GEOMETRY PART 2

Grades 8–10 · 6 pages

Logic, proofs, parallel lines, congruent and similar triangles, centers, quadrilaterals, circles, transformations, coordinate proofs, trig and constructions.

1 Logic · 2 Proofs · 3 Parallel Lines · 4 Triangle Facts · 5 Congruent Triangles · 6 Similar Triangles · 7 Triangle Centers · 8 Polygons · 9 Circles · 10 Transformations · 11 Coordinate Proofs · 12 Trig · 13 Constructions

1 LOGIC & CONDITIONALS

the grammar every proof is written in

Conditional	If **p**, then **q** (p → q). p = hypothesis, q = conclusion
Converse	If q, then p (q → p)
Inverse	If not p, then not q (~p → ~q)
Contrapositive	If not q, then not p (~q → ~p)
Biconditional	p **if and only if** q (p ↔ q): both directions true. Every good definition is one.

One statement, four forms

Form	Statement	True?
p → q	If an angle is 30°, then it is acute.	✓
Converse	If an angle is acute, then it is 30°.	✗ (50°)
Inverse	If an angle is not 30°, then it is not acute.	✗ (50°)
Contrapositive	If an angle is not acute, then it is not 30°.	✓

MEMORY TRICK **Swap, negate, both**

Converse **swaps**, **I**nverse **negates**, **C**ontrapositive does **both**.

The contrapositive is the true twin: it always shares the original's truth value; the converse and inverse share each other's.

Truth table

p	q	p → q	~q → ~p
T	T	T	T
T	F	**F**	**F**
F	T	T	T
F	F	T	T

When is p → q false?

Only when p is true and q is false. A false hypothesis makes the conditional true (a promise not tested is not broken).

Deductive laws

- **Detachment**: p → q is true and p is true, so q is true.
- **Syllogism**: p → q and q → r give p → r.
- **Counterexample**: one case with p true and q false disproves p → q.

Chain it WORKED

If two angles are vertical, they are congruent.
If two angles are congruent, they have equal measures.
∴ If two angles are vertical, they have equal measures.

syllogism

⚠ Reasoning traps

- **Affirming the conclusion**: p → q and q true does **not** give p.
- **Inductive** reasoning (patterns) suggests a conjecture; only **deductive** reasoning proves it.

2 TWO-COLUMN PROOFS

every statement needs a reason you could defend

Anatomy of a proof

- **Given**: what you may assume. **Prove**: the goal.
- Mark the figure first: ticks, arcs, right-angle boxes.
- Left column **statements**; right column **reasons**: a given, definition, postulate or proved theorem.
- Other formats: **paragraph**, **flow** (boxes and arrows), **indirect** (assume the opposite; reach a contradiction).

Properties of equality (≅ is reflexive, symmetric, transitive too)

Property	Equality form
Reflexive	a = a
Symmetric	a = b → b = a
Transitive	a = b, b = c → a = c
Substitution	a = b → b may replace a
+, −, ×, ÷	do the same to both sides (÷ by c ≠ 0)
Distributive	a(b + c) = ab + ac

MEMORY TRICK

Givens first, CPCTC last

Open with the givens, close with what was asked; a shared side or angle is free by the **reflexive** property.

Reason finder

When you see…	Cite
X shares a side or angle	reflexive property
two lines cross	vertical ∠s are ≅
a midpoint	def. of midpoint (two ≅ halves)
a bisector	def. of bisector
⊥ lines	def. of ⊥ (right ∠s); all right ∠s ≅
∥ lines + transversal	alt. interior / corresponding ∠s ≅
two ∠s forming a line	linear pair → supplementary
parts after ≅ triangles	CPCTC

Proof: M is the midpoint of $\overline{AB}$ and $\overline{CD}$. Prove $\overline{AC} \cong \overline{BD}$.

Statement	Reason
1. M is the midpoint of $\overline{AB}$ and $\overline{CD}$	given
2. $\overline{AM} \cong \overline{MB}$, $\overline{CM} \cong \overline{MD}$	def. of midpoint
3. ∠AMC ≅ ∠BMD	vertical angles are ≅
4. △AMC ≅ △BMD	SAS (2, 3, 2)
5. $\overline{AC} \cong \overline{BD}$	CPCTC

Algebraic proof WORKED

Given 3(x − 4) = 15. Prove x = 9.

