MATH · KINDERGARTEN–GRADE 5 · QUICK REFERENCE

ARITHMETIC (K–5 · Facts · Methods)

Grades K–5 | 5 pages

Counting and place value, the addition and subtraction strategies, regrouping, the times table and division, the standard algorithms and long division, fractions, decimals and money, measurement, time and the calendar, shapes and angles, perimeter, area and volume, data, ratio, percent and chance.

1 Counting & place value | 2 + and − facts | 3 Regrouping | 4 × and ÷ facts | 5 Big multiplying & dividing | 6 Fractions | 7 Decimals & money | 8 Measurement & time | 9 Geometry | 10 Data & chance

1 COUNTING & PLACE VALUE

K–5 · numbers to 20, 100, 1,000 … billions

Numbers to 20

- Count objects by touching each once; the last number said is **how many**.
- **Ten-frame:** 7 is a row of 5 and 2 more. **Teen numbers** are ten and some more: 14 = 10 + 4.
- **Number bonds:** 10 is 1+9, 2+8, 3+7, 4+6, 5+5. Know every bond to 10 by heart.
- Compare with <, >, =: 8 < 12. The open side faces the bigger number.
- **Skip count** by 2s, 5s, 10s: 5, 10, 15 … ; by 100s.
- **Even** numbers end in 0, 2, 4, 6, 8 and pair up; **odd** end in 1, 3, 5, 7, 9.

MEMORY TRICK **The hungry alligator**

The < or > sign opens its mouth toward the **bigger** number: 23 > 19.

Place value

Billions	Millions	Thousands	Hundreds	Tens	Ones
1,000,000,000	1,000,000	1,000	100	10	1

Commas every three digits. 4,305 = 4,000 + 300 + 0 + 5 (**expanded form**); "four thousand, three hundred five." 10 ones = 1 ten; 10 tens = 1 hundred; 10 hundreds = 1 thousand.

Compare and round

- Compare from the **left**: 4,305 > 4,298 because 3 hundreds > 2 hundreds.
- **Round:** find the place, look at the digit to its right; 5 or more rounds up. 3,462 → 3,500 (hundreds); 3,462 → 3,000 (thousands).
- 10 more / 10 less and 100 more / 100 less change only that digit: 347 + 10 = 357.

Decimal places (grades 4–5)

tenths 0.1, hundredths 0.01, thousandths 0.001. 2.35 = 2 + 0.3 + 0.05. Compare decimals by lining up the points; 0.5 = 0.50 > 0.45.

Roman numerals (grade 3)

I	V	X	L	C	D	M
1	5	10	50	100	500	1,000

Add when a smaller follows a larger (VI = 6, XV = 15); subtract when a smaller comes first (IV = 4, IX = 9, XL = 40, XC = 90). 2026 = MMXXVI.

Multiply and divide by 10, 100, 1,000

× 10 moves every digit one place left (adds a zero to a whole number): 47 × 100 = 4,700. ÷ 10 moves one place right: 4,700 ÷ 10 = 470; 2.5 × 10 = 25; 36 ÷ 100 = 0.36.

Below zero (grade 5)

Negative numbers sit left of 0 on the number line; −3 < 1. Temperatures, elevations and debts use them. −5 is farther from 0 than −2.

2 ADDITION & SUBTRACTION FACTS

K–2 · strategies, not just memory

Addition strategies to 20

Strategy	How	Example
Count on	start at the bigger number, count up 1, 2 or 3	8 + 3: 9, 10, **11**
Doubles	know 1+1 … 10+10	7 + 7 = **14**
Near doubles	a double, then one more	7 + 8 = 14 + 1 = **15**
Make ten	fill the ten, add the rest	8 + 5 = 8 + 2 + 3 = **13**
Add three numbers	look for a ten or a double first	6 + 4 + 7 = 10 + 7 = **17**

