GEOMETRY PART 1

Grades 8–12 · 4 pages

Formulas for perimeter, area, volume and surface area, the Pythagorean theorem, and the terms the rest of geometry stands on: points, lines, planes, space, shapes, segments, rays and angles.

1 Flat figures · 2 Solids · 3 Pythagoras · 4 Terms & shapes · 5 Lines & planes · 6 Segments & rays · 7 Angles

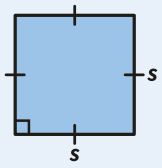
1 PERIMETER & AREA: FLAT FIGURES

P = distance around · A = square units inside

Three measures, three kinds of units

- **Perimeter (P)**: the length of the boundary. Units: in, cm, ft.
- **Circumference (C)**: the perimeter of a circle.
- **Area (A)**: how many unit squares cover the inside. Units: in², cm², ft².
- Always match units first: 2 ft × 18 in → 24 in × 18 in.

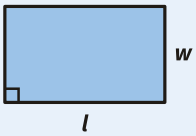
Square



P = 4s
A = s^2

EX s = 6 cm → P = 24 cm; A = 36 cm²

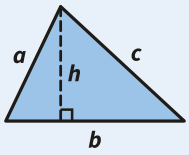
Rectangle



P = 2l + 2w
A = lw

EX l = 9, w = 4 → P = 26; A = 36 units²

Triangle



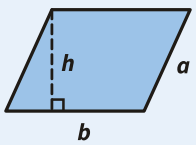
P = a + b + c
A = $\frac{1}{2}$bh

EX b = 10, h = 7 → A = ½ · 10 · 7 = 35

Height is always ⊥ to the base

The height can fall **outside** an obtuse triangle; extend the base with a dashed line and measure straight up to the top vertex. A slanted side is **not** the height.

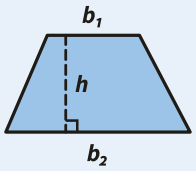
Parallelogram



P = 2a + 2b
A = bh

EX b = 12, h = 5, slant side 6 → A = 60; P = 36

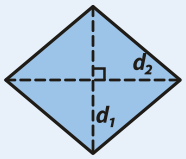
Trapezoid



A = $\frac{1}{2}(b_1 + b_2)h$
P = sum of 4 sides

EX b_1 = 6, b_2 = 10, h = 4 → A = ½ · 16 · 4 = 32

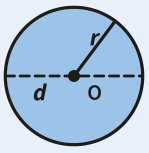
Rhombus · kite



A = $\frac{1}{2}d_1d_2$
P (rhombus) = 4s

EX d_1 = 8, d_2 = 6 → A = 24; each side = $\sqrt{(4^2 + 3^2)}$ = 5, so P = 20

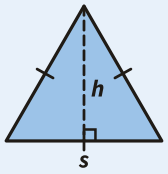
Circle



d = 2r
C = 2πr = πd
A = πr^2

EX r = 5 → C = 10π ≈ 31.4; A = 25π ≈ 78.5

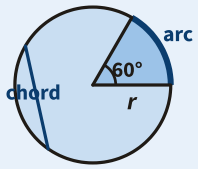
Equilateral triangle



h = $\frac{\sqrt{3}}{2}$s
A = $\frac{\sqrt{3}}{4}s^2$

EX s = 6 → h = 3√3 ≈ 5.2; A = 9√3 ≈ 15.6

Arcs & sectors (a slice of the circle)

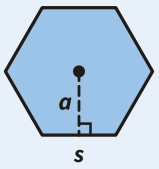


arc = $\frac{\theta}{360} \cdot 2\pi r$
sector = $\frac{\theta}{360} \cdot \pi r^2$

EX r = 6, θ = 60° → arc = 2π ≈ 6.28; sector = 6π ≈ 18.8

Radius	center to the circle
Diameter	chord through the center (2r)
Chord	segment joining two points of the circle
Arc	a piece of the circle; a semicircle is 180°
Sector	region between two radii and their arc

Regular polygon (n equal sides, apothem a)



P = ns
A = $\frac{1}{2}$aP

EX hexagon, s = 4, a = 2√3 ≈ 3.46 → P = 24; A = ½ · 3.46 · 24 ≈ 41.6

MEMORY TRICK **Fence or carpet?**

Perimeter is the **fence** around the yard (a length). **Area** is the **carpet** that covers the floor (square units). Ask "am I walking around it or covering it?"

⚠ Common slips

- Using the **diameter** where the formula wants the **radius** (halve it first).
- Taking a **slanted side** as the height.
- Mixing units (feet with inches) or forgetting the ² on area.
- Rounding π too early: keep π to the end, then round once.

