ALGEBRA II · UNITS 1–9 · QUICK REFERENCE

College Entrance Math Review (SAT · ACT · PSAT)

Grades 9–12 · 8 pages

Algebra, advanced math, problem solving and data analysis, and geometry and trigonometry, with the three test formats side by side, strategies and original practice. Not affiliated with or endorsed by any test maker. SAT, PSAT and CLEP are trademarks of College Board; ACT is a trademark of ACT, Inc.

1 The three tests · 2 Strategies · 3 Numbers · 4 Linear · 5 Systems · 6 Functions · 7 Function families · 8 Quadratics · 9 Exponents & polynomials · 10 Polynomial graphs · 11 Exponentials & logs · 12 Ratios & percents · 13 Word problems · 14 Statistics · 15 Probability & inference · 16 Angles & triangles · 17 Area, volume, circles · 18 Trigonometry · 19 Number & quantity · 20 Formulas · 21 Practice

1 THE THREE TESTS AT A GLANCE

confirm details before test day

Formats, from public information (Oct 2026)

Math section	SAT	PSAT/NMSQT · PSAT 10 · PSAT 8/9	ACT
Questions	44 (40 scored + 4 unscored pretest)	44 (40 scored + 4 unscored pretest)	45 (41 scored)
Time and pace	70 min: two 35-min modules (≈ 1.6 min each)	same as the SAT	50 min (≈ 1.1 min each)
Format	digital, adaptive (module 2 depends on module 1); easy → hard in each module	same as the SAT	paper or online; not adaptive
Answer types	multiple choice (4 choices) + typed answers (about 1 in 4)	same as the SAT	multiple choice, 4 choices
Calculator	whole section: built-in graphing calculator or your own approved one	same as the SAT	your own 4-function, scientific or graphing (no CAS); built in online
Formulas	reference sheet provided	reference sheet provided	none given: memorize them
Math score	200–800	160–760; PSAT 8/9: 120–720	1–36

What is tested (approximate share of scored math questions)

SAT and PSAT domain	Share	See	ACT reporting category	Share	See
Algebra (linear)	≈ 35 %	§4–5	Number & Quantity	10–12 %	§3, §9, §19
Advanced Math (nonlinear)	≈ 35 %	§6–11	Algebra; Functions	17–20 % each	§4–11
Problem Solving & Data Analysis	≈ 15 %	§12–15	Geometry (with trig)	17–20 %	§16–18
Geometry & Trigonometry	≈ 15 %	§16–18	Statistics & Probability	12–15 %	§14–15
(PSAT tests: same four domains)			Integrating Essential Skills	≈ 20 %	§3, §12–13

CLEP College Algebra (college credit)

- About **60 questions in 90 minutes**, some unscored.
- Built-in scientific (non-graphing) calculator only.
- Scored 20–80; ACE recommends credit at **50** (3 semester hours); colleges set their own policy.
- Study §4–11 and §19.

Scoring facts

- No wrong-answer penalty on any of these tests: **never leave a blank**.
- SAT and PSAT modules run easy → hard: bank the early points.
- Practice with the free full-length tests each test maker posts.

2 STRATEGIES THAT WORK

plug in, backsolve, graph it

Plug in numbers

- When the answer choices are expressions, pick an easy value ($x = 5$; avoid 0 and 1), work out the target number, then test each choice.

EX Which equals $\frac{x^2 - 9}{x - 3}$ $(x \neq 3)$? Try $x = 5$: $\frac{16}{2} = 8$. Only **x + 3** gives 8.

MEMORY TRICK **Reread the last line**

Before you choose, read the question's last line again: it may want 3x, the units or the total, not x.

Backsolve

- When the choices are numbers, start with a middle choice, test it in the problem, then go up or down.

EX A number plus 40 % of itself is 84. Try 60: 60 + 24 = 84 ✓, so the number is **60**.

Use the graphing calculator

- **Equations, systems:** graph each side (or each equation); read where they cross.
- **Quadratics:** tap the graph for the vertex and zeros.
- **Data:** enter a table and fit a line or curve.

Student-produced responses (SAT, PSAT)

- Type a fraction or a decimal: up to 5 characters, 6 with a minus sign.
- No mixed numbers (type 7/2, not 3 1/2), and no %, $ or commas.
- If several answers work, enter one.

⚠ Common traps

- Watch units and the words **not**, **except** and **least**.
- Check for extraneous roots after squaring or clearing fractions.

3 NUMBERS & OPERATIONS

the arithmetic every test assumes

Kinds of numbers

Set	Members	Example
Integers	…, −2, −1, 0, 1, 2, …	−7, 0, 12
Rational	a ratio of integers $\frac{p}{q}$ ($q \neq 0$)	$\frac{3}{4}$, −2.5, $0.\overline{3}$
Irrational	not a ratio: decimal never ends or repeats	$\sqrt{2}$, π
Prime	exactly two factors, 1 and itself	2, 3, 5, 7, 11, 13

2 is the only even prime; 1 is not prime. Even × anything = even; odd × odd = odd.

