ALGEBRA 1 · COLLEGE

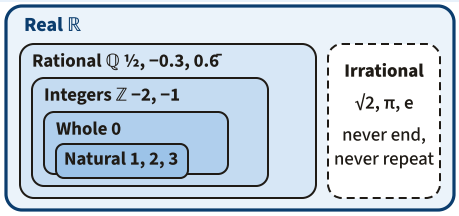
Grades 8–12 · College · 6 pages

Real numbers, exponents and radicals, equations and inequalities, polynomials and factoring, rational expressions, lines and systems, quadratics and complex numbers, functions, exponentials and logs, sequences, conics and word problems.

1 Real numbers · 2 Exponents · 3 Linear eq. · 4 Polynomials · 5 Rational · 6 Lines & systems · 7 Word problems · 8 Quadratics · 9 Conics · 10 Functions · 11 Sequences

1 REAL NUMBERS & OPERATIONS

the number system, its laws, and one agreed order



Every set sits inside the next. A real number is rational **or** irrational, never both.

- $\in$ / $\notin$ — is / is not an element of: $3 \in \mathbb{Z}$, $\frac{1}{2} \notin \mathbb{Z}$
- $\subset$ — is a subset of: $\mathbb{N} \subset \mathbb{Z}$
- $\cup$ / $\cap$ — union (in either) / intersection (in both)
- $\varnothing$ — the empty set, $\{\}$
- $\{x \mid x > 2\}$ — set-builder: "all x such that x > 2"

Properties of real numbers

Property	Addition	Multiplication
Commutative	a + b = b + a	ab = ba
Associative	(a + b) + c = a + (b + c)	(ab)c = a(bc)
Identity	a + 0 = a	a · 1 = a
Inverse	a + (−a) = 0	$a \cdot \frac{1}{a} = 1, a \neq 0$
Distributive	a(b + c) = ab + ac	

Zero product: if ab = 0, then a = 0 or b = 0. Division by 0 is undefined.

MEMORY TRICK

Commuters move, associates regroup

Commutative: the numbers *commute*, trading places (order changes). **Associative**: the same numbers change who they *associate* with (parentheses move).

Signed numbers

- **Add, same signs**: add, keep the sign. −4 + (−7) = −11
- **Add, different signs**: subtract, keep the sign of the larger |n|. −9 + 4 = −5
- **Subtract** = add the opposite: 5 − (−3) = 5 + 3 = 8
- **× and ÷**: same signs → +, different → −. (−6)(−4) = 24; −18 ÷ 3 = −6
- An **even** count of negative factors gives +; odd gives −.
- |−7| = 7: absolute value is distance from 0.

Order of operations

P parentheses & other groupings ( ) [ ] fraction bars, radicals · **E** exponents · **MD** multiply/divide, left → right · **AS** add/subtract, left → right

MEMORY TRICK

Please Excuse My Dear Aunt Sally

The traditional PEMDAS line. M and D are **partners** (whichever comes first, left to right); so are A and S. 18 ÷ 3 × 2 is 12, not 3.

Order of operations WORKED

3 + 4 × 5 = 3 + 20 = **23**	*multiply first*
18 ÷ 3 × 2 = 6 × 2 = **12**	*left → right*
2[5 + (8 − 2)²] = 2[5 + 36] = **82**	*inside out*
20 − 2³ ÷ 4 + 1 = 20 − 2 + 1 = **19**	
$\frac{12+8}{9-5} = \frac{20}{4} = 5$	*bar groups*

Fractions

- $\frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd}$: $\frac{2}{3} + \frac{1}{4} = \frac{8+3}{12} = \frac{11}{12}$
- $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$; to divide, multiply by the reciprocal: $\frac{3}{4} \div \frac{9}{8} = \frac{3}{4} \cdot \frac{8}{9} = \frac{2}{3}$

MEMORY TRICK **Keep · Change · Flip**

To divide fractions: **keep** the first, **change** ÷ to ×, **flip** the second.

Translating words

Words	Algebra
sum, more than, increased by	x + 4
5 less than x	x − 5 (order flips!)
product, of, times	3x
quotient, per, ratio	$\frac{x}{2}$
twice, double / half	2x / $\frac{x}{2}$
is, equals, results in	=
at least / at most	≥ / ≤

Fraction · decimal · percent

	½	¼	¾	⅕	⅛	⅓
decimal	0.5	0.25	0.75	0.2	0.125	$0.\overline{3}$
percent	50%	25%	75%	20%	12.5%	33⅓%

Divisibility (for factoring fast)

Divisible by	if…
2	last digit even
3	digit sum divisible by 3 (471: 12 ✓)
4	last two digits divisible by 4
5	ends in 0 or 5
6	divisible by 2 and 3
9	digit sum divisible by 9
10	ends in 0

GCF and LCM by primes WORKED

$24 = 2^3 \cdot 3$ and $36 = 2^2 \cdot 3^2$

GCF: lowest powers of shared primes = $2^2 \cdot 3$ = **12**

LCM: highest powers of all primes = $2^3 \cdot 3^2$ = **72**

check: 12 · 72 = 24 · 36 = 864 ✓

Ratios, proportions & percent

- $\frac{a}{b} = \frac{c}{d} \iff ad = bc$ (cross-multiply)
- part = percent × whole: 15% of 60 = 0.15 × 60 = **9**
- 42 is what % of 56? $\frac{42}{56} = 0.75$ = **75%**
- Properties of equality: add, subtract, multiply or divide both sides by the same number (not 0 for ÷) and the equation stays true.