MEMORY TRICK **Pie for circles**

"**C**herry pie's **d**elicious" → **C = πd**
"**A**pple pies **r 2**" → **A = πr²**
Area answers always wear a **square hat** (units²).

Triangle by three sides (Heron)

$s = \frac{a+b+c}{2}$, $A = \sqrt{(s(s-a)(s-b)(s-c))}$

EX 13, 14, 15 → s = 21; A = √(21 · 8 · 7 · 6) = √7056 = **84**

Composite figures: add or subtract — WORKED

10 × 6 rectangle; a 2 × 2 square cut from one corner

A = 60 − 4 = **56** units² — *subtract the piece*

P = **32** units (unchanged) — *the two new edges replace two lost ones*

Circle of r = 3 removed from a 10 × 10 square: A = 100 − 9π ≈ **71.7**

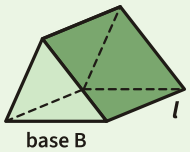
2 VOLUME & SURFACE AREA: SOLIDS

V = cubes inside · SA = total area of every face

Two measures for solids

- **Volume (V)**: unit cubes that fill it. Units: in^3, cm^3, ft^3.
- **Surface area (SA)**: add the areas of all faces (the "net"). Units^2
- **Lateral area (LA)**: sides only, no bases.
- **B** = area of a base; **h** = height ⊥ to the base; **ℓ** = slant height.

Any prism



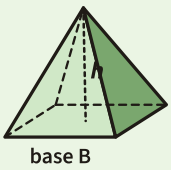
V = Bh

SA = 2B + Ph

(P = perimeter of base)

EX base = right △ 3-4-5, length 10 → B = 6; V = 60; SA = 12 + 12 · 10 = 132

Pyramid

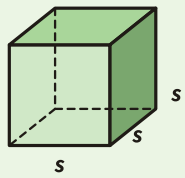


V = $\frac{1}{3}$Bh

SA = B + $\frac{1}{2}$Pℓ

EX square base 6, h = 4 → ℓ = 5; V = ⅓ · 36 · 4 = 48; SA = 36 + ½ · 24 · 5 = 96

Cube

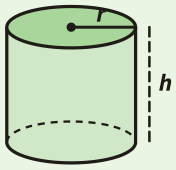


V = s^3

SA = $6s^2$

EX s = 4 → V = 64; SA = 6 · 16 = 96

Cylinder



V = $\pi r^2 h$

SA = $2\pi r^2 + 2\pi rh$

EX r = 3, h = 5 → V = 45π ≈ 141.4; SA = 18π + 30π = 48π ≈ 150.8

Sphere

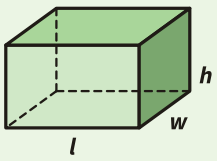


V = $\frac{4}{3}\pi r^3$

SA = $4\pi r^2$

EX r = 3 → V = 36π ≈ 113.1; SA = 36π ≈ 113.1 (equal only at r = 3)

Rectangular prism

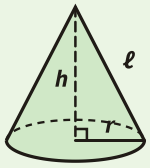


V = lwh

SA = 2(lw + lh + wh)

EX 5 × 3 × 2 → V = 30; SA = 2(15 + 10 + 6) = 62

Cone



V = $\frac{1}{3}\pi r^2 h$

SA = $\pi r^2 + \pi r\ell$

ℓ = $\sqrt{(r^2 + h^2)}$

EX r = 3, h = 4 → ℓ = 5; V = 12π ≈ 37.7; SA = 9π + 15π = 24π ≈ 75.4

⚠ Scale factor k

Double every length (k = 2): perimeter ×2, area ×4, volume ×8. In general: lengths ×k, area ×k^2, volume ×k^3.

Euler: F + V − E = 2

Cube: 6 + 8 − 12 = **2** (any convex polyhedron)

Composite solid WORKED

cylinder r = 3, h = 10, topped by a hemisphere

V = π · 9 · 10 + $\frac{2}{3}$π · 27 = 90π + 18π *half of $\frac{4}{3}\pi r^3$*

= **108π ≈ 339.3**

MEMORY TRICK — Pointy solids pay one-third

A **cone** or **pyramid** holds exactly **⅓** of the cylinder or prism with the same base and height. Sharpen it to a point → divide by 3.

