LOGIC · UNITS 1–6 AND 9 · QUICK REFERENCE

LOGIC

Grades 8–12+ | 8 pages

Terms and definitions; categorical propositions, the square of opposition and immediate inferences; syllogisms with the six rules and Venn tests; informal fallacies; propositional logic with truth tables, valid forms and short proofs; the classical tradition.

1 What logic is · 2 Terms · 3 A E I O · 4 Square · 5 Immediate inferences · 6 Syllogisms · 7 Six rules · 8 Venn tests · 9 Named forms · 10 Relevance · 11 Presumption · 12 Ambiguity · 13 Truth tables · 14 Conditionals · 15 Testing validity · 16 Valid forms · 17 Proofs · 18 Machines · 19 Tradition · 20 Dialectic · 21 Argument maps · 22 Practice

1 WHAT LOGIC IS

the art of right reasoning

The three acts of the mind

Act of the mind	Product	Ask of it
Simple apprehension: grasping what a thing is	**Term** (a word or phrase)	Is it clear?
Judgment: affirming or denying	**Proposition** (a statement)	Is it true?
Reasoning: moving from known to new	**Argument**	Does it follow?

Reading an argument

- **Premises** give the reasons; the **conclusion** is the claim they support.
- Premise indicators: because, since, for, given that.
- Conclusion indicators: therefore, so, thus, hence.
- No reasons offered means no argument, only an assertion.

Two kinds of argument

	Deduction	Induction
Claims	the conclusion follows **necessarily**	the conclusion is **probable**
Good form	**valid**	**strong**
Good form + true premises	**sound**	**cogent**

Valid but unsound WORKED

All fish can fly. A salmon is a fish. So a salmon can fly.

the form is **valid**: if both premises were true, the conclusion would be

form

premise 1 is false, so the argument is **unsound**

facts

Why reason well

"Come now, and let us reason together, saith the LORD" (Isaiah 1:18, KJV).

First principles of thought

- **Identity**: a thing is what it is.
- **Non-contradiction**: nothing can be and not be at the same time and in the same respect.
- **Excluded middle**: a statement is either true or false; there is no third option.

Truth and validity

Form	Premises	Conclusion
Valid	all true	**must** be true (sound)
Valid	one or more false	true or false
Invalid	any	true or false; nothing guaranteed

MEMORY TRICK

Form first, facts second

Ask "Does it follow?" before "Is it true?" Valid plus true premises gives sound.

2 TERMS & DEFINITIONS

first act of the mind

How a term is used

Use	Meaning	Example
Univocal	one sense every time	"triangle" in any geometry book
Equivocal	unrelated senses	"pitcher" on a mound, "pitcher" of lemonade
Analogical	different but related senses	"bright" lamp, "bright" student

Five rules of a good definition

- **Convertible**: not too broad, not too narrow.
- **States the essence**, by genus and difference.
- **Not circular.**
- **Not negative** when it can be positive.
- **Not obscure** or figurative.

Rules of division

- One principle at each step.
- Parts do not overlap and leave nothing out.
- No leaps: divide into the next kinds down.

Extension and intension

- **Extension**: the members a term covers.
  Intension: what the term means.
- Add to the meaning and the class shrinks: vehicle, car, electric car.

Porphyry's five predicables

Predicable	Says of a thing	For "square"
Genus	the wider kind	rectangle
Difference	what sets it apart in the genus	with four equal sides
Species	genus + difference	square
Property	follows from the essence, not part of it	diagonals that are equal and perpendicular
Accident	may come and go	drawn in red

MEMORY TRICK

Genus, then difference

Name the family, then the trait no other member of the family has.

Aristotle's ten categories

Substance, quantity, quality, relation, place, time, position, state, action, passion.

Testing a definition WORKED

"A pencil is a writing tool."

pens and crayons fit too: **too broad**

counterexample

"A pencil is a writing tool with a graphite core in a wooden case."

genus + difference

mechanical pencils are left out: **too narrow**; drop "in a wooden case"

fix

A faulty division WORKED

Books: fiction, nonfiction, paperbacks

two principles at once (content and binding): **cross-division**

parts overlap

fix: by content, fiction and nonfiction; by binding, hardcover and paperback

one principle each

3 CATEGORICAL PROPOSITIONS — the four forms

A, E, I, O

Form	Standard form	Quantity and quality	Distributed
A	All S are P	universal affirmative	S
E	No S are P	universal negative	S and P
I	Some S are P	particular affirmative	neither
O	Some S are not P	particular negative	P

> ⚠ **"All … are not"**
> - "All birds are not flightless" could mean E or O. Rewrite it before you test it.

