PHYSICS · UNITS 9–11 · QUICK REFERENCE

PHYSICS 2

Grades 11–12+ · 8 pages

Electric charge, fields and potential; capacitors; circuits with Ohm's law, Kirchhoff's rules and RC timing; magnetism and induction; waves and sound; mirrors, lenses and wave optics; photons, atoms, nuclear decay and half-life.

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1 CONSTANTS & KEY EQUATIONS

values used on every page

Physical constants

Constant	Value
k (Coulomb)	8.99×10^9 N·m²/C²
ε_0 (free space)	8.85×10^{-12} C²/(N·m²)
e (elementary charge)	1.602×10^{-19} C
m_e; m_p	9.11×10^{-31} kg; 1.67×10^{-27} kg
μ_0 (magnetic)	$4\pi \times 10^{-7}$ T·m/A
c (light)	3.00×10^8 m/s
h (Planck)	6.626×10^{-34} J·s
hc	1240 eV·nm
1 eV	1.602×10^{-19} J
1 u (mass unit)	931.5 MeV of energy
I_0 (faintest sound)	1.0×10^{-12} W/m²

Key equations

Topic	Equations
Charge	$F = k\frac{q_1 q_2}{r^2}$; $E = \frac{F}{q} = k\frac{Q}{r^2}$
Potential	$V = k\frac{Q}{r}$; $\Delta U = q\Delta V$; $E = \frac{\Delta V}{d}$
Capacitors	$C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$; $U = \frac{1}{2}CV^2$
Circuits	$V = IR$; $P = IV$; $R = \frac{\rho L}{A}$; $\tau = RC$
Magnetism	$F = qvB \sin\theta$; $r = \frac{mv}{qB}$; $B = \frac{\mu_0 I}{2\pi r}$; $B = \mu_0 nI$
Induction	$\Phi = BA\cos\theta$; $\mathscr{E} = -N\frac{\Delta\Phi}{\Delta t}$; $\mathscr{E} = BLv$
Waves	$v = f\lambda$; $v = \sqrt{(\frac{F_T}{\mu})}$; $\beta = 10 \log \frac{I}{I_0}$
Optics	$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$; $m = -\frac{d_i}{d_o}$; $d \sin\theta = m\lambda$
Modern	$E = hf = \frac{hc}{\lambda}$; $KE = hf - \phi$; $\lambda = \frac{h}{p}$; $E = mc^2$

Units of electricity and magnetism

Quantity	Unit	In base units
charge q	coulomb (C)	A·s
potential V	volt (V)	J/C
resistance R	ohm (Ω)	V/A
capacitance C	farad (F)	C/V
magnetic field B	tesla (T)	N/(A·m)
flux Φ	weber (Wb)	T·m²
energy	joule or eV	N·m

Reading the symbols

- $\mathscr{E}$: emf, the voltage a battery or coil supplies. **Φ**: magnetic flux. **μ**: mass per meter of a string. **ϕ**: work function.
- Prefixes: μ = 10^{-6}, n = 10^{-9}, p = 10^{-12}; k = 10^3, M = 10^6.

Vector or scalar?

- **Vectors** (add with directions): F, E, B, v.
- **Scalars** (add with signs): q, V, U, I.

Powers of ten on a calculator

- Enter 8.99×10^9 as 8.99 **EE** 9 (or EXP), not 8.99 × 10 ^ 9 typed by hand.
- Wrap fraction bottoms in parentheses: $\frac{kq}{r^2}$ is kq ÷ (r²).

MEMORY TRICK **Prefix first, then plug**

Turn μC, nF, mm and kΩ into base units before you multiply; most wrong answers are off by a power of ten.

2 CHARGE & COULOMB'S LAW

forces between charges

Electric charge

- Two kinds, + and −; like charges **repel**, unlike **attract**.
- Charge is **conserved**: rubbing moves electrons, it never makes charge.
- Charge comes in whole multiples of **e**.

Strong for its size

- Between a proton and an electron, the electric pull is about **2×10^{39}** times their gravity.
- Two 1 μC charges 30 cm apart push with about the weight of 10 g.

MEMORY TRICK **Sizes in, signs out**

Put only sizes into Coulomb's law. Then draw the arrow: like charges push apart, unlike pull together.

Charging objects

- **Friction**: electrons move from one material to another.
- **Contact**: a charged object shares charge with a conductor it touches.
- **Induction**: bring a charge near, ground the far side, remove the ground; the object keeps the **opposite** charge.

Forces add as vectors WORKED

+3.0 μC at x = 0, −5.0 μC at x = 0.40 m; +2.0 μC at x = 0.20 m

from +3.0: $\frac{(8.99 \times 10^9)(3.0 \times 10^{-6})(2.0 \times 10^{-6})}{0.20^2}$ = 1.35 N, pushed toward +x *repel*

from −5.0: 2.25 N, pulled toward +x *attract*

net: 1.35 + 2.25 = **3.60 N** toward the −5.0 μC *same direction, so add*

Coulomb's law

Coulomb measured it with a torsion balance (1785). Use sizes in the formula; get the direction from like or unlike.

$$F = k\frac{q_1 q_2}{r^2}$$

Counting electrons WORKED

a balloon picks up −8.0 nC

$n = \frac{8.0 \times 10^{-9}\ \text{C}}{1.602 \times 10^{-19}\ \text{C}}$ = **4.99×10^{10}** extra electrons

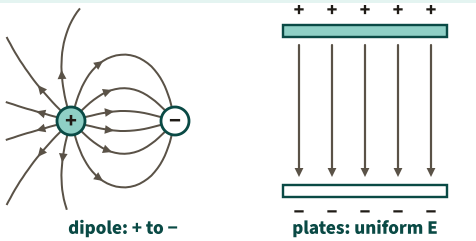
Sharing charge WORKED

identical metal spheres: +6.0 nC and −2.0 nC touch, then part

total stays +4.0 nC, split evenly: **+2.0 nC** each *charge is conserved*

3 ELECTRIC FIELDS

force per unit charge



Left: field lines leave + and end on −. Right: between parallel plates the field is uniform.

