PRE-ALGEBRA · GRADES 6–8 · QUICK REFERENCE

PRE-ALGEBRA (Arithmetic → Algebra)

Grades 6–8 · 6 pages

Place value and properties, divisibility and primes, fractions, decimals, integers, exponents and roots, ratios, percent, expressions and equations, the coordinate plane, geometry formulas, and data and probability: every rule and formula from the thirteen units, with worked examples and memory tricks.

1 Number sense · 2 Factors & primes · 3 Fractions · 4 Decimals · 5 Integers · 6 Exponents & roots · 7 Ratios · 8 Percent · 9 Expressions & equations · 10 Coordinate plane · 11 Geometry · 12 Data & probability

1 WHOLE NUMBERS & NUMBER SENSE

Unit 1 · place value, properties, order

Place value

Billions	Millions	Thousands	Ones
1,000,000,000	1,000,000	1,000	1
1 billion = 1,000 millions	1 million = 1,000 thousands		4,056 = 4 × 1,000 + 0 × 100 + 5 × 10 + 6 × 1

Read in groups of three: 4,056,210 is "four million, fifty-six thousand, two hundred ten." **Round:** look one place right; 5 or more rounds up: 3,462 → 3,500. **Estimate first:** 48 × 21 ≈ 50 × 20 = 1,000.

Properties

Property	Addition	Multiplication
Commutative	$a + b = b + a$	$a \cdot b = b \cdot a$
Associative	$(a + b) + c = a + (b + c)$	$(a \cdot b) \cdot c = a \cdot (b \cdot c)$
Identity	$a + 0 = a$	$a \cdot 1 = a$
Zero	—	$a \cdot 0 = 0$
Distributive	—	$a(b + c) = ab + ac$

Mental math by distributing: 7 × 98 = 7(100 − 2) = 700 − 14 = **686**; 6 × 43 = 6(40 + 3) = 240 + 18 = **258**.

Order of operations

G grouping ( ) [ ] fraction bars · **E** exponents · **MD** multiply and divide, left → right · **AS** add and subtract, left → right

$5^2 = 5 \times 5 = 25 \cdot 2^3 = 2 \times 2 \times 2 = 8$

Order of operations WORKED

$3 + 4 \times 5 = 3 + 20 = \mathbf{23}$	*multiply first*
$18 \div 3 \times 2 = 6 \times 2 = \mathbf{12}$	*left → right*
$2[5 + (8 - 2)^2] = 2[5 + 36] = \mathbf{82}$	*inside out*

MEMORY TRICK **Partners, not a line**

M and D are **partners**, left to right; so are A and S. 18 ÷ 3 × 2 is 12, not 3.

2 FACTORS, MULTIPLES & PRIMES

Unit 2 · divisibility, GCF, LCM

Divisibility rules

By	Test	Example
2	last digit even	3,8**4**
3	digit sum divisible by 3	5,1**3**: 5+1+3 = 9
4	last two digits divisible by 4	7,3**16**
5	ends in 0 or 5	2,3**5**
6	divisible by 2 *and* 3	4**2**: even, 4+2 = 6
9	digit sum divisible by 9	7,2**9**: 7+2+9 = 18
10	ends in 0	9**0**

Primes under 50

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47. A prime has exactly two factors, 1 and itself; 1 is neither prime nor composite; 2 is the only even prime.

Prime factorization WORKED

$360 = 2 \times 180 = 2 \times 2 \times 90 = 2 \times 2 \times 2 \times 45$ *divide by small primes*

$= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = \mathbf{2^3 \times 3^2 \times 5}$ *factor tree*