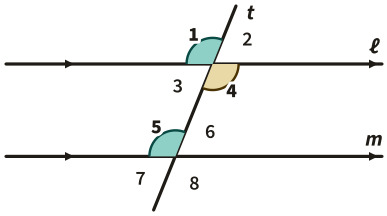
3x − 12 = 15	*distributive*
3x = 27	*addition prop.*
x = 9	*division prop.*

⚠ Proof slips

- Using what you must prove as a reason.
- Reading facts off the picture that are not marked or given.
- Citing **SSA** or **AAA**: neither proves congruence.

3 PARALLEL LINES & TRANSVERSALS

one crossing angle gives all eight



Numbering: 1–4 at the top crossing, 5–8 at the bottom (UL, UR, LL, LR). Shaded: ∠1 ≅ ∠5 (corresponding) and ∠4 (vertical to ∠1).

Transversal a line that crosses two or more lines
Interior between ℓ and m: ∠3, ∠4, ∠5, ∠6
Exterior outside ℓ and m: ∠1, ∠2, ∠7, ∠8
Alternate opposite sides of t
Same-side the same side of t

⚠ Only for parallel lines

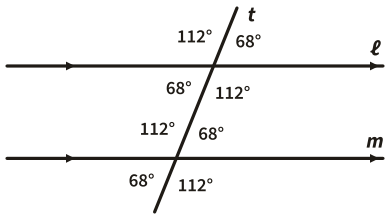
The ≅ and 180° rules need ℓ ∥ m. If the lines are not parallel, only vertical angles and linear pairs still work.

Converses prove lines parallel

If corresponding ∠s are ≅, **or** alternate interior ∠s are ≅, **or** same-side interior ∠s are supplementary, then ℓ ∥ m.

Angle pairs

Pair (ℓ ∥ m)	Example	Rule
Corresponding	1 & 5, 2 & 6, 3 & 7, 4 & 8	≅
Alternate interior	3 & 6, 4 & 5	≅
Alternate exterior	1 & 8, 2 & 7	≅
Same-side interior	3 & 5, 4 & 6	sum **180°**
Same-side exterior	1 & 7, 2 & 8	sum **180°**
Vertical	1 & 4, 2 & 3	≅ (any lines)
Linear pair	1 & 2, 5 & 7	sum 180° (any lines)



Given ∠2 = 68°: the four acute angles are 68°, the four obtuse 112°.

MEMORY TRICK **Letters F, Z, C**

Trace an **F** for corresponding angles, a **Z** for alternate interior, a **C** (or U) for same-side interior.

Only **two sizes** appear: every acute angle is equal, every obtuse angle is equal, and acute + obtuse = 180°.

Same-side interior WORKED

(3x + 10) + (5x − 6) = 180 *supplementary*
8x + 4 = 180 → x = **22**
angles **76°** and **104°**

Alternate interior WORKED

4x − 7 = 2x + 35 *congruent*
2x = 42 → x = **21**
each angle **77°**

Parallel & perpendicular facts

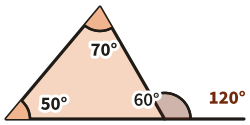
- Two lines ∥ to the same line are ∥ to each other.
- In a plane, two lines ⊥ to the same line are ∥.
- A transversal ⊥ to one of two ∥ lines is ⊥ to the other: all eight angles are 90°.

4 TRIANGLE FACTS & INEQUALITIES

what any three sides or angles can and cannot do

Angle facts

- Interior angles sum to **180°**.
- Exterior angle = sum of the two **remote** interior angles.
- Isosceles: base angles ≅ (and the converse). Equilateral ⟺ equiangular (60° each).



Exterior 120° = 50° + 70°.

Isosceles WORKED

vertex angle 40°
base angles (180 − 40) ÷ 2 = **70°** each

Exterior angle algebra WORKED

exterior 4x + 10 = (x + 20) + (2x + 15) *remote interiors*
x = 25 → exterior **110°**

Triangle inequality

Each side is **shorter** than the sum of the other two and **longer** than their difference.

EX sides 3 and 5: **2 < x < 8**. Can 4, 5, 10 be a triangle? 4 + 5 = 9 < 10: **no**.

MEMORY TRICK

Two short sides must reach

The two shorter sides together must out-reach the longest one; if they only tie, the "triangle" lies flat.