Mental math is algebra

- $(a - b)(a + b) = a^2 - b^2$: 49 · 51 = $50^2 - 1^2$ = **2,499**
- $(10n + 5)^2 = 100n(n + 1) + 25$: 65^2 → 6 · 7 = 42, then 25 → **4,225**
- a(b − c) = ab − ac: 7 × 98 = 700 − 14 = **686**
- x% of y = y% of x: 8% of 25 = 25% of 8 = **2**
- × 11 (two digits): 36 × 11 → 3 | 3+6 | 6 = **396**

Averages · estimating roots WORKED

mean = $\frac{\text{sum}}{\text{count}}$; scores 82, 90, 85 → sum 257

4th score for a mean of 88: 88 · 4 − 257 = **95**

need = target × n − sum

√50 lies between 7 and 8 (49 < 50 < 64), near 7: ≈ **7.07**

estimate roots

2 EXPONENTS & RADICALS

$a \neq 0$ throughout

Law	Rule	Example
Product	$a^m \cdot a^n = a^{m+n}$	$2^3 \cdot 2^4 = 2^7 = 128$
Quotient	$\frac{a^m}{a^n} = a^{m-n}$	$\frac{5^6}{5^2} = 5^4 = 625$
Power of a power	$(a^m)^n = a^{mn}$	$(3^2)^4 = 3^8 = 6{,}561$
Power of a product	$(ab)^n = a^n b^n$	$(2x^3)^2 = 4x^6$
Power of a quotient	$(\frac{a}{b})^n = \frac{a^n}{b^n}$	$(\frac{2}{3})^3 = \frac{8}{27}$
Zero	$a^0 = 1$	$7^0 = 1$
Negative	$a^{-n} = \frac{1}{a^n}$	$2^{-3} = \frac{1}{8}$
Rational	$a^{m/n} = (\sqrt[n]{a})^m$	$8^{2/3} = 2^2 = 4$

MEMORY TRICK

One rung down the ladder

Same base: **multiply** the powers → **add** exponents; **divide** → **subtract**; **power of a power** → **multiply**. Each operation drops one rung: ^ → ×, × → +, ÷ → −.

Simplify with the laws WORKED

$(2x^3y)^2 \cdot x^{-4} = 4x^6y^2 \cdot x^{-4} = \mathbf{4x^2y^2}$

$\frac{12a^5b^2}{4a^2b^5} = \mathbf{\frac{3a^3}{b^3}}$ *subtract*

⚠ Sign & sum traps

- $-3^2 = -9$ but $(-3)^2 = 9$: the exponent touches only what is beside it.
- $(a+b)^2 \neq a^2 + b^2$: $(3+4)^2 = 49$, not 25.
- 2^{-3} is positive: $\frac{1}{8}$, not -8.

Scientific notation

$c \times 10^n$ with $1 \leq c < 10$.

- $4{,}500{,}000 = 4.5 \times 10^6 \cdot 0.0032 = 3.2 \times 10^{-3}$
- $(3 \times 10^4)(2 \times 10^5) = 6 \times 10^9$
- $\frac{8 \times 10^7}{4 \times 10^{-2}} = 2 \times 10^9$

Squares & cubes to know

n	1	2	3	4	5	6	7	8	9	10	11	12
n^2	1	4	9	16	25	36	49	64	81	100	121	144
n^3	1	8	27	64	125	216	343	512	729	1000	1331	1728

Radical rules (a, b ≥ 0)

- $\sqrt{(ab)} = \sqrt{a} \cdot \sqrt{b}$; $\sqrt{(\frac{a}{b})} = \frac{\sqrt{a}}{\sqrt{b}}$
- $\sqrt{(x^2)} = |x|$; an even root of a negative is not real
- $\sqrt[n]{(a^m)} = a^{m/n}$
- $\sqrt[3]{(-8)} = -2$ (odd roots of negatives are real); $\sqrt[4]{(-16)}$ is not real

Simplify & combine WORKED

$\sqrt{72} = \sqrt{(36 \cdot 2)} = \mathbf{6\sqrt{2}}$ *largest square*

$\sqrt[3]{54} = \sqrt[3]{(27 \cdot 2)} = \mathbf{3\sqrt[3]{2}}$

$\sqrt{12} + \sqrt{27} = 2\sqrt{3} + 3\sqrt{3} = \mathbf{5\sqrt{3}}$ *like radicals*