Parts and distribution

- **Quantifier** + **subject** + **copula** (are, are not) + **predicate**.
- **Distributed**: the statement speaks of every member. Universals distribute the subject; negatives, the predicate.

Putting sentences in standard form — WORKED

"Not every bus is on time."

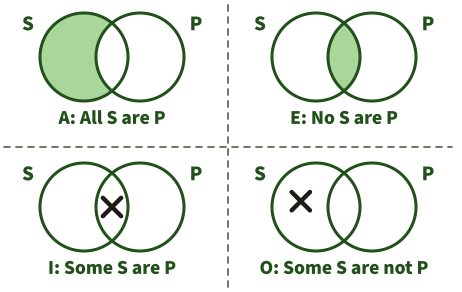
Some buses are not on-time vehicles (**O**)

not every = some … not

"Only ticket holders may enter."

All who enter are ticket holders (**A**)

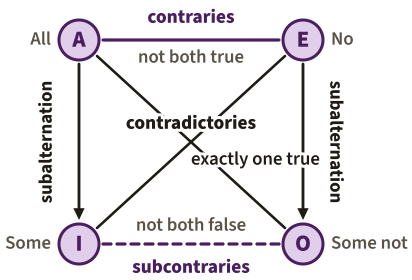
only reverses the terms



Shading = empty; X = at least one member

> **MEMORY TRICK**
> **AffIrmo, nEgO (Traditional)**
> From the Latin: A and I affirm, E and O deny.

4 SQUARE OF OPPOSITION — how the four forms relate



Same subject and predicate at all four corners

The modern (Boolean) square

- Universals do not say anything exists, so only the **contradictories** survive.
- With no dragons in the castle, "All dragons in the castle are asleep" is true; "Some … are asleep" is false.

The four relations

Relation	Pair	Rule
Contradictories	A–O, E–I	exactly one is true
Contraries	A–E	can both be false, never both true
Subcontraries	I–O	can both be true, never both false
Subalternation	A→I, E→O	truth flows down; falsity flows up

Reading the square — WORKED

Given: "All the eggs in the carton are cracked" is false.

A false

O, "Some eggs in the carton are not cracked," is **true**

contradictory

E and I stay **undetermined**: maybe none cracked, maybe some

?

What follows (? = undetermined)

Given	A	E	I	O
A true	T	F	T	F
A false	F	?	?	T
E true	F	T	F	T
E false	?	F	T	?
I true	?	F	T	?
I false	F	T	F	T
O true	F	?	?	T
O false	T	F	T	F

> **MEMORY TRICK**
> **Diagonals always disagree**
> Corners joined by a diagonal never share a truth value.

5 IMMEDIATE INFERENCES — one premise, new proposition

Three immediate inferences

Inference	How	Valid for
Conversion	swap S and P	E, I; A only by limitation (to I)
Obversion	change the quality; P becomes non-P	all four
Contraposition	swap S and P; make both non-	A, O; E only by limitation (to O)

> ⚠ **Illicit conversion**
> - "Some birds are not penguins" is true; its converse "Some penguins are not birds" is false. Never convert O.

The forms side by side

Given	Obverse	Converse	Contrapositive
A All S are P	No S are non-P	Some P are S (limit.)	All non-P are non-S
E No S are P	All S are non-P	No P are S	Some non-P are not non-S (limit.)
I Some S are P	Some S are not non-P	Some P are S	not valid
O Some S are not P	Some S are non-P	not valid	Some non-P are not non-S

A string-instrument test — WORKED

All violins are string instruments. *A*

convert: "All string instruments are violins" is **false** (guitars)

invalid for A

by limitation: Some string instruments are violins

valid

contrapose: All non-string instruments are non-violins

valid for A

> **MEMORY TRICK** **Obvert all, convert EI**
> Obvert anything. Convert E and I. Contrapose A and O.

6 CATEGORICAL SYLLOGISMS

three terms, three propositions

The three terms

- **Major (P)**: predicate of the conclusion; in the **major premise**, written first.
- **Minor (S)**: subject of the conclusion. **Middle (M)**: in both premises only.