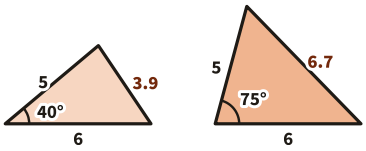
Side–angle order

The **largest angle** is opposite the **longest side**; the smallest faces the shortest.

EX sides 7, 9, 12 → the angle opposite 12 is the largest.

MEMORY TRICK **Big faces big**

The biggest angle always faces the biggest side.



Same two sides; the wider included angle gives the longer third side.

Hinge theorem (SAS inequality)

Two sides of one triangle ≅ two sides of another: the larger **included angle** sits across from the longer third side. The converse also holds (SSS inequality).

MEMORY TRICK **Open the jaw**

Open the jaw wider and the gap between the teeth gets longer.

Midsegment theorem

The segment joining two sides' midpoints is ∥ to the third side and **half** as long.

EX third side 14 → midsegment **7**.

Sides 7, 9, 12 at a glance WORKED

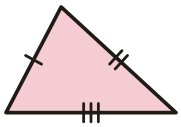
7 + 9 = 16 > 12 → a triangle ✓ · largest angle faces 12 · $7^2 + 9^2 = 130 < 144 = 12^2$ → **obtuse** (Part 1 converse)

5 CONGRUENT TRIANGLES

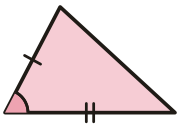
same size and shape: every matching part equal

Congruence statements

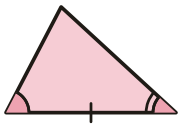
△ABC ≅ △DEF matches **in order**: A↔D, B↔E, C↔F, so $\overline{AB} \cong \overline{DE}$ and ∠C ≅ ∠F. The order tells you which parts match.



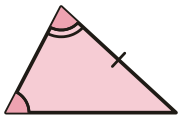
SSS
3 sides



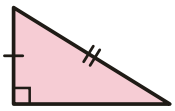
SAS
2 sides + angle **between**



ASA
2 angles + side **between**



AAS
2 angles + a side **not** between



HL
right △: hypotenuse + leg

The five shortcuts

Shortcut	Needs	Note
SSS	three pairs of sides	
SAS	two sides and the included angle	angle must be between
ASA	two angles and the included side	
AAS	two angles and a non-included side	third angles match anyway
HL	hypotenuse and one leg	right triangles only

MEMORY TRICK

SSA swings, AAA scales

SSA can swing the side into two different triangles; **AAA** only fixes the shape, not the size. Neither proves ≅.

MEMORY TRICK **CPCTC**

Corresponding **P**arts of **C**ongruent **T**riangles are **C**ongruent: prove the triangles first, then the parts.

Proof strategy

- Mark everything given; look for shared sides (reflexive) and vertical angles.
- Count S and A marks; match one of the five shortcuts.
- Overlapping triangles: redraw them apart.

Isosceles & bisector facts

- The bisector of an isosceles vertex angle is also the ⊥ bisector of the base.
- Points on a segment's ⊥ bisector are equidistant from its endpoints.
- Points on an angle bisector are equidistant from its sides.

Solve with CPCTC WORKED

△PQR ≅ △XYZ, PQ = 2x + 3, XY = 15

2x + 3 = 15 → x = **6** *CPCTC*

Which shortcut? WORKED

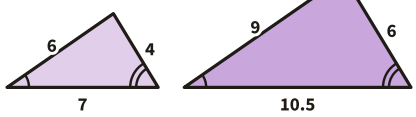
$\overline{AB} \cong \overline{DE}$, ∠A ≅ ∠D, ∠B ≅ ∠E — ***ASA***

$\overline{AB} \cong \overline{DE}$, ∠A ≅ ∠D, ∠C ≅ ∠F — ***AAS***

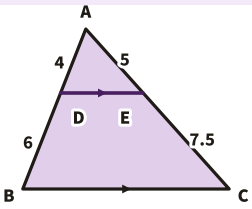
$\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$, ∠A ≅ ∠D — *SSA:* ***none***

6 SIMILAR TRIANGLES

same shape, sizes scaled by k



Sides 4, 6, 7 and 6, 9, 10.5: every ratio is k = 1.5.



$\overline{DE} \parallel \overline{BC}$: $\frac{4}{6} = \frac{5}{7.5}$.

MEMORY TRICK **AA is all you need**

Two matching angles force the third, so two angles prove similarity.