$(2+\sqrt{3})(2-\sqrt{3}) = 4 - 3 = \mathbf{1}$ *conjugates*

Rationalize the denominator WORKED

$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{3}}{3} = \mathbf{2\sqrt{3}}$

$\frac{4}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{4(3+\sqrt{5})}{4} = \mathbf{3+\sqrt{5}}$ *conjugate*

Radical equation: check! WORKED

$\sqrt{(x+3)} = x - 3$

$x + 3 = x^2 - 6x + 9 \rightarrow x^2 - 7x + 6 = 0$ *square both sides*

$x = 1$ or $x = 6$

$x = 1$: $\sqrt{4} = 2$ but $1 - 3 = -2$ ✗ · $x = 6$: $3 = 3$ ✓ → **$x = 6$**

3 LINEAR EQUATIONS & INEQUALITIES

do the same thing to both sides

Solving a first-degree equation

1. Clear fractions (× LCD) · 2. Distribute · 3. Combine like terms · 4. Variables to one side · 5. Isolate · 6. **Check** in the original.

Solve WORKED

$3(x-4) + 2 = 5x - 18$

$3x - 10 = 5x - 18 \rightarrow 8 = 2x \rightarrow \mathbf{x = 4}$

Check: $3(0) + 2 = 2$; $20 - 18 = 2$ ✓

$\frac{x}{3} + \frac{x}{4} = 7 \rightarrow 4x + 3x = 84 \rightarrow \mathbf{x = 12}$ *× 12*

$2x + 1 = 2x + 5 \rightarrow 1 = 5$: **no solution**

$3(x+2) = 3x + 6$: **all reals** *identity*

Literal equations (solve for a letter) WORKED

$A = \frac{1}{2}bh \rightarrow \mathbf{h = \frac{2A}{b}}$

$F = \frac{9}{5}C + 32 \rightarrow \mathbf{C = \frac{5}{9}(F - 32)}$ *F = 212 → C = 100*

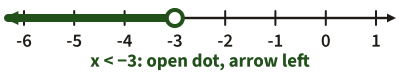
Clear decimals first WORKED

$0.2x + 1.5 = 3.1 \rightarrow \times 10 \rightarrow 2x + 15 = 31 \rightarrow \mathbf{x = 8}$

Inequalities

Same steps, **but** multiplying or dividing by a negative **flips** the sign.

EX $-3x + 5 > 14 \rightarrow -3x > 9 \rightarrow \mathbf{x < -3}$



x < −3: open dot, arrow left

MEMORY TRICK

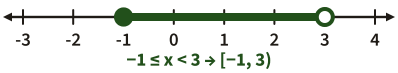
Flip the sign, flip the sign

Multiply or divide both sides by a **negative sign**? Then **flip the inequality sign**. (−2 < 3, but 2 > −3.)

Interval notation

Inequality	Interval	Graph
$x > 2$	$(2, \infty)$	open dot →
$x \leq -1$	$(-\infty, -1]$	closed dot ←
$-1 \leq x < 3$	$[-1, 3)$	closed · open
all reals	$(-\infty, \infty)$	whole line

Parentheses = open dot (not included); brackets = closed dot. ∞ always gets a parenthesis.



−1 ≤ x < 3 → [−1, 3)

Compound inequalities WORKED

$-4 < 2x + 2 \leq 8 \rightarrow -6 < 2x \leq 6 \rightarrow \mathbf{-3 < x \leq 3}$ *and: between*

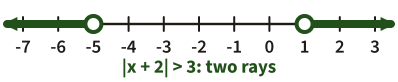
$x < -1$ or $x \geq 4 \rightarrow (-\infty, -1) \cup [4, \infty)$ *or: union*

Absolute value

- $|x - 3| = 5 \rightarrow x - 3 = 5$ or $-5 \rightarrow \mathbf{x = 8}$ **or** $\mathbf{-2}$
- $|A| < c \rightarrow -c < A < c$: $|2x - 1| \leq 7 \rightarrow \mathbf{-3 \leq x \leq 4}$
- $|A| > c \rightarrow A < -c$ or $A > c$: $|x + 2| > 3 \rightarrow \mathbf{x < -5}$ **or** $\mathbf{x > 1}$
- $|A|$ = negative → no solution

MEMORY TRICK **Less thAND, greatOR**

$|A|$ **less than** c joins into one **and** interval (between). $|A|$ **greater than** c splits into two pieces joined by **or**.



|x + 2| > 3: two rays

4 POLYNOMIALS & FACTORING

multiply out · factor back

Term	a number, a variable, or their product: $4x^3$
Degree	highest exponent: $4x^3 - 2x + 7$ has degree 3
Leading coef.	coefficient of the highest term (4)
Mono/bi/trinomial	1 / 2 / 3 terms
Standard form	terms in descending powers