Mood

- The **mood** is the three letters (major, minor, conclusion): EAE, AII, …
- 64 moods × 4 figures = 256 forms.
- **15** are valid on any reading; the traditional reading adds **9** more, for 24.

The four figures

Figure	Major premise	Minor premise	Middle term is
1	M–P	S–M	subject, then predicate
2	P–M	S–M	predicate twice
3	M–P	M–S	subject twice
4	P–M	M–S	predicate, then subject

MEMORY TRICK

Find the conclusion first

Its predicate is P and its subject is S. The term left over is M.

Naming mood and figure WORKED

No planets are stars. All gas giants are planets. So no gas giants are stars.

P = stars, S = gas giants, M = planets

from the conclusion

major: No M are P (**E**); minor: All S are M (**A**); conclusion **E**

mood EAE

M is subject, then predicate: figure **1**

***EAE-1**, Celarent: valid*

Standard order

Major premise, then minor premise, then conclusion. Rewrite before naming.

7 RULES OF VALIDITY

six rules, named faults

The six rules

#	Rule	Breaking it
1	Exactly three terms, each in one sense	**four terms** (often an equivocation)
2	The middle term is distributed at least once	**undistributed middle**
3	A term distributed in the conclusion is distributed in its premise	**illicit major** or **illicit minor**
4	Not both premises negative	**exclusive premises**
5	A negative premise requires a negative conclusion, and the reverse	negative and affirmative mismatched
6	Two universal premises cannot yield a particular conclusion	**existential fallacy** (Boolean)

Undistributed middle WORKED

All poets are dreamers. Some dreamers are night owls. So some poets are night owls.

form IAI-1: Some M are P; All S are M; Some S are P

"dreamers" (M) is distributed in neither premise

rule 2 broken

Illicit major WORKED

All cardinals are birds. No robins are cardinals. So no robins are birds.

AEE-1

the conclusion (E) distributes "birds"; the major premise (A) does not

rule 3 broken

Four terms WORKED

Every bank holds money. The grassy edge of the creek is a bank. So it holds money.

"bank" means a business, then a riverside: four terms

rule 1 broken

Passing the rules

- A form that breaks none of rules 2 to 6 is valid; the rules and Venn diagrams always agree.
- Mark each term d or u before testing.

MEMORY TRICK

Middle once, nothing extra

Spread the middle term at least once. The conclusion may not claim more than the premises gave.

8 VENN DIAGRAM TESTS

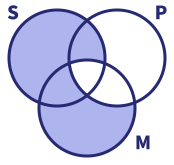
three circles, drawn by rule

The method

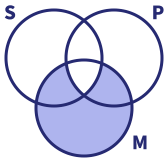
- Draw three overlapping circles: S and P on top, M below.
- Diagram the **universal** premise first (shade what is empty), then the **particular** (X).
- If an X could go in either of two regions, put it **on the line**.
- Valid when the conclusion is **already drawn**. Never draw the conclusion itself.

Testing on the traditional reading

- Put an X in the one open region of the class assumed to have members, then read again.
- Darapti: the X lands in the center, so "Some S are P" shows.

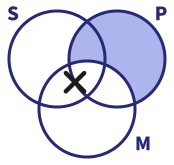


AAA-1
valid

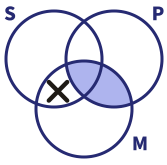


AAI-3
valid if M exists

Left: Barbara. Right: Darapti



AII-2
invalid



EIO-2
valid

Left: undistributed middle. Right: Festino

Reading the diagrams WORKED

AAA-1: all of S outside P is shaded — ***valid***

AAI-3: M is shaded except the center, but nothing says the center has a member

valid only if M exists

AII-2: the X sits on the P line, so it may be outside P

invalid

EIO-2: the X lands in S outside P: "Some S are not P"

valid

MEMORY TRICK **Shade first, X second**

Universals shade, particulars mark. An X with two possible homes goes on the line.

9 NAMED FORMS & CHAINS

the medieval names

The 24 named forms

Fig.	Valid on any reading	Only if a class has members
1	Barbara, Celarent, Darii, Ferio	Barbari (S), Celaront (S)
2	Cesare, Camestres, Festino, Baroco	Cesaro (S), Camestrop (S)
3	Disamis, Datisi, Bocardo, Ferison	Darapti (M), Felapton (M)
4	Camenes, Dimaris, Fresison	Bamalip (P), Fesapo (M), Camenop (S)

Carroll's puzzles

Lewis Carroll's Symbolic Logic (1896) turned sorites into playful puzzles: line up the premises so each predicate meets the next subject.