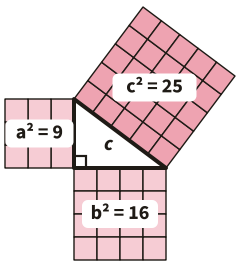
Prism and **c**ylinder: **V = Bh** ("**B**ase times **h**eight") for both.

Converting square & cubic units

- Square the length factor for area, cube it for volume.
- 1 ft = 12 in → 1 ft^2 = 144 in^2; 1 ft^3 = 1,728 in^3
- 1 yd = 3 ft → 1 yd^2 = 9 ft^2; 1 yd^3 = 27 ft^3
- 1 m = 100 cm → 1 m^2 = 10,000 cm^2; 1 m^3 = 1,000,000 cm^3

3 THE PYTHAGOREAN THEOREM

right triangles only



The square on the hypotenuse equals the two leg squares together: 9 + 16 = 25.

In any right triangle

a, b = **legs** (form the right angle) · c = **hypotenuse** (longest side, across from the right angle)

$$a^2 + b^2 = c^2$$

Distance = Pythagoras

d = $\sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$; (1, 2) to (4, 6): **5**

Find a missing side WORKED

Hypotenuse: legs 6 and 8

$c^2 = 36 + 64 = 100$ → **c = 10**

Leg: c = 13, a = 5

$b^2 = 169 - 25 = 144$ → **b = 12** *subtract for a leg*

Ladder 10 ft, base 6 ft from wall: top at $\sqrt{(100 - 36)}$ = **8 ft**

Converse: classify by sides (c longest)

- $c^2 = a^2 + b^2$ → **right**
- $c^2 < a^2 + b^2$ → **acute**
- $c^2 > a^2 + b^2$ → **obtuse**
- **EX** 6, 7, 9: 81 < 36 + 49 = 85 → acute

Diagonal of a box WORKED

$d = \sqrt{(l^2 + w^2 + h^2)}$

3 × 4 × 12 box: $\sqrt{(9 + 16 + 144)} = \sqrt{169}$ = **13**

Common triples (and multiples)

a	b	c
3	4	5
5	12	13
8	15	17
7	24	25
6	8	10

MEMORY TRICK — "3-4-5, stay alive"

Learn **3-4-5** and **5-12-13**; every multiple works too (6-8-10, 9-12-15, 10-24-26). Builders square a corner by measuring **3 ft, 4 ft** and checking for **5 ft**.

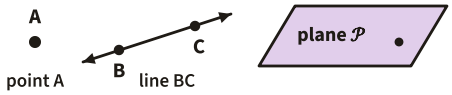
Special right triangles

- **45°-45°-90°**: legs x, x; hypotenuse x√2
- **30°-60°-90°**: short leg x; long leg x√3; hypotenuse 2x
- **EX** 30-60-90 with hypotenuse 14 → short leg 7; long leg 7√3 ≈ 12.1

4 BUILDING BLOCKS: TERMS, SPACE & SHAPES

the words every proof is built from

Undefined terms



Named, not defined: we describe them and build on them.

Point an exact location; no length, width or thickness. Drawn as a dot, named by a capital letter: point A.

Line straight, endless both ways, no thickness. Named by two of its points ($\overleftrightarrow{BC}$) or a script letter (line ℓ).

Plane a flat surface endless in every direction, no thickness. Named by a script capital (plane $\mathcal{P}$) or three noncollinear points (plane ABC).

MEMORY TRICK

Dot · Dash · Floor · Room

0-D point = a dot · **1-D** line = a dash that never ends · **2-D** plane = an endless floor · **3-D** space = the whole room. Each adds one direction.

Defined terms

Space the set of **all** points; it has three dimensions (length, width, height).

Collinear points on one line.

Coplanar points or lines in one plane.

Intersection the set of points two figures share.

Postulate a statement accepted without proof (also called an axiom).

Theorem a statement proved from definitions, postulates and earlier theorems.

Figure any set of points; a **shape** is a figure with a boundary.

Conjecture & counterexample

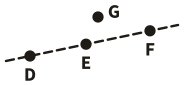
One **counterexample** disproves a **conjecture**: "all primes are odd" fails at **2**.

What makes a good definition

It names the **class** and the **difference**, and it reverses. *A right angle is an angle* (class) *that measures 90°* (difference). Reversed: an angle that measures 90° is a right angle. If the reverse fails, the definition is too loose.

Ruler & protractor postulates

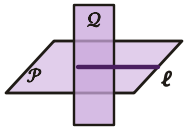
- Points on a line match real numbers; the distance between them is $|b - a|$.
- Rays from one point match the degrees 0 to 180; $m\angle$ = |difference of the readings|.



