fox got in, there are tracks in the snow by the gate. There are no tracks in the snow by the gate. (a) What follows? Name each step. (b) Suppose instead there had been tracks by the gate. What could you conclude about the fox and the raccoon?

    ▼ **Show answer**

    (a) By modus tollens: there are no tracks, so the fox did not get in. Then by disjunctive syllogism: the fox did not get in, so the raccoon did. (b) Nothing certain. Concluding that the fox got in would be affirming the consequent (the tracks could have been made another way), and even if the fox did get in, concluding that the raccoon did not would be affirming a disjunct, since the premise allows both. The raccoon might or might not have got in.

COMMON MISTAKES

- Affirming the consequent: treating "If p, then q" as if it also meant "If q, then p."
- Denying the antecedent: thinking that if p is false, q must be false too.
- Naming the form by the word "not." Ask what the second premise does to the antecedent or the consequent: affirms or denies it.
- Reading "p only if q" as "If q, then p." The words after "only if" are the consequent.
- Affirming a disjunct when both parts could be true.

> **Scripture.** "And if Christ be not risen, then is our preaching vain, and your faith is also vain." (1 Corinthians 15:14, KJV)
>
> Paul reasons with a conditional, tracing honestly what would follow if Christ were not risen. A few verses later, in 1 Corinthians 15:20, he declares, "But now is Christ risen from the dead." The Christian faith does not hide from hard "if" questions; it rests on what is true.

UNIT 6

Propositional (Symbolic) Logic

In Units 4 and 5 you looked *inside* statements, at their subject and predicate terms, and you tested syllogisms by how those terms are linked. This unit takes a different

view. **Propositional logic** treats each simple statement as a single unit and studies how statements are joined into larger ones by five connectives: *not*, *and*, *or*, *if…then* and *if and only if*. Because it writes these connectives as symbols (~, •, ∨, ⊃ and ≡), it is also called **symbolic logic**. The symbols are not a code for its own sake. They strip away everything but the form of an argument, so that you can see at a glance whether the conclusion follows.

Students learn to translate English into symbols, including tricky phrases such as "only if," "unless" and "neither…nor." They learn the **truth table** of each connective and use truth tables to classify compound statements as tautologies, contradictions or contingencies, to test two statements for equivalence (including **De Morgan's laws**), and to test whole arguments for validity, by the full method and by the shortcut method. They learn the valid forms (modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism and constructive dilemma), the two formal fallacies that imitate them (affirming the consequent and denying the antecedent), and how to write short step-by-step **proofs.** The unit ends by showing how the same tables, through Leibniz, Boole and Shannon, became the switching circuits and logic gates inside every computer.

This unit builds on Lesson 5.8, where you met modus ponens, modus tollens and disjunctive syllogism in words; now you will see why they work and how to prove it. The idea of a counterexample, a case where the premises are true and the conclusion false, becomes exact here: it is a single row of a truth table. Unit 7 will return to arguments that are strong or weak rather than valid, and Unit 9 will use the valid forms and the formal fallacies in mapping and answering real arguments. Careful, patient work pays off in this unit: every table and every proof can be checked line by line.

Review day and test. On the review day, use the mastery checklist and the review sheet (with its answer key). The unit test is below; a printable copy is ready for test day.

PREREQUISITES

- Tell a statement from a question or a command, and find the premises and conclusion of an argument (Unit 1).
- Use the words valid, invalid and sound correctly, and explain what a counterexample shows (Unit 1).
- Recognize modus ponens, modus tollens and disjunctive syllogism stated in words (Lesson 5.8).
- Use parentheses to group operations, as in arithmetic: 2 × (3 + 4) is not (2 × 3) + 4 (Pre-Algebra).

MASTERY CHECKLIST

- ☐ I can tell simple from compound statements and symbolize English sentences with ~, •, ∨, ⊃ and ≡, using parentheses correctly.
- ☐ I can write from memory the truth table of each connective, with the rows in the standard order.
- ☐ I can tell necessary from sufficient conditions and translate "only if," "if" and "unless."
- ☐ I can write the converse, inverse and contrapositive of a conditional and say which one is equivalent to it.
- ☐ I can build a truth table for a compound statement and classify it as a tautology, a contradiction or a contingency.
- ☐ I can test two statements for equivalence and state De Morgan's laws.
- ☐ I can test an argument for validity with a full truth table and with the shortcut method.
- ☐ I can name modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism and constructive dilemma, and spot affirming the consequent and denying the antecedent in everyday speech.
- ☐ I can write a short numbered proof, justifying each line with a rule of inference and line numbers.

- [ ] I can explain how Leibniz, Boole and Shannon connected logic to binary numbers, switching circuits and logic gates, and work out the output of a simple gate circuit.

LESSONS IN THIS UNIT

6.1	**Statements and Connectives**	40 minutes
6.2	**Truth Tables for the Connectives**	40 minutes
6.3	**Conditionals**	40 minutes
6.4	**Compound Statements and Equivalence**	45 minutes
6.5	**Testing Validity with Truth Tables**	45 minutes
6.6	**The Valid Forms**	40 minutes
6.7	**The Formal Fallacies**	40 minutes
6.8	**Short Proofs**	45 minutes
6.9	**Logic and Machines**	40 minutes

Unit 6 test

You have 50 minutes. No notes or books. Use the symbols ~, •, ∨, ⊃ and ≡, and say what each capital letter stands for whenever you choose your own letters. Write truth tables with T and F and with the rows in the standard order. In proofs, number every line and justify it with a rule and line numbers. For each classification question, give the name and a one-sentence reason. The test is worth 100 points.

1. **6 pts**

   Let S = “The soup is hot,” B = “The bread is fresh” and C = “Caleb sets the table.” Symbolize: (a) The soup is hot, but the bread is not fresh. (b) Neither is the soup hot nor is the bread fresh. (c) If Caleb sets the table, then the soup is hot and the bread is fresh. (d) Caleb sets the table if and only if the soup is hot.

2. **6 pts**

   Using the letters of question 1, name the main connective of each statement and translate it into English: (a) ~(S • B) (b) ~S ∨ B (c) (C ∨ S) ⊃ B

3. **6 pts**

   Suppose A is true, B is false and C is true. Find the truth value of each: (a) A ⊃ B (b) ~A ∨ C (c) (A • B) ≡ ~C (d) ~(B ∨ ~C) ⊃ B

4. **6 pts**

   (a) Write the truth table for p ⊃ q in the standard order. (b) Mr. Ortiz says, “If the hay is dry by Friday, we will bale it on Saturday.” Which of these situations would show his statement false: the hay is dry and they bale it; the hay is dry and they do not bale it; the hay is wet and they bale it anyway; the hay is wet and they do not bale it? Explain.

5. **8 pts**

   Let F = “The seeds sprout,” W = “The seeds are watered,” S = “The soil is warm” and G = “Hannah forgets the garden.” Symbolize: (a) The seeds sprout only if they are watered. (b) Warm soil is a sufficient condition for the seeds to sprout. (c) Watering is a necessary condition for the seeds to sprout. (d) The seeds will sprout unless Hannah forgets the garden. Then answer: (e) According to (c), is watering enough to make the seeds sprout?

6. **6 pts**

   Write the converse, the inverse and the contrapositive of “If the animal is a calf, then it is a mammal.” Say which are true, and which one is equivalent to the original.

7. **8 pts**

   Build truth tables and classify each statement as a tautology, a contradiction or a contingency: (a) (p ⊃ q) • (p • ~q) (b) (p • q) ⊃ (p ∨ q) (c) q ⊃ (p • q)

8. **8 pts**

(a) Use a truth table to decide whether ~(p • q) is equivalent to ~p • ~q. (b) Use De Morgan's laws to restate in English: "It is not true that both the barn door and the gate were left open."

9. **8 pts**

Let C = "The hens are in the coop," O = "The hens are in the orchard" and E = "The hens are eating the apples." Test this argument with a full truth table: "Either the hens are in the coop or they are in the orchard. If they are in the orchard, they are eating the apples. They are not eating the apples. So they are in the coop."

10. **8 pts**

Use the shortcut method to test each argument. If it is invalid, give the counterexample row. (a) A ⊃ (B ∨ C), ~B, A / ∴ C (b) A ∨ B, B ⊃ C / ∴ A ⊃ C

11. **8 pts**

Name the form of each argument. (a) If the ice is thin, skating is unsafe. The ice is thin. So skating is unsafe. (b) Either the cake is done or the oven is too cool. The cake is not done. So the oven is too cool. (c) If the dog is hungry, he whines. He is not whining. So he is not hungry. (d) If we sell the calves, we can buy a tractor; and if we sell the hay, we can fix the barn. We will sell the calves or the hay. So we can buy a tractor or fix the barn.

12. **8 pts**

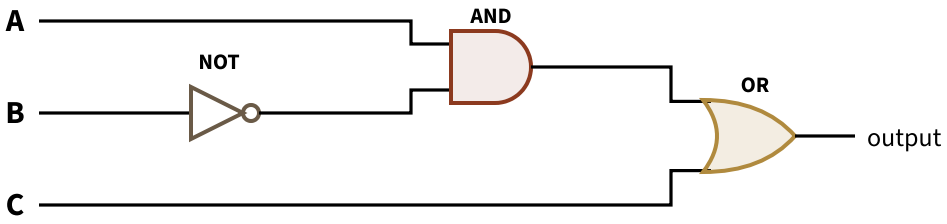
Valid or invalid? Name the form or the fallacy. (a) If Hannah practiced, she played well. She played well. So she practiced. (b) A ladder is safe to climb only if its feet are level. This ladder's feet are level. So it is safe to climb. (c) If the furnace is out, the house is cold. The furnace is not out. So the house is not cold. (d) For (c), describe a situation in which both premises are true and the conclusion false.

13. **8 pts**

Write proofs. Number each line and justify it. (a) 1. A ⊃ B 2. B ⊃ C 3. A • D / ∴ C • D (b) 1. ~E ∨ F 2. E / ∴ F ∨ G

14. **6 pts**

(a) Find the output of this circuit for A = 1, B = 0, C = 0, and for A = 1, B = 1, C = 0.



(b) In one sentence each, say what Boole contributed and what Shannon showed in 1937.

ANSWER KEY

1. (a) S • ~B. (b) ~S • ~B (also correct: ~(S ∨ B)). (c) C ⊃ (S • B). (d) C ≡ S (or S ≡ C). Accept the parts of a conjunction in either order, such as ~B • S.
2. (a) Negation: It is not the case that both the soup is hot and the bread is fresh. (b) Disjunction: Either the soup is not hot or the bread is fresh. (c) Conditional: If Caleb sets the table or the soup is hot, then the bread is fresh. Accept any wording with the same meaning.
3. (a) T ⊃ F = F. (b) ~A is F, so F ∨ T = T. (c) A • B is F and ~C is F, so F ≡ F = T. (d) ~C is F, so B ∨ ~C is F ∨ F = F; its negation is T; T ⊃ B is T ⊃ F = F.
4. (a)

p	q	p ⊃ q
T	T	T
T	F	F
F	T	T
F	F	T

   (b) Only "the hay is dry and they do not bale it." A conditional is false only when the antecedent is true and the consequent false; when the hay is wet, he has said nothing about what will happen, so his statement is not broken.
5. (a) F ⊃ W. (b) S ⊃ F. (c) F ⊃ W. (d) ~G ⊃ F (also correct: F ∨ G, G ∨ F, or ~F ⊃ G). (e) No. A necessary condition is required, not enough; (c) says the seeds cannot sprout without water, not that water guarantees sprouting.
6. Converse: If the animal is a mammal, then it is a calf (false: a horse is a mammal). Inverse: If the animal is not a calf, then it is not a mammal (false: a horse again). Contrapositive: If the animal is not a mammal, then it is not a calf (true). The contrapositive is equivalent to the original.
7.

p	q	p ⊃ q	p • ~q	(a)	p • q	p ∨ q	(b)	(c)	q ⊃ (p • q)
T	T	T	F		F	T	T	T	T
T	F	F	T		F	F	T	T	T
F	T	T	F		F	F	T	T	F
F	F	T	F		F	F	F	T	T

   (a) Contradiction: all F (p • ~q is the contradictory of p ⊃ q, so they can never both be true). (b) Tautology: all T. (c) Contingency: T, T, F, T.
8. (a) Not equivalent.

p	q	~(p • q)	~p • ~q
T	T	F	F
T	F	T	F
F	T	T	F
F	F	T	T

The columns differ in rows 2 and 3. (The correct equivalent of ~(p • q) is ~p ∨ ~q.) (b) Either the barn door was not left open or the gate was not left open (at least one of them was closed).

9. C ∨ O, O ⊃ E, ~E / ∴ C.

C	O	E	C ∨ O	O ⊃ E	~E	∴ C
T	T	T	T	T	F	T
T	T	F	T	F	T	T
T	F	T	T	T	F	T
T	F	F	T	T	T	T
F	T	T	T	T	F	F
F	T	F	T	F	T	F
F	F	T	F	T	F	F
F	F	F	F	T	T	F

Only row 4 makes all three premises true, and its conclusion is true. No counterexample: the argument is valid.

10. (a) Valid. Assume C is false. ~B is true, so B is F; A is T. Then B ∨ C is F, so A ⊃ (B ∨ C) is T ⊃ F, false, though it was assumed true. Contradiction. (b) Invalid. Assume A ⊃ C false: A is T and C is F. B ⊃ C true with C false: B is F. A ∨ B is T ∨ F, true. No contradiction. Counterexample: A true, B false, C false (premises T, T; conclusion F).

11. (a) Modus ponens: the antecedent is affirmed. (b) Disjunctive syllogism: one disjunct is denied, so the other follows. (c) Modus tollens: the consequent is denied, so the antecedent is denied. (d) Constructive dilemma: two conditionals and a choice between their antecedents give a choice between their consequents.

12. (a) Invalid: affirming the consequent. (b) Invalid: affirming the consequent. “Only if” makes the rule S ⊃ L (safe only if level), and the argument affirms L to conclude S; level feet are necessary but not sufficient (a rung could be broken). (c) Invalid: denying the antecedent. (d) The furnace is running, but a door has

been left open on a winter day, so the house is cold. Accept any other cause of a cold house with the furnace working.

13. (a) 4. A (3, Simp)
    5. A ⊃ C (1, 2, HS)
    6. C (5, 4, MP)
    7. D (3, Simp)
    8. C • D (6, 7, Conj)
    (Also correct: 4. A (3, Simp); 5. B (1, 4, MP); 6. C (2, 5, MP); 7. D (3, Simp); 8. C • D (6, 7, Conj).)
    (b) 3. ~~E (2, DN)
    4. F (1, 3, DS)
    5. F ∨ G (4, Add)
    (Accept DS directly from lines 1 and 2 if the student notes that E denies ~E.)
14. (a) A = 1, B = 0, C = 0: NOT B = 1; 1 AND 1 = 1; 1 OR 0 = 1. Output 1. A = 1, B = 1, C = 0: NOT B = 0; 1 AND 0 = 0; 0 OR 0 = 0. Output 0. (b) Boole (1847, 1854) turned logic into an algebra in which statements take the values 1 and 0 and AND works like multiplication. Shannon showed in his 1937 master's thesis that Boole's algebra describes circuits of switches and relays (series as AND, parallel as OR), so that circuits can be designed and simplified by logic.

Printable answer key (PDF, for the parent)

UNIT 6 · PROPOSITIONAL (SYMBOLIC) LOGIC · LESSON 6.1 OF 69 · 40 MINUTES

6.1 Statements and Connectives

OBJECTIVES

- Tell simple statements from compound statements.
- Name the five connectives and write each with its symbol: ~, •, ∨, ⊃ and ≡.
- Symbolize English sentences, including "but," "neither...nor" and "not both," using capital letters and parentheses.
- Find the main connective of a compound statement.

KEY TERMS

Simple statement

A statement that makes one claim and contains no other statement as a part, such as “The barn is red.”

Compound statement

A statement built from one or more other statements by a connective, such as “The barn is red and the silo is white” or “The barn is not red.”

Connective

A word or symbol that builds a compound statement from simpler ones: not (~), and (•), or (∨), if...then (⊃), if and only if (≡).

Negation

A statement that denies another, written ~p and read “not p” or “it is not the case that p.”

Conjunction

An “and” statement, written p • q; its two parts are called conjuncts.

Disjunction

An “or” statement, written p ∨ q; its parts are called disjuncts. In logic “or” is inclusive: “p or q, or both.”

Conditional

An “if...then” statement, written p ⊃ q. The “if” part (p) is the antecedent; the “then” part (q) is the consequent.

Biconditional

An “if and only if” statement, written p ≡ q, claiming that p and q go together: each holds if the other does.

CORE IDEAS

Whole statements as building blocks. In Units 4 and 5 you looked inside statements. “All beagles are dogs” has a subject term and a predicate term, and a syllogism works by linking terms through a middle term. Propositional logic takes a different view. It treats each simple statement as a single unbroken unit, and it studies how statements are joined into larger ones by words such as *not*, *and*, *or*, *if...then* and *if and only if*. These joining words are called **connectives**. Many everyday arguments depend on them and not on terms at all: “If the creek rises, the bridge will close. The creek is rising. So the bridge will close.” That argument is about whole statements, not about classes of things.

Simple and compound statements. A **simple statement** makes one claim and contains no other statement as a part: “The barn is red.” A **compound statement**

contains at least one other statement joined by a connective: “The barn is red and the silo is white” contains two. Even “The barn is not red” counts as compound, because it is built from the simple statement “The barn is red” by denying it. Be careful with a sentence like “Hannah and Caleb are cousins.” It has the word *and*, but it makes one claim about a relation between two people. It cannot be split into “Hannah is a cousin and Caleb is a cousin” without changing its meaning, so it is a simple statement. Contrast “Hannah and Caleb are tall,” which does split: “Hannah is tall and Caleb is tall.”

Five connectives, five symbols. This course uses the symbols below. Learn them now; every later lesson uses them.

Connective	Symbol	Example	Read as
Negation	~	~R	not R; it is not the case that R
Conjunction	•	R • S	R and S
Disjunction	∨	R ∨ S	R or S (or both)
Conditional	⊃	R ⊃ S	if R, then S
Biconditional	≡	R ≡ S	R if and only if S

The parts of a conjunction are its **conjuncts**, and the parts of a disjunction are its **disjuncts**. In a conditional, the “if” part is the **antecedent** and the “then” part is the **consequent**.

Capital letters for statements, small letters for patterns. To symbolize a particular statement, choose a capital letter that reminds you of it, and write down what each letter means: R = “It rains,” G = “The game is canceled.” Then “If it rains, the game is canceled” becomes R ⊃ G. A letter always stands for a whole statement, never for a single word, and it should stand for the positive statement; a denial is shown with ~. So if R means “It rains,” then “It does not rain” is ~R, not a new letter. When we want to describe a pattern that fits any statements at all, we use small letters p, q and r. The form p ⊃ q fits R ⊃ G, and it also fits (A • B) ⊃ C, with p standing for the whole compound A • B.

Different words, same connective. English has many ways to say *and*: “but,” “yet,” “although,” “however,” “while,” “also” and “moreover.” “The pie is warm, but the crust is soggy” is W • S. The word *but* adds a note of contrast, and that note is real in English; yet the claim is true exactly when both parts are true, so logic writes it with

•. Negation, too, comes in many forms: “not,” “isn’t,” “never,” “it is false that” and “it is not the case that.” Watch for words that hide a negation inside them, such as “unlocked” or “impossible.” If a passage speaks of the gate being locked in one place and unlocked in another, let L = “The gate is locked” and write “unlocked” as ~L, so that the two claims stay connected.

The inclusive “or.” In logic, R ∨ S means “R or S, or both.” This is the *inclusive* sense of “or,” and it is the only sense the symbol ∨ ever has. When a pool sign says “Members or their guests may swim,” it does not mean that a member who brings a guest is turned away. Sometimes we do mean “one or the other but not both,” as when a menu says “Soup or salad comes with your meal.” That is the *exclusive* sense, and we write it out in full: (S ∨ L) • ~(S • L), “soup or salad, and not both.” Two related phrases need care. “Neither R nor S” means both are false: ~R • ~S. “Not both R and S” means at least one is false: ~(R • S).

If, then, and if and only if. A conditional claims a link: if the antecedent holds, the consequent holds too. In English the “if” clause can come first or last. “If the creek rises, the bridge closes” and “The bridge closes if the creek rises” say the same thing, C ⊃ B. The rule of thumb is that whatever follows the word *if* is the antecedent, wherever it sits in the sentence. (Lesson 6.3 adds phrases such as “only if” and “unless,” which need special care.) A biconditional claims a link in both directions. “The bell rings if and only if the gate opens” means that the bell rings if the gate opens, and also that the bell rings only when the gate opens. We write it B ≡ G, or G ≡ B; for ≡, the order does not matter.

Parentheses and the main connective. In arithmetic, 2 × (3 + 4) is not the same as (2 × 3) + 4. Logic uses parentheses the same way. Let H = “Hannah bakes” and C = “Caleb cooks.” Then ~(H • C) says “Not both: Hannah bakes and Caleb cooks,” while ~H • C says “Hannah does not bake, and Caleb cooks.” A negation sign applies only to the letter or the parenthesis right after it. In every compound statement, the **main connective** is the one that governs the whole statement, the one you would apply last if you built the statement up from its parts. In ~(H • C) the main connective is the ~; in ~H • C it is the •. In (R • F) ⊃ G, “If it rains and the field floods, the game is canceled,” it is the ⊃. The main connective tells you what kind of statement you have: a negation, a conjunction, a disjunction, a conditional or a biconditional.

From history. The logic of whole statements is old. The Stoic philosophers of ancient Greece, above all Chrysippus (about 279–206 B.C.), studied arguments built from "if," "and," "or" and "not," and they listed basic valid patterns that you will meet in Lesson 6.6. Almost all of Chrysippus's many books are lost; we know his teaching mainly through later writers such as Sextus Empiricus and Diogenes Laertius. For centuries Aristotle's logic of terms overshadowed the Stoic logic of statements. The symbols came much later. The horseshoe ⊃ goes back to the Italian mathematician Giuseppe Peano, and ⊃, ~ and ∨ were made standard by Whitehead and Russell's *Principia Mathematica* (1910–1913). The wedge ∨ is often said to recall the Latin word *vel*, an "or" that allows both.

Practice habit. When you read a sentence with "and," "or," "if" or "not," find its main connective before you decide what it claims.

WORKED EXAMPLES

Example 1. Let B = "The bus is late," W = "Caleb walks to school" and R = "It is raining." Symbolize: (a) The bus is late, but Caleb walks to school anyway. (b) Caleb walks to school if the bus is late. (c) Caleb walks to school if and only if the bus is late and it is not raining. (d) Neither is the bus late nor is it raining.

▼ **Show solution**

1. (a) "But" is a conjunction in logic; "anyway" adds only tone. Both parts are claimed: B • W.
2. (b) Whatever follows "if" is the antecedent, even when it comes last. The antecedent is "the bus is late": B ⊃ W.
3. (c) The main connective is "if and only if." One side is W; the other side is the conjunction "the bus is late and it is not raining," B • ~R, which needs parentheses: W ≡ (B • ~R).
4. (d) "Neither…nor" denies both parts: the bus is not late, and it is not raining: ~B • ~R.

Answer: (a) B • W. (b) B ⊃ W. (c) W ≡ (B • ~R). (d) ~B • ~R (also correct: ~(B ∨ R)).

Example 2. Let F = "The field is dry" and P = "The match is played." Name the main connective of each statement and translate it into English: (a) ~(F ⊃ P) (b) ~F ⊃ ~P (c) F ∨ ~P

▼ **Show solution**

1. (a) The ~ stands outside the parentheses, so it governs the whole statement. The main connective is ~ (a negation): "It is not the case that if the field is dry, the match is played."
2. (b) Each ~ applies only to the letter after it. The connective that joins the two parts is ⊃ (a conditional): "If the field is not dry, the match is not played."
3. (c) The ~ applies only to P. The main connective is ∨ (a disjunction): "Either the field is dry or the match is not played (or both)."

Answer: (a) Negation: "It is not the case that if the field is dry, the match is played." (b) Conditional: "If the field is not dry, the match is not played." (c) Disjunction: "Either the field is dry or the match is not played."

PRACTICE

1. Easy Name the five connectives, give the symbol for each, and give one English word or phrase that each one translates.

   ▼ **Show answer**

   Negation, ~ ("not"); conjunction, • ("and"); disjunction, ∨ ("or"); conditional, ⊃ ("if...then"); biconditional, ≡ ("if and only if"). Accept any correct English equivalent, such as "but" for • or "it is not the case that" for ~.

2. Easy Is each statement simple or compound? If compound, name its main connective. (a) The barn is red. (b) The barn is red and the silo is white. (c) It is not the case that the gate is locked. (d) Hannah and Caleb are cousins.

   ▼ **Show answer**

   (a) Simple: one claim with no statement inside it. (b) Compound, a conjunction (•). (c) Compound, a negation (~). (d) Simple: it states one relation between two people and cannot be split into "Hannah is a cousin and Caleb is a cousin" without changing its meaning.

3. **Easy** Let H = “The hens are laying” and E = “Eggs are cheap.” Symbolize: (a) The hens are not laying. (b) The hens are laying and eggs are cheap. (c) Either the hens are laying or eggs are not cheap. (d) If the hens are laying, then eggs are cheap.

   ▼ **Show answer**

   (a) ~H. (b) H • E (or E • H). (c) H ∨ ~E (or ~E ∨ H). (d) H ⊃ E.

4. **Easy** Name the antecedent and the consequent of each: (a) If the creek rises, the bridge will close. (b) The bridge will close if the creek rises.

   ▼ **Show answer**

   In both, the antecedent is “the creek rises” and the consequent is “the bridge will close.” Whatever follows “if” is the antecedent, wherever it sits in the sentence. Both are C ⊃ B.

5. **Medium** Symbolize, saying what each letter stands for: (a) The pie is warm, but the crust is soggy. (b) Although Caleb practiced every day, he missed the free throw.

   ▼ **Show answer**

   (a) Let W = “The pie is warm,” S = “The crust is soggy”: W • S. (b) Let P = “Caleb practiced every day,” M = “Caleb missed the free throw”: P • M. “But” and “although” both claim that each part is true, so both are conjunctions. Accept any letters, clearly defined.

6. **Medium** Let C = “The cow is in the barn” and K = “The calf is in the barn.” Symbolize: (a) Neither the cow nor the calf is in the barn. (b) The cow and the calf are not both in the barn.

   ▼ **Show answer**

   (a) ~C • ~K (also correct: ~(C ∨ K)). Both are out of the barn. (b) ~(C • K) (also correct: ~C ∨ ~K). At least one is out of the barn, perhaps both.

7. **Medium** Let S = “The sky is clear” and T = “We see the stars.” Translate into English: (a) ~(S • T) (b) ~S • T (c) S ⊃ ~T (d) S ≡ T

   ▼ **Show answer**

   (a) It is not the case that both the sky is clear and we see the stars (not both). (b) The sky is not clear, and we see the stars. (c) If the sky is clear, then we do not see the stars. (d) The sky is clear if and only if we see the stars. Accept any wording with the same meaning.

8. **Medium** Name the main connective of each: (a) ~A ∨ B (b) ~(A ∨ B) (c) (A • B) ⊃ C (d) A • (B ⊃ C)

   ▼ **Show answer**

   (a) ∨ (a disjunction; the ~ applies only to A). (b) ~ (a negation of the whole disjunction). (c) ⊃ (a conditional whose antecedent is A • B). (d) • (a conjunction whose second conjunct is B ⊃ C).

9. **Hard** Let R = “It rains,” F = “The field floods” and G = “The game is canceled.” Symbolize, using parentheses: (a) If it rains and the field floods, the game is canceled. (b) It rains, and if the field floods, the game is canceled. Then explain in one sentence how the two statements differ.

   ▼ **Show answer**

   (a) (R • F) ⊃ G. (b) R • (F ⊃ G). Statement (a) is a conditional that claims nothing about whether it rains, only what happens if both conditions hold; statement (b) is a conjunction that asserts that it rains, and adds that a flood would cancel the game.

10. **Hard** Let H = “Hannah bakes bread” and C = “Caleb makes soup.” Symbolize: “Either Hannah bakes bread or Caleb makes soup, but not both.”

    ▼ **Show answer**

    (H ∨ C) • ~(H • C). The “but not both” makes the “or” exclusive, so it must be written out; H ∨ C alone would allow both. Accept any equivalent, such as (H • ~C) ∨ (~H • C).

11. Hard Let P = “The match is played,” D = “The field is dry” and R = “It is raining.” Symbolize: “The match is played if and only if the field is dry and it is not raining.”

▼ **Show answer**

P ≡ (D • ~R). The main connective is ≡; the parentheses show that the whole conjunction “the field is dry and it is not raining” is the other side. (Accept (D • ~R) ≡ P.)

12. Challenge Let A = “Hannah went to the fair” and B = “Caleb went to the fair.” Explain why ~(A • B) and ~A • ~B do not say the same thing. Describe a situation in which one is true and the other false.

▼ **Show answer**

~(A • B) says only that they did not both go (at least one stayed home), while ~A • ~B says that neither went. Situation: Hannah went and Caleb stayed home. Then ~(A • B) is true (they did not both go), but ~A • ~B is false (Hannah did go). Accept any situation in which exactly one of them went.

COMMON MISTAKES

- Giving a negative statement its own letter. If L = “The gate is locked,” write “The gate is not locked” as ~L, so the two claims stay connected.
- Splitting a relation into two statements. “Hannah and Caleb are cousins” is one simple statement, not a conjunction.
- Confusing “neither…nor” (~A • ~B, both false) with “not both” (~(A • B), at least one false).
- Taking whatever comes first in the sentence as the antecedent. The antecedent is whatever follows “if,” wherever it appears.
- Leaving out parentheses, so that ~A • B is written when ~(A • B) is meant.

> **Scripture.** “A word fitly spoken is like apples of gold in pictures of silver.” (Proverbs 25:11, KJV)
>
> Symbolizing a sentence is the art of the fitting word: write exactly what the sentence says, no more and no less.

6.2 Truth Tables for the Connectives

OBJECTIVES

- Explain what a truth value is and what it means for a connective to be truth-functional.
- Write from memory the truth tables for ~, •, ∨, ⊃ and ≡, with the rows in the standard order.
- Explain, using a promise, why a conditional is false only when its antecedent is true and its consequent false.
- Find the truth value of a compound statement when the truth values of its letters are given.

KEY TERMS

Truth value

The truth or falsity of a statement, written T (true) or F (false).

Truth-functional

Said of a connective when the truth value of the compound depends only on the truth values of its parts. All five connectives in this course are truth-functional.

Truth table

A chart that lists every possible combination of truth values for the letters in a statement and shows the statement's truth value in each one.

Row

One line of a truth table: one possible situation, an assignment of T or F to each letter.

Standard order

The fixed order for listing rows. With two letters: TT, TF, FT, FF. With three, the first column is four Ts then four Fs, the second alternates in pairs, and the third alternates T, F.

Material conditional

The conditional as defined by its truth table: p ⊃ q is false only when p is true and q is false, so it claims no more than "not both p and not q."

CORE IDEAS

Every statement is true or false. Propositional logic begins from the fact that every statement has a **truth value**: it is either true (T) or false (F). Logic does not ask

whether “The barn is red” is true; that is a job for your eyes. It asks a different question: *if* we know the truth values of the simple statements, what are the truth values of the compound statements built from them? For the five connectives in this course, the answer is always fixed. The truth value of a compound depends only on the truth values of its parts. Connectives that work this way are called **truth-functional**, and the chart that shows how each one works is its **truth table**.

The master table. Here are all five connectives in one table. The first two columns list every possible combination of truth values for p and q; each row is one possible situation.

p	q	~p	p • q	p ∨ q	p ⊃ q	p ≡ q
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Memorize the table by its pattern, not row by row. Negation flips the value. A conjunction is true only when both parts are true. A disjunction is false only when both parts are false. A conditional is false only when the antecedent is true and the consequent false. A biconditional is true when the two sides match and false when they differ.

Negation and conjunction. Negation is the simplest connective. If “The milk is fresh” is true, “The milk is not fresh” is false, and the other way around. A conjunction claims both of its parts, so it is true only when both are true. If Mrs. Ortiz says, “I bought flour and I bought sugar,” and she bought only flour, she has said something false, even though half of it was true. One false conjunct makes the whole conjunction false.

Disjunction. A disjunction claims that at least one of its parts is true. “The key is in the drawer or in my coat pocket” is true if the key is in the drawer, true if it is in the pocket, and false only if it is in neither place. Because ∨ is inclusive, the first row, where both parts are true, gets T. If you need the exclusive sense, “one but not both,” write it out as (p ∨ q) • ~(p • q), as you learned in Lesson 6.1.

The conditional: think of a promise. The conditional’s table puzzles most students at first, so think of a promise. Caleb’s father says, “If you finish your chores, I will take

you fishing." There are four possible situations. Caleb finishes and they go fishing: the promise is kept (T). Caleb finishes and they do not go: the promise is broken (F). Caleb does not finish, and his father takes him fishing anyway, or does not: in neither case has the father broken his word, because he promised nothing about what would happen if the chores were left undone. A promise that has not been broken counts as kept, so both of the last two rows get T. A conditional p ⊃ q is false in exactly one case: p true and q false.

What p ⊃ q claims, and what it does not. The table shows that p ⊃ q claims exactly this much: it is not the case that p is true and q is false. Logicians call this the **material conditional**. It says nothing about *why* p and q are linked, and in ordinary speech we usually mean more than that: a cause, a rule or a promise. But the "more" always includes this minimum, and the minimum is all that is needed to test the valid forms of later lessons. A conditional whose antecedent is false is sometimes called "true by default." That sounds odd, but it matches the promise: no one can accuse the father of breaking his word on a day when the chores were not done.

The biconditional. p ≡ q claims that p and q have the same truth value: both true or both false. "The porch light is on if and only if the switch is up" is true when the light is on and the switch is up, true when the light is off and the switch is down, and false in the two mixed cases. That is why ≡ is read "if and only if": it is a conditional running both ways, (p ⊃ q) • (q ⊃ p). Lesson 6.4 will show with a table that these two have the same truth value in every row.

How many rows, and in what order? Each simple statement can be T or F, so one letter needs 2 rows, two letters need 4, three need 8 and four need 16: the number of rows doubles with each new letter. Always list the rows in the **standard order**, so that tables can be compared and checked. With two letters, the first column is T, T, F, F and the second is T, F, T, F. With three letters (p, q, r), the first column is four Ts and then four Fs; the second is two Ts, two Fs, two Ts, two Fs; the third alternates T, F all the way down. Every possible combination then appears exactly once.

Working out a compound, inside out. To find the truth value of a longer statement when the values of its letters are given, work from the inside out, as in arithmetic. Suppose A is true, B is false and C is true, and you want (A ∨ B) ⊃ ~C. First the parentheses: A ∨ B is T ∨ F, which is T. Next the negation: ~C is ~T, which is F. Last

the main connective: T ⊃ F is F. So the whole statement is false. Writing the value under each connective as you go keeps the work neat and lets you check it.

> **From history.** The rule behind the conditional's table is ancient. The Greek logician Philo the Dialectician (around 300 B.C.) taught that a conditional is false only when it begins with a truth and ends with a falsehood, exactly the rule of this lesson. His teacher Diodorus Cronus disagreed and demanded more, and the debate became so famous that the poet Callimachus joked that even the crows on the rooftops were cawing about which conditionals are true. Truth tables as a working method came much later. The American logician Charles Sanders Peirce worked out formulas case by case, true and false, in the 1880s, and in 1921 Emil Post and Ludwig Wittgenstein each published tables in nearly the form used today.

> **Practice habit.** When someone says "if...then," ask what situation would prove it false. There is only one: the "if" part true and the "then" part false.

WORKED EXAMPLES

Example 1. Suppose A is true, B is false and C is false. Find the truth value of each: (a) ~A ∨ C (b) A ⊃ (B ∨ C) (c) ~(A • B) ≡ C

▼ **Show solution**

1. (a) ~A is ~T, which is F. C is F. So the disjunction is F ∨ F, which is F (a disjunction is false only when both parts are false).
2. (b) Inside the parentheses first: B ∨ C is F ∨ F, which is F. Then A ⊃ F is T ⊃ F, which is F (true antecedent, false consequent).
3. (c) Inside first: A • B is T • F, which is F. The negation ~(A • B) is ~F, which is T. Then T ≡ C is T ≡ F; the sides differ, so it is F.

Answer: (a) F. (b) F. (c) F.

Example 2. Mrs. Ortiz tells the class on Monday afternoon, "If it snows tonight, school will start late tomorrow." Consider four possible Tuesday mornings: (1) it snowed and school started late; (2) it snowed and school started on time; (3) it did not snow and school started late anyway, because a pipe burst; (4) it did not snow and school started on time. On which mornings, if any, did Mrs. Ortiz speak falsely?

▼ **Show solution**

1. Let S = "It snows tonight" and L = "School starts late tomorrow." Her statement is $S \supset L$.
2. Morning (1) is the row S true, L true: $T \supset T$ is T. Her statement held.
3. Morning (2) is S true, L false: $T \supset F$ is F. This is the one case that breaks a conditional.
4. Mornings (3) and (4) have S false. She said nothing about what would happen without snow, so her statement was not broken: $F \supset T$ and $F \supset F$ are both T.

Answer: Only on morning (2), when it snowed and school started on time, was her statement false. On the other three mornings it was true.

PRACTICE

1. Easy Complete each sentence. (a) A conjunction is true only when … (b) A disjunction is false only when … (c) A conditional is false only when … (d) A biconditional is true when …

   ▼ **Show answer**

   (a) both conjuncts are true. (b) both disjuncts are false. (c) the antecedent is true and the consequent is false. (d) both sides have the same truth value (both true or both false).

2. **Easy** Write the full truth table for p ⊃ q, with the rows in the standard order.

▼ **Show answer**

p	q	p ⊃ q
T	T	T
T	F	F
F	T	T
F	F	T

3. **Easy** How many rows does a truth table need for a statement with (a) three different letters, (b) four, (c) five?

▼ **Show answer**

(a) 8. (b) 16. (c) 32. The number doubles with each new letter (2 to the power of the number of letters).

4. **Easy** Suppose A is true and B is false. Give the truth value of: (a) A • B (b) A ∨ B (c) A ⊃ B (d) B ⊃ A (e) A ≡ B

▼ **Show answer**

(a) F. (b) T. (c) F. (d) T (a false antecedent makes a conditional true). (e) F.

5. **Medium** Suppose A is true, B is false and C is true. Give the truth value of: (a) ~A ∨ B (b) (A • C) ⊃ B (c) ~(B ∨ C) (d) A ≡ ~B

▼ **Show answer**

(a) ~A is F and B is F, so F ∨ F = F. (b) A • C is T, so T ⊃ F = F. (c) B ∨ C is T, so its negation is F. (d) ~B is T, so T ≡ T = T.

6. **Medium** Hannah's aunt says, "If it is sunny on Saturday, we will go to the lake." Which of these situations would show that her statement was false? (a) Sunny, and they went. (b) Sunny, and they did not go. (c) Cloudy, and they went anyway. (d) Cloudy, and they did not go.

▼ **Show answer**

Only (b). A conditional is false only when the antecedent is true (it was sunny) and the consequent false (they did not go). In (a) it was kept, and in (c) and (d) she promised nothing about a cloudy day, so the statement is true.

7. **Medium** Caleb says, “Hannah will bring a pie or a cake to the picnic,” and Hannah brings both. Using the logician’s ∨, was Caleb’s statement true or false? How would you symbolize the statement if he had meant “one but not both”? (Let P = “Hannah brings a pie” and C = “Hannah brings a cake.”)

▼ **Show answer**

True: ∨ is inclusive, so P ∨ C is true when both disjuncts are true. The exclusive sense would be (P ∨ C) • ~(P • C). Accept any equivalent, such as (P • ~C) ∨ (~P • C).

8. **Medium** Write the eight rows of a truth table for the letters p, q and r in the standard order.

▼ **Show answer**

p	q	r
T	T	T
T	T	F
T	F	T
T	F	F
F	T	T
F	T	F
F	F	T
F	F	F

9. **Hard** Suppose C is false, but you do not know the truth value of X. For each statement, give its truth value, or say that it cannot be determined: (a) C ⊃ X (b) X • C (c) X ∨ C (d) C ≡ X

▼ **Show answer**

(a) T: a conditional with a false antecedent is true whatever X is. (b) F: a conjunction with a false conjunct is false. (c) Cannot be determined: it is true if X is true and false if X is false. (d) Cannot be determined: it is true if X is false and false if X is true.

10. Hard Suppose D is true, but you do not know the truth value of X. For each statement, give its truth value, or say that it cannot be determined: (a) X ∨ D (b) X ⊃ D (c) D ⊃ X (d) ~D • X

▼ **Show answer**

(a) T: one true disjunct is enough. (b) T: a conditional with a true consequent cannot have the one false row (T, F). (c) Cannot be determined: it has the same value as X. (d) F: ~D is false, so the conjunction is false.

11. Hard Suppose A and B are true, and C and D are false. Find the truth value of: (a) (A ⊃ C) ∨ (B ≡ D) (b) ~(A • ~D) ⊃ (C ∨ ~B)

▼ **Show answer**

(a) A ⊃ C is T ⊃ F = F; B ≡ D is T ≡ F = F; so F ∨ F = F. (b) ~D is T, so A • ~D is T and ~(A • ~D) is F. A conditional with a false antecedent is true, so the whole statement is T. (Check: C ∨ ~B is F ∨ F = F, and F ⊃ F = T.)

12. Challenge Suppose someone objects to the last two rows of the table for p ⊃ q (the rows where p is false) and wants them changed. Rows 1 and 2 must stay T and F. Work out the three other ways rows 3 and 4 could be filled in, and show that each one turns p ⊃ q into a statement we already have. Then explain, using a promise, why T, T is the right choice.

▼ **Show answer**

The column would be T, F, then (F, T), (T, F) or (F, F). T, F, F, T is the column for p ≡ q. T, F, T, F is just the column for q. T, F, F, F is the column for p • q. So any change would make “if p then q” mean “p if and only if q,” or simply “q,” or “p and q,” none of which is what “if” means. With a promise such as “If you finish your chores, I will take you fishing,” a day when the chores are not done cannot break the promise, whatever happens; an unbroken promise counts as true, so both rows get T.

COMMON MISTAKES

- Marking a conditional with a false antecedent as false or “neither.” If the antecedent is false, the conditional is true.
- Treating ∨ as exclusive and marking the first row (both true) F. The logician’s “or” is inclusive.

- Listing rows out of the standard order, or skipping a combination. Use the halving pattern: half Ts in the first column, then pairs, then alternating.
- Working out the main connective first. Work from the inside out: parentheses and negations of single letters first, the main connective last.

REFERENCE GUIDE

An eight-page, color-coded Logic guide: what logic is, terms and definitions, the A E I O propositions, the square of opposition and immediate inferences, syllogisms with the six rules and Venn tests, the named forms, informal fallacies of relevance, presumption and ambiguity, propositional logic with truth tables, conditionals, valid forms and short proofs, the classical tradition, dialectic, argument maps and practice. The booklet prints on 11×17 paper and folds to letter size. Prints in color and reads clearly in black and white.

> **Scripture.** “let your yea be yea; and your nay, nay” (James 5:12, KJV)
>
> Every statement in logic is either true or false. Christ’s people are to speak the same way: a plain yes or a plain no, with nothing hidden behind it.

UNIT 6 · PROPOSITIONAL (SYMBOLIC) LOGIC · LESSON 6.3 OF 69 · 40 MINUTES

6.3 Conditionals

OBJECTIVES

- Tell a necessary condition from a sufficient condition, and identify each in a conditional.
- Translate “only if,” “if,” “provided that,” “unless,” “is necessary for” and “is sufficient for” into ⊃ statements.
- Write the converse, inverse and contrapositive of a conditional.
- Show with a truth table that a conditional is equivalent to its contrapositive but not to its converse or inverse.

KEY TERMS

Sufficient condition

A condition that is enough to guarantee another. In $p \supset q$, p is sufficient for q.

Necessary condition

A condition that is required for another: without it the other cannot be true. In $p \supset q$, q is necessary for p.

Converse

The conditional formed by swapping antecedent and consequent: the converse of $p \supset q$ is $q \supset p$.

Inverse

The conditional formed by negating both parts in place: the inverse of $p \supset q$ is $\sim p \supset \sim q$.

Contrapositive

The conditional formed by swapping and negating both parts: the contrapositive of $p \supset q$ is $\sim q \supset \sim p$. It is equivalent to the original.

Only if

A phrase that introduces a necessary condition, the consequent: "p only if q" is $p \supset q$.

Unless

A connective meaning "if not": "p unless q" is $\sim q \supset p$, which is equivalent to $p \vee q$.

CORE IDEAS

Sufficient conditions: enough to guarantee. Every conditional names two kinds of condition. Take "If Biscuit is a beagle, then Biscuit is a dog." Being a beagle is *enough* to make Biscuit a dog; nothing more is needed. We say the antecedent is a **sufficient condition** for the consequent. In $p \supset q$, p is sufficient for q: whenever p is true, q is true as well. A sufficient condition need not be the only way to get the result. Being a collie is also enough to make Biscuit a dog.

Necessary conditions: required. Look at the same conditional from the other end. Biscuit cannot be a beagle without being a dog, so being a dog is *required* for being a beagle. We say the consequent is a **necessary condition** for the antecedent. In $p \supset q$, q is necessary for p: if q is false, p must be false too. A necessary condition need not be enough. Oxygen is necessary for a campfire to burn, but oxygen alone does not

light one; you also need fuel and a spark. Keep the pair straight with two plain words: sufficient means *enough*, necessary means *required*.

"Only if" introduces the consequent. The phrase "only if" trips up many careful readers. "You may enter the rare-book room only if you have a pass" names a necessary condition: no pass, no entry. So the pass is the consequent: E ⊃ P. It does *not* say that having a pass is enough; the library may also require clean hands and a librarian's permission. Compare "You may enter if you have a pass," which is P ⊃ E and says the pass is enough. One small word, *only*, reverses the direction of the arrow. The rule: "p if q" is q ⊃ p, but "p only if q" is p ⊃ q.

A table of translations. Many English phrases come down to ⊃. These introduce the antecedent (the sufficient condition): "if," "provided that," "given that," "in case," "whenever" and "is sufficient for." These introduce the consequent (the necessary condition): "then," "only if," "is necessary for," "is required for." So "p is sufficient for q" is p ⊃ q, and "p is necessary for q" is q ⊃ p. "Whenever the rooster crows, the dog barks" is C ⊃ B. "Fresh yeast is necessary for the bread to rise" is R ⊃ Y: if the bread rises, the yeast was fresh. When in doubt, ask which part guarantees the other, and make that part the antecedent.

"Unless" means "if not." "Caleb will go to the fair unless it rains" means "If it does not rain, Caleb will go to the fair": ~R ⊃ C. The rule is that "p unless q" is ~q ⊃ p. It can also be written p ∨ q ("Caleb goes or it rains"), because the two have the same truth table, as Lesson 6.4 will show. Notice what the sentence does *not* say. It does not promise that Caleb stays home if it does rain; he might go in a raincoat. A speaker may hint at that, but the statement does not claim it, so logic does not add it.

Both necessary and sufficient. Sometimes a condition is both: enough and required. A triangle has three equal sides if and only if it has three equal angles. Equal sides guarantee equal angles, and equal angles guarantee equal sides. This is the biconditional: p ≡ q says that p is necessary and sufficient for q. It is the same as two conditionals joined, (p ⊃ q) • (q ⊃ p). A good definition, as you learned in Unit 2, works this way: the definition and the thing defined are convertible, each true of exactly what the other is true of.

Converse, inverse, contrapositive. From any conditional, three others can be formed. Start with "If it is a robin, then it is a bird" (R ⊃ B). The **converse** swaps the parts: "If it is a bird, then it is a robin" (B ⊃ R). The **inverse** negates both parts: "If it

is not a robin, then it is not a bird" (~R ⊃ ~B). The **contrapositive** swaps and negates: "If it is not a bird, then it is not a robin" (~B ⊃ ~R). The original is true. The converse and the inverse are both false (a sparrow is a bird but not a robin). The contrapositive is true, and it must be: anything that is not a bird cannot be a robin.

The table proves it. The truth table shows that the contrapositive always matches the original, row by row, and the converse and inverse do not.

p q	p ⊃ q	q ⊃ p (converse)	~p ⊃ ~q (inverse)	~q ⊃ ~p (contrapositive)
T T	T	T	T	T
T F	F	T	T	F
F T	T	F	F	T
F F	T	T	T	T

The original and its contrapositive have the same column, T, F, T, T, so they are **equivalent**: they always stand or fall together. The converse and inverse share a different column, T, T, F, T. They are equivalent to each other (the inverse is the contrapositive of the converse), but neither follows from the original. Mistaking a conditional for its converse is one of the most common errors in reasoning; Lesson 6.7 gives it a name.

> **From history.** Euclid's *Elements* (about 300 B.C.), the geometry textbook used for more than two thousand years, shows plainly that a converse needs a proof of its own. Book I, Proposition 5 proves that if a triangle has two equal sides, the angles opposite them are equal. Euclid does not assume the converse. He proves it separately as Proposition 6: if two angles of a triangle are equal, the sides opposite them are equal. In the same way, Proposition 47 is the famous theorem of Pythagoras about right triangles, and Proposition 48, which closes Book I, proves its converse. Generations of students learned from Euclid that "if p, then q" and "if q, then p" are two different claims, each needing its own reasons.

> **Practice habit.** When you hear "only if," "unless" or "required," rewrite the sentence as a plain "if...then" before you reason from it.

Example 1. Let R = "The bread rises," Y = "The yeast is fresh," W = "The kitchen is warm," B = "The loaf burns" and T = "Hannah sets the timer." Symbolize: (a) The bread rises only if the yeast is fresh. (b) Fresh yeast is necessary for the bread to rise. (c) A warm kitchen is sufficient for the bread to rise. (d) The loaf will burn unless Hannah sets the timer.

▼ **Show solution**

1. (a) "Only if" introduces the consequent, the necessary condition: $R \supset Y$.
2. (b) "Y is necessary for R" means R cannot happen without Y, so Y is the consequent: $R \supset Y$. This says the same thing as (a).
3. (c) "W is sufficient for R" means W guarantees R, so W is the antecedent: $W \supset R$.
4. (d) "p unless q" is $\sim q \supset p$. Here p is "the loaf will burn" and q is "Hannah sets the timer": $\sim T \supset B$.

Answer: (a) $R \supset Y$. (b) $R \supset Y$. (c) $W \supset R$. (d) $\sim T \supset B$ (equivalently $B \vee T$, or $\sim B \supset T$).

Example 2. Write the converse, inverse and contrapositive of "If the water is boiling, then it is hot." Say which are true.

▼ **Show solution**

1. Let B = "The water is boiling" and H = "The water is hot." The original is $B \supset H$, and it is true.
2. Converse (swap): $H \supset B$, "If the water is hot, then it is boiling." False: bath water is hot but not boiling.
3. Inverse (negate both): $\sim B \supset \sim H$, "If the water is not boiling, then it is not hot." False, for the same reason: hot bath water is not boiling.
4. Contrapositive (swap and negate): $\sim H \supset \sim B$, "If the water is not hot, then it is not boiling." True, as it must be, since it is equivalent to the original.

Answer: Converse: If the water is hot, it is boiling (false). Inverse: If the water is not boiling, it is not hot (false). Contrapositive: If the water is not hot, it is not boiling (true).

PRACTICE

1. **Easy** In "If Biscuit is a beagle, then Biscuit is a dog," which condition is sufficient, and for what? Which is necessary, and for what?

   ▼ **Show answer**

   Being a beagle is sufficient for being a dog (it is enough). Being a dog is necessary for being a beagle (it is required).

2. **Easy** Write the converse, the inverse and the contrapositive of "If it is a robin, then it is a bird." Which of the three are true?

   ▼ **Show answer**

   Converse: If it is a bird, then it is a robin (false). Inverse: If it is not a robin, then it is not a bird (false). Contrapositive: If it is not a bird, then it is not a robin (true). A sparrow shows that the converse and inverse are false.

3. **Easy** Symbolize each with ⊃ (and ~ where needed): (a) p only if q (b) p if q (c) p unless q

   ▼ **Show answer**

   (a) p ⊃ q. (b) q ⊃ p. (c) ~q ⊃ p (also correct: p ∨ q, q ∨ p, or ~p ⊃ q).

4. **Easy** Which of the converse, inverse and contrapositive is always equivalent to the original conditional? Which two are always equivalent to each other?

   ▼ **Show answer**

   The contrapositive is equivalent to the original. The converse and the inverse are equivalent to each other (the inverse is the contrapositive of the converse).

5. **Medium** Let R = "The bread rises," Y = "The yeast is fresh," W = "The oven light is on" and D = "The yeast is dead." Symbolize: (a) The bread rises only if the yeast is fresh. (b) The bread rises if the oven light is on. (c) The bread rises unless the yeast is dead.

   ▼ **Show answer**

   (a) R ⊃ Y. (b) W ⊃ R. (c) ~D ⊃ R (also correct: R ∨ D, D ∨ R, or ~R ⊃ D).

6. **Medium** Is the first condition necessary for the second, sufficient for it, both, or neither? Give a reason. (a) Oxygen, for a campfire to burn. (b) Being a square, for being a rectangle. (c) A triangle's having three equal sides, for its having three equal angles. (d) A whole number's being even, for its being divisible by 4.

▼ **Show answer**

(a) Necessary, not sufficient: no fire burns without oxygen, but oxygen alone does not make a fire. (b) Sufficient, not necessary: every square is a rectangle, but a long, thin rectangle is not a square. (c) Both: each guarantees the other. (d) Necessary, not sufficient: every number divisible by 4 is even, but 6 is even and not divisible by 4.

7. **Medium** The library rule says, "You may enter the rare-book room only if you have a pass." (a) Symbolize the rule. (b) Caleb has a pass. Does the rule guarantee that he may enter? (c) Hannah has no pass. What does the rule say about her?

▼ **Show answer**

(a) Let E = "You may enter" and P = "You have a pass": $E \supset P$. (b) No. The pass is necessary, not sufficient; the rule says nothing about what else may be required. (c) She may not enter: if $E \supset P$ is true and P is false, E must be false (by the contrapositive, $\sim P \supset \sim E$).

8. **Medium** Symbolize "Whenever the rooster crows, the dog barks," then write its contrapositive in symbols and in English.

▼ **Show answer**

Let C = "The rooster crows" and B = "The dog barks": $C \supset B$. Contrapositive: $\sim B \supset \sim C$, "If the dog does not bark, then the rooster does not crow (did not crow)."

9. **Hard** Symbolize each, saying what your letters mean: (a) The garden will be weeded provided that Hannah is home. (b) Rain is a sufficient condition for canceling the picnic. (c) Rain is a necessary condition for a good corn crop. (d) No runner gets a ribbon unless she finishes the race.

▼ **Show answer**

(a) H = "Hannah is home," W = "The garden is weeded": $H \supset W$. (b) R = "It rains," C = "The picnic is canceled": $R \supset C$. (c) R = "It rains," G = "The corn crop is good": $G \supset R$. (d) G = "A runner gets a ribbon," F = "She finishes the race": $\sim F \supset \sim G$ (equivalently $G \supset F$, or $\sim G \vee F$).

10. Hard Let C = “Caleb goes to the fair” and R = “It rains.” (a) Symbolize “Caleb will go to the fair unless it rains” as a conditional. (b) Write the same statement as a disjunction. (c) Does the statement say that Caleb will stay home if it rains? Explain.

▼ **Show answer**

(a) ~R ⊃ C (also correct: its contrapositive ~C ⊃ R). (b) C ∨ R (or R ∨ C). (c) No. It says only what happens if it does not rain; if it rains, the statement is true whether Caleb goes or stays home. A speaker may hint that he will stay home, but the statement does not claim it.

11. Hard Let H = “Hannah is chosen for the team,” T = “Hannah tries out” and P = “Hannah passes the skills test.” (a) Symbolize: “Hannah is chosen for the team if and only if she tries out and passes the skills test.” (b) Suppose the statement is true, and Hannah tries out but does not pass the test. Is she chosen? Explain.

▼ **Show answer**

(a) H ≡ (T • P). (b) No. T • P is false (she did not pass), and a biconditional is true only when both sides match, so H must be false too. Trying out and passing together are necessary and sufficient for being chosen.

12. Challenge A sign at Maple Street Hardware reads, “Only customers with a receipt may return an item.” Mr. Reyes brings a receipt, but the clerk refuses his return because he has clearly used the saw for a month. Mr. Reyes says, “Your sign says I may return it, because I have a receipt.” Is he right? Symbolize the sign and explain.

▼ **Show answer**

He is not right. “Only customers with a receipt may return” means “A customer may return an item only if he has a receipt”: M ⊃ R, where M = “The customer may return the item” and R = “The customer has a receipt.” The receipt is a necessary condition, not a sufficient one. Mr. Reyes is reasoning from R to M, the converse (R ⊃ M), which the sign never states. The clerk has not broken the sign’s rule.

COMMON MISTAKES

- Reversing “only if.” “p only if q” is p ⊃ q, not q ⊃ p.

- Treating a necessary condition as sufficient: having a pass is required for entry, but the rule does not say it is enough.
- Thinking the converse or the inverse follows from a conditional. Only the contrapositive is equivalent to it.
- Reading “unless” as a two-way promise. “p unless q” is ~q ⊃ p; it says nothing about what happens when q is true.