Factors and multiples WORKED

GCF(36, 60) = **12**; LCM(36, 60) = **180**

GCF × LCM = 36 × 60

$36 = 2^2 \cdot 3^2$; $60 = 2^2 \cdot 3 \cdot 5$ *prime factorizations*

47 ÷ 6 = 7 remainder **5** *47 = 6 · 7 + 5*

Fractions and order of operations WORKED

$\frac{2}{3} + \frac{5}{8} = \frac{16+15}{24} = \frac{31}{24}$ *common denominator*

$\frac{3}{4} \div \frac{9}{10} = \frac{3}{4} \times \frac{10}{9} = \frac{5}{6}$ *flip and multiply*

$20 - 3(4-6)^2 \div 2 = 20 - 3 \cdot 4 \div 2 = \mathbf{14}$

parentheses, powers, × ÷, + −

Scientific notation

- $a \times 10^n$ with $1 \le a < 10$.
- Multiply: multiply the a's, add the exponents.

EX $(3 \times 10^4)(6 \times 10^{-7}) = 18 \times 10^{-3} = \mathbf{1.8 \times 10^{-2}}$; $0.00042 = \mathbf{4.2 \times 10^{-4}}$.

Number sense

Absolute value |x| is the distance from 0: |−8| = 8; |x − 3| ≤ 2 means 1 ≤ x ≤ 5

Mean of a group total ÷ count; the total = mean × count

Digits 3,482.6: 8 is in the tens place, 6 in the tenths

4 LINEAR EQUATIONS & INEQUALITIES

slope, intercepts, forms

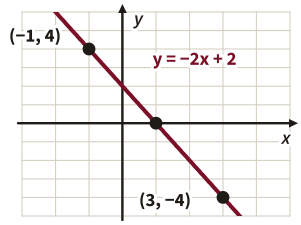
Forms of a line

Form	Equation	Shows
Slope-intercept	y = mx + b	slope m, y-intercept b
Point-slope	$y - y_1 = m(x - x_1)$	slope and one point
Standard	Ax + By = C	slope $-\frac{A}{B}$; intercepts $\frac{C}{A}$, $\frac{C}{B}$
Horizontal; vertical	y = k; x = h	slope 0; undefined

Slope

(−1, 4) and (3, −4): $m = \frac{-8}{4} = \mathbf{-2}$, so y = −2x + 2 (x-intercept 1).

$$\mathbf{m = \frac{y_2 - y_1}{x_2 - x_1}}$$



Down 2 for every 1 to the right: slope −2.

Parallel and perpendicular

- Parallel lines: **equal** slopes.
- Perpendicular lines: slopes are **negative reciprocals** (product −1).

EX Through (2, 5), perpendicular to $y = \frac{2}{3}x + 1$: $m = -\frac{3}{2}$, so $\mathbf{y = -\frac{3}{2}x + 8}$.

Solve WORKED

$\frac{2x-1}{3} = \frac{x+4}{2} \rightarrow 4x - 2 = 3x + 12 \rightarrow \mathbf{x = 14}$

cross-multiply

$-2(x - 5) \ge 14 \rightarrow x - 5 \le -7 \rightarrow \mathbf{x \le -2}$

dividing by −2 flips ≥

$|2x - 3| = 7 \rightarrow 2x - 3 = 7$ or $-7 \rightarrow \mathbf{x = 5}$ **or** $\mathbf{-2}$

two cases

Read the meaning

Linear model C = 45 + 30h: **45** is the starting fee (h = 0); **30** is the added cost per hour (the slope)

How many solutions? ax + b = cx + d has one if a ≠ c, none if a = c and b ≠ d, infinitely many if a = c and b = d

5 SYSTEMS & LINEAR MODELS

two equations, two unknowns

Elimination WORKED

3x + 2y = 16 and x − 2y = 0

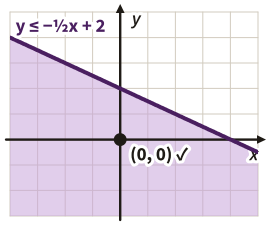
add: 4x = 16 → **x = 4**, then **y = 2**

the y-terms cancel

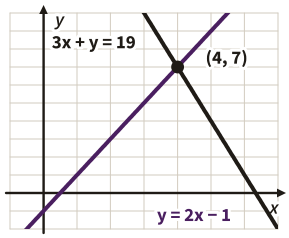
Substitution WORKED

y = 2x − 1 and 3x + y = 19

3x + 2x − 1 = 19 → **x = 4**, **y = 7** *replace y*



0 ≤ 2 is true, so shade the side with (0, 0).



The solution is where the lines cross.

Set up a word problem WORKED

Adult tickets $12, student $7; 50 tickets brought in $460

a + s = 50 and 12a + 7s = 460

one equation per fact

12a + 7(50 − a) = 460 → 5a = 110 → **a = 22, s = 28**

How many solutions?

Lines	Solutions
different slopes	exactly one
same slope, different intercepts (parallel)	none
same line (one equation is a multiple of the other)	infinitely many

2x + 3y = 6 and 4x + ky = 5 have no solution when k = **6**: same slope, but 4x + 6y = 12 ≠ 5.

Inequalities in two variables

- Solid boundary for ≤ or ≥, dashed for < or >.
- Test (0, 0): if it works, shade its side.
- A system's solutions are where the shadings overlap.

