PHYSICS · UNITS 1–10 · QUICK REFERENCE

PHYSICS 1

Grades 10–12+ · 8 pages

Units, vectors and significant figures; motion and projectiles; Newton's laws, friction and pulleys; circular motion and gravity; energy, momentum and rotation; oscillations, waves and sound; fluids; heat; electricity, magnetism and optics.

1 Measurement · 2 Constants · 3 1D motion · 4 Projectiles · 5 Newton · 6 Friction · 7 Circles & gravity · 8 Energy · 9 Momentum · 10 Rotation · 11 Oscillations · 12 Waves & sound · 13 Fluids · 14 Heat · 15 Electricity · 16 Magnetism · 17 Optics

1 MEASUREMENT, UNITS & VECTORS

The seven SI base units

Quantity	Unit	Symbol
length	meter	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
temperature	kelvin	K
amount	mole	mol
brightness	candela	cd

Metric prefixes

G, M, k 10^9, 10^6, 10^3 (giga, mega, kilo)
c, m 10^{-2}, 10^{-3} (centi, milli)
μ, n 10^{-6}, 10^{-9} (micro, nano)

MEMORY TRICK
Gentle Mice Keep Cheese Mostly Under Nightstands
Prefixes from big to small: G, M, k, c, m, μ, n.

Derived units

Unit	Equals	Unit	Equals
N	kg·m/s²	Pa	N/m²
J	N·m	Hz	1/s
W	J/s	C	A·s
V	J/C	Ω	V/A

Convert with unit fractions WORKED

$65 \text{ mi/h} \times \frac{1609 \text{ m}}{1 \text{ mi}} \times \frac{1 \text{ h}}{3600 \text{ s}}$ *units cancel*

$= 29.05 \text{ m/s} \approx$ **29 m/s** *2 significant figures*

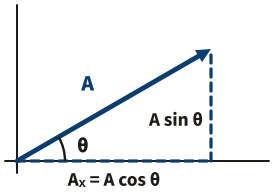
Significant figures

- Nonzero digits and zeros **between** them count; **leading** zeros never do (0.00340 has 3).
- **Trailing** zeros count only with a decimal point (2.50 has 3).
- × and ÷: fewest **significant figures**. + and −: fewest **decimal places**.
- Counted and defined numbers are exact.

Rounding the answer WORKED

12.0 cm × 3.5 cm = 42 cm² *not 42.00*

12.11 m + 0.3 m = **12.4 m** *tenths place*



Resolve a vector into perpendicular components.

Components and resultant

Scalar: size only (12 s). **Vector**: size and direction (5 m north). Add x parts and y parts separately.

$$A_x = A\cos\theta,\ A_y = A\sin\theta$$
$$R = \sqrt{(R_x^2 + R_y^2)}$$
$$\theta = \tan^{-1}(R_y \div R_x)$$

Walk east, then north WORKED

3.0 km E + 4.0 km N: $R = \sqrt{(3.0^2 + 4.0^2)} =$ **5.0 km**

$\theta = \tan^{-1}(\frac{4.0}{3.0}) =$ **53°** north of east

MEMORY TRICK **Cos is close**
The **cos**ine goes with the side **close** to the angle, so with θ measured from the x-axis, A cos θ is the x part.

Honest measure

- **Accurate**: near the true value. **Precise**: repeats agree.
- Percent error = |measured − accepted| ÷ accepted × 100 %.
- "A false balance is abomination to the LORD: but a just weight is his delight" (Proverbs 11:1, KJV).

2 CONSTANTS & KEY EQUATIONS

Physical constants

Constant	Value
g (Earth's surface)	9.8 m/s²
G (gravitation)	6.674×10^{-11} N·m²/kg²
k (Coulomb)	8.99×10^9 N·m²/C²
e (elementary charge)	1.602×10^{-19} C
c (light in vacuum)	3.00×10^8 m/s
μ_0 (magnetic)	$4\pi \times 10^{-7}$ T·m/A
Earth: mass, radius	5.97×10^{24} kg, 6371 km

Solving any problem

- **Sketch**; list knowns with units; name the unknown.
- Pick an equation where **only the unknown** is missing; solve with symbols, then numbers.
- **Check** units, size and sig figs.

Handy values

Quantity	Value
water density	1000 kg/m³
water specific heat	4186 J/(kg·°C)
ice melts (L_f)	334 kJ/kg
water boils (L_v)	2260 kJ/kg
1 atm	101.3 kPa = 760 mm Hg
sound in air, 20 °C	343 m/s
n: air, water, glass	1.00, 1.33, ≈ 1.50
absolute zero	0 K = −273.15 °C
1 cal; 1 hp	4.186 J; 746 W
1 kWh	3.6×10^6 J

MEMORY TRICK **Units ride along**
Carry units through every line; wrong units in the answer mean an algebra slip.