Add & subtract: combine like terms WORKED

$(3x^2 + 2x - 5) - (x^2 - 4x + 1)$

$= 3x^2 + 2x - 5 - x^2 + 4x - 1 = \mathbf{2x^2 + 6x - 6}$ *distribute the −*

	x	2
x	x^2	2x
3	3x	6

Box method: $(x + 3)(x + 2) = x^2 + 2x + 3x + 6 = x^2 + 5x + 6$

MEMORY TRICK **FOIL**

First · **O**uter · **I**nner · **L**ast for two binomials: $(2x - 3)(x + 4) = 2x^2 + 8x - 3x - 12 = \mathbf{2x^2 + 5x - 12}$. For bigger products, use the box.

Special products

- $(a + b)^2 = a^2 + 2ab + b^2$
- $(a - b)^2 = a^2 - 2ab + b^2$
- $(a + b)(a - b) = a^2 - b^2$
- $(a \pm b)^3 = a^3 \pm 3a^2b + 3ab^2 \pm b^3$

Dividing polynomials

	1	-6	11	-6
		1	-5	6
	1	-5	6	0

$(x^3 - 6x^2 + 11x - 6) \div (x - 1) = x^2 - 5x + 6$, remainder 0

Long division (any divisor) WORKED

$(2x^2 + 3x - 5) \div (x + 2)$

$2x^2 \div x = 2x$; $2x(x + 2) = 2x^2 + 4x$; subtract → $-x - 5$

$-x \div x = -1$; $-1(x + 2) = -x - 2$; subtract → -3

$= \mathbf{2x - 1 - \frac{3}{x + 2}}$ *remainder −3*

Rational root theorem

Any rational root of a polynomial with integer coefficients is $\pm \frac{\text{factor of constant}}{\text{factor of leading coef.}}$. For $x^3 - 6x^2 + 11x - 6$, try ±1, ±2, ±3, ±6: 1, 2 and 3 all work.

Synthetic division (divisor x − r)

- Write r and the coefficients (0 for any missing power).
- Bring down; multiply by r; add down the column; repeat.
- Last number = **remainder** = f(r) (remainder theorem). Remainder 0 → (x − r) is a factor.

Factoring: a step-by-step path

- **1. GCF first**: $6x^3 + 9x^2 = \mathbf{3x^2(2x + 3)}$
- **2. Count terms.** Two → squares or cubes. Three → trinomial. Four → grouping.
- **3. Factor each factor again** until nothing factors.

MEMORY TRICK **GCF goes first**

Before any pattern, pull out the **G**reatest **C**ommon **F**actor. It makes every later step smaller.

Trinomials WORKED

$x^2 + 7x + 12$: two numbers × to 12, + to 7 → 3, 4

$= \mathbf{(x + 3)(x + 4)}$

$6x^2 + 7x - 3$: ac = −18 → 9 and −2 *ac method*

$6x^2 + 9x - 2x - 3 = 3x(2x + 3) - 1(2x + 3)$ *split, group*

$= \mathbf{(3x - 1)(2x + 3)}$

Binomials

- $a^2 - b^2 = (a - b)(a + b)$: $9x^2 - 25 = \mathbf{(3x - 5)(3x + 5)}$
- $a^2 + b^2$ does **not** factor over the reals.
- $a^3 \pm b^3 = (a \pm b)(a^2 \mp ab + b^2)$: $x^3 - 8 = \mathbf{(x - 2)(x^2 + 2x + 4)}$

MEMORY TRICK **SOAP for cubes**

Signs in $a^3 \pm b^3 = (a \circ b)(a^2 \circ ab \circ b^2)$: **S**ame, **O**pposite, **A**lways **P**ositive.

Grouping (four terms) WORKED

$x^3 + 3x^2 - 4x - 12 = x^2(x + 3) - 4(x + 3)$

$= (x + 3)(x^2 - 4) = \mathbf{(x + 3)(x - 2)(x + 2)}$ *factor again*

Solve by factoring WORKED

$2x^2 = 8x \rightarrow 2x^2 - 8x = 0$ *never ÷ x*

$2x(x - 4) = 0 \rightarrow \mathbf{x = 0 \text{ or } 4}$ *zero product*

5 RATIONAL EXPRESSIONS

fractions with polynomials

Domain first

$\frac{P}{Q}$ is undefined where Q = 0. $\frac{x + 2}{x^2 - 9}$: $x \neq 3, -3$. Keep these restrictions through every step.