Electric field

In N/C, the same as V/m. E points the way a + charge is pushed.

$$E = \frac{F}{q} = k\frac{Q}{r^2}$$

$$E = \frac{V}{d} \text{ (parallel plates)}$$

Field lines

- Start on **+**, end on **−**; they never cross.
- Closer lines mean a **stronger** field.
- A conductor at rest has **E = 0** inside and its extra charge on the outside (a Faraday cage).

⚠ Watch out

- Electrons accelerate **against** the field arrows.
- Fields from several charges add as **vectors**; draw each before adding.

MEMORY TRICK

Follow the plus charge

To find a field's direction, imagine a tiny + charge there and ask which way it would be pushed.

Field of a point charge WORKED

+4.0 nC; point 0.30 m away

$E = \frac{(8.99 \times 10^9)(4.0 \times 10^{-9})}{0.30^2} =$ **400 N/C**, pointing away

an electron there: $F = eE =$ **6.41×10^{-17} N**, toward the charge

opposite E

Floating an oil drop WORKED

plates 1.0 cm apart at 245 V; drop mass 8.0×10^{-15} kg hangs still

$E = \frac{245}{0.010} = 2.45 \times 10^4$ N/C

$qE = mg \rightarrow q = \frac{(8.0 \times 10^{-15})(9.8)}{2.45 \times 10^4} =$ **3.2×10^{-18} C**

$= \frac{3.2 \times 10^{-18}}{1.602 \times 10^{-19}} =$ **20** electron charges

Millikan's idea (1909)

4 POTENTIAL & ENERGY

volts are joules per coulomb

Potential V	energy per charge at a point, $\frac{U}{q}$ (volts = J/C)
Potential difference ΔV	what a voltmeter reads; work per charge to move between two points
Electron-volt (eV)	energy one electron gains crossing 1 V = 1.602×10^{-19} J
Equipotential	surface of equal V; it crosses field lines at **90°**, so moving along it takes no work

Uniform field WORKED

9.0 V across plates 3.0 mm apart

$E = \frac{9.0}{3.0 \times 10^{-3}} =$ **3.0×10^3 V/m** *V/m = N/C*

Potential and energy

V is a **scalar**: add the potentials of several charges with their signs.

$$V = k\frac{Q}{r} \text{ (point charge)}$$

$$\Delta U = q\Delta V,\ E = \frac{\Delta V}{d}$$

Electron gun WORKED

an electron speeds up from rest across 500 V

KE = 500 eV = **8.01×10^{-17} J** *qΔV*

$v = \sqrt{(\frac{2KE}{m_e})} =$ **1.33×10^7 m/s** *about 4% of c*

Potential of two charges WORKED

+6.0 nC at 0.20 m and −3.0 nC at 0.30 m from point P

$V = k(\frac{6.0 \times 10^{-9}}{0.20} - \frac{3.0 \times 10^{-9}}{0.30}) = (8.99 \times 10^9)(3.0 \times 10^{-8} - 1.0 \times 10^{-8})$

= **180 V** *scalars, signs kept*

bring +2.0 μC from far away: $W = qV =$ **3.6×10^{-4} J**

MEMORY TRICK

Charges roll downhill

A + charge set free moves from high V to low V, like a ball rolling downhill; an electron rolls "uphill" to higher V.

5 CAPACITORS

storing charge and energy

Capacitance

In farads (F). Two plates hold +Q and −Q. A dielectric (κ > 1) between the plates raises C.

$$C = \frac{Q}{V},\ C = \kappa\frac{\varepsilon_0 A}{d}$$

$$U = \tfrac{1}{2}CV^2 = \tfrac{1}{2}QV$$

Capacitors at work

- Touchscreens sense a finger by its capacitance; flashes and defibrillators dump stored charge fast.

Parallel plates WORKED

$A = 0.040$ m², $d = 1.0$ mm, air, on 9.0 V

$C = \frac{(8.85 \times 10^{-12})(0.040)}{1.0 \times 10^{-3}} =$ **354 pF**

$Q = CV =$ **3.19 nC**; $U = \tfrac{1}{2}CV^2 =$ **14.3 nJ**

fill the gap with κ = 2.5: C = **885 pF**

MEMORY TRICK

Capacitors flip the rules

Capacitors combine the opposite way from resistors: in **parallel** they simply add; in **series** use reciprocals.