Reading a name

- Its first three vowels give the mood: Ce**la**r**e**nt = EAE.
- The letter in parentheses is the class that must have members.

Supplying an enthymeme WORKED

"The bakery is closed, since today is a holiday."

missing: "Every holiday is a day the bakery is closed."

unstated major premise

completed: All M are P; today is M; so today is P

Barbara, valid

MEMORY TRICK

The memory verse (Traditional)

"Barbara, Celarent, Darii, Ferioque prioris": the four forms of the first figure.

A sorites (chain) WORKED

All tigers are cats. All cats are mammals. All mammals are animals.

link 1: so all tigers are mammals; link 2: so all tigers are animals

each link AAA-1

10 FALLACIES OF RELEVANCE

true but beside the point

Kinds of fallacy

A **fallacy** only seems to support its conclusion. **Formal** flaws lie in the shape (§16); **informal** ones in the content: relevance, presumption, ambiguity.

Attacking the person

Fallacy	Original example
Abusive ad hominem: attacks the person	"Don't take Mia's advice on bike repair; her room is a mess."
Circumstantial ad hominem: blames the situation	"Of course the coach says drills help; drills are his job."
Tu quoque ("you too")	"Why should I floss? You skipped it last night."

MEMORY TRICK **Aim at the argument**

Ask what the reasons prove, not who gave them. A real expert in the field is fair evidence.

Judging the source

Fallacy	Original example
Poisoning the well: discredits in advance	"Before Sam describes the field trip, remember he exaggerates."
Genetic fallacy: judges by origin	"That recipe came off a cereal box, so it can't be good."
Appeal to authority outside the field	"A famous goalkeeper says this vitamin cures colds."

The crowd and the calendar

Fallacy	Original example
Ad populum (bandwagon)	"Everyone in class has these sneakers, so they must be the best."
Appeal to tradition or **novelty**	"We have always mowed on Saturday." "It's the newest case, so it protects better."

Appeals to feeling and force

Fallacy	Original example
Appeal to pity	"I deserve an A on this quiz; I was up all night."
Appeal to fear	"Skip this brand of sunscreen and your skin will be ruined forever."
Appeal to force (ad baculum)	"Say my drawing is best, or you can't sit with us."

Dodging the point

Fallacy	Original example
Red herring: changes the subject	Asked why the dishes aren't done: "That kitchen light has flickered for weeks."
Straw man: refutes a weaker version	"You want a shorter recess? So you think kids shouldn't play at all."

11 FALLACIES OF PRESUMPTION

a hidden, unproved assumption

Assuming too much

Fallacy	Original example
Begging the question: assumes the conclusion	"This is the most exciting book, because no book is more thrilling."
Question-begging word: a loaded label	"We should drop this useless spelling rule."
Complex question: hides an assumption	"Have you stopped leaving your bike out in the rain?"
False dilemma: only two options offered	"Either you join the soccer team or you don't care about fitness."

Generalizing badly

Fallacy	Original example
Hasty generalization: too few cases	"Two mangoes from this store were sour, so all its fruit is bad."
Accident: a rule pushed past its purpose	"Running is healthy, so Grandpa should jog on his sprained ankle."
Slippery slope: an unproved chain	"Let the puppy on the couch once and soon he'll eat at the table."

Answering a presumption

- Split a complex question; name a third option; look for a common cause.

Mistaking causes

Fallacy	Original example
Post hoc: after, so because	"I wore my blue socks and we won, so the socks won the game."
False cause: ignores a common cause	"On days the pool sells more lemonade, more swimmers get sunburned, so lemonade causes sunburn." (Hot sun causes both.)

MEMORY TRICK

What is assumed here?

Before you answer, find the claim the speaker slipped in without proof.

12 AMBIGUITY & FAIR PLAY

words that shift meaning

Shifting words

Fallacy	Original example
Equivocation: one word, two senses	"The sign says 'Fine for parking here,' so parking here must be fine."
Amphiboly: loose grammar	"Grandma saw the hiker with binoculars." Who had them?
Accent: stress or quoting out of context	A review said "surprisingly good for a cafeteria"; the poster quotes "surprisingly good."

Parts and wholes

Fallacy	Original example
Composition: parts to whole	"Every brick is light, so the wall is light."
Division: whole to parts	"The orchestra is loud, so each violinist plays loudly."