Simplify: factor, then cancel factors WORKED

$\frac{x^2 - 9}{x^2 + 5x + 6} = \frac{(x - 3)(x + 3)}{(x + 2)(x + 3)}$

$= \mathbf{\frac{x - 3}{x + 2}}$, $x \neq -3, -2$

Negative exponents become fractions WORKED

$x^{-1} + y^{-1} = \frac{1}{x} + \frac{1}{y} = \mathbf{\frac{x + y}{xy}}$

MEMORY TRICK

Only factors may leave

Cancel **factors** (things multiplied), never **terms** (things added). $\frac{x + 5}{5}$ is not x: the 5 on top is a term.

Multiply, divide, add WORKED

$\frac{x}{3} \cdot \frac{6}{x^2} = \frac{6x}{3x^2} = \mathbf{\frac{2}{x}}$

$\frac{x^2 - 1}{4} \div \frac{x + 1}{2} = \frac{(x - 1)(x + 1)}{4} \cdot \frac{2}{x + 1} = \mathbf{\frac{x - 1}{2}}$ *flip*

$\frac{3}{x} + \frac{2}{x + 1} = \frac{3(x + 1) + 2x}{x(x + 1)} = \mathbf{\frac{5x + 3}{x(x + 1)}}$ *LCD x(x + 1)*

Complex fraction WORKED

$\frac{1 + 1/x}{1 - 1/x^2} \cdot \frac{x^2}{x^2} = \frac{x^2 + x}{x^2 - 1}$ *× LCD x^2*

$= \frac{x(x + 1)}{(x - 1)(x + 1)} = \mathbf{\frac{x}{x - 1}}$

Rational equations: check for extraneous roots WORKED

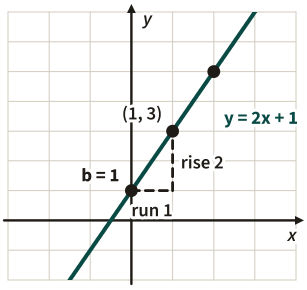
$\frac{2}{x - 1} = \frac{3}{x + 1} \rightarrow 2x + 2 = 3x - 3 \rightarrow \mathbf{x = 5}$ *cross-multiply*

$\frac{x}{x - 3} = \frac{3}{x - 3} + 2 \rightarrow x = 3 + 2(x - 3) \rightarrow x = 3$ *× (x − 3)*

x = 3 makes a denominator 0 → **no solution** *extraneous*

6 LINES, GRAPHS & SYSTEMS

slope is rise over run

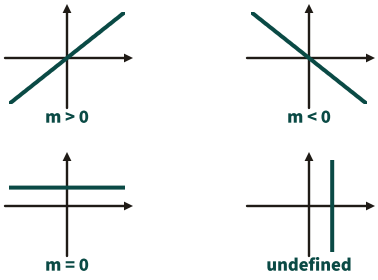


Start at the y-intercept b, rise 2, run 1: m = 2.

Slope

(1, 3) and (4, 9): $m = \frac{9-3}{4-1} = \frac{6}{3} = \mathbf{2}$

$$\mathbf{m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}}$$



MEMORY TRICK **HOY-VUX**

Horizontal line: slope **0**, equation **y** = c.
Vertical line: slope **U**ndefined, equation **x** = c.

Equations of a line

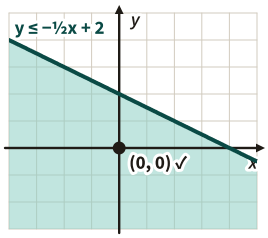
Form	Equation	Best when you know
Slope-intercept	y = mx + b	slope and y-intercept
Point-slope	$y - y_1 = m(x - x_1)$	slope and a point
Standard	Ax + By = C	intercepts; systems

Line through (1, 3) and (4, 9) WORKED

m = 2 → y − 3 = 2(x − 1) *point-slope*
y = 2x + 1 *slope-intercept*
2x − y = −1 *standard*
x-int: −½; y-int: 1 *set y = 0, x = 0*

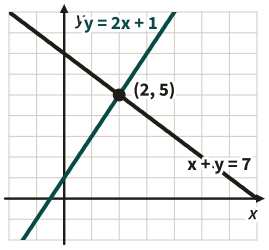
Parallel & perpendicular

- Parallel: **same slope**.
- Perpendicular: slopes are **negative reciprocals** (product −1): 2 and −½.
- **EX** ⊥ to y = 2x + 1 through (2, 5): y − 5 = −½(x − 2) → **y = −½x + 6**



Solid line for ≤ or ≥ (dashed for < or >). Test (0, 0): 0 ≤ 2 ✓, so shade the side with the origin.

Systems of two equations



The solution is the point both lines share.

Substitution WORKED

y = 2x + 1 and x + y = 7
x + (2x + 1) = 7 → 3x = 6 → x = 2 *replace y*
y = 2(2) + 1 = 5 → **(2, 5)**

Elimination WORKED

3x + 2y = 16 and x − 2y = 0
add: 4x = 16 → x = 4; then 4 − 2y = 0 → y = 2
(4, 2); check 12 + 4 = 16 ✓

How many solutions?