GCF and LCM from factorizations WORKED

$48 = 2^4 \times 3 \quad 72 = 2^3 \times 3^2$

GCF = $2^3 \times 3 = \mathbf{24}$ *lowest power of each shared prime*

LCM = $2^4 \times 3^2 = \mathbf{144}$ *highest power of every prime*

GCF × LCM = 24 × 144 = 3,456 = 48 × 72 *check*

Euclidean algorithm for GCF WORKED

GCF(252, 105): 252 = 2 × 105 + 42 *larger = q × smaller + r*

105 = 2 × 42 + 21 *GCF(larger, smaller) = GCF(smaller, r)*

42 = 2 × 21 + 0 → GCF = **21** *last nonzero remainder*

Words

factor a number that divides evenly: factors of 12 are 1, 2, 3, 4, 6, 12

multiple the times table: multiples of 4 are 4, 8, 12, …

GCF greatest common factor, the largest shared factor

LCM least common multiple, the smallest shared multiple

composite has more than two factors

MEMORY TRICK

GCF goes down, LCM goes up

The GCF is never bigger than the smaller number; the LCM is never smaller than the larger number. If one number divides the other, that number *is* the GCF and the other is the LCM.

⚠ Traps

- A number is not prime just because it is odd: 9 = 3 × 3, 51 = 3 × 17.
- GCF(a, b) × LCM(a, b) = a × b works for two numbers only.

3 FRACTIONS

Unit 3 · the piece is named by its denominator

Rules

- **Equivalent:** $\frac{a}{b} = \frac{a \times k}{b \times k}$; **simplify** by dividing top and bottom by their GCF: $\frac{18}{24} = \frac{3}{4}$.
- **Compare:** $\frac{a}{b} > \frac{c}{d}$ exactly when $ad > bc$. $\frac{5}{8}$ vs $\frac{2}{3}$: $15 < 16$, so $\frac{5}{8} < \frac{2}{3}$.
- **Add or subtract:** same denominator first (the LCD = LCM of the denominators), then add the tops. $\frac{a}{c} \pm \frac{b}{c} = \frac{a \pm b}{c}$.
- **Multiply:** tops × tops, bottoms × bottoms; cancel first. $\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$.
- **Divide:** keep, change, flip. $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$.
- **Mixed numbers:** $a\frac{b}{c} = \frac{ac + b}{c}$; $3\frac{2}{5} = \frac{17}{5}$.

Adding unlike fractions WORKED

$\frac{1}{4} + \frac{2}{3}$ → LCD of 4 and 3 is 12

$\frac{3}{12} + \frac{8}{12} = \mathbf{\frac{11}{12}}$ *multiply top and bottom alike*

Estimate: $\frac{1}{4}$ + a bit over $\frac{1}{2} \approx$ just under 1 ✓

Multiply and divide WORKED

$\frac{3}{4} \times \frac{8}{9} = \frac{1}{1} \times \frac{2}{3} = \mathbf{\frac{2}{3}}$ *cancel 3 and 4 first*

$\frac{5}{6} \div \frac{2}{3} = \frac{5}{6} \times \frac{3}{2} = \frac{15}{12} = \mathbf{1\frac{1}{4}}$ *keep, change, flip*

$2\frac{1}{2} \times 1\frac{1}{3} = \frac{5}{2} \times \frac{4}{3} = \frac{20}{6} = \mathbf{3\frac{1}{3}}$ *improper first*

MEMORY TRICK

Common LCDs by heart

2 and 3 → 6 · 3 and 4 → 12 · 4 and 6 → 12 · 6 and 8 → 24 · 3 and 5 → 15 · 8 and 12 → 24. Estimate against the benchmarks 0, ½ and 1 before you compute.

⚠ Traps

- Never add denominators: $\frac{1}{3} + \frac{1}{4} \neq \frac{2}{7}$.
- Change the top when you change the bottom.
- Simplify every answer; a mixed number should have a proper fraction part.

Word problems

Recipes: scale factor = $\frac{\text{servings wanted}}{\text{servings the recipe makes}}$; new amount = scale factor × original. "Of" means multiply: $\frac{2}{3}$ of 24 = 16.