Key equations

Topic	Equations
Motion	$v = v_0 + at$; $\Delta x = v_0 t + \frac{1}{2}at^2$; $v^2 = v_0^2 + 2a\Delta x$
Forces	$F_{net} = ma$; $W = mg$; $f = \mu N$; $F = kx$
Gravity	$a_c = v^2/r$; $F = Gm_1m_2/r^2$
Energy	$W = Fd\cos\theta$; $KE = \frac{1}{2}mv^2$; $PE = mgh$; $P = W/t$
Momentum	$p = mv$; $F\Delta t = \Delta p$
Rotation	$\tau = rF\sin\theta$; $\tau = I\alpha$; $L = I\omega$
Waves	$v = f\lambda$; $T = 2\pi\sqrt{(L/g)}$; $T = 2\pi\sqrt{(m/k)}$
Fluids	$P = F/A$; $P = P_0 + \rho gh$; $F_b = \rho Vg$
Heat	$Q = mc\Delta T$; $Q = mL$; $\Delta U = Q - W$
Electricity	$F = kq_1q_2/r^2$; $V = IR$; $P = IV$
Magnetism	$F = BIL\sin\theta$; $F = qvB\sin\theta$
Optics	$n_1\sin\theta_1 = n_2\sin\theta_2$; $1/f = 1/d_o + 1/d_i$

3 MOTION IN ONE DIMENSION

Displacement Δx change in position, with direction (vector)
Distance total path length (scalar)
Velocity v Δx ÷ Δt, with direction
Speed size of velocity
Acceleration a Δv ÷ Δt (m/s²): speeding up, slowing down or turning

The kinematic equations

Equation (constant a)	Leaves out
$v = v_0 + at$	Δx
$\Delta x = v_0 t + \frac{1}{2}at^2$	v
$v^2 = v_0^2 + 2a\Delta x$	t
$\Delta x = \frac{1}{2}(v_0 + v)t$	a

MEMORY TRICK **Skip the stranger**
List the five letters v_0, v, a, t, Δx. Choose the equation that **leaves out** the one you neither know nor want.

Speeding up WORKED

car: 0 → 27 m/s in 9.0 s

$a = \frac{27 - 0}{9.0} = \mathbf{3.0\ m/s^2}$

$\Delta x = \frac{1}{2}(0 + 27)(9.0) = 121.5 \approx \mathbf{120\ m}$

Braking distance WORKED

25 m/s, a = −5.0 m/s², to rest

$\Delta x = \frac{0^2 - 25^2}{2(-5.0)} = \mathbf{62.5\ m}$ *$v^2 = v_0^2 + 2a\Delta x$*

$t = \frac{0 - 25}{-5.0} = \mathbf{5.0\ s}$

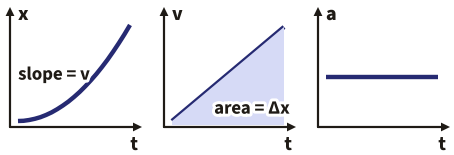
Free fall

- Take up as +: a = −g = **−9.8 m/s²**, going up or coming down.
- Without air, all objects fall alike (Apollo 15 hammer and feather, 1971).
- At the top v = 0 but a is still −g.

Thrown straight up at 19.6 m/s WORKED

top: $t = \frac{19.6}{9.8} = \mathbf{2.0\ s}$ *v = 0*

height $= \frac{19.6^2}{2(9.8)} = \mathbf{19.6\ m}$ *back down at 4.0 s*



Steady acceleration from rest: the x–t curve steepens, v–t is a straight line, a–t is flat.

Dropped from 45 m WORKED

$t = \sqrt{(\frac{2h}{g})} = \sqrt{(\frac{2(45)}{9.8})} = \mathbf{3.0\ s}$

$v = gt = (9.8)(3.03) = \mathbf{30\ m/s}$

Reading a v–t graph WORKED

v rises 0 → 12 m/s in 4.0 s, then holds 3.0 s

$a = \frac{12}{4.0} = \mathbf{3.0\ m/s^2}$ *slope*

$\Delta x = \frac{1}{2}(4.0)(12) + (12)(3.0) = \mathbf{60\ m}$ *area*

Reading motion graphs

Graph	Slope gives	Area gives
x–t	velocity	—
v–t	acceleration	displacement
a–t	—	change in v

Round trip WORKED

60 km out at 60 km/h, back at 30 km/h

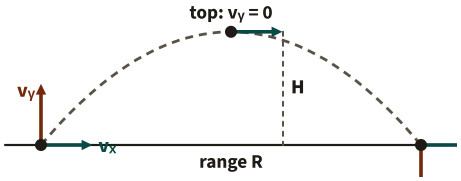
average speed $= \frac{120\ \text{km}}{1\ \text{h} + 2\ \text{h}} = \mathbf{40\ km/h}$ *not 45*

average velocity = **0** *back where it started*

⚠ Watch out

- Negative a does not always mean slowing; it means a points in the − direction.
- A flat x–t line means **at rest**; a flat v–t line means **steady velocity**.

4 PROJECTILES



Horizontal velocity stays the same; vertical velocity shrinks to 0 at the top, then grows downward.

The key idea

- Horizontal and vertical motions are **independent**; time links them.
- **Horizontal**: no force (ignoring air), so v_x stays constant.
- **Vertical**: free fall, a = −g.

Galileo showed the path is a parabola (Two New Sciences, 1638).