Lines	Slopes	Solutions
cross	different	one
parallel	same, different b	none
same line	same, same b	infinitely many

MEMORY TRICK

Lonely letter → substitute

If a variable is already **alone** (y = …), **substitute**. If coefficients **match or are opposites**, **eliminate**.

7 WORD PROBLEMS

name it · frame it · solve it · check it

MEMORY TRICK

Name it, frame it, solve it, check it

Name the unknown (let x = what is asked). **Frame** an equation from the facts. **Solve**. **Check** that the answer fits the story, with units.

Type	Model
Distance	d = rt
Work	$\frac{1}{a} + \frac{1}{b} = \frac{1}{t}$
Mixture	amount × rate, before = after
Simple interest	I = Prt
Percent change	$\frac{new - old}{old} \times 100\%$
Direct / inverse	$y = kx$ / $y = \frac{k}{x}$
Joint	y = kxz

Distance WORKED

Trains leave opposite ways at 50 and 70 mph. When are they 300 mi apart?
50t + 70t = 300 → **t = 2.5 h**

Work WORKED

One pump fills a tank in 3 h, another in 6 h. Together?
$\frac{1}{3} + \frac{1}{6} = \frac{1}{t} \rightarrow \frac{1}{2} = \frac{1}{t} \rightarrow$ **2 h**

Mixture WORKED

Liters of 40% acid to add to 10 L of 10% for 20%?
0.10(10) + 0.40x = 0.20(10 + x) → **x = 5 L**

Interest · percent · variation WORKED

$2,000 at 5% simple for 3 yr: I = 2000(0.05)(3) = **$300**

80 → 92: $\frac{12}{80}$ = **15% increase**

y varies inversely with x; y = 6 at x = 4 → k = 24; x = 8 → **y = 3**

Integers & proportions WORKED

Three consecutive odd integers sum to 75: x + (x + 2) + (x + 4) = 75 → **23, 25, 27**

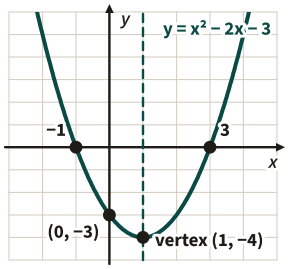
3 cups per 4 servings → 10 servings: $\frac{3}{4} = \frac{c}{10} \rightarrow$ **7.5 cups**

Tickets WORKED

30 sold: adults $8, kids $5, total $189
8a + 5(30 − a) = 189 → **13 adults, 17 kids**

8 QUADRATICS & COMPLEX NUMBERS

$ax^2 + bx + c$, $a \neq 0$



Dashed axis of symmetry x = 1. Roots −1 and 3 sit the same distance from it.

Three forms (same parabola: $x^2 - 2x - 3 = (x-1)^2 - 4 = (x+1)(x-3)$)

Form	Shows
$y = ax^2 + bx + c$	y-intercept c; vertex $x = \frac{-b}{2a}$
$y = a(x-h)^2 + k$	vertex (h, k)
$y = a(x-r)(x-s)$	roots r, s

Four ways to solve WORKED

Factor: $x^2 - 5x + 6 = 0 \rightarrow (x-2)(x-3) = 0 \rightarrow$ **2, 3**

Square root: $(x-2)^2 = 9 \rightarrow x - 2 = \pm 3 \rightarrow$ **5, −1**

Complete the square: $x^2 + 6x + 1 = 0 \rightarrow (x+3)^2 = 8 \rightarrow$ **$-3 \pm 2\sqrt{2}$**

+9 both sides

Formula: $2x^2 + 3x - 2 = 0 \rightarrow \frac{-3 \pm \sqrt{25}}{4} \rightarrow$ **½, −2**

Quadratic formula

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

MEMORY TRICK **Sing it**

To the public-domain tune *Pop Goes the Weasel*: "x equals negative b, plus or minus the square root, of b squared minus four a c, all over two a."

MEMORY TRICK

Half it, square it, add it

To complete the square on $x^2 + bx$: take **half** of b, **square** it, **add** it to both sides. $x^2 + 6x \rightarrow (6/2)^2 = 9 \rightarrow (x+3)^2$.

Discriminant

$b^2 - 4ac$	Roots	Graph
> 0, perfect square	2 rational	crosses x-axis twice
> 0, not a square	2 irrational	crosses twice
= 0	1 (double)	touches once
< 0	2 complex	misses the axis

a tells the shape

- a > 0 opens **up** (minimum); a < 0 opens **down** (maximum).
- |a| > 1 narrower; |a| < 1 wider.