> **Scripture.** “If ye love me, keep my commandments.” (John 14:15, KJV)
>
> Christ himself speaks in a conditional. Love for him is joined to keeping his commandments, so that obedience is the sure sign of love, the necessary condition that love always brings with it.

UNIT 6 · PROPOSITIONAL (SYMBOLIC) LOGIC · LESSON 6.4 OF 69 · 45 MINUTES

6.4 Compound Statements and Equivalence

OBJECTIVES

- Build a truth table for a compound statement, column by column, from the inside out.
- Classify a statement as a tautology, a contradiction or a contingency.
- Test two statements for logical equivalence, and tell equivalent statements from contradictory ones.
- State De Morgan’s laws, prove them with a truth table and use them to restate English sentences.

KEY TERMS

Tautology

A statement that is true in every row of its truth table, true by its form alone, such as p ∨ ~p.

Contradiction

A statement that is false in every row of its truth table, such as p • ~p.

Contingency

A statement that is true in at least one row and false in at least one row; its truth depends on the facts, such as p ⊃ q.

Contradictory statements

Two statements that have opposite truth values in every row, such as p ⊃ q and p • ~q.

Double negation

The equivalence of ~~p and p: denying a denial affirms.

Logically equivalent

Said of two statements that have the same truth value in every row of a truth table. Then p ≡ q is a tautology.

De Morgan's laws

~(p • q) is equivalent to ~p ∨ ~q, and ~(p ∨ q) is equivalent to ~p • ~q.

CORE IDEAS

Building a table, column by column. In Lesson 6.2 you found the truth value of a statement for one set of values. A full truth table does the same thing for every row at once. List the letters on the left in the standard order. Then give a column to each part of the statement, working from the inside out, and finish with a column for the main connective. For (p ∨ q) ⊃ ~p:

p	q	p ∨ q	~p	(p ∨ q) ⊃ ~p
T	T	T	F	F
T	F	T	F	F
F	T	T	T	T
F	F	F	T	T

The last column is the truth table of the whole statement: F, F, T, T. Every other column is scaffolding, but write it down. A neat table is a table you can check.

Three kinds of statement. Look at the final column. If it is all Ts, the statement is a **tautology**: true in every possible situation, true by its form alone. "Either it will rain tomorrow or it will not" (R ∨ ~R) is a tautology. If the final column is all Fs, the statement is a **contradiction**: false in every possible situation, such as "It is raining and it is not raining" (R • ~R). If the column has at least one T and at least one F, the

statement is a **contingency**: whether it is true depends on the facts. Most statements we care about, like "If the creek rises, the bridge closes," are contingencies.

Why tautologies tell us nothing about the weather. A tautology can never be false, but that is because it rules nothing out. A forecaster who says, "Tomorrow it will rain or it will not rain," cannot be wrong, and for exactly that reason he has told you nothing. A contradiction rules out everything, so it can never be true. You met these ideas in Unit 1: R ∨ ~R is the law of excluded middle written in symbols, and the fact that R • ~R is false in every row is the law of non-contradiction. Truth tables let you see these first principles at work in every statement you test.

Equivalence: same column, every row. Two statements are **logically equivalent** when they have the same truth value in every row. Then they say the same thing in different words, and either may replace the other in any argument. You saw one case in Lesson 6.3: p ⊃ q and its contrapositive ~q ⊃ ~p. Here is another.

p	q	p ⊃ q	~p	~p ∨ q
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

The columns match, so "If the hens are fed, they lay" says the same as "Either the hens are not fed or they lay." Another way to put the test: two statements are equivalent exactly when the biconditional joining them is a tautology.

De Morgan's laws. Two equivalences are so useful that they carry a name. **De Morgan's laws** say: ~(p • q) is equivalent to ~p ∨ ~q, and ~(p ∨ q) is equivalent to ~p • ~q. In words, to deny that two things are both true is to say that at least one is false; to deny that either is true is to say that both are false. "It is not the case that both the gate is open and the dog is out" means "Either the gate is not open or the dog is not out." "It is not true that Hannah or Caleb broke the window" means "Hannah did not break it, and Caleb did not break it." This is the logic behind Lesson 6.1's warning: "not both" and "neither…nor" are different claims.

Moving a negation inside. De Morgan's laws give a simple rule. When you move a ~ from outside a pair of parentheses to the inside, negate each part *and* flip the connective: • becomes ∨, and ∨ becomes •. The most common mistake is to forget

the flip and write ~(p • q) as ~p • ~q. That changes the meaning. “Hannah and Caleb did not both go to the fair” is true if only Hannah went; “Hannah did not go and Caleb did not go” is false in that case. When two negations meet, as in ~~p, they cancel: “It is not the case that the barn is not red” says simply that the barn is red. This is the law of **double negation**.

Equivalent, contradictory or neither. Two statements may stand in three relations. They are **equivalent** when their columns match in every row. They are **contradictory** when their columns are opposite in every row, so that exactly one of them is true in every situation. They may also be neither, as p ⊃ q and its converse are: they agree in some rows and differ in others. Knowing a statement’s contradictory tells you how to deny it. The contradictory of p ⊃ q is p • ~q. So to deny “If it snows, school starts late,” you must say “It snows and school does not start late,” not “If it snows, school does not start late.”

A small toolbox of equivalences. By now you have several equivalences to keep at hand: double negation (~~p and p); De Morgan’s two laws; the contrapositive (p ⊃ q and ~q ⊃ ~p); the conditional as a disjunction (p ⊃ q and ~p ∨ q); “unless” (~q ⊃ p and p ∨ q); and the biconditional as two conditionals (p ≡ q and (p ⊃ q) • (q ⊃ p)). You need not take any of them on trust. Each one can be checked with a four-row table in a few minutes, and checking them yourself is the best way to remember them.

> **From history.** The laws in this lesson are named for Augustus De Morgan (1806–1871), the first professor of mathematics at the newly founded London University (now University College London) and a leading English logician. His book *Formal Logic* appeared in 1847, the same year as George Boole’s first small book on the algebra of logic, and the two men corresponded and encouraged each other. De Morgan did not discover the laws. Medieval logicians, among them William of Ockham in the fourteenth century, had already stated rules of this kind in words: the denial of “both” is “at least one not,” and the denial of “either” is “neither.” De Morgan’s contribution was to place them within a new, more mathematical way of treating logic.

> **Practice habit.** Before you deny a compound statement, work out with De Morgan’s laws exactly what the denial says.

Example 1. Build one truth table to classify each statement as a tautology, a contradiction or a contingency: (a) (p • q) ⊃ p (b) p • ~(p ∨ q) (c) (p ∨ q) ⊃ ~p

▼ **Show solution**

1. List the four rows for p and q in the standard order, then add a column for each part, inside out.

p	q	p • q	(a) (p • q) ⊃ p	p ∨ q	~(p ∨ q)	(b) p • ~(p ∨ q)	~p	(c) (p ∨ q) ⊃ ~p
T	T	T	T	T	F	F	F	F
T	F	F	T	T	F	F	F	F
F	T	F	T	T	F	F	T	T
F	F	F	T	F	T	F	T	T

2. (a) The column is all Ts. It makes sense: if p and q are both true, then certainly p is true.
3. (b) The column is all Fs. When p is true, p ∨ q is true, so ~(p ∨ q) is false; when p is false, the conjunction has a false first part. Either way it is false.
4. (c) The column is F, F, T, T: some Ts and some Fs.

Answer: (a) Tautology. (b) Contradiction. (c) Contingency.

Example 2. Use a truth table to show that ~(p ∨ q) is equivalent to ~p • ~q. Then use the law to restate "It is not true that Hannah or Caleb broke the window."

▼ **Show solution**

1. Build both columns.

p	q	p ∨ q	~(p ∨ q)	~p	~q	~p • ~q
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

2. Compare the two final columns: ~(p ∨ q) is F, F, F, T, and ~p • ~q is F, F, F, T. They match in every row, so the statements are equivalent.
3. Let H = "Hannah broke the window" and C = "Caleb broke the window." The sentence is ~(H ∨ C). By De Morgan it is ~H • ~C.

Answer: The columns match (F, F, F, T), so the two are equivalent. "It is not true that Hannah or Caleb broke the window" means "Hannah did not break the window, and Caleb did not break it either."

PRACTICE

1. Easy Define tautology, contradiction and contingency in terms of a truth table.

   ▼ **Show answer**

   A tautology is true in every row. A contradiction is false in every row. A contingency is true in at least one row and false in at least one row.

2. Easy Classify each, with a one-sentence reason: (a) p ∨ ~p (b) p • ~p (c) p ⊃ q

   ▼ **Show answer**

   (a) Tautology: whichever value p has, one of the two disjuncts is true. (b) Contradiction: p and ~p can never both be true. (c) Contingency: it is false in the row p T, q F and true in the other three.

3. Easy State De Morgan's two laws in symbols.

   ▼ **Show answer**

   ~(p • q) is equivalent to ~p ∨ ~q. ~(p ∨ q) is equivalent to ~p • ~q.

4. **Easy** Use De Morgan's laws to restate each in English without "it is not the case that": (a) It is not the case that both the gate is open and the dog is out. (b) It is not the case that either the hay is wet or the barn leaks.

▼ **Show answer**

(a) Either the gate is not open or the dog is not out (or both). (b) The hay is not wet, and the barn does not leak. Accept equivalent wordings, such as "Neither is the hay wet nor does the barn leak."

5. **Medium** Use a truth table to show that ~(p • q) is equivalent to ~p ∨ ~q.

▼ **Show answer**

p	q	p • q	~(p • q)	~p	~q	~p ∨ ~q
T	T	T	F	F	F	F
T	F	F	T	F	T	T
F	T	F	T	T	F	T
F	F	F	T	T	T	T

Both final columns are F, T, T, T, so the statements are equivalent.

6. **Medium** Use a truth table to decide whether p ⊃ q is equivalent to ~p ∨ q.

▼ **Show answer**

Yes.

p	q	p ⊃ q	~p	~p ∨ q
T	T	T	F	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

Both columns are T, F, T, T in every row, so they are equivalent.

7. **Medium** Build a truth table for p ⊃ (q ⊃ p) and classify it.

▼ **Show answer**

p	q	q ⊃ p	p ⊃ (q ⊃ p)
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Tautology: the final column is all T. (When p is true, q ⊃ p has a true consequent; when p is false, the whole conditional has a false antecedent.)

8. **Medium** Build a truth table for ~(p ⊃ q) • q and classify it.

▼ **Show answer**

p	q	p ⊃ q	~(p ⊃ q)	~(p ⊃ q) • q
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

Contradiction: the final column is all F. (~(p ⊃ q) is true only when q is false, so it can never be true together with q.)

9. **Hard** Are p ⊃ q and q ⊃ p equivalent, contradictory or neither? Show the table and explain.

▼ **Show answer**

p	q	p ⊃ q	q ⊃ p
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

Neither. They differ in rows 2 and 3, so they are not equivalent; they agree in rows 1 and 4 (both true), so they are not contradictory.

10. Hard Build a truth table for $(p \supset q) \vee (q \supset p)$ and classify it. Explain the result in a sentence.

▼ **Show answer**

p	q	p ⊃ q	q ⊃ p	(p ⊃ q) ∨ (q ⊃ p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Tautology. Each conditional is false in only one row, and they are false in different rows, so in every row at least one disjunct is true.

11. Hard Use an eight-row truth table to decide whether $p \cdot (q \vee r)$ is equivalent to $(p \cdot q) \vee (p \cdot r)$.

▼ **Show answer**

p	q	r	q ∨ r	p • (q ∨ r)	p • q	p • r	(p • q) ∨ (p • r)
T	T	T	T	T	T	T	T
T	T	F	T	T	T	F	T
T	F	T	T	T	F	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Yes: both final columns are T, T, T, F, F, F, F, F, so they are equivalent. (Like multiplication over addition in arithmetic, • distributes over ∨.)

12. **Challenge** (a) Write, in simplest form, the denial of "Neither Hannah nor Caleb was late," and say it in English. (b) Caleb says the denial of "If it snows, school starts late" is "If it snows, school does not start late." Is he right? Give the correct denial and check it with a table.

▼ **Show answer**

(a) The statement is ~H • ~C. Its denial is ~(~H • ~C), which by De Morgan is ~~H ∨ ~~C, and by double negation H ∨ C: "Hannah or Caleb was late (at least one of them)." (b) He is not right. The contradictory of S ⊃ L is S • ~L, "It snows and school does not start late."

S	L	S ⊃ L	S • ~L	S ⊃ ~L
T	T	T	F	F
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

S • ~L is opposite to S ⊃ L in every row, but Caleb's S ⊃ ~L is true together with S ⊃ L in rows 3 and 4 (whenever it does not snow), so it is not a denial.

COMMON MISTAKES

- Moving a negation inside parentheses without flipping the connective: ~(p • q) is ~p ∨ ~q, not ~p • ~q.
- Calling a statement a tautology after checking only some rows. Every row must be T.
- Thinking a contingency is false because it is false in some rows. Its truth depends on which row is the real situation.
- Denying "if p then q" with "if p then not q." The denial is p • ~q.

> **Scripture.** "But as God is true, our word toward you was not yea and nay." (2 Corinthians 1:18, KJV)
>
> A word that is both yes and no at once is a contradiction, false in every case. Paul appeals to the faithfulness of God: because God is true, his messengers are not to speak in contradictions.

6.5 Testing Validity with Truth Tables

OBJECTIVES

- State the truth-table test: an argument is invalid if and only if some row makes every premise true and the conclusion false.
- Test an argument for validity with a full truth table and identify any counterexample row.
- Test an argument with the shortcut method by assuming the conclusion false and the premises true.
- Explain what a truth-table test shows and what it does not show (the truth of the premises).

KEY TERMS

Valid

Said of a deductive argument when it is impossible for all its premises to be true and its conclusion false.

Argument form

The pattern of an argument written with statement variables, such as $p \supset q$, p / $\therefore q$. Every argument with a valid form is valid.

Counterexample row

A row of a truth table in which every premise is true and the conclusion is false. One such row proves the argument invalid.

Full truth-table method

Testing an argument by writing out every row and checking each for a counterexample.

Shortcut method

Testing an argument by assuming the conclusion false and every premise true, then working out the forced values. If this can be done consistently, the argument is invalid; if it forces a contradiction, it is valid.

CORE IDEAS

Validity, made exact. In Unit 1 you learned that an argument is **valid** when it is impossible for its premises to be true and its conclusion false. Until now,

“impossible” had to be judged by imagination: can you picture a case where the premises hold and the conclusion fails? A truth table removes the guesswork. Each row is one possible situation, and the table lists every possible situation for the letters involved. So an argument in propositional logic is valid exactly when no row makes all the premises true and the conclusion false. If even one row does, the argument is invalid, and that row is a **counterexample**.

The full truth-table method. First symbolize the argument and write it on one line, with a slash before the conclusion: H ⊃ L, ~L / ~H. Then build a table with a column for each premise and one for the conclusion. Finally, scan the rows. Ignore every row in which some premise is false; those situations do not matter. Look only at the rows in which every premise is true, and ask whether the conclusion is true in each of them. If it is, the argument is valid. If any of those rows has a false conclusion, the argument is invalid.

A valid example. “If the hens are fed, they lay. They are not laying. So they were not fed.” Let H = “The hens are fed” and L = “The hens lay.”

H	**L**	**H ⊃ L**	**~L**	**∴ ~H**
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

Only row 4 makes both premises true, and in row 4 the conclusion is true. No row is a counterexample, so the argument is valid. This is the form called modus tollens, which you will study by name in Lesson 6.6.

An invalid example. Now change the argument: “If the hens are fed, they lay. They are laying. So they were fed.” The premises are H ⊃ L and L, and the conclusion is H.

H	**L**	**H ⊃ L**	**L**	**∴ H**
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Rows 1 and 3 make both premises true. In row 1 the conclusion is true, but in row 3 it is false: the hens were not fed, and yet they are laying (perhaps they foraged for themselves in the yard). Row 3 is a counterexample, so the argument is invalid. Notice that row 1 proves nothing. A valid argument needs a true conclusion in *every* row where the premises are all true, not just in some.

What the table tests, and what it does not. A truth table tests the *form* of an argument, and so it settles the question for every argument of that form at once, whatever the letters stand for. It does not test whether the premises are actually true. That question belongs, as always, to the field the argument is about. An argument can pass the table test and still have a false premise, in which case it is valid but unsound. The table answers the second of the two questions from Unit 1, "Does the conclusion follow?", and it answers it with certainty.

The shortcut method. Full tables grow quickly: three letters need 8 rows, four need 16 and five need 32. The **shortcut method** goes straight for a counterexample. Assume the conclusion is false and every premise is true, and work out what those assumptions force. If you can give every letter a value without contradiction, you have built a counterexample row, and the argument is invalid. If the assumptions force some letter to be both T and F, then no counterexample row exists, and the argument is valid.

A shortcut that ends in contradiction. Test p ⊃ q, q ⊃ r / p ⊃ r. Suppose the conclusion p ⊃ r is false. A conditional is false only when its antecedent is true and its consequent false, so p is T and r is F. Now the second premise, q ⊃ r, must be true with r false, so q must be F. The first premise, p ⊃ q, must be true with p true, so q must be T. But q cannot be both F and T. The assumption that the conclusion is false while the premises are true has led to a contradiction, so the argument is valid.

A shortcut that finds a counterexample. Test A ∨ B, A ⊃ C / C. Suppose the conclusion C is false. The premise A ⊃ C must be true with C false, so A is F. The premise A ∨ B must be true with A false, so B is T. Every letter now has a value, A false, B true and C false, with no contradiction. Check it: A ∨ B is T, A ⊃ C is T, and C is F. This is a counterexample row, so the argument is invalid. Sometimes the assumptions do not force a single value; a premise such as A ∨ B could be true in more than one way. Then try each possibility in turn. The argument is invalid if any one of them works without contradiction, and valid only if every one of them fails.

From history. The English logician and economist William Stanley Jevons (1835–1882) saw that testing an argument could be done by a fixed, mechanical procedure, and he built a machine to prove it. His "logical piano," finished in 1869, had keys for the terms of an argument. As the premises were entered, the machine struck out every combination of terms that disagreed with them and displayed the combinations that remained possible. Jevons described it to the Royal Society in London in 1870. It handled up to four terms, so it worked through sixteen combinations, the same number as the rows of a truth table with four letters. Lesson 6.9 follows this idea of logic done by machine to the switching circuits of the twentieth century.

Practice habit. Before accepting a conclusion, ask: is there any possible situation in which every reason given is true and this conclusion false?

Example 1. Test each argument with a full truth table. (a) Either the pump is broken or the well is dry. The pump is not broken. So the well is dry. (b) Either the pump is broken or the well is dry. The pump is broken. So the well is not dry.

▼ **Show solution**

1. Let P = "The pump is broken" and W = "The well is dry." (a) is P ∨ W, ~P / W. (b) is P ∨ W, P / ~W.
2. Table for (a):

P	W	P ∨ W	~P	∴ W
T	T	T	F	T
T	F	T	F	F
F	T	T	T	T
F	F	F	T	F

   Only row 3 makes both premises true, and its conclusion is true. No counterexample: valid.
3. Table for (b):

P	W	P ∨ W	P	∴ ~W
T	T	T	T	F
T	F	T	T	T
F	T	T	F	F
F	F	F	F	T

   Rows 1 and 2 make both premises true. In row 1 the conclusion is false: the pump is broken *and* the well is dry. That row is a counterexample: invalid.
4. The difference comes from the inclusive "or." Denying one disjunct lets you conclude the other, but affirming one disjunct does not let you deny the other, because both may be true.

Answer: (a) Valid: the only row with both premises true has a true conclusion. (b) Invalid: row 1 (pump broken and well dry) makes both premises true and the conclusion false.

Example 2. Use the shortcut method. (a) If Hannah studies, she passes. If Caleb studies, he passes. Hannah or Caleb studies. So Hannah passes or Caleb passes. (b) The same three premises, with the conclusion: So Hannah passes.

▼ **Show solution**

1. Let H = "Hannah studies," P = "Hannah passes," C = "Caleb studies," Q = "Caleb passes." Premises: $H \supset P$, $C \supset Q$, $H \vee C$. A full table would need 16 rows.
2. (a) Assume the conclusion $P \vee Q$ is false: P is F and Q is F. $H \supset P$ must be true with P false, so H is F. $C \supset Q$ must be true with Q false, so C is F. But then $H \vee C$ is $F \vee F$, which is false, though we assumed it true. Contradiction: no counterexample exists, so (a) is valid.
3. (b) Assume the conclusion P is false. $H \supset P$ must be true, so H is F. $H \vee C$ must be true with H false, so C is T. $C \supset Q$ must be true with C true, so Q is T. No contradiction: H false, P false, C true, Q true.
4. Check (b): $H \supset P$ is $F \supset F$, true; $C \supset Q$ is $T \supset T$, true; $H \vee C$ is $F \vee T$, true; the conclusion P is false. This is a counterexample: Caleb studied and passed, and Hannah did neither.

Answer: (a) Valid (the assumptions force a contradiction). (b) Invalid: counterexample H false, P false, C true, Q true.

PRACTICE

1. Easy State the truth-table test for validity in one sentence.

   ▼ **Show answer**

   An argument is invalid if some row of its truth table makes every premise true and the conclusion false; if no row does, it is valid.

2. Easy An argument uses the letters A, B and C. How many rows does its full truth table need? How many if a fourth letter, D, is added?

   ▼ **Show answer**

   8 rows for three letters; 16 rows for four.

3. **Easy** Test this form with a full truth table: p ⊃ q, p / q.

▼ **Show answer**

p	q	p ⊃ q	p	∴	q
T	T	T	T		T
T	F	F	T		F
F	T	T	F		T
F	F	T	F		F

Valid: only row 1 makes both premises true, and its conclusion is true.

4. **Easy** In the truth table for the form p ⊃ q, q / p, which row shows that the form is invalid? What values does that row give p and q?

▼ **Show answer**

Row 3, where p is false and q is true. Then p ⊃ q is true and q is true (both premises true), but the conclusion p is false.

5. **Medium** Test with a full truth table: p ∨ q, p / ~q.

▼ **Show answer**

p	q	p ∨ q	p	∴	~q
T	T	T	T		F
T	F	T	T		T
F	T	T	F		F
F	F	F	F		T

Invalid: row 1 (p and q both true) makes both premises true and the conclusion false.

6. **Medium** Test with a full truth table: ~(p • q), p / ~q.

▼ **Show answer**

p	q	~(p • q)	p	∴	~q
T	T	F	T		F
T	F	T	T		T
F	T	T	F		F
F	F	T	F		T

Valid: only row 2 makes both premises true, and its conclusion is true. (If two things are not both true and one is true, the other is false.)

7. **Medium** Test with a full truth table: p ⊃ q, ~p / ~q.

▼ **Show answer**

p	q	p ⊃ q	~p	∴	~q
T	T	T	F		F
T	F	F	F		T
F	T	T	T		F
F	F	T	T		T

Invalid: row 3 (p false, q true) makes both premises true and the conclusion false.

8. **Medium** Test with a full truth table: p ≡ q, ~q / ~p.

▼ **Show answer**

p	q	p ≡ q	~q	∴	~p
T	T	T	F		F
T	F	F	T		F
F	T	F	F		T
F	F	T	T		T

Valid: only row 4 makes both premises true, and its conclusion is true.

9. Hard Use the shortcut method to test: p ⊃ (q • r), ~r / ~p.

▼ **Show answer**

Valid. Assume the conclusion ~p is false: p is T. The premise ~r is true: r is F. The premise p ⊃ (q • r) is true with p true, so q • r must be T, which needs r to be T. But r is F. Contradiction, so no counterexample exists.

10. Hard Use the shortcut method to test: p ⊃ (q ∨ r), ~q / r. If it is invalid, give the counterexample row.

▼ **Show answer**

Invalid. Assume the conclusion r is false: r is F. The premise ~q is true: q is F. Then q ∨ r is F, so for p ⊃ (q ∨ r) to be true, p must be F. No contradiction. Counterexample: p false, q false, r false. (Check: F ⊃ F is T; ~q is T; r is F.)

11. Hard Symbolize and test by the shortcut method: “If the hens are fed, they lay. If they lay, we sell eggs at the market. We are not selling eggs. So the hens are not being fed.”

▼ **Show answer**

Let H = “The hens are fed,” L = “They lay,” S = “We sell eggs.” H ⊃ L, L ⊃ S, ~S / ~H. Assume ~H false: H is T. ~S true: S is F. L ⊃ S true with S false: L is F. H ⊃ L true with H true: L is T. Contradiction (L both F and T), so the argument is valid.

12. **Challenge** "If the hens are fed and the coop is warm, they lay. They are not laying. So they are not fed." Let H = "The hens are fed," W = "The coop is warm," L = "They lay." Symbolize the argument, test it with a full eight-row truth table, and describe the counterexample, if there is one, in plain words.

▼ **Show answer**

(H • W) ⊃ L, ~L / ~H.

H	W	L	H • W	(H • W) ⊃ L	~L	∴ ~H
T	T	T	T	T	F	F
T	T	F	T	F	T	F
T	F	T	F	T	F	F
T	F	F	F	T	T	F
F	T	T	F	T	F	T
F	T	F	F	T	T	T
F	F	T	F	T	F	T
F	F	F	F	T	T	T

Both premises are true in rows 4, 6 and 8. Rows 6 and 8 have a true conclusion, but row 4 (H true, W false, L false) has a false one, so the argument is invalid. In plain words: the hens are fed, but the coop is cold, and so they are not laying. The premises allow this, so "not laying" does not prove "not fed." (Valid conclusion instead: ~(H • W), they are not both fed and warm.)

COMMON MISTAKES

- Calling an argument invalid because some row has a false conclusion, without checking that every premise is true in that row.
- Calling an argument valid because one row has all premises true and a true conclusion. Every such row must have a true conclusion.
- Thinking a truth table shows that the premises are actually true. It tests only whether the conclusion follows.
- In the shortcut method, stopping after one possibility when a premise could be true in several ways. Try each one before you call the argument valid.

> **Scripture.** “searched the scriptures daily, whether those things were so” (Acts 17:11, KJV)
>
> The Bereans did not accept even an apostle’s teaching untested. A truth table is a humble tool of the same spirit: it checks every case before it accepts a conclusion.

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6.6 The Valid Forms

OBJECTIVES

- State the forms of modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism and constructive dilemma.
- Name the form of an argument stated in English or in symbols, including arguments whose parts are negated or compound.
- Supply the conclusion that a valid form allows from given premises.
- Explain the two traditional ways of answering a dilemma.

KEY TERMS

Modus ponens (MP)

“The mode that affirms”: p ⊃ q, p / ∴ q. Affirming the antecedent proves the consequent.

Modus tollens (MT)

“The mode that denies”: p ⊃ q, ~q / ∴ ~p. Denying the consequent proves the denial of the antecedent.

Hypothetical syllogism (HS)

A chain of conditionals: p ⊃ q, q ⊃ r / ∴ p ⊃ r.

Disjunctive syllogism (DS)

Proof by elimination: p ∨ q, ~p / ∴ q (or p ∨ q, ~q / ∴ p).

Constructive dilemma (CD)

(p ⊃ q) • (r ⊃ s), p ∨ r / ∴ q ∨ s. Two conditionals and a choice between their antecedents prove a choice between their consequents.

Substitution instance

An argument obtained from a form by putting statements, simple or compound, in place of the variables p, q, r and s, the same statement for the same variable each time.

CORE IDEAS

Five forms worth knowing by heart. A truth table can test any argument, but it is slow. Fortunately, a handful of forms turn up again and again in ordinary reasoning,

and each has been tested once and for all. Learn their shapes, and you will recognize a valid argument the moment you hear it. This lesson names five: modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism and constructive dilemma. Every argument that fits one of these forms is valid, whatever its subject. You can check each of them yourself with a truth table, as you did for several in Lesson 6.5.

Modus ponens: affirm the antecedent. The form is p ⊃ q, p / ∴ q. “If the frost comes early, the tomatoes will die. The frost came early. So the tomatoes will die.” The first premise links two statements; the second affirms the first of them; the conclusion affirms the second. The Latin name means “the mode that affirms.” It is the most natural argument in the world, and it is the backbone of every rule applied to a case: “If a player touches the ball with his hands, the referee awards a free kick. Caleb touched the ball with his hands. So the referee awards a free kick.”

Modus tollens: deny the consequent. The form is p ⊃ q, ~q / ∴ ~p. “If the milk is fresh, it smells sweet. It does not smell sweet. So it is not fresh.” The Latin name means “the mode that denies.” Modus tollens works because a conditional is equivalent to its contrapositive: if p ⊃ q is true, so is ~q ⊃ ~p, and then ~q gives ~p by modus ponens. Modus tollens is the form of every test that can fail. A farmer reasons, “If this field drains well, there will be no standing water after the storm. There is standing water. So it does not drain well.”

Hypothetical syllogism: the chain. The form is p ⊃ q, q ⊃ r / ∴ p ⊃ r. “If Caleb oversleeps, he misses the bus. If he misses the bus, he is late for school. So if Caleb oversleeps, he is late for school.” The consequent of the first premise is the antecedent of the second, and the chain links the first antecedent to the last consequent. A chain may have many links. Notice what the conclusion is: another conditional. Hypothetical syllogism does not prove that Caleb is late, or even that he overslept; it proves only that the one leads to the other.

Disjunctive syllogism: elimination. The form is p ∨ q, ~p / ∴ q, or with the disjuncts the other way, p ∨ q, ~q / ∴ p. “Either the fuse blew or the bulb burned out. The bulb did not burn out. So the fuse blew.” A disjunction says at least one part is true; rule one out, and the other must be the true one. This is the reasoning of every detective and every mechanic. Remember the warning of Lesson 6.5: the second premise

must *deny* a disjunct. Because "or" is inclusive, affirming one disjunct ("The bulb did burn out") does not prove the other false.

Constructive dilemma: two horns. The form is (p ⊃ q) • (r ⊃ s), p ∨ r / ∴ q ∨ s. "If we plant corn, we will need a fence. If we plant beans, we will need poles. We will plant corn or beans. So we will need a fence or poles." Two conditionals are joined to a choice between their antecedents, and the conclusion is a choice between their consequents. A dilemma is called "two-horned," as if you were facing a bull. There are two honest ways to answer one. You may *grasp a horn*, by showing that one of the conditionals is false. Or you may *go between the horns*, by showing that the choice is not complete, because there is a third option. If neither can be done, the dilemma stands.

Spotting forms in disguise. The letters p, q, r and s in a form may stand for any statements at all, simple or compound, so long as the same statement replaces the same letter each time. Each argument so obtained is a **substitution instance** of the form. So ~A ⊃ B, ~A / ∴ B is modus ponens, with p = ~A and q = B. And (A • B) ⊃ ~C, ~~C / ∴ ~(A • B) is modus tollens, with p = A • B and q = ~C; its second premise denies ~C, as modus tollens requires. The order of the premises does not matter either: ~q and p ⊃ q, given in that order, still make modus tollens. To find a form, look at the main connective of each premise first.

Listening for the form in speech. People rarely speak in standard form. They put the conclusion first, use "since" and "so," and leave the plainest premise unspoken. "The fuse must have blown, since the bulb is fine" is a disjunctive syllogism with its disjunction ("the fuse blew or the bulb burned out") left unstated. When you hear an argument, first find the conclusion, then write out the premises, including any that are clearly assumed, and then compare the shape with the five forms. If it matches, it is valid; whether it is sound still depends on the truth of its premises.

From history. Chrysippus and the Stoics listed five basic argument forms that they called "indemonstrables," because they were evident and needed no proof. Writing "the first" and "the second" where we write p and q, they gave: "If the first, then the second; the first; therefore the second" (our modus ponens), and "If the first, then the second; not the second; therefore not the first" (modus tollens). Their fifth was: "Either the first or the second; not the first; therefore the second," the pattern of disjunctive syllogism. The Latin names came from medieval logicians: *modus ponendo ponens*, "the mode that affirms by affirming," and *modus tollendo tollens*, "the mode that denies by denying," shortened today to modus ponens and modus tollens.

Practice habit. When you hear an argument, listen for its shape: is it a ponens, a tollens, a chain, an elimination or a dilemma?

WORKED EXAMPLES

Example 1. Name the form of each argument. (a) If the frost comes early, the tomatoes will die. The frost came early. So the tomatoes will die. (b) Either the fuse blew or the bulb burned out. The bulb did not burn out. So the fuse blew. (c) If Caleb oversleeps, he misses the bus. If he misses the bus, he is late. So if Caleb oversleeps, he is late. (d) If the milk is fresh, it smells sweet. It does not smell sweet. So it is not fresh.

▼ **Show solution**

1. (a) F ⊃ D, F / ∴ D. The second premise affirms the antecedent: modus ponens.
2. (b) B ∨ U, ~U / ∴ B. A disjunction, the denial of one disjunct, and the other disjunct as conclusion: disjunctive syllogism.
3. (c) O ⊃ M, M ⊃ L / ∴ O ⊃ L. The consequent of the first conditional is the antecedent of the second: hypothetical syllogism.
4. (d) F ⊃ S, ~S / ∴ ~F. The second premise denies the consequent: modus tollens.

Answer: (a) Modus ponens. (b) Disjunctive syllogism. (c) Hypothetical syllogism. (d) Modus tollens. All four are valid.

Example 2. Name the form of each, saying what p, q (and r, s) stand for. (a) (A • B) ⊃ ~C, ~~C / ∴ ~(A • B) (b) (G ∨ H) ⊃ K, G ∨ H / ∴ K (c) (A ⊃ B) • (~A ⊃ C), A ∨ ~A / ∴ B ∨ C

▼ **Show solution**

1. (a) The first premise is a conditional with antecedent A • B and consequent ~C. The second premise, ~~C, denies the consequent. With p = A • B and q = ~C, this is p ⊃ q, ~q / ∴ ~p: modus tollens.
2. (b) The first premise is a conditional with antecedent G ∨ H; the second premise affirms that antecedent. With p = G ∨ H and q = K, this is modus ponens. The ∨ inside does not make it a disjunctive syllogism; the main connective of the first premise is ⊃.
3. (c) Two conditionals joined by •, and a disjunction of their antecedents. With p = A, q = B, r = ~A, s = C, this is constructive dilemma. (Its disjunction, A ∨ ~A, is a tautology, so the second premise is certainly true.)

Answer: (a) Modus tollens (p = A • B, q = ~C). (b) Modus ponens (p = G ∨ H, q = K). (c) Constructive dilemma (p = A, q = B, r = ~A, s = C).

PRACTICE

1. Easy Write the forms of modus ponens and modus tollens in symbols.

   ▼ **Show answer**

   Modus ponens: p ⊃ q, p / ∴ q. Modus tollens: p ⊃ q, ~q / ∴ ~p.

2. Easy Name the form: “If the dog barks, someone is at the door. The dog is barking. So someone is at the door.”

   ▼ **Show answer**

   Modus ponens: the second premise affirms the antecedent, and the conclusion affirms the consequent.

3. Easy Name the form: “The key is in the drawer or in the car. It is not in the drawer. So it is in the car.”

   ▼ **Show answer**

   Disjunctive syllogism: one disjunct is denied, so the other is concluded.

4. **Easy** Name the form: “If we leave at noon, we reach the farm by three. If we reach the farm by three, we can help with the milking. So if we leave at noon, we can help with the milking.”

   ▼ **Show answer**

   Hypothetical syllogism: two conditionals linked in a chain, with a conditional as the conclusion.

5. **Medium** Supply the conclusion and name the form: “If the creek is high, the ford is closed. The ford is not closed.”

   ▼ **Show answer**

   Conclusion: The creek is not high. Form: modus tollens (H ⊃ C, ~C / ∴ ~H).

6. **Medium** Supply the conclusion and name the form: “If we plant corn, we will need a fence. If we plant beans, we will need poles. We will plant corn or beans.”

   ▼ **Show answer**

   Conclusion: We will need a fence or poles. Form: constructive dilemma ((C ⊃ F) • (B ⊃ P), C ∨ B / ∴ F ∨ P).

7. **Medium** Name the form of each: (a) (A • B) ⊃ C, ~C / ∴ ~(A • B) (b) ~F ∨ G, ~~F / ∴ G (c) ~H ⊃ ~K, ~H / ∴ ~K

   ▼ **Show answer**

   (a) Modus tollens (p = A • B, q = C). (b) Disjunctive syllogism (p = ~F, q = G; the second premise denies the first disjunct, ~F). (c) Modus ponens (p = ~H, q = ~K).

8. **Medium** Which of these is a disjunctive syllogism, and which is not a valid form at all? (a) P ∨ Q, ~Q / ∴ P (b) P ∨ Q, P / ∴ ~Q. Explain.

   ▼ **Show answer**

   (a) is a disjunctive syllogism: it denies one disjunct and concludes the other. (b) is not a valid form: it affirms a disjunct and denies the other, but since ∨ is inclusive both may be true (the row P true, Q true is a counterexample).

9. Hard “If it freezes tonight, the pipes will burst. If the pipes burst, the kitchen will flood. The kitchen did not flood. So it did not freeze.” Symbolize the argument and show how it can be reached in two steps using the valid forms.

▼ **Show answer**

Let F = “It freezes,” B = “The pipes burst,” K = “The kitchen floods.” Premises: F ⊃ B, B ⊃ K, ~K. Step 1: from F ⊃ B and B ⊃ K, hypothetical syllogism gives F ⊃ K. Step 2: from F ⊃ K and ~K, modus tollens gives ~F. (Also correct: from B ⊃ K and ~K, modus tollens gives ~B; then from F ⊃ B and ~B, modus tollens gives ~F.)

10. Hard Explain why disjunctive syllogism is valid even though ∨ is inclusive, but “p ∨ q, p / ∴ ~q” is not. When could a speaker fairly reason, “It is one or the other; it is this one; so it is not the other”?

▼ **Show answer**

p ∨ q says at least one is true. If p is denied, q must be the true one, so disjunctive syllogism is valid. But affirming p tells us nothing about q, because the inclusive “or” allows both to be true. The second kind of reasoning is fair only when the “or” is exclusive and that is stated or plainly understood, (p ∨ q) • ~(p • q), as with “The switch is either up or down.” Then the extra premise ~(p • q) and p together give ~q.

11. Hard Symbolize and name the form: “Either Hannah feeds the calves or Caleb does. If Hannah feeds them, she will be late for breakfast. If Caleb feeds them, he will miss the bus. So Hannah will be late for breakfast or Caleb will miss the bus.”

▼ **Show answer**

Let H = “Hannah feeds the calves,” L = “Hannah is late for breakfast,” C = “Caleb feeds the calves,” M = “Caleb misses the bus.” (H ⊃ L) • (C ⊃ M), H ∨ C / ∴ L ∨ M. Constructive dilemma. (Accept the two conditionals written as separate premises.)

12. **Challenge** Caleb broke a bat at practice. He reasons: “If I tell the coach, I will lose my place in the lineup. If I keep quiet, I will have a guilty conscience. I must either tell or keep quiet. So I will lose my place or have a guilty conscience.” (a) Name the form. (b) Explain the two traditional ways of answering a dilemma, and say which, if either, works here.

▼ **Show answer**

(a) Constructive dilemma. (b) To grasp a horn is to show that one of the conditionals is false; to go between the horns is to show that the disjunction leaves out a third option. Going between the horns fails here, because “tell or keep quiet” (T ∨ ~T) covers every case. But Caleb may grasp the first horn: an honest player who reports an accident is not likely to lose his place for it, so “If I tell, I will lose my place” is doubtful. Accept any well-reasoned answer that correctly explains both methods; a student may also argue that losing his place would be a smaller loss than a guilty conscience, which does not refute the dilemma but helps him choose.

COMMON MISTAKES

- Using disjunctive syllogism after affirming a disjunct. Only denying one disjunct proves the other.
- Thinking hypothetical syllogism proves its last consequent. It proves only a new conditional, p ⊃ r.
- Missing a form because its parts are negated or compound. Look at the main connective of each premise and let p and q stand for whole parts.
- Confusing modus tollens (deny the consequent) with denying the antecedent, which is a fallacy (Lesson 6.7).

REFERENCE GUIDE

An eight-page, color-coded Logic guide: what logic is, terms and definitions, the A E I O propositions, the square of opposition and immediate inferences, syllogisms with the six rules and Venn tests, the named forms, informal fallacies of relevance, presumption and ambiguity, propositional logic with truth tables, conditionals, valid forms and short proofs, the classical tradition, dialectic, argument maps and practice. The booklet prints on 11×17 paper and folds to letter size. Prints in color and reads clearly in black and white.

> **Scripture.** “And if Christ be not risen, then is our preaching vain, and your faith is also vain.” (1 Corinthians 15:14, KJV)
>
> Paul reasons in a chain of conditionals: if there is no resurrection of the dead, Christ is not risen (verse 13); if Christ is not risen, the faith is vain. He follows the chain to its end to show how much rests on the resurrection, and then he denies its starting point: Christ is risen (verse 20).

UNIT 6 · PROPOSITIONAL (SYMBOLIC) LOGIC · LESSON 6.7 OF 69 · 40 MINUTES

6.7 The Formal Fallacies

OBJECTIVES

- Define a formal fallacy and tell it from an informal fallacy.
- State the forms of affirming the consequent and denying the antecedent, and show with a truth table why each is invalid.
- Spot both fallacies in everyday speech and advertising, including when “only if” disguises them.
- Refute a fallacious argument with a counterexample of the same form.

KEY TERMS

Formal fallacy

An argument whose form is invalid, so that the error lies in the pattern of reasoning itself, whatever the subject.

Affirming the consequent

The invalid form $p \supset q,\ q\ /\ \therefore p$. It imitates modus ponens but affirms the wrong part.

Denying the antecedent

The invalid form $p \supset q,\ \sim p\ /\ \therefore \sim q$. It imitates modus tollens but denies the wrong part.

Affirming a disjunct

The invalid form $p \vee q,\ p\ /\ \therefore \sim q$, which treats an inclusive “or” as exclusive.

Refutation by counterexample

Showing that an argument’s form is invalid by giving another argument of exactly the same form whose premises are plainly true and whose conclusion is plainly false.

Errors in the pattern itself. In Unit 3 you studied informal fallacies, errors that arise from the content of an argument: an irrelevant appeal, a shifted meaning, a loaded question. A **formal fallacy** is different. It is an argument whose form is invalid, so the mistake would be a mistake whatever the statements were about. Two formal fallacies are so common, and so easily mistaken for valid forms, that every student of logic should be able to spot them at once: **affirming the consequent** and **denying the antecedent**. Each one is a counterfeit of a valid form you learned in Lesson 6.6.

Affirming the consequent. The form is $p \supset q$, q / $\therefore p$. "If it rained last night, the street is wet. The street is wet. So it rained last night." It looks like modus ponens, but it affirms the consequent instead of the antecedent. The street may be wet because the street sweeper came by, or a hydrant was opened, or a neighbor washed his car. The truth table shows the flaw in the row p false, q true: there $p \supset q$ is true and q is true, but p is false. A conditional says that p leads to q; it does not say that q can come only from p.

Denying the antecedent. The form is $p \supset q$, $\sim p$ / $\therefore \sim q$. "If it rained last night, the street is wet. It did not rain last night. So the street is not wet." It looks like modus tollens, but it denies the antecedent instead of the consequent. Once again the sweeper may have wet the street. The same row, p false and q true, is the counterexample: both premises are true and the conclusion is false. The rule to remember is short: *affirm the antecedent, deny the consequent*. Those two moves are valid. The other two, affirming the consequent and denying the antecedent, are not.

Why the counterfeits fool us. Both fallacies come from the confusion you met in Lesson 6.3. Affirming the consequent reasons as though the converse, $q \supset p$, were true. Denying the antecedent reasons as though the inverse, $\sim p \supset \sim q$, were true. In ordinary talk we often hear "if" as "if and only if." When a mother says, "If you finish your homework, you may go outside," her children understand that unfinished homework means staying in. But that extra meaning comes from the setting, not from the word "if." Whenever the reasoning depends on it, the speaker owes us the stronger premise, stated plainly.

In everyday speech and in advertising. The fallacies are everywhere once you look. "If you have the flu, you will run a fever. You are running a fever. So you have the flu" (affirming the consequent; many illnesses bring fever). "If the battery is dead, the

car will not start. The battery is fine. So the car will start" (denying the antecedent; the tank may be empty). Advertisers lean on the converse all the time. A poster for Glimmerwax says, "Every prize-winning car at the county show was polished to a mirror shine." The hope is that you will reason, "If a car wins a prize, it shines. My car will shine with Glimmerwax. So my car will be a prize-winner," which affirms the consequent. Watch also for "only if." "You may take the canoe out only if you have passed the swim test" is $C \supset S$, so "Caleb passed the swim test, so he may take the canoe out" affirms the consequent.

Refuting by counterexample. The quickest way to show someone that a form is invalid is to give another argument of exactly the same form, with premises that are plainly true and a conclusion that is plainly false. Against affirming the consequent: "If Biscuit is a cat, then Biscuit is an animal. Biscuit is an animal. So Biscuit is a cat." But Biscuit is a beagle. Against denying the antecedent: "If a fruit is a lemon, it is sour. This green apple is not a lemon. So it is not sour." But green apples can be very sour. If the form allows true premises and a false conclusion even once, it cannot be trusted anywhere.

When it is not a fallacy. Two cautions keep you fair. First, if the premise really is a biconditional, the reasoning is valid: "Hannah gets the ribbon if and only if she finishes in the top three. She did not finish in the top three. So she does not get the ribbon." That is not denying the antecedent, because $p \equiv q$ includes $q \supset p$. Second, reasoning from an effect to a likely cause is not always foolish. A doctor who sees a fever and a certain rash may judge that measles is the likely cause and then test her guess. That is inductive reasoning to a probable explanation, the subject of Unit 7. It becomes a fallacy only when it claims to *prove* its conclusion.

A bad argument does not make a false conclusion. Remember the fallacy fallacy from Unit 3. Showing that an argument affirms the consequent proves that the argument fails, not that its conclusion is false. Perhaps it did rain last night; the wet street just does not prove it. The fair response is "Your reason does not show that," not "So you are wrong." Then, if the question matters, look for better evidence: a rain gauge, a neighbor's report, puddles in the field.

From history. Aristotle described affirming the consequent more than two thousand years ago. In his short book *On Sophistical Refutations*, he lists a fallacy that arises from "the consequent": people suppose that a consequence runs both ways. Because the ground becomes wet when it rains, they think that if the ground is wet, it must have rained. He notes that our senses mislead us in the same way, as when someone takes yellow bile for honey because honey is yellow. Medieval logicians, who studied this book closely, called the error the *fallacia consequentis*, "the fallacy of the consequent," and it has kept a place in nearly every logic textbook since.

Practice habit. When someone reasons from a result back to a cause, ask: what else could have produced the same result?

WORKED EXAMPLES

Example 1. Is each argument valid or invalid? Name its form. (a) If the pond is frozen, the ducks have left. The ducks have left. So the pond is frozen. (b) If the pond is frozen, the ducks have left. The pond is not frozen. So the ducks have not left. (c) If the pond is frozen, the ducks have left. The ducks have not left. So the pond is not frozen.

▼ **Show solution**

1. Let F = "The pond is frozen" and D = "The ducks have left." The first premise in each is F ⊃ D.
2. (a) The second premise is D, the consequent, and the conclusion is F. That is affirming the consequent: invalid. The ducks may have left for another reason, such as a hawk nearby.
3. (b) The second premise is ~F, a denial of the antecedent, and the conclusion is ~D. That is denying the antecedent: invalid. Again the ducks may have left for another reason.
4. (c) The second premise is ~D, a denial of the consequent, and the conclusion is ~F. That is modus tollens: valid.

Answer: (a) Invalid: affirming the consequent. (b) Invalid: denying the antecedent. (c) Valid: modus tollens.

Example 2. A sign at the Brightloaf Bakery reads: "If bread is baked by a prize-winning baker, it is made with fine flour. Brightloaf bread is made with fine flour. So Brightloaf bread is baked by prize-winning bakers!" Name the fallacy and refute it with a counterexample of the same form.

▼ **Show solution**

1. Let P = "The bread is baked by a prize-winning baker" and F = "The bread is made with fine flour." The argument is $P \supset F$, F / $\therefore P$.
2. The second premise affirms the consequent, and the conclusion affirms the antecedent. This is affirming the consequent.
3. Build an argument of the same form with plainly true premises and a plainly false conclusion: "If an animal is a horse, it has four legs. Biscuit the beagle has four legs. So Biscuit is a horse."
4. Since the form can lead from true premises to a false conclusion, the bakery's argument proves nothing. Plenty of ordinary bakers use fine flour too.

Answer: Affirming the consequent. Counterexample: "If an animal is a horse, it has four legs. Biscuit has four legs. So Biscuit is a horse." True premises, false conclusion, same form. (Accept any counterexample of the same form with plainly true premises and a plainly false conclusion.)

PRACTICE

1. Easy Write the forms of affirming the consequent and denying the antecedent in symbols.

   ▼ **Show answer**

   Affirming the consequent: $p \supset q$, q / $\therefore p$. Denying the antecedent: $p \supset q$, $\sim p$ / $\therefore \sim q$.

2. Easy Which valid form does each fallacy imitate? (a) Affirming the consequent (b) Denying the antecedent

   ▼ **Show answer**

   (a) Modus ponens (which affirms the antecedent). (b) Modus tollens (which denies the consequent).

3. **Easy** Name the fallacy: “If it rained, the street is wet. The street is wet. So it rained.”

▼ Show answer

Affirming the consequent: the second premise affirms the consequent (the street is wet), and the conclusion affirms the antecedent.

4. **Easy** Name the fallacy: “If Hannah studies, she will pass. She did not study. So she will not pass.”

▼ Show answer

Denying the antecedent: the second premise denies the antecedent (she did not study), and the conclusion denies the consequent. She might pass anyway, for instance because she already knew the material.

5. **Medium** Valid or invalid? Name the form of each. (a) If the bread is moldy, it is not fit to eat. It is not moldy. So it is fit to eat. (b) If the bread is moldy, it is not fit to eat. It is moldy. So it is not fit to eat. (c) If the battery is dead, the car will not start. The car will not start. So the battery is dead. (d) If the battery is dead, the car will not start. The car started. So the battery is not dead.

▼ Show answer

(a) Invalid: denying the antecedent (stale or burnt bread is unfit too). (b) Valid: modus ponens. (c) Invalid: affirming the consequent. (d) Valid: modus tollens (“the car started” denies the consequent “the car will not start”).

6. **Medium** For argument (c) in the previous problem (“If the battery is dead, the car will not start. The car will not start. So the battery is dead.”), describe a situation in which both premises are true and the conclusion is false.

▼ Show answer

The battery is fine, but the car is out of gas, so it will not start. Then “If the battery is dead, the car will not start” is still true, and “The car will not start” is true, but “The battery is dead” is false. Accept any other cause, such as a broken starter.

7. **Medium** Use the truth table for p ⊃ q to show that affirming the consequent is invalid. Which row is the counterexample?

▼ **Show answer**

p	q	p ⊃ q	q ∴ p
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	F

Row 3 (p false, q true) makes both premises true and the conclusion false, so the form is invalid.

8. **Medium** An advertisement says: “If you use Glimmerwax, your car will shine like new. Mr. Patel’s car shines like new. He must use Glimmerwax!” Name the fallacy and explain the error in one or two sentences.

▼ **Show answer**

Affirming the consequent. The ad claims only that Glimmerwax leads to a shine, not that a shine can come only from Glimmerwax; Mr. Patel’s car may shine because it is new, or because he used another wax.

9. **Hard** Write an argument of the same form as this one, with plainly true premises and a plainly false conclusion: “If the cook is a trained chef, the soup will be good. The cook is not a trained chef. So the soup will not be good.” Name the form.

▼ **Show answer**

The form is denying the antecedent. Sample counterexample: “If Biscuit is a cat, then Biscuit is an animal. Biscuit is not a cat (he is a beagle). So Biscuit is not an animal.” The premises are true and the conclusion is false. Accept any argument of the form p ⊃ q, ~p / ∴ ~q with plainly true premises and a plainly false conclusion.

10. Hard Dr. Lin reasons: “If a child has chicken pox, she will have an itchy, spotted rash. This child has an itchy, spotted rash. So she probably has chicken pox, and I will examine her further.” Is Dr. Lin committing a fallacy? Explain carefully.

▼ **Show answer**

As a deductive proof, the reasoning would be affirming the consequent, because other illnesses can cause a rash. But Dr. Lin does not claim to prove it. She reasons inductively to a likely explanation (“probably”) and plans to test it further, which is fair reasoning. It would be a fallacy only if she claimed the rash proves chicken pox. Accept an answer that distinguishes the invalid deduction from a reasonable, tentative inference.

11. Hard “Hannah will get the blue ribbon if and only if she finishes first. She did not finish first. So she will not get the blue ribbon.” Is this denying the antecedent? Is it valid?

▼ **Show answer**

It is not the fallacy, and it is valid. The premise is a biconditional, $R \equiv F$, which includes $R \supset F$. From $R \supset F$ and $\sim F$, modus tollens gives $\sim R$. (By a truth table: only the row R false, F false makes both premises true, and there the conclusion $\sim R$ is true.)

12. Challenge The camp rule is: “A camper may take a canoe out only if he has passed the swim test.” Symbolize the rule, then judge each argument valid or invalid and name its form. (a) Caleb has passed the swim test. So he may take a canoe out. (b) Caleb has not passed the swim test. So he may not take a canoe out. (c) Caleb may take a canoe out. So he has passed the swim test.

▼ **Show answer**

Let C = “Caleb may take a canoe out” and S = “Caleb has passed the swim test.” “Only if” introduces the consequent, so the rule is $C \supset S$. (a) $C \supset S$, S / $\therefore C$: affirming the consequent, invalid (the test is necessary, not sufficient; he may also need a life jacket or a counselor’s permission). (b) $C \supset S$, $\sim S$ / $\therefore \sim C$: modus tollens, valid. (c) $C \supset S$, C / $\therefore S$: modus ponens, valid.

COMMON MISTAKES

- Thinking an argument is valid because its conclusion happens to be true. A wet street does not prove rain, even on a night when it did rain.

- Confusing denying the antecedent with modus tollens. Modus tollens denies the consequent; the fallacy denies the antecedent.
- Missing the fallacy when "only if" reverses the conditional. Rewrite "p only if q" as p ⊃ q before judging.
- Calling an argument fallacious when its premise really is a biconditional. With "if and only if," both directions hold.
- Concluding that a fallacious argument's conclusion is false. A bad argument fails to prove its conclusion; it does not disprove it.

> **Scripture.** "believe not every spirit, but try the spirits whether they are of God" (1 John 4:1, KJV)
>
> John tells believers to test what they hear rather than accept it because it sounds right. A counterfeit argument, like a counterfeit coin, can look almost exactly like the real thing; only testing tells them apart.

UNIT 6 · PROPOSITIONAL (SYMBOLIC) LOGIC · LESSON 6.8 OF 69 · 45 MINUTES

6.8 Short Proofs

OBJECTIVES

- State the eight rules of inference (MP, MT, HS, DS, CD, Simp, Conj, Add) and the replacement rules DN and DeM.
- Write a proof in numbered lines, justifying each new line with a rule and the numbers of the lines it uses.
- Spot errors in a proof, especially a rule applied to part of a line.
- Plan a proof by working backward from the conclusion.

KEY TERMS

Proof

A numbered sequence of statements, beginning with the premises, in which each new line follows from earlier lines by a rule, and the last line is the conclusion.

Rule of inference

A valid argument form used to add a new line to a proof. It applies only to whole lines.

Simplification (Simp)

p • q / ∴ p (or ∴ q). From a conjunction, either conjunct may be written alone.

Conjunction (Conj)

p, q / ∴ p • q. Two lines may be joined with •.

Addition (Add)

p / ∴ p ∨ q. To any line, any statement may be added with ∨.

Replacement rule

A rule that lets you replace a statement, or any part of one, with an equivalent: double negation (DN), ~~p for p, and De Morgan (DeM).

Justification

The note beside a proof line giving the rule used and the line numbers it comes from, such as "1, 3, MP."

CORE IDEAS

Why prove instead of tabulate? A truth table can test any argument, but it grows fast. Five letters need 32 rows, and ten letters need 1,024. A table also tells you only *that* an argument is valid, not *why*. A **proof** shows why. It moves from the premises to the conclusion in small steps, and each step uses a valid form you already trust. Since each step is valid, the whole chain is valid: if the premises are true, every line that follows from them is true, and so is the last.

The eight rules. Here are the rules of inference for this course. The first five you know from Lesson 6.6.

Rule	Abbreviation	Form
Modus ponens	MP	p ⊃ q, p / ∴ q
Modus tollens	MT	p ⊃ q, ~q / ∴ ~p
Hypothetical syllogism	HS	p ⊃ q, q ⊃ r / ∴ p ⊃ r
Disjunctive syllogism	DS	p ∨ q, ~p / ∴ q
Constructive dilemma	CD	(p ⊃ q) • (r ⊃ s), p ∨ r / ∴ q ∨ s
Simplification	Simp	p • q / ∴ p (or q)
Conjunction	Conj	p, q / ∴ p • q
Addition	Add	p / ∴ p ∨ q

In DS, either disjunct may be the one denied. In Simp, either conjunct may be the one kept.