Combining capacitors

	Parallel	Series
Same on each	V	Q
Total	$C_1 + C_2 + \ldots$	$\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$
6.0 μF with 3.0 μF	**9.0 μF**	**2.0 μF**

Camera flash WORKED

150 μF charged to 300 V

$U = \tfrac{1}{2}(150 \times 10^{-6})(300^2) =$ **6.75 J**

dumped in a millisecond

6 CURRENT, RESISTANCE & POWER

Ohm's law at work

Current I — $\frac{\Delta q}{\Delta t}$ in amperes; 1 A = 1 C/s

Resistance R — $\frac{V}{I}$ in ohms; $R = \frac{\rho L}{A}$

Resistivity ρ — a property of the material (Ω·m); in metals it rises with temperature

emf $\mathscr{E}$ — energy per charge a source supplies

Internal resistance r — resistance inside the source: $V_{terminal} = \mathscr{E} - Ir$

MEMORY TRICK

Watts are volts times amps

P = IV. Swap in V = IR to get I^2R or V^2/R, then use the form with the two numbers you know.

Ohm's law and power

Ohm (1827). Energy = Pt; utilities bill in kWh.

$$V = IR$$

$$P = IV = I^2R = \frac{V^2}{R}$$

Resistance of a wire WORKED

copper, $\rho = 1.68 \times 10^{-8}$ Ω·m; 20 m long, 1.0 mm across

$R = \frac{(1.68 \times 10^{-8})(20)}{\pi(0.50 \times 10^{-3})^2} = \mathbf{0.428\ \Omega}$ $A = \pi r^2$

Space heater WORKED

1500 W on 120 V

$I = \frac{1500}{120} = \mathbf{12.5\ A}$; $R = \frac{120}{12.5} = \mathbf{9.6\ \Omega}$

3.0 h a day for 30 days: 1.5 kW × 90 h = 135 kWh

at $0.15 per kWh: **$20.25**

A real battery WORKED

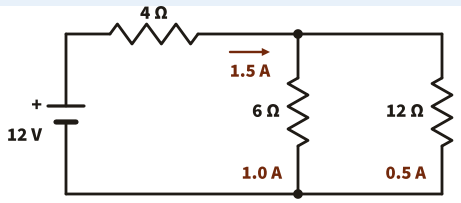
$\mathscr{E}$ = 9.0 V, r = 0.50 Ω, load 4.0 Ω

$I = \frac{9.0}{4.0 + 0.50} = \mathbf{2.0\ A}$

terminal voltage = (2.0)(4.0) = **8.0 V**; 1.0 V is lost inside

7 SERIES-PARALLEL & KIRCHHOFF

solving any network



4 Ω in series with 6 Ω and 12 Ω in parallel, on 12 V.

Combination circuit WORKED

parallel part: $\frac{(6)(12)}{6 + 12} = 4\ \Omega$; total 4 + 4 = 8 Ω

innermost first

$I = \frac{12}{8} = \mathbf{1.5\ A}$

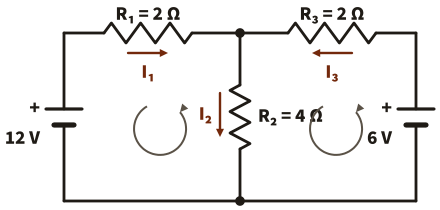
4 Ω drops 6.0 V, so 6.0 V is left across the pair

branches: $\frac{6.0}{6} = \mathbf{1.0\ A}$, $\frac{6.0}{12} = \mathbf{0.5\ A}$ *sum 1.5 A*

battery power: (12)(1.5) = **18 W**

Kirchhoff's rules (1845)

- **Junction**: current in = current out (charge is conserved).
- **Loop**: the voltage changes around any closed loop add to **zero** (energy is conserved).
- Across a resistor in the current's direction: −IR. Across a battery from − to +: $+\mathscr{E}$.



Two batteries, three branches: guess current directions, then let the algebra decide.

Two-loop circuit WORKED

junction: $I_1 + I_3 = I_2$

left loop: $12 - 2I_1 - 4I_2 = 0$

right loop: $6 - 2I_3 - 4I_2 = 0$

$I_1 = \mathbf{2.4\ A}$, $I_2 = \mathbf{1.8\ A}$, $I_3 = \mathbf{-0.6\ A}$ *solve together*

the minus sign: I_3 really flows down, charging the 6 V battery

Meters

- **Ammeter**: in series, nearly 0 Ω.
- **Voltmeter**: in parallel, very high resistance.

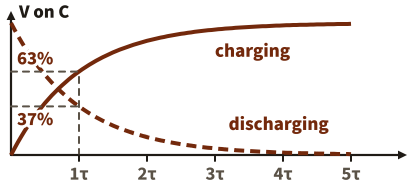
MEMORY TRICK

Junctions count, loops balance

At a junction count the amps in and out; around a loop the volts gained and lost balance to zero.

8 RC CIRCUITS

charging takes time



Charging rises toward the battery voltage; discharging falls toward zero. Marks at one time constant.

Discharging to half WORKED

same 10 s time constant

$\frac{1}{2} = e^{-t/10} \rightarrow t = 10 \ln 2 = \mathbf{6.93\ s}$ *ln 2 = 0.693*

RC circuits

τ in seconds when R is in ohms and C in farads. The current falls as $I_0e^{-t/RC}$ in both cases.

$$\tau = RC$$

$$\text{charging: } V = V_0(1 - e^{-t/RC})$$

$$\text{discharging: } V = V_0e^{-t/RC}$$

MEMORY TRICK

Sixty-three up, thirty-seven down

After one time constant a charging capacitor is 63% full; a discharging one keeps 37%.