Off a cliff WORKED

15 m/s horizontally from 20 m up

fall time $t = \sqrt{(\frac{2(20)}{9.8})} = \mathbf{2.02\ s}$ *same as a drop*

$x = (15)(2.02) = \mathbf{30\ m}$

Projectile equations

R and H formulas: level ground, no air.

$$v_x = v_0 \cos\theta,\ v_{0y} = v_0 \sin\theta$$
$$x = v_x t,\ y = v_{0y} t - \tfrac{1}{2}gt^2$$
$$R = \frac{v_0^2 \sin 2\theta}{g},\ H = \frac{(v_0 \sin\theta)^2}{2g}$$

Launched at an angle WORKED

20 m/s at 30° on level ground

$v_x = 17.3$ m/s, $v_{0y} = 10.0$ m/s

time aloft $= \frac{2(10.0)}{9.8} = \mathbf{2.04\ s}$

$R = \frac{20^2 \sin 60°}{9.8} = \mathbf{35.3\ m}$; $H = \frac{10.0^2}{2(9.8)} = \mathbf{5.10\ m}$

⚠ Watch out

- At the top the velocity is v_x, not zero.
- The acceleration is g downward at every point of the flight.

Facts worth knowing

- On level ground **45°** gives the longest range (no air).
- Angles that add to 90° (30° and 60°) give the **same range**.
- A ball dropped and one thrown sideways from the same height land **together**.

Crossing a river WORKED

boat 4.0 m/s straight across; current 3.0 m/s; river 80 m wide

speed over ground $\sqrt{(4.0^2 + 3.0^2)} = \mathbf{5.0\ m/s}$, 37° downstream

time $\frac{80}{4.0} = \mathbf{20\ s}$; drift $(3.0)(20) = \mathbf{60\ m}$

velocities add as vectors

MEMORY TRICK
Ruler across, stone down
Sideways the motion is steady as a ruler; up and down it falls like a dropped stone. Solve each, then join them by t.

5 NEWTON'S LAWS & FREE-BODY DIAGRAMS

Newton's three laws (Principia, 1687)

Law	In words	Example
1st (inertia)	An object keeps its velocity (or rest) unless a net force acts.	riders lurch forward when a car stops
2nd	$F_{net} = ma$: acceleration follows the net force.	twice the force, twice the acceleration
3rd	Forces come in equal, opposite pairs on **two different** objects.	a rocket pushes gas down; gas pushes it up

Force a push or pull, in newtons (N)

Mass m amount of matter; measures inertia (kg)

Weight W gravity's pull: $W = mg$ (N)

Equilibrium $\Sigma F = 0$: at rest or at constant velocity

Drawing a free-body diagram

- Draw the object alone as a box or dot.
- Draw **every** force on it as an arrow from it; label each.
- Choose axes, one along the acceleration.
- Write $\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$; solve.

Forces to look for

Force	Direction
Weight $W = mg$	straight down
Normal N	perpendicular out of a surface
Tension T	along a rope, pulling
Friction f	along a surface, against sliding
Spring $F = kx$	back toward rest length (Hooke, 1678)
Air drag	against the motion

F = ma WORKED

1200 kg car, net force 3600 N

$a = \frac{3600}{1200} = \mathbf{3.0\ m/s^2}$

Riding an elevator WORKED

60 kg person, $a = 2.0\ m/s^2$ up

$N - mg = ma \rightarrow N = 60(9.8 + 2.0) = \mathbf{708\ N}$

feels heavier

speeding up downward: $N = 60(9.8 - 2.0) = \mathbf{468\ N}$

feels lighter

Weight on the Moon WORKED

60 kg astronaut; Moon $g = 1.62\ m/s^2$

Earth: $(60)(9.8) = \mathbf{588\ N}$; Moon: $(60)(1.62) = \mathbf{97\ N}$

same 60 kg of mass

Hanging sign WORKED

20 kg sign, two cables each 30° above level

$2T \sin 30° = mg \rightarrow T = \frac{(20)(9.8)}{2(0.500)} = \mathbf{196\ N}$

each cable

Terminal velocity

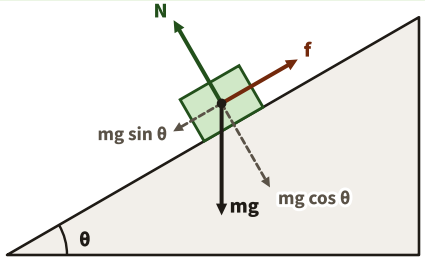
- Air drag grows with speed until it equals the weight; then $\Sigma F = 0$ and the speed stays constant.

MEMORY TRICK

Two objects, two forces

A third-law pair acts on **two different** objects, so the pair never cancels. Forces that cancel act on the **same** object.

6 FRICTION, INCLINES & PULLEYS



Block on an incline: weight splits into mg sin θ down the slope and mg cos θ into it; friction points up the slope against sliding.

Friction

Static friction grows to match the push, up to its limit; usually $\mu_s > \mu_k$. Slipping angle: $\mu_s = \tan\theta$.

$$f_s \le \mu_s N,\ f_k = \mu_k N$$

Incline (angle θ)

$$F_{\parallel} = mg\sin\theta,\ N = mg\cos\theta$$
$$a = g\sin\theta \text{ (no friction)}$$
$$a = g(\sin\theta - \mu_k\cos\theta) \text{ (sliding down)}$$

Typical coefficients (approximate)

Surfaces	μ_s	μ_k
rubber on dry concrete	1.0	0.8
wood on wood	0.25–0.5	0.2
steel on steel (dry)	0.74	0.57
ice on ice	0.1	0.03

Pushing a crate WORKED

20 kg, $\mu_k = 0.30$, push 100 N level

$f = 0.30(20)(9.8) = 58.8\ N$

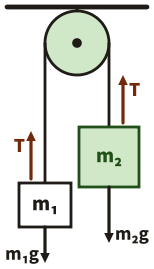
$a = \frac{100 - 58.8}{20} = \mathbf{2.06\ m/s^2}$

Sliding down a ramp WORKED

30°, $\mu_k = 0.20$

$a = 9.8(0.500 - 0.20 \times 0.866) = \mathbf{3.20\ m/s^2}$

mass cancels



Atwood machine: the heavier side falls.