How a proof is laid out. Number the premises 1, 2, 3 and so on, and write the conclusion to be proved after a slash on the last premise line: "/ ∴ R." Each new line gets the next number, a statement, and a **justification**: the line numbers it comes from and the rule used. Here is a short proof.
1. P ⊃ Q (premise)
2. Q ⊃ R (premise)
3. P (premise) / ∴ R
4. Q (1, 3, MP)
5. R (2, 4, MP)
Line 4 comes from lines 1 and 3 by modus ponens; line 5 from lines 2 and 4 the same way. When the conclusion appears on a line of its own, the proof is done. Another route would work too (HS on lines 1 and 2, then MP); a proof is correct if every line is justified, not only if it is the shortest.

Three small rules. Simplification, conjunction and addition look almost too simple, but proofs need them. **Simplification** takes a conjunction apart: if "The barn is red and the silo is white" is true, so is "The silo is white." **Conjunction** puts two lines together: if each is true, so is the pair. **Addition** surprises students most. From "The barn is red," you may write "The barn is red or the moon is cheese." That is valid, because a disjunction needs only one true disjunct. Addition is useful when a premise such as (A ∨ B) ⊃ C waits for its antecedent: from A, addition gives A ∨ B, and then modus ponens gives C.

Rules of inference work on whole lines. A rule of inference applies only when the *main connective* of the line fits the rule. From (A • B) ⊃ C and A, you may not conclude C: the antecedent is A • B, and you have only A. From (A • B) ⊃ C you may not "simplify" to A ⊃ C either, because the line is a conditional, not a conjunction. These are not just technicalities. If A is true, B false and C false, then (A • B) ⊃ C and A are both true and C is false, so the step would lead from truth to falsehood. Before using a rule, find the main connective of each line you are citing.

Two replacement rules. Two equivalences from Lesson 6.4 are also allowed in proofs. **Double negation** (DN) lets you write p for ~~p, or ~~p for p. **De Morgan** (DeM) lets you write ~p ∨ ~q for ~(p • q), ~p • ~q for ~(p ∨ q), and the reverse of each.

Because these replace a statement with an equivalent one, they may be used on a whole line or on any part of a line. For example, from ~~A ⊃ B, DN gives A ⊃ B. Rules of inference, by contrast, never work on parts.

Strategy: work backward, then forward. Proofs reward a plan. Start with the conclusion and ask where its letters appear in the premises. Suppose you must prove M • ~J. You will probably finish with conjunction, so you need M and ~J on separate lines. Where can ~J come from? If a premise says J ⊃ something, and you can deny that something, modus tollens will give ~J. Where can M come from? If a premise says J ∨ M, disjunctive syllogism will give M once you have ~J. Now work forward, writing each line with its justification. Look first for the simple moves: a conjunction to simplify, a conditional whose antecedent you already have, a disjunction with one part denied.

Checking a proof. A proof is only as good as its weakest line, so check each one. Does the rule named really have the form of the lines cited? Are the line numbers right? Was a rule of inference used on part of a line? Was disjunctive syllogism used with a disjunct affirmed instead of denied, or modus tollens with the antecedent denied instead of the consequent? Those two errors are the formal fallacies of Lesson 6.7 hiding inside a proof. A proof that passes this check shows beyond doubt that the conclusion follows from the premises.

> **From history.** Proving a conclusion step by step from accepted starting points is as old as Euclid's *Elements* (about 300 B.C.). There each proposition is proved from definitions, postulates and earlier propositions, and every step is given its reason. Proofs in symbolic logic follow the same plan. Gottlob Frege, in 1879, and later Whitehead and Russell built proof systems that were exact but used few rules and many axioms, and their proofs were long and hard to follow. The style used in this lesson is called *natural deduction*, because its rules imitate the natural steps of ordinary reasoning. It was worked out in 1934 by two logicians working separately: Gerhard Gentzen in Germany and Stanisław Jaśkowski in Poland.

> **Practice habit.** Before you write a proof line, name the rule and point to the lines it comes from.

Example 1. Write a proof.

1. (A ∨ B) • C
2. ~A / ∴ B • C

▼ **Show solution**

1. Plan backward: the conclusion B • C is a conjunction, so get B and C on separate lines and join them with Conj. C comes from line 1 by Simp. B could come by DS from A ∨ B and ~A, and A ∨ B also comes from line 1 by Simp.
2. 3. A ∨ B (1, Simp). Line 1 is a conjunction; its first conjunct is A ∨ B.
3. 4. B (3, 2, DS). Line 3 is a disjunction and line 2 denies its first disjunct.
4. 5. C (1, Simp). The second conjunct of line 1.
5. 6. B • C (4, 5, Conj).

Answer: 1. (A ∨ B) • C (premise)
2. ~A (premise) / ∴ B • C
3. A ∨ B (1, Simp)
4. B (3, 2, DS)
5. C (1, Simp)
6. B • C (4, 5, Conj)

Example 2. Write a proof.

1. J ⊃ K
2. K ⊃ L
3. ~L
4. J ∨ M / ∴ M • ~J

▼ **Show solution**

1. Plan backward: finish with Conj, so you need M and ~J. M can come from line 4 by DS once you have ~J. ~J can come by MT from a conditional J ⊃ L and ~L. And J ⊃ L comes from lines 1 and 2 by HS.
2. 5. J ⊃ L (1, 2, HS).
3. 6. ~J (5, 3, MT). Line 5 is a conditional, and line 3 denies its consequent.
4. 7. M (4, 6, DS). Line 4 is a disjunction, and line 6 denies its first disjunct.
5. 8. M • ~J (7, 6, Conj).

Answer: 1. J ⊃ K (premise)
2. K ⊃ L (premise)
3. ~L (premise)
4. J ∨ M (premise) / ∴ M • ~J
5. J ⊃ L (1, 2, HS)
6. ~J (5, 3, MT)
7. M (4, 6, DS)
8. M • ~J (7, 6, Conj)

(Another correct route: from 2 and 3, MT gives ~K; from 1 and ~K, MT gives ~J; then DS and Conj as above.)

PRACTICE

1. Easy Name the rule of inference used in each step: (a) A • B / ∴ A (b) A, B / ∴ A • B (c) A / ∴ A ∨ C (d) A ⊃ B, B ⊃ C / ∴ A ⊃ C

   ▼ **Show answer**

   (a) Simplification (Simp). (b) Conjunction (Conj). (c) Addition (Add). (d) Hypothetical syllogism (HS).

2. **Easy** Give the justification for line 3.

   1. P ⊃ Q
   2. ~Q / ∴ ~P
   3. ~P

   ▼ Show answer

   1, 2, MT (modus tollens: line 2 denies the consequent of line 1).

3. **Easy** Complete the proof.

   1. R ∨ S
   2. ~R / ∴ S

   ▼ Show answer

   3. S (1, 2, DS).

4. **Easy** Write a proof.

   1. A • B / ∴ B ∨ C

   ▼ Show answer

   2. B (1, Simp)
   3. B ∨ C (2, Add)

5. **Medium** Write a proof.

   1. P ⊃ Q
   2. Q ⊃ R
   3. ~R / ∴ ~P

   ▼ Show answer

   4. P ⊃ R (1, 2, HS)
   5. ~P (4, 3, MT)

   (Also correct: 4. ~Q (2, 3, MT); 5. ~P (1, 4, MT).)

6. **Medium** Write a proof.

   1. (A ∨ B) ⊃ C
   2. A / ∴ C

   ▼ Show answer

   3. A ∨ B (2, Add)
   4. C (1, 3, MP)

7. **Medium** Write a proof.

   1. D • E
   2. D ⊃ F / ∴ F • E

   ▼ **Show answer**

   3. D (1, Simp)
   4. F (2, 3, MP)
   5. E (1, Simp)
   6. F • E (4, 5, Conj)

8. **Medium** Find the error in this "proof," and explain why the step is not allowed.

   1. (A • B) ⊃ C
   2. A / ∴ C
   3. C (1, 2, MP)

   ▼ **Show answer**

   Line 3 is not justified. Modus ponens needs the whole antecedent of line 1, which is A • B, but line 2 gives only A. The step is not just unjustified but invalid: if A is true, B false and C false, both premises are true and C is false. (So no correct proof of C from these premises exists.)

9. **Hard** Write a proof.

   1. ~A ⊃ B
   2. ~B
   3. A ⊃ C / ∴ C

   ▼ **Show answer**

   4. ~~A (1, 2, MT)
   5. A (4, DN)
   6. C (3, 5, MP)

   (Accept a proof that goes from line 4 straight to MP with a note that ~~A is A, but the careful form uses DN.)

10. Hard Write a proof.

1. (H ⊃ L) • (C ⊃ M)
2. H ∨ C
3. ~L / ∴ M

▼ **Show answer**

4. L ∨ M (1, 2, CD)
5. M (4, 3, DS)

11. Hard Symbolize and prove: “If the gate is open, the goats get out. If the goats get out, they eat the roses. Either the gate is open or the fence is broken. The fence is not broken. So the goats eat the roses.” (Use G, O, R, F.)

▼ **Show answer**

Let G = “The gate is open,” O = “The goats get out,” R = “The goats eat the roses,” F = “The fence is broken.”

1. G ⊃ O
2. O ⊃ R
3. G ∨ F
4. ~F / ∴ R
5. G (3, 4, DS)
6. O (1, 5, MP)
7. R (2, 6, MP)

(Also correct: 5. G ⊃ R (1, 2, HS); 6. G (3, 4, DS); 7. R (5, 6, MP).)

12. Challenge Write a proof, using De Morgan and double negation where needed.

1. ~(A • B)
2. A
3. B ∨ C / ∴ C

▼ **Show answer**

4. ~A ∨ ~B (1, DeM)
5. ~~A (2, DN)
6. ~B (4, 5, DS)
7. C (3, 6, DS)

(Line 6: line 4 is a disjunction whose first disjunct is ~A, and line 5 denies it.)

COMMON MISTAKES

- Applying a rule of inference to part of a line, such as using A with (A • B) ⊃ C for modus ponens.
- Leaving out the line numbers or citing the wrong lines in a justification.
- Hiding a formal fallacy inside a proof: DS with a disjunct affirmed, or MT with the antecedent denied.
- Using Simp on a disjunction. From A ∨ B you cannot write A; only a conjunction can be simplified.

> **Scripture.** “Ponder the path of thy feet, and let all thy ways be established.” (Proverbs 4:26, KJV)
>
> A proof ponders every step. Each line is made sure before the next is taken, so that the end of the path is as firmly established as its beginning.

UNIT 6 · PROPOSITIONAL (SYMBOLIC) LOGIC · LESSON 6.9 OF 69 · 40 MINUTES

6.9 Logic and Machines

OBJECTIVES

- Explain how Boole’s algebra treats true and false as 1 and 0, with AND behaving like multiplication.
- Relate switches in series and in parallel to AND and OR, and describe what Shannon showed in 1937.
- Work out the output of a circuit of AND, OR and NOT gates for given inputs, and write its table.
- Describe the contributions of Leibniz, Boole and Shannon in their order.

WATCH

Understanding Logic Gates

Spanning Tree · 7:27

https://www.youtube.com/watch?v=INEtYZqtjTo

> **WHEN TO WATCH**
> After the history note: see the truth tables you built turned into circuits.

Note: Parent preview: An animated explanation of how transistors act as on/off switches and how NOT, AND, OR, NAND, NOR, XOR and XNOR gates follow truth tables. No people, no opinions; purely technical. Nothing objectionable; transcript checked October 18, 2026.

KEY TERMS

Binary numeral

A way of writing numbers with only two digits, 0 and 1, each place worth twice the place to its right: 101 in binary is 4 + 0 + 1 = 5.

Boolean algebra

An algebra, begun by George Boole, in which letters take only the values 1 (true) and 0 (false), and logical connectives work like arithmetic operations.

Switching circuit

A circuit of switches or relays, each either closed (1, current flows) or open (0, no current). Switches in series act like AND; switches in parallel act like OR.

Logic gate

An electronic device that takes inputs of 1 or 0 and gives an output of 1 or 0 by a fixed rule, such as AND, OR or NOT.

Input and output

The signals going into a gate or circuit (the values of A, B, C) and the signal that comes out (the value of the whole expression).

Relay

An electrically operated switch, used in telephone exchanges and early computing machines: current in one circuit opens or closes a switch in another.

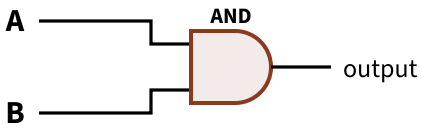
From tables to machines. Look back over this unit and notice something about truth tables: they can be filled in without understanding what the letters mean. Each column follows a fixed rule from the columns before it. Anything that can be done by fixed rules can, in principle, be done by a machine, provided the machine has some way to stand for "true" and "false." This lesson tells how that idea grew, through three thinkers spread over more than two centuries, into the logic gates inside every computer, calculator and microwave oven.

Leibniz and the two digits. The German philosopher and mathematician Gottfried Wilhelm Leibniz showed that every whole number can be written with only two digits, 0 and 1. In binary, each place is worth twice the place to its right: 1, 2, 4, 8 and so on. So 101 means 4 + 0 + 1, which is 5, and 110 means 4 + 2 + 0, which is 6. Leibniz published an account of this arithmetic in 1703. He also dreamed of a precise language of reasoning in which disputes could be settled by calculation. He did not live to see either dream put to work, but both pointed toward the same goal: thought carried out by exact rules.

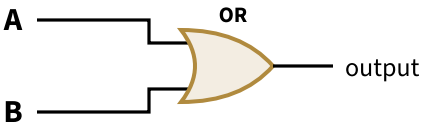
Boole's algebra of logic. George Boole, an English mathematician, made logic into a kind of algebra. In his books of 1847 and 1854, letters stand for classes or statements, and the numbers 1 and 0 play special roles. In the form engineers use today, 1 stands for true and 0 for false. Then AND behaves like multiplication: $1 \times 1 = 1$, but 1×0, 0×1 and 0×0 are all 0, exactly the table for •. NOT p behaves like $1 - p$: it turns 1 into 0 and 0 into 1. OR gives 1 when at least one input is 1. Boole noticed a law that ordinary algebra lacks: for his letters, $x \times x = x$, since $1 \times 1 = 1$ and $0 \times 0 = 0$. Saying "It is raining and it is raining" says no more than "It is raining."

Switches in series and in parallel. A switch is either closed, letting current through (call it 1), or open, stopping it (0). Put two switches in a row, one after the other: this is called *series*. The lamp lights only if both are closed, so the circuit acts like AND. Put two switches side by side, each on its own path to the lamp: this is called *parallel*. The lamp lights if either one is closed, or both, so the circuit acts like OR (the inclusive "or" of this unit). In 1937 Claude Shannon, a student at the Massachusetts Institute of Technology, showed in his master's thesis that Boole's algebra could describe any circuit of switches and relays, and could be used to design and simplify such circuits. The logic of the Stoics and of Boole had become a tool for engineers.

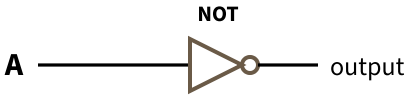
The three basic gates. In modern electronics the switches are tiny, but the rules are the same. A **logic gate** takes inputs of 1 or 0 and gives an output by a fixed rule. An AND gate gives 1 only when both inputs are 1:



An OR gate gives 1 when at least one input is 1:

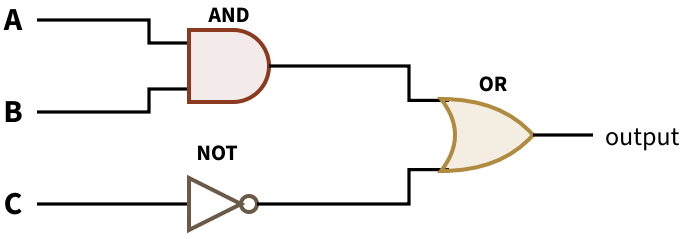


A NOT gate has one input and flips it:



Their tables are the tables of •, ∨ and ~ from Lesson 6.2, with 1 written for T and 0 for F.

Circuits of gates. Gates can be wired together, the output of one becoming the input of the next, just as connectives combine into compound statements. The circuit below computes (A AND B) OR (NOT C), which in our symbols is (A • B) ∨ ~C.



To find its output, work from the inputs forward, as you worked inside out in Lesson 6.2. If A = 1, B = 0 and C = 1, the AND gate gives 0, the NOT gate gives 0, and the OR gate gives 0. If C is changed to 0, the NOT gate gives 1, and the output becomes 1.

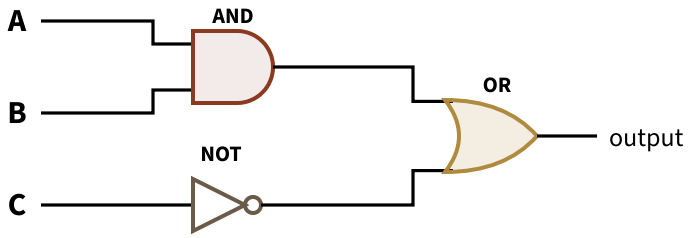
Equivalent circuits, fewer parts. Because a circuit is a compound statement, the equivalences of Lesson 6.4 apply to it. By De Morgan's laws, NOT (A OR B) gives the same output as (NOT A) AND (NOT B) for every input, so an engineer may build whichever uses fewer or cheaper parts. This was one of Shannon's great points: an engineer can write a circuit as an expression, simplify the expression by the laws of logic, and then build the simpler circuit, sure that it will behave exactly like the original.

Machines that follow rules. Gates can even do arithmetic. Adding two one-digit binary numbers needs only two outputs: a sum digit that is 1 when exactly one input is 1, and a carry digit that is 1 when both are (in binary, 1 + 1 = 10). Each can be built from AND, OR and NOT. Join millions of such circuits and you have a computer. Yet a computer does not reason; it carries out rules that people discovered and built into it. The laws of logic were not invented by the engineers. They were found, as Boole believed, in the order of the mind, and the Christian sees that order as the gift of the God who made both mind and world.

> **From history.** Three thinkers, spread over more than two centuries, connected logic to machines. Gottfried Wilhelm Leibniz (1646–1716) worked out arithmetic with only 0 and 1 and published an account of it in 1703. George Boole (1815–1864), a largely self-taught English schoolmaster who became professor of mathematics at Queen's College, Cork, in Ireland, published *The Mathematical Analysis of Logic* in 1847 and *An Investigation of the Laws of Thought* in 1854, treating logic as an algebra of 1 and 0. Claude Shannon (1916–2001), a student at the Massachusetts Institute of Technology, showed in his 1937 master's thesis, *A Symbolic Analysis of Relay and Switching Circuits*, that Boole's algebra could describe and simplify electrical switching circuits. Every digital machine since rests on that link.

> **Practice habit.** When you flip a light switch or press a button today, picture the 1s and 0s behind it, and remember that they follow the same laws you used in this unit.

Example 1. For the circuit (A AND B) OR (NOT C), find the output for each set of inputs: (a) A = 1, B = 0, C = 0 (b) A = 1, B = 1, C = 1 (c) A = 0, B = 1, C = 1.

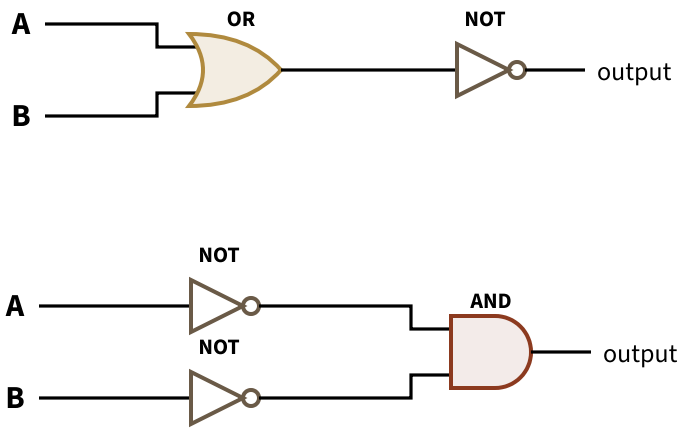


▼ **Show solution**

1. (a) AND gate: A = 1, B = 0, so 1 AND 0 = 0. NOT gate: C = 0, so NOT 0 = 1. OR gate: 0 OR 1 = 1.
2. (b) AND gate: 1 AND 1 = 1. NOT gate: NOT 1 = 0. OR gate: 1 OR 0 = 1.
3. (c) AND gate: 0 AND 1 = 0. NOT gate: NOT 1 = 0. OR gate: 0 OR 0 = 0.
4. Check with symbols: (c) is (F • T) ∨ ~T, which is F ∨ F, false. The circuit and the truth table agree.

Answer: (a) 1. (b) 1. (c) 0.

Example 2. Show that these two circuits give the same output for every input, and name the law that explains it.



▼ **Show solution**

1. Make a table with 1 for true and 0 for false, in the standard order.

A	B	A OR B	NOT (A OR B)	NOT A	NOT B	(NOT A) AND (NOT B)
1	1	1	0	0	0	0
1	0	1	0	0	1	0
0	1	1	0	1	0	0
0	0	0	1	1	1	1

2. The two output columns are both 0, 0, 0, 1, so the circuits are equivalent.
3. In symbols, the first is ~(A ∨ B) and the second is ~A • ~B. Their equivalence is De Morgan's second law.

Answer: Both circuits output 0, 0, 0, 1 for the four inputs, so they are equivalent, by De Morgan's law ~(A ∨ B) ≡ ~A • ~B. The first uses two gates and the second three, so the first is simpler to build.

PRACTICE

1. Easy In Boole's algebra as engineers use it, what do 1 and 0 stand for? Which arithmetic operation does AND behave like?

   ▼ **Show answer**

   1 stands for true and 0 for false. AND behaves like multiplication: the product is 1 only when both factors are 1.

2. Easy Name the gate: (a) its output is 1 only when both inputs are 1; (b) its output is 1 when at least one input is 1; (c) it has one input and gives the opposite value.

   ▼ **Show answer**

   (a) AND. (b) OR. (c) NOT.

3. Easy Two switches control a lamp. Which connective does the circuit act like if the switches are (a) in series, (b) in parallel?

   ▼ **Show answer**

   (a) In series, the lamp lights only when both are closed: AND (•). (b) In parallel, it lights when either or both are closed: OR (∨).

4. Easy Write the numbers 5 and 6 in binary. Who published an account of binary arithmetic in 1703?

   ▼ **Show answer**

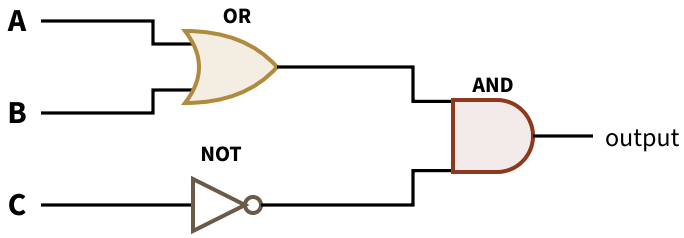
   5 is 101 (4 + 0 + 1) and 6 is 110 (4 + 2 + 0). Gottfried Wilhelm Leibniz.

5. Medium Write the tables for A AND B and for A OR B, using 1 and 0, for all four pairs of inputs in the standard order.

   ▼ **Show answer**

A	B	A AND B	A OR B
1	1	1	1
1	0	0	1
0	1	0	1
0	0	0	0

6. **Medium** Find the output of this circuit for (a) A = 0, B = 1, C = 0 and (b) A = 1, B = 1, C = 1.



▼ **Show answer**

(a) A OR B = 0 OR 1 = 1; NOT C = NOT 0 = 1; 1 AND 1 = 1. Output 1. (b) A OR B = 1; NOT C = NOT 1 = 0; 1 AND 0 = 0. Output 0.

7. **Medium** In Boole's algebra, (a) find $x \times y$ when $x = 1$ and $y = 0$; (b) find $1 - x$ when $x = 1$; (c) explain why $x \times x = x$ for every value of x, and what this law says in plain words.

▼ **Show answer**

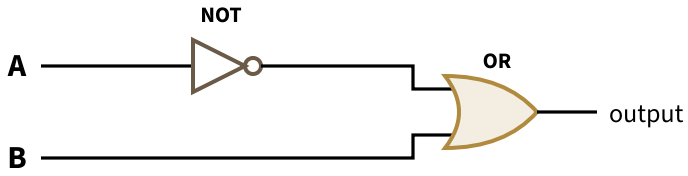
(a) 0, which matches "true and false" being false. (b) 0, which matches "not true" being false. (c) x can be only 1 or 0, and $1 \times 1 = 1$, $0 \times 0 = 0$. In words: saying a statement twice with "and" ("It is raining and it is raining") says no more than saying it once.

8. **Medium** A lamp lights when switch A is closed and either switch B or switch C is closed. Write this as a gate expression and in the symbols of this unit. How many gates does it need?

▼ **Show answer**

A AND (B OR C); in symbols $A \cdot (B \vee C)$. It needs two gates: an OR gate for B and C, and an AND gate joining A with that output. (In switches: A in series with B and C, which are in parallel with each other.)

9. Hard Write the full table for this circuit. Which connective from this unit has the same table?



▼ **Show answer**

A	B	NOT A	(NOT A) OR B
1	1	0	1
1	0	0	0
0	1	1	1
0	0	1	1

The output is 1, 0, 1, 1, the table of the conditional A ⊃ B. (This is the equivalence of p ⊃ q and ~p ∨ q from Lesson 6.4.)

10. Hard An engineer must build NOT (A AND B), but she has only NOT gates and OR gates. Using De Morgan's law, write an expression she can build, and check it for the inputs A = 1, B = 0.

▼ **Show answer**

(NOT A) OR (NOT B), from ~(A • B) ≡ ~A ∨ ~B. Check with A = 1, B = 0: NOT (1 AND 0) = NOT 0 = 1; (NOT 1) OR (NOT 0) = 0 OR 1 = 1. Both give 1. (A full check of all four rows also matches: 0, 1, 1, 1.)

11. Hard In three or four sentences, explain what Claude Shannon showed in 1937, and why the earlier work of Boole made it possible.

▼ **Show answer**

Sample answer: In his 1937 master's thesis, Shannon showed that circuits of switches and relays could be described by Boole's algebra, with closed and open switches standing for 1 and 0, series for AND and parallel for OR. This let engineers write a circuit as an expression, simplify it by the laws of logic, and build the simpler circuit. It was possible because Boole, in 1847 and 1854, had already turned logic into an exact algebra of two values. Shannon saw that the two values of logic and the two states of a switch obey the same rules.

12. Challenge When two one-digit binary numbers A and B are added, the sum digit is 1 when exactly one of them is 1, and the carry digit is 1 when both are 1 (1 + 1 = 10 in binary). (a) Write a gate expression for the sum digit using AND, OR and NOT, and give its full table. (b) Write a gate expression for the carry digit. (c) Use your answers to add 1 + 1.

▼ **Show answer**

(a) (A OR B) AND (NOT (A AND B)): "one or the other, but not both."

A	B	A OR B	A AND B	NOT (A AND B)	Sum
1	1	1	1	0	0
1	0	1	0	1	1
0	1	1	0	1	1
0	0	0	0	1	0

Accept any equivalent expression, such as (A AND (NOT B)) OR ((NOT A) AND B). (b) A AND B. (c) With A = 1 and B = 1, the sum digit is 0 and the carry digit is 1, so the answer is written 10, which is binary for two.

COMMON MISTAKES

- Confusing series with parallel. Switches in series act like AND; switches in parallel act like OR.
- Thinking an OR gate gives 0 when both inputs are 1. Like `∨`, it is inclusive.
- Reading the binary numeral 10 as "ten." In binary, 10 means one two and no ones: the number two.

- Thinking that a computer reasons. It carries out, very fast, rules of logic that people discovered and built into it.

> **Scripture.** "It is the glory of God to conceal a thing: but the honour of kings is to search out a matter." (Proverbs 25:2, KJV)
>
> The laws of logic were there long before anyone wrote them as algebra or wired them into a circuit. Leibniz, Boole and Shannon had the honor of searching out what God had set in the order of things.

UNIT 7

Induction and Scientific Reasoning

Most of what we know about the world was not proved by deduction. We know that seeds sprout in warm soil, that a certain medicine eases a headache, that a bridge will hold a loaded truck and that Caesar crossed the Rubicon, but none of these follows *necessarily* from premises we already hold. They follow *probably*, from samples, comparisons, experiments and witnesses. This unit studies that kind of reasoning, **induction**, and the way careful people, and especially natural scientists, use it to reach conclusions they can trust without claiming more certainty than the evidence gives.

Students learn to judge a generalization by the size, variety and fairness of its sample, and to test an **argument from analogy** by its relevant likenesses and differences. They practice **Mill's methods** for finding causes (agreement, difference, the joint method, residues and concomitant variation), and they learn how a **hypothesis** is tested by its predictions, together with Francis Bacon's four "idols" that lead the mind astray. They use simple probability to avoid two classic errors about chance, weigh **testimony** by the tests of competence, honesty, consistency and independence, and compare rival explanations by fit, scope and simplicity.

The unit ends by asking what science can and cannot do. Some claims can be observed and repeated in the laboratory or the field; others are inferences about an unobserved past, which rest on traces, testimony and assumptions that cannot be rerun. All of science rests on the confidence that nature is orderly and that the future will resemble the past in its laws, a confidence that induction itself cannot prove but that a Christian receives from the faithfulness of the Creator ("seedtime

and harvest, and cold and heat, and summer and winter, and day and night shall not cease"). Unit 8 builds directly on this unit by studying how statistics can be used to deceive.

Review day and test. On the review day, use the mastery checklist and the review sheet (with its answer key). The unit test is below; a printable copy is ready for test day.

PREREQUISITES

- Tell a deductive argument from an inductive one, and use the words valid, sound, strong and cogent correctly (Unit 1).
- Recognize the informal fallacies of hasty generalization, false cause, weak analogy and appeal to authority (Unit 3).
- Recognize modus ponens, modus tollens and the fallacy of affirming the consequent (Units 5 and 6).
- Work with fractions: simplify them, compare them, and multiply two of them (Pre-Algebra).
- Read a short passage and list the facts it reports separately from the conclusions it draws.

MASTERY CHECKLIST

- ☐ I can judge an inductive generalization by the size, variety and fairness of its sample and by how much its conclusion claims.
- ☐ I can test an argument from analogy by its relevant likenesses, its relevant differences and the number of cases compared, and answer a weak analogy with a counter-analogy.
- ☐ I can name and apply Mill's five methods (agreement, difference, joint method, residues, concomitant variation) and state a limit of each.
- ☐ I can describe how a hypothesis is tested by its predictions, explain why a failed prediction counts against it more decisively than a passed one counts for it, and design a fair controlled test.
- ☐ I can name Bacon's four idols (tribe, cave, marketplace, theatre) and give an example of each.
- ☐ I can find simple probabilities as fractions, and explain the fallacy of the maturity of chances and the conjunction error.
- ☐ I can weigh a witness or a source by competence, honesty, consistency and independence, and tell a primary source from a secondary one.
- ☐ I can compare rival explanations by fit, scope, simplicity and

agreement with what is already well known.

- ☐ I can tell a claim that is observed and repeatable from an inference about the unobserved past, and explain why science assumes an orderly creation.

LESSONS IN THIS UNIT

7.1	**Strong and Weak Inductive Arguments**	40 minutes
7.2	**Argument from Analogy**	40 minutes
7.3	**Finding Causes: Mill's Methods**	45 minutes
7.4	**Hypotheses and Testing**	45 minutes
7.5	**Reasoning about Chance**	45 minutes
7.6	**Testimony and Evidence**	40 minutes
7.7	**Best Explanation, and the Reach of Science**	45 minutes

Unit 7 test

You have 45 minutes. No notes or books; you may use scratch paper for the probability questions. Answer each question in complete sentences unless it asks for a number, a list or a single word. For classification questions, name the category and give a one-sentence reason. Give probabilities as fractions in lowest terms. The test is worth 100 points.

1. **6 pts**

   To learn how students get to Hillcrest School, Ellie asked the 15 students waiting at the bus stop one morning. All 15 said they ride the bus. She concluded, “Every student at Hillcrest rides the bus.” (a) Name the sample, the population and the property. (b) Is the argument strong or weak? Give two reasons.

2. **5 pts**

   A fair sample of 150 eggs, taken from every day of one week’s collection at Brookside Farm, contained 6 cracked eggs. Rank these conclusions from strongest to weakest and explain briefly: (a) About 4 percent of the farm’s eggs that week were cracked. (b) Some of the farm’s eggs that week were cracked. (c) Exactly 4 eggs in every 100 were cracked on every day of that week.

3. **7 pts**

   “My neighbor’s apple tree and mine are the same variety, were planted the same year, grow in the same kind of soil and both get full sun. Her tree gave three bushels of apples this fall. So mine will probably give about three bushels next fall.” (a) Name the primary and secondary analogues. (b) Is the argument strong or weak? Explain using the tests for analogy. (c) Name one fact that would weaken it.

4. **6 pts**

   “Running shoes are like car tires: both wear out with use. Car tires can go about 50,000 miles before they need replacing. So a pair of running shoes can go about 50,000 miles too.” Name the fallacy and explain what is wrong with the analogy.

5. **9 pts**

   Name the Mill’s method used in each case, with a one-sentence reason.

   1. The more salt Mrs. Ortiz adds to the pot, the higher the temperature at which the water boils.
   2. A full rain barrel holds 55 gallons. Mr. Okafor knows that 40 gallons came from last night’s rain and 10 from the roof drain on Tuesday. He looks for the source of the other 5 gallons.
   3. Two identical bird feeders hang side by side, with the same seed, height and shade. One has a small roof over it. Only the feeder without the roof gets moldy seed.

6. **7 pts**

These shrubs wilted: the rose (watered from the rain barrel, mulched, sunny spot), the lilac (rain barrel, not mulched, shady spot) and the hydrangea (rain barrel, mulched, shady spot). These stayed healthy: the peony (watered from the hose, mulched, sunny spot), the hosta (hose, not mulched, shady spot) and the fern (hose, mulched, shady spot). (a) What is the probable cause of the wilting? (b) Which method shows it? (c) Why is the conclusion still only probable?

7. **8 pts**

Caleb's hypothesis: "The kitchen faucet drips because its washer is worn out." His prediction: "If I replace the washer, the dripping will stop." (a) He replaces the washer and the faucet still drips. Write his reasoning, name its form and say what he may conclude. (b) Suppose instead the dripping stops. Write that reasoning, name its form and say what he may and may not conclude.

8. **7 pts**

The makers of Rootwell plant food say it increases the number of tomatoes a plant produces. Design a fair controlled experiment to test the claim. Name the control group, the one circumstance you change, how you will choose which plants get Rootwell, and what you will measure.

9. **6 pts**

Name the idol (Bacon's idols of the tribe, the cave, the marketplace or the theatre) shown in each example.

1. A carpenter who loves his lathe thinks every problem in the house can be solved by turning a new part.
2. Two farmers argue about whether a field is "wet" before realizing that one means "too wet to plow" and the other means "has standing water."
3. People everywhere tend to notice the times a dream "came true" and forget the many times it did not.
4. A student accepts an entire old system of nature because a famous school taught it, without checking it against observation.

10. **8 pts**

A jar holds 4 red marbles and 6 white marbles. (a) One marble is drawn without looking. What is the probability it is red? (b) What is the probability it is not red? (c) A marble is drawn, put back, and the jar shaken; then a second marble is drawn. What is the probability that both draws are red? (d) Two fair dice are rolled in a board game. What is the probability that both show a 1?

11. **6 pts**

(a) Joel's team has lost the coin toss before each of its last six games. He says, "We're due to win the toss this time." What is the probability his team wins the next toss with a fair coin, and what is his error called? (b) Which is more probable, and why? "The bus will be late tomorrow" or "The bus will be late tomorrow and it will be snowing."

12. **7 pts**

A bicycle was taken from outside the library. Weigh these three witnesses, using the tests of competence, honesty and consistency, and say which testimony deserves the most weight: (1) Mr. Gray, the librarian, who saw a boy in a red jacket ride off on it while he was standing at the front window; he does not know the boy. (2) Ryan, who says his friend Sam told him he saw the theft. (3) Ben, the bicycle's owner, who was inside and did not see the theft, but says it must have been a boy he quarreled with last week.

13. **5 pts**

(a) Is a letter written by a farmer during the drought of 1934, describing his own fields, a primary or a secondary source? (b) Four neighbors all report that the old mill pond froze solid last January. You learn that three of them heard it from the fourth, Mrs. Hale, who skated on it. How many independent witnesses are there? Explain.

14. **7 pts**

In the morning the Fieldings find their trash can tipped over, garbage scattered across the yard and small, hand-shaped muddy prints on the lid. Compare two explanations by at least three tests: (A) a raccoon got into the trash during the night; (B) a strong wind blew the can over. Which is better?

15. **6 pts**

Short written answer (four to six sentences): Explain the difference between a claim that is observed and repeatable and an inference about the unobserved past, with one example of each. Then explain why all scientific reasoning assumes that nature is orderly, and what reason a Christian has for that assumption.

ANSWER KEY

1. (a) Sample: the 15 students at the bus stop. Population: all students at Hillcrest School. Property: riding the bus to school. (b) Weak. Any two reasons: the sample is biased, because students waiting at a bus stop are exactly the ones who ride the bus (walkers and students driven by parents had no chance to be asked); it lacks variety (one place, one morning); it is small for a whole school; and the conclusion ("every student") claims far too much. (2 points for (a), 2 points for each reason.)
2. (b), then (a), then (c). (b) is nearly certain, since cracked eggs were actually found. (a) matches the sample rate (6 out of 150 is 4 percent) and claims only "about." (c) claims an exact rate on every single day, which the sample cannot show. The more a conclusion claims, the weaker the argument.
3. (a) Primary analogue: the neighbor's tree. Secondary analogue: my tree. (b) Fairly strong: the likenesses (variety, age, soil, sun) are all relevant to how much fruit a tree bears, there are several of them, and the conclusion is modest ("probably," "about"). A student may note that it compares this fall with next fall; accept "moderately strong" if that is the reason given. (c) Any relevant difference, such as: my tree was damaged by a late frost, has a disease or pests, was pruned hard, or gets less water; or apple trees often bear heavily one year and lightly the next. (2 points for (a), 3 for (b), 2 for (c).)
4. Weak (false) analogy. The one likeness, that both wear out with use, does not support the specific conclusion about how many miles they last. The relevant differences are large: tires are made of thick rubber built to roll, while shoes are made of light foam and fabric that are struck by a runner's whole weight at every step. Nothing in the likeness gives a reason to expect the same mileage. (Accept any relevant difference.)
5. 1: concomitant variation; the boiling temperature rises as the amount of salt rises. 2: residues; he subtracts the part of the total explained by known causes (40 + 10 = 50 gallons) and looks for the cause of the 5 gallons left over. 3: difference; the two feeders are alike in every listed circumstance except the roof, and the moldy seed appears only where the roof is absent, so rain reaching the seed is probably the cause. (3 points each.)
6. (a) Watering from the rain barrel. (b) The joint method of agreement and difference: every wilted shrub got rain-barrel water, and no healthy shrub did, while mulch and sun or shade appear among both wilted and healthy shrubs. (c) A circumstance that was not listed might be shared by the wilted shrubs (for

example, they might all be in the same bed or planted the same week), so the methods give probable, not certain, conclusions. A further test, such as checking or changing the barrel water, would help. (3 points for (a) with (b), 2 for the reason in (b), 2 for (c).)

7. (a) “If the worn washer caused the drip, replacing it would stop the drip. It did not stop. So the worn washer was not the cause.” This is modus tollens, a valid form; he may conclude that the washer was not the cause (or not the only cause), provided his background assumptions hold, for example that he installed the new washer correctly. (b) “If the worn washer caused the drip, replacing it would stop the drip. It stopped. So the worn washer was the cause.” This is affirming the consequent, which is invalid as a deduction. The result confirms the hypothesis and makes it much more probable, but it does not strictly prove it; something else he did while taking the faucet apart (such as tightening a loose part) might have stopped the drip. (4 points each.)
8. Sample answer: Take 20 tomato plants of the same variety and size. Assign them at random (for example, by drawing numbers) to two groups of 10. The experimental group gets Rootwell; the control group gets plain water on the same schedule. Everything else is kept the same: soil, pot size, sun, amount of water. At the end of the season count (or weigh) the tomatoes from each plant, ideally by someone who does not know which plants got Rootwell, and compare the groups; repeat if possible. Grading: control group (2), one changed circumstance with others held the same (2), random assignment (2), a clear measure (1). Accept any equivalent design.
9. 1: cave (one person’s own bent, from his trade and habits). 2: marketplace (confusion caused by a loose word used in two senses). 3: tribe (a tendency common to all human beings). 4: theatre (a received system accepted on authority without testing). (1½ points each.)
10. (a) 4/10 = 2/5. (b) 1 − 2/5 = 3/5 (6 of the 10 marbles are white). (c) The draws are independent because the marble is put back: 2/5 × 2/5 = 4/25. (d) 1/6 × 1/6 = 1/36. (2 points each.)
11. (a) 1/2: the tosses are independent and the coin has no memory. His error is the fallacy of the maturity of chances (the “it’s due” fallacy). (b) “The bus will be late tomorrow” is more probable (or at least as probable): every case in which the bus is late and it snows is also a case in which the bus is late, so adding “and” can never make a claim more probable. Choosing the second is the conjunction error. (3 points each.)

12. Mr. Gray deserves the most weight: he was present and watching (competent), has no apparent interest in blaming anyone (honest), and reports only what he saw. Ryan's report is hearsay: he did not see the theft, and the real witness, Sam, should be asked directly. Ben did not see the theft at all, so he is not competent to say who took the bicycle, and his quarrel gives him a reason to suspect one boy; his claim is a guess, not testimony. Further evidence, such as whether the boy Ben names owns a red jacket, could check the accounts for consistency. (3 points for Mr. Gray with reasons, 2 for Ryan, 2 for Ben.)
13. (a) Primary: it was written at the time by someone in a position to know. (b) One: the other three are repeating Mrs. Hale's report (hearsay), so all four reports rest on her single observation. (2 points for (a), 3 for (b).)
14. (A) is better. Fit: a raccoon explains the tipped can, the scattered garbage and the small hand-shaped prints; wind could tip the can and scatter some garbage but does not explain the prints. Scope: (A) covers every fact, while (B) leaves the prints unexplained. Simplicity and agreement with what is known: raccoons are known to raid trash cans at night and leave hand-shaped prints; (B) would need a separate explanation for the prints. Testability: (A) predicts tracks elsewhere in the mud or a repeat visit, which the Fieldings can check (for example, whether a neighbor heard wind in the night). Any three tests correctly applied earn full credit.
15. Sample answer: An observed and repeatable claim can be tested today and tested again by others, such as "Water at sea level boils at 212 degrees Fahrenheit." An inference about the past concerns an event that happened once and cannot be rerun, such as "The barn burned because of a lightning strike in 1910"; it rests on traces, testimony and assumptions about how things went. All scientific reasoning assumes that nature follows the same laws in unobserved cases as in observed ones, and induction cannot prove this without arguing in a circle. A Christian has a reason for this assumption: a faithful Creator made and upholds an orderly world, as God promised in Genesis 8:22 that seedtime and harvest, cold and heat, and day and night shall not cease. Grading: 2 points for the distinction with examples, 2 for the uniformity of nature and why induction cannot prove it, 2 for the Christian ground.

Printable answer key (PDF, for the parent)

7.1 Strong and Weak Inductive Arguments

OBJECTIVES

- Identify the sample, the population and the property in an inductive generalization.
- Judge a generalization by the size, variety and fairness of its sample.
- Explain why a more modest conclusion makes an inductive argument stronger.
- Recognize a hasty generalization and a biased sample.

KEY TERMS

Inductive generalization

An argument that reasons from what is true of a sample to what is probably true of the whole group the sample came from.

Sample

The members of a group that were actually observed, counted or tested.

Population

The whole group the conclusion is about (also called the target group).

Representative sample

A sample that resembles the population in the ways that matter to the property being studied.

Random sample

A sample chosen so that every member of the population had an equal chance of being picked.

Biased sample

A sample chosen in a way that makes some kinds of members more likely to be included than others, so that it misrepresents the population.

Hasty generalization

The fallacy of drawing a general conclusion from a sample that is too small or too unusual to support it.

CORE IDEAS

Induction reaches beyond the evidence. In Unit 1 you learned that an inductive argument claims only that its conclusion is probable. That is not a weakness to be ashamed of; it is the price of learning something new. A deductive conclusion never

says more than its premises already contain. An inductive conclusion always does: it moves from the cases we have seen to cases we have not. Because it reaches beyond the evidence, an inductive argument is never valid or invalid. It is **strong** or **weak**, and strength comes in degrees. This lesson gives you tests for measuring it.

The parts of a generalization. The most common kind of induction is the **inductive generalization**. Caleb picks 50 apples from his family's orchard and finds 3 with worm holes. He concludes, "About 6 percent of the apples in our orchard are wormy." The apples he actually checked are the **sample**. The whole crop is the **population**. Being wormy is the **property** he is measuring. Every generalization has these three parts, and every test of strength asks how well the sample stands in for the population with respect to that property.

Test one: size. A sample must be large enough. If Caleb had checked only two apples and found one wormy, he could not sensibly conclude that half the orchard is wormy. Drawing a broad conclusion from too few cases is the fallacy of **hasty generalization**. Larger samples are better, but size is not everything. A thousand apples taken from one diseased tree tell you less about the orchard than a hundred taken from every row. Once a sample is fairly chosen, adding more cases helps less and less.

Test two: variety. A good sample includes the different kinds of members the population contains. If the orchard has early and late varieties, trees on the hill and trees by the creek, and apples high in the branches and low near the ground, a strong sample includes all of them. A sample that resembles the population in the ways that matter is called **representative**. How much variety you need depends on how much the population varies. A cook tastes one spoonful of a well-stirred pot of soup and knows how salty the whole pot is, because stirring has made the soup the same throughout.

Test three: fairness. A sample must be chosen in a way that does not favor some members over others. The surest way is a **random sample**, in which every member of the population has an equal chance of being picked, like drawing names from a hat. A **biased sample** is chosen so that some kinds of members are more likely to be included. If Hannah wants to know how many students at her school like to read, she should not ask only the students she finds in the library. People who answer a survey

because they feel strongly about it, or customers a company chose to quote, also make a biased sample.

Test four: how much the conclusion claims. The same evidence supports a modest conclusion more strongly than a bold one. From Caleb's 50 apples, "Most of our apples are free of worms" is very strong; "About 6 percent of our apples are wormy" is fairly strong; "Exactly 3 apples in every 50 on every tree are wormy" is weak. A conclusion about this year's crop is stronger than one about every crop forever. Watch especially for the word "all." A generalization about all members can be overturned by a single counterexample, while "most" or "nearly all" can survive a few exceptions.

Background knowledge matters. Sometimes a very small sample is enough. A chemist who measures the melting point of one piece of pure copper is confident that every piece of pure copper melts at the same temperature, because long experience shows that pure metals are uniform in that respect. But one friendly dog does not show that every dog of its breed is friendly, because we know temperament varies from animal to animal. The tests of size, variety and fairness always work together with what we already know about how much the population varies.

Strong is not the same as cogent. Remember the two questions from Unit 1. Strength asks whether the conclusion would probably be true *if* the premises were true. Cogency also asks whether the premises really are true: whether Caleb counted carefully and reported honestly. And a cogent argument must not ignore evidence against its conclusion. If Caleb knows that the trees by the creek had a bad worm problem last spring and skips them, his argument is not cogent, however many apples he counts. Honest induction looks at all the evidence it can reasonably get, not only the evidence it likes.

From history. For centuries Europeans used the phrase "black swan" for something that did not exist. The Roman poet Juvenal called a rare thing a bird as rare on earth as a black swan, and every swan anyone in Europe had seen was white. In 1697 a Dutch expedition led by Willem de Vlamingh explored the coast of western Australia and saw black swans on a river there, which is still called the Swan River. A generalization that rested on many centuries of observation was overturned in a single afternoon. Logicians ever since have used the swans to show that no number of agreeing cases can make an inductive generalization certain, and that a single counterexample can refute a claim about "all."

Practice habit. When someone says "everyone" or "all," ask quietly: How many did they actually look at, and how were those chosen?

WORKED EXAMPLES

Example 1. Evaluate this argument: "Hannah asked the twelve members of the robotics club whether the school should spend its gift money on new computers. Eleven said yes. So nearly every student at Brookside Middle School wants the money spent on computers."

▼ **Show solution**

1. Identify the parts. Sample: the twelve robotics club members. Population: all students at the school. Property: wanting the gift money spent on computers.
2. Size: twelve students is small for a conclusion about a whole school.
3. Variety: all twelve belong to one club with one shared interest. The sample leaves out students in sports, music, art and every other activity.
4. Fairness: robotics club members are more likely than most students to want computers, so the way the sample was chosen favors one answer. The sample is biased.
5. Claim: "nearly every student" is a bold conclusion.

Answer: Weak. The sample is small, lacks variety and is biased toward students who like computers, and the conclusion claims a great deal. It is a hasty generalization from a biased sample.

Example 2. Mr. Okafor scoops 100 bean seeds from different depths of a 50-pound sack, after mixing the sack well, and plants them. Ninety-two sprout. Rank these conclusions from strongest to weakest: (a) Every seed in the sack will sprout. (b) Most seeds in the sack will sprout. (c) About 9 out of 10 seeds in the sack will sprout.

▼ **Show solution**

1. The sample is fairly large, was taken from different depths after mixing (variety and fairness), and comes from the very sack the conclusion is about. The evidence is good.
2. (b) "Most" only needs more than half to sprout. With 92 out of 100 sprouting in a fair sample, this is very likely.
3. (c) "About 9 out of 10" is more precise, so it claims more than (b), but 92 out of 100 supports it well.
4. (a) "Every seed" is already contradicted by the evidence: 8 of the sample seeds did not sprout.

Answer: (b) is strongest, (c) is next and still strong, and (a) is not merely weak but refuted by the sample itself.

PRACTICE

1. Easy Name the sample, the population and the property: "Mrs. Ortiz tasted three peaches from the bushel basket, and all three were sweet. So the peaches in the basket are probably sweet."

   ▼ **Show answer**

   Sample: the three peaches she tasted. Population: all the peaches in the bushel basket. Property: being sweet.

2. Easy What is a random sample?

   ▼ **Show answer**

   A sample chosen so that every member of the population had an equal chance of being picked (for example, by drawing names from a hat).

3. **Easy** Name the three tests for a sample given in this lesson, and the fourth test that concerns the conclusion.

   ▼ **Show answer**

   The sample tests are size (is it large enough?), variety (does it include the different kinds of members?) and fairness (was it chosen without favoring some members?). The fourth test asks how much the conclusion claims; a more modest conclusion is stronger.

4. **Easy** Can an inductive argument be valid? Explain in one sentence.

   ▼ **Show answer**

   No: an inductive argument claims only that its conclusion is probable, so it is called strong or weak (and cogent or not cogent), never valid or invalid.

5. **Medium** Strong or weak? Give a reason. “I have ridden the school bus twice, and both times it was late. The school bus is always late.”

   ▼ **Show answer**

   Weak: two rides is far too small a sample for a conclusion about every ride, and “always” is a bold claim that one on-time bus would refute. It is a hasty generalization.

6. **Medium** A cereal maker, Sunnyfield Oats, prints letters from five happy customers on the back of the box and says, “Families everywhere love Sunnyfield!” Which test does this sample fail most clearly? Explain.

   ▼ **Show answer**

   Fairness: the company chose which letters to print, so the sample is biased toward happy customers (it also fails size, since five letters is very few). Accept either fairness or size as the main failure if the reason is given; the best answer mentions that the letters were hand-picked.

7. Medium Why can a cook judge the saltiness of a whole pot of soup from one spoonful, while a farmer cannot judge a whole field of wheat from one stalk?

▼ **Show answer**

The soup has been stirred, so it is the same throughout and one small sample represents it. A field of wheat varies from place to place (soil, water, pests, shade), so a single stalk cannot represent the whole field; the farmer needs a larger and more varied sample.

8. Medium Rank from strongest to weakest, using the same evidence (in a fair sample of 200 library books, 30 had torn pages): (a) Some books in the library have torn pages. (b) About 15 percent of the library's books have torn pages. (c) Exactly 15 percent of the books on every shelf have torn pages.

▼ **Show answer**

(a) strongest, (b) next, (c) weakest. (a) is nearly certain, since torn books were actually found. (b) matches the sample rate of 30 out of 200 (15 percent) and claims only "about." (c) claims an exact rate on every single shelf, which the sample cannot show.

9. Hard Joel wants to know how many people in his town walk for exercise. He stands at the entrance to the town walking trail at 7 a.m. on a Saturday and asks everyone who passes. Of the 40 people he asks, 38 say they walk for exercise at least three times a week. Evaluate his conclusion that "95 percent of people in our town walk for exercise at least three times a week."

▼ **Show answer**

Weak: the sample is badly biased, because people at a walking trail early on a Saturday are far more likely than the average townsperson to walk for exercise. It also lacks variety (one place, one time, one kind of person). The sample size of 40 is a lesser problem; even a much larger sample taken at the trail would still be biased. A fair sample would choose townspeople at random, for example from a town directory.

10. Hard A chemist tests the boiling point of one sample of pure water at sea level and concludes that all pure water at sea level boils at that temperature. A shopper tastes one grape from a crate and concludes that all the grapes in the crate are sweet. Is either argument strong? Explain the difference.

▼ **Show answer**

The chemist's argument is strong, because long experience and background knowledge show that pure water is uniform in its boiling point, so one careful test represents every sample. The shopper's argument is weak, because grapes vary in ripeness from one to another, so one grape is far too small a sample for a varied population. (Accept the answer that the chemist should still repeat the test to rule out a measuring error, as long as the student sees that the chemist's argument is much stronger.)

11. Hard Caleb counts 200 apples for worms but skips the trees by the creek, which he knows had worm trouble last spring. He finds 4 wormy apples and concludes that only about 2 percent of the orchard is wormy. Is his argument cogent? Explain, using the idea of total evidence.

▼ **Show answer**

No. Even if his count is accurate, he deliberately left out the part of the orchard most likely to be wormy, so his sample is biased and his argument ignores evidence he knew about. A cogent argument must be strong, have true premises and not leave out relevant evidence; his conclusion probably understates the true rate.

12. **Challenge** Design a sampling plan for this question: "What fraction of the 600 students at Riverside School eat breakfast before school?" Say how many students you would ask, how you would choose them, and what conclusion you would be entitled to draw. Name one way your plan could still go wrong.

▼ **Show answer**

Sample answer: Ask about 60–100 students chosen at random from the school roster (for example, every sixth name on an alphabetical list, or names drawn by lot), making sure every grade is included. Ask the question the same way to each, privately, so no one feels pressure. Conclude only "about such-and-such a fraction" of students eat breakfast, not an exact figure. It could still go wrong if students answer untruthfully, if absent students are skipped (they may differ from others), or if the question is worded vaguely (does a granola bar count as breakfast?). Any plan that is reasonably large, random or varied across grades, and modest in its conclusion earns full credit, with any sensible weakness named.

COMMON MISTAKES

- Thinking a large sample is always a good sample. A huge sample taken in a biased way can be worse than a small fair one.
- Calling a strong inductive argument "valid." Induction is strong or weak, never valid.
- Drawing an "all" conclusion when the evidence supports only "most" or "about." A bolder conclusion is a weaker argument.
- Forgetting background knowledge. Whether a small sample is enough depends on how much the population varies.
- Ignoring evidence that goes against the conclusion. An argument that leaves out known evidence is not cogent.

> **Scripture.** "He that answereth a matter before he heareth it, it is folly and shame unto him." (Proverbs 18:13, KJV)
>
> A hasty generalization answers the matter before the evidence has been heard; patient induction waits until it has a fair sample.

7.2 Argument from Analogy

OBJECTIVES

- Put an argument from analogy into its standard pattern and find the likenesses it relies on.
- Test an analogy by the relevance and number of its likenesses, its relevant differences and the number of cases compared.
- Tell an analogy used to argue from an analogy used only to illustrate.
- Answer a weak analogy with a counter-analogy or by pointing out a relevant difference.

KEY TERMS

Analogy

A comparison between two or more things based on likenesses they share.

Argument from analogy

An inductive argument that reasons that because two things are alike in certain ways, they are probably alike in a further way.

Primary analogue

The thing (or things) already known to have the property in question; the case we argue from.

Secondary analogue

The thing the conclusion is about; the case we argue to.

Relevant likeness

A likeness that is connected to the property in the conclusion, so that it gives a real reason to expect that property.

Disanalogy

A relevant difference between the things compared, which weakens an argument from analogy.

Weak analogy

The fallacy of resting a conclusion on likenesses that are few, irrelevant or outweighed by relevant differences (also called false analogy).

Reasoning from one case to another. Hannah's older brother loved Mr. Lindqvist's history class, read the same kind of books she reads, and enjoys the same sports. Hannah reasons that she will probably love the class too. She is arguing from **analogy**: because two things are alike in some ways, they are probably alike in another. We reason this way all the time. A gardener expects a new tomato variety to need the same care as its close cousin. A doctor expects a medicine that helped one patient to help another with the same illness. Like a generalization, an argument from analogy is inductive: it can be strong or weak, never valid.

The pattern. Every argument from analogy can be put in this form:
1. A and B are alike in having features *f*, *g* and *h*.
2. A also has feature *z*.
Therefore, B probably has feature *z* too.
A is the **primary analogue**, the case we already know about; B is the **secondary analogue**, the case the conclusion is about. Writing the argument this way shows you exactly which likenesses are doing the work, and what is being inferred from them.