Similarity tests

Test	Needs
AA~	two pairs of ≅ angles
SAS~	two side ratios equal + included ∠ ≅
SSS~	all three side ratios equal

Shadow problem WORKED

A 6-ft post casts a 4-ft shadow; a tree casts 18 ft.

AA~ (same sun angle)

$\frac{6}{4} = \frac{x}{18}$ → 4x = 108 → x = **27 ft**

Side splitter

A line ∥ to one side cuts the other two sides **proportionally**: $\frac{AD}{DB} = \frac{AE}{EC}$. Converse: proportional cuts mean the line is ∥.

Angle bisector theorem

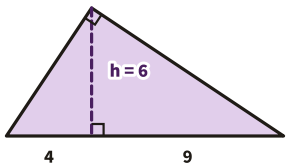
An angle bisector splits the opposite side in the ratio of the adjacent sides.

EX sides 6 and 9, opposite side 10 → pieces **4** and **6**.

Scale factor

Corresponding sides are proportional: $\frac{6}{4} = \frac{9}{6} = \frac{10.5}{7}$ = **1.5**. Perimeters scale by k; areas by k^2 (Part 1).

EX perimeters 17 and 25.5 (ratio 1.5); areas in ratio 1.5^2 = **2.25**.



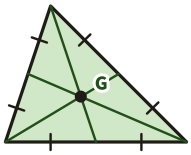
$h^2 = 4 \cdot 9$, so h = 6.

Altitude to the hypotenuse

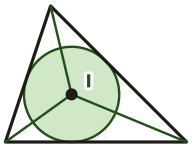
- Makes three similar right triangles.
- $h = \sqrt{(\text{piece}_1 \cdot \text{piece}_2)}$ (geometric mean).
- Each leg = √(its piece · hypotenuse): $\sqrt{(4 \cdot 13)} = 2\sqrt{13}$, $\sqrt{(9 \cdot 13)} = 3\sqrt{13}$.

7 TRIANGLE CENTERS & SEGMENTS

four special points where three lines meet



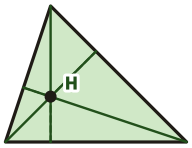
centroid



incenter



circumcenter



orthocenter

The four centers

Center	Where 3 … meet	Special property	Location
Centroid G	medians (vertex to opposite midpoint)	balance point; 2 : 1 split of each median	always inside
Incenter I	angle bisectors	equidistant from the **sides**; center of the inscribed circle	always inside
Circumcenter O	⊥ bisectors of the sides	equidistant from the **vertices**; center of the circumscribed circle	in, on (right △: midpoint of hypotenuse), or out
Orthocenter H	altitudes	—	in, at the right-angle vertex, or out

MEMORY TRICK

Cool Ice Cream Overflows; Mom Always Packs Aprons

Centroid–**M**edians · **I**ncenter–**A**ngle bisectors · **C**ircumcenter–**P**erpendicular bisectors · **O**rthocenter–**A**ltitudes.

MEMORY TRICK

Two-thirds to the vertex

The centroid sits **⅔** of the way from each vertex to the opposite midpoint.

Centroid split WORKED

median 12 long

vertex to G = ⅔ · 12 = **8**; G to midpoint = **4**

Centers of A(2,6), B(0,0), C(8,0) WORKED

G = $(\frac{2+0+8}{3}, \frac{6+0+0}{3})$ = **(10/3, 2)** *average*

O = **(4, 2)**, R = √20 ≈ 4.47 *OA = OB = OC*

H = **(2, 2)** *altitudes*

I ≈ **(2.92, 2.10)**, r ≈ 2.10 $r = \frac{Area}{s} = \frac{24}{11.40}$

Euler line

O, G and H always lie on one line, with HG = 2 · GO. Above: all three have y = 2; HG = $\frac{4}{3}$, GO = $\frac{2}{3}$.

⚠ **Altitude ≠ median**

An **altitude** is ⊥ to the opposite side (it may fall outside); a **median** hits the midpoint. They agree only from the vertex of an isosceles triangle.

8 POLYGONS & QUADRILATERALS

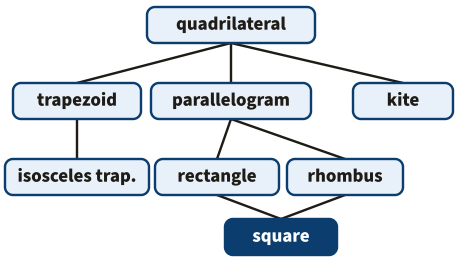
each step down the tree adds properties

Exterior angles

- One exterior angle per vertex always sums to **360°**, for any convex polygon.
- Regular n-gon: each exterior = $\frac{360}{n}$, each interior = 180 − $\frac{360}{n}$.