4 DECIMALS

Unit 4 · fractions with 10, 100, 1,000 underneath

Rules

- $0.1 = \frac{1}{10}$, $0.01 = \frac{1}{100}$, $0.001 = \frac{1}{1{,}000}$; $0.7 = 0.70 = 0.700$. Read 3.25 as "three and twenty-five hundredths."
- **Add or subtract:** line up the decimal points; fill with zeros. 4.5 + 0.37 = 4.50 + 0.37 = 4.87.
- **Multiply:** multiply as whole numbers, then count the decimal places in both factors and put that many in the product. 0.3 × 0.2 = 0.06.
- **Divide:** move the point in the divisor until it is a whole number and move the dividend's point the same way. 4.8 ÷ 0.6 = 48 ÷ 6 = 8.
- × 10, 100, 1,000 moves the point 1, 2, 3 places right; ÷ moves it left.

Fraction ↔ decimal ↔ percent

Fraction	Decimal	Percent
$\frac{1}{2}$	0.5	50%
$\frac{1}{4}$	0.25	25%
$\frac{3}{4}$	0.75	75%
$\frac{1}{5}$	0.2	20%
$\frac{1}{8}$	0.125	12.5%
$\frac{1}{10}$	0.1	10%
$\frac{1}{3}$	$0.\overline{3}$	33⅓%
$\frac{2}{3}$	$0.\overline{6}$	66⅔%

$\frac{a}{b} = a \div b$. In lowest terms, a fraction **terminates** only when its denominator is $2^m \times 5^n$; otherwise it repeats.

Change and unit price WORKED

Paid $20 for $13.45: 20.00 − 13.45 = **$6.55**
line up points

12 oz for $2.88: 2.88 ÷ 12 = **$0.24 per oz**
unit price = total ÷ units

0.25 × 1.2 = 25 × 12 = 300 → **0.300 = 0.3**
3 decimal places

MEMORY TRICK

Points stay put

Adding and subtracting: points in a **column**. Multiplying: **count** places. Dividing: **move** both points the same number of places.

5 INTEGERS & RATIONAL NUMBERS

Unit 5 · signs and the number line

Signed numbers

- $|a|$ is the distance from a to 0: $|-7| = 7$. Farther right on the number line is greater: $-2 > -5$.
- **Add, same signs:** add the absolute values, keep the sign. $-4 + (-7) = -11$.
- **Add, different signs:** subtract the absolute values, keep the sign of the larger. $-9 + 4 = -5$.
- **Subtract** = add the opposite: $a - b = a + (-b)$. $5 - (-3) = 5 + 3 = 8$; $-2 - 6 = -8$.
- **Multiply or divide:** same signs → +, different signs → −. $(-6)(-4) = 24$; $-18 \div 3 = -6$.
- An **even** count of negative factors gives +, an odd count gives −.

Mixed signs WORKED

$-3 + 8 - (-2) = 5 + 2 = \mathbf{7}$ *change − (−2) to + 2*

$(-2)(-3)(-4) = 6 \times (-4) = \mathbf{-24}$ *three negatives: odd*

$\frac{-3}{4} + \frac{1}{2} = \frac{-3}{4} + \frac{2}{4} = \mathbf{\frac{-1}{4}}$ *rational numbers, same rules*

$(-5)^2 = 25$ but $-5^2 = \mathbf{-25}$ *the exponent touches only the 5*

Change

change = new value − old value. A temperature from −4° to 7° changes by $7 - (-4) = \mathbf{+11°}$.

MEMORY TRICK

Same signs, happy sign

For × and ÷: signs **alike** give a **positive**; signs **different** give a **negative**. Adding is different: look at which absolute value is bigger.

⚠ Zero

- $0 \div a = 0$ ($a \neq 0$), but $a \div 0$ is **undefined**.
- $\frac{-a}{b} = \frac{a}{-b} = -\frac{a}{b}$: one negative sign may sit anywhere.