Test one: are the likenesses relevant? The most important test is whether the shared features are connected to the feature in the conclusion. Two used trucks of the same make, model, year and engine, both driven gently on country roads, are likely to get similar gas mileage, because each of those likenesses affects mileage. But if the trucks are alike only in color and in having the same first owner's name, the likenesses tell you nothing about mileage. A single relevant likeness outweighs a dozen irrelevant ones. Ask of each likeness: does it give any reason to expect *z*?

Test two: how many relevant likenesses? Other things being equal, the more relevant likenesses there are, the stronger the argument. If the two trucks share make, model, year, engine size, tire size, and driving conditions, the argument is stronger than if they share only make and model. But counting is no substitute for judging relevance; ten irrelevant likenesses add nothing.

Test three: are there relevant differences? A **disanalogy** is a relevant difference between the things compared, and it weakens the argument. If one truck has been hauling heavy loads up mountain roads for years while the other has not, that difference matters for mileage. Hannah's argument weakens if her brother had a

different teacher the year he took the class, or if the course has been rewritten since. To criticize an argument from analogy fairly, look for differences that bear on the conclusion, not just any differences. Every two things differ somehow; only the relevant differences count.

Test four: how many cases, and how much is claimed? An argument from several primary analogues is often stronger than an argument from one, especially if those cases differ among themselves while still sharing the important features. If five different families, with different children and different schedules, have all found that a certain chore chart works, that is better evidence for your family than one family's success. As with generalizations, a modest conclusion is stronger: "Hannah will probably enjoy the class" is stronger than "Hannah will get the top grade in the class."

Illustration is not argument. Not every analogy is an argument. Teachers, preachers and storytellers use comparisons to make an idea clear or vivid, not to prove it. When a coach says, "A team is like a body; every part needs the others," he is helping the players picture something, not offering evidence. Our Lord's parables, such as the mustard seed, illustrate truths about the kingdom of heaven; they are not arguments from analogy to be scored for strength. Before you judge an analogy, ask whether it is being offered as a reason to believe a conclusion or only as a picture of an idea.

Answering a weak analogy. When an argument rests on a **weak analogy** (also called false analogy), there are two good replies. First, name a relevant difference: "Our garden is in full shade, but Mrs. Ortiz's is in full sun, so her tomato schedule may not work for us." Second, offer a **counter-analogy**: an equally good comparison that points the other way. If someone argues that a new student will surely do well because she resembles last year's top student in being quiet and neat, you might reply that she also resembles last year's struggling student in exactly the same ways. The counter-analogy shows that those likenesses do not settle anything.

From history. Analogy has a long history in Christian thought. In 1736 Bishop Joseph Butler published *The Analogy of Religion*, arguing that the difficulties people raise against revealed religion are like difficulties we already accept in the ordinary course of nature. In 1802 William Paley opened his *Natural Theology* with a famous comparison: if you found a watch lying on a heath, its parts fitted together for a purpose would lead you to conclude that it had a maker; and living things, he argued, show far more fitting of parts to purposes than any watch. Readers have debated these arguments ever since, and the debate is a good place to practice the tests of this lesson: which likenesses are relevant, and are there relevant differences?

Practice habit. When someone says "It's just like …," ask: Alike in what ways, and do those ways have anything to do with the point?

Enrichment: Paley's watch. Read the first paragraph of chapter one of William Paley's *Natural Theology* (1802), which is in the public domain. Paley compares a stone and a watch found on a heath. Write the argument in the standard pattern of this lesson: what are the likenesses, and what is inferred? Then list one likeness that you think is clearly relevant and one difference that a critic might raise. How would you answer that critic?

Example 1. Put this argument into the standard pattern and evaluate it: "Brookside Dairy's new chocolate milk comes in the same blue carton as its plain milk, is sold at the same stores, and costs the same. Everyone likes Brookside's plain milk. So everyone will like the chocolate milk."

▼ **Show solution**

1. Pattern: 1. The plain milk (A) and the chocolate milk (B) are alike in carton, stores and price. 2. The plain milk is liked by everyone. Therefore, the chocolate milk will probably be liked by everyone.
2. Relevance: carton color, stores and price have little to do with whether people like the taste. These likenesses are irrelevant to the conclusion.
3. Differences: the most relevant feature, the flavor, is exactly where the two products differ.
4. Claim: "everyone" is a bold conclusion.

Answer: Weak. The likenesses are irrelevant to taste, the key difference (flavor) is ignored, and the conclusion claims too much. It is a weak analogy.

Example 2. Mr. Okafor wants to know whether his new field will grow good potatoes. He reasons: "My north field and the new field have the same sandy loam soil, the same drainage, the same amount of sun and the same rainfall. My north field grows excellent potatoes. So the new field will probably grow good potatoes too." Evaluate the argument, and name one fact that would weaken it.

▼ **Show solution**

1. Pattern: A (north field) and B (new field) share soil type, drainage, sun and rainfall; A grows excellent potatoes; so B probably will.
2. Relevance: every likeness listed affects how potatoes grow. These are relevant likenesses.
3. Number: there are four relevant likenesses, which is a good number.
4. Claim: "good potatoes," with "probably," is modest (he does not claim the same record harvest).
5. A weakening disanalogy would be any relevant difference: for example, if the new field was planted in potatoes last year and may carry potato diseases in the soil, or if it lies in a frost pocket.

Answer: Strong: the likenesses are many and relevant, and the conclusion is modest. It would be weakened by a relevant difference, such as disease in the new field's soil or a frost pocket. (Any relevant difference is a correct answer.)

PRACTICE

1. Easy Write the standard pattern of an argument from analogy.

   ▼ **Show answer**

   1. A and B are alike in having features f, g and h. 2. A also has feature z. Therefore, B probably has feature z too.

2. Easy Is an argument from analogy deductive or inductive? Can it be valid?

   ▼ **Show answer**

   Inductive. It claims only that its conclusion is probable, so it is strong or weak, never valid or invalid.

3. **Easy** What is a disanalogy?

▼ Show answer

A relevant difference between the things compared, which weakens an argument from analogy.

4. **Easy** In this argument, name the primary analogue, the secondary analogue and the feature inferred: “Last year’s Fairview Hawks had the same coach and the same starting five as this year’s team, and they won the league. So this year’s team will probably win the league.”

▼ Show answer

Primary analogue: last year’s team. Secondary analogue: this year’s team. Feature inferred: winning the league.

5. **Medium** Relevant or irrelevant to the conclusion “This sweater will keep me as warm as my old one”? (a) Both are made of thick wool. (b) Both are green. (c) Both were bought at the same store. (d) Both are knit tightly.

▼ Show answer

(a) Relevant: thick wool affects warmth. (b) Irrelevant: color does not affect warmth. (c) Irrelevant: the store does not affect warmth (unless the student explains that the same store might sell the same maker’s goods; accept “weakly relevant” with that reason). (d) Relevant: a tight knit keeps in heat.

6. **Medium** Strong or weak? Give a reason. “Grandpa’s old pickup and this one are both red and both have a dent in the tailgate. Grandpa’s pickup ran for 300,000 miles. So this one will too.”

▼ Show answer

Weak: the likenesses (color and a dent) are irrelevant to how long an engine lasts, and the conclusion (300,000 miles) claims a great deal. It is a weak analogy.

7. Medium Is this analogy used as an argument or only as an illustration? Explain. "A good habit is like a path across a field: the more often you walk it, the easier it is to follow."

▼ **Show answer**

An illustration: the speaker is making the idea of habit clear and vivid, not offering evidence that a conclusion is true. (Accept "argument" only if the student explains that the comparison is being used to persuade someone to keep practicing; even then it gives a picture more than evidence.)

8. Medium Caleb argues, "My cousin's family keeps a beagle in a small apartment and it is perfectly happy, so a beagle will be happy in our small apartment." Name one relevant likeness he could add to strengthen the argument and one relevant difference that would weaken it.

▼ **Show answer**

Sample answer: Strengthen: both families take the dog for long daily walks (or both have someone home most of the day). Weaken: his cousin's family is home all day, while Caleb's family is gone ten hours a day (or the cousin's apartment is next to a park and Caleb's is not). Any likeness or difference that bears on a beagle's happiness is correct.

9. Hard Write a counter-analogy to answer this argument: "A school is like a business. A business fires workers who do not earn a profit. So a school should send home students who earn low grades."

▼ **Show answer**

Sample answer: "A school is also like a hospital: it exists to help those who need it most. A hospital does not send home its sickest patients; it gives them more care. So a school should give students with low grades more help." A good answer may also name the key disanalogy: students are not employees paid to produce something for the school; they are the people the school exists to serve. Any fair counter-analogy that points toward a different conclusion earns credit.

10. Hard Which argument is stronger, and why? (a) “The Hendersons tried the Clearwater chore chart with their four children, and it worked well. So it will work well for our family.” (b) “Six families we know, with children of different ages and different schedules, all tried the Clearwater chore chart, and it worked well for each. So it will probably work well for our family.”

▼ **Show answer**

(b) is stronger. It has six primary analogues instead of one, and they differ among themselves (different ages and schedules), which shows that the chart’s success does not depend on one family’s special situation. It also says “probably,” a more modest claim than (a).

11. Hard Mrs. Ortiz reasons: “The recipe for oatmeal cookies worked perfectly at my sister’s house, so it will work perfectly here.” Her sister lives at sea level; Mrs. Ortiz lives high in the mountains. Explain why this difference is relevant, and restate the argument so that it is stronger.

▼ **Show answer**

Altitude is relevant because baking works differently at high elevation (water boils at a lower temperature and dough rises faster), so a recipe that works at sea level may need changes. A stronger version: “The recipe worked perfectly for several friends who live at about my altitude and use ovens like mine, so it will probably work well here.” Any restatement that adds relevant likenesses or a more modest conclusion is correct.

12. Challenge Write your own argument from analogy about gardening, sports or cooking that is strong. Then change one premise so that it becomes weak, and explain which test the weak version fails.

▼ **Show answer**

Sample answer: Strong: “My brother’s bicycle and mine are the same model and size, with the same tires, and both are ridden on the same gravel roads. His tires lasted about two years. So mine will probably last about two years.” Weak: “My brother’s bicycle and mine are both blue and both have bells. His tires lasted about two years. So mine will too.” The weak version fails the relevance test: color and a bell have nothing to do with how long tires last. Any pair in which the strong version has relevant likenesses and a modest claim, and the weak version fails a named test, earns full credit.

COMMON MISTAKES

- Counting likenesses without asking whether they are relevant. One relevant likeness outweighs many irrelevant ones.
- Rejecting an analogy because of any difference at all. Every two things differ; only differences that bear on the conclusion weaken the argument.
- Treating an illustration as an argument, or an argument as a mere illustration. Ask what the comparison is being used to do.
- Calling a strong analogy "valid." Arguments from analogy are inductive.

> **Scripture.** "Go to the ant, thou sluggard; consider her ways, and be wise:" (Proverbs 6:6, KJV)
>
> Scripture often teaches by comparison: the ant, who gathers in summer for the winter, is like the wise worker in the way that matters, so the sluggard can learn from her.

UNIT 7 · INDUCTION AND SCIENTIFIC REASONING · LESSON 7.3 OF 69 · 45 MINUTES

7.3 Finding Causes: Mill's Methods

OBJECTIVES

- State Mill's methods of agreement, difference, the joint method, residues and concomitant variation.
- Apply each method to a set of cases to find the probable cause of an effect.
- Name a limit of each method and explain why the methods give probable, not certain, conclusions.
- Connect the method of difference to the controlled experiment and the false cause fallacy.

KEY TERMS

Method of agreement

If two or more cases of an effect have only one relevant circumstance in common, that circumstance is probably the cause (or part of the cause).

Method of difference

If a case in which the effect occurs and a case in which it does not are alike in every relevant circumstance but one, that one circumstance, present only with the effect, is probably the cause (or part of it).

Joint method

The method of agreement and difference used together: the circumstance present in every case with the effect and absent in every case without it is probably the cause.

Method of residues

Subtract from an effect the part already explained by known causes; the part left over is probably due to the remaining circumstance.

Method of concomitant variation

If one thing rises or falls whenever another rises or falls, the two are probably connected by cause and effect (one causes the other, or both come from a common cause).

Circumstance

Any condition or factor present in a case that might be connected with the effect.

CORE IDEAS

The search for causes. Why did the bread fail to rise? Why are the beans in one row taller than the beans in the next? Why did four campers get sick? Questions about causes are among the most practical in life, and they are inductive: we cannot see a cause the way we see a color, so we reason to it from cases. In 1843 the English philosopher John Stuart Mill set out five rules, now called **Mill's methods**, for finding causes from cases. They do not give certainty, but they turn guessing into careful reasoning, and they show exactly what went wrong in the false cause fallacy you met in Unit 3.

The method of agreement. Look at several cases where the effect happened, and find what they have in common. Four campers at Pine Lake got sick on the same night. They ate different suppers, slept in different cabins and swam at different

times, but all four drank water from the stream behind the camp. If the cases agree in only one relevant **circumstance**, that circumstance is probably the cause, or part of it. The weakness of this method is that the cases may share a circumstance nobody thought to list. Perhaps all four also used the same tin cup.

The method of difference. Compare a case where the effect happened with a case where it did not, alike in every relevant circumstance but one. Mr. Okafor plants two rows of beans side by side, from the same packet, in the same soil, with the same water and sun. He works bone meal into one row only. If that row grows taller, the bone meal is probably the cause. This method is the heart of the controlled experiment, which you will study in Lesson 7.4. Its weakness is that in real life it is hard to be sure that only one circumstance differs.

The joint method. The **joint method** combines the first two. Look at several cases with the effect and several without it. If one circumstance is present in every case with the effect and absent in every case without it, it is very probably the cause. After a church picnic, eight people had stomachaches and twenty did not. All eight ate the potato salad; none of the twenty did. The cases with the effect agree in the potato salad, and the cases without the effect agree in its absence. The joint method is stronger than either method alone, because it checks the cause from both sides.

The method of residues. Sometimes most of an effect is already explained, and only a part is left over. Subtract what known causes account for, and look for a cause of the remainder. A farmer weighs his loaded wagon at 2,600 pounds. The wagon alone weighs 900 pounds, and the hay he loaded weighs 1,500 pounds. That accounts for 2,400 pounds; the remaining 200 pounds must come from something else, and he finds a forgotten sack of feed under the hay. The method is only as good as our knowledge of the known causes; if the wagon's weight was wrong, the residue is wrong too.

The method of concomitant variation. "Concomitant" means "going along with." If one thing rises or falls whenever another does, they are probably connected by cause and effect. The colder the night, the thicker the ice on the water trough. The more Ellie practices free throws each week, the higher her shooting percentage. This method is useful when a circumstance cannot be removed entirely, only increased or decreased. But it has two dangers. The two things may both be effects of a third

cause, and the pattern may hold only within a range: a little fertilizer helps a plant, but a great deal burns its roots.

Cause, correlation and the false cause fallacy. When two things go together, we say they are *correlated*. Correlation is a clue to cause, not a proof of it. In early summer, ice cream sales and sunburns rise together, but ice cream does not cause sunburn; hot, sunny weather causes both. The false cause fallacy (Unit 3) jumps from "together" or "after" to "because of." Mill's methods slow that jump down: they ask whether we have looked at cases without the effect, whether other circumstances were held the same, and whether a common cause could explain the pattern.

What the methods cannot do. Mill's methods test the circumstances we think to list; they cannot tell us which circumstances to list in the first place. That requires knowledge and imagination. If no one thinks of the tin cup, no method will find it. The methods also give probable conclusions, not certain ones, because a hidden circumstance can always be at work. That is why careful investigators combine the methods, repeat their tests and remain willing to change their minds. As Proverbs 25:2 says, searching out a matter is an honorable labor, and it is rarely finished in one step.

> **From history.** In 1854 a terrible outbreak of cholera struck the Soho district of London. A physician named John Snow marked each death on a street map and found that the deaths clustered around a public water pump on Broad Street. Households nearer other pumps were mostly spared, and he learned that several victims who lived farther away had used water from the Broad Street pump. Snow persuaded the local officials to remove the pump's handle. In a larger study he compared households supplied by two water companies, one drawing its water from a part of the Thames fouled by the city's sewage and one drawing cleaner water upstream, and found far more cholera deaths among the first. His reasoning is a classic use of agreement and difference.

> **Practice habit.** When someone says "X causes Y," ask: What happened when X was absent, and could something else cause both?

Example 1. Three loaves of Hannah's bread failed to rise this month. For the first she used Brand A flour, the yellow mixing bowl and yeast from the open jar. For the second, Brand B flour, the glass bowl and yeast from the open jar. For the third, Brand A flour, the glass bowl and yeast from the open jar. Which method applies, and what is the probable cause? What is a weakness of this conclusion?

▼ **Show solution**

1. All three cases have the effect (bread that failed to rise), so look for what they agree in. This is the method of agreement.
2. Flour: A, B, A. Not common to all.
3. Bowl: yellow, glass, glass. Not common to all.
4. Yeast: open jar every time. This is the only listed circumstance common to all three.
5. Weakness: there may be a common circumstance that was not listed, such as the same cold kitchen or the same thermometer for the water.

Answer: Method of agreement: the yeast from the open jar is probably the cause (perhaps it is old or was spoiled by moisture). The conclusion is only probable, because an unlisted circumstance shared by all three loaves could be the real cause.

Example 2. A loaded farm truck weighs 9,400 pounds. The empty truck weighs 6,000 pounds, and the 40 bags of feed in it weigh 50 pounds each. The driver weighs 180 pounds. How much weight is unexplained, and which method is this?

▼ **Show solution**

1. The feed weighs $40 \times 50 = 2{,}000$ pounds.
2. Known causes: $6{,}000 + 2{,}000 + 180 = 8{,}180$ pounds.
3. Residue: $9{,}400 - 8{,}180 = 1{,}220$ pounds.
4. Subtracting the part of an effect explained by known causes to find what is left for an unknown cause is the method of residues.

Answer: 1,220 pounds are unexplained; this is the method of residues, and something else in the truck (or an error in one of the known weights) accounts for it.

PRACTICE

1. Easy Name Mill's five methods.

   ▼ **Show answer**

   The method of agreement, the method of difference, the joint method (of agreement and difference), the method of residues and the method of concomitant variation.

2. Easy Which method compares a case with the effect to a case without it, alike in everything but one circumstance?

   ▼ **Show answer**

   The method of difference.

3. Easy Which method is used? "The more hours the stove burns, the warmer the cabin gets."

   ▼ **Show answer**

   The method of concomitant variation: one quantity (warmth) rises as the other (hours of burning) rises.

4. **Easy** What does “concomitant” mean?

   ▼ **Show answer**

   “Going along with” or “accompanying”: in the method of concomitant variation, one thing changes along with another.

5. **Medium** Name the method and the probable cause: “Three of our hens stopped laying this week. One is a Leghorn, one a Rhode Island Red and one a Plymouth Rock. They roost in different corners of the coop, but all three were moved to the new run where the neighbor’s dog barks all day.”

   ▼ **Show answer**

   Method of agreement: the only listed circumstance the three cases share is being moved to the new run near the barking dog, so the move (perhaps stress from the dog) is the probable cause. (Accept “the new run” or “the barking dog” as the cause, since both are the shared circumstance.)

6. **Medium** Name the method and the probable cause: “Two identical seed trays were filled with the same soil, planted with the same seeds and watered the same. One sat on the sunny windowsill and one in the dark pantry. Only the windowsill tray sprouted green, healthy seedlings.”

   ▼ **Show answer**

   Method of difference: the trays were alike in every listed circumstance except light, so the sunlight is probably the cause of the healthy seedlings (or the lack of light the cause of the poor ones).

7. **Medium** After a school lunch, these students felt sick: Ana (ate soup, salad, pudding), Ben (soup, sandwich, pudding), Cara (salad, sandwich, pudding). These did not: Dov (soup, salad, fruit), Eli (sandwich, salad, fruit), Fay (soup, sandwich, fruit). Which food is probably the cause, and which method shows it?

   ▼ **Show answer**

   The pudding. Every sick student ate pudding, and no student who stayed well ate it; every other food was eaten by some who stayed well. This is the joint method of agreement and difference.

8. Medium A family's electric bill was $140 in March. The heater usually adds $60, the water heater $35, and lights and appliances $25. Using the method of residues, how much of the bill is left to explain?

▼ **Show answer**

$60 + $35 + $25 = $120 is explained; $140 − $120 = $20 is left over, to be explained by some other cause (for example, a new freezer or a space heater left on).

9. Hard In one town, the more fire trucks sent to a fire, the more damage the fire does. Does this show that fire trucks cause damage? Use the idea of concomitant variation to explain.

▼ **Show answer**

No. The two vary together because both are effects of a common cause: the size of the fire. Big fires bring more trucks and also do more damage. Concomitant variation shows that two things are connected, but the connection may be a common cause rather than one causing the other (it could also run in the reverse direction).

10. Hard Caleb says, "Every time I wore my blue socks this season, we won. My blue socks cause our wins." Which of Mill's methods is he trying to use, and what does it leave out? Name the fallacy.

▼ **Show answer**

He is using a rough method of agreement (the winning games share the blue socks). It leaves out many other circumstances common to those games (the team's skill, the opponents, the starting players) and ignores cases without the effect: did the team lose any games when he wore them, or win when he did not? He has no reason to think socks could affect the score. The fallacy is false cause (accept post hoc, "after this, therefore because of this," or cum hoc, "with this, therefore because of this").

11. Hard Mr. Okafor finds that tomato plants given 1, 2 and 3 scoops of fertilizer grow taller with each extra scoop. He concludes that 10 scoops will make them grow taller still. What is wrong with his reasoning?

▼ **Show answer**

Concomitant variation shows a connection only within the range observed. Beyond some point more fertilizer stops helping and can burn the roots, so the pattern from 1–3 scoops cannot be stretched to 10. He should test larger amounts before concluding.

12. Challenge Hannah's geraniums are wilting. She suspects either the new potting soil or the new spot by the radiator. Design a test that uses the method of difference to find out which is the cause, and explain what results would point to each.

▼ **Show answer**

Sample answer: Use four similar geraniums from the same batch. Plant two in the new soil and two in the old soil. Put one of each pair by the radiator and the other in the old spot, watering all four the same. If the two by the radiator wilt and the others do not, the radiator spot is probably the cause. If the two in new soil wilt wherever they sit, the soil is probably the cause. If only the plant with both new soil and radiator wilts, the two may work together. Any design that changes one circumstance at a time while holding the others the same earns full credit.

COMMON MISTAKES

- Thinking the method of agreement proves a cause. The cases may share a circumstance no one listed.
- Ignoring cases without the effect. Agreement alone looks only at the cases with the effect; the joint method checks both sides.
- Treating "they rise together" as "one causes the other." A common cause may explain both, or the cause may run the other way.
- Stretching concomitant variation beyond the range observed. More of a good thing is not always better.
- Forgetting that the methods test the circumstances we think to list. They do not supply the ideas.

> **Scripture.** “It is the glory of God to conceal a thing: but the honour of kings is to search out a matter.” (Proverbs 25:2, KJV)
>
> God has made a world worth investigating; searching out causes patiently and carefully is honorable work.

UNIT 7 · INDUCTION AND SCIENTIFIC REASONING · LESSON 7.4 OF 69 · 45 MINUTES

7.4 Hypotheses and Testing

OBJECTIVES

- Describe how a hypothesis is tested by deducing a prediction and checking it.
- Explain, using modus tollens and affirming the consequent, why a failed prediction counts against a hypothesis more decisively than a passed prediction counts for it.
- Design a fair controlled test with a control group and one changed circumstance, and recognize an ad hoc rescue.
- Name Bacon’s four idols (tribe, cave, marketplace, theatre) and give an example of each.

WATCH

The Scientific Method

MooMooMath and Science · 4:37

https://www.youtube.com/watch?v=c6Gp2R0wzhU

> **WHEN TO WATCH**
> Before Bacon’s idols: a simple picture of testing a hypothesis by its prediction.

Note: Parent preview: A cartoon “soggy sandwich” example walks through observation, hypothesis, test and conclusion; the first hypothesis fails and a second one passes. Good discussion point: one passed test supports a hypothesis but does not prove it (the video says the hypothesis “held true” after one try; the lesson explains why that is not proof). Ends with “kindness multiplies kindness.” Nothing objectionable; transcript checked October 18, 2026.

KEY TERMS

Hypothesis

A proposed explanation or answer that can be tested by checking what it predicts.

Prediction

A statement of what we should observe under certain conditions if the hypothesis is true.

Controlled experiment

A test that compares an experimental group, which receives the change being tested, with a control group that is alike in every other relevant way but does not receive it.

Confirmation

Support a hypothesis gains when one of its predictions comes true; it raises the hypothesis's probability but does not prove it.

Ad hoc rescue

An excuse invented only to save a hypothesis from a failed prediction, with no independent reason to believe it.

Idols of the mind

Francis Bacon's name for four kinds of false notions that lead us astray: idols of the tribe, the cave, the marketplace and the theatre.

CORE IDEAS

A hypothesis is a testable guess. Hannah's sourdough starter has stopped bubbling. She thinks of three possible reasons: the kitchen is too cold, the flour is different, or the starter has died. Each of these is a **hypothesis**, a proposed explanation that can be tested. A good hypothesis is not just any idea that pops into your head. It must be specific enough to tell you what you should see if it is true, and so it must be capable of turning out to be wrong.

From hypothesis to prediction. To test a hypothesis, reason forward from it to a **prediction**: "If the kitchen is too cold, then moving the starter to a warm spot near the oven should make it bubble again within a day." This step is deductive: the prediction follows from the hypothesis together with what we already know about yeast. Then comes the observation, which is the inductive step. Hannah moves the jar and watches. The prediction is the bridge between an idea in the mind and the world outside it.

Failed predictions and passed predictions. Suppose the starter stays flat in the warm spot. Then Hannah can reason: "If the cold was the cause, the starter would bubble when warm. It did not bubble. So the cold was not the cause." That is modus tollens, a valid form. Now suppose the starter does bubble. It is tempting to reason: "If the cold was the cause, it would bubble when warm. It bubbled. So the cold was the cause." That is affirming the consequent, which is invalid: something else might have changed too. A passed prediction gives **confirmation**; it makes the hypothesis more probable, especially if the result would have been surprising otherwise. But it does not prove the hypothesis.

The fine print in every test. A failed prediction is not quite as simple as it looks. Every prediction depends on the hypothesis *and* on other assumptions: that the oven spot is really warm, that the thermometer works, that the jar was not knocked over. If the prediction fails, one of those assumptions might be the problem. Honest testers check them. But there is a dishonest way to do this, called an **ad hoc rescue**: inventing an excuse only to save a favorite hypothesis, with no independent reason to believe the excuse. "My lucky pencil did not work on the test because it was tired" is an ad hoc rescue. A hypothesis that can be rescued from every failure has stopped saying anything at all.

The controlled experiment. The surest test uses Mill's method of difference. Divide the cases into two groups that are alike in every relevant way. The **experimental group** receives the change being tested; the **control group** does not. Then compare. To test whether a new seed coating, Sproutwell, helps beans sprout, plant 100 coated and 100 uncoated seeds from the same packet in the same soil, water and light. Choose which seeds get the coating at random, so that no one picks the best-looking ones for one group. Use enough cases, and repeat the test. Where people are judging the results, keep them from knowing which group is which, so their hopes cannot color what they see.

Bacon's idols: tribe and cave. In the *Novum Organum* (1620), Francis Bacon warned that the mind is crowded with "idols," false images that keep us from seeing nature as it is. The **idols of the tribe** are errors common to the whole human race: we see patterns that are not there, remember the hits and forget the misses, and believe what we wish were true. The **idols of the cave** belong to each person's own private den: our upbringing, our temperament, our favorite subject, our habits. A boy who

loves baseball explains everything with a baseball comparison; a farmer who distrusts anything new rejects a better plow without trying it.

Bacon's idols: marketplace and theatre. The **idols of the marketplace** come from words, which we pick up in the everyday talk of the market. Loose, vague words can make us think we understand something when we do not, or make us argue about a word instead of a thing. Calling a weed "healthy" or a storm "unnatural" settles nothing. The **idols of the theatre** come from systems of thought that we accept whole, like the plot of a stage play, simply because they are taught by a famous school or a respected authority, without testing them. Bacon wanted every claim about nature to face observation and fair trial.

Testing as humility. Hypothesis testing is a discipline of humility. It asks us to say in advance what would show us wrong, to look for evidence against our ideas and not only for them, and to accept the result. That is why careful investigators write down their predictions before they look, count the misses as carefully as the hits, and let others repeat their work. Daniel showed the same spirit when he asked the steward to "prove" him and his friends for ten days on a plain diet and then compare them with the other young men. He proposed a fair test and let the result speak.

> **From history.** For many centuries people believed that maggots arose on their own from rotting meat. In 1668 the Italian physician Francesco Redi tested this idea. He placed pieces of meat in several jars, leaving some open and covering others. Flies landed on the meat in the open jars, and maggots appeared there. In the jars covered with fine gauze, no maggots appeared on the meat, although flies, drawn by the smell, laid eggs on the gauze itself. Redi had a control group (the open jars) and an experimental group (the covered jars) that differed in one circumstance: whether flies could reach the meat. His experiment is still told as a model of a fair, controlled test.

> **Practice habit.** Before you test an idea, say out loud what result would show that you are wrong.

Enrichment: Bacon's idols in his own words. Bacon's *Novum Organum* (1620) was written in Latin and has been translated many times; older English translations are in the public domain. Find aphorisms 38–44 of Book One, where Bacon introduces the four idols. Choose the idol you think most often leads you astray, and write a short paragraph giving an example from your own week and one habit that could guard against it.

WORKED EXAMPLES

Example 1. Caleb's hypothesis: "Our basketball goes flat because the valve leaks." (a) State a prediction that follows from it. (b) Suppose the prediction fails. Write the reasoning and name its form. (c) Suppose the prediction comes true. What may Caleb conclude, and what may he not conclude?

▼ **Show solution**

1. (a) A prediction: if the valve leaks, then putting soapy water on the valve of a fully pumped ball should produce bubbles.
2. (b) If the valve leaks, bubbles will form at the valve. No bubbles formed. So the valve does not leak (or at least no leak showed). This is modus tollens, a valid form, though Caleb should check his assumptions (was the ball fully pumped? did he wait long enough?).
3. (c) If bubbles form, the hypothesis is confirmed: it is now much more probable. But reasoning "If the valve leaks, bubbles form; bubbles formed; so the valve leaks" is affirming the consequent. Here the result is strong confirmation because bubbles at the valve are hard to explain any other way, but strictly it is not a proof; there could also be a second leak elsewhere on the ball.

Answer: (a) Soapy water on the valve of a pumped ball should bubble. (b) Modus tollens: no bubbles, so (unless a background assumption failed) the valve is not leaking. (c) Bubbles confirm the hypothesis and make it very probable, but they do not prove it in the deductive sense, and they do not rule out a second leak.

Example 2. Mrs. Ortiz wants to know whether Greenleaf plant food makes African violets bloom more. Design a controlled experiment.

▼ **Show solution**

1. Choose a fair number of similar plants, say 20 African violets of the same variety and age.
2. Assign them at random to two groups of 10, for example by drawing numbers from a hat.
3. Experimental group: water with Greenleaf plant food. Control group: water with plain water. Keep everything else the same: pots, soil, window, amount and timing of water.
4. Count the blooms on each plant after eight weeks. Ideally, have someone who does not know which plants got the food do the counting.
5. Compare the groups, and repeat the test with a new set of plants if possible.

Answer: Twenty similar violets, split at random into a Greenleaf group and a plain-water control group, treated alike in every other way, with blooms counted after eight weeks by someone who does not know which group is which. (Any design with a control group, random assignment, one changed circumstance and a fair measure is correct.)

PRACTICE

1. Easy What is a hypothesis?

   ▼ **Show answer**

   A proposed explanation or answer that can be tested by checking what it predicts.

2. Easy In a controlled experiment, what is the control group?

   ▼ **Show answer**

   The group that is alike in every relevant way to the experimental group but does not receive the change being tested, so that the two can be compared.

3. Easy Name Bacon's four idols.

   ▼ **Show answer**

   The idols of the tribe, the cave, the marketplace and the theatre.

4. **Easy** In what book, and in what year, did Francis Bacon describe the four idols?

   ▼ **Show answer**

   The *Novum Organum* ("New Instrument"), 1620.

5. **Medium** Name the form and say whether it is valid: "If the seeds are dead, then none will sprout in warm, wet soil. Some sprouted in warm, wet soil. So the seeds are not all dead."

   ▼ **Show answer**

   Modus tollens (denying the consequent), which is valid: "some sprouted" denies the consequent "none will sprout," so the antecedent "the seeds are (all) dead" is denied.

6. **Medium** Name the form and say whether it is valid: "If the new feed makes the calves gain weight, they will weigh more next month. They weighed more next month. So the new feed made them gain weight."

   ▼ **Show answer**

   Affirming the consequent, which is invalid: the calves might have gained weight anyway, because they are growing or for some other reason. The result confirms the hypothesis a little but does not prove it; a control group of calves on the old feed would be needed.

7. **Medium** Which of Bacon's idols is shown? Give a reason. "Joel always remembers the times his hunch about the weather was right and forgets the times it was wrong."

   ▼ **Show answer**

   The idol of the tribe: remembering hits and forgetting misses is a tendency common to all human beings, not just to Joel. (Accept "cave" only if the student argues it is Joel's personal habit; the best answer is tribe.)

8. **Medium** Which of Bacon's idols is shown? Give a reason. "Two neighbors argue for an hour about whether a tomato is a 'vegetable' before they realize that one means the cook's sense of the word and the other the botanist's."

   ▼ **Show answer**

   The idol of the marketplace: the trouble comes from a loose word used in two senses, so they argued about a word instead of a thing.

9. Hard Grandpa says his copper bracelet keeps his knees from aching. When his knees ache anyway on a rainy day, he says, “The bracelet works, but rain is too strong for it.” What is this kind of reply called, and why is it a problem?

▼ **Show answer**

An ad hoc rescue: the excuse (that rain is too strong for the bracelet) was invented only to save the hypothesis after a failed prediction, with no independent reason to believe it. If every failure can be explained away like this, the hypothesis can never be tested and no longer tells us anything.

10. Hard A company claims that its Pellbrook vitamin gummies help students score higher on tests. It gave the gummies to 30 students for a month, and 25 of them scored higher at the end of the month. Name two things missing from this test and explain why each matters.

▼ **Show answer**

Any two: (1) A control group: students often score higher at the end of a month anyway, through study and practice, so we need similar students who did not take the gummies for comparison. (2) Random assignment: if the company chose eager students for the gummies, the result reflects the students, not the gummies. (3) Blinding: students who know they are taking a “brain” gummy may try harder; a look-alike gummy with no vitamins would rule this out. (4) Independent repetition: the test was run by the company that sells the product. (5) More cases: 30 students is a small sample.

11. Hard Which of Bacon’s idols is shown? Explain. (a) “Mrs. Lin was raised on a dairy farm, so she assumes every farm problem must come from the cows.” (b) “For centuries, scholars accepted a famous teacher’s whole system of nature without checking it against observation.”

▼ **Show answer**

(a) The idol of the cave: her mistake comes from her own particular upbringing and experience, not from human nature in general. (b) The idol of the theatre: they accepted a received system of thought on authority, as an audience accepts a play, without testing it.

12. **Challenge** Hannah thinks her houseplants grow better when she plays music for them. Design a fair test, name the control group, and state what result would count against her hypothesis. Then name one of Bacon's idols that might tempt her to misjudge the result, and explain how her design guards against it.

▼ **Show answer**

Sample answer: Take 12 similar plants of one kind and assign them at random to two groups in two rooms (or two ends of the house) with the same light, temperature, soil and watering. Play music in one room for an hour a day (experimental group); keep the other room quiet (control group). After six weeks, measure the height or count the leaves. If the music plants do no better than the quiet plants, that counts against her hypothesis. The idol of the tribe (seeing what she wishes to see) or the idol of the cave (her love of music) might tempt her to judge the music plants more kindly; she can guard against this by measuring with a ruler, recording numbers before comparing, or having someone who does not know which plants heard music do the measuring. Accept any design with a control group, one changed circumstance and a fair measure, with a reasonable idol named.

COMMON MISTAKES

- Thinking a passed test proves a hypothesis. It confirms it (makes it more probable); reasoning from a true prediction to a true hypothesis is affirming the consequent.
- Forgetting the control group. Without one, you cannot tell whether the change caused the result or would have happened anyway.
- Rescuing a favorite idea with an invented excuse after every failed prediction.
- Mixing up the idols: tribe is human nature in general; cave is one person's own bent; marketplace is words; theatre is received systems accepted on authority.

> **Scripture.** "Prove thy servants, I beseech thee, ten days; and let them give us pulse to eat, and water to drink." (Daniel 1:12, KJV)
>
> Daniel proposed a fair test with a comparison group, the young men who ate the king's meat, and let the result after ten days decide the matter.

7.5 Reasoning about Chance

OBJECTIVES

- Find the probability of an event with equally likely outcomes as a fraction, including the probability that it does not happen.
- Find the probability that two independent events both happen, by listing outcomes or by multiplying.
- Explain the fallacy of the maturity of chances (the "it's due" fallacy) and when it does and does not apply.
- Explain the conjunction error: "A and B" can never be more probable than "A" alone.

KEY TERMS

Probability

A number from 0 (impossible) to 1 (certain) that measures how likely an event is.

Outcome

One possible result of a chance process, such as rolling a 4 on a die.

Complement

The event that a given event does not happen. Its probability is 1 minus the probability of the event.

Independent events

Events in which the result of one does not change the probability of the other, such as two flips of a fair coin.

Fallacy of the maturity of chances

The error of thinking that a result is "due" because it has not happened lately, when the events are independent (also called the "it's due" fallacy).

Conjunction error

The error of judging that two things together (A and B) are more probable than one of them (A) alone.

CORE IDEAS

Measuring the likely. Induction deals in probability, and some probabilities can be measured with numbers. A **probability** is a number from 0 to 1. An impossible event has probability 0; a certain event has probability 1; everything else falls between. A probability can be written as a fraction, a decimal or a percent: a probability of 1/4 is

the same as 0.25 or 25 percent. In this lesson we use fractions, and we keep to simple cases: coins, dice in board games, spinners, marbles in a jar and seeds in a packet.

Counting equally likely outcomes. When every **outcome** is equally likely, the probability of an event is the number of outcomes that make it happen divided by the total number of outcomes. A die has six faces, so the probability of rolling a 5 is 1/6. Three faces are even (2, 4 and 6), so the probability of rolling an even number is 3/6, which is 1/2. A jar holds 3 red marbles and 7 blue ones; the probability of drawing a red marble without looking is 3/10. If two outcomes cannot happen together, the probability of one *or* the other is the sum: rolling a 1 or a 2 has probability 1/6 + 1/6 = 2/6 = 1/3.

The complement. Either an event happens or it does not, so the two probabilities add up to 1. The probability that something does *not* happen, its **complement**, is 1 minus the probability that it does. The probability of not rolling a 5 is 1 − 1/6 = 5/6. If the forecast gives a 3/10 chance of rain, it gives a 7/10 chance of no rain. The complement is often the easy way in. To find the chance of at least one six in two rolls, first find the chance of no six at all.

Two events at once. Flip two coins. The possible outcomes are heads-heads, heads-tails, tails-heads and tails-tails: four equally likely outcomes. So the probability of two heads is 1/4, and the probability of exactly one head is 2/4 = 1/2. Listing works, but there is a shortcut when the events are **independent**, meaning that one result does not change the chances of the other. Then the probability that both happen is the product: 1/2 × 1/2 = 1/4. For two dice, there are 6 × 6 = 36 outcomes, so the chance of rolling two sixes is 1/36.

Coins have no memory. Caleb flips a fair coin and gets heads five times in a row. He says, “Tails is due.” But the coin does not know what it did before. Each flip is independent, and the chance of tails on the next flip is still 1/2. This mistake is called the **fallacy of the maturity of chances**, or more simply the “it’s due” fallacy. It is true that six heads in a row is unlikely from the start (1/2 multiplied six times, or 1/64). But once five heads have already happened, only the sixth flip is in doubt, and its probability is 1/2. The same mistake runs the other way when someone says a fair die is “hot” and will keep rolling sixes.

When the past does matter. The fallacy applies only to independent events. Many events are not independent. If you draw marbles from a jar without putting them

back, each draw changes what is left. A pitcher who has thrown a hundred pitches is more likely to tire. And a long, strange run can be evidence that the process is not what you thought. If a coin lands heads twenty times in a row, the wise conclusion is not "tails is due" but "this coin may not be fair." If ten seeds in a row fail to sprout, they are not "due" to sprout; perhaps the seed is old or the soil is too cold.

The conjunction error. Read this: "Samuel is tall, practices his jump shot every afternoon and owns three basketballs." Which is more probable: (a) Samuel plays on a basketball team, or (b) Samuel plays on a basketball team and is the team captain? Many people choose (b), because the story makes it feel fitting. But (b) cannot be more probable than (a). Every case in which Samuel is a team captain is also a case in which he plays on a team. In general, the probability of "A and B" is never greater than the probability of A alone. Judging otherwise is the **conjunction error**.

Details make a story vivid, not likely. The conjunction error matters outside puzzles. A detailed story with many parts feels more believable than a plain one, but each added detail is one more thing that must be true, so the whole becomes less probable, not more. When a salesman, a rumor or our own imagination adds detail after detail, logic asks us to multiply the risks, not the confidence. As Ecclesiastes says, "time and chance happeneth to them all": we cannot know every outcome in advance, so we should reason about chances with care and humility, trusting the One who does.

> **From history.** The mathematics of probability began in 1654 with letters between two Frenchmen, Blaise Pascal and Pierre de Fermat. They solved an old puzzle: how to divide a prize fairly between two players when their game is stopped before anyone has won. Their answer depended on counting the ways the game could still have ended. Pascal was also a devout Christian thinker whose notes on faith were gathered after his death as the *Pensées*. In 1814 the French astronomer Pierre-Simon Laplace wrote an essay on probability for general readers, in which he said, roughly, that the theory of probability is at bottom only common sense reduced to calculation.

> **Practice habit.** When you hear "it's due" or a story full of details, ask: Are these events independent, and does each added detail make the whole more likely or less?

WORKED EXAMPLES

Example 1. A jar holds 10 marbles: 3 red, 5 green and 2 yellow. One marble is drawn without looking. Find the probability that it is (a) green, (b) not red, (c) red or yellow.

▼ **Show solution**

1. There are 10 equally likely outcomes.
2. (a) Green: 5 of the 10 marbles. 5/10 = 1/2.
3. (b) Not red: use the complement. The probability of red is 3/10, so not red is 1 − 3/10 = 7/10. (Check: 5 green + 2 yellow = 7 marbles.)
4. (c) Red or yellow: a marble cannot be both, so add: 3/10 + 2/10 = 5/10 = 1/2.

Answer: (a) 1/2. (b) 7/10. (c) 1/2.

Example 2. In a board game, Hannah needs to roll a 6 to move her piece out of the starting square. She has failed on her last four turns. Her brother says, "You're due for a six this time." (a) What is the probability she rolls a 6 on her next turn? (b) What fallacy did her brother commit? (c) What is the probability she fails on both of her next two turns?

▼ **Show solution**

1. (a) The die has no memory; each roll is independent. The probability of a 6 is 1/6, the same as on any other roll.
2. (b) Thinking a six is more likely because it has not come up lately is the fallacy of the maturity of chances (the "it's due" fallacy).
3. (c) The probability of not rolling a 6 on one turn is 5/6. The two turns are independent, so multiply: 5/6 × 5/6 = 25/36.

Answer: (a) 1/6. (b) The fallacy of the maturity of chances ("it's due"). (c) 25/36.

PRACTICE

1. Easy A fair die is rolled once. What is the probability of rolling a 4? Of not rolling a 4?

   ▼ **Show answer**

   Rolling a 4: 1/6. Not rolling a 4: 1 − 1/6 = 5/6.

2. **Easy** A spinner in a board game has 8 equal sections: 3 blue, 4 yellow and 1 white. What is the probability of landing on blue? On blue or white?

   ▼ **Show answer**

   Blue: 3/8. Blue or white: 3/8 + 1/8 = 4/8 = 1/2.

3. **Easy** What does it mean for two events to be independent? Give an example.

   ▼ **Show answer**

   The result of one does not change the probability of the other. Examples: two flips of a fair coin; rolling one die and then another; drawing a marble, putting it back, and drawing again.

4. **Easy** The forecast says there is a 2/5 chance of snow tomorrow. What is the chance of no snow?

   ▼ **Show answer**

   1 − 2/5 = 3/5.

5. **Medium** Two fair coins are flipped. List all the outcomes, and find the probability of (a) two tails, (b) exactly one tail, (c) at least one tail.

   ▼ **Show answer**

   Outcomes: HH, HT, TH, TT. (a) Two tails: 1/4. (b) Exactly one tail (HT, TH): 2/4 = 1/2. (c) At least one tail (HT, TH, TT): 3/4 (or 1 − 1/4 for HH).

6. **Medium** Two fair dice are rolled in a board game. What is the probability of rolling doubles (both dice showing the same number)?

   ▼ **Show answer**

   There are 6 × 6 = 36 equally likely outcomes, and 6 of them are doubles (1-1, 2-2, 3-3, 4-4, 5-5, 6-6). So the probability is 6/36 = 1/6.

7. **Medium** Caleb's little sister has flipped a fair coin four times and gotten tails each time. She says, "The next one has to be heads!" Is she right? Name the error.

   ▼ **Show answer**

   No. The flips are independent, so the probability of heads on the next flip is still 1/2. Thinking heads is "due" is the fallacy of the maturity of chances (the "it's due" fallacy).

8. Medium Which is more probable? (a) It will rain on Saturday. (b) It will rain on Saturday and the soccer game will be canceled. Explain.

▼ **Show answer**

(a). Every case in which it rains and the game is canceled is also a case in which it rains, so "rain and canceled" can never be more probable than "rain" alone. Choosing (b) would be the conjunction error. (They would be equally probable only if rain made cancellation certain.)

9. Hard A seed packet says that 4 out of 5 of its seeds sprout. Hannah plants two seeds in separate pots; assume each sprouts or fails independently. Find the probability that (a) both sprout, (b) neither sprouts, (c) at least one sprouts.

▼ **Show answer**

(a) $4/5 \times 4/5 = 16/25$. (b) Each fails with probability $1/5$, so $1/5 \times 1/5 = 1/25$. (c) At least one is the complement of neither: $1 - 1/25 = 24/25$.

10. Hard Mr. Okafor's nephew flips a coin 20 times and gets 20 heads. The nephew says tails is surely due. What would a careful reasoner say instead?

▼ **Show answer**

If the coin is fair, the flips are independent and the next flip is still 1/2 heads and 1/2 tails, so tails is not "due." But 20 heads in a row from a fair coin is extremely unlikely (1/2 multiplied 20 times, about 1 chance in a million), so the run is strong evidence that the coin is not fair, or that it is being flipped in a way that is not random. A careful reasoner would suspect the coin, and if anything would expect more heads, not tails.

11. Hard "Miriam is quiet, loves books and spends every Saturday at the library." Rank these from most probable to least probable, and explain: (a) Miriam is a teacher who reads mystery novels and lives in Ohio. (b) Miriam is a teacher. (c) Miriam is a teacher who reads mystery novels.

▼ **Show answer**

(b), then (c), then (a). Each added detail is one more thing that must be true, so (c) can be no more probable than (b), and (a) can be no more probable than (c). This holds whatever the story says about Miriam; the description may make (a) feel fitting, but ranking it higher would be the conjunction error. (Strictly, two could be equal, but none can be higher than the one with fewer conditions; accept "(b) at least as probable as (c), and (c) at least as probable as (a).")

12. Challenge In a board game you need to roll a 6 on one die to start. (a) What is the probability that you do not start within your first two turns? (b) What is the probability that you do start within your first two turns? (c) After missing on your first five turns, what is your chance of a 6 on the sixth turn? Explain why (c) does not contradict the fact that missing six times in a row is less likely than not.

▼ **Show answer**

(a) Missing on one turn has probability 5/6, so missing twice is $5/6 \times 5/6 = 25/36$. (b) The complement: $1 - 25/36 = 11/36$. (c) 1/6, because the rolls are independent. Before any rolls, missing six times in a row has probability 5/6 multiplied six times (15,625/46,656, about 1 in 3). But once five misses have already happened, they are no longer uncertain; only the sixth roll is in doubt, and its chance of a 6 is 1/6 like any other roll. Full credit for (c) needs the 1/6 and the idea that past rolls are already settled; the exact fraction for six misses is not required.

COMMON MISTAKES

- Thinking a result is "due" after a run of other results, when the events are independent.
- Forgetting that a very long run can be evidence that a process is not fair or not random.
- Judging a detailed story more probable than a simpler part of it. Adding "and" can never raise a probability.

- Adding probabilities of events that can happen together, or multiplying probabilities of events that are not independent.
- Forgetting the complement: the probability that something does not happen is 1 minus the probability that it does.

> **Scripture.** “the race is not to the swift, nor the battle to the strong, neither yet bread to the wise, nor yet riches to men of understanding, nor yet favour to men of skill; but time and chance happeneth to them all.” (Ecclesiastes 9:11, KJV)
>
> The Preacher saw that outcomes under the sun are often uncertain to us; reasoning well about chance is part of living wisely in a world we do not control, under a God who does.

UNIT 7 · INDUCTION AND SCIENTIFIC REASONING · LESSON 7.6 OF 69 · 40 MINUTES

7.6 Testimony and Evidence

OBJECTIVES

- Explain why accepting testimony is reasonable, and why it must be weighed.
- Weigh a witness by competence, honesty and consistency, and tell when a witness’s interest matters.
- Explain why two witnesses add strength only if they are independent, and recognize signs of copying or collusion.
- Tell a primary source from a secondary source, and trace a claim back to its source.

WATCH

Evaluating a source's reasoning and evidence

Khan Academy · 7:22

https://www.youtube.com/watch?v=v4HhzvSgUpU

> **WHEN TO WATCH**
> After the rules for weighing witnesses: claim, evidence and reasoning, and why independent sources matter.

Note: Parent preview: A Khan Academy reading teacher contrasts a joke conspiracy ("goblins live in the moon," proof only he can see) with a passage that defends the 1969 moon landings using independent, non-NASA sources (observatories in the UK and Germany, later photographs). A brief neutral mention of the Cold War and Vietnam as background. No partisan content. Nothing objectionable; transcript checked October 18, 2026.

KEY TERMS

Testimony

A report by a witness, spoken or written, that something is so; evidence that comes to us through another person.

Competence

A witness's ability to know the thing reported: being present, able to observe it, attentive, and (for expert matters) trained in the field.

Honesty (sincerity)

A witness's willingness to report what he or she believes to be true, judged by character, record and any reason to deceive.

Corroboration

Support for one witness or source from another independent witness, source or piece of physical evidence.

Independent witnesses

Witnesses who learned what they report separately, each from his or her own observation, not from each other or from a common source.

Primary source

A record made at or near the time by someone in a position to know, such as a letter, diary, log book or eyewitness report.

Secondary source

A later account that draws on primary sources, such as a history book or an encyclopedia article.

Hearsay

A report of what someone else said, rather than of what the witness personally saw or heard happen.

CORE IDEAS

We live by testimony. You believe that you were born on a certain day, that Rome is in Italy and that George Washington led the Continental Army, though you saw none of these things. You know them by **testimony**: the word of parents, maps, books and teachers. Augustine noticed this in his *Confessions*: he believed, roughly, a great many things he had never seen, about the history of nations and about places he had never visited, and he could not even know who his parents were except by believing what he was told. Trusting testimony is not gullible. Refusing all testimony would leave us knowing almost nothing. But testimony must be weighed, and logic gives us the scales.

Test one: competence. Was the witness in a position to know? A witness must have been present, able to observe (with good light, a clear view and working eyes and ears), and paying attention. Mrs. Diaz, who watched the collision from her porch twenty feet away, is a better witness than Mr. Hale, who heard a crash from inside his house and looked out afterward. On matters that need training, competence includes expertise: a veterinarian is competent to say whether a horse is lame, but on a question about bridges she is simply another witness. Be wary of **hearsay**, a report of what someone else said, because it adds another link where error can enter.

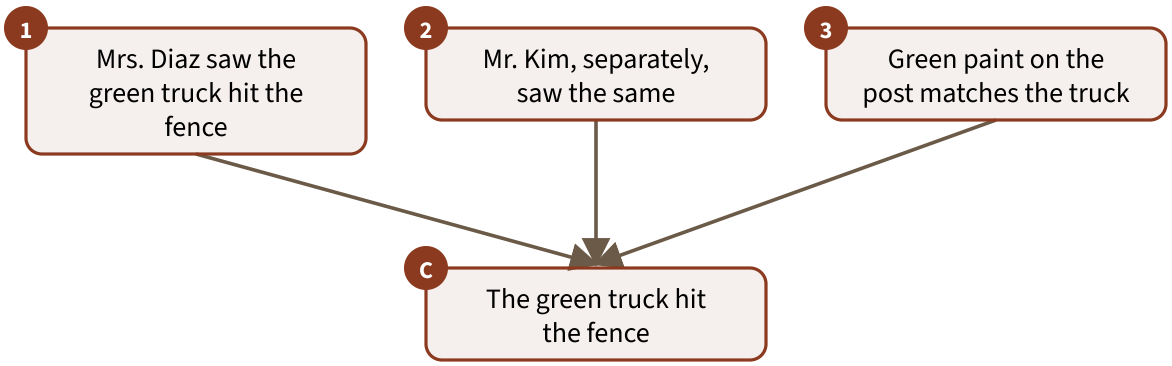
Test two: honesty. Was the witness willing to tell the truth? Look at the witness's character and record, and ask whether the witness has a reason to deceive. A witness's *interest*, what he gains or loses by the testimony, is a fair question here; Unit 3 showed that this is not an ad hominem attack when the issue is whether to trust a witness's word. But interest is a reason for care, not proof of a lie. Testimony that goes *against* the witness's own interest deserves special weight: when the boy who broke the window admits it, his testimony is all the more believable.

Test three: consistency. Does the testimony hang together? It should agree with itself (a witness who says the truck was green and later that it was red has a problem),

with other witnesses, and with facts already known. But notice a surprising point. Honest independent witnesses usually agree in the main facts and differ in small details, because each saw from a different place and remembers differently. Stories that match word for word, even in trivial details, suggest that the witnesses rehearsed together or copied one source. Agreement in substance with variety in detail is a mark of honest, independent testimony.

Two or three witnesses. The law of Moses required more than one witness to establish a matter. Deuteronomy 19:15 says that "at the mouth of two witnesses, or at the mouth of three witnesses, shall the matter be established." The logic is sound. One witness may be mistaken or dishonest; two **independent witnesses** who agree are much less likely to be both mistaken in the same way. But the key word is independent. If Hannah tells Caleb what she saw, and then both report it, there is only one witness, repeated twice. Ten newspapers printing the same story from one reporter are one source, not ten.

Mapping the support. Independent witnesses each give a separate reason for the conclusion; if one is discredited, the others still stand. An argument map shows this: each witness has its own arrow to the conclusion, rather than being joined to the others.



Physical evidence, such as the paint on the post, is another independent line, and it can **corroborate** testimony or count against it.

Primary and secondary sources. Historians weigh written testimony the same way. A **primary source** was made at or near the time by someone in a position to know: a soldier's letter home, a ship's log, a farmer's account book. A **secondary source** is a later account built on primary sources, like a history textbook. Secondary sources

are useful, but the closer a source is to the event in time and place, the less room for error. When you meet a startling claim, trace it back. Who first said it, and how did that person know? “Everyone says” and “I read somewhere” are not sources.

Unusual claims and honest weighing. The more surprising a claim, the stronger the testimony we should ask for before believing it. A neighbor’s report that her hens laid eggs this morning needs little support; a report that her hen laid a golden egg needs a great deal. But we must not swing to the opposite error and reject well-attested events simply because they are unusual. Many true events in history were unusual. The honest course is to apply the same tests every time: competence, honesty, consistency and independence, whether or not we like the conclusion.

> **From history.** In 1819 Richard Whately, who later wrote a widely used textbook of logic, published a short satire called *Historic Doubts Relative to Napoleon Buonaparte*. Napoleon was still alive at the time. Whately applied to the reports about Napoleon the same extreme doubts that some skeptics of his day applied to the testimony of the Bible, and showed that by those standards one would have to doubt that Napoleon ever existed. His point was that the skeptics’ rules proved too much. In 1846 Simon Greenleaf, a Harvard law professor and an authority on the law of evidence, published a book examining the testimony of the four Gospel writers by the rules of evidence used in courts of law.

> **Practice habit.** When you hear a claim, ask: Who first said this, how did they know, and is there anyone independent who agrees?

> **Enrichment: Whately’s Napoleon.** Richard Whately’s *Historic Doubts Relative to Napoleon Buonaparte* (1819) is short and in the public domain. Read its opening pages and notice how Whately pretends to reason as a skeptic would. Then write a paragraph answering: What tests of testimony from this lesson would show that Napoleon really existed? Why did Whately think a fair reader would also have to apply those tests to the testimony of Scripture?

Example 1. Two players say the ball crossed the goal line; the referee says it did not. Weigh the testimony: Player A plays for the team that would gain the goal and was 40 yards away. Player B plays for the other team and was standing on the goal line. The referee was 15 yards away, at a poor angle.

▼ **Show solution**

1. Competence: B was in the best position (on the line). A was 40 yards away, a poor position. The referee was closer than A but at a poor angle.
2. Honesty: A has an interest in the goal counting, so A's word needs care. B's testimony goes against B's own team's interest, which gives it extra weight. The referee has no stake either way.
3. Consistency: A and B agree on the main fact. They are on opposite teams, so they are unlikely to have arranged their stories together.
4. Overall: B is both the most competent witness and testifying against interest; A adds a little support. The referee is honest but poorly placed.