Atwood machine

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2},\ T = \frac{2m_1m_2g}{m_1 + m_2}$$

3.0 kg and 5.0 kg WORKED

$a = \frac{(5.0 - 3.0)(9.8)}{8.0} = \mathbf{2.45\ m/s^2}$

$T = \frac{2(3.0)(5.0)(9.8)}{8.0} = \mathbf{36.8\ N}$

between the two weights

Two blocks pulled together WORKED

30 N pulls 2.0 kg + 4.0 kg on a frictionless floor

$a = \frac{30}{6.0} = \mathbf{5.0\ m/s^2}$ *treat them as one*

rope between them pulls the 2.0 kg: $T = (2.0)(5.0) = \mathbf{10\ N}$

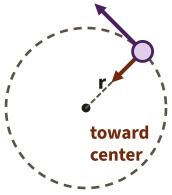
if the 30 N pulls the 4.0 kg

MEMORY TRICK

Friction lies down, normal stands up

Friction runs **along** the surface against sliding; the normal force stands **perpendicular** to it. On a slope, N is less than mg.

7 CIRCULAR MOTION & GRAVITATION



Velocity is tangent; acceleration and net force point to the center.

Uniform circular motion

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$F_c = \frac{mv^2}{r},\ v = \frac{2\pi r}{T}$$

What supplies the force?

- Centripetal force is not a new force: it is the **net inward** force from tension, gravity, friction or the normal force.
- The outward "pull" riders feel is their own inertia.

Earth's g from G WORKED

$$g = \frac{(6.674 \times 10^{-11})(5.97 \times 10^{24})}{(6.371 \times 10^{6})^2}$$

$= \mathbf{9.82\ m/s^2}$ *matches 9.8*

Car on a flat curve WORKED

1000 kg at 20 m/s, r = 50 m

$a_c = \frac{20^2}{50} = 8.0\ m/s^2$; $F =$ **8000 N** *from friction*

needs $\mu_s \geq \frac{8.0}{9.8} =$ **0.82**

Universal gravitation

r is center to center. Cavendish measured G with a torsion balance (1798).

$$F = \frac{Gm_1m_2}{r^2},\ g = \frac{GM}{r^2}$$

MEMORY TRICK

Twice as far, a quarter the pull

Gravity and the electric force fall off as $1/r^2$: double r and the force drops to ¼; triple it, to 1/9.

Kepler's laws of planets

Kepler's law	Says
1st (1609)	Planets move in **ellipses** with the Sun at one focus.
2nd (1609)	A line to the Sun sweeps **equal areas in equal times**: faster when closer.
3rd (1619)	$T^2 \propto a^3$; around the Sun, $T^2 = a^3$ (T in years, a in AU).

Circular orbits

$GM_E = 3.986 \times 10^{14}\ m^3/s^2$.

$$v = \sqrt{\frac{GM}{r}},\ T = 2\pi\sqrt{\frac{r^3}{GM}}$$

Space station orbit WORKED

410 km up: r = 6371 + 410 = 6781 km

$v = \sqrt{\frac{3.986 \times 10^{14}}{6.781 \times 10^{6}}} =$ **7.67 km/s**

$T = \frac{2\pi r}{v} = 5557\ s \approx$ **93 min**

Loop-the-loop WORKED

loop radius r = 10 m

at the top gravity alone supplies F_c: $v \geq \sqrt{(gr)} =$ **9.9 m/s**

frictionless start height $\geq 2.5r =$ **25 m**

Kepler's third law WORKED

Mars: a = 1.52 AU

$T = \sqrt{(1.52^3)} =$ **1.87 years**

Held in place

- "He stretcheth out the north over the empty place, and hangeth the earth upon nothing" (Job 26:7, KJV).

8 WORK, ENERGY & POWER

Work

In joules. θ is between the force and the motion. A force at 90° to the motion does **no work**.

$$W = Fd\cos\theta$$

Pulling a wagon WORKED

50 N at 30° above level, 10 m

$W = (50)(10)\cos 30° =$ **433 J**

Work–energy theorem

$$W_{net} = \Delta KE = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$

Roller coaster drop WORKED

from rest, 20 m down, no friction

$mgh = \frac{1}{2}mv^2 \rightarrow v = \sqrt{(2gh)} = \sqrt{(2(9.8)(20))} =$ **19.8 m/s**

mass cancels

Kinds of energy

Energy	Formula
Kinetic	$KE = \frac{1}{2}mv^2$
Gravitational potential	$PE = mgh$
Spring (elastic)	$PE = \frac{1}{2}kx^2$
Thermal, chemical, electrical	stored or moving in particles and fields

Conservation of energy

- Without friction: $KE_i + PE_i = KE_f + PE_f$.
- With friction, the lost mechanical energy becomes **thermal** energy; the total stays the same.