Charging a capacitor WORKED

50 kΩ and 200 μF on 9.0 V, starting empty

$\tau = (50 \times 10^3)(200 \times 10^{-6}) = \mathbf{10\ s}$

at 10 s: $V = 9.0(1 - e^{-1}) = \mathbf{5.69\ V}$ *63%*

at 50 s: **8.94 V** (99.3%), "full" *5τ*

current at the start: $\frac{9.0}{50 \times 10^3} = \mathbf{0.18\ mA}$

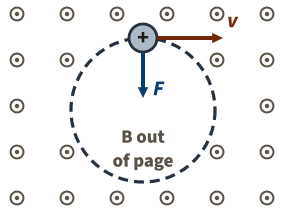
C acts like a wire

Two moments

- **Just switched on**: an empty capacitor passes current like a plain wire.
- **Long after**: a full capacitor blocks steady current like a gap.
- Uses: camera flashes, blinking lights, timers, filters.

9 MAGNETIC FORCES & FIELDS

moving charges and currents



A + charge moving right through B out of the page (dots) is pushed down and circles clockwise.

Force on moving charge

F is perpendicular to both v and B, so it bends the path but does **no work**: the speed stays the same.

$$F = qvB \sin\theta$$

$$r = \frac{mv}{qB}$$

MEMORY TRICK

Right-hand rule (Traditional)

Fingers along v, curl them toward B; your thumb gives F on a + charge. Flip it for an electron.

Fields and forces of currents

Source	Field B
long straight wire	$\frac{\mu_0 I}{2\pi r}$; circles the wire
solenoid (n turns per m)	$\mu_0 nI$ inside, nearly uniform
wire in a field	$F = BIL \sin\theta$

Solenoid WORKED

2000 turns on 0.40 m; 1.5 A

$n = \frac{2000}{0.40} = 5000$ per m

$B = (4\pi \times 10^{-7})(5000)(1.5) =$ **9.42 mT**

Proton in a magnet WORKED

$v = 2.0 \times 10^6$ m/s, B = 0.50 T, at 90°

$F = (1.602 \times 10^{-19})(2.0 \times 10^6)(0.50) =$ **1.60×10^{-13} N**

$r = \frac{(1.67 \times 10^{-27})(2.0 \times 10^6)}{(1.602 \times 10^{-19})(0.50)} =$ **4.17 cm**

mass spectrometer idea

one lap: $T = \frac{2\pi m}{qB} =$ **1.31×10^{-7} s**

same for any speed

Motor coil WORKED

50 turns, 0.010 m^2, 2.0 A, in 0.30 T, plane along B

$\tau = NIAB \sin\theta = (50)(2.0)(0.010)(0.30) =$ **0.30 N·m**

θ from the coil's axis to B

the commutator flips the current each half turn

DC motor

Parallel wires WORKED

15 A and 10 A, same direction, 5.0 cm apart

$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi d} =$ **6.0×10^{-4} N/m**, attracting

opposite currents repel

Field directions

- **Wire**: grip it with your right thumb along I; fingers curl the way B circles.
- **Solenoid**: curl right fingers with the current; the thumb points to the **north** end.

Velocity selector

- Crossed E and B fields let through only charges with $\mathbf{v = \frac{E}{B}}$: 3.0×10^4 V/m with 0.10 T passes **3.0×10^5 m/s**.
- Thomson used crossed fields on cathode rays (1897).

10 INDUCTION & AC

changing flux makes voltage

Faraday's law

Flux in webers (T·m^2). Faraday found induction in 1831. The minus sign is Lenz's law.

$$\Phi = BA\cos\theta$$

$$\mathcal{E} = -N\frac{\Delta\Phi}{\Delta t},\ \mathcal{E} = BLv$$

Lenz's law

- The induced current makes a field that **opposes the change** in flux.
- Push a magnet's N pole into a coil: the coil's near end becomes N and pushes back.

Sliding rod WORKED

0.50 m rod at 4.0 m/s on rails, B = 0.80 T; circuit 2.0 Ω

$\mathcal{E} = BLv =$ **1.6 V**; I = **0.80 A**

pushing force $F = BIL =$ **0.32 N** *against the motion*

power: $Fv = I^2R =$ **1.28 W** *energy is conserved*

Coil in a rising field WORKED

200 turns, 0.010 m^2; B rises 0 → 0.60 T in 0.20 s

$\mathcal{E} = (200)\frac{(0.60)(0.010)}{0.20} =$ **6.0 V**

Generator WORKED

100 turns, 0.020 m^2, 0.40 T, 60 Hz

$\omega = 2\pi(60) = 377$ rad/s

$\mathcal{E}_{max} = (100)(0.40)(0.020)(377) =$ **302 V**

Why power lines use high voltage WORKED

send 500 kW through 4.0 Ω of line

at 10 kV: I = 50 A, loss $I^2R =$ **10 kW** (2.0%)

at 100 kV: I = 5.0 A, loss = **100 W**

transformers step up, then down

MEMORY TRICK

Change makes current

A steady field induces nothing. Move, turn, grow or shrink the flux, and a current appears that resists the change.

Generators and AC

- A coil turning in B gives $\mathcal{E}_{max} =$ **NBAω**, with $\omega = 2\pi f$.
- $V_{rms} = \frac{V_{peak}}{\sqrt{2}}$: a U.S. outlet is 120 V rms, about **170 V** peak.

Transformers

- $\frac{V_s}{V_p} = \frac{N_s}{N_p} = \frac{I_p}{I_s}$ (ideal): voltage up means current down.
- They work only with **changing** current (AC).
- More on motors and generators: the School Launch **Magnetism and Induction** sheet (Lesson 10.4).