6 EXPONENTS & SQUARE ROOTS

Unit 6 · powers, scientific notation, roots

Laws of exponents

Law	Rule	Example
Product	$a^m \cdot a^n = a^{m+n}$	$2^3 \cdot 2^4 = 2^7$
Quotient	$\frac{a^m}{a^n} = a^{m-n}$	$5^6 \div 5^2 = 5^4$
Power	$(a^m)^n = a^{mn}$	$(3^2)^4 = 3^8$
Zero	$a^0 = 1$ ($a \neq 0$)	$7^0 = 1$
Negative	$10^{-n} = \frac{1}{10^n}$	$10^{-3} = \frac{1}{1{,}000} = 0.001$

a^n means n factors of a: $2^5 = 32$. 10^n is 1 followed by n zeros. $(-3)^2 = 9$ but $-3^2 = -9$.

Scientific notation

$a \times 10^n$ with $1 \leq a < 10$. $4{,}500{,}000 = 4.5 \times 10^6$; $0.0032 = 3.2 \times 10^{-3}$. Multiply: $(3 \times 10^4)(2 \times 10^5) = 6 \times 10^9$.

Square roots

- $\sqrt{a} = b$ means $b \times b = a$ and $b \geq 0$. $\sqrt{49} = 7$; $\sqrt{(a^2)} = a$ for $a \geq 0$.
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$: $\sqrt{\frac{9}{16}} = \frac{3}{4}$.
- **Estimate:** if $a^2 < n < b^2$, then $a < \sqrt{n} < b$. $7^2 = 49 < 55 < 64 = 8^2$, so $\sqrt{55} \approx 7.4$.
- $\sqrt{2}$, $\sqrt{3}$, π are **irrational**: their decimals never end or repeat.

MEMORY TRICK

The number family

counting ⊂ whole ⊂ integers ⊂ **rational** ⊂ real; the irrationals fill the rest of the real line.

Perfect squares to 20^2

n	1	2	3	4	5	6	7	8	9
n^2	1	4	9	16	25	36	49	64	81
n	11	12	13	14	15	16	17	18	19
n^2	121	144	169	196	225	256	289	324	361

Cubes: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1,000.

Estimating a root WORKED

$\sqrt{40}$: $36 < 40 < 49 \rightarrow 6 < \sqrt{40} < 7$ *between perfect squares*

40 is 4 past 36 of a 13 gap → $\sqrt{40} \approx \mathbf{6.3}$ *$6.3^2 = 39.69$ ✓*

7 RATIOS, RATES & PROPORTIONS

Unit 7 · comparing by division

Ratios and rates

- a to b = a : b = $\frac{a}{b}$; scale a ratio by multiplying both parts: 2:3 = 6:9.
- **Unit rate:** $\frac{\text{first quantity}}{\text{second quantity}}$ with a second quantity of 1. 150 miles in 3 hours = 50 mi/h.
- speed = $\frac{\text{distance}}{\text{time}}$; distance = speed × time; unit price = $\frac{\text{total price}}{\text{number of units}}$.
- **Proportion:** $\frac{a}{b} = \frac{c}{d}$ exactly when ad = bc (cross products).

Solving a proportion WORKED

$\frac{3}{4} = \frac{x}{20} \rightarrow 4x = 3 \times 20 = 60$ *cross multiply*

$x = 60 \div 4 =$ **15**

Scale drawings

scale = $\frac{\text{drawing length}}{\text{actual length}}$, both in the same unit. Map scale 1 in : 50 mi; 3.5 in on the map = 3.5 × 50 = **175 mi**.

Unit conversion WORKED

5 yd to inches: $5 \text{ yd} \times \frac{3 \text{ ft}}{1 \text{ yd}} \times \frac{12 \text{ in}}{1 \text{ ft}} =$ **180 in**

multiply by fractions equal to 1

2.5 km to m: 2.5 × 1,000 = **2,500 m**

metric: move the point

Conversions

Length	Weight / Capacity
12 in = 1 ft · 3 ft = 1 yd · 5,280 ft = 1 mi	16 oz = 1 lb · 2,000 lb = 1 ton
1 km = 1,000 m · 1 m = 100 cm · 1 cm = 10 mm	8 fl oz = 1 c · 2 c = 1 pt · 2 pt = 1 qt · 4 qt = 1 gal
1 in = 2.54 cm · 1 mi ≈ 1.61 km	1 kg = 1,000 g · 1 L = 1,000 mL · 1 kg ≈ 2.2 lb

Set up each factor so the unit you are leaving cancels.