Subtraction strategies

Strategy	How	Example
Count back	back 1, 2 or 3	11 − 2: 10, **9**
Think addition	what plus the part makes the whole?	13 − 8: 8 + ? = 13 → **5**
Back to ten	take to 10, then the rest	15 − 7 = 15 − 5 − 2 = **8**
Fact family	one triangle, four facts	4 + 9 = 13, 9 + 4 = 13, 13 − 4 = 9, 13 − 9 = 4
Missing number	use the family	? + 6 = 14 → 14 − 6 = **8**

Story problems (bar models)

- **Join**: part + part = whole. 5 birds, 3 more come: 5 + 3.
- **Separate**: whole − part = part. 9 cookies, eat 4: 9 − 4.
- **Compare**: bigger − smaller = difference. 12 and 7: 12 − 7 = 5 more.
- **Part-part-whole**: draw a bar for the whole, split it into parts; the missing piece tells which operation to use.
- Two steps: solve the first question, then use its answer.

MEMORY TRICK **+ and − are a family**

Every addition fact hides two subtraction facts. If you know 6 + 7 = 13, you know 13 − 6 and 13 − 7. The **=** sign means "is the same as," not "the answer is."

3 ADDING & SUBTRACTING LARGER NUMBERS

grades 2–4 · line up the places, regroup

Adding with regrouping: 478 + 265

	H	T	O
carry	1	1	
	4	7	8
+	2	6	5
=	**7**	**4**	**3**

8 + 5 = 13: write 3, carry 1 ten. 1 + 7 + 6 = 14: write 4, carry 1 hundred. 1 + 4 + 2 = 7.

Subtracting with regrouping: 627 − 349

	H	T	O
regroup	5	11	17
	6	2	7
−	3	4	9
=	**2**	**7**	**8**

7 − 9 won't go: take 1 ten → 17 − 9 = 8. Now 1 − 4 won't go: take 1 hundred → 11 − 4 = 7. Then 5 − 3 = 2.

Steps

1. Line up ones under ones, tens under tens.
2. Add the ones. Ten or more? Write the ones digit, **regroup** 10 ones as 1 ten.
3. Add the tens (with the carried ten), then the hundreds.
4. Estimate to check: 478 + 265 ≈ 500 + 300 = 800 ✓

Across zeros WORKED

400 − 167: regroup 400 as 3 hundreds, 9 tens, 10 ones

then 10 − 7 = 3, 9 − 6 = 3, 3 − 1 = 2

400 − 167 = **233** *check: 233 + 167 = 400 ✓*

Mental math

- **Compensate:** 299 + 456 = 300 + 456 − 1 = 755.
- **Break apart:** 64 + 28 = 64 + 20 + 8 = 92.
- **Count up to subtract:** 83 − 58: 58 → 60 (2), → 80 (20), → 83 (3): 25.
- **Same change:** 72 − 29 = 73 − 30 = 43.
- Adding tens: 40 + 30 = 70 (4 tens + 3 tens).

MEMORY TRICK

Check by going backward

Subtraction is checked by adding: difference + what you took away = the start. Addition is checked by subtracting or by estimating.

⚠ Common slips

- Forgetting the carried 1.
- Subtracting the smaller digit from the larger in the wrong direction (7 − 9 is not 2).

4 MULTIPLICATION & DIVISION FACTS

grades 3–4 · equal groups, arrays, families

Times table to 12 × 12

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

What they mean

- 3 × 4 = 3 groups of 4 = 4 + 4 + 4 = 12; an **array** of 3 rows and 4 columns.
- **Commutative:** 3 × 4 = 4 × 3, so the table is a mirror. 1 × n = n; 0 × n = 0.
- **Division** undoes multiplication: 12 ÷ 4 = 3 asks "4 times what is 12?" Sharing: 12 cookies, 4 children, 3 each. Grouping: 12 cookies, 4 in a bag, 3 bags.
- **Fact family:** 6 × 7 = 42, 7 × 6 = 42, 42 ÷ 6 = 7, 42 ÷ 7 = 6.
- Division by 0 is not allowed; 0 ÷ 5 = 0.