Collective or distributive?

- **Distributive**: said of each member ("The players are tall").
- **Collective**: said of the group as a whole ("The players ate a mountain of pasta").
- Composition and division slide from one to the other.

Naming fallacies fairly

- **Charity**: read an argument in its most reasonable sense first.
- Answer the **strongest form** of the other side.
- The **fallacy fallacy**: a bad argument does not make its conclusion false.
- Avoid **fallacy hunting**; aim to understand, then repair.

In advertising

- **Weasel words**: "helps fight," "up to," "may."
- **Unfinished comparison**: "Now crunchier!" Than what?
- **Puffery**: "simply the best," which no one can test.

Repairing a weak argument WORKED

"Everyone loves this trail, so it's the best hike."

ad populum

repair: "This trail has shade, water and views of the falls, so it's the best hike for summer."

reasons, not a crowd

MEMORY TRICK **Pin the word down**

If a key word could mean two things, define it before you argue.

13 CONNECTIVES & TRUTH TABLES

building compound statements

The five connectives

Connective	Symbol	Read	True when
Negation	~p	not p	p is false
Conjunction	p • q	p and q	both are true
Disjunction	p ∨ q	p or q (or both)	at least one is true
Conditional	p ⊃ q	if p, then q	not (p true, q false)
Biconditional	p ≡ q	p if and only if q	both match

⚠ Parentheses matter

- ~(p • q) means "not both." ~p • q means "not p, and q."

Truth table for every connective

p	q	~p	p • q	p ∨ q	p ⊃ q	p ≡ q
T	T	F	T	T	T	T
T	F	F	F	T	F	F
F	T	T	F	T	T	F
F	F	T	F	F	T	T

Evaluating a compound WORKED

p true, q false, r true: (p ∨ q) ⊃ ~r

p ∨ q = T; ~r = F — *inside first*

T ⊃ F = **F** — *main connective ⊃*

Building a table

- Rows = 2 to the number of letters: 2 letters, 4 rows; 3 letters, 8 rows.
- First letter: half T, then half F; each next letter halves the run.
- Work from the inside out; the **main connective** is computed last.
- Other books write ¬, ∧, → and ↔ for ~, •, ⊃ and ≡.

MEMORY TRICK

The conditional fails once

p ⊃ q is false in one row only: p true and q false.

14 CONDITIONALS & EQUIVALENCE

if-then and its relatives

Four related conditionals

Name	Form	Same as p ⊃ q?
Conditional	p ⊃ q	(itself)
Converse	q ⊃ p	no
Inverse	~p ⊃ ~q	no (it matches the converse)
Contrapositive	~q ⊃ ~p	**yes**

A geometry conditional WORKED

"If a figure is a square, then it is a rectangle." — *true*

converse: if a rectangle, then a square — ***false***

contrapositive: if not a rectangle, then not a square — ***true***

Sufficient and necessary

- In p ⊃ q, p is **sufficient** for q, and q is **necessary** for p.
- "p only if q" is p ⊃ q. "p unless q" is ~q ⊃ p, the same as p ∨ q.

Equivalences checked by truth table

Equivalence	Statement
Double negation	~~p ≡ p
De Morgan	~(p • q) ≡ ~p ∨ ~q; ~(p ∨ q) ≡ ~p • ~q
Material implication	p ⊃ q ≡ ~p ∨ q
Contraposition	p ⊃ q ≡ ~q ⊃ ~p
Biconditional	p ≡ q is (p ⊃ q) • (q ⊃ p)
Exportation	(p • q) ⊃ r ≡ p ⊃ (q ⊃ r)

De Morgan's laws: matching columns

p	q	~(p • q)	~p ∨ ~q	~(p ∨ q)	~p • ~q
T	T	F	F	F	F
T	F	T	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

Classifying a statement

- **Tautology**: true in every row (p ∨ ~p).
- **Contradiction**: false in every row (p • ~p).
- **Contingent**: some of each (p ⊃ q).

MEMORY TRICK **Flip and negate both**

The contrapositive always agrees with the conditional. Flip only, or negate only, and it may not.

15 TESTING VALIDITY

hunt for a bad row

The full method

- Give each premise and the conclusion a column.
- An argument is **invalid** if some row has every premise T and the conclusion F; otherwise **valid**.