8 PERCENT

Unit 8 · per hundred

Rules

- n% = $\frac{n}{100}$. Percent → decimal: divide by 100 (35% = 0.35). Decimal → percent: multiply by 100 (0.08 = 8%).
- n% of A = $\frac{n}{100}$ × A. 10% of A = A ÷ 10; 1% = A ÷ 100; 5% is half of 10%; 25% = ¼; 50% = ½.
- **part = percent × whole**; $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$; percent = $\frac{\text{part}}{\text{whole}} \times 100\%$; whole = part ÷ percent.
- **Percent change** = $\frac{\text{amount of change}}{\text{original amount}} \times 100\%$.

MEMORY TRICK

Three questions, one triangle

Part = % × whole. Cover the one you want: part ÷ whole gives the percent; part ÷ percent gives the whole.

The three kinds WORKED

What is 30% of 80? 0.30 × 80 = **24** *part*

18 is what % of 24? 18 ÷ 24 = 0.75 = **75%** *percent*

12 is 40% of what? 12 ÷ 0.40 = **30** *whole*

Price 40 → 50: change 10; 10 ÷ 40 = **25% increase**

original on the bottom

Money

- sale price = (100% − discount rate) × regular price: 20% off $45 = 0.80 × 45 = $36.
- total with tax = (100% + tax rate) × price: 6% tax on $36 = 1.06 × 36 = $38.16.
- tip: 15% of $40 = 10% + 5% = 4 + 2 = $6.
- selling price = (100% + markup rate) × cost.
- **Simple interest:** I = Prt; balance = P + I. $500 at 4% for 3 years: I = 500 × 0.04 × 3 = $60.
- Compound interest adds r × balance each year, so the balance grows faster.

9 VARIABLES, EXPRESSIONS & EQUATIONS

Units 9–10 · the language of algebra

Expressions

- In $4x^2 - 7x + 9$: **4** is a coefficient, **9** a constant, $4x^2$, $-7x$ and 9 are **terms**. $x = 1x$; $-x = -1x$.
- **Like terms** have the same variable part: $ax + bx = (a + b)x$; $5y - 2y + 3 = 3y + 3$.
- **Distribute:** $a(b + c) = ab + ac$; $-(b - c) = -b + c$; $3(2x - 5) = 6x - 15$. **Factor** backward: $4x + 12 = 4(x + 3)$.
- **Evaluate** by substituting, then order of operations: $2l + 2w$ with $l = 7$, $w = 3$ is $14 + 6 = 20$.

Words → symbols

Words	Symbols
b less than a	$a - b$
b more than a	$a + b$
k times the sum of a and b	$k(a + b)$
the quotient of a and b	$\frac{a}{b}$
at least / at most	$\geq$ / $\leq$
more than / fewer than	$>$ / $<$
consecutive integers	n, n + 1, n + 2
consecutive even or odd	n, n + 2, n + 4

Solving equations

Whatever you do to one side, do to the other: if $a = b$ then $a + c = b + c$, $a - c = b - c$, $ac = bc$, $a \div c = b \div c$ ($c \neq 0$). Undo + with −, × with ÷.

1. Simplify each side (distribute, combine like terms).
2. Get the variable terms on one side, constants on the other.
3. Undo + or −.
4. Undo × or ÷.
5. **Check** in the original equation.