Fact strategies

Facts	Trick
×2	double it
×4	double twice
×5	half of ×10; ends in 0 or 5
×9	×10 minus one group: 9 × 7 = 70 − 7 = 63; digits add to 9
×6, ×7, ×8	split: 7 × 8 = 7 × 5 + 7 × 3 = 35 + 21 (distributive)
×11	11 × 6 = 66
×12	×10 + ×2: 12 × 7 = 70 + 14

MEMORY TRICK

Remainders

17 ÷ 5 = 3 R2: 3 groups of 5 with 2 left. Check: 3 × 5 + 2 = 17. The remainder is always smaller than the divisor.

5 MULTIPLYING & DIVIDING LARGER NUMBERS

grades 4–5 · partial products, the algorithm, long division

Partial products / area model WORKED

$23 \times 47 = (20 + 3)(40 + 7)$

$20 \times 40 = 800 \cdot 20 \times 7 = 140$ *top row*

$3 \times 40 = 120 \cdot 3 \times 7 = 21$ *bottom row*

$800 + 140 + 120 + 21 = \mathbf{1{,}081}$ *add the four boxes*

Standard algorithm: 47 × 23

	Th	H	T	O
			4	7
×			2	3
		1	4	1
+		9	4	0
=	**1**	**0**	**8**	**1**

47 × 3 = 141; 47 × 20 = 940 (write the 0 first); add.

Estimate first

47 × 23 ≈ 50 × 20 = 1,000. Multiplying by 10, 100, 1,000: add that many zeros to a whole number: 36 × 1,000 = 36,000.

Long division: 586 ÷ 4 WORKED

4 into 5 goes **1**; 1 × 4 = 4; 5 − 4 = 1 *divide, multiply, subtract*

bring down the 8 → 18; 4 into 18 goes **4**; 16; 18 − 16 = 2 *bring down, repeat*

bring down the 6 → 26; 4 into 26 goes **6**; 24; 26 − 24 = 2 *remainder 2*

586 ÷ 4 = **146 R2** *check: 146 × 4 + 2 = 586 ✓*

MEMORY TRICK

Does McDonald's Sell Burgers?

Divide, **M**ultiply, **S**ubtract, **B**ring down, then repeat. Every remainder must be smaller than the divisor, or the quotient digit was too small.

Zeros and remainders

- **Zero in the quotient:** if the divisor will not go into the number you brought down, write 0 and bring down the next digit: 816 ÷ 4 = 204.
- **Two-digit divisors (grade 5):** estimate with the front digits: 1,254 ÷ 38 ≈ 1,200 ÷ 40 = 30; try 33: 38 × 33 = 1,254 ✓
- **Interpreting the remainder:** 26 people, vans hold 8: 26 ÷ 8 = 3 R2 → **4 vans** (round up); 26 pencils, 8 each: 3 full boxes (drop it); 26 ÷ 8 = 3¼ dollars each (fraction).

Order of operations (grade 5)

Parentheses first, then × and ÷ left to right, then + and − left to right: 3 + 4 × 2 = 11; (3 + 4) × 2 = 14; 20 − 8 ÷ 4 = 18.

Letters for numbers (grade 5)

A letter stands for a number: P = 2l + 2w; if l = 7 and w = 3, P = 14 + 6 = 20. V = l × w × h. Write the formula, substitute, compute.

6 FRACTIONS

grades 1–5 · equal parts of a whole or a set

What a fraction is

- $\frac{3}{4}$: the **denominator** 4 names the size of the piece (fourths); the **numerator** 3 counts the pieces. A **unit fraction** has numerator 1.
- Halves, thirds, fourths are **equal shares** of a whole or of a set (⅓ of 12 = 4).
- On the number line, $\frac{3}{4}$ is 3 jumps of $\frac{1}{4}$ from 0; $\frac{4}{4} = 1$; $\frac{6}{4} = 1\frac{2}{4}$.
- **Equivalent:** multiply or divide top and bottom by the same number: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$; **simplest form** when no common factor is left: $\frac{6}{8} = \frac{3}{4}$.
- **Mixed ↔ improper:** $2\frac{3}{5} = \frac{13}{5}$ $(2 \times 5 + 3)$; $\frac{11}{4} = 2\frac{3}{4}$.