Height of a thrown ball WORKED

$h = -16t^2 + 64t$ (feet, seconds)

vertex $t = \frac{-64}{2(-16)} = 2 \rightarrow h =$ **64 ft** *max*

lands: $-16t(t-4) = 0 \rightarrow$ **$t = 4$ s**

Complex numbers

- $i = \sqrt{(-1)}$, $i^2 = -1$; a + bi (a real part, b imaginary part)
- $\sqrt{(-16)} = 4i$ · conjugate of a + bi is a − bi
- $x^2 + 4x + 13 = 0 \rightarrow \frac{-4 \pm \sqrt{(-36)}}{2} =$ **$-2 \pm 3i$**

Operate like binomials WORKED

$(3 + 2i) + (1 - 5i) =$ **$4 - 3i$**

$(2 + 3i)(1 - i) = 2 + i - 3i^2 =$ **$5 + i$** *$i^2 = -1$*

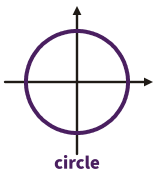
$\frac{1+i}{1-i} \cdot \frac{1+i}{1+i} = \frac{2i}{2} =$ **i** *conjugate*

MEMORY TRICK **i runs in a circle of 4**

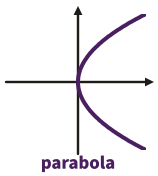
i, −1, −i, 1, then repeat. For i^n divide n by 4 and use the remainder: $i^{23} \rightarrow$ remainder 3 → **−i**.

9 CONIC SECTIONS

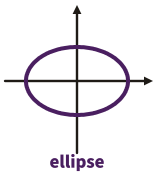
slices of a cone



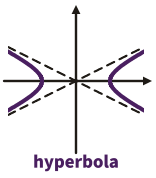
$x^2 + y^2 = 9$



$x = \frac{1}{4}y^2$



$\frac{x^2}{9} + \frac{y^2}{4} = 1$



$\frac{x^2}{4} - y^2 = 1$

Conic	Standard form	Key facts
Circle	$(x-h)^2 + (y-k)^2 = r^2$	center (h, k), radius r
Parabola	$y - k = a(x-h)^2$	vertex (h, k); or $x - h = a(y-k)^2$
Ellipse	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	center (h, k); half-axes a, b
Hyperbola	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	asymptotes $y - k = \pm\frac{b}{a}(x-h)$

Circle from general form WORKED

$x^2 + y^2 - 4x + 6y - 12 = 0$

$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 12 + 4 + 9$ *complete both squares*

$(x-2)^2 + (y+3)^2 = 25$: **center (2, −3), r = 5**

MEMORY TRICK

Plus hugs, minus splits

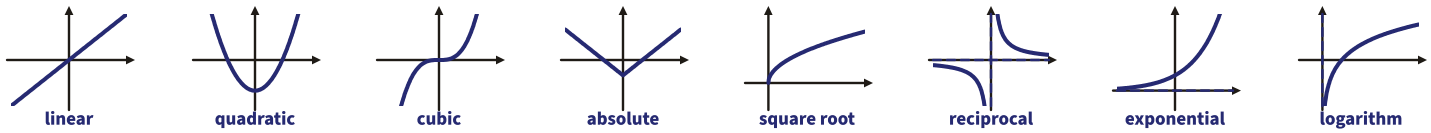
Two squared terms **added** (same signs) → a closed curve: circle if the denominators match, ellipse if not. **Subtracted** → a hyperbola, split in two. Only **one** squared variable → parabola.

Which conic? $Ax^2 + Cy^2 + Dx + Ey + F = 0$

- A = C → **circle** · A, C same sign → **ellipse**
- A, C opposite signs → **hyperbola** · A or C = 0 → **parabola**
- Foci: ellipse $c^2 = a^2 - b^2$; hyperbola $c^2 = a^2 + b^2$. **EX** $\frac{x^2}{9} + \frac{y^2}{4} = 1$: $c =$ **$\sqrt{5}$**

10 FUNCTIONS, EXPONENTIALS & LOGS

one input, exactly one output



Function	each input x has exactly **one** output f(x)
Domain	allowed inputs
Range	outputs produced
Vertical line test	a graph is a function if no vertical line hits it twice

Evaluate $f(x) = x^2 - 3x$ WORKED

$f(-2) = 4 + 6 = \mathbf{10}$

$f(a + 1) = a^2 + 2a + 1 - 3a - 3 = \mathbf{a^2 - a - 2}$

Finding the domain

- No **zero denominators**: $g(x) = \frac{1}{x+1} \to x \neq -1$
- No **negatives under even roots**: $\sqrt{(x-4)} \to [4, \infty)$
- Logs need **positive** inputs: $\ln(x-2) \to (2, \infty)$

Combine & compose ($f = 2x + 1$, $g = x^2$) WORKED

$(f + g)(x) = x^2 + 2x + 1$

$(f \circ g)(3) = f(9) = \mathbf{19}$ *inside first*

$(g \circ f)(3) = g(7) = \mathbf{49}$ *order matters*

Inverse: swap x and y, solve WORKED

$y = 2x + 1 \to x = 2y + 1 \to \mathbf{f^{-1}(x) = \frac{x-1}{2}}$

check $f(f^{-1}(5)) = 2(2) + 1 = 5$ ✓

Average rate of change

$\frac{f(b) - f(a)}{b - a}$, the slope between two points of the graph.