6 FUNCTIONS — notation, graphs, transformations

Function language

Domain; range — all allowed inputs; all outputs

Zero; y-intercept — an x with f(x) = 0; the value f(0)

Average rate of change — $\frac{f(b)-f(a)}{b-a}$: for x^2 from 1 to 4, $\frac{16-1}{3} = \mathbf{5}$

Evaluate and compose WORKED

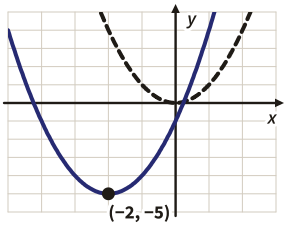
$f(x) = x^2 - 3x$, $g(x) = 2x + 1$

$f(-2) = 4 + 6 = \mathbf{10}$

$f(g(1)) = f(3) = \mathbf{0}$; $g(f(1)) = g(-2) = \mathbf{-3}$ *inside first*

Transformations

Change	Effect on the graph of y = f(x)
f(x) + k	up k (down if k < 0)
f(x − h)	right h (f(x + h): left h)
−f(x); f(−x)	reflect over the x-axis; over the y-axis
a·f(x)	vertical stretch (a > 1) or shrink (0 < a < 1)
f(bx)	horizontal shrink by a factor of b (b > 1)



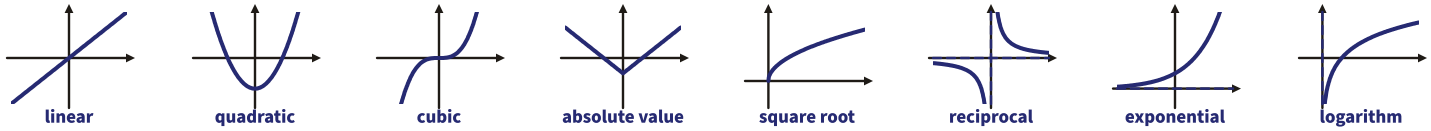
Dashed $y = x^2$; solid $y = (x + 2)^2 - 5$: left 2, down 5, so the vertex moves from (0, 0) to (−2, −5).

Inverse WORKED

$f(x) = 3x - 6$: swap x and y, solve: $\mathbf{f^{-1}(x) = \frac{x+6}{3}}$

check: $f(4) = 6$ and $f^{-1}(6) = 4$ ✓

7 FUNCTION FAMILIES & GRAPHS — know the shapes on sight



Domain and range

Family	Domain	Range	Key feature
$y = x$, $y = x^3$	all reals	all reals	through (0, 0)
$y = x^2$; $y = \lvert x \rvert$	all reals	$y \geq 0$	vertex (0, 0)
$y = \sqrt{x}$	$x \geq 0$	$y \geq 0$	starts at (0, 0)
$y = \frac{1}{x}$	$x \neq 0$	$y \neq 0$	asymptotes x = 0, y = 0
$y = 2^x$	all reals	$y > 0$	asymptote y = 0; (0, 1)
$y = \log_2 x$	$x > 0$	all reals	asymptote x = 0; (1, 0)

Reading a graph

- f(a) is the **height** of the graph at x = a.
- Solutions of f(x) = k: the x-values where the graph meets the line y = k.
- f(x) > 0 where the graph is **above** the x-axis.
- Number of real zeros = number of x-intercepts.

More function terms

Vertical line test — a graph is a function if no vertical line meets it twice

Piecewise — different rules on different intervals: f(x) = x + 1 for x < 2, 7 − x for x ≥ 2 gives f(1) = 2, f(4) = 3

Increasing — rises left to right; decreasing falls

Domain from a formula WORKED

$g(x) = \frac{1}{x+1}$: $\mathbf{x \neq -1}$ *no zero denominator*

$h(x) = \sqrt{(x-4)}$: $\mathbf{x \geq 4}$ *no negative under √*

$k(x) = \ln(x-2)$: $\mathbf{x > 2}$ *log of a positive*

8 QUADRATICS — three forms, roots, vertex

Three forms

Form	Equation	Shows
Standard	$ax^2 + bx + c$	y-intercept c; axis $x = -\frac{b}{2a}$
Vertex	$a(x-h)^2 + k$	vertex (h, k)
Factored	$a(x-r)(x-s)$	zeros r and s; axis halfway

a > 0 opens up (minimum); a < 0 opens down (maximum).

Vertex and tangency WORKED

$y = -2x^2 + 8x + 1$: $x = -\frac{8}{-4} = 2$, $y = \mathbf{9}$ (maximum)

$y = 2x + k$ touches $y = x^2$ once when $x^2 - 2x - k = 0$ has $4 + 4k = 0$: $\mathbf{k = -1}$

discriminant = 0

Quadratic formula

Discriminant $b^2 - 4ac$: **> 0** two real roots, **= 0** one (double) root, **< 0** no real roots (two complex).

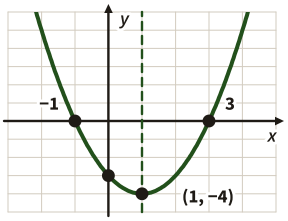
$$\mathbf{x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}}$$

Solve three ways WORKED

$2x^2 - 3x - 5 = 0 \rightarrow (2x - 5)(x + 1) = 0 \rightarrow \mathbf{x = \frac{5}{2}}$ **or −1** *factor*

$x^2 + 6x + 2 = 0 \rightarrow (x + 3)^2 = 7 \rightarrow \mathbf{x = -3 \pm \sqrt{7}}$ *complete the square*

$x^2 - 4x + 1 = 0 \rightarrow x = \frac{4 \pm \sqrt{12}}{2} = \mathbf{2 \pm \sqrt{3}}$ *formula*



$y = x^2 - 2x - 3$: zeros −1 and 3; vertex (1, −4) on the axis x = 1; y-intercept −3.

Sum and product of roots

- Sum $= -\frac{b}{a}$; product $= \frac{c}{a}$.

EX $2x^2 - 3x - 5$: sum $\frac{3}{2}$, product $-\frac{5}{2}$ (check: $\frac{5}{2} + (-1)$, $\frac{5}{2} \times (-1)$).