Joule's falling-weight paddle wheel (1845) showed heat is a form of energy.

Spring launcher WORKED

k = 200 N/m pressed 0.10 m

$PE = \frac{1}{2}(200)(0.10)^2 =$ **1.0 J**

0.050 kg ball: $v = \sqrt{\frac{2(1.0)}{0.050}} =$ **6.3 m/s**

Power

In watts (J/s). James Watt's unit: 1 hp = 746 W. Energy bills use 1 kWh = 3.6×10^6 J.

$$P = \frac{W}{t} = Fv$$

Climbing stairs WORKED

60 kg rises 3.0 m in 4.0 s

$P = \frac{(60)(9.8)(3.0)}{4.0} =$ **441 W** ≈ 0.59 hp

Efficiency $\frac{\text{useful energy out}}{\text{energy in}} \times 100\ \%$; always under 100 %

Kilowatt-hour a 100 W bulb lit for 10 h uses 1 kWh

MEMORY TRICK **Push along the path**

Work needs force **along** the motion. Carrying a tray across a level room, or gravity on a circular orbit, does zero work.

9 MOMENTUM & COLLISIONS

Momentum and impulse

In kg·m/s (= N·s). Momentum is a vector: pick a + direction.

$$p = mv$$

$$J = F_{avg}\Delta t = \Delta p$$

Batting a ball WORKED

0.15 kg ball: +40 m/s in, −50 m/s out; contact 1.5 ms

$\Delta p = 0.15(-50 - 40) =$ **-13.5 kg·m/s**

$F = \frac{13.5}{0.0015} =$ **9000 N**

Stretch the time

- The same Δp over a **longer** time means a **smaller** force: air bags, seat belts, crumple zones, bent knees on landing.

Conservation of momentum

Holds whenever no outside net force acts on the system.

$$m_1v_1 + m_2v_2 = m_1v_1' + m_2v_2'$$

Truck meets car; they stick WORKED

2000 kg at 10 m/s hits 1000 kg at rest

$v' = \frac{(2000)(10)}{3000} =$ **6.67 m/s**

KE: 100 kJ → 66.7 kJ; **⅓ lost**

Kinds of collisions

Collision	Momentum	Kinetic energy
Elastic	conserved	conserved (equal billiard balls swap speeds)
Inelastic	conserved	some lost to heat, sound, bending
Perfectly inelastic	conserved	most lost: objects stick

MEMORY TRICK

Add up before, add up after

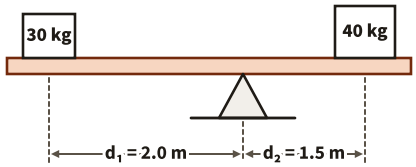
Total momentum **before** equals total **after**. Choose + once, give each velocity its sign, then solve.

10 ROTATION, TORQUE & BALANCE

Angle θ in radians: 1 rev = 2π rad = 360°

Angular velocity ω Δθ ÷ Δt (rad/s); v = ωr

Angular acceleration α Δω ÷ Δt (rad/s²)



Balanced when the torques match: (30)(9.8)(2.0) = (40)(9.8)(1.5).

Equilibrium

An object tips once its center of mass passes outside its base.

$$\Sigma F = 0 \text{ and } \Sigma\tau = 0$$

$$F_1d_1 = F_2d_2 \text{ (lever)}$$

Torque

Lever arm: the perpendicular distance from the pivot to the force's line.

$$\tau = rF\sin\theta = F \times \text{lever arm}$$

Wrench WORKED

40 N, 0.30 m from the bolt

at 90°: $\tau = (0.30)(40) =$ **12 N·m**

at 30°: $\tau = 12 \sin 30° =$ **6.0 N·m**

Each linear idea has a turning twin

Linear	Rotational
x, v, a	θ, ω, α
mass m	rotational inertia I
F = ma	τ = Iα
KE = ½mv²	KE = ½Iω²
p = mv	L = Iω

Rotational inertia

Shape (mass M, radius R)	I
hoop or ring	MR^2
solid disk	$\frac{1}{2}MR^2$
solid sphere	$\frac{2}{5}MR^2$
rod (length L) about center	$\frac{1}{12}ML^2$
rod about one end	$\frac{1}{3}ML^2$

Mass far from the axis makes I larger: a hoop rolls downhill slower than a disk.

Spinning skater WORKED

arms in: I drops from 4.0 to 2.0 kg·m²

$I_1\omega_1 = I_2\omega_2$: 2.0 rev/s → **4.0 rev/s** *L conserved*

MEMORY TRICK **Leverage is length**

The same push gives more torque farther from the pivot: grab a door at the handle, not near the hinge, and push at right angles.

11 OSCILLATIONS & PENDULUMS

Period T time for one full cycle (s)

Frequency f cycles per second, f = 1/T (Hz)

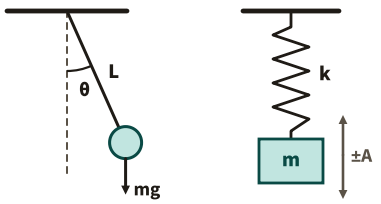
Amplitude A largest distance from rest; restoring force F = −kx

Periods

Pendulum: small swings; mass and amplitude do not matter.

$$\text{pendulum: } T = 2\pi\sqrt{\left(\frac{L}{g}\right)}$$

$$\text{spring: } T = 2\pi\sqrt{\left(\frac{m}{k}\right)}$$



A pendulum and a mass on a spring: both are simple harmonic motion.