Answer: The testimony that the ball crossed the line is stronger, chiefly because Player B was in the best position and spoke against his own team's interest. Player A's report adds little because of distance and interest; the referee is disinterested but poorly placed.

Example 2. Three neighbors report that the creek flooded the Miller road on Tuesday night. Later you learn that two of them did not see the flood; they heard about it from the third, Mr. Shaw, who drove through it. How many independent witnesses are there, and what kind of testimony do the other two give?

▼ **Show solution**

1. Ask how each witness learned what he reports.
2. Mr. Shaw saw the flood himself: a firsthand witness.
3. The other two learned it from Mr. Shaw. Their reports are hearsay and are not independent of his.
4. So the three reports rest on one observation.

Answer: Only one independent witness, Mr. Shaw. The other two give hearsay that repeats his testimony and adds no independent support (though it does show that he told the same story to more than one person).

PRACTICE

1. Easy Name four tests for weighing a witness given in this lesson.

   ▼ **Show answer**

   Competence, honesty (sincerity), consistency and independence. (Accept closely related tests from the lesson, such as firsthand knowledge rather than hearsay, expertise in the field, or corroboration by other evidence, in place of one of these.)

2. Easy Primary or secondary source? (a) A diary kept by a soldier during the war. (b) A history of the war written eighty years later. (c) A ship's log written by the captain during the voyage.

   ▼ **Show answer**

   (a) Primary: made at the time by someone in a position to know. (b) Secondary: a later account built on earlier sources. (c) Primary: written during the voyage by someone aboard.

3. **Easy** What is hearsay, and why is it weaker than firsthand testimony?

▼ Show answer

Hearsay is a report of what someone else said, rather than of what the witness personally saw or heard happen. It is weaker because it adds another link where error or distortion can enter, and the original witness cannot be questioned.

4. **Easy** Write out the part of Deuteronomy 19:15 that tells how many witnesses are needed to establish a matter.

▼ Show answer

"at the mouth of two witnesses, or at the mouth of three witnesses, shall the matter be established."

5. **Medium** Which witness is more competent to say whether a dog bit the mail carrier? (a) A neighbor who watched from across the street. (b) A neighbor who heard barking from her kitchen and came out a minute later. Explain.

▼ Show answer

(a): she was present and watching when it happened, while (b) only heard barking and arrived afterward, so she did not observe the bite itself.

6. **Medium** A veterinarian says, "This horse has a stone bruise in its left front hoof," and later says, "The new bridge on Route 4 is unsafe." Which statement carries the weight of expert testimony? Explain.

▼ Show answer

The first: diagnosing a horse's hoof is within a veterinarian's field. On the bridge she is not an expert, so her opinion counts only as an ordinary witness's (unless she reports something she plainly observed, such as a crack, which any witness could report).

7. **Medium** Caleb admits, "I left the gate open, and that's how the goats got out." Why is this testimony especially believable?

▼ Show answer

It goes against his own interest: he gains nothing and may be blamed or punished by saying it, so he has no motive to invent it.

8. Medium The Fernhill Gazette and four other papers all report that a famous ballplayer will visit the county fair. You learn that all five printed the same short notice sent out by one writer. How many independent sources are there? Explain.

▼ **Show answer**

One: all five papers repeat the same notice from one writer, so they are not independent. Five printings of one report add no more support than the report itself.

9. Hard Two witnesses to a barn fire tell the police exactly the same story, word for word, including that the smoke was “the color of a storm cloud at dusk.” A third witness agrees on the main facts but describes the smoke differently and is unsure of the time. Which pattern is more typical of honest, independent testimony? Explain.

▼ **Show answer**

The third witness’s pattern, agreement on the main facts with differences in small details, is typical of honest independent witnesses, who see from different places and remember differently. Word-for-word agreement, even in an unusual phrase, suggests that the first two rehearsed together or copied one account, so they may count as only one source. (This does not prove the first two are lying, but it is a reason to question them separately.)

10. Hard Mrs. Lin owns the Lakeside Orchard and testifies that her neighbor’s cattle broke through the fence and trampled her trees. Her neighbor says, “Of course she says that; she wants me to pay for the trees, so we can ignore her.” Is the neighbor’s reply a fair point, a fallacy, or partly both? Explain.

▼ **Show answer**

Partly both. It is fair to note that Mrs. Lin has an interest in the outcome, so her testimony should be checked with care. But it is a fallacy (ad hominem, circumstantial) to conclude that her testimony can be ignored or is false simply because she has an interest. Her account should be weighed by the usual tests and checked against other evidence, such as hoof prints, the broken fence and other witnesses.

11. Hard A website says, “Experts agree that Glimmer toothpaste whitens teeth in three days.” Write three questions you would ask to weigh this testimony.

▼ **Show answer**

Any three, such as: Which experts, by name, and are they dentists or others trained in this field? Did they examine Glimmer themselves, or is this a report of what someone else said? Were they paid by the company or do they have some other interest? Do other independent experts agree? Is there a published test, with a control group, that I can check? Who runs the website?

12. Challenge Joel says the old Brookside covered bridge was built in 1868. Describe three different kinds of evidence that could support or test his claim, rank them from strongest to weakest, and explain your ranking.

▼ **Show answer**

Sample answer: (1) A primary source, such as the county’s 1868 record of payment to the builders or a dated builder’s plate on the bridge (strongest: made at the time by people in a position to know, with little reason to err). (2) A letter or diary from someone in town at the time mentioning the new bridge (strong, especially if independent of the county record; it corroborates it). (3) A county history written in 1950 that gives the date (secondary: useful, but only as good as the sources it used, and errors can be copied). Weakest of all would be “everyone around here says so,” which may be a single story repeated. Any ranking that puts primary, independent evidence above later or repeated reports, with reasons, earns full credit.

COMMON MISTAKES

- Counting repeated reports as independent witnesses. Ask how each witness learned what he reports.
- Treating a witness’s interest as proof that he is lying. Interest is a reason for care, not a refutation.
- Trusting an expert outside his or her field as if it were inside it.
- Expecting honest witnesses to agree in every detail. Identical stories can be a warning sign; agreement in substance with variety in detail is normal.
- Rejecting well-attested events simply because they are unusual, or accepting surprising claims on weak testimony.

> **Scripture.** “at the mouth of two witnesses, or at the mouth of three witnesses, shall the matter be established.” (Deuteronomy 19:15, KJV)
>
> God’s law guarded justice by requiring more than one witness; two independent witnesses are far less likely than one to be mistaken or false in the same way.

UNIT 7 · INDUCTION AND SCIENTIFIC REASONING · LESSON 7.7 OF 69 · 45 MINUTES

7.7 Best Explanation, and the Reach of Science

OBJECTIVES

- Reason to the best explanation by listing rival explanations and comparing them by fit, scope, simplicity and agreement with what is well known.
- Tell a claim that is observed and repeatable from an inference about the unobserved past, and name the assumptions such an inference depends on.
- Explain why every inductive argument assumes that nature is orderly, and why induction cannot prove that assumption without arguing in a circle.
- Explain how the doctrine of creation gives a reason to expect an orderly, knowable world.

KEY TERMS

Inference to the best explanation

Reasoning from a set of facts to the explanation that, among the available rivals, accounts for them best (also called abduction).

Fit

How well an explanation accounts for all the facts without conflicting with any of them.

Scope

How many facts, and how many kinds of facts, an explanation accounts for.

Simplicity

Making the fewest unsupported assumptions and adding no ad hoc parts.

Observed and repeatable

Said of a claim that can be checked by observation or experiment now, and checked again by others.

Inference about the past

A conclusion about an event that cannot be observed or repeated now, reached from present traces and testimony together with assumptions about how things went.

Uniformity of nature

The assumption, made in every inductive argument, that nature follows the same orderly laws in unobserved cases as in observed ones.

CORE IDEAS

Reasoning to an explanation. Caleb finds the lettuce in the garden nibbled to the ground. There are small round droppings between the rows and a gap, about four inches high, under the fence. He concludes that a rabbit got in. He did not see a rabbit. He reasoned from the facts to the explanation that accounts for them best. This kind of reasoning is called **inference to the best explanation**, and the American logician Charles Sanders Peirce called it *abduction*. Detectives, doctors, mechanics, historians and natural scientists use it every day. It is inductive: the best explanation is probably true, not certainly.

First, list the rivals. The most common mistake in reasoning to an explanation is to stop at the first one that comes to mind. Before choosing, list the reasonable rival explanations. For Caleb's lettuce: a rabbit, a deer, the neighbor's goat, or his little brother picking it for a salad. A good explanation is not merely one that could account for the facts; it is the one that accounts for them better than its rivals. If you have considered only one explanation, you have not yet found the best one.

Fit and scope. The first test is **fit**: does the explanation account for all the facts, and does it conflict with none? A deer could eat lettuce, but a deer cannot crawl through a four-inch gap, and deer droppings are larger. The second test is **scope**: how many facts, and how many kinds of facts, does the explanation cover? The rabbit explains the nibbling, the droppings and the gap. The little brother explains the missing

lettuce but not the droppings or the gap. Other things being equal, prefer the explanation that fits all the facts and covers the most.

Simplicity and agreement with what we know. The third test is **simplicity**: prefer the explanation that makes the fewest unsupported assumptions. “A rabbit got in” is simpler than “a goat got in, squeezed under the fence somehow, and a rabbit happened to leave droppings on the same night.” Isaac Newton made this his first rule of reasoning: admit, roughly, no more causes than are true and sufficient to explain what we see. The fourth test is agreement with what is already well known. An explanation that requires us to give up many well-tested beliefs needs much stronger evidence. Finally, a good explanation can be tested: it suggests new predictions (rabbit tracks in the morning dew), as Lesson 7.4 showed.

Two kinds of questions. Science answers two different kinds of questions, and they reach their answers in different ways. Some claims are **observed and repeatable**: “Pure water at sea level boils at 212 degrees Fahrenheit,” or “This bean variety sprouts in about eight days in warm soil.” Anyone with the right equipment can test them again today, and if they are wrong, repeated tests will show it. Other claims are **inferences about the past**: “The stone wall on the Miller farm was built around 1820,” or “The fire in the old mill began in the boiler room.” Those events happened once. No one can rerun them; we can only reason back from present traces and testimony.

Every inference about the past rests on assumptions. Suppose a candle that was 8 inches tall now measures 6 inches, and candles of this kind burn about 1 inch per hour. Was it lit two hours ago? Only if it started at full height, burned at a steady rate the whole time, was never blown out and relit, and was never trimmed. Those assumptions may be very reasonable, but they are assumptions; the burning itself was not observed. Inferences about the past can be well supported, especially when several independent lines of evidence agree and when there is reliable eyewitness testimony (Lesson 7.6). But they should be held with the confidence their evidence and assumptions allow, and the assumptions should be stated openly.

The assumption behind all induction. Every inductive argument assumes that nature is orderly: that the unobserved cases will follow the same laws as the observed ones, that water will boil tomorrow as it did today. This is the **uniformity of nature**. Can it be proved by induction? Only by arguing, “Nature has been orderly

in the past, so it will probably be orderly in the future," and that argument itself assumes the very order it is trying to prove. It argues in a circle. Induction cannot supply its own foundation. Every scientist must simply assume an orderly world before the first experiment begins.

Science assumes an orderly creation. The Christian has a reason for that assumption. The world is not an accident; it was made by a wise God who upholds it faithfully. After the flood, God promised, "While the earth remaineth, seedtime and harvest, and cold and heat, and summer and winter, and day and night shall not cease" (Genesis 8:22). Through Jeremiah he spoke of the "ordinances of heaven and earth" (Jeremiah 33:25) as something he had appointed. Many of the founders of modern science, such as Johannes Kepler, Robert Boyle and Isaac Newton, were believers who expected to find order in nature because they believed in the Creator. Logic and science, rightly used, are ways of reading a world that God made readable.

> **From history.** In the *Principia* (first published in 1687), Isaac Newton set down "rules of reasoning" for natural science, which he revised in later editions. The first rule says, roughly, that we should admit no more causes of natural things than are both true and sufficient to explain them. The second says that the same natural effects should, as far as possible, be assigned the same causes. A later rule says that conclusions gathered by induction from what we observe should be held as accurately or very nearly true until other observations make them more exact or show exceptions. Newton, a lifelong student of Scripture, added to the second edition a closing section on the God whose wisdom and power he believed the order of nature displayed.

> **Practice habit.** When you meet a puzzling fact, list at least two explanations before you choose one, and ask which assumptions each one needs.

Example 1. Mrs. Ortiz comes home to find the kitchen window open, the curtains wet, a puddle under the window and papers scattered on the floor. The sky is clear now, but the street is wet. Compare two explanations: (A) A thunderstorm with wind blew rain and air through the open window while she was out. (B) Someone broke in, spilled water and threw the papers around.

▼ **Show solution**

1. Fit: (A) fits the wet curtains, the puddle under the window, the scattered papers (blown by wind) and the wet street. (B) fits the papers and the water, but not the wet street, and nothing is missing or broken.
2. Scope: (A) explains every fact listed, including the wet street outside. (B) explains only the indoor facts.
3. Simplicity: (A) needs only a passing storm, which a wet street already suggests, and an open window. (B) needs an intruder with a strange purpose and no sign of entry or theft.
4. Agreement with what is known: storms are common; burglars who spill water and take nothing are not.
5. Testability: (A) predicts that neighbors saw rain and that other houses' yards are wet; she can check.

Answer: (A) is the better explanation: it fits all the facts, covers more of them (including the wet street), needs fewer unsupported assumptions and agrees with ordinary experience. It is probably, not certainly, true.

Example 2. Label each claim as observed and repeatable or as an inference about the past, and for the inference, name one assumption it depends on. (a) A ball of this clay shrinks about one-tenth when fired in a kiln. (b) The clay pot found in the Fieldings' attic was made in the 1880s.

▼ **Show solution**

1. (a) Anyone can fire another ball of the same clay today and measure it. Observed and repeatable.
2. (b) The making of the pot happened once, long ago; no one can watch it again. We reason from traces (style, maker's mark, family records) and testimony. Inference about the past.
3. Assumptions for (b): for example, that the maker's mark is genuine and not added later, that the style was used only in the 1880s, or that the family's records are accurate.

Answer: (a) Observed and repeatable. (b) Inference about the past; it assumes, for example, that the maker's mark is genuine or that the style was used only in that decade. (Any reasonable assumption is correct.)

PRACTICE

1. Easy Name four tests for comparing rival explanations given in this lesson.

   ▼ **Show answer**

   Any four of: fit (accounts for all the facts, conflicts with none), scope (covers more facts and kinds of facts), simplicity (fewest unsupported assumptions), agreement with what is already well known, and testability (suggests new predictions).

2. Easy What did Charles Sanders Peirce call inference to the best explanation?

   ▼ **Show answer**

   Abduction.

3. **Easy** Observed and repeatable, or an inference about the past? (a) Ice melts faster in warm water than in cold water. (b) The old oak by the church was planted by the town's first pastor.

▼ Show answer

(a) Observed and repeatable: anyone can test it again today. (b) An inference about the past: the planting happened once and can only be reasoned to from records, testimony or other traces.

4. **Easy** What is the uniformity of nature?

▼ Show answer

The assumption, made in every inductive argument, that nature follows the same orderly laws in unobserved cases (including the future) as in observed ones.

5. **Medium** The car will not start. The headlights are very dim, and the car was parked outside all night in bitter cold. Compare two explanations: (A) the battery is weak or dead; (B) the car is out of gas. Which is better, and why?

▼ Show answer

(A) is better. It fits and explains both facts: a weak battery causes dim headlights, and cold weakens batteries. (B) might explain why the car will not start, but it does not explain the dim headlights (the lights do not depend on gas). (A) has greater fit and scope.

6. **Medium** Hannah finds the cookie jar empty and crumbs on her brother's bed. Her brother says, "A mouse must have carried the cookies onto my bed and eaten them there." Which test of a good explanation does his explanation fail most clearly?

▼ Show answer

Simplicity (and agreement with what is known): it adds an unlikely assumption (a mouse carrying cookies onto a bed) with no evidence for it, when a simpler explanation fits the facts. Accept fit or agreement with what is known if the student explains, for example, that mice do not carry whole cookies onto beds or that there are no other signs of a mouse.

7. Medium A candle that was 10 inches tall is now 7 inches tall. Candles of this kind burn about 1 inch per hour. Caleb concludes that it was lit 3 hours ago. Name two assumptions his conclusion depends on.

▼ **Show answer**

Any two: that the candle was 10 inches tall when it was lit (no one burned it earlier); that it burned at a steady rate the whole time; that it was never blown out and relit; that no one trimmed or broke it; that drafts did not make it burn faster or slower.

8. Medium Why can induction not prove the uniformity of nature?

▼ **Show answer**

Any inductive argument for it would run, "Nature has been orderly in the past, so it will probably be orderly in the future." But that is itself an inductive argument, which already assumes that the future will resemble the past. It assumes what it is trying to prove, so it argues in a circle.

9. Hard The Fieldings' well water suddenly tastes of iron. They list three explanations: (A) a rusty old pipe broke and was replaced last week, and some rust was stirred loose; (B) the underground water has changed for some unknown reason; (C) a neighbor secretly added iron to their well. Rank the explanations and give reasons, using at least three tests.

▼ **Show answer**

Best is (A): it fits the timing (the taste began after the pipe work), agrees with what is known (disturbing old pipes stirs up rust), is simple, and is testable (the taste should fade after flushing the pipes, and the new pipe will be free of rust). Next is (B): it might be true, but it explains nothing in particular ("some unknown reason"), so it has little scope and is hard to test. Last is (C): it needs an unlikely and unsupported assumption (a secret, strange act with no motive or evidence), so it fails simplicity and agreement with what is known. Accept a ranking of (C) above (B) only with a sensible reason; (A) must be first.

10. Hard Explain the difference between how we know “This steel beam bends under a load of 2,000 pounds” and how we know “The old mill burned down because a lantern was knocked over.” Is one of these claims necessarily false or unreasonable?

▼ **Show answer**

The first is observed and repeatable: engineers can load another beam of the same steel today and watch it bend, and others can check the result. The second is an inference about a past event that happened once; it rests on traces (where the fire started, a melted lantern) and testimony, together with assumptions about what happened that night. Neither is necessarily false or unreasonable. The second can be well supported, but it rests on assumptions and cannot be tested again directly, so it should be held with the confidence its evidence allows.

11. Hard A student says, “Science has proved that nature is orderly, so we don’t need any other reason to trust it.” Answer in three or four sentences.

▼ **Show answer**

Sample answer: Science does not prove that nature is orderly; it assumes it. Every experiment assumes that the same causes will produce the same effects tomorrow as today, and any argument that nature will stay orderly because it always has been orderly is itself inductive and so argues in a circle. The Christian has a reason for the assumption: a faithful Creator made and upholds the world, as Genesis 8:22 promises that seedtime and harvest, cold and heat, and day and night shall not cease. Many founders of modern science expected order in nature for exactly this reason.

12. **Challenge** On a winter morning Joel finds the barn door open, the hay bale torn apart, large hoofprints in the snow leading from the pasture gate, and the gate latch hanging loose. Write two rival explanations, compare them by at least three tests, and state the best explanation with one prediction that would test it.

▼ **Show answer**

Sample answer: (A) The horse worked the loose gate latch open, walked to the barn, pushed the door open and ate the hay. (B) Someone left both the gate and the barn door open last night, and a passing deer tore the hay apart. Fit: (A) explains the loose latch, the large hoofprints from the pasture and the torn hay; (B) explains the open door, but deer prints are small and split, and the prints come from the pasture gate. Scope: (A) covers every fact; (B) leaves the large hoofprints from the pasture unexplained. Simplicity: (A) needs one animal and one loose latch; (B) needs two separate mistakes and a deer. Best: (A). Prediction: the horse will be found near the barn or with hay on its muzzle, and the hoofprints will match its shoes. Any two sensible rivals compared by at least three named tests, with a testable prediction, earns full credit.

COMMON MISTAKES

- Stopping at the first explanation that fits. Always list the rivals before choosing.
- Thinking the best explanation is certainly true. It is the most probable of the explanations considered.
- Treating an inference about the past as if it had been observed. Name the assumptions it depends on.
- Treating every inference about the past as worthless. Such inferences can be well supported; they simply rest on evidence and assumptions of a different kind.
- Thinking science proves that nature is orderly. Science assumes it.

> **Scripture.** “While the earth remaineth, seedtime and harvest, and cold and heat, and summer and winter, and day and night shall not cease.” (Genesis 8:22, KJV)
>
> God’s promise of a faithful, orderly world is the ground on which every farmer plants and every scientist expects tomorrow to keep today’s laws.

Statistical Deception

Numbers carry a special authority. A claim with a percent, a graph or an average in it sounds as if it has already been checked, and that is exactly why numbers are so easily used to mislead. Most statistical deception uses no false numbers at all. It uses true numbers framed so that they seem to say more than they do. This unit teaches you to see through that framing and to answer it: **how can a claim be true in every number and still leave a false impression, and how do you show that plainly and fairly?**

Lessons 8.1–8.7 are shared with the Statistics course, where they are Lessons 14.1–14.7. You will read each one on the Statistics pages and do its **Practice Set L**, the lighter set written for logic students. Those lessons cover graphs that deceive (8.1), maps and scales (8.2), who got counted (8.3), averages and percents (8.4), relative risk and base rates (8.5), correlation and cause (8.6), and regression to the mean and streaks (8.7). Each one ties a statistical trick to a fallacy you already know: accent, composition and division, hasty generalization and suppressed evidence, equivocation, the base-rate fallacy and the appeal to fear, false cause, and the regression fallacy and the fallacy that a chance result is "due." Read the core ideas and worked examples with care; the Statistics practice set (Set S) is not required.

Lesson 8.8, written for this course alone, gathers the whole unit into one skill: **refuting a statistical claim**. You will learn to restate a claim fairly, lay it out as an argument, name the fallacy and show it with numbers, say what evidence would settle the question, and write a short, courteous refutation that claims no more than it has shown. The unit test covers all eight lessons with light arithmetic. This skill leads straight into Unit 9, where you will map, answer and debate arguments of every kind.

Review day and test. On the review day, use the mastery checklist and the review sheet (with its answer key). The unit test is below; a printable copy is ready for test day.

PREREQUISITES

- Find the premises and conclusion of an argument and write it in standard form (Unit 1).
- Name and explain the common informal fallacies, including equivocation, accent, composition, division, hasty generalization, false cause and the straw man (Unit 3).
- Compute a percent, a percent change and a simple fraction of a group (Pre-Algebra).
- Explain why a comparison group and a fair sample matter in testing a cause (Unit 7).

MASTERY CHECKLIST

- ☐ I can spot a cut-off axis, a hand-picked time window or a swollen picture on a graph, find how much it exaggerates, and explain how it commits the fallacy of accent.
- ☐ I can tell totals from rates on a map, compute a rate per 1,000, and recognize the fallacies of composition and division.
- ☐ I can recognize self-selected samples, survivorship bias and cherry-picked evidence, and name the missing cases that would settle a claim.
- ☐ I can choose between the mean and the median, tell percentage points from percent change, and spot equivocation on “average.”
- ☐ I can turn a relative risk into absolute numbers out of 1,000 and use a frequency tree to show why most positive results for a rare condition are false (the base-rate fallacy).
- ☐ I can name reverse cause, a common cause and coincidence as alternatives to “A causes B,” and explain a simple Simpson’s paradox.
- ☐ I can explain regression to the mean and the error of thinking a chance result is “due,” and say why a group chosen for being extreme needs a comparison group.

- ☐ I can restate a statistical claim fairly and lay it out as an argument in standard form with its hidden premise.
- ☐ I can write a short, courteous refutation that names the fallacy, shows it with numbers, says what evidence would settle the question and gives a measured verdict.

LESSONS IN THIS UNIT

8.1	**Graphs That Deceive**	45 minutes
8.2	**Maps and Scales That Deceive**	45 minutes
8.3	**Who Got Counted?**	45 minutes
8.4	**Averages and Percents That Mislead**	45 minutes
8.5	**Risk: Relative, Absolute and Base Rates**	45 minutes
8.6	**Correlation, Cause and Lurking Variables**	45 minutes
8.7	**Regression to the Mean and Streaks**	45 minutes
8.8	**Refuting a Statistical Claim**	45 minutes

Unit 8 test

You have 50 minutes. No notes or books; you may use a calculator, though the arithmetic is light. Show your work for every calculation. When a question asks you to name a fallacy, name it and give a one-sentence reason. Question 14 is a written refutation of four to six sentences. The test covers Lessons 8.1–8.8 and is worth 100 points.

1. **6 pts**

   (Lesson 8.1) A lawn-mower ad compares how long two battery mowers run on one charge: the Fernbrook mower 55 minutes and a rival 50 minutes. The bars are drawn on a vertical axis that starts at 45 minutes. (a) How many times as tall as the rival's bar is Fernbrook's bar? (b) What percent longer does the Fernbrook mower really run? (c) Name the fallacy and say in one sentence how the graph commits it.

2. **6 pts**

   (Lesson 8.1) A pumpkin farm's visitors tripled, from 2,000 one fall to 6,000 the next. Its flyer shows a pumpkin for each year, the second drawn three times as tall and three times as wide as the first. (a) How many times the area does the second pumpkin cover? (b) Find the exaggeration factor (picture ratio divided by data ratio). (c) How should the growth be drawn honestly?

3. **7 pts**

   (Lesson 8.2) Last year Ashford County had 120 lost-dog reports among 60,000 people, and Birchwood County had 45 among 9,000 people. A newspaper map shaded by total reports makes Ashford the darkest county. (a) Find each county's reports per 1,000 people. (b) In which county is a person more likely to report a lost dog? (c) A reader says, "Ashford has the most lost-dog reports, so Ashford dog owners must be careless." Name two problems with this reasoning.

4. **6 pts**

   (Lesson 8.2) Composition or division? Give a reason for each. (a) "Each player on the Wildcats is an excellent shooter, so the Wildcats must be an excellent team." (b) "Maple County has the highest average farm income in the state, so the Pikes, who farm in Maple County, must have a high income."

5. **7 pts**

   (Lesson 8.3) A music school's brochure says, "30 of our 40 graduates now play in a community orchestra!" Over the same years, 200 students enrolled in its four-year program. (a) What percent of the graduates play in an orchestra? (b) What percent of all the students who enrolled play in an orchestra? (c) Name the error in the brochure and the missing data that would matter.

6. **6 pts**

   (Lesson 8.3) A radio station asks listeners to call in and answer, “Should the town build a new swimming pool?” Of 2,000 callers, 85% say yes. (a) What kind of sample is this, and why is it biased? (b) Would 20,000 callers fix the problem? (c) How should the town find out what its residents think?

7. **8 pts**

   (Lesson 8.4) The seven farms in Cedar Valley have 40, 45, 50, 55, 60, 70 and 600 acres. A real-estate flyer says, “The average farm in Cedar Valley is over 130 acres!” (a) Find the mean and the median. (b) Is the flyer true? Which number better describes a typical farm, and why? (c) A buyer reads “average” as “typical” and expects most farms to be about 130 acres. Name the fallacy.

8. **6 pts**

   (Lesson 8.4) (a) The share of students at Lakeview School who walk to school rose from 10% to 15%. One newsletter says “up 5 points” and another says “up 50%.” Which is right? Explain. (b) A bicycle cost $200. Its price rose 25% and then fell 25%. Find the final price, and explain why it is not $200.

9. **8 pts**

   (Lesson 8.5) Cinderline Chimney Service advertises, “Yearly cleaning cuts your risk of a chimney fire in half!” Suppose the risk of a chimney fire in a year is 6 in 10,000 homes without yearly cleaning and 3 in 10,000 with it. (a) Is “in half” true? (b) Find the absolute difference in risk, as a count per 10,000 homes. (c) About how many homes would need yearly cleaning to prevent one chimney fire in a year? (d) The ad also shows a house in flames and the words “Don’t wait until it’s too late!” What fallacy does that add?

10. **8 pts**

    (Lesson 8.5) A sorting machine at an orchard flags bruised apples. Of the apples it sorts, 4% are bruised. The machine flags 90% of bruised apples, and it also flags 5% of good apples. Its maker says, “When our machine flags an apple, it’s bruised!” Use 1,000 apples. (a) How many apples are bruised, and how many of those are flagged? (b) How many good apples are flagged? (c) What fraction of the flagged apples are really bruised? (d) Name the fallacy the maker’s slogan invites.

11. **8 pts**

(Lesson 8.6) (a) “Homes with more books have children who read better, so buying more books will make children better readers.” Give one common-cause explanation and one reverse-cause explanation for the link. (b) “I started wearing my blue socks on game days, and we won the next three games.” Name the fallacy, including its Latin name.

12. **8 pts**

(Lesson 8.7) (a) A coach puts the four slowest runners from Monday’s timed mile on an extra stretching routine, and on Friday their average time improves by 20 seconds. Why is this weak evidence that the routine works, and what would make the evidence stronger? (b) In a board game, Hannah has rolled a fair die six times without a 6. Her brother says, “A 6 is due.” What is the chance of a 6 on her next roll, and what fallacy has he committed?

13. **6 pts**

(Lesson 8.8) “Players at our summer basketball camp raised their free-throw percentage from 60% to 70% during the camp. Sign up and improve your game!” Write the ad’s argument in standard form, including its hidden premise, and say whether the hidden premise is believable.

14. **10 pts**

(Lesson 8.8) Write a refutation of four to six sentences: “Brightcard flashcards work! Nine of the ten highest scorers on Riverside High’s exams last year used Brightcard.” (Riverside High has 400 students, and 300 of them use Brightcard.) Include these parts: restate the claim fairly, name the fallacy and show it with numbers, say what evidence would settle the question, and give a measured verdict.

ANSWER KEY

1. (a) Heights above 45: $55 - 45 = 10$ and $50 - 45 = 5$, so $10 \div 5 = 2$ times as tall. (b) $\dfrac{55 - 50}{50} = 0.1$, or 10% longer. (c) The fallacy of accent: the numbers are true, but cutting off the axis stresses the difference, so a 10% advantage looks like double.

2. (a) $3 \times 3 = 9$ times the area. (b) $9 \div 3 = 3$: the picture makes the growth look three times as large as it was. (c) As two plain bars from 0, the second three times as tall as the first (accept: pumpkins of one size, one pumpkin for each 1,000 visitors; or a second pumpkin whose area, not its height, is three times the first, so each side is about 1.7 times as long).

3. (a) Ashford $120 \div 60 = 2$; Birchwood $45 \div 9 = 5$ reports per 1,000 people. (b) Birchwood, at more than twice Ashford's rate. (c) Any two: the map shows totals, which mostly reflect how many people live in each county, not a rate; the reader applies a fact about the county's total to each dog owner (the fallacy of division); and a report count does not measure carelessness at all (dogs may stray for many reasons, and some owners report more readily than others).

4. (a) Composition: it reasons from the parts (each player) to the whole (the team), but a team's quality also depends on defense, passing and playing together. (b) Division: it reasons from the whole (the county's average) to one part (the Pikes); a county average says nothing certain about any one farm, and a few very large farms may pull the average up.

5. (a) $30 \div 40 = 0.75$, or 75%. (b) $30 \div 200 = 0.15$, or 15%. (c) Survivorship bias (accept suppressed evidence or cherry-picking): only the students who finished are counted. The missing data are what happened to the 160 who enrolled and did not graduate, and how many students like these would have joined an orchestra without the school (accept either).

6. (a) A self-selected (voluntary response) sample: only listeners of that station who chose to call are counted, and people with strong feelings, especially those who want the pool, are the most likely to call. (b) No. A bigger biased sample only measures its bias more precisely; size cannot cure bias. (c) Ask a random sample of the town's households (or residents), so that every resident has a fair chance to be chosen.

7. (a) Mean $920 \div 7 \approx 131.4$ acres; median 55 acres (the 4th of the 7 values in order). (b) True for the mean, but misleading: six of the seven farms have 70 acres or less, and the one 600-acre farm pulls the mean far up. The median, 55

acres, better describes a typical farm. (c) Equivocation: “average” means the mean in the flyer but is heard as “typical.”

8. (a) Both: the share rose 5 percentage points, and $5 \div 10 = 50\%$ of the old share. A careful writer says which one is meant. (b) $200 \times 1.25 = 250$ dollars, then $250 \times 0.75 = 187.50$ dollars. The fall is 25% of \$250, a larger base than the \$200 the rise was taken from.

9. (a) Yes, as a relative risk: $3 \div 6 = \frac{1}{2}$. (b) $6 - 3 = 3$ fewer fires per 10,000 homes a year. (c) $10{,}000 \div 3 \approx 3{,}333$ homes (accept about 3,300). (d) The appeal to fear: alarm and urgency are used in place of reasons. (The claim may still be true; the honest version gives the absolute numbers.)

10. (a) $1000 \times 0.04 = 40$ bruised, and $40 \times 0.9 = 36$ of them flagged. (b) $960 \times 0.05 = 48$ good apples flagged. (c) $\dfrac{36}{36 + 48} = \dfrac{36}{84} = \dfrac{3}{7} \approx 0.43$, about 43%: more than half of the flagged apples are good. (d) The base-rate fallacy: the slogan ignores that bruised apples are rare, so false flags among the many good apples outnumber the true ones.

11. (a) Common cause (accept any reasonable one): parents who love reading both buy more books and spend more time reading with their children, and that time builds reading skill. Reverse cause: children who already read well ask for, and are given, more books. (b) False cause, in the form *post hoc, ergo propter hoc* (“after this, therefore because of this”): the wins came after the socks, which does not show the socks caused them.

12. (a) The runners were chosen because they had an extremely slow Monday, which likely included some bad luck (a poor night’s sleep, a bad start), so they would tend to run faster on Friday anyway: regression to the mean. Stronger evidence: compare them with a group of similarly slow runners who did not do the routine, ideally assigned at random, and time them over more than one run. Crediting the routine for the whole gain would be the regression fallacy. (b) Still $\frac{1}{6}$; the die has no memory. He has committed the fallacy of thinking an independent chance event is “due” (the fallacy of the maturity of chances, met in Lesson 8.7 under another name).

13. 1. Players at the camp raised their free-throw percentage from 60% to 70% during the camp.
    2. (Hidden) The camp caused the improvement; the players would not have improved as much without it.
    Therefore, the camp will improve your game.
    The hidden premise is doubtful: players who shoot every day all summer may

improve anyway, and players who choose a camp may be unusually eager (a self-selected group). The ad gives no comparison group. Accept also the hidden premise "the campers are like you, so what happened to them will happen to you," which is doubtful for the same reasons.

14. Model answer: "The ad says nine of Riverside's ten top scorers used Brightcard, and it wants us to conclude that the flashcards raise scores. But 300 of the school's 400 students, or 75%, use Brightcard, so we would expect about 7 or 8 of any ten students, top scorers or not, to be users. Nine of ten is little more than that, and the ad leaves out how many of the lowest scorers also used Brightcard (suppressed evidence; the inference from the link to a cause is false cause). A fair test would compare the scores of users and non-users of similar ability, or better, students assigned at random to use the cards or not. Until then, the ad has not shown that Brightcard raises scores." Scoring: fair restatement (2), the base rate shown with numbers (3: 75%, about 7.5 of 10), a named fallacy (2: accept false cause, suppressed evidence or the base-rate fallacy), settling evidence (2), measured, courteous verdict (1). Deduct for claiming the cards do not work.

> Printable answer key (PDF, for the parent)

UNIT 8 · STATISTICAL DECEPTION · LESSON 8.1 OF 69 · 45 MINUTES

8.1 Graphs That Deceive

OBJECTIVES

- Recognize five ways a graph can mislead while every number on it is true: a cut-off vertical axis, a stretched scale or hand-picked time window, two vertical scales, pictures or 3-D effects scaled by area or volume, and steering labels.
- Compute the picture ratio, the data ratio and the exaggeration factor of a graph.
- Redraw a deceptive graph honestly and state in one sentence what the data really show.
- Explain how a deceptive graph commits the fallacy of accent.

CORE IDEAS

This lesson is taught from the Statistics course. Logic Lesson 8.1 is the same lesson as Statistics Lesson 14.1. Read Statistics Lesson 14.1: Graphs That Deceive (its video and figures are there), then do its *Practice Set L*, the lighter set written for Logic

students. The printable worksheets for that lesson include Set L: Worksheet A (key) and Worksheet B (key).

The logic in it: the fallacy of accent (Lesson 3.7). A graph with a cut-off axis or a swollen picture puts false emphasis on a true number, just as accent puts false stress on a true sentence. The words of the claim may be accurate; the picture says more than the data do.

UNIT 8 · STATISTICAL DECEPTION · LESSON 8.2 OF 69 · 45 MINUTES

8.2 Maps and Scales That Deceive

OBJECTIVES

- Explain how a shaded map can mislead by showing totals in place of rates, by its class breaks and by its colors.
- Compute rates per 1,000 people (or per acre) and choose totals or rates to fit a question.
- Use the factor $1/\cos^2\varphi$ to explain how a Mercator map enlarges areas far from the equator.
- Explain how reading a map of a region's totals or averages can commit the fallacies of composition and division.

CORE IDEAS

This lesson is taught from the Statistics course. Logic Lesson 8.2 is the same lesson as Statistics Lesson 14.2. Read Statistics Lesson 14.2: Maps and Scales That Deceive (its video and figures are there), then do its *Practice Set L*, the lighter set written for Logic students. The printable worksheets for that lesson include Set L: Worksheet A (key) and Worksheet B (key).

The logic in it: composition and division (Lesson 3.8). A map can shade a whole county by a total that belongs to a few towns in it, or let a large, empty region look more important than a small, crowded one. Reading a total as if it were a rate, or a region's figure as if it described every place inside it, is reasoning from parts to whole or whole to parts without warrant.

UNIT 8 · STATISTICAL DECEPTION · LESSON 8.3 OF 69 · 45 MINUTES

8.3 Who Got Counted?

OBJECTIVES

- Recognize self-selected samples, survivorship bias and cherry-picked evidence, and name the missing data that would settle a claim.
- Explain the wartime study of returning bombers and Bacon's votive tablets as the same error.
- Explain why a larger biased sample is not a better sample, using the 1936 *Literary Digest* poll.
- Compute how unequal response rates distort a poll.
- Identify hasty generalization and suppressed evidence in claims built on missing cases.

CORE IDEAS

This lesson is taught from the Statistics course. Logic Lesson 8.3 is the same lesson as Statistics Lesson 14.3. Read Statistics Lesson 14.3: Who Got Counted? Cherry-Picking, Biased Samples and Survivorship (its video and figures are there), then do its *Practice Set L*, the lighter set written for Logic students. The printable worksheets for that lesson include Set L: Worksheet A (key) and Worksheet B (key).

The logic in it: hasty generalization and suppressed evidence (Lessons 3.6 and 7.1). A sample that picked itself, a list of only the successes, or a survey of the people who happened to answer is a weak inductive base. The conclusion may reach far past the evidence, and the missing cases are evidence too.

UNIT 8 · STATISTICAL DECEPTION · LESSON 8.4 OF 69 · 45 MINUTES

8.4 Averages and Percents That Mislead

OBJECTIVES

- Choose between the mean and the median for a skewed set of values and explain how the choice can mislead.
- Compute percent change, tell percentage points from percent change, and see how a small base inflates a percent.
- Explain why a rise and an equal-percent fall do not cancel, and why averaging percents can mislead.
- Recognize equivocation on the word "average" and other words with more than one meaning.

CORE IDEAS

This lesson is taught from the Statistics course. Logic Lesson 8.4 is the same lesson as Statistics Lesson 14.4. Read Statistics Lesson 14.4: Averages and Percents That Mislead (its video and figures are there), then do its *Practice Set L*, the lighter set written for Logic students. The printable worksheets for that lesson include Set L: Worksheet A (key) and Worksheet B (key).

The logic in it: equivocation (Lesson 3.7). "Average" can mean the mean or the median, and "up 5 percent" can mean five percentage points or five percent of the old figure. An argument that slides from one meaning to the other uses one word as two terms.

UNIT 8 · STATISTICAL DECEPTION · LESSON 8.5 OF 69 · 45 MINUTES

8.5 Risk: Relative, Absolute and Base Rates

OBJECTIVES

- Convert a relative-risk claim into absolute risks and an absolute risk difference, and write risks as natural frequencies.
- Compute and interpret the number needed to treat or to harm.
- Use a frequency tree for a test of a rare condition to find the probability that a positive result is true.
- Name and explain the base-rate fallacy and the appeal to fear.

CORE IDEAS

This lesson is taught from the Statistics course. Logic Lesson 8.5 is the same lesson as Statistics Lesson 14.5. Read Statistics Lesson 14.5: Risk: Relative, Absolute and Base Rates (its video and figures are there), then do its *Practice Set L*, the lighter set written for Logic students. The printable worksheets for that lesson include Set L: Worksheet A (key) and Worksheet B (key).

The logic in it: the base-rate fallacy (Lesson 7.5). "Doubles the risk" is true but empty until we know the risk it doubles, and a test that is right most of the time can still be wrong about most of the people it flags when the condition is rare. Ignoring how common something is to begin with is a failure of inductive reasoning about chance.

UNIT 8 · STATISTICAL DECEPTION · LESSON 8.6 OF 69 · 45 MINUTES

8.6 Correlation, Cause and Lurking Variables

OBJECTIVES

- Name the alternatives to “A causes B”: reverse cause, a common cause and coincidence.
- Work a Simpson’s paradox reversal from a two-way table and name the lurking variable.
- State Mill’s methods of agreement and difference and say what evidence would show cause.
- Recognize the false-cause fallacy, including *post hoc, ergo propter hoc*.

CORE IDEAS

This lesson is taught from the Statistics course. Logic Lesson 8.6 is the same lesson as Statistics Lesson 14.6. Read Statistics Lesson 14.6: Correlation, Cause and Lurking Variables (its video and figures are there), then do its *Practice Set L*, the lighter set written for Logic students. The printable worksheets for that lesson include Set L: Worksheet A (key) and Worksheet B (key).

The logic in it: false cause (Lessons 3.6 and 7.3). Two things that rise and fall together may share a cause, may run the other way, or may meet by chance. Mill’s methods ask what else differs between the cases; a lurking variable is the “something else” a careful reasoner looks for before saying “A causes B.”

UNIT 8 · STATISTICAL DECEPTION · LESSON 8.7 OF 69 · 45 MINUTES

8.7 Regression to the Mean and Streaks

OBJECTIVES

- Explain why an extreme result is usually followed by a less extreme one, and how this makes useless remedies look effective.
- Separate a real effect from regression to the mean using a comparison group.
- Predict a second score from a first using $\hat{z}_y = r\, z_x$.
- Explain the gambler’s fallacy and compute how often streaks happen by chance.

CORE IDEAS

This lesson is taught from the Statistics course. Logic Lesson 8.7 is the same lesson as Statistics Lesson 14.7. Read Statistics Lesson 14.7: Regression to the Mean and Streaks (its video and figures are there), then do its *Practice Set L*, the lighter set

written for Logic students. The printable worksheets for that lesson include Set L: Worksheet A (key) and Worksheet B (key).

The logic in it: the regression fallacy and the gambler's fallacy (Lesson 7.5). Extreme results tend to be followed by more ordinary ones, so whatever was done right after an extreme (a new remedy, a lucky charm, a scolding) seems to work. Mistaking that ordinary return for an effect, or expecting chance to "even out" a streak, is reasoning from a pattern that is not there.

UNIT 8 · STATISTICAL DECEPTION · LESSON 8.8 OF 69 · 45 MINUTES

8.8 Refuting a Statistical Claim

OBJECTIVES

- Restate a statistical claim fairly, in plain words, without making it stronger or weaker than it is.
- Lay out the claim as an argument in standard form and find the hidden premise where the flaw usually lies.
- Name the fallacy behind a misleading statistical claim, using the tools of Lessons 8.1–8.7, and show it with numbers.
- Say what evidence would settle the claim, and write a short, courteous refutation with a measured verdict.

WATCH

Misleading line graphs

Khan Academy · 4:53

https://www.youtube.com/watch?v=gs-OPF3KEGU

> **WHEN TO WATCH**
> Before writing your own refutation: a worked example of an honest-looking but misleading advertisement.

Note: Parent preview: Sal Khan shows how a soda company's ad uses two graphs with different scales (one starting at 50%) to make a losing brand look like a winner, even though every data point is accurate. Made-up brands ("Thrill Soda," "Yummy Cola"). Nothing objectionable; transcript checked October 18, 2026.

KEY TERMS

Refutation

A reasoned reply showing that a claim has not been proved by the evidence offered for it, or that the claim is false.

Hidden premise

A premise an argument needs but does not state, such as "the people surveyed represent all buyers."

Settling evidence

The fair evidence that would actually decide a question, such as a random sample, a comparison group or the full set of data.

Measured verdict

A conclusion that claims exactly what the evidence shows, usually "not shown" rather than "false."

Fair restatement

Saying in plain words exactly what a claim says, no more and no less, in a form its maker would accept.

Straw man

The fallacy of attacking a weaker or different claim than the one actually made.

Fallacy fallacy

The error of concluding that a claim is false because one argument for it is bad.

CORE IDEAS

From spotting to answering. In Lessons 8.1–8.7 you learned to spot the tricks numbers can play: the cut-off axis, the map of totals, the sample that chose itself, the mean pulled up by one large value, the doubled risk with no baseline, the correlation taken for a cause and the extreme result that fades. Spotting a trick is half the work. The other half is to answer it in words a neighbor can follow. That answer is a **refutation**: a short, reasoned reply showing that a claim has not been proved, or that it is false. This lesson gives you a method in four steps and a model for writing it.

Step 1: Restate the claim fairly. Begin by saying exactly what the claim says, no more and no less. Copy its numbers and key words, then put it in plain language. Who or what is being compared, with what, and by how much? An ad for Voltara batteries says, "Up to 3 times longer life!" Fairly restated, it claims that in at least one test a Voltara battery lasted about three times as long as some other battery; "up to"

promises only the best case. If you answer a claim the ad never made, such as "Voltara batteries always last three times as long as every other brand," you commit the **straw man** fallacy from Unit 3, and your refutation misses its target. Give the claim the strongest reading its words allow, the reading its maker would accept.

Step 2: Lay out the argument. A statistical claim is usually the conclusion of an argument, and the ad's number is its premise. Write them in standard form and supply the premise the ad leaves unstated. Crestline Tutoring says, "Our students raised their math grades an average of 12 points!" It wants you to conclude that its tutoring raises grades. In standard form:

1. Students who took Crestline tutoring scored an average of 12 points higher afterward.
2. (Hidden) They would not have improved that much without the tutoring.

Therefore, Crestline tutoring raises grades by about 12 points.

The flaw in a statistical argument usually hides in the premise nobody stated. Here it is the claim that nothing else would have raised the scores.

Step 3: Name the fallacy and show it with numbers. Find where the argument breaks, give the error its name, and show it with the ad's own numbers when you can. Most statistical tricks fall under a few names you already know:

- A graph that stresses a small difference: *accent* (8.1).
- A total read as a rate, or a fact about a group pinned on each member: *composition* or *division* (8.2).
- A sample that chose itself, survived or was picked by hand: *hasty generalization* or *suppressed evidence* (8.3).
- "Average" or "percent" used in two senses: *equivocation* (8.4).
- A frightening relative risk, or a test result read without the base rate: the *appeal to fear* or the *base-rate fallacy* (8.5).
- A correlation or a sequence in time taken for a cause: *false cause* (8.6).
- A change in a group chosen for being extreme, or a result said to be "due": the *regression fallacy*, or the fallacy of the *maturity of chances* (8.7).

In the tutoring ad, students often sign up right after a bad test. Students chosen for a low score tend to score higher next time anyway, so part of the gain may be regression to the mean, a kind of false cause.

Step 4: Say what evidence would settle it. A refutation that only says "flawed" is half finished. Tell the reader what fair evidence would look like, so that the question can actually be decided. For the tutoring ad, the settling evidence is a comparison group: students with the same low scores who did not take tutoring, ideally assigned at random, measured over the same weeks. If the tutored students gained 12 points and the comparison group gained 7, the tutoring earns credit for about 5. For a graph, the settling evidence is the honest redraw; for a sample, a random one; for a risk, the absolute numbers out of 1,000; for a combined rate, the rates within each group.

Do not claim more than you have shown. A bad argument does not prove its conclusion false. If Voltara's graph exaggerates, that shows the ad has not proved its batteries are much better. It does not show that they are worse, or even that they are no better. Concluding "false" from "badly argued" is the **fallacy fallacy** you met in Unit 3. So the usual verdict of a statistical refutation is "not shown," not "false." You may say "false" only when the evidence itself shows it, as when the ad's own numbers contradict its headline. Overstating your case is its own kind of carelessness with the truth, and it hands the other side an easy reply.

Writing the refutation. A good written refutation is one paragraph of four to six sentences, in this order: the claim, fairly restated; the evidence offered; the flaw, named and shown with numbers; the evidence that would settle the question; and a measured verdict. For example: "Crestline says its students raised their math grades an average of 12 points, which suggests that its tutoring caused the gain. But students usually sign up after a bad test, and students chosen for a low score tend to score higher next time anyway, so part of the gain may be regression to the mean. The ad gives no comparison group. A fair test would compare tutored students with similar students who were not tutored, over the same weeks. Until then, the ad shows that its students improved, not how much the tutoring helped."

Answer the claim, not the person. Write courteously. Do not call the advertiser a liar; most misleading numbers are true numbers badly framed, and the writer of the ad may not have seen the problem. Answer the reasoning, credit what is true ("the 12-point gain is real"), and leave room to be shown wrong. Proverbs 14:15 sets the standard: "The simple believeth every word: but the prudent man looketh well to his going." The prudent man is not the man who believes nothing. He looks well, weighs the evidence, and then believes what the evidence supports. A refutation is simply that looking, written down so that others can check it.

From history. The word *refutation* has a long history in logic. The last treatise of Aristotle's *Organon* is called *On Sophistical Refutations*. A refutation, in Aristotle's sense, is reasoning that proves the contradictory of someone's claim; a "sophistical" refutation only seems to do so. Aristotle sorted the tricks that make bad reasoning look good into thirteen kinds: six that depend on language, among them equivocation, combination (composition), division and accent, and seven that do not, among them treating what is not a cause as a cause. Several of the fallacies in this unit therefore carry names that Aristotle sorted more than two thousand years ago, long before anyone drew a bar graph. Medieval students studied the book closely, because it taught them to answer a bad argument by naming exactly where it went wrong.

Practice habit. When you see a number in an ad this week, say the claim back in plain words before you judge it, then ask: What evidence would settle this?

Enrichment: Bacon on the mind that wants to believe. In the same aphorism of the *Novum Organum* (1620) that tells the story of the votive tablets (Lesson 8.3), Francis Bacon warns about a habit of every human mind. As Bacon put it, roughly, once the understanding has adopted an opinion, it draws everything else in to support it, and it passes over or brushes aside the cases on the other side, however many and weighty they are. Bacon's warning cuts both ways. It explains why an advertiser counts only the happy customers. It also warns the person writing a refutation, who may be just as eager to count only the evidence against the ad. How could you guard against this habit in yourself when you answer a claim you already dislike?

Example 1. An ad for Voltara batteries shows two bars under the headline "Three Times the Life!" Voltara's bar is labeled 12 hours and Brand X's 10 hours, and the vertical axis starts at 9 hours. Refute the headline using the four steps.

▼ **Show solution**

1. Restate fairly. The ad claims that a Voltara battery lasts three times as long as a Brand X battery, and it supports this with a test in which Voltara lasted 12 hours and Brand X lasted 10.
2. Lay out the argument.
   1. On the graph, Voltara's bar is three times as tall as Brand X's.
   2. (Hidden) The height of each bar is in proportion to the battery life.

   Therefore, Voltara lasts three times as long as Brand X.

   The bars rise $12 - 9 = 3$ and $10 - 9 = 1$ above the axis, so the picture ratio really is 3.
3. Name the fallacy and show it. Premise 2 is false, because the axis starts at 9, not 0. The real ratio is $12 \div 10 = 1.2$: Voltara lasted 2 hours, or 20%, longer. The graph gives true numbers a false emphasis, which is the fallacy of accent.
4. Say what would settle it. The honest graph draws both bars from 0, where they look nearly alike. A fair reader would also want to know how many batteries were tested, in what device, and who ran the test.
5. Give a measured verdict. Here the ad's own numbers contradict its headline, so we may say plainly that "three times the life" is false as stated, while granting that Voltara did last longer in this test.

Answer: "The Voltara ad claims its batteries last three times as long as Brand X, but its own labels say 12 hours against 10. That is 1.2 times as long, or 20% longer. The bars look three times as tall only because the axis starts at 9 hours instead of 0, which gives true numbers a false emphasis (the fallacy of accent). Drawn from 0, the bars look nearly equal, and we would still want to know how many batteries were tested and how. Voltara did last 2 hours longer in this test, but the ad's own numbers show that "three times the life" is false as stated."

Example 2. Tillwell fertilizer's flyer says, "Farms that used Tillwell harvested 50% more corn last year than farms that did not!" The fine print says the 20 Tillwell farms averaged 300 acres of corn and 51,000 bushels, and the 20 other farms averaged 200 acres and 34,000 bushels. (a) Check the 50%. (b) Find each group's yield per acre. (c) Write a short refutation.

▼ **Show solution**

1. Restate fairly: Tillwell farms harvested half again as much corn as other farms, and the flyer wants you to conclude that Tillwell raised the harvest.
2. Check the percent: $\dfrac{51{,}000 - 34{,}000}{34{,}000} = \dfrac{17{,}000}{34{,}000} = 0.5$, so the 50% is correct.
3. Per acre: Tillwell farms $51{,}000 \div 300 = 170$ bushels; other farms $34{,}000 \div 200 = 170$ bushels. The yields per acre are the same.
4. The flaw: the flyer compares totals, not rates (8.2), and the larger total has a common cause, farm size (8.6). Bigger farms grow more corn, and perhaps bigger farms are also more likely to buy Tillwell. Taking the total for proof that the fertilizer worked is false cause.
5. Settling evidence: a fair trial on the same farms, with strips of each field chosen at random to get Tillwell or not, compared in bushels per acre (Mill's method of difference, Unit 7).

Answer: (a) Correct: 17,000 more bushels is 50% of 34,000. (b) 170 bushels per acre for both groups. (c) For example: "Tillwell's flyer says its farms harvested 50% more corn, and that is true of the totals. But the Tillwell farms were also 50% larger, and per acre both groups harvested 170 bushels, so the extra corn came from extra acres, not from the fertilizer. Reading a total as if it were a rate here is false cause, with farm size as the common cause. A fair test would compare strips of the same fields, chosen at random to get Tillwell or not, in bushels per acre. On the flyer's own numbers, Tillwell has not been shown to raise yields."

PRACTICE

1. Easy List the four steps of refuting a statistical claim, in order. With what does a written refutation end?

   ▼ **Show answer**

   (1) Restate the claim fairly. (2) Lay out the argument in standard form, including its hidden premise. (3) Name the fallacy and show it with numbers. (4) Say what evidence would settle the question. The written refutation ends with a measured verdict, usually that the claim is "not shown" (or "false" only if the evidence shows it false). Accept equivalent wording, such as "find the hidden premise" for step 2 or "explain what data are missing" for step 4.

2. Easy A cereal box says, "Crunchwell: up to 40% more fiber than other leading brands!" Restate the claim fairly in plain words, and name one thing it does *not* claim.

   ▼ **Show answer**

   Fair restatement: compared with at least one other leading brand, a serving of Crunchwell has as much as 40% more fiber; "up to" means 40% is the best case. It does not claim 40% more fiber than every leading brand (against some it may be much less), and it does not claim that Crunchwell is healthier or better in any other way. Accept any one of these, or another thing the words plainly do not say.

3. **Easy** Name the fallacy most clearly at work in each claim. (a) An orchard's sales doubled, and its poster shows this with an apple drawn twice as tall and twice as wide. (b) "Our 500 online reviews average 4.8 stars, so nearly everyone who buys a SnugNest pillow loves it." (c) "Runners who wear SwiftStride racing shoes win more races, so the shoes make you faster." (d) "Our town's 4-H club has the largest membership in the state, so every member must be very active."

▼ **Show answer**

(a) Accent: the apple covers $2 \times 2 = 4$ times the area, so a doubling looks like a fourfold rise. (b) Hasty generalization from a self-selected sample: reviewers chose themselves (accept "biased sample" or "suppressed evidence"). (c) False cause: a common cause, such as serious, fast runners being the ones who buy racing shoes, could explain the link (accept a reverse-cause explanation: winning runners can afford or are given such shoes). (d) Division: a fact about the club as a whole (its size) is pinned on each member.

4. **Easy** Caleb finds that a battery ad's graph uses a cut-off axis. He writes, "So the batteries are no better than any other brand." What has he done wrong, and what should he conclude instead?

▼ **Show answer**

He has claimed more than he has shown: a bad argument does not prove its conclusion false (the fallacy fallacy). The cut-off axis shows only that the ad exaggerates the difference. He should conclude that the ad has not shown the batteries to be much better, and that the real difference is whatever the actual numbers show, which may be small but may still be real.

5. **Medium** "Of the customers who mailed back our survey card, 9 in 10 said they love Crunchwell cereal. So 9 in 10 cereal eaters love Crunchwell." (a) Write the argument in standard form, including the hidden premise. (b) Explain why the hidden premise is doubtful. (c) What evidence would settle the question?