9 EXPONENTS, POLYNOMIALS & RADICALS

equivalent expressions and equations

Exponents and radicals

Rule	Example	Rule	Example
$a^m \cdot a^n = a^{m+n}$	$x^3 \cdot x^4 = x^7$	$a^0 = 1$; $a^{-n} = \frac{1}{a^n}$	$5^{-2} = \frac{1}{25}$
$\frac{a^m}{a^n} = a^{m-n}$	$\frac{x^9}{x^3} = x^6$	$a^{m/n} = (\sqrt[n]{a})^m$	$8^{2/3} = 4$; $27^{-1/3} = \frac{1}{3}$
$(a^m)^n = a^{mn}$; $(ab)^n = a^n b^n$	$(2x^3)^2 = 4x^6$	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$	$\sqrt{72} = 6\sqrt{2}$

Factoring patterns

Pattern	Factored
$a^2 - b^2$	$(a + b)(a - b)$
$a^2 + 2ab + b^2$; $a^2 - 2ab + b^2$	$(a + b)^2$; $(a - b)^2$
$a^3 + b^3$; $a^3 - b^3$	$(a + b)(a^2 - ab + b^2)$; $(a - b)(a^2 + ab + b^2)$
$x^3 + 2x^2 - 9x - 18$ (grouping)	$(x + 2)(x + 3)(x - 3)$

Remainder and factor theorems

- $p(x) \div (x - a)$ leaves remainder **p(a)**.
- $(x - a)$ is a factor exactly when **p(a) = 0**.

EX $p(x) = x^3 - 4x^2 + x + 6$: $p(2) = 0$, so $p(x) = (x - 2)(x^2 - 2x - 3) =$ **$(x - 2)(x - 3)(x + 1)$**.

Radical equation WORKED

$\sqrt{(x + 7)} = x + 1 \rightarrow x + 7 = x^2 + 2x + 1 \rightarrow x^2 + x - 6 = 0$

square both sides

$x = 2$ or -3; $x = -3$ gives $\sqrt{4} = -2$ ✗, so **$x = 2$**

check each root

Rational expressions WORKED

$\frac{x^2 - 9}{x^2 + x - 6} = \frac{(x - 3)(x + 3)}{(x + 3)(x - 2)} =$ **$\frac{x - 3}{x - 2}$** $x \neq -3, 2$

$\frac{x^2}{x - 2} = \frac{4}{x - 2} \rightarrow x^2 = 4 \rightarrow x = \pm 2$; $x = 2$ divides by 0, so **$x = -2$**

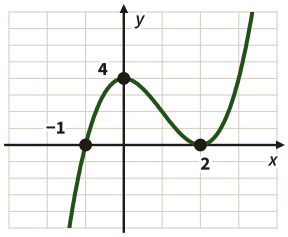
extraneous root

⚠ Check in the original

- Squaring both sides or clearing denominators can add roots that do not work.

10 POLYNOMIAL & RATIONAL GRAPHS

zeros, end behavior, asymptotes



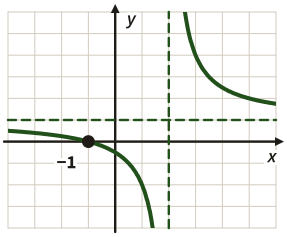
$y = (x + 1)(x - 2)^2$ crosses at -1 (single zero) and touches at 2 (double zero); y-intercept 4.

End behavior

Leading term	Left end	Right end
even degree, $a > 0$	up	up
even degree, $a < 0$	down	down
odd degree, $a > 0$	down	up
odd degree, $a < 0$	up	down

Zeros and multiplicity

- A factor $(x - r)$ gives a zero at $x = r$.
- Odd multiplicity: the graph **crosses**. Even multiplicity: it **touches** and turns back.



$y = \frac{x + 1}{x - 2}$: vertical asymptote $x = 2$, horizontal asymptote $y = 1$, zero at -1.

Rational functions

Vertical asymptote a zero of the denominator that does not cancel

Hole a factor that cancels: $\frac{(x - 3)(x + 1)}{x - 3}$ has a hole at $x = 3$

Horizontal asymptote degree top < bottom: $y = 0$; equal: y = ratio of leading coefficients; top > bottom: none

11 EXPONENTIAL MODELS & LOGARITHMS

growth, decay, logs

Exponential model

a = starting amount; $b > 1$ grows by $r = b - 1$ per step; $0 < b < 1$ decays.

$$y = a \cdot b^x = a(1 \pm r)^t$$

Models WORKED

$2,000 at 5 % a year for 3 years: $2{,}000(1.05)^3 =$ **$2,315.25**

600 mg, losing 20 % per hour, after 2 h: $600(0.8)^2 =$ **384 mg**

50 cells doubling every 4 h: $50 \cdot 2^{t/4}$; at $t = 12$, **400**

Linear or exponential?

Linear adds the same amount each step: 5, 8, 11, 14 (differences 3)

Exponential multiplies by the same factor: 3, 6, 12, 24 (ratios 2)

Compound interest

Second form: continuous. $1,000 at 6 % monthly for 2 years: $1{,}000(1.005)^{24} \approx$ **$1,127.16**.