MEMORY TRICK

Four times the string, twice the swing

T grows with √L: make a pendulum 4× longer and its period doubles.

Two periods WORKED

1.00 m pendulum: $T = 2\pi\sqrt{\left(\frac{1.00}{9.8}\right)} =$ **2.01 s**

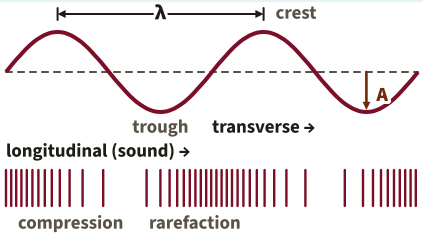
≈ 1 s each way

0.50 kg on k = 200 N/m: $T = 2\pi\sqrt{\left(\frac{0.50}{200}\right)} =$ **0.314 s**; f = **3.18 Hz**

Energy in SHM

- Ends: speed 0; PE and acceleration greatest. Center: speed and KE greatest.
- Huygens built the first pendulum clock (1656).

12 WAVES & SOUND



Transverse waves shake across the travel direction; longitudinal waves (sound) shake along it.

Mechanical wave needs a medium (sound, water, string)
Electromagnetic wave needs none (light, radio)
Wavelength λ crest to crest (m)

Wave speed

The medium sets v; the source sets f.

$$v = f\lambda$$

What waves do

- **Reflect** (echo), **refract** (bend entering a new medium), **diffract** (spread around edges).
- **Interfere**: in step add (constructive); out of step cancel (destructive).
- Two waves crossing back and forth make **standing waves** with nodes (still) and antinodes.

Concert A WORKED

440 Hz in air at 343 m/s

$\lambda = \frac{343}{440} =$ **0.780 m**

Standing waves

String fixed at both ends, n = 1, 2, 3, … Tube closed at one end: $\lambda_1 = 4L$.

$$\lambda_n = \frac{2L}{n},\ f_n = \frac{nv}{2L}$$

Sound in brief

Sound	Fact
speed	≈ (331 + 0.6T) m/s; 343 m/s at 20 °C
pitch	set by frequency
loudness	set by amplitude; +10 dB = 10× the intensity
human range	about 20 to 20,000 Hz

Light is a wave too

Young's two-slit experiment (early 1800s): bright fringes where waves from the slits arrive in step.

$$d \sin\theta = m\lambda,\ \Delta y = \frac{\lambda L}{d}$$

Guitar string WORKED

L = 0.65 m, v = 286 m/s

$f_1 = \frac{286}{2(0.65)} =$ **220 Hz**; $f_2 =$ **440 Hz**

Red laser, two slits WORKED

λ = 650 nm, d = 0.20 mm, screen L = 2.0 m

$\Delta y = \frac{(650 \times 10^{-9})(2.0)}{0.20 \times 10^{-3}} =$ **6.5 mm** *fringe spacing*

Doppler effect

Upper signs when moving **toward** each other. Doppler (1842); Buys Ballot tested it with horn players on a train (1845).

$$f' = f\,\frac{v \pm v_o}{v \mp v_s}$$

Passing horn WORKED

500 Hz horn, 25 m/s, still listener

coming: $500 \times \frac{343}{343 - 25} =$ **539 Hz**

going: $500 \times \frac{343}{343 + 25} =$ **466 Hz**

MEMORY TRICK

Coming close, pitch goes up

Waves bunch up ahead of a moving source (higher f) and spread out behind it (lower f).

13 FLUIDS

Density and pressure

Pressure in pascals (N/m²) acts in every direction and grows with depth.

$$\rho = \frac{m}{V},\ P = \frac{F}{A}$$

$$P = P_0 + \rho g h$$

Ten meters down WORKED

P = 101.3 kPa + (1000)(9.8)(10) Pa

= 101.3 + 98 = **199 kPa** ≈ 2 atm

Pascal's principle

- Pressure on a confined fluid passes **undiminished** to every part: $\frac{F_1}{A_1} = \frac{F_2}{A_2}$.

At Pascal's request, Périer carried a barometer up the Puy de Dôme (1648); the mercury fell with height.

MEMORY TRICK

Ten meters, one atmosphere

Each 10 m of water adds about one atmosphere of pressure.