MEMORY TRICK

Same bottom, count the tops

Pieces can only be added when they are the **same size**. The denominator is the piece's *name*; it never gets added.

Comparing fractions

Case	Rule	Example
Same denominator	more pieces is more	$\frac{5}{8} > \frac{3}{8}$
Same numerator	bigger pieces (smaller bottom) is more	$\frac{1}{3} > \frac{1}{5}$
Benchmarks	compare to 0, ½, 1	$\frac{5}{9} > \frac{1}{2} > \frac{2}{5}$
Common denominator	rewrite, then compare	$\frac{2}{3} = \frac{8}{12}$, $\frac{3}{4} = \frac{9}{12}$

Adding and subtracting WORKED

$\frac{2}{7} + \frac{3}{7} = \mathbf{\frac{5}{7}}$ *like: add the tops*

$\frac{1}{2} + \frac{1}{3} = \frac{3}{6} + \frac{2}{6} = \mathbf{\frac{5}{6}}$ *unlike: common denominator 6*

$3\frac{1}{4} - 1\frac{3}{4} = 2\frac{5}{4} - 1\frac{3}{4} = \mathbf{1\frac{2}{4}} = 1\frac{1}{2}$ *regroup 1 as 4/4*

Multiplying and dividing WORKED

$3 \times \frac{2}{5} = \frac{6}{5} = \mathbf{1\frac{1}{5}}$ *whole × fraction*

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \mathbf{\frac{1}{2}}$ *tops × tops, bottoms × bottoms*

$\frac{2}{3}$ of $24 = 24 \div 3 \times 2 = \mathbf{16}$ *"of" means ×*

$4 \div \frac{1}{3} = \mathbf{12}$ *how many thirds in 4?*

$\frac{1}{2} \div 4 = \mathbf{\frac{1}{8}}$ *half shared 4 ways*

Area with fractional sides

A rectangle $\frac{1}{2}$ by $\frac{3}{4}$ has area $\frac{3}{8}$: tile it with $\frac{1}{4} \times \frac{1}{2}$ pieces… or just multiply.

⚠ Slips

- $\frac{1}{2} + \frac{1}{3}$ is not $\frac{2}{5}$.
- Simplify every answer; keep a mixed number's fraction part proper.

7 DECIMALS & MONEY

grades 4–5 · tenths, hundredths, thousandths

Decimals, fractions and money

Decimal	Fraction	Money	Words
0.1	$\frac{1}{10}$	a dime	one tenth
0.01	$\frac{1}{100}$	a penny	one hundredth
0.25	$\frac{1}{4}$	a quarter	twenty-five hundredths
0.5	$\frac{1}{2}$	a half dollar	five tenths
0.75	$\frac{3}{4}$	three quarters	seventy-five hundredths
1.001	$1\frac{1}{1000}$		one and one thousandth

$3.47 = 3 dollars + 4 dimes + 7 pennies. Money is decimals with two places.

Rules

- **Compare:** line up the points; add zeros to the right: 0.6 = 0.60 > 0.58.
- **Round:** 3.746 → 3.75 (hundredths), → 3.7 (tenths), → 4 (whole).
- **Add or subtract:** points in a column, then add as usual: 4.5 + 0.37 = 4.87; $20.00 − $13.45 = $6.55.
- **Multiply:** ignore the points, multiply, then count the decimal places in both factors: 0.3 × 0.2 = 0.06; 1.5 × 4 = 6.0.
- **Divide by a whole number:** put the point straight up: 7.2 ÷ 4 = 1.8.
- **Divide by a decimal:** move both points until the divisor is whole: 4.8 ÷ 0.6 = 48 ÷ 6 = 8.