▼ **Show answer**

(a) 1. Nine in ten customers who mailed back the survey card said they love Crunchwell.
2. (Hidden) The customers who mailed back cards represent all cereal eaters.
Therefore, nine in ten cereal eaters love Crunchwell.
(b) The people who answered are doubly unrepresentative: they are customers who already buy Crunchwell, and they chose to mail the card back, which pleased customers are more likely to do. This is hasty generalization from a self-selected sample. (c) A random sample of all cereal eaters, not only Crunchwell buyers, asked the same question, ideally with a blind taste test against other cereals.

6. **Medium** "SureStep trail shoes cut ankle sprains in half!" In the maker's season-long test, 12 of 400 hikers in ordinary boots and 6 of 400 hikers in SureStep shoes sprained an ankle. (a) Find each group's risk as a percent. (b) How many fewer sprains is that per 1,000 hikers? (c) Write an honest one-sentence version of the claim.

▼ **Show answer**

(a) Ordinary boots: $12 \div 400 = 0.03$, or 3%. SureStep: $6 \div 400 = 0.015$, or 1.5%.
(b) Per 1,000 hikers, 30 and 15, so 15 fewer sprains (1.5 percentage points). (c) For example: "In the maker's test, about 15 of every 1,000 hikers in SureStep sprained an ankle, against 30 of every 1,000 in ordinary boots." "In half" is true as a relative risk; the honest version adds the absolute numbers. (A careful reader would also ask whether hikers were assigned their footwear at random.)

7. **Medium** "Larkfield Tutoring: our pass rate is up 50%!" Last year 20% of Larkfield's students passed the state algebra exam; this year 30% did. (a) By how many percentage points did the pass rate rise? (b) Is "up 50%" correct? (c) What might a careless reader think the ad means? Write a fairer sentence. (d) Name one more thing you would want to know before giving Larkfield the credit.

▼ **Show answer**

(a) $30 - 20 = 10$ percentage points. (b) Yes, as a percent change: $\frac{10}{20} = 50\%$. (c) A reader might think the pass rate rose by 50 points, to 70%, or that half of the students now pass. Fairer: "Our pass rate rose from 20% to 30%." (d) Any reasonable answer: whether pass rates rose for all students in the state that year (an easier exam), how many students Larkfield had, whether the same kind of students enrolled both years, or how a comparison group did.

8. **Medium** Two students refute the ad "Kids who eat Sunnyfield Oats for breakfast get better grades!"
Refutation A: "This ad is a lie from a greedy company, and anyone who believes it is foolish. Sunnyfield Oats do nothing for grades."
Refutation B: "The ad says children who eat Sunnyfield Oats get better grades, suggesting the oats cause the better grades. But it reports only a link. Families who serve a hot breakfast may also check homework and keep regular bedtimes, and those habits could explain the grades. A fair test would compare similar children assigned at random to different breakfasts. Until then, the ad has not shown that the oats raise grades."
Which is better? Name three faults in the other one.

▼ **Show answer**

B is better. Faults in A (accept any three): it attacks the company and the readers instead of the reasoning (ad hominem); it never restates the claim; it names no fallacy and gives no numbers or reasons; it says nothing about what evidence would settle the question; and it overreaches, claiming the oats "do nothing" when it has shown at most that the ad has not proved its point (the fallacy fallacy).

9. Hard Write a refutation of four to six sentences: “Since the Hendersons installed a Brookmere water softener, their plumbing repairs fell from 6 last year to 1 this year! Brookmere protects your pipes.” (The Hendersons bought the softener because last year was the worst they had ever had.)

▼ **Show answer**

Model answer: “The ad says the Hendersons’ plumbing repairs fell from 6 to 1 after they installed a Brookmere softener, and it concludes that the softener protects pipes. But the repairs fell after the softener was installed, which does not show they fell because of it (post hoc, false cause). The Hendersons bought it after their worst year ever, and an unusually bad year tends to be followed by a more ordinary one anyway: regression to the mean. One family’s two years is also far too small a sample (hasty generalization). A fair test would compare many similar homes with and without the softener, over several years. Until then, the ad has not shown that Brookmere prevents repairs.” Full credit for a fair restatement, at least one named fallacy shown with the numbers (post hoc, regression or hasty generalization), settling evidence and a measured verdict.

10. Hard The box of a BlightCheck test kit says, “Catches 9 of 10 blighted trees and wrongly flags only 1 healthy tree in 20. If BlightCheck says blight, take the tree down!” In one orchard region, 2% of apple trees have the blight. (a) Use a frequency tree for 1,000 trees: how many are blighted and flagged, and how many are healthy and flagged? (b) What fraction of flagged trees really have the blight? (c) Name the fallacy in the slogan and say what a grower should do instead.

▼ **Show answer**

(a) 20 trees are blighted, and $20 \times 0.9 = 18$ are flagged. 980 are healthy, and $980 \div 20 = 49$ are flagged. (b) $\dfrac{18}{18 + 49} = \dfrac{18}{67} \approx 0.27$, about 27%: nearly three of every four flagged trees are healthy. (c) The base-rate fallacy: the slogan ignores how rare the blight is, so most positives are false alarms. A grower should confirm a positive with a second, more exact test (or an expert’s inspection) before cutting down the tree.

11. Hard ReadRight Learning Center advertises, "80% of our students pass the reading exam, against only 71% at Northgate!" The full figures:

Center	Advanced students passing	Beginning students passing
ReadRight	144 of 160	16 of 40
Northgate	38 of 40	104 of 160

(a) Check both combined rates. (b) Find each center's pass rate for advanced students and for beginners. (c) Which center does better? Name the lurking variable and write one sentence refuting the ad.

▼ **Show answer**

(a) ReadRight: $\frac{144+16}{200} = \frac{160}{200} = 80\%$. Northgate: $\frac{38+104}{200} = \frac{142}{200} = 71\%$. Both are correct. (b) ReadRight: advanced $144/160 = 90\%$, beginners $16/40 = 40\%$. Northgate: advanced $38/40 = 95\%$, beginners $104/160 = 65\%$. (c) Northgate does better with both kinds of students. The lurking variable is the students' starting level: most of ReadRight's students are advanced and most of Northgate's are beginners (Simpson's paradox). For example: "ReadRight's higher combined rate comes from teaching mostly advanced students; with students of the same level, Northgate's pass rates are higher, so the ad does not show that ReadRight teaches better."

12. **Challenge** An ad for Meadowlark Plant Food says, "Yields up 300%! Since Mr. Dunn switched to Meadowlark last spring, his test plot went from 15 bushels to 60. And 9 out of 10 gardeners who returned our postcard say they would buy it again." The year before last spring was the worst drought in the county in a decade. (a) Check the 300%. (b) Name at least three separate problems with the ad, with the fallacy or error behind each. (c) For each problem, say what evidence would settle it. (d) Write a one-sentence measured verdict.

▼ **Show answer**

(a) $\frac{60 - 15}{15} = \frac{45}{15} = 3$, so 300% is correct. (b) Accept any three, well explained: (1) False cause (post hoc): the yield rose after the switch, but the weather also changed, from a drought year to an ordinary one. (2) Regression to the mean: the 15-bushel year was an extreme low, so a better year was likely anyway. (3) A huge percent from a small, unusual base: 300% measured from a drought year says little (8.4). (4) Hasty generalization: one plot of one farmer is far too small a sample. (5) A self-selected sample: gardeners who returned the postcard chose themselves and are already buyers. (6) The postcard question ("would buy again") measures liking, not yield. (c) For (1)–(4): side-by-side plots in the same season, on many farms, with plots chosen at random to get Meadowlark or not, compared in bushels per acre. For (5)–(6): a random sample of buyers, and measured yields instead of opinions. (d) For example: "The ad shows that one plot did far better in an ordinary year than in a drought year, not that Meadowlark raised its yield."

COMMON MISTAKES

- Answering a claim the ad never made, such as reading "up to" as "always." That is a straw man, and it lets the ad off the hook.
- Naming a fallacy without showing it. Use the ad's own numbers to show where the reasoning breaks.
- Stopping at "flawed." A finished refutation says what evidence would settle the question.
- Concluding that a claim is false when you have shown only that it is unproved (the fallacy fallacy).
- Insulting the advertiser instead of answering the reasoning. Most misleading numbers are true numbers badly framed.

> **Scripture.** "The simple believeth every word: but the prudent man looketh well to his going." (Proverbs 14:15, KJV)
>
> The prudent man neither swallows every claim nor scorns every claim; he looks well, weighs the evidence, and believes what it supports. A careful refutation is that looking, written down.

UNIT 9

Logic in Argument and Debate

Everything in this course has been building toward one moment: two people who disagree sitting down to reason together. This last unit takes the tools of the earlier units (terms and definitions, fallacies, propositions, syllogisms, symbolic logic and induction) and puts them to work in real argument. It asks a practical question: **how do we argue well, honestly and fairly, about things that matter?**

The unit moves from reading arguments to making them. Students learn to **map** an argument so its structure lies open, to find **hidden premises**, and to state the other side in **its strongest form** before answering it. They practice the two great classical methods of dialectic: **Socratic questioning**, as in Plato's *Euthyphro* and *Republic* I, and the **disputed question** of the medieval schools, as in Thomas Aquinas's *Summa Theologica*. They study how logic serves rhetoric in Aristotle's ethos, pathos and logos, how the old doctrine of **stasis** finds the real point of a disagreement, and how to cross-examine and refute. The unit ends with the common debate formats, including Lincoln-Douglas and the formal disputatio, and a practice round.

This unit prepares directly for the capstone, a formal disputatio with a written portfolio. The disputed question of Lesson 9.4 is the form of the capstone; the mapping, charity and refutation of Lessons 9.1, 9.2 and 9.7 are the skills it tests. Through it all runs the warning of Proverbs 18:17: the first speaker always sounds right until someone else comes and examines him. A Christian argues to find and serve the truth, not merely to win.

Review day and test. On the review day, use the mastery checklist and the review sheet (with its answer key). The unit test is below; a printable copy is ready for test day.

PREREQUISITES

- Find the premises and conclusion of an argument and write it in standard form (Unit 1).
- Define a term by genus and difference and tell a good definition from a bad one (Unit 2).
- Name and spot the common informal fallacies, such as the straw man and the ad hominem (Unit 3).
- Test a categorical syllogism for validity and recognize modus ponens and modus tollens (Units 5–6).
- Tell a strong inductive argument from a weak one and weigh testimony and evidence (Unit 7).

MASTERY CHECKLIST

- ☐ I can map an argument, showing its premises, sub-conclusions and main conclusion, and tell joint premises from independent ones.
- ☐ I can find the hidden premise that makes an argument valid and judge whether it is true.
- ☐ I can restate an opponent's argument in its strongest form before I answer it.
- ☐ I can test a definition by Socratic questioning, using counterexamples and consequences.
- ☐ I can write a short disputed question with objections, "On the contrary," "I answer that" and replies.
- ☐ I can explain ethos, pathos and logos, and build an enthymeme and an argument from example.
- ☐ I can use the four stases (fact, definition, quality and procedure) to find where a disagreement really lies.
- ☐ I can ask short, purposeful cross-examination questions and refute a point in four steps: they say, but, because, therefore.
- ☐ I can describe the Lincoln-Douglas, team policy, parliamentary and disputatio formats and flow a practice round.

LESSONS IN THIS UNIT

9.1	**Mapping Arguments**	40 minutes
9.2	**Hidden Premises and the Strongest Form**	40 minutes
9.3	**Dialectic: Socratic Questioning**	45 minutes
9.4	**The Disputed Question**	45 minutes
9.5	**Logic in Rhetoric**	40 minutes
9.6	**Finding the Real Question**	40 minutes
9.7	**Cross-Examination and Refutation**	45 minutes
9.8	**Debate Formats and a Practice Round**	45 minutes

Unit 9 test

You have 45 minutes. No notes or books. Answer each question in complete sentences unless it asks for a list, a map or a label. For classification questions, name the category and give a one-sentence reason. For written answers (objections, replies, refutations), any answer that follows the form taught in the lesson and reasons fairly earns full credit. The test is worth 100 points.

1. **8 pts**

   Map this argument. Show joint and independent premises, and name any sub-conclusion and the main conclusion. “(1) Hannah’s soccer team has won all six of its games this season. (2) Its goalkeeper has not let in a goal since the second game. (3) So the team is the strongest in the league. (4) The championship will be played on the team’s home field, (5) and teams usually play better at home. (6) Therefore Hannah’s team will probably win the championship.”

2. **6 pts**

   Are the premises joint or independent? Give a reason for each. (a) “No one under twelve may drive the garden tractor. Caleb is ten. So Caleb may not drive the garden tractor.” (b) “We should repaint the barn this fall: the old paint is peeling badly, and the color has faded so much that the barn looks neglected.”

3. **6 pts**

   Supply the hidden premise that makes each argument valid, and say whether it is true. (a) “That plant is a fern, so it does not produce flowers.” (b) “Every member of the choir must come to Saturday’s rehearsal, so Ruth must come to Saturday’s rehearsal.”

4. **8 pts**

   Leah says, “We should shut the chickens in their coop every night, because a fox has been seen near the farm.” Her brother replies, “So you think a fox is going to break into our house!” (a) Name the mistake her brother makes. (b) Restate Leah’s argument in its strongest form, including a hidden premise. (c) Write one sentence that answers her strongest form fairly, whether you agree or not.

5. **6 pts**

   (a) Give a counterexample to the definition “A fruit is anything sweet that grows on a plant,” and say whether it shows the definition is too broad or too narrow. (b) In Plato’s *Euthyphro*, what is Euthyphro’s first answer to “What is piety?”, and why does Socrates reject it?

6. **6 pts**

   Use the consequence tool against the claim “Anything that makes you happy is good for you.” Then name one other Socratic tool and use it against the same claim.

7. **8 pts**

   Put these parts of an article in order and label each part. (a) “Reply to Objection 1. Some games are won by chance, but chess and checkers are won by thinking, so these at least are not a waste of time.” (b) “I answer that, we must distinguish games played only to pass time from games that train the mind. Games that require planning and foresight exercise the mind and teach patience, so they are not a waste of time when played in moderation.” (c) “Whether board games are a waste of time.” (d) “On the contrary, the wise have always valued recreation that refreshes the mind for work.” (e) “Objection 1. It would seem that board games are a waste of time, for they are won by luck, and nothing is learned from luck.”

8. **8 pts**

   Write a reply to this objection using *distinguo*, and explain in one sentence what makes it a distinction: “Objection. It would seem that students should not ask questions in class, for asking questions interrupts the teacher.”

9. **8 pts**

   Label each sentence from a speech as ethos, pathos or logos, and then say whether sentence (b) is an honest appeal or a fallacy. (a) “I have kept bees for fifteen years and sold honey at this market every summer.” (b) “Imagine a garden in June with no bees humming: no squash, no apples, no berries.” (c) “Bees pollinate many of our fruit and vegetable crops, so a town beehive program would help every garden here.” (d) “I am asking for this because I want every family in town to have a good harvest.”

10. **6 pts**

    (a) Supply the hidden premise of this enthymeme: “The bread will not rise; the yeast is three years old.” (b) Write an argument from example for the claim “Our family should start a compost pile.”

11. **8 pts**

    (a) Name the stasis of each question: (1) Did the hailstorm damage the barn roof? (2) Is a pumpkin a fruit? (3) Was it right for Caleb to skip practice to help his neighbor fix a fence? (4) Should the coach or the team captain decide who plays goalkeeper? (b) Find the real question: Ruth: “You ate my piece of pie!” Leah: “Yes, I ate it, but Mom said any pie left after supper was for anyone.”

12. **8 pts**

Use the four steps of refutation to answer this argument: “Our class should not memorize poems, because we can always look them up.”

13. **6 pts**

(a) Rewrite this loaded question as a fair cross-examination question: “Why do you keep ignoring the cost of your plan?” (b) Give the purpose (clarify, test or gain an admission) of each question: (1) “By ‘every student,’ do you mean every student in the school, or only those in your grade?” (2) “Would you agree that the plan needs at least one adult volunteer each week?”

14. **8 pts**

(a) List the speeches of a Lincoln-Douglas round in order with their times. (b) On a flow, the affirmative made Contention 2 in the AC, and the column beside it for the NC is empty. What does this mean, and what should the affirmative do in the 1AR?

ANSWER KEY

1. 1 → 3 and 2 → 3 (independent: each record alone is some evidence of strength); 3 → 6; 4 + 5 → 6 (joint: the home field matters only because teams play better at home, and vice versa). Claim 3 is a sub-conclusion; claim 6 is the main conclusion.



   Accept 1 and 2 as joint if the student gives a reason using the removal test. Deduct if 4 and 5 are drawn as independent without a reason.

2. (a) Joint: neither premise says anything about Caleb driving the tractor without the other (it is a valid syllogism). (b) Independent: either reason alone, the peeling paint or the faded look, gives some reason to repaint. (Accept joint only if the student gives a removal-test reason, such as that neither problem alone would justify the work.)

3. (a) "No ferns produce flowers" (or "Ferns do not produce flowers"). It is true: ferns reproduce by spores, not flowers. (b) "Ruth is a member of the choir." Its truth depends on the facts about Ruth; it is presumably what the speaker knows to be true.

4. (a) A straw man: Leah said nothing about the house; he replaced her argument with a weaker, silly one. (b) Sample: "Foxes hunt at night and kill chickens that are left outside. A fox has been seen near our farm, so our chickens are in danger at night. Shutting them in the coop protects them. Therefore we should shut the chickens in every night." (Hidden premise: foxes kill chickens left outside at night, or the coop keeps foxes out.) (c) Sample: "You are right that the

fox is a danger, and the coop is the surest protection, so let's make shutting it part of the evening chores." Or a fair disagreement such as "The coop door is broken, so we need to fix it first or the plan won't work." Accept any fair answer aimed at her real argument.

5. (a) Too broad: sweet potatoes, carrots or sugar beets are sweet and grow on plants but are not fruits. (Also accept too narrow: lemons and cranberries are fruits but are not sweet.) (b) Euthyphro says piety is doing what he is doing, prosecuting a wrongdoer (even his own father). Socrates rejects it because it is only an example of a pious act, not a definition of what makes every pious act pious.
6. Consequence: if the claim were true, then eating a whole cake every day, or staying up all night playing games, would be good for you as long as it made you happy; but these things harm you, so the claim leads to a false result. Other tool, counterexample: a child is made happy by skipping his medicine, yet skipping it is bad for him. Or request for definition: "What do you mean by 'happy': a passing pleasure, or a lasting good?" Accept any correct use of a second tool.
7. (c) question; (e) objection; (d) sed contra ("On the contrary"); (b) respondeo ("I answer that"); (a) reply. Order: c, e, d, b, a.
8. Sample: "Reply to the Objection. I distinguish: questions asked at any moment, about anything, do interrupt the teacher, and I grant that these should wait. But questions asked at a fitting time, about the lesson, do not interrupt the teaching; they help it, by showing what needs to be explained. So students should ask questions, at the right time." It is a distinction because it shows the premise ("asking questions interrupts the teacher") is true in one sense and false in another, and that the objection needs the false sense. Accept any reply that clearly distinguishes two senses or cases.
9. (a) Ethos: experience gives the speaker good sense. (b) Pathos: it stirs concern and loss. (c) Logos: it gives a reason (pollination) for the conclusion. (d) Ethos: it shows goodwill. Sentence (b) is honest: the feeling fits the facts (many crops depend on pollination), and a real reason remains when the emotion is taken away. Accept a second label where explained, such as logos for (b).
10. (a) Hidden premise: "Yeast that is three years old is too old to make bread rise" (or "Old yeast no longer works"). (b) Sample: "When the Hollands started a compost pile, their garden soil became richer and their tomatoes grew twice as big. Our garden has the same heavy clay soil, so a compost pile would help us too." Accept any argument from a relevantly similar case.

11. (a) (1) Fact (conjecture); (2) definition; (3) quality; (4) procedure (translation). (b) The fact is agreed (Leah ate the pie). The real question is whether it was wrong, given Mom's rule: a question of quality. (Accept definition, if the student explains that the dispute is whether it was "Ruth's piece" at all.)
12. Sample: "They say we should not memorize poems because we can always look them up. But looking up a poem is not the same as knowing it, because a memorized poem stays with us when no book is at hand and shapes how we think and speak. Therefore memorizing poems gives us something that looking them up cannot." Accept any refutation that states the point fairly (they say), gives an answer (but), a reason (because) and what follows (therefore).
13. (a) Sample: "How much will your plan cost?" or "Have you included the cost of supplies?" (The original assumes the speaker is ignoring the cost.) Accept any short, neutral question on the same point. (b) (1) Clarify; (2) gain an admission.
14. (a) AC 6 minutes; CX (negative questions affirmative) 3; NC 7; CX (affirmative questions negative) 3; 1AR 4; NR 6; 2AR 3. (b) The negative dropped Contention 2: it was never answered, so it is usually treated as granted. In the 1AR, the affirmative should point out the drop, extend Contention 2 (remind the judge what it says) and explain why it helps the affirmative win.

Printable answer key (PDF, for the parent)

UNIT 9 · LOGIC IN ARGUMENT AND DEBATE · LESSON 9.1 OF 69 · 40 MINUTES

9.1 Mapping Arguments

OBJECTIVES

- Number the claims in a passage and set aside sentences that are not part of the argument.
- Tell a premise, a sub-conclusion and a main conclusion apart.
- Tell joint premises (which work only together) from independent premises (which each give support alone).
- Draw an argument map with arrows and use it to see where an argument can be challenged.

KEY TERMS

Argument map

A diagram of an argument in which each claim is a numbered box and arrows run from the premises to the claims they support.

Main conclusion

The claim the whole passage is trying to prove; on a map it is the claim that no arrow leaves.

Sub-conclusion

A claim that is supported by some premises and then serves as a premise for a further conclusion (also called an intermediate conclusion).

Joint premises

Premises that support a conclusion only when taken together; if one is removed, the others give little or no support (also called linked premises).

Independent premises

Premises that each give some support to a conclusion by themselves, so that removing one leaves the others still standing (also called convergent premises).

Chain argument

An argument in which one claim supports a second, which supports a third, and so on, like links in a chain.

CORE IDEAS

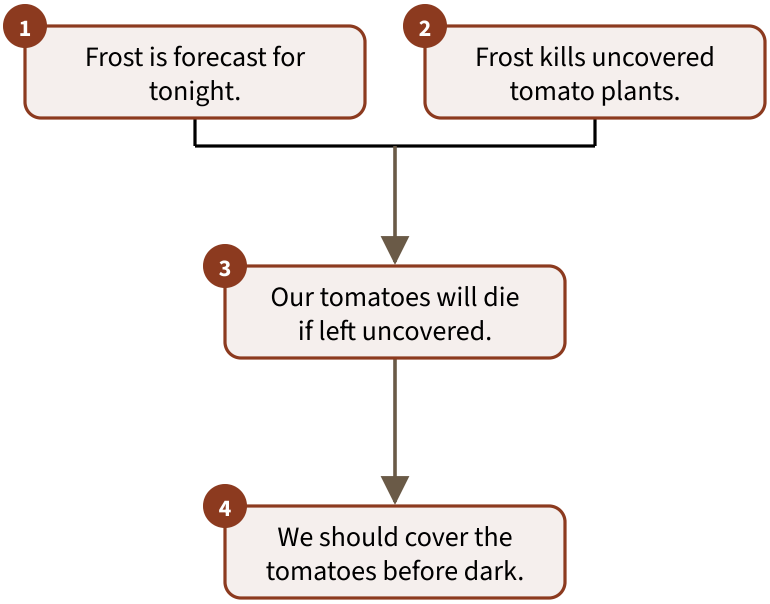
Long arguments need a picture. In Unit 1 you learned to find the premises and conclusion of a short argument and write it in standard form. Real arguments are seldom that tidy. A letter to a town newspaper, a chapter of history or a speech in a debate may offer a dozen claims, some supporting others, some repeated, some mere background. An **argument map** is a simple diagram that shows how the claims fit together. Each claim gets a number, and an arrow runs from every premise to the claim it supports. Once the map is drawn, you can see the whole argument at a glance and know exactly where it stands or falls.

Step one: number the claims and find the main conclusion. Read the whole passage first. Then number each separate claim, even when two claims share one sentence ("The field is flooded, and the bus broke down" holds two claims). Next, find the **main conclusion**, the point the whole passage is driving at. Indicator words help: "therefore," "so" and "thus" point to conclusions; "because," "since" and "for" point

to premises. If there are no indicator words, ask, “What is the writer trying to get me to believe, and what is offered as a reason for it?”

Step two: set aside what is not argument. Not every sentence in a passage is a premise. Some sentences give background (“Henley’s Hardware opened in 1931”), some repeat a claim in other words, some are illustrations, and some report the other side’s view only to answer it. Leave these off the map, or note them to one side. A repeated claim gets the same number both times. A view the writer rejects is not one of the writer’s premises, though the writer’s answer to it may be.

Step three: find the sub-conclusions. Many arguments move in stages. Look at this one: (1) The forecast calls for frost tonight. (2) Frost kills uncovered tomato plants. (3) So our tomatoes will die if we leave them uncovered. (4) Therefore we should cover them before dark. Claim 3 is a conclusion (it is supported by 1 and 2), but it is also a premise (it supports 4). A claim like that is a **sub-conclusion**. An argument in which each claim supports the next is a **chain argument**. The **main conclusion** is the one claim that supports nothing further; on the map, it is the box no arrow leaves.



Step four: joint or independent? When two premises point to the same claim, ask how they work. **Joint premises** work only together. In the tomato argument, claim 1 alone (frost is coming) says nothing about tomatoes, and claim 2 alone (frost kills tomatoes) says nothing about tonight; together they prove claim 3. On the map, joint premises are grouped and share one arrow (in plain text, write “1 + 2 → 3”).

Independent premises each give some support alone. “The game will be canceled: the field is under water, and the other team’s bus broke down.” Either reason by itself would point to a cancellation. Independent premises get separate arrows (“1 → C” and “2 → C”).

The removal test. To tell the two kinds apart, imagine one premise is false or missing. If the other premise still gives some reason for the conclusion, the premises are independent. If the other premise now gives no reason at all, they are joint. Every valid syllogism has joint premises: in “All the pears in this crate came from the Hollis orchard; this pear came from this crate; so it came from the Hollis orchard,” neither premise proves anything without the other. Sometimes careful readers disagree about a pair of premises. That is acceptable, so long as each can give a reason for the choice.

Maps show where to push. A map is not only a picture; it is a plan for testing. An argument can fail in two places: a box (a premise is false) or an arrow (the claim does not really support the next one). If premises are joint, showing that one is false knocks down the whole group. If premises are independent, knocking down one leaves the others standing, so an honest critic must answer each one. And if a sub-conclusion falls, everything resting on it falls too. Before you answer anyone’s argument in a discussion or a debate, map it. You will answer what was actually said, and you will answer the part that matters.

Mapping is a form of fairness. A careful map often shows that an argument is better than it first sounded, or that two people were never really disagreeing about the main point. In Acts 17:11, Scripture praises the Bereans because they “searched the scriptures daily, whether those things were so.” They neither swallowed Paul’s claims whole nor brushed them aside; they examined them. Mapping an argument is that same patient habit applied to any claim: lay out exactly what is said and why, and then judge it.

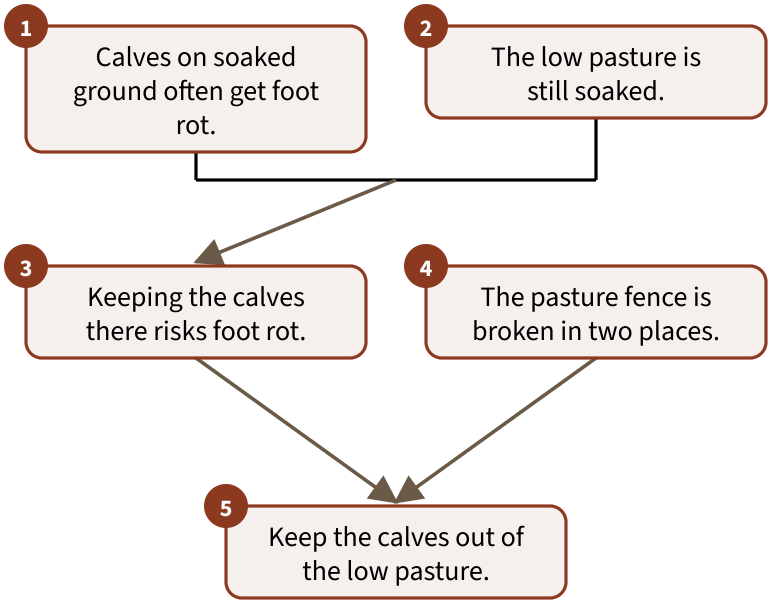
From history. Drawing diagrams of reasoning is newer than most of logic. In 1913 the American law professor John Henry Wigmore published *The Principles of Judicial Proof*, in which he drew charts of the evidence in a trial, with symbols for each piece of testimony and lines showing what supported what. Lawyers could see at a glance which facts held up the case. In 1950 the philosopher Monroe Beardsley taught students to diagram ordinary arguments with numbered claims and arrows, the method used in this lesson. Older logicians did the same work in words, setting out each step of a long proof in order so that every premise could be checked.

Practice habit. Before you answer an argument, number its claims and sketch its map, even if only in your head: what is the conclusion, and what holds it up?

Example 1. Map this argument: "(1) Calves that stand on soaked ground for days often get foot rot. (2) The low pasture is still soaked from the spring floods. (3) So keeping the calves there would risk foot rot. (4) Besides, the fence along that pasture is broken in two places. (5) Therefore the calves should stay out of the low pasture this week."

▼ **Show solution**

1. Find the main conclusion. "Therefore" points to claim 5, and 5 supports nothing further. It is the main conclusion.
2. Claim 3 begins with "So," and it is a reason for 5. It is a sub-conclusion.
3. What supports 3? Claims 1 and 2. Apply the removal test: claim 1 alone says nothing about this pasture, and claim 2 alone says nothing about foot rot. They are joint: 1 + 2 → 3.
4. What supports 5? Claim 3 (the risk of foot rot) and claim 4 (the broken fence). "Besides" signals an extra reason. Either one alone would give some reason to keep the calves out, so they are independent: 3 → 5 and 4 → 5.
5. Draw the map.



Answer: 1 + 2 → 3 (joint); 3 → 5 and 4 → 5 (independent). Claim 3 is a sub-conclusion; claim 5 is the main conclusion.

Example 2. Use the removal test to decide whether the premises are joint or independent. (a) "Caleb should take the bus today, since his bike has a flat tire and it is pouring rain." (b) "Every member of the choir must attend Thursday's rehearsal. Hannah is a member of the choir. So Hannah must attend Thursday's rehearsal."

▼ **Show solution**

1. (a) Remove "his bike has a flat tire." Pouring rain alone still gives Caleb a reason to take the bus. Remove "it is pouring rain." A flat tire alone still gives a reason. Each premise works alone, so they are independent.
2. (b) Remove "Hannah is a member of the choir." The rule about choir members now says nothing about Hannah. Remove the rule instead. Hannah's membership alone says nothing about Thursday. Neither works alone, so they are joint.
3. Notice that (b) is a valid deductive argument. The premises of a valid syllogism are always joint.

Answer: (a) Independent: each reason alone supports taking the bus. Map: 1 → C, 2 → C. (b) Joint: neither premise supports the conclusion without the other. Map: 1 + 2 → C.

PRACTICE

1. Easy What is a sub-conclusion?

▼ **Show answer**

A claim that is supported by some premises and then serves as a premise for a further conclusion. It is a conclusion and a premise at the same time.

2. Easy Explain the difference between joint premises and independent premises.

▼ **Show answer**

Joint premises support the conclusion only together; remove one and the others give little or no support. Independent premises each give some support by themselves; remove one and the others still stand.

3. Easy On a finished argument map, how can you tell which box is the main conclusion?

▼ **Show answer**

It is the claim that no arrow leaves: arrows point into it, but it supports nothing further.

4. Easy Name the premise and the conclusion, and say which word told you: "The library closes early on Saturdays, so we should go on Friday."

▼ **Show answer**

Premise: The library closes early on Saturdays. Conclusion: We should go on Friday. The word "so" signals the conclusion.

5. Medium Map this argument and name any sub-conclusion: "(1) Biscuit has not touched his food all day. (2) He has been hiding under the porch since morning. (3) So he may be sick. (4) Therefore we should call the veterinarian."

▼ **Show answer**

1 → 3 and 2 → 3 (independent: each sign alone gives some reason to think Biscuit is sick); 3 → 4. Claim 3 is a sub-conclusion; claim 4 is the main conclusion. Accept "1 + 2 → 3" (joint) if the student explains that neither sign alone would be enough, though independent is the better reading.

6. Medium Map this argument and name the sub-conclusions: "(1) The bridge on Mill Road is closed for repairs. (2) So the school bus must go the long way around by the quarry. (3) So the bus will reach Caleb's stop later than usual. (4) Therefore Caleb has time to finish his breakfast."

▼ **Show answer**

1 → 2 → 3 → 4, a chain argument. Claims 2 and 3 are sub-conclusions; claim 4 is the main conclusion.

7. **Medium** Are the premises joint or independent? Give a reason. (a) “All the eggs in this basket came from the Weller farm. This egg came from this basket. So this egg came from the Weller farm.” (b) “We should hold the picnic in the church hall instead of the park: thunderstorms are forecast for the afternoon, and the park’s picnic tables are all being repainted this week.”

▼ **Show answer**

(a) Joint: neither premise says anything about where this egg came from without the other (it is a valid syllogism). (b) Independent: either reason alone, the storms or the tables being repainted, gives some reason to move the picnic out of the park.

8. **Medium** Which numbered sentences are not part of the argument? Then map the rest. “(1) Our garden is on the south side of the house. (2) The tomato plants are already four feet tall and leaning over. (3) Tomato plants that lean over will snap in the next storm. (4) So we need to stake them this weekend. (5) Dad bought the stakes at Henley’s Hardware, which opened in 1931.”

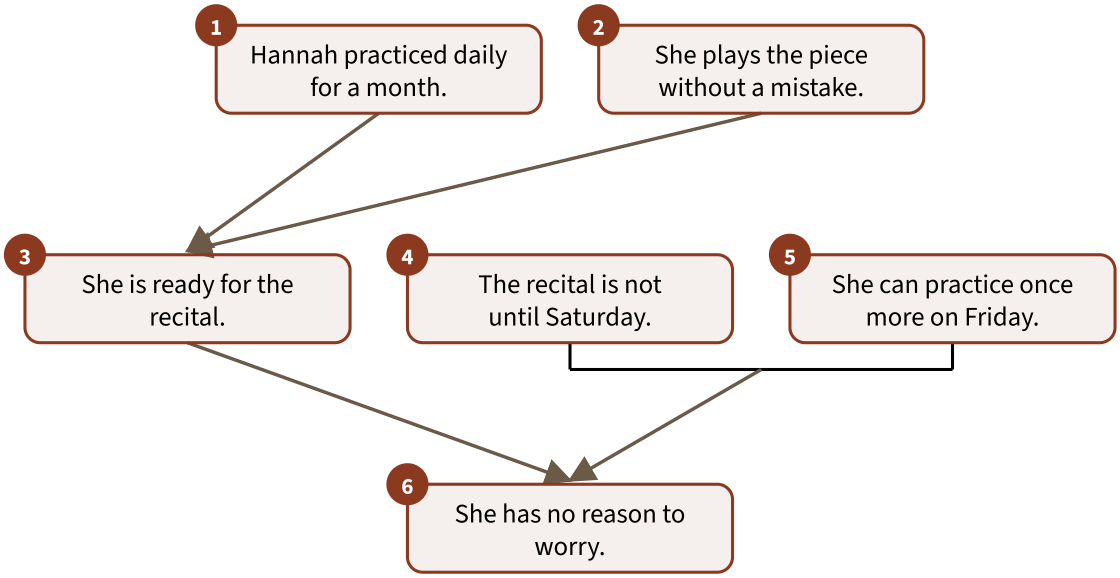
▼ **Show answer**

Sentences 1 and 5 are background; they give no reason for the conclusion. Map: 2 + 3 → 4 (joint: the plants’ leaning matters only because leaning plants snap, and vice versa). Accept a student who links 1 if they give a sensible reason (for example, that a sunny garden grows tall plants), but 1 and 5 are best left off.

9. Hard Map this argument: "(1) Hannah has practiced her piano piece every day for a month, (2) and her teacher says she now plays it without a mistake. (3) So she is ready for the recital. (4) Besides, the recital is not until Saturday, (5) and she can practice once more on Friday. (6) So she has no reason to worry."

▼ **Show answer**

1 → 3 and 2 → 3 (independent: each is some evidence she is ready); 3 → 6; 4 + 5 → 6 (joint: together they say she has time for one more practice). Claim 3 is a sub-conclusion; 6 is the main conclusion.



Accept 4 and 5 as independent, or 1 and 2 as joint, if the student gives a reason using the removal test.

10. Hard Write a short argument of your own (four claims, numbered) that fits this map: 1 + 2 → 3; 3 → C; 4 → C. Label C as claim 5 if you like.

▼ **Show answer**

Sample answer: (1) Seeds need warm soil to sprout. (2) The soil in our garden is still cold. (3) So seeds planted today would not sprout. (4) Besides, more frost is forecast for next week. (5) Therefore we should wait to plant the beans. Any passage is correct if 1 and 2 work only together to support 3, and 3 and 4 each support the conclusion on their own.

11. Hard You want to answer an argument fairly and effectively. Why does it matter whether its premises are joint or independent?

▼ **Show answer**

If the premises are joint, showing that any one of them is false (or does not connect) defeats the whole group, so one good answer may be enough. If they are independent, defeating one leaves the others standing, so you must answer each premise; attacking only one and declaring victory would be unfair and would not refute the argument. (Answers may also note that a fallen sub-conclusion takes down everything resting on it.)

12. Challenge Map this passage. Say which sentence is not one of the writer's premises, and why. "(1) Some say our class should take its field trip to the city museum. (2) But the trip to Brook Farm costs half as much. (3) It also fits our science unit on soil and crops, (4) and Mr. Hale, the farmer, has offered to give the tour himself. (5) So the farm trip would teach us more. (6) Since it is both cheaper and more useful, the class should choose Brook Farm."

▼ **Show answer**

Sentence 1 reports the other side's view, which the writer rejects; it is not one of the writer's premises. Map: 3 → 5 and 4 → 5 (independent: each is a separate reason the farm trip would teach more); 2 → 6 and 5 → 6 (independent: cheaper and more useful are two separate reasons). Claim 5 is a sub-conclusion; 6 is the main conclusion. Accept 3 + 4 as joint if the student gives a reason.

COMMON MISTAKES

- Putting every sentence on the map. Background facts, repetitions and the other side's view are not the writer's premises.
- Missing a sub-conclusion. If a claim is both supported and supporting, it is a sub-conclusion; don't draw it as a plain premise or as the main conclusion.
- Drawing joint premises as independent. If a premise says nothing about the conclusion without its partner, they must share one arrow.
- Treating the first sentence as the conclusion. The main conclusion can come first, last or in the middle; find it by asking what the passage is trying to prove.

REFERENCE GUIDE

An eight-page, color-coded Logic guide: what logic is, terms and definitions, the A E I O propositions, the square of opposition and immediate inferences, syllogisms with the six rules and Venn tests, the named forms, informal fallacies of relevance, presumption and ambiguity, propositional logic with truth tables, conditionals, valid forms and short proofs, the classical tradition, dialectic, argument maps and practice. The booklet prints on 11×17 paper and folds to letter size. Prints in color and reads clearly in black and white.

> **Scripture.** “they received the word with all readiness of mind, and searched the scriptures daily, whether those things were so.” (Acts 17:11, KJV)
>
> The Bereans neither swallowed nor dismissed what they heard; they examined it. Mapping an argument is the same patient examination.

UNIT 9 · LOGIC IN ARGUMENT AND DEBATE · LESSON 9.2 OF 69 · 40 MINUTES

9.2 Hidden Premises and the Strongest Form

OBJECTIVES

- Find the hidden premise that an argument needs in order to be valid.
- Choose the most reasonable version of a hidden premise, following the principle of charity.
- Judge whether a hidden premise is true, and see why arguments are often weakest there.
- Restate an opponent’s argument in its strongest form before answering it.

KEY TERMS

Hidden premise

A premise an argument needs but does not state; also called an unstated, missing or suppressed premise.

Enthymeme

In logic, an argument with a premise (or sometimes the conclusion) left unstated. Lesson 9.5 gives Aristotle’s fuller meaning of the word.

Bridge premise

The hidden premise that connects what is stated to the conclusion, often in the form "All A are B" or "If p, then q."

Principle of charity

The rule that we should read another person's argument in the most reasonable way their words allow before we judge it.

Strongest form

An opponent's argument restated as clearly and persuasively as possible, with its best evidence and most reasonable premises, so that the opponent would accept it as their own.

Straw man

The fallacy of answering a weaker or distorted version of an argument instead of the one actually given (Unit 3).

CORE IDEAS

Most arguments leave something out. "Whales are mammals, so whales breathe air." No one talks in full syllogisms, and no one needs to. The speaker expects you to supply the missing piece: all mammals breathe air. A premise that an argument needs but does not state is a **hidden premise**. An argument with a part left unstated is often called an **enthymeme**. Leaving out the obvious is not a fault; it saves time. But the unstated part is still part of the argument, and it may be false. Learning to find it is one of the most useful skills in logic.

Find the bridge. To find a hidden premise, ask: "What would I have to add to make the conclusion follow?" Look for a term in the conclusion that the stated premise never mentions, and build a bridge to it. In "This is a Fernwood bicycle, so it will last for years," the stated premise talks about Fernwood bicycles and the conclusion talks about lasting for years. The bridge is "All Fernwood bicycles last for years." The same works for conditionals. "The streets are wet, so it must have rained" needs the bridge "If the streets are wet, it rained." A hidden premise that makes the argument valid is the first one to try.

Charity: choose the most reasonable bridge. Often more than one bridge will do. "Mrs. Ortiz runs the Brookside Bakery, so she must be a good cook." We could supply "Everyone who runs a bakery is a good cook," which makes the argument valid but is plainly false. Or we could supply "People who run bakeries are usually good cooks," which makes the argument inductive but is far more believable. The **principle of**

charity says to choose the reading the speaker most likely meant and that gives the argument its best chance. Don't hand the speaker a silly premise just so you can knock it down.

Arguments are often weakest where they are silent. People rarely state the premise they are least sure of. An advertisement says, "Zestara cereal has more added vitamins than any other brand, so it is the healthiest breakfast you can buy." The hidden bridge is "The cereal with the most added vitamins is the healthiest breakfast." Put into words, it is easy to doubt: a cereal could be loaded with vitamins and also with sugar. Bringing the hidden premise into the light is often all it takes to show that an argument fails. It also works the other way: sometimes the hidden premise is obviously true, and the argument is better than you thought.

Hidden conclusions. Sometimes it is the conclusion that is left unsaid. "Only players who come to practice may play on Saturday, and you missed practice." The speaker does not need to finish the thought; you may not play on Saturday. Advertisements love hidden conclusions because an unspoken claim is harder to challenge. "Nine out of ten trainers wear Quillfoot shoes" invites you to conclude, without anyone saying it, that you should buy them. State the hidden conclusion, and then ask whether the premises support it.

The strongest form. In Unit 3 you learned the straw man fallacy: answering a weaker version of an argument than the one given. The cure is to do the opposite on purpose. Before you answer someone, restate the argument in its **strongest form**: put it in the clearest words, supply the most reasonable hidden premises, and add the best evidence its defenders would offer. A good test is whether the other person would say, "Yes, that is exactly what I mean; you put it better than I did." Only then do you answer it.

Why the strongest form helps you. Answering the strongest form is fair, but it is also wise. If you defeat a weak version, the other side simply restates the real argument, and you have gained nothing. If you defeat the strongest form, you have really answered the question. And if you find that you cannot answer it, you have learned something important: perhaps the other side is right, or partly right. Listeners notice, too. A speaker who states the other side fairly earns trust, while a speaker who twists it loses trust the moment anyone checks.

The Golden Rule in argument. In Matthew 7:12 Jesus taught, "all things whatsoever ye would that men should do to you, do ye even so to them." We all want our own arguments heard at their best, not mocked in a garbled form. So we owe others the same courtesy. Charity in reading is not softness, and it does not mean agreeing. It means aiming at the truth by testing the best case each side can make. The rest of this unit, from Socratic questioning to formal debate, depends on this habit.

> **From history.** Logicians have long noticed that ordinary arguments leave premises unstated. The *Port-Royal Logic* (1662), written by Antoine Arnauld and Pierre Nicole at the abbey of Port-Royal near Paris, described the enthymeme as a syllogism that is complete in the mind but incomplete in the words, because the speaker trusts the hearer to supply what is missing. Leaving out the obvious, they saw, is usually no fault at all: a speech that spelled out every premise would be tiresome to hear. Richard Whately, in his *Elements of Logic* (1826), taught students to supply the suppressed premise of an enthymeme before judging it, so that every part of the argument could be examined in the open. That is exactly the skill of this lesson.

> **Practice habit.** Before you disagree with someone, say their argument back to them in its strongest form and ask, "Is that what you mean?"

Example 1. Find the hidden premise in "Mrs. Ortiz runs the Brookside Bakery, so she must be a good cook." Give the version that makes the argument valid and the most charitable version, and judge each.

▼ **Show solution**

1. Find the gap. The premise is about running the bakery; the conclusion is about being a good cook. The bridge must connect the two.
2. Valid version: "Everyone who runs a bakery is a good cook." With it, the argument is valid. But the premise is false: someone could own and manage a bakery while hiring others to do the baking.
3. Charitable version: "People who run bakeries are usually good cooks." With it, the argument is inductive rather than valid, but the premise is much more believable, and it is probably what the speaker meant.
4. Judge. The charitable reading gives a fairly strong inductive argument, which is the fair thing to evaluate.

Answer: Valid version: "Everyone who runs a bakery is a good cook" (valid but false). Charitable version: "People who run bakeries are usually good cooks" (inductive and reasonably strong). Evaluate the charitable version.

Example 2. Caleb wants a puppy. His older sister Leah says, "We shouldn't get a puppy; puppies are a lot of work." Caleb replies, "So you think we should never have any pet that takes work!" Explain what went wrong, restate Leah's argument in its strongest form, and say how Caleb could answer it.

▼ **Show solution**

1. Caleb's reply is a straw man. Leah said nothing about never having any pet that takes work; Caleb replaced her argument with a weaker one.
2. Find Leah's hidden premise. Her stated reason is "puppies are a lot of work." The bridge to "we shouldn't get one" is something like "We don't have time right now for the work a puppy needs."
3. Add her best evidence. A puppy must be fed, walked and trained several times a day for months, and the family's school and work schedules are full.
4. Strongest form: "A puppy needs feeding, walking and training several times a day for months. With everyone's schedule as full as it is, no one has the time to do that well right now. So getting a puppy now would be unfair to the puppy and to us."
5. Now Caleb can answer the real argument: for example, by showing a written plan for who will walk and train the puppy and when, or by proposing to wait until summer.

Answer: Caleb attacked a straw man. Strongest form: a puppy needs daily care for months, the family has no time to give it right now, so getting one now would be unfair to the puppy and the family. Caleb should answer that, perhaps with a care schedule or a plan to wait until summer.

PRACTICE

1. Easy What is a hidden premise?

▼ **Show answer**

A premise that an argument needs in order for its conclusion to follow but that the speaker does not state.

2. **Easy** What does it mean to restate an argument in its strongest form, and how is that the opposite of a straw man?

   ▼ **Show answer**

   It means putting the other person's argument in its clearest words, with its most reasonable premises and best evidence, so that they would accept it as their own. A straw man does the opposite: it replaces the real argument with a weaker or distorted one that is easier to knock down.

3. **Easy** Supply the hidden premise: "Whales are mammals, so whales breathe air."

   ▼ **Show answer**

   All mammals breathe air.

4. **Easy** Supply the hidden premise: "This is a Fernwood bicycle, so it will last for years."

   ▼ **Show answer**

   All Fernwood bicycles last for years. (Accept the more charitable "Fernwood bicycles usually last for years," which makes the argument inductive.)

5. **Medium** Supply the hidden premise as an "if ... then" statement, and say whether it is true: "The sidewalk is wet this morning, so it must have rained last night."

   ▼ **Show answer**

   Hidden premise: "If the sidewalk is wet this morning, then it rained last night." It is doubtful: the sidewalk could be wet from sprinklers, dew, melting frost or someone washing a car. Accept a charitable "usually" version, which makes the argument inductive and only moderately strong.

6. **Medium** Find the hidden premise and judge it: "Zestara cereal has more added vitamins than any other brand, so it is the healthiest breakfast you can buy."

   ▼ **Show answer**

   Hidden premise: "The breakfast with the most added vitamins is the healthiest breakfast." It is false or at least doubtful: healthfulness depends on more than vitamins (sugar, fiber, protein and so on), so the argument fails at its hidden premise.

7. Medium Which hidden premise is more charitable for "Grandpa has farmed for fifty years, so he knows when to plant the corn"? (a) "Anyone who has farmed for fifty years knows everything about farming." (b) "A farmer with fifty years of experience usually knows when to plant his main crop." Explain.

▼ **Show answer**

(b) is more charitable. It is believable and is probably what the speaker meant; it makes the argument a strong inductive one. (a) is far more than the speaker needs and is plainly false (no one knows everything), so supplying it would set up a straw man.

8. Medium State the hidden conclusion: "Only players who come to practice may play on Saturday, and you missed practice."

▼ **Show answer**

You may not play on Saturday. ("Only players who come to practice may play" means "All who may play are players who came to practice"; you did not come, so you may not play.)

9. Hard Hannah argues, "Our family should eat supper together every night, because meals together are the best time to talk." Her brother answers, "So Hannah wants us to sit at the table and do nothing but talk for three hours every night!" (a) Name the mistake. (b) Restate Hannah's argument in its strongest form, with its hidden premise.

▼ **Show answer**

(a) A straw man: Hannah said nothing about three hours or doing nothing but talk. (b) Sample strongest form: "Families grow closer when they talk together regularly. Supper is the one time of day when everyone is home and not busy with other things, so it is the best time to talk. Therefore our family should eat supper together every night." Accept any version that keeps her point and supplies a reasonable hidden premise such as "We should make regular time to talk as a family."

10. Hard Supply the hidden premise that makes each argument valid. (a) “No reptiles have fur, so no snakes have fur.” (b) “Every member of the Willow Creek Junior Farmers raises an animal, so Hannah raises an animal.”

▼ **Show answer**

(a) All snakes are reptiles (true; the argument is sound). (b) Hannah is a member of the Willow Creek Junior Farmers. (Accept equivalent wordings.)

11. Hard A poster says, “Hollow Oak Academy students score higher on reading tests than students at any other school in the county. Enroll your child at Hollow Oak!” State the hidden conclusion and two hidden premises, and say which premise is most open to doubt.

▼ **Show answer**

Hidden conclusion: Hollow Oak would raise your child’s reading scores (and so you should enroll). (Also accept “Hollow Oak is a better school than the others” or “your child would do better there.”) Hidden premises, for example: (1) The high scores are caused by Hollow Oak’s teaching, not by which students it admits. (2) Higher reading scores show that a school is better for your child. Premise (1) is most open to doubt: the school may simply enroll students who already read well. Accept other reasonable hidden premises if the student explains them.

12. Challenge Choose one of these claims you might disagree with: (a) “Students should not be given homework on weekends,” or (b) “Every family should keep a vegetable garden.” Write its strongest form in three or four sentences, including the hidden premise. Then write one sentence that answers it fairly.

▼ **Show answer**

Sample for (a): “Weekends are the main time families have for rest, worship and time together. Homework takes hours away from those things, and tired students learn less on Monday. Rest is necessary for learning, not opposed to it. So schools should not assign homework on weekends.” Fair answer: “Rest matters, but a small weekend assignment, such as one chapter of reading, keeps skills fresh without taking much family time.” Accept any version that a defender of the claim would accept as their own and any courteous answer aimed at its real premises.

COMMON MISTAKES

- Supplying a hidden premise that is stronger than needed ("all" when the speaker meant "most"), and then knocking it down. That is a straw man.
- Forgetting that a hidden premise is still part of the argument. If it is false, the argument fails, even though no one said it.
- Thinking the strongest form means agreeing. You state the other side at its best so that you can test it fairly, not so that you must accept it.
- Restating an opponent's argument in words they would never use or accept. The test is whether they would say, "Yes, that is what I mean."

> **Scripture.** "all things whatsoever ye would that men should do to you, do ye even so to them" (Matthew 7:12, KJV)
>
> We want our own arguments heard at their best; the Golden Rule asks us to hear others' arguments the same way.

UNIT 9 · LOGIC IN ARGUMENT AND DEBATE · LESSON 9.3 OF 69 · 45 MINUTES

9.3 Dialectic: Socratic Questioning

OBJECTIVES

- Explain dialectic as the testing of a claim by question and answer.
- Use the three Socratic tools: the request for a definition, the counterexample and the consequence.
- Follow Socrates' reasoning in Plato's *Euthyphro* and *Republic* I.
- Question a definition courteously and revise it in light of the answers.

WATCH

The Socratic Method

Ligonier Ministries · 4:40

https://www.youtube.com/watch?v=aOdjpByHLEQ

> **WHEN TO WATCH**
> Before reading the Euthyphro excerpt: what Socrates was doing when he asked questions.

Note: Parent preview: R. C. Sproul explains Socrates' method of dialogue, the admission of ignorance as the start of learning, and Socrates' concern to define virtues such as justice and honesty. He notes that Socrates did not hold the biblical doctrine of original sin. One small slip: Sproul ties "educate" to "el duce"; the Latin root is educere, "to lead out." Nothing objectionable; transcript checked October 18, 2026.

KEY TERMS

Dialectic

The art of testing a claim or definition by question and answer between two or more people, aiming at the truth.

Socratic method

Dialectic as practiced by Socrates: asking someone for a definition, then testing it with questions, counterexamples and consequences until it stands or falls.

Counterexample

A clear case that fits a definition or general claim but not the thing defined (or the reverse), showing that the definition or claim is false.

Consequence (drawing out)

Showing what follows from a claim; if what follows is false, absurd or contradicts the claim, the claim must be given up (reduction to absurdity).

Aporia

Greek for "no way through": the puzzled state at the end of many Socratic dialogues, when every proposed definition has failed.

Piety

Holiness, or right reverence toward God (in the *Euthyphro*, toward the gods); the virtue Socrates asks Euthyphro to define.

Reasoning out loud with another person. So far you have mostly tested arguments on paper. **Dialectic** tests them in conversation. One person makes a claim; the other asks questions about it; together they find out whether it holds. The great master of this art was Socrates of Athens, who wrote nothing himself but appears as the questioner in the dialogues of his student Plato. His way of questioning, the **Socratic method**, uses three tools: he asks for a **definition**, he tests it with **counterexamples**, and he draws out its **consequences**.

Tool one: "What is it?" Socrates begins by asking for a definition. In the *Euthyphro*, he meets a man named Euthyphro outside the courthouse, just as Socrates himself is about to face trial. Euthyphro is prosecuting his own father and is sure that this is the pious, or holy, thing to do. Socrates asks him what piety is. Euthyphro answers that piety is doing what he is doing: prosecuting a wrongdoer. Socrates points out that this is only an **example**. There are many pious acts; what he wants is the one thing that makes all of them pious. As you learned in Unit 2, a definition must fit every case of the thing defined and nothing else.

Tool two: the counterexample. In Book I of the *Republic*, Socrates asks what justice is. The old man Cephalus suggests, roughly, that justice is telling the truth and paying back what you owe. Socrates answers with a case. Suppose a friend lent you a weapon while he was in his right mind, and now he asks for it back after he has gone mad. Should you return it? Everyone agrees you should not, and that withholding it is not unjust. So the definition is too broad: returning what you owe is not always just. A good **counterexample** is like that. It is clear, it fits the definition, and everyone can see it does not fit the thing defined.

Tool three: the consequence. The third tool asks, "If that is true, what follows?" Euthyphro's second definition is that piety is what is dear to the gods. But Euthyphro believes in many gods who quarrel among themselves. Socrates draws out the consequence: the same act could then be loved by one god and hated by another, and so be both pious and impious at once. A definition that leads to a contradiction cannot be right. Later in *Republic* I, Thrasymachus claims that justice is whatever benefits the stronger, and that the just man obeys the rulers' laws. Socrates gets him to admit that rulers sometimes make mistakes about their own benefit. Then

obeying them would sometimes harm the stronger, and justice would be both for and against the stronger's benefit. The definition refutes itself.

The famous question. Euthyphro then revises: piety is what *all* the gods love. Socrates asks a deeper question. Is the pious loved by the gods because it is pious, or is it pious because they love it? If the gods love it because it is pious, then being loved by the gods is only a result of piety, not what piety is. The definition names a **property**, something that follows from the essence, not the essence itself. The question has been discussed ever since. Christian thinkers such as Augustine and Aquinas taught that God is goodness itself, so that goodness is neither an arbitrary choice of God nor a standard outside him; it is rooted in his own nature.

The puzzled ending. Euthyphro tries several more definitions, and each one fails. At last he says he is in a hurry and must go, and the dialogue ends without an answer. This puzzled state is called **aporia**, "no way through." It can seem like a failure, but it is real progress. Euthyphro began sure of himself and ignorant of his ignorance; the reader ends knowing several wrong answers to avoid and having a clearer idea of what a right answer would need. Socrates thought that knowing you do not know is the beginning of wisdom.