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

Logarithms (ACT, CLEP)

Log rule	Example
$\log_b x = y$ means $b^y = x$	$\log_2 32 = 5$; $\log_3 \frac{1}{9} = -2$
$\log(xy) = \log x + \log y$	$\log 4 + \log 25 = \log 100 = 2$
$\log(x/y) = \log x - \log y$	$\log_2 48 - \log_2 3 = \log_2 16 = 4$
$\log(x^n) = n \log x$	$\log 10^5 = 5$
$\log_b x = \frac{\ln x}{\ln b}$	$3^x = 20 \rightarrow x = \frac{\ln 20}{\ln 3} \approx 2.727$

12 RATIOS, RATES, PERCENTS & UNITS

proportional reasoning

Percent

part = percent × whole

$$\%\ \text{change} = \frac{\text{new} - \text{old}}{\text{old}} \times 100$$

MEMORY TRICK

Multiply by the multiplier

Up 7 % means × 1.07; down 7 % means × 0.93. Chain several changes by multiplying the multipliers.

Equivalents to know

Fraction	Decimal	Percent
$\frac{1}{8}$; $\frac{1}{5}$	0.125; 0.2	12.5 %; 20 %
$\frac{1}{4}$; $\frac{1}{3}$	0.25; $0.3\overline{3}$	25 %; $33\frac{1}{3}$ %
$\frac{1}{2}$; $\frac{2}{3}$	0.5; $0.6\overline{6}$	50 %; $66\frac{2}{3}$ %
$\frac{3}{4}$; $\frac{3}{2}$	0.75; 1.5	75 %; 150 %

Percent problems WORKED

$80 jacket, 25 % off, then 8 % tax: 80(0.75)(1.08) = **$64.80**

Up 20 %, then down 20 %: 1.2 × 0.8 = 0.96 → **4 % lower**

not back to even

After a 15 % raise the pay is $27.60/h; before: 27.60 ÷ 1.15 = **$24.00**

divide by the multiplier

Ratios and variation

- a : b = 3 : 5 means a = 3k and b = 5k (total 8k).
- Direct variation y = kx; inverse variation $y = \frac{k}{x}$.

EX Split 96 in a 3 : 5 ratio: k = 12, so **36 and 60**. Inverse: y = 12 when x = 3 gives k = 36, so y = **4** at x = 9.

Proportions and rates WORKED

3 cups of flour for 24 cookies; for 60: $3 \times \frac{60}{24}$ = **7.5 cups**

18 oz for $4.50 ($0.25/oz) or 24 oz for $5.76 ($0.24/oz): **24 oz** is cheaper per ounce

unit price

map 1 in : 15 mi; 3.2 in = **48 mi**

Percent shortcuts

- 10 %: move the decimal point one place left; 5 % is half of that.

EX 15 % tip on $42: 4.20 + 2.10 = **$6.30**.

Units WORKED

60 mi/h = $\frac{60 \times 5{,}280\ \text{ft}}{3{,}600\ \text{s}}$ = **88 ft/s**

cancel units

3 ft × 4 ft = 36 in × 48 in = **1,728 in²**

1 ft² = 144 in²

250 g in 100 cm³: density **2.5 g/cm³**

Time and rate conversions WORKED

$3.20 per pound = 3.20 ÷ 16 = **$0.20 per ounce**

2.25 h = **2 h 15 min**; 135 min = **2.25 h**

0.25 h = 15 min

13 WORD PROBLEMS & MODELING

turn words into equations

Translate

Words	Math	Words	Math
is, equals	=	of (with a percent)	×
more than, increased by	+	per, for each	÷ (a rate)
less than, fewer	−	at least; at most	≥; ≤
5 less than x	x − 5	twice the sum of x and 3	2(x + 3)

Distance, rate, time

Two cars leave the same town in opposite directions at 50 and 60 mi/h: they are 330 mi apart after 330 ÷ 110 = **3 h**.

d = rt

Work together WORKED

A fills a tank in 3 h, B in 6 h: $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$ tank per hour → **2 h**

add the rates

Build a model WORKED

A plan costs $25 plus $0.10 per text: C = 25 + 0.10t; at 340 texts, C = **$59**

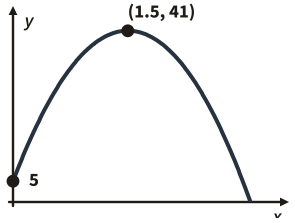
linear

A town of 8,000 grows 3 % a year: P = 8,000(1.03)^t; after 10 years ≈ **10,751**

exponential

A ball thrown up at 48 ft/s from 5 ft: h = −16t² + 48t + 5; highest at t = **1.5 s**, h = **41 ft**

quadratic



h = −16t² + 48t + 5: the vertex (1.5, 41) is the highest point; the graph starts at 5 ft.

Mixture WORKED

How many liters of 40 % acid with 6 L of 10 % acid make a 20 % mix?

0.40x + 0.10(6) = 0.20(x + 6) → 0.2x = 0.6 → **x = 3 L**

acid in = acid out

Integers and ages WORKED

Three consecutive even integers sum to 78: 3n + 6 = 78 → **24, 26, 28**

n, n + 2, n + 4

Ana is 3 times Ben's age; in 8 years she will be twice his: 3b + 8 = 2(b + 8) → Ben **8**, Ana **24**

Money WORKED

Simple interest I = Prt: $1,500 at 4 % for 3 years = **$180**

6 % commission on a $240,000 sale = **$14,400**

$18 after a 25 % markup: cost = 18 ÷ 1.25 = **$14.40**

Interpret the pieces

- In y = mx + b: m is the rate, b the starting value.
- In y = a(b)^x: a is the start, b the growth factor per period.
- In h = −16t² + vt + h_0: v is the launch speed, h_0 the launch height.