Shopping WORKED

3 pens at $1.25: 1.25 × 3 = **$3.75**

Paid with $5: 5.00 − 3.75 = **$1.25 change**

count up: 3.75 → 4 (25¢) → 5 ($1)

12 oz for $2.88: 2.88 ÷ 12 = **$0.24 an ounce**

unit price

Coins and bills

penny 1¢ · nickel 5¢ · dime 10¢ · quarter 25¢ · half dollar 50¢ · dollar 100¢. Count coins from the largest down: quarter, dime, nickel, penny = 25, 35, 40, 41¢. **Make change** by counting up from the price to the amount paid.

MEMORY TRICK **Points stay put**

Add and subtract: points in a **column**. Multiply: **count** places. Divide: **move** both points the same way.

8 MEASUREMENT, TIME & THE CALENDAR

K–5 · units and conversions

Customary units

Measure	Units
Length	12 in = 1 ft · 3 ft = 1 yd · 5,280 ft = 1 mi
Weight	16 oz = 1 lb · 2,000 lb = 1 ton
Capacity	8 fl oz = 1 cup · 2 c = 1 pt · 2 pt = 1 qt · 4 qt = 1 gal

Metric units

Measure	Units
Length	10 mm = 1 cm · 100 cm = 1 m · 1,000 m = 1 km
Mass	1,000 g = 1 kg
Capacity	1,000 mL = 1 L

To convert, multiply going to a smaller unit (3 ft = 36 in) and divide going to a larger unit (500 cm = 5 m). Measure from the 0 mark of the ruler, not its edge.

Time

- 60 seconds = 1 minute · 60 minutes = 1 hour · 24 hours = 1 day · 7 days = 1 week · 12 months = 1 year · 365 days (366 in a leap year).
- The short hand tells the **hour**, the long hand the **minutes**. Each number on the clock is 5 minutes: at the 4 it is :20. Half past = :30; quarter past = :15; quarter to = :45.
- **a.m.** is midnight to noon; **p.m.** is noon to midnight.
- **Elapsed time:** count up in jumps. 10:40 to 1:15: 20 min → 11:00, 2 h → 1:00, 15 min → 1:15: **2 h 35 min**.

Calendar

Days: Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, Saturday. Months: January, February, March, April, May, June, July, August, September, October, November, December. “Thirty days hath September, April, June and November; all the rest have thirty-one, except February alone.”

Measuring sense

- A paper clip is about 1 inch; a door is about 2 m or 7 ft; a liter bottle; a kilogram is a bit over 2 pounds; a gram is a paper clip.
- Length: how long or tall. Weight / mass: how heavy. Capacity: how much it holds. Temperature: °F or °C; water freezes at 32 °F (0 °C), boils at 212 °F (100 °C).
- Read a scale by finding what each tick mark counts.

MEMORY TRICK **Gallon land**

A **G**allon holds 4 **Q**ueens, each queen 2 **P**rinces, each prince 2 **C**hildren: 4 qt, 8 pt, 16 cups in a gallon.

Conversions WORKED

2 yd 1 ft = 6 ft + 1 ft = **7 ft** = 84 in

3.5 kg = **3,500 g** *× 1,000*

2 h 15 min = **135 min** *60 × 2 + 15*

9 SHAPES, ANGLES & MEASURING FIGURES

K–5 · geometry

Flat shapes (polygons)

Shape	Sides	Note
Triangle	3	angles add to 180°
Quadrilateral	4	angles add to 360°
Pentagon	5	home plate in baseball
Hexagon	6	honeycomb
Octagon	8	stop sign

A circle has no sides: center, radius (to the edge), diameter (across, = 2 radii).