How to question well. To use the method yourself, ask one short question at a time, and let the other person answer. Begin with what they believe, not what you believe. When they give an example, ask what all such examples share. When they give a definition, look for a clear counterexample, and ask what would follow if it were true. Keep the aim in view: you are hunting for the truth together, not trying to embarrass anyone. Proverbs 20:5 says, "Counsel in the heart of man is like deep water; but a man of understanding will draw it out." Good questions draw out what a person really thinks.

Turn the method on yourself. The Socratic method is not only for testing other people. Before you defend a claim in a debate or an essay, ask yourself: What exactly do I mean by my key word? Can I think of a counterexample? What follows if I am right? A definition that survives your own questions will survive an opponent's. For more on dialectic and the great books, visit the Classics Library Logic & Debate page.

From history. Socrates (about 470–399 B.C.) taught in the streets and markets of Athens by asking questions, and he left no writings. We know him chiefly through Plato, whose dialogues show him questioning generals, poets and teachers about courage, piety and justice. In 399 B.C. Socrates was tried by an Athenian court, condemned and put to death; the *Euthyphro* is set just before that trial. Aristotle later wrote that two things could fairly be credited to Socrates: inductive arguments and the search for universal definitions. The English scholar Benjamin Jowett published his translation of Plato's dialogues in 1871, and it is still widely read.

Practice habit. When someone uses an important word in an argument today, ask yourself what they mean by it, and think of one case that would test their meaning.

Enrichment: reading the *Euthyphro*. The *Euthyphro* is one of Plato's shortest dialogues, and Jowett's 1871 translation is in the public domain. Read it in one sitting with a pencil in hand. Each time Euthyphro offers a definition of piety, write it down and note which tool Socrates uses against it: counterexample, consequence, or the complaint that it is only an example. How many definitions does Euthyphro offer? Which one comes closest, in your judgment, and why?

Example 1. Hannah says, "A friend is someone who is nice to you." Test this definition with the three Socratic tools, and suggest a better one.

▼ **Show solution**

1. Ask for clarity. "Do you mean anyone who is nice to you, even once?" Suppose Hannah says yes.
2. Counterexample. A salesman at the Fairview store is very nice to you in order to sell you a coat, and a stranger holds a door for you. Both are nice, but neither is your friend. The definition is too broad.
3. Consequence. If the definition were right, a friend who told you a hard truth you did not want to hear (that would not feel "nice") would stop being your friend at that moment. But we usually think such a friend is a true friend. The definition leads to a result Hannah herself rejects.
4. Revise. A better attempt: "A friend is someone who knows you well and wants what is good for you, and whom you treat the same way." This excludes the salesman and the stranger and includes the friend who tells hard truths. It could be tested further.

Answer: The definition is too broad (the salesman and the stranger are nice but not friends) and has a consequence Hannah would reject (an honest friend would stop being a friend). Revised: a friend is someone who knows you well and wants your good, and whom you treat the same way.

Example 2. In *Republic* I, Cephalus suggests that justice is telling the truth and paying back what you owe, and Socrates replies with the case of a friend who has gone mad and asks for his weapon back. Explain which Socratic tool Socrates uses and what fault in the definition it shows.

▼ **Show solution**

1. Identify the definition: justice = telling the truth and paying back what you owe.
2. Identify the case: returning a weapon to a friend who has gone mad. It fits the definition (you would be paying back what you owe).
3. Everyone agrees that returning it would not be just, and that refusing would not be unjust. The case fits the definition but not the thing defined.
4. Name the fault. A case that fits the definition but not the thing defined shows the definition is too broad (it is not convertible with "justice").

Answer: Socrates uses a counterexample. It shows that the definition is too broad: paying back what you owe is not always just.

PRACTICE

1. Easy What is dialectic?

   ▼ **Show answer**

   The art of testing a claim or definition by question and answer between two or more people, aiming at the truth.

2. Easy Name the three tools of the Socratic method.

   ▼ **Show answer**

   Asking for a definition ("What is it?"), testing it with counterexamples, and drawing out its consequences.

3. Easy In the *Euthyphro*, what does Socrates ask Euthyphro to define, and why does Euthyphro's first answer fail?

   ▼ **Show answer**

   Piety (holiness). Euthyphro's first answer, "doing what I am doing, prosecuting a wrongdoer," is only an example of a pious act, not a definition of what makes every pious act pious.

4. **Easy** In *Republic* I, what definition of justice does Cephalus suggest, and what case does Socrates use against it?

   ▼ **Show answer**

   Cephalus suggests that justice is telling the truth and paying back what you owe. Socrates asks whether you should return a weapon to a friend who has gone mad; everyone agrees you should not, so the definition fails.

5. **Medium** Give a counterexample to the definition "A bird is an animal that flies," and say whether it shows the definition is too broad or too narrow.

   ▼ **Show answer**

   Too narrow: penguins and ostriches are birds that do not fly. (Also accept: too broad, because bats and bees fly but are not birds. The definition fails both ways.)

6. **Medium** Give a counterexample to the definition "A chair is something you sit on," and say which way the definition fails.

   ▼ **Show answer**

   Too broad: you can sit on a bench, a stool, a log or a horse, and none of them is a chair. (Accept any clear case of something you sit on that is not a chair. Also accept "too narrow" with a case such as a doll's chair or a museum chair no one may sit on, though "too broad" is the main fault.)

7. **Medium** Use the consequence tool against the claim "Whatever most people believe is true."

   ▼ **Show answer**

   Sample answer: Most people once believed that the sun goes around the earth, and later most people believed the opposite. If the claim were right, both beliefs would be true, so the earth would both go around the sun and not. Or: the truth would change every time the majority changed its mind. These consequences are absurd, so the claim must be given up. Accept any consequence that is clearly false or contradictory.

8. Medium Caleb says, “Courage is when a firefighter runs into a burning house to save a child.” What would Socrates say about this answer?

▼ **Show answer**

That it is an example of courage, not a definition. Socrates would ask what that act shares with every other courageous act (a soldier holding his post, a patient bearing pain, a student telling the truth when it costs her) that makes all of them courageous.

9. Hard Write three Socratic questions to test the definition “A good student is one who gets good grades.” Label which tool each question uses.

▼ **Show answer**

Sample questions: (1) Definition: “What do you mean by ‘good’: good at learning, or good in character, or both?” (2) Counterexample: “Could a student get high grades by copying a friend’s work? Would that make him a good student?” or “Could a hardworking student who struggles with math and earns Cs be a good student?” (3) Consequence: “If grades make a good student, does a student become a worse student when he moves to a school with harder grading?” Accept any three fair questions that use the tools correctly.

10. Hard Explain how Socrates uses the consequence tool against Thrasymachus’s claim that justice is whatever benefits the stronger.

▼ **Show answer**

Thrasymachus also says the just man obeys the rulers’ laws. Socrates gets him to admit that rulers sometimes make mistakes and pass laws that harm themselves. Then obeying those laws, which is just, would harm the stronger. So justice would be both what benefits the stronger and what harms the stronger, a contradiction. The definition refutes itself.

11. Hard In your own words, explain the question Socrates asks Euthyphro about piety and the gods' love, and why it shows that "what the gods love" is not a good definition.

▼ **Show answer**

Socrates asks whether the gods love the pious because it is pious, or whether it is pious because they love it. Euthyphro agrees the gods love it because it is pious. Then the gods' love is a result of piety, not what piety is; "loved by the gods" names something that follows from piety (a property) rather than its essence, so it is not a proper definition.

12. Challenge Write a short dialogue (six to eight lines) in which one person defines "a hero" and the other tests the definition with all three Socratic tools. End with a revised definition.

▼ **Show answer**

Sample: A: "A hero is someone who is famous for doing something brave." B: "What do you mean by famous?" A: "Lots of people know about him." B: "What about a mother who quietly runs into a flooded creek to save her child, and no one ever hears of it? Is she a hero?" A: "Yes, she is." B: "Then fame is not needed. And if fame made heroes, a man who did something brave but foolish, and became famous, would be a hero. Is he?" A: "Not really. A hero is someone who bravely risks something to do good for others." Accept any dialogue that uses a request for definition, a clear counterexample and a consequence, and ends with a better definition.

COMMON MISTAKES

- Giving an example when asked for a definition. Socrates wants what all the examples have in common.
- Using a counterexample that is doubtful. A good counterexample is plainly true to everyone.
- Treating questioning as a way to trap or embarrass. Dialectic is a shared search for the truth.
- Thinking a dialogue that ends in puzzlement has achieved nothing. Learning which answers are wrong is real progress.

> **Scripture.** "Counsel in the heart of man is like deep water; but a man of understanding will draw it out." (Proverbs 20:5, KJV)
>
> Good questions draw out what a person really thinks, which is the heart of the Socratic method.

UNIT 9 · LOGIC IN ARGUMENT AND DEBATE · LESSON 9.4 OF 69 · 45 MINUTES

9.4 The Disputed Question

OBJECTIVES

- Name and explain the five parts of a disputed question: the question, the objections, "On the contrary," "I answer that" and the replies.
- State a question in the "Whether ..." form and write fair objections to your own view.
- Write a respondeo that defines terms and draws distinctions, and reply to each objection by conceding, denying or distinguishing.
- Explain why the disputed question is a school of fairness as well as of logic.

KEY TERMS

Disputed question

A medieval form of written or oral argument in which a yes-or-no question is settled by stating objections, a contrary consideration, the author's answer and replies to each objection (Latin *quaestio disputata*).

Article

One disputed question within a larger work such as Aquinas's *Summa Theologica*, beginning "Whether ...".

Objection

An argument for the view the author will reject, stated fairly at the start of the article.

Sed contra ("On the contrary")

A brief consideration, often a quotation from Scripture or a respected authority, that points against the objections and toward the author's answer.

Respondeo ("I answer that")

The body of the article, where the author defines terms, draws distinctions and gives his own reasoned answer.

Reply

The author's answer to one objection, showing where it goes wrong or what part of it is true.

Concedo, nego, distinguo

Latin for "I grant, I deny, I distinguish": the three classic ways to answer a premise, by accepting it, rejecting it, or showing it is true in one sense and false in another.

CORE IDEAS

Argument in the medieval schools. In the universities of the Middle Ages, such as Paris and Oxford, the chief way of teaching was the **disputation**. A master posed a question, students argued both sides, and the master gave his answer and replied to the arguments against it. The written form of this exercise is the **disputed question**. Its most famous user was Thomas Aquinas, whose *Summa Theologica* is built entirely of short disputed questions called **articles**. The form has five parts, always in the same order.

Part one: the question. Every article begins with a yes-or-no question introduced by "Whether": "Whether God exists," "Whether it is better to tell a hard truth than a kind lie," "Whether a farmer should rotate his crops." Stating the question this way forces clarity. There are only two answers, so both sides know exactly what is in dispute. A good question is about one thing, uses clear terms, and can be answered by reasoning.

Part two: the objections. Next come the **objections**, usually two to four. They argue for the side the author will reject, and they begin, "Objection 1. It would seem that ..." This is the most surprising feature of the form: the author starts by stating the other side's case, and he states it well. Each objection is a real argument, often with a premise from a respected source. Aquinas's objections are so fair that careless readers sometimes quote them as if they were his own views. Writing objections is practice in the strongest form from Lesson 9.2.

Part three: "On the contrary." The **sed contra** gives a brief consideration on the other side, usually a sentence of Scripture or a respected authority. In the article "Whether God exists," for example, Aquinas cites the verse in which God names himself to Moses (Exodus 3:14). The sed contra does not prove the answer; it shows that the objections cannot be the whole story and points toward the answer to come.

Part four: "I answer that." The **respondeo** is the heart of the article. Here the author gives his own answer, and he gives reasons for it. A good respondeo often begins by defining the key terms or drawing a **distinction**: a word may be used in two senses, or a claim may be true in one way and false in another. Then it argues for the answer. In the English Dominican translation of 1920, the respondeo of "Whether God exists" begins: "I answer that, The existence of God can be proved in five ways." Aquinas then gives the five arguments.

Part five: the replies. Finally the author answers every objection, one by one: "Reply to Objection 1. ..." No objection is left hanging. There are three classic ways to answer a premise. You may grant it (**concedo**) and show that the conclusion still does not follow. You may deny it (**nego**) and say why it is false. Or you may distinguish (**distinguo**): show that the premise is true in one sense and false in another, and that the objection needs the false sense. Distinctions are often the most illuminating replies, because they show what was right in the objection as well as what was wrong.

A short example. Whether it is better to tell a hard truth than a kind lie. *Objection 1.* It would seem that a kind lie is better, for kindness spares a friend pain, and sparing pain is good. *On the contrary,* Scripture says in Colossians 3:9, "Lie not one to another." *I answer that,* we must distinguish what is said from how it is said. A lie damages trust and puts a falsehood in a friend's mind, so it harms him, even when it soothes him for a moment. But a hard truth can be told kindly: with care in its words, its timing and its tone. So it is better to tell the truth, kindly. *Reply to Objection 1.* Kindness is indeed good, but it is shown in how we tell the truth, not in replacing truth with falsehood.

A school of fairness. The disputed question trains the mind in three ways at once. It makes you state the other side first and fairly. It makes you give reasons, not just opinions. And it makes you answer every objection, not only the easy ones. Paul's command fits the form well: "Prove all things; hold fast that which is good" (1

Thessalonians 5:21). The capstone of this course is a disputation in exactly this form, so practice it now. You can find more on the disputatio on the Classics Library Logic & Debate page.

> **From history.** The disputed question grew out of the twelfth-century schools. Around 1120, Peter Abelard compiled *Sic et Non* ("Yes and No"), a collection of passages from the Church Fathers that seemed to disagree, and challenged students to reconcile them. In the universities of the thirteenth century, public disputations became a regular part of study, and masters wrote up their answers. Thomas Aquinas (about 1225–1274), a Dominican friar who taught at Paris, wrote the *Summa Theologica* as a guide for beginners in theology; it was unfinished at his death. The English translation by the Fathers of the English Dominican Province is now in the public domain.

> **Practice habit.** When you form an opinion on a question this week, write it as "Whether ..." and list the best objection to your own answer before you defend it.

> **Enrichment: read one article of the Summa.** Find Aquinas's article "Whether God exists" (*Summa Theologica*, First Part, Question 2, Article 3) in the English Dominican translation. It is only a few pages long. Mark the two objections, the sed contra, the respondeo with its five ways, and the two replies. Notice how carefully the objections are stated. Then ask: does each reply answer its objection directly?

Example 1. Label each part of this short article. (a) “I answer that, a student must memorize some things in order to think about them: no one can reason about the history of Rome without remembering its main events. But memorizing is the beginning of learning, not the whole of it.” (b) “Whether students should memorize facts.” (c) “Reply to Objection 1. Books hold facts, but a mind that has none of them cannot even tell what to look up.” (d) “On the contrary, the wise teacher of Proverbs urges his son to keep his words and write them on the table of his heart.” (e) “Objection 1. It would seem that students should not memorize facts, for every fact can be looked up in a book.”

▼ **Show solution**

1. Find the question: it begins with “Whether.” That is (b).
2. Find the objection: it begins “Objection 1. It would seem that …” and argues for the side to be rejected. That is (e).
3. Find the sed contra: it begins “On the contrary” and cites an authority. That is (d).
4. Find the respondeo: it begins “I answer that” and gives the author’s answer with a distinction (memorizing as beginning, not whole). That is (a).
5. Find the reply: it begins “Reply to Objection 1.” That is (c).

Answer: (b) question; (e) objection; (d) sed contra; (a) respondeo; (c) reply. In order: b, e, d, a, c.

Example 2. Write two objections and the replies to them for the article "Whether a family should eat supper together," where the author's answer is yes. Label each reply as concedo, nego or distinguo.

▼ **Show solution**

1. The objections must argue for the side the author rejects, so they argue that a family should not (or need not) eat supper together. State them as fairly as possible.
2. Objection 1. It would seem that a family need not eat supper together, for family members have different schedules, and forcing one time on everyone causes stress.
3. Objection 2. Further, families can talk at other times, such as in the car, so supper together adds nothing.
4. Reply to Objection 1 (distinguo). Schedules differ on some nights, and on those nights flexibility is wise; but on most nights a family can choose one time if it makes supper a priority. The objection is true of some nights, not of most.
5. Reply to Objection 2 (nego). It is not true that supper adds nothing: in the car someone is driving and not everyone is present, while at the table everyone can face one another and talk without hurry.

Answer: Objection 1: different schedules make a shared supper stressful. Reply (distinguo): true of some nights, not of most. Objection 2: families can talk at other times. Reply (nego): supper is the one time everyone is present and can talk without hurry. Accept any fair objections and replies that fit the labels.

PRACTICE

1. Easy List the five parts of a disputed question in order.

▼ **Show answer**

(1) The question ("Whether ..."); (2) the objections; (3) "On the contrary" (sed contra); (4) "I answer that" (respondeo); (5) the replies to the objections.

2. **Easy** With what word does the question of an article begin, and why is that form useful?

   ▼ **Show answer**

   "Whether." It turns the topic into a yes-or-no question, so both sides know exactly what is in dispute.

3. **Easy** What does *sed contra* mean in English, and what does it usually contain?

   ▼ **Show answer**

   "On the contrary" (literally "but against"). It usually contains a brief consideration against the objections, often a quotation from Scripture or a respected authority.

4. **Easy** In an article, which side do the objections argue for?

   ▼ **Show answer**

   The side the author will reject (the opposite of his final answer).

5. **Medium** Put these parts of an article in the correct order and label each. (a) "Reply to Objection 1. Rest does not waste time when it restores strength for work." (b) "Objection 1. It would seem that a farmer should not rest one day in seven during harvest, for a day lost may cost the crop." (c) "Whether a farmer should rest one day in seven even during harvest." (d) "I answer that, rest is part of the order God gave to work, and a rested worker works better. Danger to the crop in a true emergency is another matter." (e) "On the contrary, the commandment to remember the sabbath day calls for rest on the seventh day."

   ▼ **Show answer**

   (c) question; (b) objection; (e) sed contra; (d) respondeo; (a) reply. Order: c, b, e, d, a.

6. **Medium** Rewrite each topic as the question of an article. (a) Should students learn Latin? (b) Is honesty more valuable than cleverness? (c) Do animals feel pain?

   ▼ **Show answer**

   (a) Whether students should learn Latin. (b) Whether honesty is more valuable than cleverness. (c) Whether animals feel pain. (Accept a closing question mark, as in many translations.)

7. Medium The author's answer to "Whether students should learn Latin" is yes. Write two fair objections.

▼ **Show answer**

Sample: Objection 1. It would seem that students should not learn Latin, for no one speaks it today, and time is better spent on living languages. Objection 2. Further, most great Latin books can be read in translation, so learning Latin adds little. Accept any two fair arguments for the answer "no," each beginning in the style "It would seem that …".

8. Medium Is each reply an example of concedo, nego or distinguo? (a) "It is true that Latin is no longer spoken; still, it trains the mind in grammar, so the conclusion does not follow." (b) "It is false that most Latin books are translated well; much is lost in translation." (c) "If 'useful' means 'useful for ordering food abroad,' Latin is not useful; if it means 'useful for understanding English words and law and Scripture,' it is very useful."

▼ **Show answer**

(a) Concedo: it grants the premise but denies that the conclusion follows. (b) Nego: it denies the premise. (c) Distinguo: it shows the premise is true in one sense of "useful" and false in another.

9. Hard Write a respondeo (three to five sentences) for "Whether a good athlete must be a good teammate," answering yes. Include a distinction.

▼ **Show answer**

Sample: "I answer that, we must distinguish two meanings of 'good athlete.' If it means only a player with great physical skill, then a selfish player can be a good athlete. But in a team sport, being a good athlete means playing well for the team, since the game is won by the team. A player who will not pass or encourage others plays worse in the very respect that counts. Therefore, in the full sense, a good athlete in a team sport must be a good teammate." Accept any reasoned answer with a clear distinction.

10. Hard Answer this objection with a reply that uses distinguo: “Objection. It would seem that one should never change one’s mind, for a person who changes his mind is unreliable.”

▼ **Show answer**

Sample: “Reply to the Objection. I distinguish: a person who changes his mind for no reason, or to please whoever is in the room, is unreliable; I grant that. But a person who changes his mind because he has met better evidence or a better argument is not unreliable but honest; I deny that such a person is unreliable. So one should change one’s mind when the truth requires it.” Accept any reply that shows the premise is true in one sense and false in another.

11. Hard Why does the disputed question put the objections first? Give two reasons, and connect one to Lesson 9.2.

▼ **Show answer**

Sample reasons: (1) It makes the author state the other side fairly and fully before answering, which is the strongest form of Lesson 9.2; an answer that ignores the best objections has not settled the question. (2) It shows the reader exactly what the answer must overcome, so the respondeo can be judged. (3) It trains humility and charity: the author must understand why someone might disagree. Accept any two sound reasons, one tied to stating the other side at full strength.

12. **Challenge** Write a complete short article on “Whether it is better to read a book slowly than quickly.” Include two objections, a sed contra, a respondeo with a distinction, and two replies.

▼ **Show answer**

Sample: Whether it is better to read a book slowly than quickly. Objection 1. It would seem that it is better to read quickly, for a quick reader reads more books. Objection 2. Further, reading slowly lets the mind wander. On the contrary, Proverbs 19:2 warns that “he that hasteth with his feet sinneth,” and careful work is everywhere praised. I answer that, we must distinguish kinds of books and purposes. A newspaper or a list may be read quickly. But a book worth understanding (a great book, or Scripture) must be read slowly enough to grasp its arguments and remember them; reading it quickly gains pages but loses understanding. So for books worth reading, slow reading is better. Reply to Objection 1. Reading more books is good only if each is understood; one book understood is worth more than five skimmed. Reply to Objection 2. The mind wanders when reading is careless, not when it is slow; slow reading with a pencil keeps the mind at work. Accept any article that has all five parts in order, fair objections, a reasoned respondeo with a distinction, and a reply to each objection. (If a verse is quoted, it must be exact KJV.)

COMMON MISTAKES

- Writing objections that argue for your own view. Objections argue for the side you will reject.
- Writing weak objections so they are easy to answer. The form works only if the objections are the strongest you can find.
- Treating the sed contra as the proof. It points toward the answer; the respondeo gives the reasons.
- Leaving an objection without a reply. Every objection gets its own reply.
- Giving opinions in the respondeo without reasons. “I answer that” must be followed by an argument.

> **Scripture.** “Prove all things; hold fast that which is good.” (1 Thessalonians 5:21, KJV)
>
> The disputed question tests every claim by its best objections and then holds fast to what survives.

9.5 Logic in Rhetoric

OBJECTIVES

- Explain Aristotle's view of rhetoric as the counterpart of dialectic.
- Define the three proofs of rhetoric, ethos, pathos and logos, and recognize each in a speech or advertisement.
- Build an enthymeme (a rhetorical syllogism) and an argument from example (a rhetorical induction).
- Tell an honest appeal to character or feeling from a fallacy that replaces reasons.

WATCH

Ethos, Pathos & Logos

Texas A&M University Writing Center · 3:16

https://www.youtube.com/watch?v=aUpiy67_nt4

> **WHEN TO WATCH**
> At the start: Aristotle's three appeals, before the enthymeme and the example.

Note: Parent preview: A university writing center introduces Aristotle's three appeals (ethos, pathos, logos) with short skits, including a brief mock-dramatic movie scene ("No, no, please"). Says logos is the strongest support in academic writing. Nothing objectionable; transcript checked October 18, 2026.

KEY TERMS

Rhetoric

The art of persuasion; for Aristotle, the ability to see, in any case, the available means of persuading.

Ethos

Persuasion through the speaker's character, as shown in the speech: good sense, good character and goodwill toward the hearers.

Pathos

Persuasion through the hearers' feelings, by moving them to an emotion that fits the matter, such as pity, fear, hope or anger at a wrong.

Logos

Persuasion through the argument itself: reasons that prove, or seem to prove, the point.

Enthymeme (in rhetoric)

Aristotle's "rhetorical syllogism": a short deductive argument from what is usually true or from signs, often with a premise left for the audience to supply.

Example (in rhetoric)

Aristotle's "rhetorical induction": an argument from one or more similar cases, real or invented, to the case at hand.

CORE IDEAS

Logic and rhetoric belong together. In the trivium, rhetoric follows logic: first learn to reason rightly, then learn to speak persuasively. Some people think of rhetoric as tricks for winning over a crowd. Aristotle disagreed. He opened his *Rhetoric* by calling rhetoric the counterpart of dialectic. Dialectic tests arguments between two people by question and answer; rhetoric presents arguments to an audience in a continuous speech. Both deal with questions that ordinary people argue about, and both depend on reasoning. A speaker who does not know logic may be persuasive, but he has no way to make sure he is persuading people of the truth.

Three kinds of proof. Aristotle taught that a speech persuades in three ways. **Ethos** persuades through the speaker's character: we believe a speaker we trust. **Pathos** persuades through the hearers' feelings: a speech that moves us to pity, hope or concern can lead us to act. **Logos** persuades through the argument itself: the reasons given for the claim. Every good speech uses all three, but logos is the backbone. Ethos and pathos should support the reasons, not replace them.

Ethos: the speaker's character. Aristotle said that hearers trust a speaker who shows three things: good sense (he knows what he is talking about), good character (he is honest) and goodwill (he wants what is good for them). A speaker shows these in the speech itself, by being accurate, fair to the other side and careful with claims. When the veterinarian Dr. Lin explains why the calves need a booster shot, her years of training and her plain, careful explanation give her ethos. Ethos is earned by honesty. One exaggeration discovered can destroy it.

Pathos: fitting feelings. Emotions are not enemies of reason. It is right to feel pity for a hungry child, concern about a weak bridge and gratitude for a gift. A speech that moves its hearers to feelings that fit the facts helps them act on the truth. Pathos becomes a fallacy when the feeling replaces the reasons or does not fit the facts: the appeals to pity and fear from Unit 3. The test is simple. Take away the emotion and ask: Is there still a good reason here? If so, the emotion is helping. If not, it is doing the work an argument should do.

Logos, first form: the enthymeme. Aristotle called the **enthymeme** a rhetorical syllogism. It is a deductive argument shaped for an audience. Its premises are often things that are usually true rather than always true, or signs ("there is smoke, so there is fire"). And it is usually short, because the audience can supply a premise everyone knows. "We should leave early; the roads are icy" is an enthymeme. The hidden premise, "When the roads are icy, it is wise to allow extra time," goes without saying. Spelling it out would make the speech slow and dull. Leaving it out lets the hearers complete the reasoning themselves, which often makes them more convinced.

Logos, second form: the example. Aristotle called the **example** a rhetorical induction. Instead of reasoning from a general rule, it reasons from a similar case. "When the Holland family planted pines along their north fence, the wind stopped drying out their garden. A row of pines on our north side would help our garden too." Aristotle noted that examples may be real events from history or invented ones, such as comparisons and fables. An example is as strong as the likeness between the cases. As you learned in Unit 7, ask whether the cases are alike in the ways that matter.

Why ethos depends on logos. Logically, the truth of a claim does not depend on who says it; attacking the speaker instead of the argument is the ad hominem fallacy. But in practice, an audience cannot check every fact, so it must decide whom to trust. That is why character matters. And the surest way to gain lasting ethos is sound logos: hearers who find that your reasons hold up when they check them will trust you the next time. Rhetoric without logic becomes flattery; logic without rhetoric is often unheard.

Speech seasoned with salt. Paul writes in Colossians 4:6, "Let your speech be alway with grace, seasoned with salt, that ye may know how ye ought to answer every

man." Grace and salt together: kindness and substance. That is a fair summary of honest rhetoric. Speak with goodwill toward your hearers, move their feelings only toward what is true and good, and give them real reasons. Persuasion that rests on truth honors both the truth and the people who hear it.

> **From history.** Aristotle (384–322 B.C.) studied for about twenty years in Plato's Academy. Plato, in his dialogue *Gorgias*, had sharply criticized the rhetoric teachers of Athens for caring more about winning than about truth. Aristotle's *Rhetoric* answered by rooting persuasion in reasoning, with the enthymeme as its chief tool. Later Roman teachers built on his work. Cicero (106–43 B.C.) wrote several books on oratory, and Quintilian (about A.D. 35–100) wrote a long guide for training a speaker from childhood. Quintilian took up the old Roman ideal, which he credits to Cato the Elder, that the orator is "a good man skilled in speaking."

> **Practice habit.** When an advertisement or speech moves you, ask three questions: Do I trust the speaker, and why? What feeling is being stirred, and does it fit? What is the actual reason given?

Example 1. Coach Barrett speaks to the parents: “I have coached this team for twelve years, and I care about every one of these players. Last season, three of our runners were injured in the final month because they were worn out. Injuries are heartbreaking for a young athlete who has worked all year. Runners who rest one day a week recover better and get injured less. So this season, Sunday and Wednesday will be rest days.” Find the ethos, pathos and logos.

▼ **Show solution**

1. Ethos: “I have coached this team for twelve years” shows experience (good sense); “I care about every one of these players” shows goodwill.
2. Pathos: “Injuries are heartbreaking for a young athlete who has worked all year” moves the parents to concern for their children.
3. Logos: the reasons. Three runners were injured last season from fatigue (an example), and runners who rest recover better and get injured less (a general premise). So the team will have rest days (conclusion).
4. Check the pathos: take away the emotion, and good reasons remain. The emotion fits the facts and supports the argument.

Answer: Ethos: twelve years of coaching and care for the players. Pathos: the heartbreak of injury. Logos: last season’s injuries and the general fact that rest reduces injuries, leading to the rest-day plan. The emotional appeal is honest because it fits the reasons.

Example 2. Support the claim "We should plant a windbreak of trees on the north side of the farm" with (a) an enthymeme and (b) an argument from example. Then write out the enthymeme's hidden premise.

▼ **Show solution**

1. (a) An enthymeme gives a short reason and leaves a premise unstated: "We should plant a windbreak, because the north wind dries out our soil every spring."
2. Hidden premise: "Whatever would stop the north wind from drying out our soil is worth planting" (or "A windbreak would stop the north wind from drying out our soil"). The audience supplies it without being told.
3. (b) An example argues from a similar case: "When the Hollands planted a row of pines along their north fence, their soil stopped drying out in the spring. Our farm sits on the same open ridge, so a windbreak would help us the same way."
4. Check the example: the cases are alike in what matters (same ridge, same north wind), so the example is fairly strong.

Answer: (a) "We should plant a windbreak, because the north wind dries out our soil every spring." Hidden premise: a windbreak would stop the wind from drying out our soil (and so is worth planting). (b) "The Hollands' pines stopped their soil from drying out; our farm is on the same ridge, so a windbreak will help us too." Accept any short enthymeme and any relevant example.

PRACTICE

1. Easy Define ethos, pathos and logos in one sentence each.

▼ **Show answer**

Ethos is persuasion through the speaker's character (good sense, good character, goodwill). Pathos is persuasion through the hearers' feelings. Logos is persuasion through the argument itself, the reasons given.

2. **Easy** What did Aristotle call the enthymeme and the example?

▼ **Show answer**

He called the enthymeme a rhetorical syllogism (rhetorical deduction) and the example a rhetorical induction.

3. **Easy** According to Aristotle, what three qualities make hearers trust a speaker?

▼ **Show answer**

Good sense (practical wisdom), good character (virtue, honesty) and goodwill toward the hearers.

4. **Easy** Aristotle called rhetoric the counterpart of dialectic. What do the two have in common, and how do they differ?

▼ **Show answer**

Both deal with questions people commonly argue about and both rest on reasoning. Dialectic tests arguments by question and answer between people; rhetoric presents arguments to an audience in a continuous speech.

5. **Medium** Label each sentence from a speech as mainly ethos, pathos or logos. (a) “As a nurse for twenty years, I have seen this problem many times.” (b) “Picture a child waiting alone in the rain for a bus that never comes.” (c) “The bus route was cut from ten stops to six, so four neighborhoods now have no stop.” (d) “I am speaking tonight because I want what is best for every family on this road.”

▼ **Show answer**

(a) Ethos: experience gives the speaker good sense. (b) Pathos: it moves the hearers to pity and concern. (c) Logos: it gives a reason (the cut stops). (d) Ethos: it shows goodwill. Accept a second label where a student explains it (for example, (b) also hints at the logos that children are left without a bus).

6. **Medium** This enthymeme leaves out a premise. Supply it: “Caleb must have been in the barn; there is hay all over his coat.”

▼ **Show answer**

Hidden premise: “Anyone with hay all over his coat has been in the barn” (or, more charitably, “Hay on someone’s coat is a sign he has been in the barn”). It is an argument from a sign, so it is probable, not certain; he might have been in the hayloft of another building or carried hay elsewhere.

7. **Medium** Write an argument from example to support the claim "Our class should start studying for the final exam two weeks early."

▼ **Show answer**

Sample: "Last spring, Mrs. Ortiz's class began reviewing two weeks before the final, and nearly every student said the exam felt manageable. Our class is taking the same course with the same exam, so starting two weeks early should help us too." Accept any argument from a relevantly similar case.

8. **Medium** Is the appeal to feeling honest or a fallacy? Explain. (a) A fire chief shows photos of a house destroyed by a chimney fire and urges families to have their chimneys cleaned every year. (b) A salesman says, "Think how sad your children will be if you don't buy them a Frostrunner sled today!"

▼ **Show answer**

(a) Honest: the feeling of concern fits the facts, and a real reason remains (chimney fires are a real danger that cleaning reduces). (b) A fallacy (appeal to pity or emotion): the sadness replaces any reason that this sled is worth buying; take away the emotion and no argument is left.

9. **Hard** An advertisement says: "Mark Penner, the famous actor who plays Dr. Hale on a TV series, says: 'Your family deserves the best. Give them Mellowbrook tea: it tastes wonderful, and you'll love seeing their smiles.'" Evaluate its ethos, pathos and logos.

▼ **Show answer**

Ethos: weak or false; he is an actor, not a real doctor, so his "doctor" image gives no real good sense about tea (an improper appeal to authority). Pathos: it plays on love for one's family and the pleasure of their smiles, but these feelings are not tied to any reason the tea is better. Logos: almost none; the only reason is that it tastes wonderful, which is a mere claim with no evidence or comparison. The ad leans on borrowed ethos and pathos in place of logos.

10. Hard Judge this argument from example: “The Greenvale bakery doubled its sales after it began staying open until nine at night. So the hardware store on Main Street should stay open until nine, too.”

▼ **Show answer**

Weak to moderate. The cases are alike in being shops in a town, but they differ in what matters: people often buy bread or treats in the evening on impulse, while hardware needs are often planned for the daytime, and the bakery’s sales might have risen for other reasons. The example suggests trying it, but it does not show the hardware store would gain. Accept a judgment of “weak” or “moderate” if the student names relevant differences.

11. Hard Hannah says, “If the truth of a claim doesn’t depend on who says it, then ethos shouldn’t matter at all.” Answer her in three or four sentences.

▼ **Show answer**

Sample: Hannah is right that a claim’s truth does not depend on the speaker, and that attacking a person instead of the argument is the ad hominem fallacy. But hearers often cannot check every fact for themselves, so they must decide whose testimony to trust, and for that a speaker’s good sense, honesty and goodwill are real evidence. Ethos matters for whether we should believe what someone reports, while the validity of an argument stands or falls on its own. And honest reasoning is itself the best way to earn ethos.

12. **Challenge** Write a short speech (five or six sentences) urging your family or class to do something good and practical. Use ethos, pathos and logos honestly, include one enthymeme, and label each appeal.

▼ **Show answer**

Sample: “I have helped at the Riverside food pantry every Saturday this year, so I know what it needs [ethos]. Every week, families wait in line for food, and some go home with half-empty bags [pathos]. The pantry runs short in winter, when donations drop, so a winter food drive would help the most [logos, enthymeme; hidden premise: help is most useful when the need is greatest]. Last year the Brookside church held a drive in January and collected over a thousand cans [logos, example]. I want every family who comes to the pantry to leave with enough [ethos, goodwill]. Let’s hold our food drive in January.” Accept any speech with honest, labeled appeals and an enthymeme whose hidden premise is reasonable.

COMMON MISTAKES

- Thinking rhetoric is the same as trickery. For Aristotle, rhetoric rests on reasoning, and its main proof is the enthymeme.
- Thinking every appeal to emotion is a fallacy. It is a fallacy only when the feeling replaces the reasons or does not fit the facts.
- Confusing ethos with fame. Ethos is earned by good sense, honesty and goodwill, not by being well known.
- Using an example that differs in the ways that matter. An argument from example is only as strong as the likeness between the cases.

> **Scripture.** “Let your speech be alway with grace, seasoned with salt, that ye may know how ye ought to answer every man.” (Colossians 4:6, KJV)
>
> Grace and salt together, kindness and substance, describe honest rhetoric: goodwill toward the hearers and real reasons for them.

9.6 Finding the Real Question

OBJECTIVES

- Explain what a stasis is and why disagreements stall when people argue different questions.
- Name the four stases (fact, definition, quality and procedure) and the question each asks.
- Classify a point of disagreement by its stasis.
- Work through the stases in order to find where a disagreement really lies.

KEY TERMS

Stasis

Greek for "a standing" or "a stand": the point where two sides of a dispute meet and clash; the real question at issue (Latin *status* or *constitutio*).

Stasis of fact (conjecture)

The question "Did it happen? Is it so?": a dispute about what is or was the case.

Stasis of definition

The question "What is it? What should we call it?": a dispute about how an agreed fact should be named or classified.

Stasis of quality

The question "What kind is it? Was it good or bad, justified or not, serious or slight?": a dispute about the value or rightness of an agreed, defined act.

Stasis of procedure (translation)

The question "Is this the right person, place, time or way to decide it?": a dispute about who should judge the matter and how.

Arguing past each other

A stalled disagreement in which the two sides are answering different questions, often at different stases.

CORE IDEAS

Why some arguments go nowhere. Two people can argue for an hour and settle nothing, not because either is unreasonable but because they are answering different questions. One says, "You broke the rake!" The other says, "It was already cracked!" The first is arguing about what happened; the second is arguing about how much blame is deserved. Until they notice, they will talk past each other forever. The

ancient teachers of rhetoric developed a tool for this problem: the doctrine of **stasis**, which helps you find the real question.

What a stasis is. The Greek word *stasis* means "a standing": the place where two sides take their stand against each other, like two wrestlers locked together. The teachers of the courts noticed that every dispute stands on one of four kinds of question, and that the kinds come in a natural order. They are usually called **fact**, **definition**, **quality** and **procedure**.

First: fact (conjecture). The first question is "Did it happen? Is it so?" Did the dog dig up the tulips? Is the creek higher than it was last spring? Did Caleb take the last apple? A dispute of fact is settled by evidence: witnesses, signs, records, measurements. The old name for this stasis is **conjecture**, because when no one saw what happened, each side must reason from signs to what probably happened, as you learned in Unit 7.

Second: definition. Suppose both sides agree on the facts: Caleb did take the last apple. Now the question may be "What should we call it?" Was it "taking his share" or "taking what belonged to someone else"? Is a tomato a fruit or a vegetable? Is a picture traced from a book "your own drawing"? Disputes of **definition** turn on the meaning of a word, and they are settled by the skills of Unit 2: genus and difference, clear cases, and definitions that are neither too broad nor too narrow.

Third: quality. Suppose the facts and the name are agreed: Caleb took the apple that was saved for his sister. Now the question may be "How should we judge it?" Was it wrong, or was there a good reason (he was faint from hunger after a long hike)? Was it a serious matter or a slight one? Disputes of **quality** concern value, motive and circumstances: whether an agreed act was right, justified, excusable, wise or more important than something else. Many of the deepest disputes, such as whether honesty is more valuable than cleverness, are questions of quality.

Fourth: procedure (translation). Finally, a side may say, "This is not the right way to decide this." Should Caleb's older sister settle the matter, or should their mother? Should a disputed goal be decided by the players or by the referee? Is the student council the right body to change the school dress code, or is that the principal's job? Disputes of **procedure**, called **translation** in the old books (because the case is "transferred" to another judge or court), concern who should judge, when, where and by what rules.

Work through them in order. The four stases build on one another. There is no point arguing whether an act was justified (quality) if the two sides do not agree that it happened (fact). So when a disagreement stalls, walk through the questions in order. Do we agree on the facts? Do we agree on what to call them? Do we agree on how to judge them? Do we agree on who should decide? The first question where the two sides part ways is the real question. Spend your effort there, and set aside the rest.

Hear before you answer. Proverbs 18:13 warns, "He that answereth a matter before he heareth it, it is folly and shame unto him." Stasis is a way of truly hearing. Before you defend your side, find out which question the other person is actually asking. Often you will discover that you agree on more than you thought, and that the real disagreement is narrower and easier to discuss. In a debate, stasis tells you where to aim your case. In a family or a friendship, it can turn a quarrel into a conversation.

> **From history.** The doctrine of stasis is credited to Hermagoras of Temnos, a Greek teacher of rhetoric in the second century B.C. His own books are lost, but his teaching survives through later writers. The young Cicero (106–43 B.C.) set out four kinds of issue (conjecture, definition, quality and transfer to another court) in his early work *On Invention*. More than a century later, Quintilian (about A.D. 35–100) discussed stasis at length in his *Institutes of Oratory*, comparing the views of earlier teachers. For centuries, students of rhetoric in Europe learned to begin every case by asking which stasis it stood on.

> **Practice habit.** When a disagreement starts going in circles, stop and ask: Do we agree on what happened? On what to call it? On how to judge it? On who should decide?

Example 1. Name the stasis of each disputed question. (a) Did the frost last night damage the peach blossoms? (b) Is a whale a fish? (c) Was it right for Mrs. Ortiz to keep the store open late during the snowstorm so stranded travelers could get warm? (d) Should the dispute over the property line be settled by the two neighbors or by the county surveyor?

▼ **Show solution**

1. (a) asks whether something happened. That is fact (conjecture), settled by examining the blossoms.
2. (b) asks what to call an agreed thing. That is definition, settled by the definitions of "fish" and "mammal."
3. (c) agrees on what she did and asks whether it was right. That is quality.
4. (d) asks who should decide. That is procedure (translation).

Answer: (a) Fact; (b) definition; (c) quality; (d) procedure.

Example 2. Hannah and Caleb disagree. Hannah: "You broke Dad's rake." Caleb: "I did use it, and the handle did snap, but it was already cracked, so it wasn't my fault." Use the stases in order to find the real question.

▼ **Show solution**

1. Fact: Did Caleb use the rake, and did it snap while he used it? Caleb agrees that it did. No dispute of fact.
2. Definition: Is this "breaking the rake"? Caleb does not really deny the word; he admits the handle snapped while he used it.
3. Quality: Was it his fault? Here they part ways. Hannah thinks he is to blame; Caleb says the earlier crack excuses him. This is the real question.
4. Procedure: Neither has raised who should decide (perhaps Dad), so this is not yet in dispute.
5. So the conversation should turn to quality: Was the crack the main cause? Did Caleb use the rake roughly or properly?

Answer: The facts and the name are agreed. The real question is one of quality: whether Caleb is to blame, given that the handle was already cracked.

PRACTICE

1. Easy What does the Greek word *stasis* mean, and to whom is the doctrine of stasis usually credited?

   ▼ **Show answer**

   "A standing" (or "a stand"): the point where two sides take their stand and clash. It is credited to Hermagoras of Temnos, a Greek teacher of rhetoric in the second century B.C.

2. Easy Name the four stases in order and the question each asks.

   ▼ **Show answer**

   Fact (conjecture): Did it happen? Is it so? Definition: What is it? What should we call it? Quality: What kind is it; was it right, justified or serious? Procedure (translation): Who should decide, and how, when or where?

3. Easy Which stasis? "Did the neighbor's dog dig up our tulips?"

   ▼ **Show answer**

   Fact (conjecture): it asks whether something happened.

4. Easy Which stasis? "Is a tomato a fruit or a vegetable?"

   ▼ **Show answer**

   Definition: it asks how to classify an agreed thing.

5. Medium Name the stasis of each question, with a reason. (a) Is a hot dog a sandwich? (b) Was the bridge on Mill Road built before 1900? (c) Was it wise for the coach to rest his best pitcher in the final game? (d) Should the dispute about the broken window be settled by the boys' parents or by the principal?

   ▼ **Show answer**

   (a) Definition: it asks what to call an agreed thing. (b) Fact: it asks what was the case, settled by records. (c) Quality: the act is agreed, and the question is whether it was wise. (d) Procedure: it asks who should decide.

6. **Medium** In a soccer game, a shot hits the crossbar and bounces down near the goal line. Write one disputed question about this play for each of the four stases.

▼ **Show answer**

Sample: Fact: Did the whole ball cross the goal line? Definition: Under the rules, does a ball on the line count as "over the line"? Quality: Even if it was a goal, was the referee right to allow it after a foul in the build-up? Procedure: Should the referee decide alone, or should he consult the assistant referee? Accept any four questions that fit their stases.

7. **Medium** Leah: "You took my sweater without asking!" Ruth: "I only borrowed it, and I was going to give it back." Are they disagreeing about fact, definition or quality? Explain.

▼ **Show answer**

Ruth agrees she took the sweater without asking, so the fact is agreed. Her reply turns on what to call it ("borrowed," not "took") and on whether it was wrong (she meant to return it). The main stasis is definition (borrowing versus taking), with a quality question close behind (was it wrong to take it without asking?). Accept either definition or quality with a clear reason.

8. **Medium** Why should the stases be worked through in order, starting with fact?

▼ **Show answer**

Each stasis assumes the earlier ones are agreed. It makes no sense to argue about what to call an act, or whether it was right, until both sides agree that it happened. Going in order finds the first point of real disagreement, which is the real question.

9. **Hard** Find the real question: Mr. Hale: "Your cows got into my cornfield and ate half a row." Mr. Dunn: "Yes, they did; I saw it myself. But the fence they came through is yours to keep up, and you left the gate open. And besides, half a row is hardly worth fussing about."

▼ **Show answer**

Fact is agreed (the cows ate half a row), and the name is not disputed. The disagreement is one of quality: who is to blame (Mr. Dunn says the open gate and the fence make it Mr. Hale's fault) and how serious the harm is ("hardly worth fussing about"). Accept "quality" with either or both of these points.

10. Hard A student says, “The question ‘Was the umpire’s call fair?’ could belong to more than one stasis.” Show how it could be a question of fact, of quality and of procedure.

▼ **Show answer**

Fact: if it means “Did the runner really reach the base before the tag?” (what happened). Quality: if it means “Was the umpire right to apply the rule strictly in this situation?” or “Was he biased?” (a judgment of rightness). Procedure: if it means “Should the call have been reviewed by the other umpires rather than made by one umpire alone?” (who should decide and how). The words alone do not settle the stasis; you must ask what is actually disputed.

11. Hard You will debate “Resolved: that honesty is more valuable than cleverness.” What is the main stasis of this resolution, and which other stasis must you prepare for? Explain.

▼ **Show answer**

The main stasis is quality: the resolution asks which of two goods is more valuable. You must also prepare for definition, because the debate will turn on what “honesty,” “cleverness” and “valuable” mean (for example, whether cleverness includes wisdom). Fact may enter in examples, but it is not the main question.

12. Challenge A library book comes back three weeks late with a torn cover. The librarian wants Caleb to pay for the book. Write a short dialogue (four to six lines) in which Caleb and the librarian disagree, and then use the stases in order to show where their disagreement lies.

▼ **Show answer**

Sample: Librarian: “This book is three weeks late and the cover is torn. You need to pay for it.” Caleb: “I know it’s late, but the cover was torn when I checked it out.” Librarian: “Our records don’t show any damage.” Caleb: “The clerk didn’t check it.” Analysis: The lateness is agreed. The real question is one of fact: was the cover torn before Caleb borrowed it? Once that is settled, quality (should he pay the full price, or only a fine?) and procedure (should the head librarian decide?) may follow. Accept any dialogue and analysis that correctly finds the first stasis where the two sides part ways.

COMMON MISTAKES

- Arguing about whether an act was right before agreeing on whether it happened. Start with fact.
- Thinking a dispute of definition is "just words." How we name a thing often decides how we judge it.
- Assuming the same words always mean the same stasis. Ask what is actually disputed.
- Forgetting procedure. Sometimes the real disagreement is not about the matter but about who should decide it.

> **Scripture.** "He that answereth a matter before he heareth it, it is folly and shame unto him." (Proverbs 18:13, KJV)
>
> Finding the stasis is a way of truly hearing: learning which question the other person is asking before we answer.

UNIT 9 · LOGIC IN ARGUMENT AND DEBATE · LESSON 9.7 OF 69 · 45 MINUTES

9.7 Cross-Examination and Refutation

OBJECTIVES

- State the purposes of cross-examination and ask short, purposeful questions.
- Answer cross-examination honestly, using concession and distinction.
- Refute a point in four steps: they say, but, because, therefore.
- Choose among the ways to refute: deny a premise, break the link, give a counterexample, turn the point or outweigh it.

KEY TERMS

Cross-examination

A period in which one speaker questions another about what the second has said, to clarify it, test it or gain admissions.

Admission

A point the other side grants during questioning, which the questioner can then use in a later speech.

Refutation

Answering an opponent's argument by showing that a premise is false, that the conclusion does not follow, or that it matters less than another point.

Four-step refutation

A pattern for answering a point: "They say ..." (state it fairly), "But ..." (your answer), "Because ..." (your reason), "Therefore ..." (why it matters).

Turn

A refutation that shows the opponent's own evidence or reasoning actually supports your side.

Outweigh

A refutation that grants the opponent's point but shows that your own point matters more.

CORE IDEAS

The neighbor comes and searches. In Proverbs 18:17 we read, "He that is first in his own cause seemeth just; but his neighbour cometh and searcheth him." Anyone sounds convincing when no one answers. The first speaker in a debate, a court or a family discussion lays out a case that seems complete. Only when someone else examines it do its gaps appear. This lesson teaches the two ways of examining: **cross-examination**, which tests a case by questions, and **refutation**, which answers it by argument.

What cross-examination is for. Cross-examination has three purposes. First, **clarify**: make sure you understand exactly what the other side claims ("Do you mean every student, or most students?"). Second, **test**: probe the evidence and the reasoning ("Where did that number come from?"). Third, **gain admissions**: get the other side to grant points you will use later ("So you agree the new scoreboard would cost the school something to maintain?"). Cross-examination is not a time for speeches. Your arguments come later; now you are gathering material.

How to ask good questions. Keep each question short, and ask about one thing at a time. When you want an admission, use a closed question that can be answered yes or no. When you want to understand, use an open question ("How did you measure that?"). Know why you are asking each question, and stop when you have what you need. Avoid loaded questions such as "Have you stopped ignoring the evidence?", which assume what is in dispute (the complex question fallacy of Unit 3). Be courteous: let the other person answer, and never badger. A calm, fair questioner is far more persuasive than a hostile one.

How to answer questions. When you are cross-examined, answer honestly and briefly. If a point is true, grant it; a speaker who will not admit the obvious loses all trust. Then you can explain why it does not hurt your case. If a question uses a word in two senses, distinguish them, as in the disputed question of Lesson 9.4: "In one sense yes, in another no." If a question is unclear, ask what it means. Never pretend to know what you do not; "I don't have that figure" is an honest answer. Evasion looks worse than any admission.

Refutation in four steps. To answer a point, use four steps. **They say:** state the opponent's point fairly, in its strongest form, so everyone knows exactly what you are answering. **But:** state your answer in one clear sentence. **Because:** give your reason or evidence. **Therefore:** explain what follows, why your answer matters for the whole question. For example: "They say the scoreboard will pay for itself through ticket sales. But ticket sales will not rise enough to pay for it, because Ridgeview School bought the same scoreboard and its attendance did not change. Therefore the scoreboard would cost our school money, not earn it."

Five ways to refute. There are several ways to answer an argument, and your argument map from Lesson 9.1 shows where each applies. You may **deny a premise**: show that a box on the map is false. You may **break the link**: show that the conclusion does not follow, even if the premises are true. You may give a **counterexample** to a general claim. You may **turn** the point: show that the opponent's own evidence supports your side. Or you may **outweigh** it: grant the point but show that your point matters more. Choose the answer that fits the weakness you found.

Aim at the argument, not the person. Refutation answers reasons, not people. Calling an opponent careless or foolish is the ad hominem fallacy, and it convinces no fair listener. Refute what was actually said, not a weaker version, and admit what is true in it. Use your map: if two premises are joint, refuting one defeats the pair; if a sub-conclusion falls, everything resting on it falls. When you have answered a point, move on. Repeating yourself does not make a refutation stronger.

The purpose of the contest. Cross-examination and refutation can feel like combat, and in a debate round there is a winner. But the deeper purpose is the one in Proverbs: the neighbor searches the first speaker so that the truth comes out. A good examiner helps everyone, even the person examined, see the case more clearly. If

you are shown to be wrong, the honest response is to say so. A Christian debater wants to win, but wants the truth more.

> **From history.** Questioning an opponent is as old as argument. In Plato's *Apology*, Socrates, on trial in 399 B.C., cross-examines his accuser Meletus before the court and leads him into contradicting himself. Roman orators such as Cicero questioned witnesses in the law courts. In English and American trials, cross-examination of witnesses became a central safeguard of justice. The American scholar John Henry Wigmore, whose charts of evidence you met in Lesson 9.1, called it, roughly, the greatest legal engine ever invented for discovering the truth. Formal school debates later borrowed the practice, giving each speaker a few minutes to question an opponent.

> **Practice habit.** When you disagree with someone this week, try the four steps out loud: "You say …, but …, because …, therefore …."

Example 1. The resolution is "Resolved: that a school garden is more valuable than a new scoreboard." Your opponent has argued, "The new scoreboard will pay for itself through higher ticket sales." Write three cross-examination questions and give the purpose of each.

▼ **Show solution**

1. Begin by clarifying the claim: "How much do you expect ticket sales to rise?" (open question; purpose: clarify the size of the claim).
2. Test the evidence: "Do you have a school with a new scoreboard whose attendance went up because of it?" (closed question; purpose: test whether there is evidence behind the claim).
3. Gain an admission: "Does the scoreboard need electricity and repairs every year?" (closed question; purpose: get the admission that it has ongoing costs, for use in a later speech).
4. Keep each question short and about one thing. Do not argue during cross-examination; save the conclusion for your speech.

Answer: Sample questions: (1) "How much do you expect ticket sales to rise?" (clarify). (2) "Do you have an example of a school whose attendance rose because of a new scoreboard?" (test). (3) "Does the scoreboard need electricity and repairs every year?" (gain an admission).

Example 2. Your opponent argues, “The garden is a bad idea because students will lose interest after a few weeks.” Refute this point using the four steps.

▼ **Show solution**

1. They say: state it fairly. “My opponent says students will lose interest in the garden after a few weeks.”
2. But: give your answer in one sentence. “But the garden will be part of the science class, not a voluntary club.”
3. Because: give the reason. “Because each class will be assigned beds to plant, tend and harvest as graded work, the garden does not depend on students staying interested on their own.”
4. Therefore: show why it matters. “Therefore my opponent’s main objection to the garden does not apply, and its value stands.”
5. Name the method: this refutation breaks the link. Even if student interest fades, the garden will still be kept, so the conclusion does not follow.

Answer: “They say students will lose interest after a few weeks. But the garden will be part of science class, not a voluntary club, because each class will be assigned beds to tend as graded work. Therefore the objection does not apply, and the garden’s value stands.” Method: breaking the link.

PRACTICE

1. Easy Name the three purposes of cross-examination.

   ▼ **Show answer**

   To clarify what the other side claims, to test their evidence and reasoning, and to gain admissions to use later.

2. Easy Name the four steps of refutation in order and say what each does.

   ▼ **Show answer**

   They say (state the opponent’s point fairly); But (give your answer); Because (give your reason or evidence); Therefore (show what follows and why it matters).

3. **Easy** Explain Proverbs 18:17 in your own words and say what it has to do with debate.

   ▼ **Show answer**

   Whoever speaks first seems right until someone else examines his case. In debate, a case can sound complete until the other side cross-examines and refutes it, so we should not judge until both sides have been heard and tested.

4. **Easy** Which is the better cross-examination question, and why? (a) “Isn’t it true that your evidence, which comes from a study that is very old and was done in another state where conditions are completely different, can’t really be trusted?” (b) “What year was your study done?”

   ▼ **Show answer**

   (b): it is short, asks one thing and gets a clear fact. (a) is long, asks several things at once and makes a speech instead of asking a question.

5. **Medium** Rewrite this as a fair, short question: “Have you stopped ignoring the evidence about how much the scoreboard costs?”

   ▼ **Show answer**

   Sample: “How much will the scoreboard cost?” or “Did you include the yearly repair costs in your figures?” The original is a loaded (complex) question: either answer admits that the speaker was ignoring evidence. Accept any short, neutral question on the same point.

6. **Medium** Use the four steps to refute: “We should not get a dog, because dogs make the house dirty.”

   ▼ **Show answer**

   Sample: “They say we should not get a dog because dogs make the house dirty. But a short-haired dog that is brushed and wiped off at the door makes very little mess, because most dirt comes in on muddy paws and loose fur, both of which we can control. Therefore the mess is a small cost that a family can manage, not a reason to refuse a dog.” Accept any refutation that states the point fairly and follows all four steps.

7. **Medium** Name the method of each refutation (deny a premise, break the link, counterexample, turn, or outweigh). (a) “Even if the garden takes some time from class, what students learn about growing food matters more.” (b) “My opponent’s survey shows that most parents want a garden, which supports my side, not his.” (c) “It is not true that gardens always fail; Brookside School’s garden has thrived for ten years.”