EX each interior 140° → exterior 40° → n = $\frac{360}{40}$ = **9**

EX interior sum 1440°: (n − 2)·180 = 1440 → n = **10**



Every square is a rectangle and a rhombus. In this course a kite has no congruent opposite sides, so a rhombus is not a kite.

Parallelogram family (▱ = parallelogram)

Property	▱	Rect	Rhom	Sq
Opposite sides ∥ and ≅	✓	✓	✓	✓
Opposite ∠s ≅; consecutive supp.	✓	✓	✓	✓
Diagonals bisect each other	✓	✓	✓	✓
Four right angles		✓		✓
Diagonals ≅		✓		✓
Four ≅ sides			✓	✓
Diagonals ⊥; bisect the angles			✓	✓

MEMORY TRICK **The square wins**

The square inherits every property in the table: it is the most special quadrilateral.

Prove a parallelogram (any one)

- both pairs of opposite sides ∥, or both ≅
- one pair both ∥ **and** ≅
- diagonals bisect each other
- both pairs of opposite angles ≅

Trapezoids & kites

- **Trapezoid**: exactly one pair of ∥ sides (bases). Midsegment = $\frac{b_1 + b_2}{2}$.
- **Isosceles trapezoid**: legs ≅, base angle pairs ≅, diagonals ≅.
- **Kite**: two pairs of ≅ adjacent sides; diagonals ⊥; one pair of opposite ∠s ≅.

EX bases 8 and 14 → midsegment $\frac{8+14}{2}$ = **11**

EX rhombus diagonals 10, 24 → side √(5^2 + 12^2) = **13** (halves meet at 90°)

Parallelogram & rectangle WORKED

consecutive ∠s 2x + 10, 3x − 5: 5x + 5 = 180 *supplementary*

x = 35 → **80°** and **100°**

rectangle: AC = 3x + 1, BD = 5x − 9 → x = 5, each **16** *diagonals ≅*

Kite angles WORKED

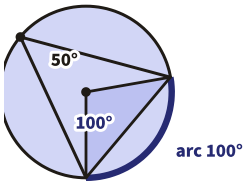
other two angles 70° and 110°

≅ pair: $\frac{360 - 70 - 110}{2}$ = **90°** each

9 CIRCLES

angles from arcs, lengths from products

Chord	both endpoints on the circle
Secant	line cutting the circle twice
Tangent	line touching it once
Central ∠	vertex at the center
Inscribed ∠	vertex on the circle
Arcs	minor < 180° < major; semicircle 180°



Central 100°, inscribed 50°: same arc.

MEMORY TRICK **Inscribed is half**

An inscribed angle is **half** the central angle (and arc) it shares.

Inscribed-angle facts

- Inscribed ∠s on the same arc are ≅.
- An angle inscribed in a semicircle is **90°**.
- Inscribed quadrilateral: opposite ∠s **supplementary** (85° ↔ 95°).

Tangent facts

- A tangent is ⊥ to the radius at the point of tangency.
- Two tangents from one outside point are ≅.
- Tangent–chord angle = ½ its arc (arc 130° → 65°).
- A radius ⊥ to a chord bisects it; ≅ chords ⟺ ≅ arcs.

Angle from arcs

Vertex	Angle	EX
center	= arc	arc 100° → 100°
on circle	½ · arc	arc 100° → 50°
inside	½ (sum of arcs)	½(80 + 40) = 60°
outside	½ (difference)	½(150 − 50) = 50°

MEMORY TRICK

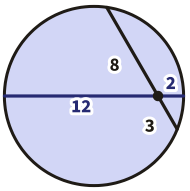
Inside: average · outside: half the difference

Vertex inside, average the two arcs; vertex outside, take half their difference.

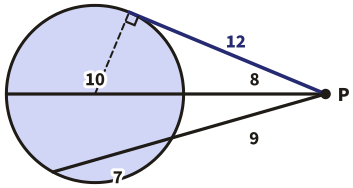
Equation of a circle

Center (h, k), radius r: $(x-h)^2 + (y-k)^2 = r^2$.