Solid shapes

Solid	Faces	Edges	Vertices
Cube	6	12	8
Rectangular prism	6	12	8
Square pyramid	5	8	5
Cylinder	2 flat + 1 curved	2	0
Cone	1 flat + 1 curved	1	1
Sphere	1 curved	0	0

Lines and angles (grades 3–5)

- **Point, line, line segment, ray.** Parallel lines never meet; perpendicular lines meet at right angles.
- **Right** angle = 90° (a square corner); **acute** < 90°; **obtuse** between 90° and 180°; **straight** = 180°. A protractor measures from its 0 line.
- **Triangles by sides:** equilateral (3 equal), isosceles (2 equal), scalene (none). **By angles:** right, acute, obtuse. The three angles always add to 180°: 60° + 70° + ? → 50°.
- **Quadrilateral family:** a square is a rectangle (4 right angles) and a rhombus (4 equal sides) and a parallelogram (2 pairs of parallel sides); a trapezoid has one pair of parallel sides.
- **Symmetry:** a line of symmetry folds a shape onto itself. **Slide, flip, turn** move a shape without changing it.

Perimeter, area, volume

Figure	Formula
Perimeter	add all the sides; rectangle P = 2l + 2w
Area of a rectangle	A = l × w, in square units (tile it with unit squares)
Area of a square	A = s × s
Volume of a box	V = l × w × h, in cubic units (count unit cubes in layers)

Rectangle 5 by 3: P = 16 units, A = 15 square units. Box 4 × 3 × 2: V = 24 cubic units. Area and perimeter can disagree: 1 × 8 and 2 × 4 have the same area but not the same perimeter.

The coordinate plane (grade 5)

A point is (x, y): go **across** x, then **up** y, from the origin (0, 0). Plot a pattern's pairs, such as (1, 2), (2, 4), (3, 6), and look for the rule: y is double x.

MEMORY TRICK

Across the hall, then up the stairs

In (x, y), x comes first and runs along the floor; y climbs. (3, 5) is not (5, 3).

10 DATA, RATIO, PERCENT & CHANCE

grades 1–5 · graphs, averages, likelihood

Showing data

- **Tally chart:** |||| = 4, a bundle of five crosses the four.
- **Pictograph:** read the key; one picture may stand for 2 or 10.
- **Bar graph:** read the scale on the axis; bars can count by 2s, 5s or 10s.
- **Line plot:** an X for each value on a number line; shows the spread.
- **Times as many (grade 4):** 12 is 3 times as many as 4; 12 ÷ 4 = 3.

Averages (grade 5)

Mean = total ÷ how many: 7, 9, 14 → 30 ÷ 3 = 10. **Median** = middle value in order. **Mode** = most common. **Range** = largest − smallest.

Ratios (grade 5)

A ratio compares two quantities: 2 red to 3 blue = 2:3 = $\frac{2}{3}$. A **ratio table** scales both parts alike: 2:3, 4:6, 6:9, 8:12. 6 red paired with 9 blue keeps the same ratio.

Percent (grades 4–5)

Percent means out of a hundred: 25% = $\frac{25}{100}$ = $\frac{1}{4}$ = 0.25. 50% is half; 10% of 80 is 8 (80 ÷ 10); 25% of 80 is 20 (80 ÷ 4); 5% is half of 10%; 100% is the whole.

Percent of a number WORKED

30% of 60: 10% is 6, so 30% is 3 × 6 = **18**

A $40 game at 25% off: 40 ÷ 4 = 10 off → **$30**

Chance

- **Certain, likely, unlikely, impossible**: a flipped coin is equally likely heads or tails.
- **Probability as a fraction** = $\frac{\text{ways it can happen}}{\text{all the equally likely ways}}$. A die: P(4) = $\frac{1}{6}$; P(even) = $\frac{3}{6}$ = $\frac{1}{2}$; P(7) = 0; P(1 to 6) = 1.
- **Experiments:** do many trials; the results tend toward the fraction. 100 spins expect about 25 reds on a four-color spinner.
- **Money and budgets (grade 5):** income − spending = savings; plan a budget before you spend.

MEMORY TRICK **Fraction of the whole**

Percent, probability and ratio are all **fractions in disguise**: a part compared to a whole (or to another part).