14 STATISTICS & DATA

center, spread, displays

Center and spread

Mean; median — sum ÷ count; middle value in order (mean of the two middles if even)

Mode; range — most frequent value; largest − smallest

Standard deviation — typical distance from the mean: bigger means more spread

IQR; outlier — Q3 − Q1; a value beyond 1.5 × IQR from Q1 or Q3

Changing every value

- Add c: mean and median go up by c; range and SD stay the same.
- Multiply by c: mean, median, range and SD are all multiplied by c.

One data set WORKED

2, 4, 4, 5, 7, 9, 11: mean **6**, median **5**, mode **4**, range **9**

add 30: mean $\frac{72}{8} = 9$, median $\frac{5+7}{2} = 6$

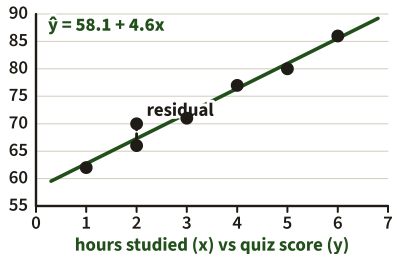
an outlier pulls the mean

mean of 5 scores is 85; four are 80, 92, 75, 88 → fifth = 425 − 335 = **90**

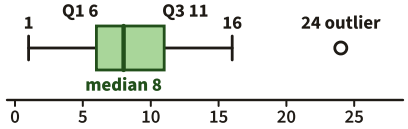
total = mean × count

Displays

Display	Read it for
Histogram; dot plot	shape (symmetric, skewed), center, gaps
Box plot	min, Q1, median, Q3, max; IQR = box width
Scatterplot	direction, form, strength, outliers
Two-way table	counts by two categories (§15)



Strong positive linear association. Slope 4.6: about 4.6 more points per hour. At x = 2 the line predicts 67.3, so the point (2, 70) has residual 70 − 67.3 = 2.7.



Q1 = 6, median 8, Q3 = 11, IQR 5; fences −1.5 and 18.5, so 24 is an outlier.

15 PROBABILITY & INFERENCE

two-way tables, samples, studies

A two-way table (original data)

Lunch choice	Hot	Packed	Total
Grade 9	42	18	60
Grade 10	30	30	60
Total	72	48	120

Read it WORKED

P(hot) = $\frac{72}{120}$ = **0.6** — *whole table*

P(hot | grade 10) = $\frac{30}{60}$ = **0.5** — *grade 10 row only*

P(grade 10 | packed) = $\frac{30}{48}$ = **0.625** — *packed column only*

Probability rules

- P(not A) = 1 − P(A).
- Independent events: P(A and B) = P(A) · P(B).
- P(A or B) = P(A) + P(B) − P(A and B).

EX Two heads and then a 6: $\frac{1}{4} \times \frac{1}{6} = \frac{1}{24}$.

Estimate from a sample WORKED

58 % ± 4 % → between **54 % and 62 %**

46 of 200 sampled prefer A; of 5,000 people, expect 5,000 × $\frac{46}{200}$ = **1,150**

same proportion

Sampling and studies

Random sample — results can be generalized to the population sampled

Random assignment — needed to conclude cause and effect

Observational study — shows association, not cause

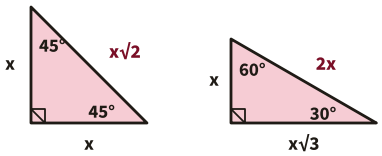
Margin of error — estimate ± MOE is the plausible range; larger samples give smaller margins

16 LINES, ANGLES & TRIANGLES

angles, similarity, Pythagoras

Angle facts

- Vertical angles are equal; a linear pair sums to 180°.
- Parallel lines and a transversal: corresponding and alternate interior angles are equal; same-side interior angles sum to 180°.
- Triangle angles sum to 180°; an exterior angle equals the two remote interior angles together.
- Polygon with n sides: interior angles sum to (n − 2) · 180° (hexagon 720°).



The two special right triangles.

Similar triangles WORKED

Corresponding sides are proportional; angles are equal

a 6-ft post casts a 4-ft shadow; a tree casts 18 ft: $\frac{6}{4} = \frac{h}{18}$ → **h = 27 ft**

scale factor k: lengths × k, areas × k^2, volumes × k^3

k = 3: area × 9

Special right triangles WORKED

30-60-90, hypotenuse 10: legs **5** and **5√3 ≈ 8.66**

45-45-90, leg 7: hypotenuse **7√2 ≈ 9.90**

square with diagonal 12: side $\frac{12}{\sqrt{2}}$ = **6√2**

Triangle toolkit

Fact	Detail
Pythagorean theorem	$a^2 + b^2 = c^2$ (c is the hypotenuse)
Common triples	3-4-5, 5-12-13, 8-15-17, 7-24-25 and multiples (6-8-10)
Triangle inequality	each side < the sum of the other two: sides 4 and 9 → third between 5 and 13
Isosceles	equal sides face equal angles

17 AREA, VOLUME & CIRCLES

formulas and circle equations

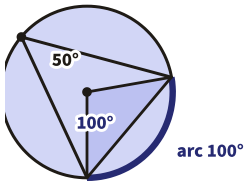
Area and volume

Figure	Formula	Figure	Formula	Solid	Volume
Rectangle	$A = lw$	Circle	$A = \pi r^2$; $C = 2\pi r$	Box	$V = lwh$
Triangle	$A = \frac{1}{2}bh$	Sector	$A = \frac{\theta}{360^\circ} \cdot \pi r^2$	Cylinder	$V = \pi r^2 h$
Parallelogram	$A = bh$	Arc length	$s = \frac{\theta}{360^\circ} \cdot 2\pi r = r\theta$ (radians)	Cone; pyramid	$V = \frac{1}{3}\pi r^2 h$; $\frac{1}{3}Bh$
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	Rectangular box	surface $2(lw + lh + wh)$	Sphere	$V = \frac{4}{3}\pi r^3$; $S = 4\pi r^2$