The shortcut method WORKED

test p ⊃ q, q ⊃ r ∴ p ⊃ r

make the conclusion F: p = T, r = F

assume the bad row

q ⊃ r true forces q = F; p ⊃ q true forces q = T

contradiction

no bad row can exist: **valid**

Testing p ⊃ q, q ∴ p

p	q	p ⊃ q	q (premise)	p (concl.)
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Reading the table WORKED

row 3: p ⊃ q is T, q is T, p is F

premises true, conclusion false

so the argument is **invalid**: affirming the consequent

one bad row is enough

MEMORY TRICK **Hunt for one bad row**

True premises with a false conclusion in any single row sinks the argument.

Full test of p ⊃ q, q ⊃ r ∴ p ⊃ r: no bad row

p	q	r	p ⊃ q	q ⊃ r	p ⊃ r
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

⚠ "Or" means "or both"

- p ∨ q, p ∴ ~q is invalid: in row 1 both are true. Only the wording "but not both" allows it.

16 VALID FORMS & FORMAL FALLACIES

patterns worth memorizing

The conditional forms

Form	Pattern	Original example
Modus ponens (MP)	p ⊃ q, p ∴ q	"If the kettle whistles, the water is boiling. It whistles. So the water is boiling."
Modus tollens (MT)	p ⊃ q, ~q ∴ ~p	"If the bread is done, the crust is brown. The crust is not brown. So the bread is not done."

The dilemma

Form	Pattern	Original example
Constructive dilemma (CD)	(p ⊃ q) • (r ⊃ s), p ∨ r ∴ q ∨ s	"If we take the bus, we wait; if we walk, we get wet. We take the bus or walk. So we wait or get wet."

Chains and choices

Form	Pattern	Original example
Hypothetical syllogism (HS)	p ⊃ q, q ⊃ r ∴ p ⊃ r	"If it snows, school closes; if school closes, we go sledding. So if it snows, we go sledding."
Disjunctive syllogism (DS)	p ∨ q, ~p ∴ q	"The key is in my coat or my backpack. It is not in my coat. So it is in my backpack."

Why the fallacies fail

- A conditional says p is enough for q, not that p is the only route to q.
- So learning q, or learning ~p, settles nothing about the other letter.

Two formal fallacies

Fallacy	Pattern	Original example
Affirming the consequent	p ⊃ q, q ∴ p	"If it rained, the grass is wet. The grass is wet. So it rained." (Sprinklers?)
Denying the antecedent	p ⊃ q, ~p ∴ ~q	"If it rained, the grass is wet. It did not rain. So the grass is dry." (Sprinklers again.)

Test what you hear

"Prove all things; hold fast that which is good" (1 Thessalonians 5:21, KJV).

MEMORY TRICK

Affirm the front, deny the back

Valid: affirm the antecedent (MP) or deny the consequent (MT). The reverse moves are fallacies.

17 SHORT PROOFS

step by valid step

How a proof works

- List the premises and number every line.
- Each new line follows from earlier lines by a named rule; cite the line numbers.
- Rules apply only to **whole lines**.

Rules of inference

Rule	Pattern
MP, MT, HS, DS, CD	the five valid forms (§16)
Simplification (Simp)	p • q ∴ p
Conjunction (Conj)	p, q ∴ p • q
Addition (Add)	p ∴ p ∨ q

Proof 1: prove r ∨ t

#	Line	Reason
1	p ⊃ q	premise
2	q ⊃ r	premise
3	p • s	premise
4	p	3, Simp
5	p ⊃ r	1, 2, HS
6	r	5, 4, MP
7	r ∨ t	6, Add

MEMORY TRICK **Cite every line**

If you cannot name the rule and the lines, the step is not yet proved.

Proof 2, in words and symbols WORKED

The cat is in the attic (p) or the basement (q). If in the basement, the laundry basket is tipped (r). It is not tipped.

1. p ∨ q; 2. q ⊃ r; 3. ~r — *premises*

4. ~q — *2, 3, MT*

5. p: the cat is in the **attic** — *1, 4, DS*

Proof strategy

- Work **backward**: what line would give the conclusion?
- Split conjunctions with Simp; build a needed disjunction with Add.

18 LOGIC & MACHINES

truth tables in circuits

Boole's algebra

- True = **1**, false = **0**. AND behaves like multiplication: 1 × 0 = 0.
- OR is x + y − xy; NOT x is 1 − x.

MEMORY TRICK

Series AND, parallel OR

Switches in series act like AND: all must close. Switches in parallel act like OR: one is enough.