Induction at work

- Generators, induction cooktops, wireless chargers, metal detectors and guitar pickups.
- **Eddy currents** in moving metal oppose the motion: magnetic brakes on trains and roller coasters.

11 WAVES & STANDING WAVES

strings, pipes and beats

Wave basics

- **Transverse**: the medium moves across the wave (string, light). **Longitudinal**: along it (sound).
- The **medium** sets the speed; the **source** sets the frequency.
- Entering a new medium, f stays; v and λ change.

Waves on a string

F_T = tension (N); μ = mass per length (kg/m). Tighter or lighter strings carry faster waves.

$$v = f\lambda,\ v = \sqrt{\left(\frac{F_T}{\mu}\right)}$$

Tension and speed

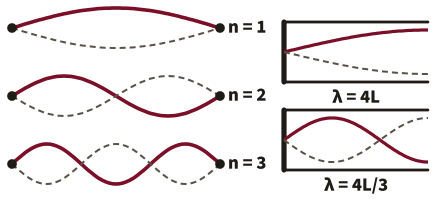
- v grows as $\sqrt{F_T}$: **4 times** the tension makes waves **2 times** as fast, raising every harmonic an octave.

Radio wavelength WORKED

FM station at 101.5 MHz

$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8}{101.5 \times 10^6} =$ **2.96 m**

light speed, not sound



string: λ = 2L/n **closed tube**

Standing waves: a string fixed at both ends (n = 1, 2, 3) and a tube closed at one end (λ = 4L, 4L/3).

Standing-wave patterns

Holder	Allowed λ	Frequencies
string, or tube open at both ends	$\frac{2L}{n}$	$f_n = n\frac{v}{2L}$, n = 1, 2, 3 …
tube closed at one end	$\frac{4L}{n}$	$f_n = n\frac{v}{4L}$, n = 1, 3, 5 …

Tuning a string WORKED

0.80 m string, mass 4.0 g, tension 80 N

$\mu = \frac{0.0040}{0.80} = 0.0050$ kg/m; $v = \sqrt{\left(\frac{80}{0.0050}\right)} =$ **126 m/s**

$f_1 = \frac{126}{2(0.80)} =$ **79.1 Hz**; $f_3 =$ **237 Hz**

Bottle-length tubes WORKED

a 0.25 m tube, closed at one end

$f_1 = \frac{343}{4(0.25)} =$ **343 Hz**; next: **1029 Hz**

odd harmonics only

open at both ends: $f_1 =$ **686 Hz**

Interference and beats

- Path difference of **mλ**: waves add. Of **(m + ½)λ**: they cancel.
- **Beats**: $f_{beat} = |f_1 - f_2|$. 330 Hz with 333 Hz throbs **3 times** a second.
- Chladni made 2-D standing waves visible with sand on a bowed plate (1787).

Resonance

- Driving a system at one of its natural frequencies builds large amplitudes: a pushed swing, a singer's glass, an instrument's body.

MEMORY TRICK **Closed ends are nodes**

A fixed string end or a closed tube end cannot move: always a node. An open tube end swings freely: an antinode.

12 SOUND

loudness, Doppler and booms

Intensity and decibels

I in W/m²; $I_0 = 1.0 \times 10^{-12}$ W/m². Each **+10 dB** is 10 times the intensity.

$$I = \frac{P}{4\pi r^2}$$

$$\beta = 10 \log \frac{I}{I_0}$$

Outdoor speaker WORKED

2.0 W of sound, spreading evenly; listener 4.0 m away

$I = \frac{2.0}{4\pi(4.0)^2} = 9.95 \times 10^{-3}$ W/m²

$\beta = 10 \log \frac{9.95 \times 10^{-3}}{10^{-12}} =$ **100 dB**

at 8.0 m: ¼ the intensity, **94 dB** *about 6 dB less*

MEMORY TRICK

Closing in, pitch climbs

When source and listener get closer, every sign choice should raise f′; when they part, lower it. Check your answer that way.

Decibel ladder (typical sounds)

Level	Intensity	Like
0 dB	10^{-12} W/m²	faintest sound heard
30 dB	10^{-9} W/m²	a whisper
60 dB	10^{-6} W/m²	conversation
90 dB	10^{-3} W/m²	a lawn mower up close
120 dB	1 W/m²	threshold of pain

Two equal sources together: +3 dB, since 10 log 2 ≈ 3.

Shock waves

- Faster than sound, wave crests pile into a cone: a **sonic boom**. Mach number = v ÷ v_{sound}.
- A jet at 515 m/s: Mach **1.50**; cone half-angle $\sin^{-1}\left(\frac{1}{1.50}\right) =$ **41.8°**.

MEMORY TRICK

Ten times means ten decibels

Every factor of 10 in intensity adds 10 dB; doubling your distance from a source drops about 6 dB.

Doppler effect

- $f' = f\frac{v \pm v_o}{v \mp v_s}$, v = 343 m/s in air.
- Upper signs when source and observer move **toward** each other.

Siren and car WORKED

700 Hz siren at 30 m/s; car at 20 m/s toward it

approaching: $700 \times \frac{343 + 20}{343 - 30} =$ **812 Hz**

after passing: $700 \times \frac{343 - 20}{343 + 30} =$ **606 Hz**

Echo WORKED

a shout returns from a cliff after 1.2 s

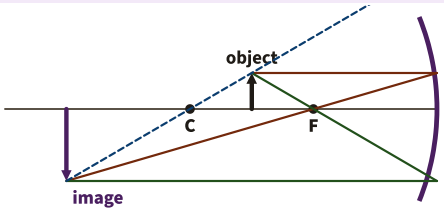
$d = \frac{(343)(1.2)}{2} =$ **206 m** *there and back*

Doppler with waves of light

- Radar guns and weather radar find speed from the frequency shift of reflected microwaves.
- Light from a receding star shifts toward red (**redshift**).