EX $x^2 + y^2 - 6x + 4y - 12 = 0$ → complete the squares: $(x-3)^2 + (y+2)^2 = 25$ → center **(3, −2)**, r = **5**.



Crossing chords: 12 · 2 = 8 · 3 = 24.



From P: $12^2 = 8 \cdot 18 = 9 \cdot 16 = 144$.

Power of a point

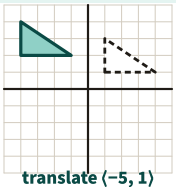
Case	Rule
two chords	part · part = part · part
two secants	outside · whole = outside · whole
tangent + secant	tangent² = outside · whole

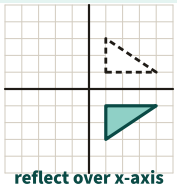
MEMORY TRICK **Part × whole**

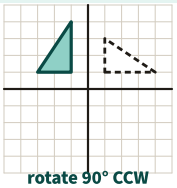
Outside the circle: outside part × whole = outside part × whole (a tangent counts as both).

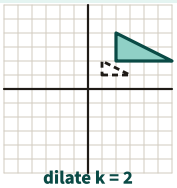
10 TRANSFORMATIONS & SYMMETRY

move it, flip it, turn it, scale it


translate ⟨−5, 1⟩


reflect over x-axis


rotate 90° CCW


dilate k = 2

Coordinate rules (dashed = preimage, shaded = image; turns about the origin)

Transformation	Rule	Image of (1, 3)
Translation ⟨a, b⟩	(x + a, y + b)	⟨−5, 1⟩: (−4, 4)
Reflect over x-axis	(x, −y)	(1, −3)
Reflect over y-axis	(−x, y)	(−1, 3)
Reflect over y = x	(y, x)	(3, 1)
Rotate 90° CCW	(−y, x)	(−3, 1)
Rotate 180°	(−x, −y)	(−1, −3)
Rotate 270° CCW	(y, −x)	(3, −1)
Dilate by k (center O)	(kx, ky)	k = 2: (2, 6)

MEMORY TRICK

The axis keeps its coordinate

Reflect over the **x**-axis: x stays, y flips. Over the **y**-axis: y stays, x flips.

MEMORY TRICK **Swap, then negate**

Rotate 90° CCW: **swap** x and y, then **negate the new first** coordinate: (1, 3) → (3, 1) → (−3, 1).

Rigid motions (isometries)

Translations, reflections, rotations and glide reflections keep **lengths and angles**: image ≅ preimage. A dilation keeps angles but scales lengths: image ~ preimage. **Rigid motions keep size and shape; dilations keep shape only.**

Compose WORKED

(1, 3): reflect over the y-axis, then translate ⟨2, −1⟩

(−1, 3) → **(1, 2)**

Compositions

- Reflect over two ∥ lines → a translation of **twice** the gap.
- Reflect over two lines meeting at θ → a rotation of **2θ**.
- Glide reflection: translate, then reflect over a line ∥ to the slide.
- Order matters: do the first one first.

Symmetry

- **Line**: a reflection maps the figure onto itself.
- **Rotational**: a turn under 360° does.
- Regular n-gon: n lines; turns of $\frac{360}{n}$°.

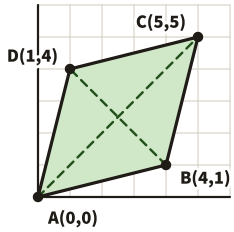
EX square 4 lines, 90° · rectangle 2 lines, 180° · regular pentagon 5 lines, 72°

11 COORDINATE PROOFS

algebra does the measuring

MEMORY TRICK **Pick the right tool**

Slopes for ∥ and ⊥ · **distance** for ≅ · **midpoint** for bisect.



Is ABCD a rhombus? A square?

Classify ABCD WORKED

AB = BC = CD = DA = $\sqrt{17}$ *distance*

diagonal slopes 1 and −1 → ⊥ *product −1*

AB slope $\frac{1}{4}$, BC slope 4: not ⊥ *no right ∠*

∴ **rhombus**, not a square

diagonals $\sqrt{50} \neq \sqrt{18}$; both midpoints (2.5, 2.5) *bisect, not ≅*

Midpoint of a hypotenuse WORKED

right △ (0, 0), (2a, 0), (0, 2b); M = (a, b) *midpoint*

M to each vertex: $\sqrt{(a^2 + b^2)}$ *distance*

∴ M is equidistant from all three vertices

Placing the figure

- Put a vertex at the origin and a side on the x-axis.
- Use variables (a, b, c), not numbers, so the proof covers every case.
- Use 2a, 2b when midpoints are coming: no fractions.