Circle equation

center (h, k), radius r. Complete the square: $x^2 + y^2 - 6x + 4y - 12 = 0 \to (x-3)^2 + (y+2)^2 = \mathbf{25}$: center **(3, −2)**, r = **5**.

$$\mathbf{(x-h)^2 + (y-k)^2 = r^2}$$



A central angle equals its arc; an inscribed angle is half of it (100° and 50° here), so one on a diameter is 90°. A tangent is perpendicular to the radius.

Worked WORKED

r = 6, θ = 120°: arc = 4π ≈ **12.57**; sector = 12π ≈ **37.70**

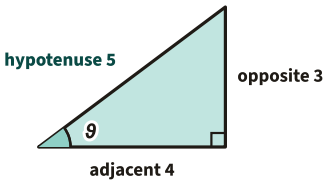
cylinder r = 3, h = 8: 72π ≈ **226.19**; sphere r = 3: 36π ≈ **113.10**

(−2, 1) to (4, 9): distance √(36 + 64) = **10**; midpoint **(1, 5)**

$d = \sqrt{(\Delta x)^2 + (\Delta y)^2}$

18 TRIGONOMETRY

SOH-CAH-TOA, radians, unit circle



Label the sides from θ's point of view.

Right-triangle ratios

- $\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$, $\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$

EX In the figure: $\sin\theta = \frac{3}{5}$, $\cos\theta = \frac{4}{5}$, $\tan\theta = \frac{3}{4}$.

MEMORY TRICK

SOH-CAH-TOA (Traditional)

Sine = **O**pposite ÷ **H**ypotenuse, **C**osine = **A**djacent ÷ **H**ypotenuse, **T**angent = **O**pposite ÷ **A**djacent.

Cofunctions and identity

- $\sin x^\circ = \cos(90^\circ - x^\circ)$: $\sin 28^\circ = \cos 62^\circ$.
- $\sin^2\theta + \cos^2\theta = 1$.

Radians

$135^\circ = \mathbf{\frac{3\pi}{4}}$; $\frac{5\pi}{6} = \mathbf{150^\circ}$. Unit circle: the point at angle θ is (cos θ, sin θ).

$$\mathbf{\pi \text{ radians} = 180^\circ}$$

Exact values

θ	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined

ACT extras

Law of sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Law of cosines $c^2 = a^2 + b^2 - 2ab\cos C$

y = A sin(Bx) + D amplitude $|A|$, period $\frac{2\pi}{|B|}$, midline y = D: $y = 3\sin 2x + 1$ has amplitude 3, period π, range −2 to 4

19 NUMBER & QUANTITY (ACT, CLEP)

beyond the SAT list

Complex numbers WORKED

$i = \sqrt{-1}$, $i^2 = -1$; powers repeat every 4: $i^{23} = i^3 = \mathbf{-i}$

$(3 + 2i)(1 - 4i) = 3 - 12i + 2i - 8i^2 = \mathbf{11 - 10i}$

$\frac{5+5i}{1+2i} \cdot \frac{1-2i}{1-2i} = \frac{15-5i}{5} = \mathbf{3 - i}$

multiply by the conjugate

More ACT topics

Matrices add entry by entry; multiply row by column; the determinant of rows (a, b), (c, d) is ad − bc

Vectors ⟨a, b⟩ has length $\sqrt{a^2 + b^2}$: ⟨−5, 12⟩ has length **13**

Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$: vertices (±a, 0), co-vertices (0, ±b)

Counting

- Permutations (order matters): ${}_nP_r = \frac{n!}{(n-r)!}$.
- Combinations (order does not): ${}_nC_r = \frac{n!}{r!(n-r)!}$.

EX 8 runners, gold-silver-bronze: 8 · 7 · 6 = **336**. A 3-person committee from 8: **56**.

Sequences

	Arithmetic (add d)	Geometric (multiply by r)
nth term	$a_n = a_1 + (n-1)d$	$a_n = a_1 r^{n-1}$
Sum of n terms	$S_n = \frac{n(a_1 + a_n)}{2}$	$S_n = \frac{a_1(1 - r^n)}{1 - r}$
Example	7, 11, 15, …: a_{20} = **83**; S_{20} = **900**	3, 6, 12, …: a_8 = **384**