D, E, F collinear; G is not. Any 3 points are coplanar.

Point-line-plane postulates

- Through any **2 points** there is exactly **one line**.
- Through any **3 noncollinear points** there is exactly **one plane**.
- If 2 points lie in a plane, the **whole line** through them lies in it.
- Two lines meet in **at most one point**.
- Two planes meet in **exactly one line** (if they meet at all).



Planes $\mathcal{P}$ and $\mathcal{Q}$ intersect in line ℓ.

Symbols to know

Sym.	Means	Sym.	Means
∠	angle	≅	congruent
m∠	measure of	~	similar
⊥	perpendicular	△	triangle
∥	parallel	≈	about
°	degrees	∈	is on / in

Polygons

A **polygon** is a closed plane figure made of segments (**sides**) that meet only at endpoints (**vertices**). **Convex**: no dent; every diagonal stays inside. **Concave**: at least one dent. **Regular**: all sides and all angles equal.

Sides	Name	Angle sum
3	triangle	180°
4	quadrilateral	360°
5	pentagon	540°
6	hexagon	720°
7	heptagon	900°
8	octagon	1080°
10	decagon	1440°
12	dodecagon	1800°
n	n-gon	(n − 2) · 180°

Each angle of a regular n-gon = (n − 2) · 180° ÷ n (hexagon: 120°).

MEMORY TRICK **Names that count**

Pentagon: the 5-sided Pentagon building · **Hexa**gon: 6-sided honeycomb cells · **Octa**gon: an **oct**opus has 8 arms · **Deca**gon: a **dec**ade has 10 years.

Triangles · Quadrilaterals

- By sides: **scalene** (none equal), **isosceles** (≥ 2), **equilateral** (3).
- By angles: **acute** (all < 90°), **right** (one 90°), **obtuse** (one > 90°).
- Square ⊂ rectangle ⊂ parallelogram; square ⊂ rhombus ⊂ parallelogram; trapezoid has exactly 1 pair of parallel sides (some texts: ≥ 1).

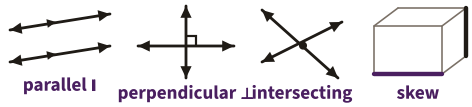
Regular hexagon WORKED

interior $\frac{4 \cdot 180°}{6} = \mathbf{120°}$; exterior $\frac{360°}{6} = \mathbf{60°}$

diagonals $\frac{n(n-3)}{2} = \frac{6 \cdot 3}{2} = \mathbf{9}$

5 LINES & PLANES

how lines and planes meet, or never meet



Matching arrowheads on two lines mark them parallel.

MEMORY TRICK **Skew lines skip**

Skew lines **sk**ip past each other in 3-D: no meeting point, no common plane. Two lines in one plane must cross or be parallel.

Parallel (∥) coplanar lines that never meet. Same slope on a graph.

Perpendicular (⊥) lines that meet at 90°. Slopes multiply to −1.

Intersecting lines with exactly one common point.

Skew lines in **different planes**: never meet, yet not parallel (a box's top front edge and a back side edge).

Parallel planes never meet (floor and ceiling).

Line ∥ plane the line never touches the plane.

Line ⊥ plane ⊥ to every line in the plane through its foot.

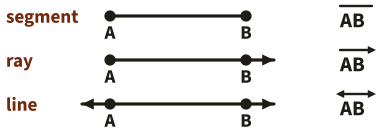
Slopes tell you WORKED

$y = 2x + 1$ vs $y = 2x - 5$: slopes 2, 2 → **parallel**

$y = 2x + 1$ vs $y = -\frac{1}{2}x + 3$: $2 \cdot (-\frac{1}{2}) = -1$ → **perpendicular**

6 SEGMENTS & RAYS

pieces of a line



Endpoints are dots; arrows mean "keeps going."

Segment $\overline{AB}$ points A and B and every point between them. Its **length** is AB (no bar).

Ray $\overrightarrow{AB}$ starts at endpoint A, passes through B, goes on forever. **Order matters**: $\overrightarrow{AB} \neq \overrightarrow{BA}$.

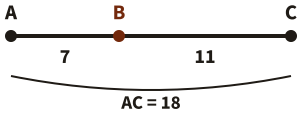
Congruent (≅) segments with equal lengths: $\overline{AB} \cong \overline{CD}$ means AB = CD.

MEMORY TRICK

Bar = thing, no bar = number

$\overline{AB}$ is the segment itself (a figure); AB is its length (a number). Figures are **congruent (≅)**; numbers are **equal (=)**.