▼ **Show answer**

(a) Outweigh: it grants the cost but argues the benefit matters more. (b) Turn: it shows the opponent’s evidence supports the other side. (c) Counterexample: one clear case defeats the general claim “gardens always fail.” (Accept “deny a premise” for (c), since the counterexample is used to deny the premise.)

8. **Medium** You argued that the school should start a garden. In cross-examination you are asked, “Won’t a garden take some work?” Write an honest answer that concedes what is true without giving up your case.

▼ **Show answer**

Sample: “Yes, it will take some work, about an hour a week for each class. But that work is part of what students learn, so it is a benefit as well as a cost.” Accept any answer that honestly grants the true point and briefly shows why it does not defeat the case.

9. **Hard** Your opponent says, “Students should be allowed to use calculators on every math test, because calculators are used in every job today.” Write three short cross-examination questions, one for each purpose, and label them.

▼ **Show answer**

Sample: (1) Clarify: “Do you mean every math test, including tests on basic multiplication?” (2) Test: “Can you name a job in which no one ever needs to estimate or work out a number without a calculator?” (3) Admission: “Would you agree that a student who cannot do arithmetic would not notice when a calculator gives a wrong answer from a mistyped number?” Accept any short, fair questions that fit the three purposes.

10. Hard Turn this argument: “We should not take the family trip to the lake this summer, because the drive takes five hours.”

▼ **Show answer**

Sample: “The five-hour drive is actually a reason to go: it gives our family five hours together without screens or chores, which is one of the things we said we wanted from a vacation.” A turn uses the opponent’s own point (the long drive) as support for the other side. Accept any reply that does this.

11. Hard Your opponent argues: “(1) Every food sold at the school fair must be homemade. (2) The Bakers’ Club cookies are not homemade. (3) So the Bakers’ Club cookies cannot be sold at the fair.” You think the cookies should be sold. Using what you know about joint premises, explain how to refute this argument and write a four-step refutation.

▼ **Show answer**

Premises 1 and 2 are joint, so refuting either one defeats the argument; the argument is valid, so you must attack a premise rather than the link. Sample (attacking premise 2): “They say the Bakers’ Club cookies cannot be sold because they are not homemade. But the cookies are homemade, because club members bake them in their own kitchens from scratch. Therefore the cookies meet the rule and may be sold.” Accept an attack on premise 1 instead (for example, showing the rule allows packaged items from local bakeries), if the student gives evidence.

12. **Challenge** Your opponent claims, “Reading novels is a waste of time, because novels are not true.” Write (a) three cross-examination questions with their purposes and (b) a four-step refutation that uses at least one admission you might gain.

▼ **Show answer**

Sample (a): (1) Clarify: “By ‘not true,’ do you mean the events did not happen?” (2) Test: “Are the parables Jesus told about events that happened, or stories told to teach truth?” (3) Admission: “Would you agree that a story can show something true about courage or honesty, even if its characters are invented?” (b) “They say novels waste time because they are not true. But a story can be invented and still teach what is true, because, as my opponent agreed, stories can show real truths about courage and honesty; our Lord himself taught in parables. Therefore novels are not a waste of time simply because their events did not happen.” Accept any fair questions that fit the purposes and a refutation that follows the four steps and uses an admission. (A good answer may distinguish two senses of “true.”)

COMMON MISTAKES

- Making speeches during cross-examination. Ask short questions and save the arguments for your speech.
- Asking loaded or multiple questions. Each question should be fair and ask one thing.
- Refusing to admit an obvious point. Grant what is true and then show why it does not defeat your case.
- Skipping “they say.” If you do not state the opponent’s point fairly, no one knows what you are answering.
- Attacking the person instead of the argument.

> **Scripture.** “He that is first in his own cause seemeth just; but his neighbour cometh and searcheth him.” (Proverbs 18:17, KJV)
>
> A case is not proved until it has been examined; cross-examination and refutation are the neighbor’s search.

9.8 Debate Formats and a Practice Round

OBJECTIVES

- Describe four debate formats: Lincoln-Douglas, team policy, parliamentary and the formal disputatio.
- Give the order and times of the speeches in a Lincoln-Douglas round.
- Write a fair resolution and a short case with definitions, a value, a criterion and contentions.
- Flow a round and use the flow to find dropped arguments and plan a rebuttal.

KEY TERMS

Resolution

The statement a debate is about, beginning "Resolved: that …"; the affirmative supports it and the negative opposes it.

Lincoln-Douglas debate

A one-on-one debate format, usually on a resolution of value, with a fixed order of constructive speeches, cross-examinations and rebuttals.

Value and criterion

In Lincoln-Douglas debate, the value is the highest good a debater's case aims to uphold (such as justice or truth); the criterion is the standard for judging which side better upholds it.

Contention

One of the main arguments of a case, usually stated as a claim with its reasons and evidence.

Flow

A debater's notes of a round, written in columns, one per speech, so that each argument and its answers can be followed across the page.

Dropped argument

An argument the other side never answered; in debate it is usually treated as granted.

Disputatio

The formal medieval disputation, in which a respondent defends a thesis, an opponent presses objections in syllogistic form, and a presiding master settles the question.

CORE IDEAS

Rules that make a fair contest. A debate is an argument with rules: a fixed question, a fixed order of speakers and fixed times. The rules make the contest fair, since each side gets equal time to make its case, question its opponent and answer. The statement being debated is the **resolution**, which begins "Resolved: that ...." The **affirmative** supports it; the **negative** opposes it. Good resolutions are clear, debatable on both sides and about one question. For practice, choose timeless questions such as "Resolved: that honesty is more valuable than cleverness" or "Resolved: that it is better to be a good friend than a popular one." The Classics Library Logic & Debate page has a resolution bank of 316 resolutions and more on each format.

Lincoln-Douglas debate. This format is named for the famous debates of 1858 between Abraham Lincoln and Stephen Douglas in Illinois. It is a one-on-one debate, usually on a question of value (the stasis of quality from Lesson 9.6). The standard order is:

Speech	Minutes
Affirmative constructive (AC)	6
Cross-examination of the affirmative by the negative (CX)	3
Negative constructive (NC)	7
Cross-examination of the negative by the affirmative (CX)	3
First affirmative rebuttal (1AR)	4
Negative rebuttal (NR)	6
Second affirmative rebuttal (2AR)	3

Each debater also has a little preparation time to use between speeches. Constructive speeches build each side's case; rebuttals answer the other side and sum up. Each side speaks for 13 minutes in all.

The Lincoln-Douglas case. A Lincoln-Douglas case usually has four parts. First, **definitions** of the key terms of the resolution, by genus and difference where possible (Unit 2). Second, a **value**: the highest good the debater's side upholds, such as justice, truth or the common good. Third, a **criterion**: the standard for measuring which side better upholds the value. Fourth, two or three **contentions**, the main arguments, each with reasons and examples. The debater who best shows that his or her side upholds the value, by the criterion, wins the round.

Team policy debate. In team policy debate, two-person teams debate a resolution of policy: whether some action should be taken. The affirmative presents a **plan** and argues that it would solve a real problem; the negative argues that the problem is not serious, that the plan would not work, or that it would cause harms that outweigh its benefits. Each debater gives a constructive speech and a rebuttal and is cross-examined. Policy debate rewards careful research and evidence, and the stases of fact and quality do much of the work.

Parliamentary debate. Parliamentary debate is modeled loosely on the debates of the British Parliament. Two teams, often called the Government and the Opposition, receive the resolution only a short time before the round, so they must rely on general knowledge and quick thinking rather than prepared evidence. Members of the other team may rise during a speech to offer a brief **point of information**, a question or comment, which the speaker may accept or decline. Parliamentary debate trains you to reason clearly on your feet.

The formal disputatio. The oldest format in this lesson is the medieval disputation, the living form of the disputed question in Lesson 9.4. A **respondent** states and defends a thesis. An **opponent** presses objections, each put as a syllogism. The respondent repeats each objection and answers it premise by premise: *concedo* (I grant), *nego* (I deny) or *distinguo* (I distinguish). A presiding master then gives the final answer. Every part of this course is at work in a disputatio: definitions, syllogisms, fallacies and charity. This is the format of the course capstone.

Flowing a round. Debaters take notes in a special way called **flowing**. Divide a sheet into columns, one for each speech. Write each argument in the column of the speech where it is made, in short words and abbreviations, and write each answer in the next column, beside the argument it answers. Then arrows and gaps tell the story. An argument with nothing beside it in the next column was never answered: it is a **dropped argument**, usually treated as granted. In your rebuttal, point out dropped arguments, answer the arguments that still stand, and explain why your side wins.

A practice round. To hold a practice round at home, choose a timeless resolution and a format; Lincoln-Douglas works well for two people. Each side writes a short case: definitions, a value and criterion, and two contentions. A parent or classmate keeps time. Both debaters flow the whole round. Afterward, talk through the flow together: which arguments were dropped, which clashed directly, and which side's

reasons held up. Proverbs 27:17 says, “Iron sharpeneth iron; so a man sharpeneth the countenance of his friend.” A good round leaves both debaters sharper, whoever wins.

> **From history.** Formal debate has deep roots. In the medieval universities, disputations were a regular part of study; at Paris, masters also held open disputations called *quodlibets*, in which anyone could raise a question on any subject, and Thomas Aquinas held several. In the English and American colleges of later centuries, students practiced disputations and then debates in literary and debating societies. The format called Lincoln-Douglas takes its name from seven famous debates held across Illinois in 1858, in which each speaker addressed the crowd at length and then answered the other. Debate as a school contest, with judges, rounds and tournaments, grew up in the early twentieth century.

> **Practice habit.** Choose one resolution this week and, before you decide what you think, write the best one-sentence argument for each side.

Example 1. Write a short affirmative Lincoln-Douglas case outline for "Resolved: that honesty is more valuable than cleverness."

▼ **Show solution**

1. Definitions. Honesty: the virtue of speaking and acting truthfully. Cleverness: quickness of mind in solving problems or getting what one wants. Valuable: worth having for a good life.
2. Value: the good life (living well as a person). The affirmative claims honesty contributes more to a good life than cleverness does.
3. Criterion: trustworthiness in relationships. Whichever quality makes a person more worthy of trust contributes more to a good life, since every friendship, family and community depends on trust.
4. Contention 1: Honesty makes a person trustworthy; cleverness alone does not, and cleverness without honesty makes a person more dangerous, not more trustworthy.
5. Contention 2: Honesty can guide cleverness, but cleverness cannot supply honesty. A clever person who lies will use his skill for harm; an honest person of ordinary ability uses what he has for good.

Answer: Definitions of honesty, cleverness and valuable; value: the good life; criterion: trustworthiness; Contention 1: honesty makes a person trustworthy and cleverness alone does not; Contention 2: honesty can direct cleverness, but not the reverse. Accept any outline with clear definitions, a value, a criterion that measures it, and contentions that link the resolution to the value.

Example 2. Here is part of a flow. Find the dropped argument and say what the negative should do in the negative rebuttal.

AC	NC	1AR
C1: Honesty builds trust	Clever people can earn trust by results	Results without honesty: trust is lost when the lie is found
C2: Honesty can guide cleverness	Cleverness is needed to solve problems	(nothing)
	NC1: Cleverness feeds families (farmers, inventors)	Honest workers feed families too

▼ **Show solution**

1. Read across each row. Row 1: the affirmative's C1 was answered by the NC and defended in the 1AR. It still clashes.
2. Row 2: the NC answered C2 (cleverness is needed to solve problems), but the 1AR wrote nothing in reply. The negative's answer to C2 has been dropped by the affirmative.
3. Row 3: the NC's own argument NC1 was answered in the 1AR, so it still clashes.
4. In the NR, the negative should point out that the affirmative dropped the response to C2, so that point stands as granted; then answer the 1AR on rows 1 and 3, and explain why the negative wins on the criterion.

Answer: The negative's answer to C2 ("Cleverness is needed to solve problems") was never answered in the 1AR, so it is dropped and stands as granted. In the NR, the negative should extend that point, answer the remaining clashes, and weigh the round.

PRACTICE

1. Easy List the speeches of a Lincoln-Douglas round in order, with their times.

   ▼ **Show answer**

   Affirmative constructive 6 minutes; cross-examination of the affirmative by the negative 3; negative constructive 7; cross-examination of the negative by the affirmative 3; first affirmative rebuttal 4; negative rebuttal 6; second affirmative rebuttal 3.

2. Easy In a Lincoln-Douglas case, what is a value and what is a criterion?

   ▼ **Show answer**

   The value is the highest good the debater's case aims to uphold (such as justice or truth). The criterion is the standard used to judge which side better upholds that value.

3. Easy Give two differences between Lincoln-Douglas debate and team policy debate.

   ▼ **Show answer**

   Lincoln-Douglas is one-on-one, while team policy has two-person teams. Lincoln-Douglas usually debates a resolution of value, while team policy debates whether to take an action, with the affirmative presenting a plan. (Also accept: policy debate relies more heavily on researched evidence.)

4. Easy What is flowing, and how is a flow sheet arranged?

   ▼ **Show answer**

   Flowing is the way debaters take notes during a round. The sheet is divided into columns, one per speech; each argument goes in the column of the speech where it is made, and each answer goes in the next column, beside the argument it answers.

5. **Medium** Which format fits best, and why? (a) Two students want to debate whether loyalty is more important than honesty. (b) A club wants practice thinking on its feet with almost no time to prepare. (c) A class wants to practice defending a thesis against syllogistic objections, answering "concedo," "nego" or "distinguo." (d) Two teams want to research whether their town library should add a seed-lending program.

▼ **Show answer**

(a) Lincoln-Douglas: one-on-one, on a question of value. (b) Parliamentary: the resolution is given shortly before the round. (c) Formal disputatio: a respondent answers syllogistic objections premise by premise. (d) Team policy: two-person teams debate whether to take an action, with research and a plan.

6. **Medium** Each resolution has a fault. Name it and rewrite the resolution. (a) "Resolved: that sports are good." (b) "Resolved: that reading is valuable and that every student should play a musical instrument." (c) "Resolved: that honesty is good."

▼ **Show answer**

(a) Too vague (good in what way, compared with what?). Sample rewrite: "Resolved: that team sports teach more virtue than individual sports." (b) Two questions in one. Rewrite as one: "Resolved: that every student should learn to play a musical instrument." (c) Not debatable; almost no one would oppose it. Sample: "Resolved: that honesty is more valuable than kindness." Accept any rewrite that is clear, about one question and debatable on both sides (and non-political).

7. **Medium** Name the three roles in a formal disputatio and say what each does.

▼ **Show answer**

The respondent states and defends the thesis, answering each objection premise by premise (concedo, nego, distinguo). The opponent presses objections against the thesis, each put as a syllogism. The presiding master oversees the disputation and gives the final answer (determination).

8. Medium On your flow, the negative made an argument in the NC, and the column for the 1AR beside it is empty. What does this mean, and what should the negative do in the NR?

▼ **Show answer**

It means the affirmative dropped the argument: it was never answered, so it is usually treated as granted. In the NR, the negative should point out the drop, extend the argument (remind the judge what it was and why it matters) and show how it helps the negative win.

9. Hard For "Resolved: that it is better to be a good friend than a popular one," write a value, a criterion and one contention for the affirmative.

▼ **Show answer**

Sample: Value: the good life (or happiness, or virtue). Criterion: lasting goods; whichever brings goods that last contributes more to the good life. Contention: Good friendship lasts through hard times, while popularity changes with the crowd; a good friend has someone to rely on in trouble, and a merely popular person may be left alone when the crowd moves on. Accept any value, a criterion that measures it, and a contention that links the resolution to the value by that criterion.

10. Hard In a Lincoln-Douglas round, how many minutes does each debater speak in speeches, and how many minutes does each spend asking cross-examination questions? Show your work.

▼ **Show answer**

Affirmative speeches: AC 6 + 1AR 4 + 2AR 3 = 13 minutes. Negative speeches: NC 7 + NR 6 = 13 minutes. Each asks questions for one 3-minute cross-examination (and answers questions in the other). So each speaks 13 minutes in speeches and asks questions for 3 minutes.

11. Hard Here is part of a flow. Which arguments were dropped, and by whom?

AC	NC	1AR	NR
C1: Friends help in trouble	Popular people have many helpers	Many acquaintances, few who stay	(nothing)
C2: Friendship builds virtue	(nothing)	Extend C2: no answer	
	NC1: Popularity gives influence for good	(nothing)	Extend NC1: dropped

▼ **Show answer**

C2 was dropped by the negative (no answer in the NC), and the affirmative rightly extended it in the 1AR. NC1 was dropped by the affirmative (no answer in the 1AR), and the negative rightly extended it in the NR. On C1, the negative did not reply in the NR to the 1AR's answer ("Many acquaintances, few who stay"), so that answer now stands; the affirmative can point this out in the 2AR.

12. Challenge Plan a practice Lincoln-Douglas round at home on "Resolved: that honesty is more valuable than cleverness." Write (a) a negative case outline (definitions, value, criterion, two contentions) and (b) two cross-examination questions the negative could ask the affirmative.

▼ **Show answer**

Sample (a): Definitions as in the affirmative case or adjusted (cleverness: the ability to see and solve problems quickly). Value: the common good. Criterion: benefit to others. Contention 1: Cleverness solves problems that harm many people, such as finding better ways to grow food or cure illness; honesty alone cannot do that. Contention 2: Honesty without cleverness often does little good, since an honest person who cannot see the way forward cannot help others much. (b) Questions: "Can an honest person who lacks skill help a sick neighbor as much as a skilled doctor can?" "Would you agree that many of the greatest benefits to others came from clever inventions?" Accept any negative case with clear definitions, a value, a criterion and two contentions linked to it, and any fair, short questions.

COMMON MISTAKES

- Choosing a resolution that is vague, two questions in one or not really debatable.
- Picking a value without a criterion. The criterion tells the judge how to measure which side upholds the value.
- Not flowing. Without a flow, you cannot tell which arguments were dropped or answered.
- Bringing up a brand-new argument in the last rebuttal, when the other side has no chance to answer it. Rebuttals answer and weigh; constructive speeches build.

> **Scripture.** "Iron sharpeneth iron; so a man sharpeneth the countenance of his friend." (Proverbs 27:17, KJV)
>
> A practice round is iron sharpening iron: both debaters leave sharper, whoever wins.

COURSE EXAM

Midterm Exam

Covers Units 1–5. Time: 60 minutes.

You have 60 minutes. No notes or books. The exam covers Units 1–5 and is worth 100 points. Answer in complete sentences unless a question asks for a list, a letter, a form or a diagram. For every classification question, give the category and a one-sentence reason. When you translate, write the full standard-form proposition. Use the traditional square of opposition unless a question says otherwise. Draw Venn diagrams with labeled circles: two circles with the subject on the left, or three circles with S at the top left, P at the top right and M at the bottom. When you test a syllogism, say "valid" or "invalid" and give the reason: the form's name, or the rule broken and the fallacy's name. Unless a question says otherwise, judge syllogisms on the Boolean reading. **Before the exam:** work the review sheet (answer key). A printable copy of the exam is ready for exam day.

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1. **3 pts** **Lesson 1.2**

   Is each a term, a proposition, an argument or none of these? For each term, proposition or argument, name the act of the mind that produces it.

   1. the bell in the church tower
   2. Ring the bell at noon.
   3. The bell in the church tower is cracked.
   4. The bell sounds flat and a thin line runs down its side, so it is cracked.

2. **5 pts** **Lesson 1.3**

   Argument or explanation? Give a reason for each. If it is an argument, write it in standard form.

   1. Caleb must have fed the horses already, since their buckets are full of oats and his boots are by the barn door.
   2. The horses are calm tonight because Caleb fed them early.

3. **6 pts** **Lesson 1.4**

   Is each argument deductive or inductive? Then judge it: valid or invalid, and sound or unsound, if it is deductive; strong or weak if it is inductive. Give a reason for each.

   1. Every planet in our solar system has rings. Mars is a planet in our solar system. So Mars has rings.
   2. Some trees lose their leaves every fall. The sugar maple is a tree. So the sugar maple loses its leaves every fall.
   3. For the last five school days, the bus has reached our stop before 7:30. So tomorrow it will probably reach our stop before 7:30.

4. **4 pts** **Lesson 1.5**

   (a) A farmer says, “This field is both plowed and not plowed.” Which law of thought does he seem to break? How might a charitable listener make sense of what he said? (b) Explain why the sentence “No sentence is true” refutes itself.

5. **3 pts** **Lesson 2.1**

Is the italic word used univocally, equivocally or analogically in each pair? Give a reason for each.

1. "a *healthy* lamb" / "a *healthy* walk"
2. "a *light* suitcase" / "a *light* in the window"
3. "Hannah's *horse*" / "the Ortizes' *horse*"

6. **5 pts** **Lesson 2.4**

(a) Write a classical definition of "kitten," and name its genus and difference. (b) Name the fault in each definition.

1. "A hammer is a tool."
2. "A circle is a figure that is perfectly circular."
3. "A square is a figure that is not a triangle."
4. "Bread is the staff of life."

7. **4 pts** **Lesson 2.6**

Mrs. Ortiz divides the books in the school library into fiction, nonfiction, hardbacks and books with red covers. (a) Name the faults in this division, with an example. (b) Does it leave any book out? (c) Write a correct division of the library's books.

8. **6 pts** **Lesson 3.2**

Name the fallacy in each, with a one-sentence reason.

1. "Don't listen to Coach Reed's advice about stretching before a race. He eats donuts every morning."
2. "Hundreds of kids at our school already carry a Pondhopper backpack. You should get one too."
3. "The forecast says there's a small chance of showers, so either we cancel the picnic or everyone gets soaked."
4. "If you let your little brother borrow your bike once, next he'll want your skates, then your room, and soon he'll have everything you own."

9. **4 pts** **Lesson 3.8**

Equivocation, composition or division? Give a reason for each.

1. “Ruth is a bright student. Bright things hurt the eyes. So Ruth hurts the eyes.”
2. “Every brick in this wall is small, so the wall is small.”
3. “The choir sang loudly, so each singer in it must have sung loudly.”

10. **4 pts** **Lesson 3.3**

(a) Ruth says, “Dr. Lin, the veterinarian who examined our dog this morning, says his teeth need cleaning.” Her brother answers, “That’s an appeal to authority, so it’s a fallacy.” Is he right? Explain. (b) Suppose Ruth had said instead, “A famous singer says our dog’s teeth need cleaning.” Would that show that the dog’s teeth do *not* need cleaning? Name the mistake of anyone who concludes that they do not.

11. **6 pts** **Lesson 4.2**

Translate each into standard form. Then name its form (A, E, I or O) and list its distributed terms.

1. Only members may swim in the club pool.
2. Not every apple in the basket is ripe.
3. None of the goats in the pen escaped.

12. **6 pts** **Lesson 4.4**

Use the traditional square. Give the truth value of the other three propositions, and name the relation you used for each. (a) “All the jars on the pantry shelf are sealed jars” is *true*. (b) “No pears in the crate are bruised pears” is *false*.

13. **5 pts** **Lesson 4.5**

Perform each inference and say whether it is valid.

1. Convert “No snakes are mammals.”
2. Obvert “Some students in the club are not swimmers.”
3. Contrapose “All squares are rectangles.”
4. Convert “Some dogs are not puppies.”

14. **5 pts** **Lesson 4.7**

(a) Draw a labeled Venn diagram for each: “Some players on the team are left-handers” and “All hawks are birds.” (b) On the Boolean reading, does your diagram for “All hawks are birds” say that any hawks exist? Explain.

15. **6 pts** **Lesson 5.2**

For each syllogism, name the minor, major and middle terms, the mood and figure, and the form’s name. Is it valid?

1. No reptiles are furry animals. Some pets are furry animals. Therefore some pets are not reptiles.
2. Some athletes are not sprinters. All athletes are hard workers. Therefore some hard workers are not sprinters.

16. **7 pts** **Lesson 5.4**

Test this syllogism with a three-circle Venn diagram, and name its mood, figure and form: “All gardeners are early risers. No early risers are night owls. Therefore no night owls are gardeners.”

17. **6 pts** **Lesson 5.3**

Each syllogism is invalid. Name the fallacy and explain it in one sentence.

1. All wrens are songbirds. All robins are songbirds. Therefore all robins are wrens.
2. All carpenters are skilled workers. No plumbers are carpenters. Therefore no plumbers are skilled workers.
3. No cats are dogs. No dogs are birds. Therefore no cats are birds.
4. All puppies are young animals. All puppies are dogs. Therefore all dogs are young animals.

18. **5 pts** **Lesson 5.6**

“That tomato plant won’t bear fruit this summer; it has never had any sun.” (a) Supply the missing premise that makes the argument valid, and name the form of the completed syllogism. (b) Is the missing premise believable? Explain.

19. **5 pts** **Lesson 5.8**

Name the form of each argument and say whether it is valid.

1. If the battery is dead, the flashlight won't turn on. The flashlight won't turn on. So the battery is dead.
2. Either the key is in the drawer or it is on the hook. It is not on the hook. So it is in the drawer.
3. If it is Saturday, the farmers' market is open. It is not Saturday. So the farmers' market is not open.

20. **5 pts** **Lesson 5.7**

Find the conclusion that uses all three premises, and show each step: (1) No one who forgets the password can open the captain's chest. (2) Every sailor on the *Gull* keeps a parrot. (3) Everyone who keeps a parrot forgets the password.

ANSWER KEY

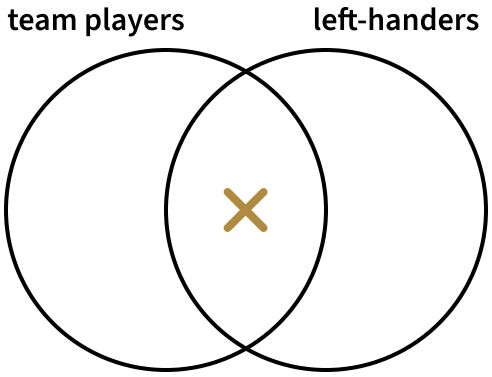
1. 1: Term (it names a thing and claims nothing), produced by simple apprehension. 2: None of these; it is a command, which is neither true nor false. 3: Proposition (it claims something true or false), produced by judgment. 4: Argument (two reasons, then "so" and a conclusion), produced by reasoning.
2. 1: Argument: "since" introduces reasons offered to prove something we did not already know, that Caleb has fed the horses. Standard form:
   1. The horses' buckets are full of oats.
   2. Caleb's boots are by the barn door.
   Therefore, Caleb has already fed the horses.
   (It is inductive: the clues make the conclusion likely, not certain.) 2: Explanation: that the horses are calm is already accepted, and "because" tells why it is so; nothing is being proved. (3 points for 1, 2 for 2.)
3. 1: Deductive; valid but unsound. If both premises were true, Mars would have to have rings, but the first premise is false (Earth, for one, has no rings). 2: Deductive; invalid, and therefore unsound. "Some trees" does not tell us that the sugar maple is one of them. The conclusion happens to be true, but it does not follow: the same form gives "Some animals are cats; a dog is an animal; so a dog is a cat." (Accept "inductive and weak" if the student explains that the premises give little support.) 3: Inductive ("probably"; it reasons from past days to the next one). Fairly strong: the record is steady and the conclusion is modest. Accept "weak" if the reason given is that five days is a short record. (2 points each.)
4. (a) The law of non-contradiction: nothing can both be and not be at the same time and in the same respect. Charitably, he means it in different respects or parts: for example, half the field is plowed and half is not, or it has been plowed but not yet harrowed. Then there is no contradiction. (Accept any reasonable reading that separates the respects.) (b) It is itself a sentence, offered as true. If it were true, at least one sentence (itself) would be true, so it would be false. It cannot be true, so it refutes itself. (2 points each.)
5. 1: Analogical: the lamb has health, and the walk helps cause health, so the second meaning points back to the first. 2: Equivocal: "not heavy" and "something that shines" are unrelated meanings that share only a spelling. 3: Univocal: "horse" means the same kind of animal in both.
6. (a) "A kitten is a young cat." Genus: cat. Difference: young. (Accept any correct genus-and-difference definition.) (b) 1: Too broad: a saw and a wrench are tools

too. 2: Circular: it uses “circular” to explain “circle.” 3: Negative (it says what a square is not; accept also “too broad,” since a circle is a figure that is not a triangle). 4: Figurative (obscure): it says what bread is like or does for us, not what it is. (1 point for (a), 1 point for each fault.)

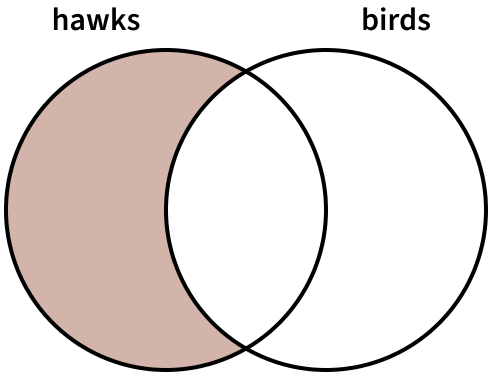
7. (a) Cross-division: it uses three principles at once (content, binding and cover color). The parts are not mutually exclusive: a hardback novel with a red cover falls into three parts. (b) No: every book is either fiction or nonfiction, so nothing is left out. But a good division must also use one principle and have no overlap, so it still fails. (c) Any division by one principle with parts that do not overlap and leave nothing out, for example “fiction and nonfiction,” or “hardbacks and softcover books.” (2 points for (a), 1 each for (b) and (c).)
8. 1: Abusive ad hominem: his breakfast has nothing to do with whether his advice about stretching is good. (Accept “ad hominem” without the subtype.) 2: Appeal to the many (ad populum, bandwagon): that many students own the backpack is no reason to think it is good or that you need one. 3: False dilemma: there are other options, such as bringing umbrellas, finding a shelter, or going ahead since the chance is small. 4: Slippery slope: no reason is given to think that one loan leads to each later step. (1½ points each.)
9. 1: Equivocation: “bright” means “clever” in the first premise and “shining” in the second, so the premises do not connect. 2: Composition: many small bricks can make a large wall, so smallness does not carry from the parts to the whole. 3: Division: the choir’s loudness comes from many voices together, so a soft voice can be part of a loud choir. (1⅓ points each; full marks for three correct answers with reasons.)
10. (a) No. This is a legitimate appeal to authority: Dr. Lin is an expert speaking within her own field, about a dog she has just examined. (b) No. The singer is not an authority on dogs’ teeth, so the argument gives no real support; but a bad argument leaves its conclusion unsupported, not disproved. Concluding that the teeth do not need cleaning is the fallacy fallacy. (2 points each.)
11. 1: “All people who may swim in the club pool are members.” A; distributes the subject, “people who may swim in the club pool.” 2: “Some apples in the basket are not ripe apples.” O; distributes the predicate, “ripe apples.” 3: “No goats in the pen are goats that escaped.” E; distributes both terms. Accept any class terms with the same meaning (for example, “animals that escaped”). (2 points each: 1 for the translation, 1 for the form and distribution.)
12. (a) E false (contrary of a true A: contraries cannot both be true). I true (subaltern: truth flows down from A to I). O false (contradictory of a true A). (b) I

true (contradictory of a false E). A undetermined (A and E are contraries, which can both be false, so a false E tells us nothing about A). O undetermined (falsity flows up, not down, so a false E tells us nothing about O). (1 point for each value with its relation.)

13. 1: “No mammals are snakes”; valid (E converts simply). 2: “Some students in the club are non-swimmers”; valid (obversion is always valid). 3: “All non-rectangles are non-squares”; valid (A contraposes). 4: “Some puppies are not dogs”; invalid (O does not convert): the premise is true, but the result is false. (1¼ points each.)
14. (a) “Some players on the team are left-handers” (I): an X in the overlap; nothing shaded.



“All hawks are birds” (A): shade the part of “hawks” outside “birds”; no X.

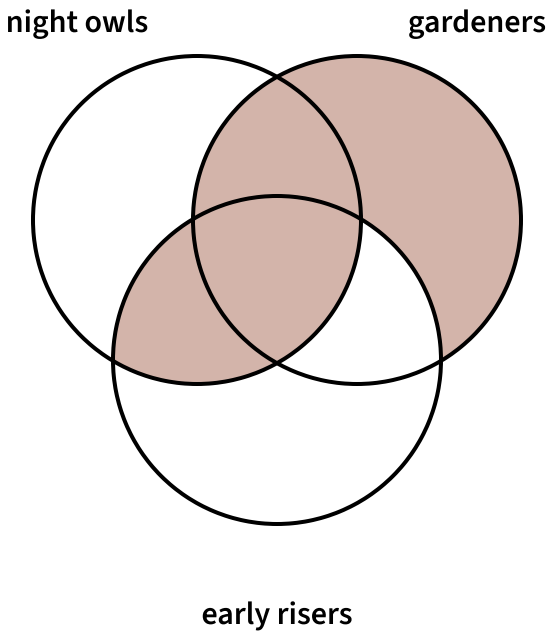


(b) No. The diagram has only shading, which says a region is empty; there is no X anywhere, so it does not say that anything exists. On the Boolean reading, a universal proposition makes no claim that its subject class has members. (3 points for (a), 2 for (b); deduct for an X in the A diagram or shading in the I diagram.)

15. 1: Minor (S) = pets; major (P) = reptiles; middle (M) = furry animals. The middle term is the predicate of both premises, so it is figure 2: EIO-2, Festino. Valid. 2:

Minor (S) = hard workers; major (P) = sprinters; middle (M) = athletes. The middle term is the subject of both premises, so it is figure 3: OAO-3, Bocardo. Valid. (3 points each.)

16. S = night owls, P = gardeners, M = early risers. Both premises are universal. "All P are M" shades the parts of P outside M: P only and SP. "No M are S" shades the overlap of M and S: SM and SPM. The conclusion "No S are P" needs SP and SPM empty: both are shaded. Valid.



The form is AEE-4 (P–M, M–S), Camenes (also spelled Calemes). The rules agree: the middle term is distributed in the minor premise, both terms distributed in the conclusion are distributed in their premises, and the one negative premise has a negative conclusion. (5 points for the diagram and verdict, 2 for the form.)

17. 1: Undistributed middle: "songbirds" is the predicate of two A propositions, so it is distributed in neither premise. 2: Illicit major: "skilled workers" is distributed in the conclusion (predicate of an E) but not in the major premise (predicate of an A). 3: Exclusive premises (two negative premises): both premises are negative, so nothing follows, even though the conclusion happens to be true. 4: Illicit minor: "dogs" is distributed in the conclusion (subject of an A) but not in the minor premise (predicate of an A). (1½ points each.)

18. (a) Missing major premise: "No plants that never have any sun are plants that bear fruit." Completed: No plants that never have any sun bear fruit; that tomato plant is a plant that never has any sun; so that tomato plant does not bear fruit. Celarent (EAE-1), valid. Accept also "All plants that bear fruit are plants that get sun," with the minor premise "That tomato plant is not a plant that gets sun,"

which gives Camestres (AEE-2), also valid; or the A form "All plants that never have any sun are plants that will not bear fruit," which gives Barbara (AAA-1, with the conclusion "That tomato plant is a plant that will not bear fruit"), also valid. (b) Yes: tomato plants need sunlight to grow and to set fruit, so a plant with no sun at all will not bear. (Accept also a student who notes that "never had any sun" may be an exaggeration, and that a plant with a little sun might bear a few tomatoes.) (3 points for (a), 2 for (b).)

19. 1: Affirming the consequent; invalid (the bulb might be broken instead). 2: Disjunctive syllogism (denying one disjunct); valid. 3: Denying the antecedent; invalid (the market might also be open on other days). (2 points each for 1 and 3, 1 point for 2; full marks for three correct answers.)
20. Classes: G = sailors on the *Gull*, K = people who keep a parrot, F = people who forget the password, O = people who can open the captain's chest. (2) All G are K. (3) All K are F. So all G are F (Barbara). (1) No F are O. With "All G are F," this gives: No G are O (Celarent). Conclusion: No sailor on the *Gull* can open the captain's chest. (Accept any equivalent wording, such as "None of the *Gull*'s sailors can open the captain's chest.")

Printable answer key (PDF, for the parent)

COURSE EXAM

Final Exam

Covers Units 1–9. Time: 75 minutes.

You have 75 minutes. No notes or books; you may use scratch paper for the arithmetic, which is light. The exam covers the whole course, Units 1–9, and is worth 100 points. Answer in complete sentences unless a question asks for a list, a number, a symbol, a table, a diagram or a map. For every classification question, give the category and a one-sentence reason. Use the symbols ~, •, ∨, ⊃ and ≡; write truth tables with T and F, rows in the standard order; and in proofs, number every line and justify it with a rule and line numbers. Draw Venn diagrams with labeled circles (three circles: S at the top left, P at the top right, M at the bottom), and judge syllogisms on the Boolean reading unless a question says otherwise. Give probabilities as fractions in lowest terms. For written answers, any answer that follows the form taught in the course and reasons fairly earns full credit. **Before the**

exam: work the review sheet (answer key). A printable copy of the exam is ready for exam day.

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1. **4 pts** **Lesson 1.4**

   (a) A deductive argument has all true premises and a false conclusion. What can you say about it for certain, and why? (b) Can an inductive argument be called valid? What words do we use instead? (c) Who first treated logic as a kind of algebra, in books published in 1847 and 1854?

2. **4 pts** **Lesson 2.3**

   A triangle is defined as a polygon with three sides. Name the predicable each statement shows, with a short reason.

   1. A triangle is a polygon.
   2. A triangle has three sides.
   3. This triangle is drawn in red chalk.
   4. The angles of a triangle on a flat surface add up to 180 degrees.

3. **5 pts** **Lesson 3.5**

   Name the fallacy in each, with a one-sentence reason.

   1. “Have you stopped spending your whole allowance on comic books?”
   2. “The first two people I met in Millbrook were rude, so Millbrook is a rude town.”
   3. “Please give my essay an A. I worked on it all weekend, and I’ll be crushed if I get anything less.”

4. **5 pts** **Lesson 4.2**

   Translate each into standard form, name its form, and list its distributed terms.

   1. None but registered players may enter the tournament.
   2. Some of the hens did not lay an egg today.
   3. Every barn on Mill Road has a red roof.

5. **5 pts** **Lesson 4.4**

(a) Use the traditional square. If “Some beehives in the orchard are not full hives” is *true*, what can you infer about the A, E and I? (b) On Boole’s reading, explain why “All students who scored 110 on the 100-point quiz received a prize” does not imply “Some students who scored 110 on the 100-point quiz received a prize.” Which relation of the traditional square fails here?

6. **4 pts** **Lesson 4.7**

(a) Draw a labeled Venn diagram for “No ferns are flowering plants.” (b) Obvert the proposition, and explain why its obverse has the same diagram.

7. **5 pts** **Lesson 5.4**

Test with a three-circle Venn diagram, and check your verdict with the rules: “All beekeepers are patient people. Some patient people are teachers. Therefore some teachers are beekeepers.”

8. **4 pts** **Lesson 5.2**

(a) Name the mood, figure and form of this syllogism: “All herbs in the window box need daily water. Some herbs in the window box are mint plants. Therefore some mint plants need daily water.” (b) Use the rules to explain why every syllogism of the form AEE-1 is invalid.

9. **4 pts** **Lesson 6.3**

Let R = “It rains,” G = “The game is canceled” and T = “Tom brings the tent.” Symbolize: (a) The game is canceled only if it rains. (b) Rain is a sufficient condition for canceling the game. (c) Tom brings the tent unless it rains. (d) It is not the case that both it rains and Tom brings the tent.

10. **6 pts** **Lesson 6.5**

Let P = “The pump is broken,” W = “The well is dry” and G = “The garden wilts.” Test this argument with a full truth table: “Either the pump is broken or the well is dry. If the well is dry, the garden wilts. The garden wilts. So the well is dry.” If it is invalid, give the counterexample row.

11. **4 pts** **Lesson 6.4**

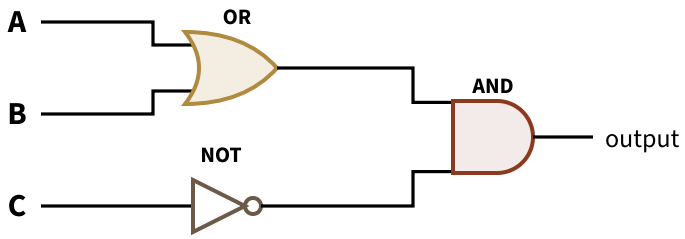
(a) Build a truth table for $(p \supset q) \vee (q \supset p)$, and classify it as a tautology, a contradiction or a contingency. (b) Use De Morgan's laws to restate in English, and give the symbols: "It is not true that Hannah or Caleb fed the pig." (Let H = "Hannah fed the pig" and C = "Caleb fed the pig.")

12. **4 pts** **Lesson 6.8**

Write a proof. Number each line and justify it. 1. $A \vee B$ 2. $A \supset C$ 3. $\sim C$ / $\therefore B \vee D$

13. **3 pts** **Lesson 6.9**

(a) Find the output of this circuit for A = 0, B = 1, C = 0, and for A = 1, B = 1, C = 1.



(b) Write the circuit as a statement in the symbols of Unit 6.

14. **4 pts** **Lesson 7.1**

Strong or weak? Give two reasons for each.

1. To learn whether the families of Pine Hollow want the town library open on Sundays, a volunteer asked 25 people leaving the library on a Saturday afternoon. Twenty-four said yes, so the volunteer concluded that nearly all the town's families want Sunday hours.
2. The Ridgeline Seed Company planted 50 seeds from each of 20 bags of one batch, taken from different parts of its warehouse; 940 of the 1,000 seeds sprouted. It printed "About 94% of these seeds sprout" on that batch's packets.

15. **4 pts** **Lesson 7.3**

Name the Mill's method in each case, give the probable cause, and say why the conclusion is only probable.

1. Four campers got stomachaches on Tuesday. They ate different suppers, slept in different cabins and hiked different trails, but all four drank from the same spring.
2. Two loaves were made with the same flour, water and salt and baked in the same oven for the same time. Only the loaf made with fresh yeast rose; the loaf made with old yeast stayed flat.

16. **4 pts** **Lesson 7.5**

(a) A packet holds 12 bean seeds, and 9 of them are good seeds that will sprout. If you plant one seed chosen at random, what is the probability it sprouts? (b) A fair coin lands heads four times in a row. Caleb says, "Tails is due." What is the probability of tails on the next toss, and what is his error called? (c) What is the probability that a fair coin lands heads on both of two tosses? (d) Which is more probable, and why: "Lena plays the piano" or "Lena plays the piano and sings in the choir"?

17. **3 pts** **Lesson 7.2**

"Our new puppy is the same breed as Aunt Rosa's dog, which is gentle with small children. So our puppy will probably be gentle with small children too." Name one relevant likeness, name one relevant difference that would weaken the argument, and say whether the argument is strong or weak as it stands.

18. **5 pts** **Lesson 8.4**

The seven players on a youth basketball team scored these season totals: 4, 6, 6, 8, 10, 12 and 102 points. The team's flyer says, "Our players average over 21 points a season!" (a) Find the mean and the median. (b) Is the flyer true? Which number better describes a typical player, and why? (c) A parent reads "average" as "typical." Name the fallacy.

19. **5 pts** **Lesson 8.5**

In a large greenhouse, 10% of the plants have a leaf disease called blight. A leaf test flags 90% of the plants that have blight, and it also flags 10% of the healthy plants. Its maker says, “If our test flags your plant, it has blight!” Use 1,000 plants. (a) How many plants have blight, and how many of those are flagged? (b) How many healthy plants are flagged? (c) What fraction of the flagged plants really have blight? (d) Name the fallacy the maker’s claim invites.

20. **4 pts** **Lesson 8.6**

(a) In a seaside town, the weeks with the most ice-cream sales are also the weeks with the most sunburns. Does eating ice cream cause sunburn? Name a lurking variable. (b) The three teams with the worst records in the league last season all hired new coaches, and all three won more games this season. Why is this weak evidence that the new coaches made the difference?

21. **5 pts** **Lesson 9.1**

Map this argument. Show which premises are joint and which are independent, and name the sub-conclusion and the main conclusion. “(1) The north field drains poorly, (2) and potatoes rot in wet soil. (3) So potatoes would do badly in the north field. (4) Corn, on the other hand, grew well there last year. (5) Therefore we should plant corn, not potatoes, in the north field.”

22. **4 pts** **Lesson 9.6**

Name the stasis of each question: fact (conjecture), definition, quality or procedure (translation).

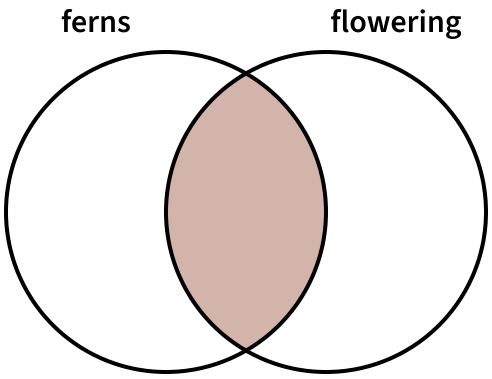
1. Did the late frost kill the peach blossoms?
2. Is a peanut really a nut?
3. Was Caleb right to miss a fishing trip he had promised his cousin, so that he could help a neighbor whose cellar was flooding?
4. Should the quarrel about the property line be settled by the two neighbors themselves or by the county surveyor?

23. **5 pts** **Lesson 9.2**

A classmate says, “We shouldn’t bother learning cursive, since everyone types now.” (a) Supply the hidden premise, and restate the argument in its strongest form. (b) Answer it with the four steps of refutation (they say, but, because, therefore).

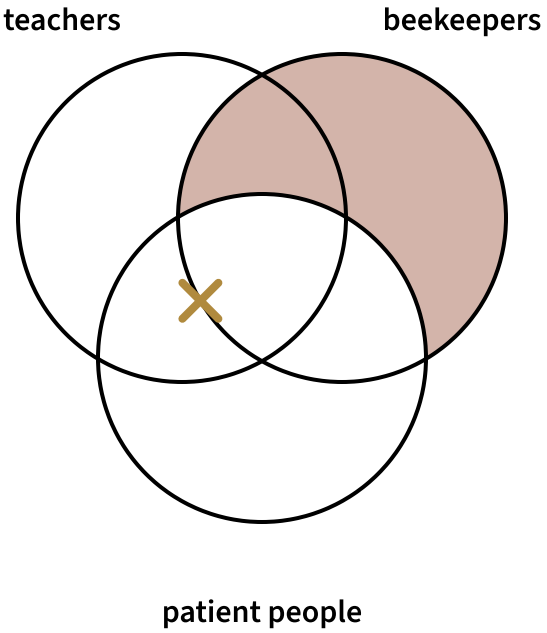
ANSWER KEY

1. (a) It is invalid (and so unsound): a valid argument can never have all true premises and a false conclusion. (b) No. An inductive argument is called strong or weak (and cogent if it is strong and its premises are true). (c) George Boole. (1½ points each for (a) and (b), 1 for (c).)
2. 1: Genus: the part of the essence a triangle shares with other polygons. 2: Difference: the part of the essence that sets triangles apart from other polygons. 3: Accident: it could be drawn in white chalk, or in ink, and still be a triangle. 4: Property: it is not part of the definition, but it follows from what a triangle is and belongs to every triangle (on a flat surface), and among polygons only to triangles. (1 point each.)
3. 1: Complex question: either "yes" or "no" admits that you have been spending your whole allowance on comic books, which has not been shown. 2: Hasty generalization: two people are far too few to judge a whole town. 3: Appeal to pity: the student's effort and feelings do not show that the essay deserves an A, which depends on its quality. (Accept "appeal to emotion" for 3.) (2 points for 1, 2 for 2, 1 for 3; full marks for three correct answers with reasons.)
4. 1: "All people who may enter the tournament are registered players." A; distributes the subject, "people who may enter the tournament." 2: "Some hens are not hens that laid an egg today." O; distributes the predicate, "hens that laid an egg today." 3: "All barns on Mill Road are barns with red roofs." A; distributes the subject, "barns on Mill Road." Accept any class terms with the same meaning. (2 points for 1, 2 for 2, 1 for 3; full marks for three correct answers.)
5. (a) A false (contradictory of a true O). E undetermined (falsity flows up, but truth does not flow up from O to E). I undetermined (subcontraries can both be true). (b) No student can score 110 on a 100-point quiz, so the class is empty. On Boole's reading a universal makes no claim that anything exists, so the A is true (nothing breaks it), while the I claims that at least one such student exists, so it is false. A true universal with a false particular shows that subalternation fails. (3 points for (a), 2 for (b).)
6. (a) Circles "ferns" (left) and "flowering plants" (right); shade the overlap; no X.



(b) Obverse: “All ferns are non-flowering plants.” It says that no fern lies outside the class of non-flowering plants, that is, no fern is inside the flowering-plants circle. That is the same region, the overlap, shaded as empty: the obverse is equivalent to the original, so it has the same diagram. (2 points each.)

7. S = teachers, P = beekeepers, M = patient people. Universal first: “All P are M” shades the parts of P outside M: P only and SP. “Some M are S” points to SM or SPM; neither is shaded, so the X goes on the line between them. The conclusion “Some S are P” needs an X in SP or SPM, but the X might be in SM (patient teachers who keep no bees). Invalid.



The rules agree: undistributed middle, since “patient people” is the predicate of an A and the subject of an I, distributed in neither. (The form is AII-4.) (3 points for the diagram, 2 for the verdict and rule.)

8. (a) S = mint plants, P = things that need daily water, M = herbs in the window box. The middle term is the subject of both premises: AII-3, Datisi; valid. (b) AEE-1 is "All M are P; No S are M; so No S are P." The E conclusion distributes P, but in the major premise P is the predicate of an A, which is undistributed. So it commits the illicit major. (2 points each.)

9. (a) G ⊃ R. (b) R ⊃ G. (c) ~R ⊃ T (also correct: T ∨ R, R ∨ T or ~T ⊃ R). (d) ~(R • T) (also correct: ~R ∨ ~T). (1 point each.)

10. P ∨ W, W ⊃ G, G / ∴ W.

P	W	G	P ∨ W	W ⊃ G	G	∴ W
T	T	T	T	T	T	T
T	T	F	T	F	F	T
T	F	T	T	T	T	F
T	F	F	T	T	F	F
F	T	T	T	T	T	T
F	T	F	T	F	F	T
F	F	T	F	T	T	F
F	F	F	F	T	F	F

All three premises are true in rows 1, 3 and 5. In row 3 the conclusion is false, so the argument is invalid. Counterexample: the pump is broken, the well is not dry, and the garden wilts (P true, W false, G true). (The second and third premises affirm the consequent.) (4 points for the table, 2 for the verdict and row.)

11. (a)

p	q	p ⊃ q	q ⊃ p	(p ⊃ q) ∨ (q ⊃ p)
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

Tautology: it is true in every row. (b) "Hannah did not feed the pig, and Caleb did not feed the pig" (neither of them fed it). ~(H ∨ C) ≡ ~H • ~C. (2 points each.)

12. 4. ~A (2, 3, MT)
    5. B (1, 4, DS)
    6. B ∨ D (5, Add)
    (Award full credit for any correct proof whose every line is justified.)

13. (a) A = 0, B = 1, C = 0: A OR B = 1; NOT C = 1; 1 AND 1 = 1. Output 1. A = 1, B = 1, C = 1: A OR B = 1; NOT C = 0; 1 AND 0 = 0. Output 0. (b) (A ∨ B) • ~C. (1 point for each output, 1 for (b).)

14. 1: Weak. Any two: the sample is biased (people leaving the library are the ones most likely to want more library hours, and families who never visit had no chance to be asked); it is small for a whole town; it lacks variety (one place, one afternoon); and the conclusion ("nearly all") claims too much. 2: Strong. Any two: the sample is large (1,000 seeds); it is varied and fair (seeds from every one of 20 bags, from different parts of the warehouse); the conclusion is about the same batch the sample came from; and it claims only "about" 94%, which matches the sample (940 ÷ 1,000 = 94%). (2 points each.)

15. 1: Method of agreement: the cases differ in every listed circumstance but agree in one, the spring water, so the spring is the probable cause. 2: Method of difference: the two loaves are alike in every listed circumstance except the yeast, and the rising appears only with fresh yeast, so the old yeast is the probable cause of the flat loaf. Both are only probable because some circumstance that was not listed might be the real cause (for example, the four campers might all have eaten the same snack, or the old-yeast dough might have been kneaded less). (1½ points for each method with its cause, 1 for the reason.)

16. (a) 9/12 = 3/4. (b) 1/2: the tosses are independent, and the coin has no memory. His error is the fallacy of the maturity of chances (the "it's due" fallacy). (c) 1/2 × 1/2 = 1/4. (d) "Lena plays the piano" is more probable (or at least as probable): every case in which she plays and sings is also a case in which she plays, so adding "and" can never make a claim more probable. Choosing the second is the conjunction error. (1 point each.)

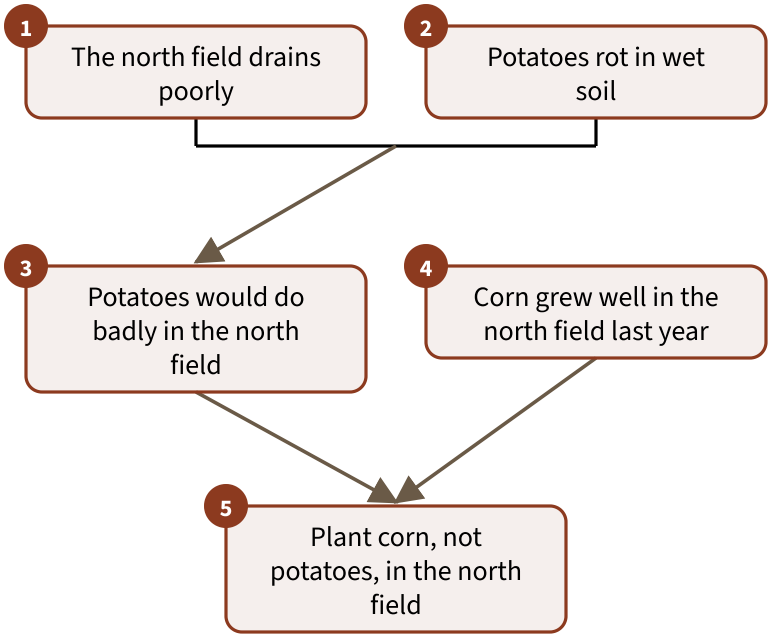
17. Likeness: the same breed, and dogs of one breed often share a temperament. Difference (any one): the puppy's training or how it is raised, its age, the temper of its own parents, or how it has been treated. As it stands it is fairly weak: it rests on one likeness and on a single dog. Accept "moderately strong" if the student argues that breed matters a great deal and the conclusion claims only "probably." (1 point each.)

18. (a) Mean: 4 + 6 + 6 + 8 + 10 + 12 + 102 = 148, and 148 ÷ 7 ≈ 21.1 points. Median: 8 points (the 4th of the 7 totals in order). (b) True for the mean, but misleading: six of the seven players scored 12 points or fewer, and one player's 102 points pulls the mean far up. The median, 8 points, better describes a typical player. (c) Equivocation: "average" means the mean in the flyer but is heard as "typical." (2 points for (a), 2 for (b), 1 for (c).)

19. (a) 1,000 × 0.10 = 100 plants have blight, and 100 × 0.9 = 90 of them are flagged. (b) 900 healthy plants × 0.10 = 90 flagged. (c) 90 ÷ (90 + 90) = 90/180 = 1/2: only half the flagged plants have blight. (d) The base-rate fallacy: the claim ignores that most plants are healthy, so false flags among the many healthy plants are as common as true ones. (1 point each for (a), (b) and (d); 2 for (c).)

20. (a) No. A lurking variable, hot sunny summer weather (accept also more visitors at the beach), causes both more ice-cream sales and more sunburns. Thinking the ice cream causes the sunburn is false cause. (b) Regression to the mean: a very poor season usually includes some bad luck (injuries, close losses), so the worst teams would tend to do better the next season even with no change. A fair test would compare them with similar teams that kept their coaches. (2 points each.)

21. 1 + 2 → 3 (joint: poor drainage matters only because potatoes rot in wet soil, and neither premise alone says anything about potatoes in that field); 3 → 5; 4 → 5 (independent: each gives its own reason, one against potatoes and one for corn). Claim 3 is a sub-conclusion; claim 5 is the main conclusion.



Accept 3 and 4 drawn as joint if the student explains that the conclusion compares the two crops and needs both reasons. Deduct if 1 and 2 are drawn as independent without a reason. (3 points for the map, 2 for the labels.)

22. 1: Fact (conjecture): whether something happened. 2: Definition: what kind of thing a peanut is (by its kind, a peanut is a legume, not a true nut). 3: Quality:

whether the act was right, given the circumstances. 4: Procedure (translation): who should decide, and how. (1 point each.)

23. (a) Hidden premise: “A skill we will seldom use is not worth the time it takes to learn.” Strongest form (sample): “School time is limited. Most writing for school and work is now done on a keyboard, so students will seldom need cursive. Time spent on it would be better spent on skills they will use often. Therefore cursive is not worth class time.” (b) Sample: “They say we should not learn cursive because everyone types now. But typing has not replaced all handwriting, because notes, cards and signatures are still written by hand, and anyone who wants to read old letters, diaries and documents must be able to read cursive. Therefore cursive still deserves at least a small place in our lessons.” Accept any refutation that states the other side fairly (they say), answers it (but), gives a reason (because) and draws a conclusion (therefore), whether or not the student agrees with the classmate. (2 points for (a), 3 for (b).)

Printable answer key (PDF, for the parent)