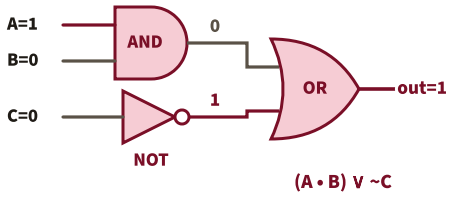
Gates in 1s and 0s

A	B	A AND B	A OR B	NOT A
1	1	1	1	0
1	0	0	1	0
0	1	0	1	1
0	0	0	0	1

Binary numerals WORKED

1101 in binary

8 + 4 + 0 + 1 = **13** *each place doubles*



Each gate applies one connective; wires carry 1 or 0

19 THE CLASSICAL TRADITION

from Aristotle to computers

The trivium

- **Trivium** ("where three roads meet"): grammar, logic, rhetoric. Then the **quadrivium**: arithmetic, geometry, music, astronomy.
- Logic sits in the middle: grammar supplies its words; rhetoric needs it so persuasion rests on truth.

Schoolmen to Watts

Who	When	Contribution
Peter of Spain	1200s	Summulae Logicales, a standard textbook
Thomas Aquinas	1225–1274	commentaries on Aristotle's logic
Port-Royal Logic	1662	conceiving, judging, reasoning, method
Isaac Watts	1724	a logic textbook used for generations

Ancient and early medieval

Who	When	Contribution
Aristotle	384–322 B.C.	founded logic; the syllogism; six works gathered as the **Organon** ("the instrument")
Chrysippus, Stoics	about 279–206 B.C.	whole propositions joined by if, and, or
Boethius	about 480–524	translated Aristotle's logic into Latin
Abelard	1079–1142	Sic et Non ("Yes and No")

MEMORY TRICK

Words, judgments, arguments

Aristotle's order still runs the course: terms, then propositions, then arguments.

Toward symbols and machines

Who	When	Contribution
Leibniz	1646–1716	dreamed of reasoning done like arithmetic
George Boole	1854	The Laws of Thought: an algebra of logic
Gottlob Frege	1879	Begriffsschrift, the start of modern logic
John Venn	1880	overlapping-circle diagrams
Claude Shannon	1937	switching circuits behave like Boole's algebra

The Organon

- Categories (terms), On Interpretation (propositions), Prior Analytics (the syllogism), Posterior Analytics (proof), Topics (probable reasoning), Sophistical Refutations (fallacies).

20 DIALECTIC & RHETORIC

logic in live argument

The Socratic method

- Ask for a **definition**; test it with **counterexamples** and **consequences**.
- A definition that fails is revised or dropped; the puzzled end is **aporia** ("no way through").

A short disputed question WORKED

Whether a tomato is a vegetable?

Objection: it is cooked and served with vegetables.

I answer that: by the botanist's definition it is a fruit (it holds seeds); by the cook's, a vegetable.

distinguo

Reply: the objection is true in the kitchen sense only.

The disputed question (Aquinas)

Part	What it does
Whether …?	a yes-or-no question
Objections	the other side, stated fairly
On the contrary (sed contra)	a brief point toward the answer
I answer that (respondeo)	define, distinguish, give the reasoned answer
Replies	answer each objection in turn

Answering a premise

- **Concedo**: I grant it. **Nego**: I deny it. **Distinguo**: true in one sense, false in another.

MEMORY TRICK **Fair before forceful**

State the other side so well its holders would sign it; then answer it.

Aristotle's three appeals

Appeal	Persuades through
Ethos	the speaker's character: good sense, good character, goodwill
Pathos	fitting feelings in the hearers
Logos	the argument itself

Finding the real question (stasis)

- **Fact**: did it happen? **Definition**: what should we call it?
- **Quality**: was it good or bad? **Procedure**: who should decide, and how?

Four-step refutation

- **They say** (state it fairly), **but** (your answer), **because** (your reason), **therefore** (why it matters).

21 ARGUMENT MAPS

seeing an argument's structure

Drawing a map

- Number each claim; draw arrows from premises to the claims they support.
- The **main conclusion** is the claim no arrow leaves.
- **Joint** premises work only together: bracket them with +.
- **Independent** premises each support the claim alone: give each its own arrow.

Chains and convergence

- **Chain**: 1 → 2 → 3, each claim supporting the next.
- **Convergent**: several independent reasons meet at one conclusion.