Hydraulic lift WORKED

200 N on a 0.010 m² piston; big piston 0.50 m²

$F_2 = 200 \times \frac{0.50}{0.010} =$ **10,000 N**

small force, long stroke

Archimedes' principle

Buoyant force = weight of the fluid pushed aside. Floats if $\rho_{object} < \rho_{fluid}$; fraction under = $\rho_{object} \div \rho_{fluid}$.

$$F_b = \rho_{fluid} V_{sub} g$$

Rock under water WORKED

V = 0.0020 m³, ρ = 2650 kg/m³

F_b = (1000)(0.0020)(9.8) = **19.6 N**

weight 51.9 N; in water it weighs **32.3 N**

Iceberg WORKED

ice 917 kg/m³ in seawater 1025 kg/m³

$\frac{917}{1025} =$ **89 %** under the surface

Moving fluids

Continuity, and Bernoulli's equation (Hydrodynamica, 1738): along a flow, faster fluid has lower pressure.

$$A_1 v_1 = A_2 v_2$$

$$P + \tfrac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Leaking tank WORKED

hole 0.80 m below the water surface

$v = \sqrt{(2gh)} = \sqrt{(2(9.8)(0.80))} =$ **4.0 m/s** *Torricelli*

Thumb on the hose WORKED

cut the opening to ¼ of its area

$v_2 = v_1 \times \frac{A_1}{A_2} =$ **4×** faster

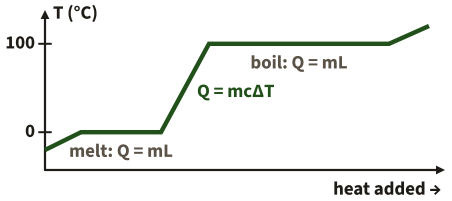
14 HEAT & THERMODYNAMICS

Temperature average kinetic energy of the particles

Heat Q energy that flows because of a temperature difference

Specific heat c J to warm 1 kg by 1 °C

Latent heat L J per kg to melt or boil, with no temperature change



Heating water: temperature climbs, then holds flat while ice melts and while water boils.

Laws of thermodynamics

Law	Says
Zeroth	Two objects in balance with a third are in balance with each other.
First	ΔU = Q − W: energy is conserved (500 J in, 200 J of work out: ΔU = +300 J).
Second	Heat flows hot → cold on its own; no engine turns all heat into work; entropy grows.
Third	Absolute zero cannot be reached.

Scales

$$T_K = T_C + 273.15$$

$$T_F = \frac{9}{5}T_C + 32$$

EX 25 °C = **298 K** = **77 °F**. Kelvin set the absolute scale in 1848.

Heat equations

$$Q = mc\Delta T,\ Q = mL$$

Boiling water for pasta WORKED

2.0 kg water, 20 → 100 °C

Q = (2.0)(4186)(80) = 669,760 J ≈ **670 kJ**

Melting ice WORKED

0.50 kg at 0 °C

Q = (0.50)(334 kJ/kg) = **167 kJ**

temperature holds at 0 °C

Mixing water WORKED

0.20 kg at 80 °C + 0.30 kg at 20 °C

heat lost = heat gained

$$T = \frac{(0.20)(80) + (0.30)(20)}{0.50} = \mathbf{44\ °C}$$

Three ways heat moves

Transfer	How	Example
Conduction	contact, particle to particle	hot pan handle
Convection	warm fluid rises and circulates	boiling pot, sea breeze
Radiation	electromagnetic waves; no medium	sunlight, campfire

Heat engines

The best possible engine: Carnot (1824).

$$e = \frac{W}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

$$e_{max} = 1 - \frac{T_C}{T_H}\ \text{(kelvins)}$$

Best possible engine WORKED

T_H = 500 K, T_C = 300 K

$e_{max} = 1 - \frac{300}{500} = \mathbf{40\ \%}$

MEMORY TRICK **Ratios need kelvins**

Use kelvins whenever temperatures are multiplied or divided (efficiency, gas laws). A change of 1 °C equals a change of 1 K.

15 ELECTRICITY: CHARGE & CIRCUITS

Electric charge

- Two kinds: + and −. Like charges **repel**; opposites **attract**.
- Charge is **conserved** and comes in whole multiples of e = 1.602 × 10^{-19} C.
- Objects charge by friction, contact or induction; **conductors** let charge move, **insulators** do not.

Coulomb's law

k = 8.99 × 10^9 N·m²/C². Coulomb measured it with a torsion balance (1785).

$$F = k\frac{q_1 q_2}{r^2}$$

Current I charge per second, q ÷ t (amperes)

Voltage V energy per charge (J/C = volts)

Resistance R opposition to current (ohms)

Conventional current flows + to −; electrons drift the other way

Two charged spheres WORKED

+2.0 μC and +2.0 μC, 0.30 m apart

$$F = \frac{(8.99\times10^9)(2.0\times10^{-6})^2}{0.30^2} = \mathbf{0.40\ N}$$ *repel*

Ohm's law and power

Ohm (1827). Resistance of a wire: R = ρL ÷ A.

$$V = IR$$

$$P = IV = I^2R = \frac{V^2}{R}$$

Series circuit WORKED

2 Ω + 4 Ω + 6 Ω on 12 V

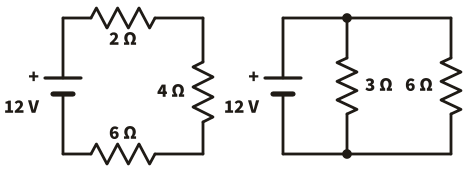
R = 12 Ω, $I = \frac{12}{12} = \mathbf{1.0\ A}$

drops: 2 V, 4 V, 6 V (sum 12 V)

MEMORY TRICK

Series shares volts, parallel shares amps

In series the voltage divides and current is the same; in parallel the current divides and voltage is the same.



series parallel

The two ways to connect resistors (values used in the examples below).