Segment addition postulate



If B is between A and C, then AB + BC = AC.

Algebra with segments WORKED

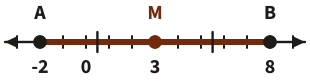
B between A, C; AB = 2x + 1, BC = 3x − 4, AC = 27

(2x + 1) + (3x − 4) = 27 *add the parts*

5x − 3 = 27 → x = 6

AB = **13**, BC = **14** (13 + 14 = 27 ✓) *check*

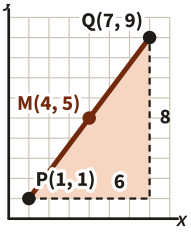
Midpoint & distance



Number line: M = (−2 + 8) ÷ 2 = 3; AB = |8 − (−2)| = 10.

Formulas

- Number line: $M = \frac{a+b}{2}$; length = |b − a|
- Plane: $M = (\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$
- $d = \sqrt{((x_2-x_1)^2 + (y_2-y_1)^2)}$

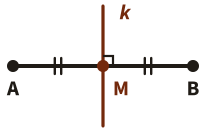


$d = \sqrt{(6^2 + 8^2)} = 10$; M = (4, 5).

MEMORY TRICK

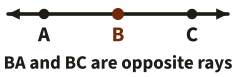
Midpoint = average · distance = Pythagoras

Add and halve to find a midpoint. The distance formula is just $a^2 + b^2 = c^2$ on a grid: run, rise, then the slanted side.



A **segment bisector** passes through the midpoint; a ⊥ bisector also makes a right angle.

Rays: opposite, union, intersection

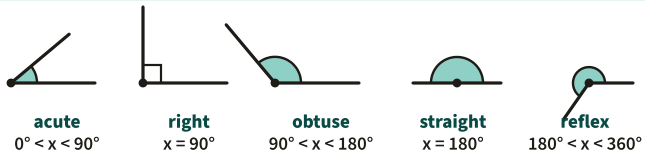


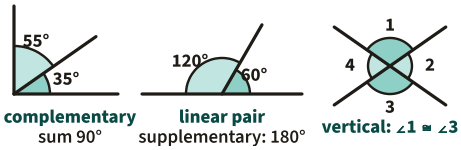
BA and BC are opposite rays

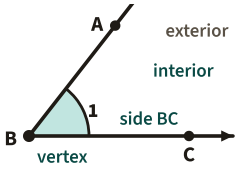
- **Opposite rays** share an endpoint and point in opposite directions; together they form a line.
- $\overrightarrow{BA} \cup \overrightarrow{BC} = \overleftrightarrow{AC}$ when they are opposite.
- $\overrightarrow{AB} \cap \overrightarrow{BA} = \overline{AB}$ (the part they share).
- Two rays with one endpoint that are **not** opposite form an **angle**.

7 ANGLES

two rays, one endpoint







Name it ∠ABC, ∠CBA, ∠B or ∠1; the vertex is always the middle letter.

Angle two rays (**sides**) with a common endpoint (**vertex**).

m∠ABC its measure in degrees (°); a full turn = 360°.

Congruent ∠s equal measures; marked with matching arcs.

Adjacent ∠s share a vertex and a side, with no interior points in common.

Angle addition postulate

If D is in the interior of ∠ABC, then m∠ABD + m∠DBC = m∠ABC.

Solve for x WORKED

m∠ABD = 3x + 5, m∠DBC = 2x + 10, ∠ABC is right

5x + 15 = 90 → x = 15

m∠ABD = **50°**, m∠DBC = **40°** *50 + 40 = 90 ✓*

Complementary two angles summing to **90°**. Complement of 62° = 28°.

Supplementary two angles summing to **180°**. Supplement of 62° = 118°.

Linear pair adjacent angles whose outer sides are opposite rays; always supplementary.

Vertical ∠s opposite angles formed by two crossing lines; always **congruent**.

Around a point angles fill **360°**.

MEMORY TRICK **Corner & Straight**

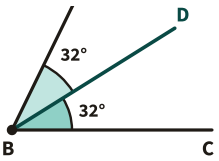
Complementary → **C**orner (90°). **S**upplementary → **S**traight (180°). C comes before S in the alphabet, and 90 comes before 180.

Vertical angles WORKED

Vertical angles: 4x − 10 and 2x + 30

4x − 10 = 2x + 30 → x = 20 *set equal*

each angle = **70°**; its neighbors = **110°** *linear pair*



$\overrightarrow{BD}$ **bisects** ∠ABC: it splits it into two ≅ angles.