Hidden premises

- A **bridge premise** joins what is stated to the conclusion: "All A are B" or "If p, then q."
- Supply the most reasonable one (charity), then judge it.

A passage to map WORKED

(1) The museum closes soon. (2) The drive takes most of an hour. So (3) we would arrive after closing. Besides, (4) we already have tickets for Saturday. So (5) we should go Saturday.

1 + 2 → 3 (joint); 3 → 5 and 4 → 5 (independent)

3 is a sub-conclusion

Reading a map aloud

- Read from the bottom up: "5, because 3 and 4; and 3, because 1 and 2 together."

Finding the bridge WORKED

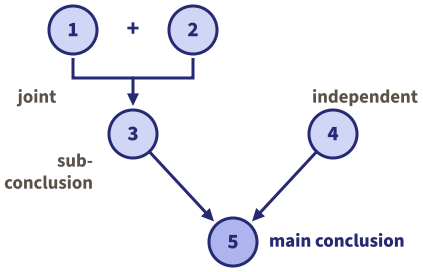
"Maya will be tired tomorrow, since she runs the race today."

bridge: anyone who runs the race is tired the next day

unstated major premise

now ask whether the bridge is true

judge it fairly



The museum argument as a map

The removal test

- Cross out one premise. If the other still gives some support, they are **independent**; if it gives none, they are **joint**.

MEMORY TRICK

No arrow out: the point

On a finished map, the claim with no arrow leaving it is what the speaker is proving.

22 PRACTICE WITH ANSWERS

check yourself

Syllogisms and the square

Problem	Answer
1. Some athletes are musicians. All musicians are early risers. So some early risers are athletes. Mood, figure, verdict?	IAI-4, **Dimaris**: valid
2. No fish are mammals. Some pets are not fish. So some pets are mammals.	EOI-1: **exclusive premises**, invalid
3. Contradictory of "Some songs are not catchy"?	"All songs are catchy"

Inferences and a proof

Problem	Answer
8. Convert "Some teachers are coaches."	"Some coaches are teachers" (valid)
9. Prove q from p ⊃ q, r ⊃ p, r.	4. p (2, 3, MP); 5. q (1, 4, MP)
10. "Some stamps are rare" is false. What follows?	E true; A false; O true

Forms, rules and definitions

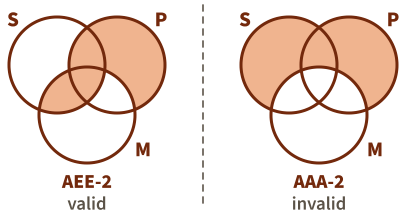
Problem	Answer
15. Test: All P are M; No S are M; so No S are P.	AEE-2, **Camestres**: valid
16. Which rule does AAA-2 break?	**undistributed middle**
17. "A clock is something that ticks." Fault?	too broad (metronomes) and too narrow (digital clocks)

Truth tables and inferences

Problem	Answer
4. Classify (p ⊃ q) ∨ (q ⊃ p).	**tautology**
5. Valid? p ⊃ q, ~p ∴ ~q	invalid: row 3 (denying the antecedent)
6. Obvert "All cars are vehicles."	"No cars are non-vehicles"
7. Equivalent to "If it's a holiday, the library is not open"?	"If the library is open, it is not a holiday"

Connectives and conditionals

Problem	Answer
18. p true, q false: value of ~(p • q)?	**T**
19. Is p ∨ ~q equivalent to q ⊃ p?	**yes** (material implication)
20. Contrapositive of "If it snows, the game is canceled"?	"If the game is not canceled, it did not snow"



Problems 15 and 16, drawn

Name the fallacy

Problem	Answer
11. "This shampoo must be great; it outsells every other brand."	**ad populum**
12. "Our neighbor's dog barked, then the power went out, so the barking cut the power."	**post hoc**
13. "Is your cat still scratching the new couch?"	**complex question**
14. "Each player is excellent, so the team must be excellent."	**composition**

Name the fallacy, part 2

Problem	Answer
21. "My uncle is a dentist, so his advice on car engines must be right."	**appeal to authority**
22. "Don't trust Leo's math answer; he's always late."	**abusive ad hominem**
23. "Either we paint the fence blue or it will look terrible."	**false dilemma**
24. "Skip one practice and you'll quit the team and never play again."	**slippery slope**

MEMORY TRICK

Name it, then explain it

A label is not an answer. Say which rule or relation applies and why.