Two-step and multi-step WORKED

$3x + 5 = 20 \rightarrow 3x = 15 \rightarrow x = \mathbf{5}$ — *undo +5, then ×3*

$\frac{x}{4} - 2 = 1 \rightarrow \frac{x}{4} = 3 \rightarrow x = \mathbf{12}$ — *undo −2, then ÷4*

$2(x - 3) = x + 4 \rightarrow 2x - 6 = x + 4 \rightarrow x = \mathbf{10}$ — *distribute, collect x*

$\frac{2}{3}x = 8 \rightarrow x = \frac{3}{2} \times 8 = \mathbf{12}$ — *multiply by the reciprocal*

Inequalities

- Solve like an equation, but **flip the sign** when you multiply or divide by a negative: $-2x < 6 \rightarrow x > -3$.
- Graph: open circle for $<$ or $>$, closed circle for $\leq$ or $\geq$, shade toward the solutions.
- "at least 12" means $x \geq 12$; "fewer than 5" means $x < 5$.

Word problem WORKED

Three consecutive integers sum to 48: $n + (n+1) + (n+2) = 48$

$3n + 3 = 48 \rightarrow 3n = 45 \rightarrow n = 15 \rightarrow$ **15, 16, 17**

check: 48 ✓

Perimeter 26, length 3 more than width: $2w + 2(w + 3) = 26 \rightarrow w = \mathbf{5}$

MEMORY TRICK

Golden rule of equations

Do unto one side as you do unto the other. Check every answer by putting it back.

10 THE COORDINATE PLANE & LINES

Unit 11 · graphs, slope, y = mx + b

The plane

- A point is (x, y): x right or left, then y up or down. Origin (0, 0).
- Quadrants: I (+, +), II (−, +), III (−, −), IV (+, −).
- Distance along a horizontal line: $|x_2 - x_1|$; along a vertical line: $|y_2 - y_1|$.
- $y = c$ is a horizontal line; $x = c$ is a vertical line. A point is on a line when its coordinates make the equation true.

Tables and rules

input x → rule → output y. If y rises by m each time x rises by 1, the rule begins $y = m \cdot x + \ldots$. Plot the pairs; a linear rule makes a straight line.

Slope

(1, 3) and (4, 9): $m = (9 - 3) \div (4 - 1) = 6 \div 3 = \mathbf{2}$

$$\mathbf{m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{rise}{run}}$$

Kinds of slope

uphill +, downhill −, horizontal 0, vertical **undefined**. Parallel lines share a slope.

MEMORY TRICK

Rise over run

Rise first (up is +, down is −), then run (right is +). Read a slope from a graph by walking from one lattice point to the next.

Slope-intercept form

m = slope, (0, b) = y-intercept. cost = fee + rate × hours.

$$\mathbf{y = mx + b}$$

Graph y = 2x − 1 WORKED

Start at (0, −1); slope 2 = rise 2, run 1 → (1, 1), (2, 3)

plot, then draw the line

Check (3, 5): $2(3) - 1 = 5$ ✓

Direct variation

$y = kx$ with $k = \frac{y}{x}$ constant; the graph is a line through the origin. If $y = 12$ when $x = 4$, $k = 3$ and $y = 3x$.

11 GEOMETRY

Unit 12 · angles, polygons, area, volume, Pythagoras

Angles

- complementary: $\angle A + \angle B = 90°$; supplementary: $\angle A + \angle B = 180°$; vertical angles are equal.
- Parallel lines with a transversal: corresponding angles are equal, alternate interior angles are equal.
- Triangle: $\angle A + \angle B + \angle C = 180°$. Polygon with n sides: $(n - 2) \times 180°$; each angle of a regular n-gon: $(n - 2) \times 180° \div n$. Hexagon: 720°, 120° each.