Series vs parallel

	Series	Parallel
Current	same everywhere	splits among branches
Voltage	shared (drops add up)	same across each branch
Total R	$R_1 + R_2 + \ldots$	$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots$
One bulb out	all go out	others stay lit (house wiring)

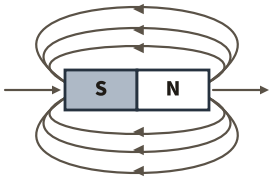
Parallel circuit WORKED

3 Ω and 6 Ω on 12 V

$R = \frac{(3)(6)}{3+6} = \mathbf{2.0\ \Omega}$; I = **6.0 A**

branches: 4.0 A and 2.0 A

16 MAGNETISM



Field lines leave the north pole and loop around into the south pole.

MEMORY TRICK **Right-hand grip**
Thumb along the current, fingers curl the way B circles the wire. (Traditional.)

Magnetic force

On a current-carrying wire, and on a moving charge. B in tesla (T); the force is perpendicular to both B and the motion.

$$F = BIL \sin\theta$$

$$F = qvB \sin\theta$$

Magnets

- Every magnet has a **north** and a **south** pole; like poles repel, unlike attract. Cutting a magnet makes two smaller magnets.
- Earth is a giant magnet (Gilbert, De Magnete, 1600); a compass's north end points toward geographic north.

Currents make magnetism

- Oersted saw a current turn a compass needle (1820).
- Straight wire: $B = \frac{\mu_0 I}{2\pi r}$; coil (solenoid): $B = \mu_0 n I$.

Induction

- A **changing** magnetic field through a coil makes a voltage (Faraday, 1831): generators turn motion into electricity.
- Transformer: $\frac{V_s}{V_p} = \frac{N_s}{N_p}$.

Force on a wire WORKED

0.50 T, 2.0 A, 0.30 m, at 90°

$F = (0.50)(2.0)(0.30) = \mathbf{0.30\ N}$

Field near a wire WORKED

10 A, 2.0 cm away

$B = \frac{(4\pi \times 10^{-7})(10)}{2\pi(0.020)} = \mathbf{1.0 \times 10^{-4}\ T}$

about twice Earth's field

Step-down transformer WORKED

120 V, 500 turns in, 50 turns out

$V_s = 120 \times \frac{50}{500} = \mathbf{12\ V}$

Go deeper

- Loops and solenoids, electromagnets, the motor effect, DC motors, and Faraday's and Lenz's laws: see the School Launch **Magnetism and Induction** sheet (Physics, Unit 10).

17 LIGHT & OPTICS

Light an electromagnetic wave; $c = 3.00 \times 10^8$ m/s in vacuum

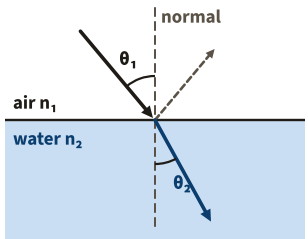
Spectrum radio, microwave, infrared, **visible** (700–400 nm), ultraviolet, X-ray, gamma: rising frequency

Index n $n = c \div v$: how much a material slows light

MEMORY TRICK **ROY G BIV**
Red, orange, yellow, green, blue, indigo, violet: longest to shortest visible wavelength. (Traditional.)

Reflection

- Angle of reflection = angle of incidence, both from the **normal**.
- A flat mirror makes an upright, same-size, **virtual** image as far behind as the object is in front.



Entering a slower medium, light bends toward the normal.

Refraction (Snell's law)

Snell (1621); Ibn Sahl found it about 984. Past the critical angle θ_c, light reflects totally (optical fibers).

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

Air into water WORKED

30° from the normal, n = 1.33

$\sin\theta_2 = \frac{\sin 30°}{1.33} \rightarrow \theta_2 = \mathbf{22.1°}$

speed in water: $\frac{3.00 \times 10^8}{1.33} = \mathbf{2.26 \times 10^8\ m/s}$

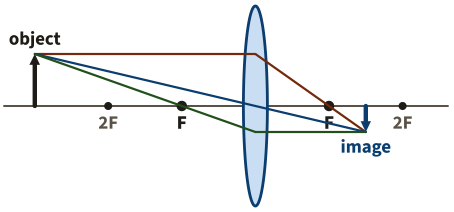
Critical angle WORKED

glass (1.50) to air

$\theta_c = \sin^{-1}(\frac{1.00}{1.50}) = \mathbf{41.8°}$

Where the image lands

Object (converging lens)	Image
beyond 2F	real, inverted, smaller (camera)
between F and 2F	real, inverted, larger (projector)
inside F	virtual, upright, larger (magnifier)
any (diverging lens)	virtual, upright, smaller



Converging lens, f = 10 cm, object at 30 cm: three rays meet at a real, inverted image.

Thin lens and mirror

f + for converging lenses and concave mirrors; d_i + real, − virtual; m − inverted.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$m = -\frac{d_i}{d_o} = \frac{h_i}{h_o}$$

Camera-style image WORKED

f = 10 cm, d_o = 30 cm: $\frac{1}{d_i} = \frac{1}{10} - \frac{1}{30} \rightarrow d_i = \mathbf{15}$ **cm**

$m = -\frac{15}{30} = \mathbf{-0.50}$: real, inverted, half size

MEMORY TRICK **Real images flip**
A **real** image is inverted and can fall on a screen; a **virtual** image is upright and only seen by looking through the lens or into the mirror.