Matrix work WORKED

det of rows (2, 1) and (3, 4) = 2 · 4 − 1 · 3 = **5**

rows (1, 2), (0, 1) times column (3, 4) = column **(11, 4)**

row · column

20 FORMULAS TO KNOW COLD

the ACT gives no sheet

Formula summary

Topic	Formula	Topic	Formula	Topic	Formula
Slope	$m = \frac{y_2 - y_1}{x_2 - x_1}$	Quadratic formula	$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$	Circle	$(x - h)^2 + (y - k)^2 = r^2$
Line	$y = mx + b$	Vertex	$x = -\frac{b}{2a}$	Arc; sector	$\frac{\theta}{360} \cdot 2\pi r$; $\frac{\theta}{360} \cdot \pi r^2$
Distance	$\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$	Growth or decay	$y = a(1 \pm r)^t$	Pythagorean	$a^2 + b^2 = c^2$
Midpoint	$(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2})$	Interest	$A = P(1 + \frac{r}{n})^{nt}$	Special triangles	x, x, x√2; x, x√3, 2x
Percent change	$\frac{\text{new} - \text{old}}{\text{old}} \times 100$	Arithmetic	$a_n = a_1 + (n - 1)d$	SOH-CAH-TOA	$\frac{\text{opp}}{\text{hyp}}, \frac{\text{adj}}{\text{hyp}}, \frac{\text{opp}}{\text{adj}}$
Mean	$\frac{\text{sum}}{\text{count}}$	Geometric	$a_n = a_1 r^{n-1}$	Cylinder; cone	$\pi r^2 h$; $\frac{1}{3}\pi r^2 h$
Probability	$\frac{\text{favorable}}{\text{total}}$	Combinations	${}_nC_r = \frac{n!}{r!(n - r)!}$	Sphere	$\frac{4}{3}\pi r^3$; $4\pi r^2$

Common mistakes

Mistake	Fix	Mistake	Fix
$(a + b)^2 = a^2 + b^2$	$(a + b)^2 = a^2 + 2ab + b^2$	$\sqrt{(9 + 16)} = 3 + 4$	$\sqrt{25} = 5$, not 7
$-3^2 = 9$	$-3^2 = -9$; $(-3)^2 = 9$	$x^2 = 25 \rightarrow x = 5$	$x = \pm 5$
$\frac{a + b}{a} = b$	$\frac{a + b}{a} = 1 + \frac{b}{a}$	up 20 %, down 20 % = no change	$1.2 \times 0.8 = 0.96$: 4 % less
flipping ≤ after × by a positive	flip only for a negative	trig in the wrong mode	check degrees or radians first

21 PRACTICE SET

original problems, answers checked

Thirty-four practice problems (original; not from any test)

#	Problem	Answer	#	Problem	Answer
1	(Algebra) Solve $5x - 7 = 2x + 11$	**$x = 6$**	9	(Adv. math) Simplify $\frac{x^2 - 16}{x + 4}$	**$x - 4$** (plug in $x = 1$: -3)
2	(Algebra) Line through (1, −2) and (4, 7)	**$y = 3x - 5$**	10	(Data) $60, up 10 %, then down 10 %	**$59.40**
3	(Algebra) $x + y = 10$ and $x - y = 4$	**(7, 3)**	11	(Data) Mean of 6, 9, 10, 15 and x is 11; find x	**15**
4	(Algebra) For which c does $3x + 6 = 3x + c$ have infinitely many solutions?	**$c = 6$**	12	(Data) From the §15 table, P(packed | grade 9)	**0.3**
5	(Adv. math) Zeros of $x^2 - 5x - 14$	**7 and −2**	13	(Geometry) Right triangle with legs 9 and 12: hypotenuse	**15** (3-4-5 × 3)
6	(Adv. math) Vertex of $y = 2(x - 3)^2 + 5$	**(3, 5)**, minimum 5	14	(Geometry) Center and radius of $x^2 + y^2 + 8x - 2y = 8$	**(−4, 1)**, r = **5**
7	(Adv. math) $f(x) = 4x - 1$; find f(f(2))	**27**	15	(Trig) sin A = $\frac{5}{13}$ in a right triangle; cos A and tan A	**$\frac{12}{13}$; $\frac{5}{12}$**
8	(Adv. math) Solve $2^{x+1} = 32$	**$x = 4$** (backsolve: $2^5 = 32$)	16	(ACT) $(2 - 3i)(2 + 3i)$	**13**

Eighteen more

#	Problem	Answer	#	Problem	Answer
17	(Numbers) GCF and LCM of 18 and 24	**6; 72**	26	(Geometry) Arc length, r = 9, central angle 40°	**$2\pi \approx 6.28$**
18	(Numbers) $(4 \times 10^5)(5 \times 10^{-2})$ in scientific notation	**2×10^4**	27	(Trig) Convert 210° to radians	**$\frac{7\pi}{6}$**
19	(Algebra) Solve $-3(x + 2) < 9$	**$x > -5$**	28	(ACT) 5th term of 2, 6, 18, …; ${}_6C_2$	**162; 15**
20	(Algebra) Slope of a line perpendicular to $4x - 2y = 7$	**$-\frac{1}{2}$**	29	(Algebra) $3x + 2y = 12$ and $x - y = -1$	**(2, 3)**
21	(Adv. math) Number of real solutions of $x^2 + 4x + 5 = 0$	**none** (discriminant −4)	30	(Adv. math) Simplify $(3x^2y)^3$	**$27x^6y^3$**
22	(Adv. math) Solve $\sqrt{(2x + 1)} = 5$	**$x = 12$**	31	(Adv. math) Zeros of $y = x(x - 4)^2$: cross or touch?	**0 crosses; 4 touches**
23	(Data) A $250 bike is marked down to $200; percent decrease	**20 %**	32	(Adv. math) Horizontal asymptote of $y = \frac{6x + 1}{2x - 5}$	**$y = 3$**
24	(Data) Pipe A fills a pool in 4 h, pipe B in 12 h; together	**3 h**	33	(Data) Mean and median of 4, 8, 8, 10, 20	**10; 8**
25	(Geometry) Interior angle sum of an octagon	**1,080°**	34	(Geometry) Volume of a cone, r = 3, h = 4	**$12\pi \approx 37.70$**