Pythagorean theorem

legs a, b; hypotenuse c (opposite the right angle). $c = \sqrt{(a^2 + b^2)}$; $a = \sqrt{(c^2 - b^2)}$. Triples: 3-4-5, 5-12-13, 8-15-17, 7-24-25 and their multiples.

$$a^2 + b^2 = c^2$$

Using it WORKED

Legs 6 and 8: $c = \sqrt{(36 + 64)} = \sqrt{100} = \mathbf{10}$

$c = 13$, $b = 5$: $a = \sqrt{(169 - 25)} = \sqrt{144} = \mathbf{12}$

Distance (1, 2) to (4, 6): $\sqrt{(3^2 + 4^2)} = \mathbf{5}$

$d = \sqrt{((x_2 - x_1)^2 + (y_2 - y_1)^2)}$

Sides 5, 7, 9: $25 + 49 \neq 81 \rightarrow$ **not right** *converse*

Perimeter and area

Shape	Area	Perimeter
Rectangle	$A = lw$	$P = 2l + 2w$
Square	$A = s^2$	$P = 4s$
Parallelogram	$A = bh$	add the sides
Triangle	$A = \frac{1}{2}bh$	add the sides
Trapezoid	$A = \frac{1}{2}(b_1 + b_2)h$	add the sides
Circle	$A = \pi r^2$	$C = \pi d = 2\pi r$

$d = 2r$; $\pi \approx 3.14 \approx 22/7$. Area is in square units, perimeter in plain units. Circle $r = 5$: $C \approx 31.4$, $A \approx 78.5$.

Volume and surface area

Solid	Volume	Surface area
Any prism or cylinder	$V = Bh$ (B = base area)	
Rectangular prism	$V = lwh$	$SA = 2lw + 2lh + 2wh$
Cylinder	$V = \pi r^2 h$	$SA = 2\pi r^2 + 2\pi rh$

Volume in cubic units; surface area in square units.

Area and volume WORKED

Trapezoid $b_1 = 6$, $b_2 = 10$, $h = 4$: $\frac{1}{2}(16)(4) = \mathbf{32}$

Box $3 \times 4 \times 5$: $V = \mathbf{60}$; $SA = 2(12) + 2(15) + 2(20) = \mathbf{94}$

Can $r = 2$, $h = 7$: $V = \pi(4)(7) \approx \mathbf{87.9}$ *28π*

MEMORY TRICK **Height stands up**

In every area and volume formula, h is the **perpendicular** height, not a slanted side. The base B of a prism is a whole area, not a length.

⚠ Units

- Convert before computing: 2 ft and 6 in → 24 in and 6 in.
- Doubling every side multiplies area by 4 and volume by 8.

12 DATA & PROBABILITY

Unit 13 · averages and chance

Measures of center

- **mean** = $\frac{\text{sum of the values}}{\text{number of values}}$; sum = mean × count.
- **median** = middle value when sorted (average the two middles for an even count).
- **mode** = most frequent value; there may be none or several.
- **range** = largest − smallest.
- An **outlier** is far from the rest; it drags the mean, so report the median then.
- Mean from a table: $\frac{\text{sum of (value} \times \text{count)}}{\text{sum of the counts}}$.

Find all four WORKED

Data: 4, 7, 7, 10, 22

mean = 50 ÷ 5 = **10**; median = **7**; mode = **7**; range = **18**

Without the outlier 22: mean = 7, median = 7

the median barely moves

Five tests average 84: need $84 \times 6 = 504$ for six; $504 - 420 =$ **84** on the next

sum = mean × count

Probability

- P(event) = $\frac{\text{outcomes in the event}}{\text{total equally likely outcomes}}$, from 0 (impossible) to 1 (certain). P(not A) = 1 − P(A).
- **Experimental:** $\frac{\text{times it happened}}{\text{number of trials}}$. Expected count = P × trials.
- **Counting principle:** m choices then n choices make m × n outcomes. 3 shirts × 4 pants = 12 outfits.
- **Independent events:** P(A and B) = P(A) × P(B). Two heads: ½ × ½ = ¼.
- One die: P(even) = $\frac{3}{6} = \frac{1}{2}$; P(5) = $\frac{1}{6}$; P(7) = 0.