13 MIRRORS

ray diagrams and images



Concave mirror, f = 10 cm, object at 15 cm: the rays meet 30 cm out in a real, inverted image twice the size.

Mirror equation

The same equation as a thin lens.

$$f = \frac{R}{2}$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i},\ m = -\frac{d_i}{d_o}$$

Sign rules

Sign	+	–
f	concave	convex
d_i	real, in front	virtual, behind
m	upright	inverted

Principal rays

Ray in	Ray out (concave)
parallel to the axis	through F
through F	parallel to the axis
through C (center)	straight back on itself
to the center of the mirror	at an equal angle below the axis

Concave mirror WORKED

R = 20 cm, so f = 10 cm; object at 15 cm

$\frac{1}{d_i} = \frac{1}{10} - \frac{1}{15} \rightarrow d_i =$ **30 cm**

$m = -\frac{30}{15} =$ **−2.0**: real, inverted, twice the size

matches the figure

Store security mirror WORKED

convex, f = −20 cm; shopper 30 cm away

$d_i =$ **−12 cm**; m = **0.40** *virtual, upright, smaller*

convex mirrors always shrink: wide view

Makeup mirror WORKED

concave, f = 20 cm; face at 10 cm

$d_i =$ **−20 cm**; m = **2.0**: upright, magnified

object inside F

Where the image lands

Object (concave mirror)	Image
beyond C	real, inverted, smaller
at C	real, inverted, same size
between C and F	real, inverted, larger
at F	none: rays leave parallel
inside F	virtual, upright, larger
any (plane mirror)	virtual, upright, same size, as far behind

Telescopes

- A spherical mirror blurs rays far from the axis; a **parabolic** mirror brings all parallel rays to one focus.
- Newton's reflecting telescope used a concave mirror instead of a lens, avoiding color fringes.

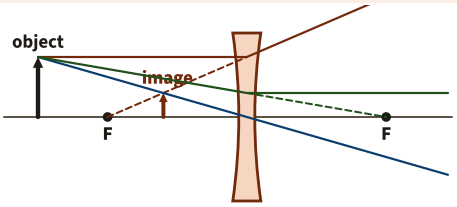
MEMORY TRICK

Concave caves in (Traditional)

A concave mirror curves inward like a cave and gathers light to a focus; a convex one bulges out and spreads it.

14 REFRACTION & LENSES

bending light to form images



Diverging lens, f = −10 cm, object at 15 cm: rays spread as if from a virtual image 6 cm in front of the lens.

Snell's law and lenses

Converging (convex) lens: f > 0. Diverging (concave) lens: f < 0, always a virtual, upright, smaller image.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

Diverging lens WORKED

f = −10 cm, object at 15 cm

$\frac{1}{d_i} = -\frac{1}{10} - \frac{1}{15} \rightarrow d_i =$ **−6.0 cm** *virtual*

$m = -\frac{-6.0}{15} =$ **0.40** *upright, smaller*

Into a glass block WORKED

light hits glass (n = 1.52) at 40° from the normal

$\sin\theta_2 = \frac{\sin 40°}{1.52} \rightarrow \theta_2 =$ **25.0°**

bends toward the normal

it leaves the far side at 40° again, shifted sideways

parallel faces

Lens power and eyes

- Power in diopters: $P = \frac{1}{f}$ (f in meters): f = 0.50 m is **+2.0 D**.
- **Farsighted** eyes need converging (+) lenses; **nearsighted**, diverging (−): a −2.5 D lens has f = **−40 cm**.

Two lenses WORKED

f_1 = 10 cm, object at 15 cm: image 30 cm past lens 1, $m_1 = -2$

first image

lens 2 (f = 5 cm) sits 40 cm from lens 1: object 10 cm away → d_i = 10 cm, $m_2 = -1$

total m = (−2)(−1) = **+2**: upright, twice the size

multiply the m's

Magnifying glass WORKED

f = 5.0 cm; stamp at 4.0 cm *inside F*

$\frac{1}{d_i} = \frac{1}{5.0} - \frac{1}{4.0} \rightarrow d_i =$ **−20 cm**

$m = -\frac{-20}{4.0} =$ **5.0** *upright, 5 times larger*

Water tricks WORKED

pool 2.0 m deep, seen from straight above

looks $\frac{2.0}{1.33} =$ **1.50 m** deep *apparent depth*

from under water, light past $\sin^{-1}(\frac{1}{1.33}) =$ **48.8°** reflects totally

critical angle

Dispersion

- n depends on color: violet bends most, red least, so a prism spreads white light into a spectrum.

MEMORY TRICK

Negative means virtual

A negative d_i is a virtual image on the object's side; a negative f is a diverging lens or a convex mirror.

15 WAVE OPTICS & LIGHT

interference, diffraction, polarization

Light is an electromagnetic wave

- Maxwell (1865) found $c = \frac{1}{\sqrt{(\mu_0 \varepsilon_0)}} = \mathbf{3.00 \times 10^8\ m/s}$; Hertz made radio waves (1887).
- "And God said, Let there be light: and there was light" (Genesis 1:3, KJV).