Right triangle? WORKED

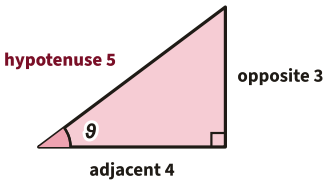
P(0, 0), Q(3, 1), R(−1, 3)

slope PQ = $\frac{1}{3}$, slope PR = −3 *slopes*

product −1 → ⊥ → **right angle at P**

12 RIGHT-TRIANGLE TRIG & THE LAWS

ratios of sides name the angles



Label sides from θ's point of view.

MEMORY TRICK **SOH-CAH-TOA**

Sine = **O**pp/**H**yp · **C**osine = **A**dj/**H**yp · **T**angent = **O**pp/**A**dj.

Exact values

θ	sin	cos	tan
30°	½	√3⁄2	√3⁄3
45°	√2⁄2	√2⁄2	1
60°	√3⁄2	½	√3

MEMORY TRICK **Count up the roots**

sin 30°, 45°, 60° = $\frac{\sqrt{1}}{2}, \frac{\sqrt{2}}{2}, \frac{\sqrt{3}}{2}$; cos runs the same list backward.

3-4-5 WORKED

sin θ = $\frac{3}{5}$ = 0.6, cos θ = $\frac{4}{5}$ = 0.8, tan θ = $\frac{3}{4}$ = 0.75

θ = $\sin^{-1}(0.6) \approx$ **36.87°** *inverse*

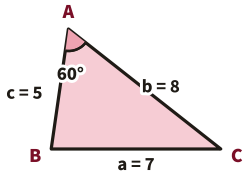
$0.6^2 + 0.8^2 = 1$ and $\frac{0.6}{0.8} = 0.75$

$\sin^2 + \cos^2 = 1$; tan = sin/cos

Angle of elevation WORKED

50 ft from a tree, elevation 35°

h = 50 tan 35° ≈ **35.0 ft** *tan = opp/adj*



Side a faces angle A.

Area with sine

Area = ½ab sin C.

EX ½ · 5 · 8 · sin 60° = 10√3 ≈ **17.32**.

Law of sines

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$: use with AAS, ASA or SSA.

EX A = 40°, B = 75°, a = 10 → C = 65°, b ≈ **15.03**, c ≈ **14.10**.

Law of cosines

$c^2 = a^2 + b^2 - 2ab \cos C$: use with SAS or SSS.

EX sides 5, 8, 7: cos θ = $\frac{25 + 64 - 49}{80} = 0.5$ → the angle facing 7 is **60°**.

MEMORY TRICK

Pythagoras with a correction

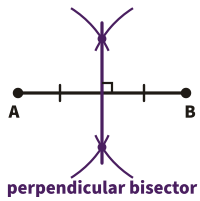
Law of cosines = Pythagoras minus 2ab cos C; at C = 90° the correction is 0.

⚠ **SSA: the ambiguous case**

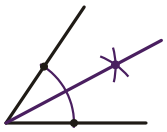
A = 30°, a = 5, b = 8: sin B = $\frac{8 \cdot 0.5}{5} = 0.8$ → B ≈ 53.13° **or** 126.87°: two triangles.

13 CONSTRUCTIONS

compass and straightedge only, no measuring



perpendicular bisector



angle bisector

MEMORY TRICK

Compass for equal, straightedge for straight

Every arc copies a length; every line just joins two points.

Why they work

The crossing points are equidistant from A and B, so they lie on the ⊥ bisector (§5). The angle bisector uses SSS.

Common constructions

Construct	Key steps
⊥ bisector of $\overline{AB}$	equal arcs from A and B (radius > ½AB) cross twice; join the crossings
Angle bisector	arc from the vertex cuts both sides; equal arcs from those points cross; join to the vertex
Copy a segment / angle	transfer the length with the compass; for an angle, copy the arc and its chord
⊥ through a point	arcs mark two points equally far on the line; bisect between them
∥ through a point	copy a corresponding angle at the point
Equilateral △	radius AB arcs from A and from B; they meet at the third vertex
Regular hexagon	step the radius 6 times around the circle