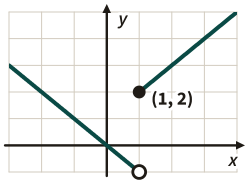
EX $f(x) = x^2$ from 1 to 4: $\frac{16 - 1}{3} = \mathbf{5}$

Inverse facts

- Exists (as a function) only if f is one-to-one: **horizontal line test**.
- Graph of f^{-1} = graph of f reflected in **y = x**; domain and range trade places.

Moving graphs: $y = a \cdot f(x - h) + k$

- h: right h · k: up k
- $|a| > 1$ stretch; $a < 0$ reflect over the x-axis
- Even: $f(-x) = f(x)$, y-axis symmetry. Odd: $f(-x) = -f(x)$, origin symmetry.



Piecewise: $f(x) = -x$ for $x < 1$; $x + 1$ for $x \geq 1$. Open dot = not included.

Exponential & logarithmic

- $f(x) = a \cdot b^x$: growth if $b > 1$, decay if $0 < b < 1$
- Compound interest $A = P(1 + \frac{r}{n})^{nt}$; continuous $A = Pe^{rt}$
- **EX** \$1,000 at 6% monthly, 5 yr: $1000(1.005)^{60} \approx$ **\$1,348.85**

Same-base equations WORKED

$4^{x+1} = 8 \to 2^{2x+2} = 2^3$

$2x + 2 = 3 \to \mathbf{x = ½}$ *match exponents*

Logarithms

- $\log_b x = y$ means $b^y = x$: $\log_2 32 = 5$, $\log 1000 = 3$, $\ln e = 1$
- $\log(MN) = \log M + \log N$ · $\log(\frac{M}{N}) = \log M - \log N$ · $\log M^p = p \log M$
- Change of base: $\log_b x = \frac{\ln x}{\ln b}$
- **EX** $3^x = 20 \to x = \frac{\ln 20}{\ln 3} \approx$ **2.727**

MEMORY TRICK **A log is an exponent**

$\log_2 32$ asks "2 to **what power** is 32?" Answer: 5. Loop it: the **base** goes to the **answer's** power and lands on the **inside**.

Rational functions: asymptotes

- Vertical: zeros of the denominator (after canceling).
- Horizontal: compare degrees of top (n) and bottom (d).
- **EX** $\frac{2x+1}{x-3}$: VA x = 3, HA y = 2

MEMORY TRICK

BOBO · BOTN · EATS-DC

Bigger **O**n **B**ottom → y = **0**. **B**igger **O**n **T**op → **N**one. **E**xponents **A**re **T**he **S**ame → **D**ivide **C**oefficients.

Half-life WORKED

80 g, half-life 5 yr; after 15 yr:

$80 \cdot (½)^{15/5} = 80 \cdot ⅛ =$ **10 g**

11 SEQUENCES & SERIES

patterns and their sums

	Arithmetic (add d)	Geometric (× r)
nth term	$a_n = a_1 + (n-1)d$	$a_n = a_1 r^{n-1}$
Sum of n	$S_n = \frac{n(a_1 + a_n)}{2}$	$S_n = \frac{a_1(1 - r^n)}{1 - r}$
Infinite sum	diverges ($d \neq 0$)	$\frac{a_1}{1 - r}$ if $\lvert r \rvert < 1$

Arithmetic: 3, 7, 11, … WORKED

$d = 4$: $a_{20} = 3 + 19 \cdot 4 =$ **79**

$S_{20} = \frac{20(3 + 79)}{2} =$ **820**

Geometric: 2, 6, 18, … WORKED

$r = 3$: $a_6 = 2 \cdot 3^5 =$ **486**

$S_6 = \frac{2(1 - 3^6)}{1 - 3} =$ **728**

$8 + 4 + 2 + \ldots = \frac{8}{1 - ½} =$ **16** *r = ½*

MEMORY TRICK **Pair the ends**

1 + 2 + … + 100: pair 1 with 100, 2 with 99, …: 50 pairs of 101 = **5,050**. That pairing is the arithmetic-sum formula.

Sigma & binomials

- $\Sigma_{k=1}^{4} k^2 = 1 + 4 + 9 + 16 =$ **30**
- Pascal row 3: 1 3 3 1 → $(x + 2)^3 = x^3 + 6x^2 + 12x + 8$
- $n! = n(n-1)\cdots 1$; $5! = 120$; $0! = 1$

Recursive form

$a_1 = 3$, $a_n = a_{n-1} + 4 \to 3, 7, 11, 15, \ldots$ (each term from the one before)