Coated lens WORKED

MgF_2 coating, n = 1.38, to cancel 550 nm reflections

$t = \frac{\lambda}{4n} = \frac{550}{4(1.38)} = \mathbf{99.6\ nm}$ *thin film*

Two polarizers WORKED

800 W/m² of unpolarized light

first filter: half passes, 400 W/m²

second at 60°: $I = I_0 \cos^2\theta = 400(0.5)^2 = \mathbf{100\ W/m^2}$

Malus's law

MEMORY TRICK

Whole waves shine bright

Bright spots come where paths differ by a whole number of wavelengths; dark ones at a half-wave extra.

Gratings and slits

d = slit spacing; a = slit width; m = 1, 2, 3 …

The central bright band of a single slit is twice as wide as the others.

grating, bright: $d \sin\theta = m\lambda$

single slit, dark: $a \sin\theta = m\lambda$

Diffraction grating WORKED

600 lines/mm; green laser, 532 nm

$d = \frac{1\ mm}{600} = 1.67 \times 10^{-6}$ m

m = 1: **18.6°**; m = 2: **39.7°**; m = 3: **73.3°**

$\sin\theta = \frac{m\lambda}{d}$

m = 4 would need sin θ > 1: none

Soap film WORKED

film n = 1.33 in air; brightest reflection of 600 nm

one reflection flips phase, so $2nt = \frac{\lambda}{2}$: $t = \frac{600}{4(1.33)} = \mathbf{113\ nm}$

thinnest film

Single slit WORKED

633 nm through a 0.10 mm slit; screen 1.5 m away

central band width $\approx \frac{2\lambda L}{a} = \mathbf{19.0\ mm}$

small angles

EM spectrum (rough bounds)

Band	Wavelength	Use
radio	longer than 1 m	broadcasting
microwave	1 mm to 1 m	radar, ovens, Wi-Fi
infrared	700 nm to 1 mm	heat lamps, remotes
visible	400 to 700 nm	sight: 3.10 to 1.77 eV
ultraviolet	10 to 400 nm	sunburn, sterilizing
X-ray	0.01 to 10 nm	medical images
gamma	under 0.01 nm	nuclear decay, cancer care

Polarized light at work

- Polarized sunglasses block glare reflected from water and roads; LCD screens use crossed polarizers.

16 PHOTONS & ATOMS

light packets and energy levels

Photons

Planck (1900): light energy comes in packets. Einstein (1905): each photon acts alone; Millikan confirmed it (1916).

$$\mathbf{E = hf = \frac{hc}{\lambda}}$$

$$\mathbf{E\ (eV) = \frac{1240}{\lambda\ (nm)}}$$

Light on a metal WORKED

φ = 2.30 eV; violet light, 400 nm

$E = \frac{1240}{400} = 3.10$ eV; $KE_{max} = 3.10 - 2.30 = \mathbf{0.80\ eV}$

stopping voltage 0.80 V

red, 700 nm: 1.77 eV < 2.30 eV: **no** electrons, however bright

cutoff: $\lambda = \frac{1240}{2.30} = \mathbf{539\ nm}$

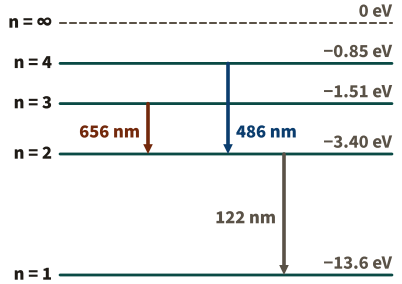
MEMORY TRICK

One number both ways

E in eV = 1240 ÷ λ in nm, and λ = 1240 ÷ E. Short waves carry big packets.

Photoelectric effect

- $KE_{max} = hf - \phi$, where φ (work function) is the least energy that frees an electron.
- Brighter light frees **more** electrons, not faster ones; below a cutoff frequency, none at all.



Hydrogen's levels (not to scale). Drops to n = 2 give visible lines; drops to n = 1 give ultraviolet.

Hydrogen's blue-green line WORKED

electron drops from n = 4 to n = 2

E = −0.85 − (−3.40) = **2.55 eV**

$\lambda = \frac{1240}{2.55} = \mathbf{486\ nm}$ *blue-green*

n = 2 → 1: 10.2 eV, **122 nm** *ultraviolet*

Photons from a laser pointer WORKED

5.0 mW, 532 nm

$E = \frac{1240}{532} = 2.33\ eV = 3.73 \times 10^{-19}$ J

photons per second: $\frac{5.0 \times 10^{-3}}{3.73 \times 10^{-19}} = \mathbf{1.34 \times 10^{16}}$

Bohr model of hydrogen

- $E_n = -\frac{13.6\ eV}{n^2}$: n = 1, 2, 3, 4 give −13.6, −3.40, −1.51, −0.85 eV.
- A drop between levels emits one photon carrying the difference; freeing the electron from n = 1 takes 13.6 eV.
- Bohr (1913) explained the lines Balmer fit in 1885.

Matter waves

- de Broglie (1920s): $\lambda = \frac{h}{p} = \frac{h}{mv}$.
- An electron after 500 V: $\lambda = \frac{h}{\sqrt{(2 m_e eV)}} = \mathbf{5.48 \times 10^{-11}\ m}$, about the spacing of atoms, so electrons diffract in crystals.
- A 0.145 kg ball at 40 m/s: $\mathbf{1.14 \times 10^{-34}\ m}$, far too small to notice.

17 NUCLEUS, DECAY & HALF-LIFE

inside the nucleus

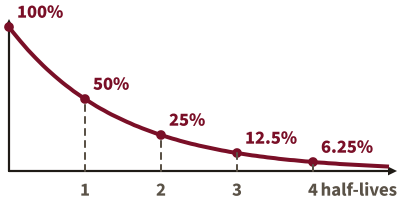
The nucleus

- Rutherford found the tiny, dense nucleus (1911); Chadwick found the neutron (1932).
- ${}^{A}_{Z}X$: A = protons + neutrons, Z = protons. **Isotopes** share Z but differ in A.
- The **strong force** holds the nucleus together against the protons' repulsion.

Half-life

T = half-life. Decay is random for one nucleus but steady for billions.

$$N = N_0(½)^{t/T}$$



Each half-life leaves half of what remained: 100, 50, 25, 12.5, 6.25 percent.

Medical tracer WORKED

technetium-99m, T = 6.0 h; 80 mg delivered

$\frac{24}{6.0}$ = 4 half-lives: 80 → 40 → 20 → 10 → **5.0 mg**

Three kinds of radiation (Becquerel 1896; the Curies 1898)

Decay	Emits	Change	Stopped by
alpha α	${}^{4}_{2}He$ nucleus	A − 4, Z − 2	paper or skin
beta β^-	electron	A same, Z + 1	a few mm of aluminum
gamma γ	photon	no change in A or Z	thick lead or concrete

E = mc²: binding energy WORKED

2 protons + 2 neutrons: 2(1.00728) + 2(1.00866) u

helium-4 nucleus: 4.00151 u; missing mass **0.0304 u**

mass defect

E = 0.0304 × 931.5 = **28.3 MeV** *binding energy*

Half-life and age

- Half-lives are **measured in labs today** and hold steady under ordinary conditions.
- Turning one into an age also needs the **starting amounts** and a **closed** sample, which no one observed. Christians agree decay is real and differ on the ages; see the course (Lesson 11.3) and High School Geology.

Balancing decays WORKED

${}^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^{4}_{2}He$ *210 = 206 + 4; 84 = 82 + 2*

${}^{14}_{6}C \rightarrow {}^{14}_{7}N + {}^{0}_{-1}e$ *charge: 6 = 7 − 1*

${}^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + {}^{0}_{-1}e + \gamma$ *sterilizes supplies*

Not a whole number WORKED

cobalt-60, T = 5.27 years; 10.0 g for 8.0 years

$N = 10.0(½)^{8.0/5.27}$ = **3.49 g** *use the y^x key*

Activity

- Decays per second, in **becquerels** (1 Bq = 1 decay/s). Activity halves every half-life, just as N does.

Radiation safety

- **Time**: less exposure, less dose. **Distance**: dose falls off like $1/r^2$. **Shielding**: match the barrier to the radiation.
- Alpha sources are dangerous mainly if swallowed or breathed in.

MEMORY TRICK

Alpha sheds four and two

Alpha: A drops 4 and Z drops 2. Beta-minus: Z rises 1, A stays. Gamma: only energy leaves.

18 FISSION, FUSION & RELATIVITY

E = mc² at work

Why both release energy

- Binding energy per nucleon peaks near **iron-56**.
- Splitting heavy nuclei (**fission**) or joining light ones (**fusion**) both move toward iron and free energy: E = Δm × 931.5 MeV per u.

Splitting uranium-235 WORKED

${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n$

236 = 141 + 92 + 3; 92 = 56 + 36

about **200 MeV** per split, millions of times a chemical reaction

the 3 new neutrons can split more nuclei: a **chain reaction**

Fermi's pile, Chicago, 1942

Fast particle WORKED

lives 2.2 μs at rest; moves at 0.980c

$\gamma = \frac{1}{\sqrt{(1 - 0.980^2)}}$ = **5.03**

lab lifetime 2.2 × 5.03 = **11.1 μs**: travels **3.25 km**, not 0.65 km

why cosmic-ray muons reach the ground

Fusing deuterium WORKED

${}^{2}H + {}^{2}H \rightarrow {}^{3}He + {}^{1}_{0}n$

Δm = 2(2.01355) − 3.01493 − 1.00866 = **0.00351 u**

E = 0.00351 × 931.5 = **3.27 MeV**

D + T gives 17.6 MeV

Fission and fusion

	Fission	Fusion
Nuclei	heavy split (U-235)	light join (H isotopes)
Needs	slow neutrons	enormous temperature
Where	power plants (Shippingport, 1957)	the Sun (Bethe, 1939); research reactors
Waste	radioactive fragments	mostly helium

MEMORY TRICK

Moving clocks run slow

A clock passing you ticks slow by a factor γ; a ruler passing you is short by γ.

Special relativity (1905)

- **Postulates**: the laws of physics are the same at every steady velocity, and light's speed c is the same for every observer.
- Michelson and Morley (1887) found no change in c with Earth's motion.

Lorentz factor

Moving clocks run slow; moving lengths shrink along the motion. Both matter only near c.

$$\gamma = \frac{1}{\sqrt{(1 - v^2/c^2)}}$$

$$t = \gamma t_0,\ L = \frac{L_0}{\gamma}$$

Clocks in orbit

- GPS satellite clocks run about **38 μs a day** fast (speed and weaker gravity together); light goes 11 km in that time, so engineers correct them.